Lecture 2: **Intermediate macroeconomics, autumn 2008**

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GDP per capita, percent of OECD average, PPP-adjusted

Position 1970		Index
1	Switzerland	154
2	USA	147
3	Luxembourg	119
4	**Sweden**	**113 (105*)**
5	Canada	111
6	Denmark	109
7	France	105
8	Australia	103
9	Netherlands	102
10	New Zeeland	100
11	Great Britain	96
12	Belgium	95
13	Germany	93
14	Italy	89
14	Austria	89
16	Norway	88
17	Japan	86
18	Finland	85
19	Iceland	83
20	Spain	66
21	**Ireland**	**55**
22	Greece	53
23	Portugal	46
24	Mexico	40
25	Turkey	28

Position 1980		Index
1	USA	140
2	Switzerland	137
3	Canada	118
4	Luxembourg	115
5	Iceland	110
6	France	109
7	Norway	107
7	**Sweden**	**107 (98*)**
9	Denmark	105
10	Belgium	104
11	Australia	101
11	Netherlands	101
11	Austria	101
14	Italy	97
14	Germany	97
16	Japan	95
17	Great Britain	93
18	Finland	92
19	New Zeeland	89
20	Spain	68
21	Greece	61
21	**Ireland**	**61**
23	Portugal	53
24	Mexico	45
25	Turkey	27

* If Mexico and Turkey are excluded.

GDP per capita, percent of OECD average, PPP-adjusted

Position 1990		Index	Position 1998		Index
1	Luxembourg	141	1	Luxembourg	156
2	USA	137	2	USA	138
3	Switzerland	131	3	Norway	124
4	Canada	114	4	Switzerland	120
5	Japan	110	5	Denmark	119
6	Norway	108	5	Iceland	119
7	France	107	7	Canada	111
7	Iceland	107	8	Belgium	109
9	Denmark	105	8	Japan	109
9	**Sweden**	**105 (94*)**	10	Austria	108
11	Belgium	103	11	Netherlands	104
11	Austria	103	12	Australia	103
13	Finland	100	12	Germany	103
13	Italy	100	**14**	**Ireland**	**102**
15	Australia	99	15	France	100
15	Germany	99	16	Finland	98
17	Netherlands	98	16	Italy	98
17	Great Britain	98	18	Great Britain	96
19	New Zeeland	82	**18**	**Sweden**	**96 (85*)**
20	Spain	73	20	New Zeeland	80
21	**Ireland**	**70**	21	Spain	76
22	Portugal	59	22	Portugal	69
23	Greece	57	23	Greece	65
24	Mexico	36	24	Mexico	36
25	Turkey	29	25	Turkey	30

* If Mexico and Turkey are excluded.

GDP per capita, US dollars, PPP-adjusted, percent of OECD average, ranking by country

		2006
1	Luxembourg	247
2	Norway	165
3	United States	138
4	**Ireland**	**129**
5	Switzerland	119
6	Canada	116
7	Iceland	115
8	Netherlands	115
9	Austria	113
10	Australia	112
11	Denmark	111
12	**Sweden**	**110**
13	Belgium	106
14	United Kingdom	104
15	Finland	103
16	Japan	101
17	Germany	101
18	France	99
19	Euro area	98
20	Spain	93
21	Italy	91
22	Greece	86
23	New Zealand	83
24	Korea	73
25	Czech Republic	69
26	Portugal	66
27	Hungary	57
28	Slovak Republic	55
29	Poland	46
30	Mexico	38
31	Turkey	36

Source: OECD

$$Y = F(K, L)$$

$$MPL = F(K, L+1) - F(K, L)$$

$$MPL = \frac{dY}{dL} = \frac{dF(K, L)}{dL} = F_L$$

$$MPK = F(K+1, L) - F(K, L)$$

$$MPK = \frac{dY}{dK} = \frac{dF(K, L)}{dK} = F_K$$

<u>First-order condition for maximum</u>

If $y = f(x, z)$

$$\frac{dy}{dx} = f_x = 0$$

$$\frac{dy}{dz} = f_z = 0$$

<u>Profit maximisation</u>

$$\pi = PY - RK - WL = PF(K, L) - RK - WL$$

$$\frac{d\pi}{dL} = PF_L - W = 0 \quad \Leftrightarrow \quad F_L = \frac{W}{P}$$

$$\frac{d\pi}{dK} = PF_K - R = 0 \quad \Leftrightarrow \quad F_K = \frac{R}{P}$$

Figure 3-3: The production function

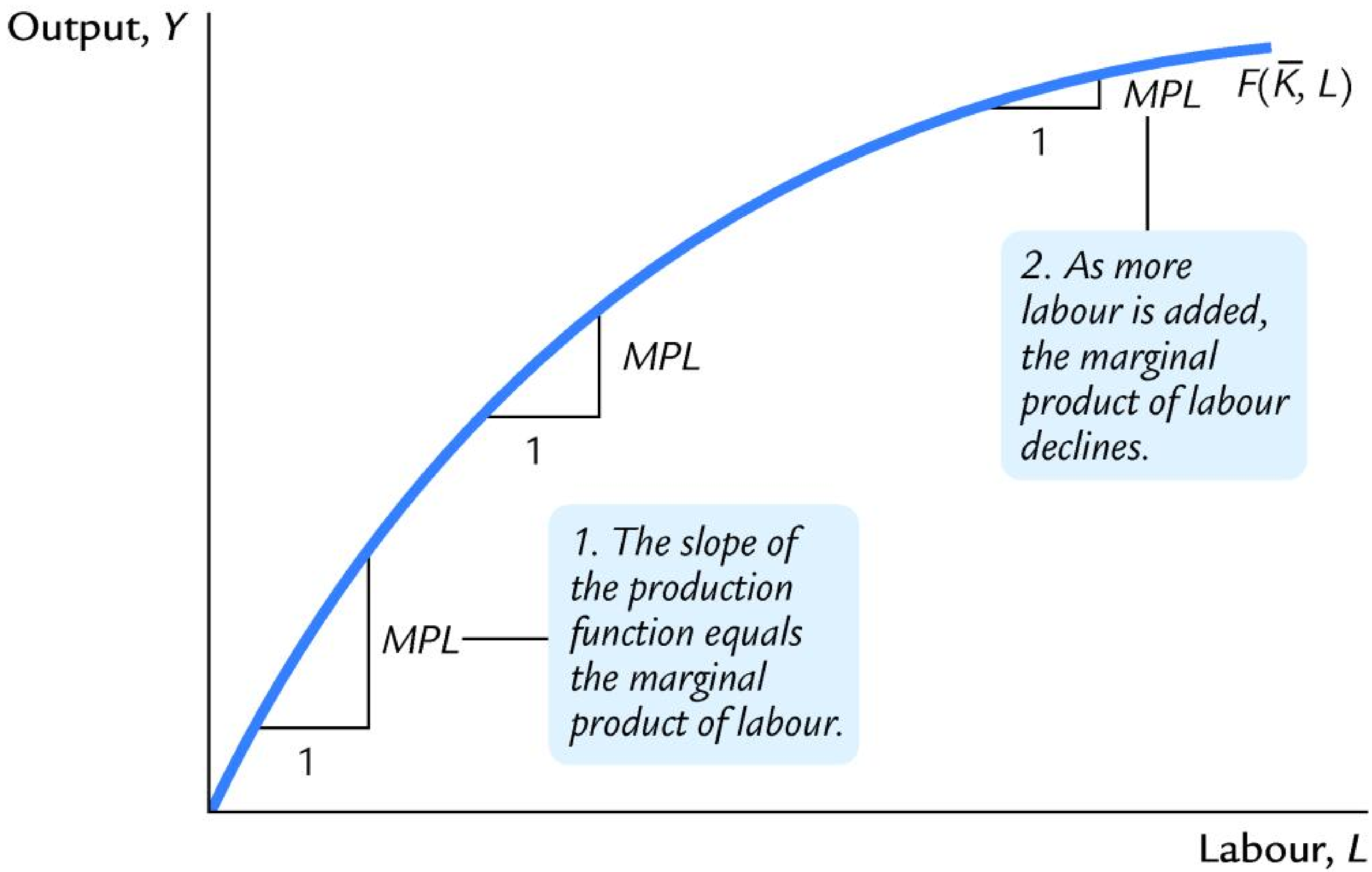
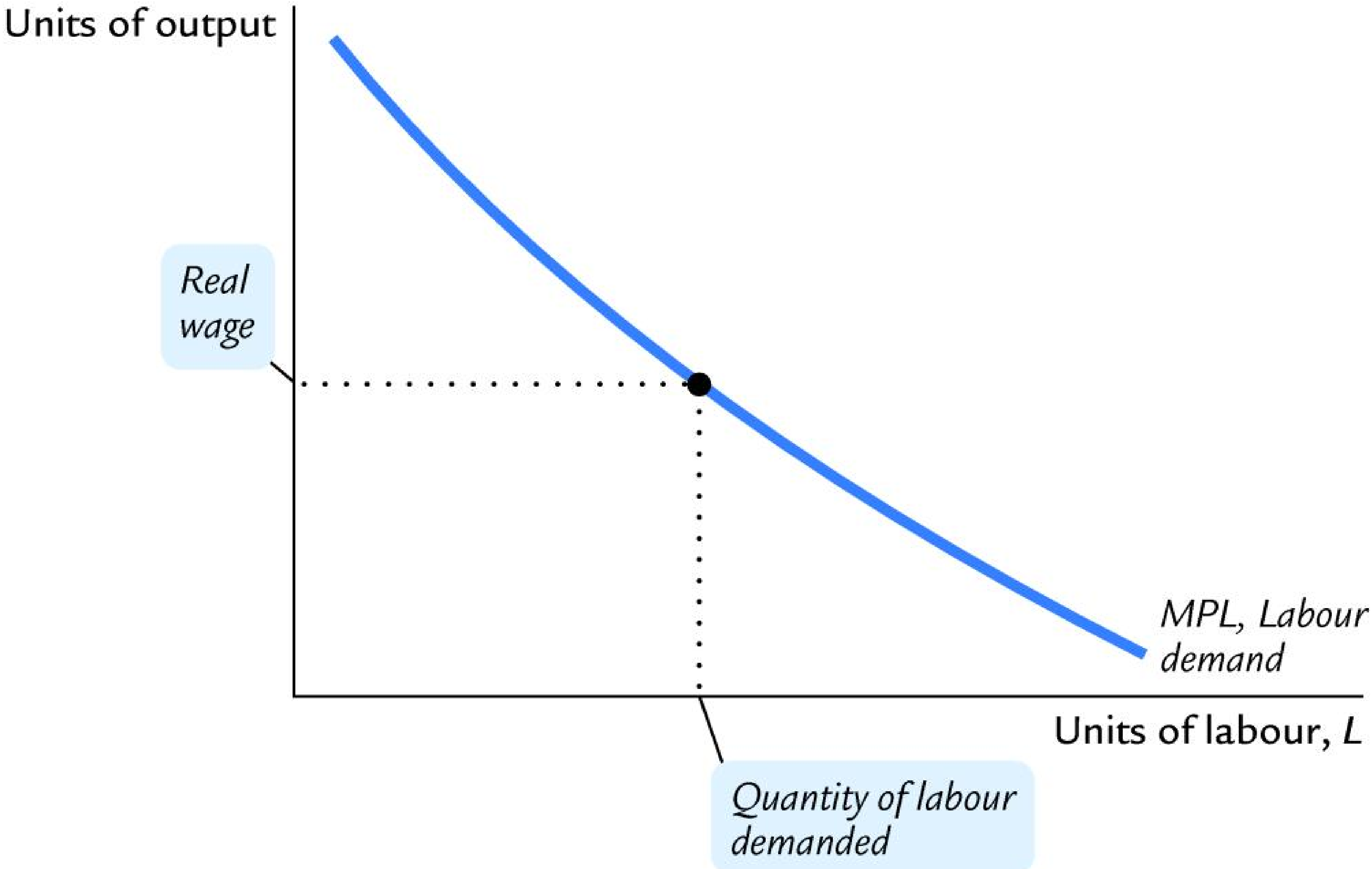


Figure 3-4: The marginal product of labour schedule



Production function

$Y = F(K, L)$

$Y = AF(K, L)$ A = total factor productivity

It holds that:

$$\frac{\Delta Y}{Y} \approx \frac{\Delta A}{A} + \alpha\frac{\Delta K}{K} + (1-\alpha)\frac{\Delta L}{L}$$

α = capital income share

1-α = labour income share

GDP growth = total factor productivity growth
+ contribution from growth of the capital stock
+ contribution from growth of the labour force

Growth accounting

The Solow-residual:

$$\frac{\Delta \mathrm{A}}{A} \approx \frac{\Delta Y}{Y} - \alpha\frac{\Delta K}{K} - (1-\alpha)\frac{\Delta L}{L}$$

Figure 3-5: The ration of labour income to total income in the US and the UK

Mathematics to derive growth equation in the case of Cobb-Douglas production function

Logarithms

$\ell n\, x$ is the natural logarithm of x. If

$x = e^{a}$, then $a = \ell n\, x$.

$$\ell n\, xy = \ell n\, x + \ell n\, y$$
$$\ell n\, x/y = \ell n\, x - \ell n\, y$$
$$\ell n\, x^{\beta} = \beta\, \ell n\, x$$

Rules of differentiation

If $y = f(g)$ and $g = g(x)$, i.e. if $y = f(g(x))$, then

$$\frac{dy}{dx} = f_g g_x$$

$$\frac{d(\ell n\, x)}{dx} = \frac{1}{x}$$

Maximisation

If $y = f(x, z)$, then the first-order conditions for a maximum are:

$$\frac{dy}{dx} = f_x = 0$$

$$\frac{dy}{dz} = f_z = 0$$

Another rule of differentiation

If $y = x^z$, then $\frac{dy}{dx} = zx^{z-1}$

Cobb-Douglas production function

$Y = AF(K, L) = AK^{\alpha}L^{1-\alpha}$

K, L and A and thus also Y are functions of time.

$\because$

$$Y(t) = A(t)K(t)^{\alpha}L(t)^{1-\alpha}$$

Taking logarithms:

$$\ell n\ Y(t) = \ell n\ A(t) + \ell n\ K(t)^{\alpha} + \ell n\ L(t)^{1-\alpha}$$
$$\ell n\ Y(t) = \ell n\ A(t) + \alpha\ \ell n\ K(t) + (1-\alpha)\ \ell n\ L(t)$$

Differentiation w.r.t. time gives:

$$\frac{d\ell n\ Y(t)}{dt} = \frac{d\ell n\ A(t)}{dt} + \alpha\frac{d\ell n\ K(t)}{dt} + (1-\alpha)\frac{d\ell n\ L(t)}{dt}$$

$$\frac{dY}{dt} \bullet \frac{1}{Y} = \frac{dA}{dt} \bullet \frac{1}{A} + \alpha\frac{dK}{dt} \bullet \frac{1}{K} + (1-\alpha)\frac{dL}{dt} \bullet \frac{1}{L}$$

Call $\frac{dY}{dt} = \dot{Y}$, $\frac{dA}{dt} = \dot{A}$, $\frac{dK}{dt} = \dot{K}$ och $\frac{dL}{dt} = \dot{L}$

$\because$

$$\frac{\dot{Y}}{Y} = \frac{\dot{A}}{A} + \alpha\frac{\dot{K}}{K} + (1-\alpha)\frac{\dot{L}}{L}$$

α = profit share
1-α = wage share

We thus have:

$$\frac{\Delta Y}{Y} \approx \frac{\Delta A}{A} + \alpha\frac{\Delta K}{K} + (1-\alpha)\frac{\Delta L}{L}$$

Profit maximisation

$$\pi = PY - RK - WL = PAK^{\alpha}L^{1-\alpha} - RK - WL$$

$$\frac{d\pi}{dK} = \alpha PAK^{\alpha-1}L^{1-\alpha} - R = 0$$

$$\frac{d\pi}{dL} = (1\text{-}\alpha)PAK^{\alpha}L^{-\alpha} - W = 0$$

$\because$

$$\alpha = \frac{R}{PAK^{\alpha-1}L^{1-\alpha}} = \frac{RK}{PAK^{\alpha}L^{1-\alpha}} = \frac{RK}{PY}$$

$$1-\alpha = \frac{W}{PAK^{\alpha}L^{-\alpha}} = \frac{WL}{PAK^{\alpha}L^{1-\alpha}} = \frac{WL}{PY}$$

Accounting for growth in labour productivity

$$\frac{\Delta Y}{Y} = \alpha \ \frac{\Delta K}{K} + (1\text{-}\alpha) \ \frac{\Delta L}{L} + \frac{\Delta A}{A}$$

GDP growth = contribution from growth of capital stock + contribution from growth of labour + total factor productivity growth

Labour productivity: Y/L

$$\frac{\Delta\left(\frac{Y}{L}\right)}{\left(\frac{Y}{L}\right)} \approx \frac{\Delta Y}{Y} - \frac{\Delta L}{L}$$

$$\frac{\Delta Y}{Y} - \frac{\Delta L}{L} = \alpha \ \frac{\Delta K}{K} + (1\text{-}\alpha) \ \frac{\Delta L}{L} + \frac{\Delta A}{A} - \frac{\Delta L}{L}$$

$$\frac{\Delta Y}{Y} - \frac{\Delta L}{L} = \alpha \ (\frac{\Delta K}{K} - \frac{\Delta L}{L}) + \frac{\Delta A}{A}$$

Growth in labour productivity = contribution from capital deepening + total factor productivity growth

Capital deepening: Increase in capital intensity (capital relative to labour)

Capital deepening can be decomposed into ICT capital deepening and non-ICT capital deepening

TABLE 8-3

The Contribution of Total Factor Productivity Growth to Growth in Real GDP per Worker

	UNITED KINGDOM		FRANCE		ITALY		UNITED STATES		JAPAN	
	Output growth per worker	Growth in total factor productivity	Output growth per worker	Growth in total factor productivity	Output growth per worker	Growth in total factor productivity	Output growth per worker	Growth in total factor productivity	Output growth per worker	Growth in total factor productivity
1966-1970	2.2	0.2	4.9	2.3	6.4	4.4	1.7	−0.3	9.5	5.4
1971-1975	1.4	−0.1	2.0	−0.1	2.1	0.2	−0.2	−0.9	3.1	−0.8
1976-1980	1.4	0.6	2.4	1.2	4.7	3.6	1.0	0.7	4.0	2.0
1981-1985	1.6	1.2	0.2	−0.4	0.3	−0.4	1.3	0.8	2.9	1.3
1986-1990	3.1	1.9	2.3	1.6	2.5	1.7	1.7	0.7	3.8	2.0

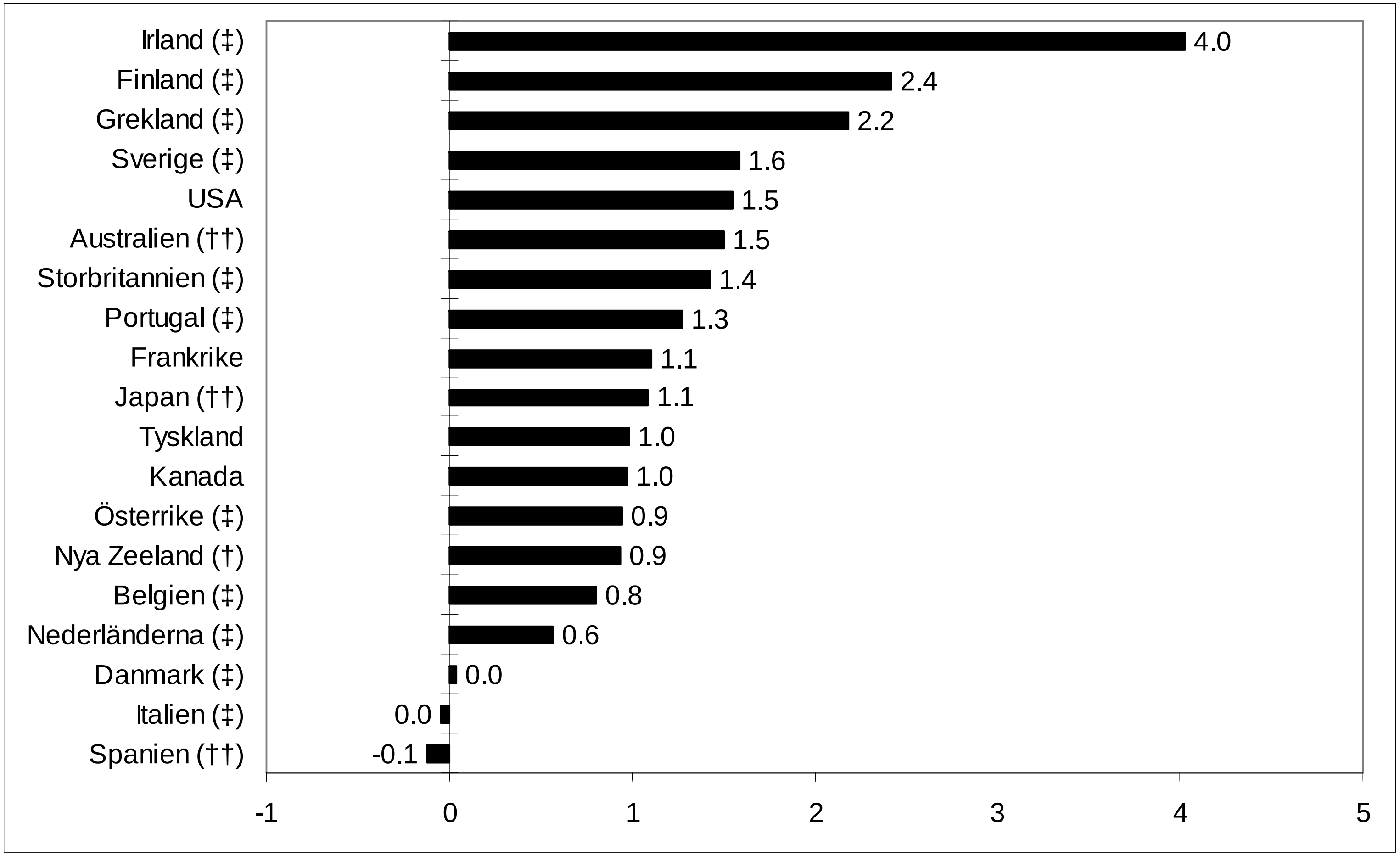
Source: Authors' calculations and Penn World Tables (Alan Heston, Robert Summers and Bettina Aten, Penn World Tables, Center for International Comparisons of Production, Income and Prices, University of Pennsylvania).

Table 4.5

Contributions to average annual growth in GDP per hour, percentage points, 1990–2004

	Growth in GDP per hour	Contribution from ICT capital deepening	Contribution from non-ICT capital deepening	Total factor productivity growth
Denmark				
1990–94	2.4	0.6	0.5	1.3
1995–99	1.8	1.0	0.5	0.3
2000–04	1.4	0.5	1.0	−0.1
Finland				
1990–94	2.1	0.5	1.1	0.5
1995–99	2.7	0.5	−0.7	2.8
2000–04	2.8	0.6	0.2	2.0
Sweden				
1990–94	2.0	0.5	0.7	0.7
1995–99	2.4	1.0	0.2	1.2
2000–04	2.6	0.4	0.3	1.9
Average Scandinavian countries				
1990–94	2.2	0.5	0.8	0.9
1995–99	2.3	0.9	0.0	1.4
2000–04	2.3	0.5	0.5	1.3
Austria				
1990–94	0.9	0.3	0.6	0.0
1995–99	3.2	0.6	0.8	1.8
2000–04	1.4	0.4	0.8	0.2
Belgium				
1990–94	2.9	0.5	0.9	1.6
1995–99	2.7	0.9	0.2	1.5
2000-04	0.6	0.4	−0.1	0.3
France				
1990–94	1.5	0.2	1.3	0.0
1995–99	2.1	0.4	0.6	1.1
2000–04	1.5	0.2	0.9	0.5
Germany				
1990–94	3.0	0.4	0.9	1.8
1995–99	1.9	0.5	0.4	1.0
2000–04	1.2	0.3	0.3	0.6

Annual growth of total factor productivity in OECD countries, 1995-2005



Explanations of high productivity growth in Sweden

- **Large contributions from both ICT-producing and ICT-using sectors**
- **Encompassing deregulations of product and service markets**
  - **- low level of regulation**
  - **- early deregulations**
- **High educational level (complementarity between ICT technology and high-skilled labour)**
- **High R&D expenditures (Research and development)**
- **Creative destruction in the 1990s**

Constant returns to scale

Y = F(K, L)

zY = zF(K, L) = F(zK, zL)

10 % larger input of capital and labour raises output also by 10 %.

$$z = \frac{1}{L} \Rightarrow$$

$$\frac{Y}{L} = F(\frac{K}{L}, 1)$$

$\frac{Y}{L} = y =$ **output per capita**

$\frac{K}{L} = k =$ **capital intensity (capital stock per capita)**

$$y = F(k, 1) = f(k)$$

Output per capita is a function of capital intensity

$$y = Y/L = K^{\alpha}L^{1-\alpha}/L = K^{\alpha}L^{-\alpha} = (K/L)^{\alpha} = k^{\alpha}$$

$$y = Y/L = AK^{\alpha}L^{1-\alpha}/L = AK^{\alpha}L^{-\alpha} = A(K/L)^{\alpha} = Ak^{\alpha}$$

The Solow model

(1)	$y = c + i$	Goods market equilibrium
(2)	$c = (1\text{-}s)\ y$	Consumption function, s is the savings rate
(3)	$y = f(k)$	Production function
(4)	$d = \delta k$	Capital depreciation, δ is the rate of depreciation
(5)	$\Delta k = i - \delta k$	Change in the capital stock

Change in the capital stock = Gross investment – Depreciation

$$y = (1\text{-}s)y + i$$

$$i = sy$$

Investment = Saving

$$i = sf(k)$$

$\Delta k = i - \delta k \;= sf(k) - \delta k$

In a steady state, the capital stock is unchanged from period to period, i.e. $\Delta k = 0$ and thus:

$$sf(k) = \delta k$$

<u>Convergence of GDP per capita</u>

- Countries with different initial GDP per capita will converge (if they have the same production function, the same savings rate and the same depreciation rate).
- The catch-up factor
- Strong empirical support for the hypothesis that GDP growth is higher the lower is initial GDP per capita

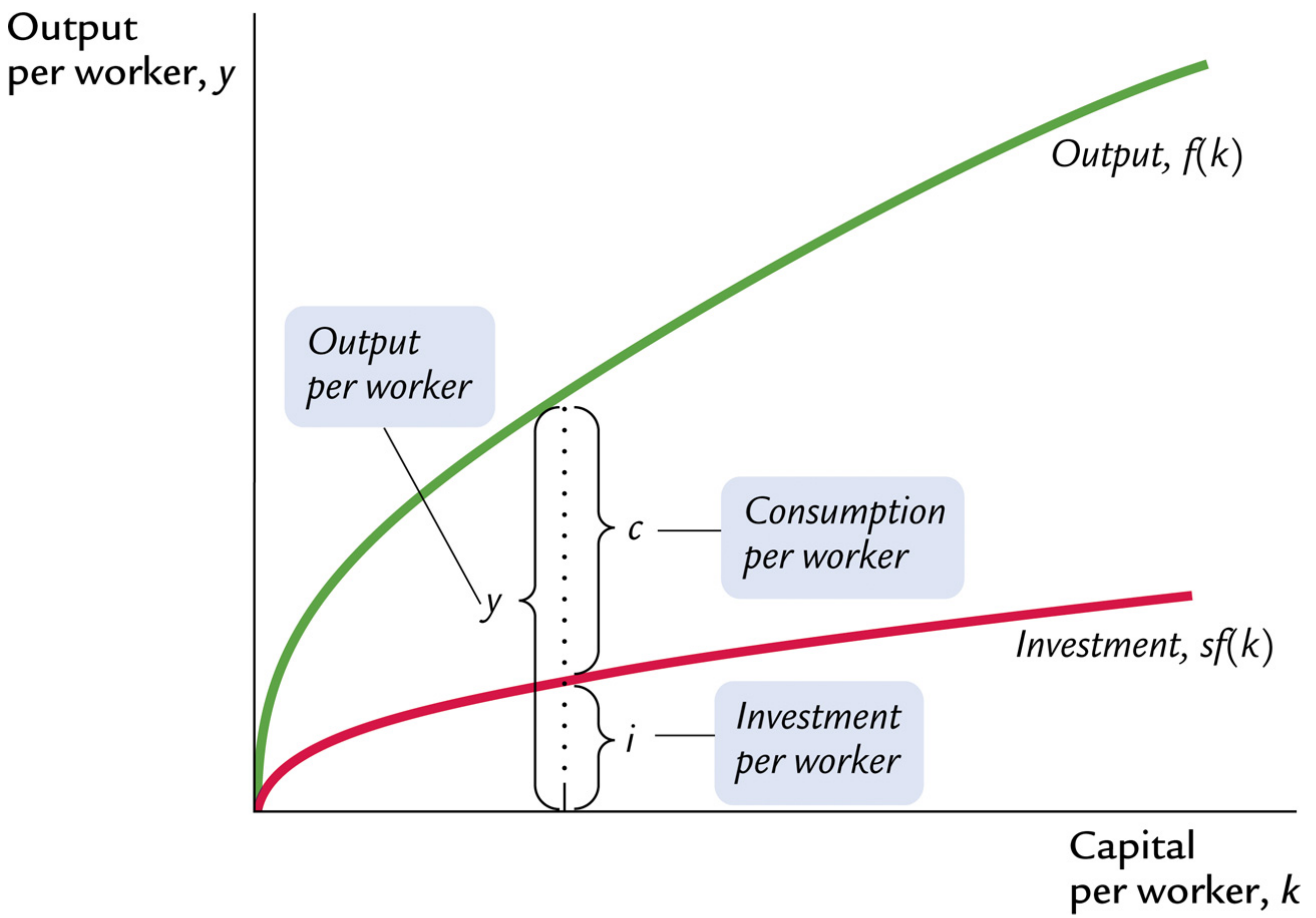


Figure 7-2: Output, consumption and investment

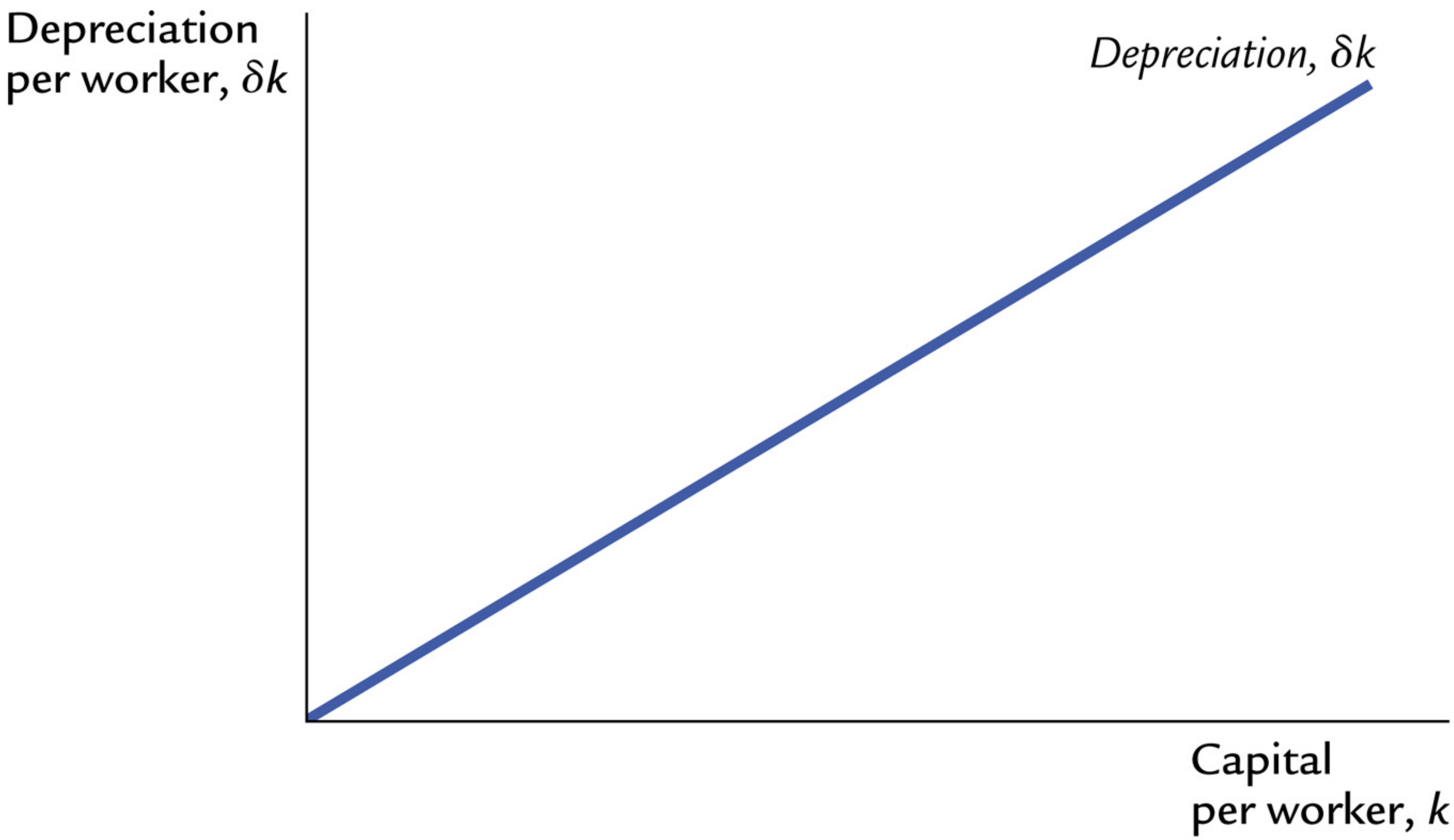


Figure 7-3: Depreciation

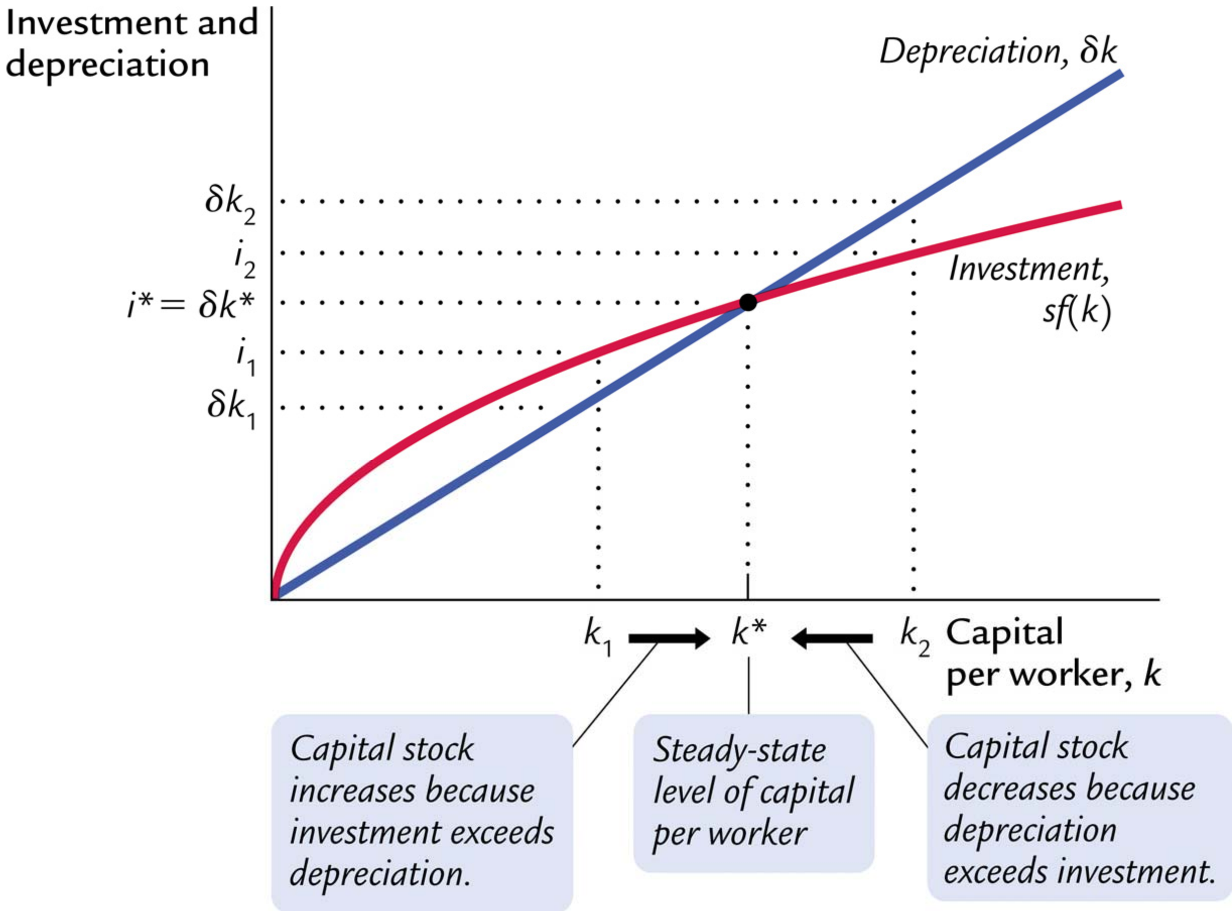


Figure 7-4: Investment, depreciation and the steady state

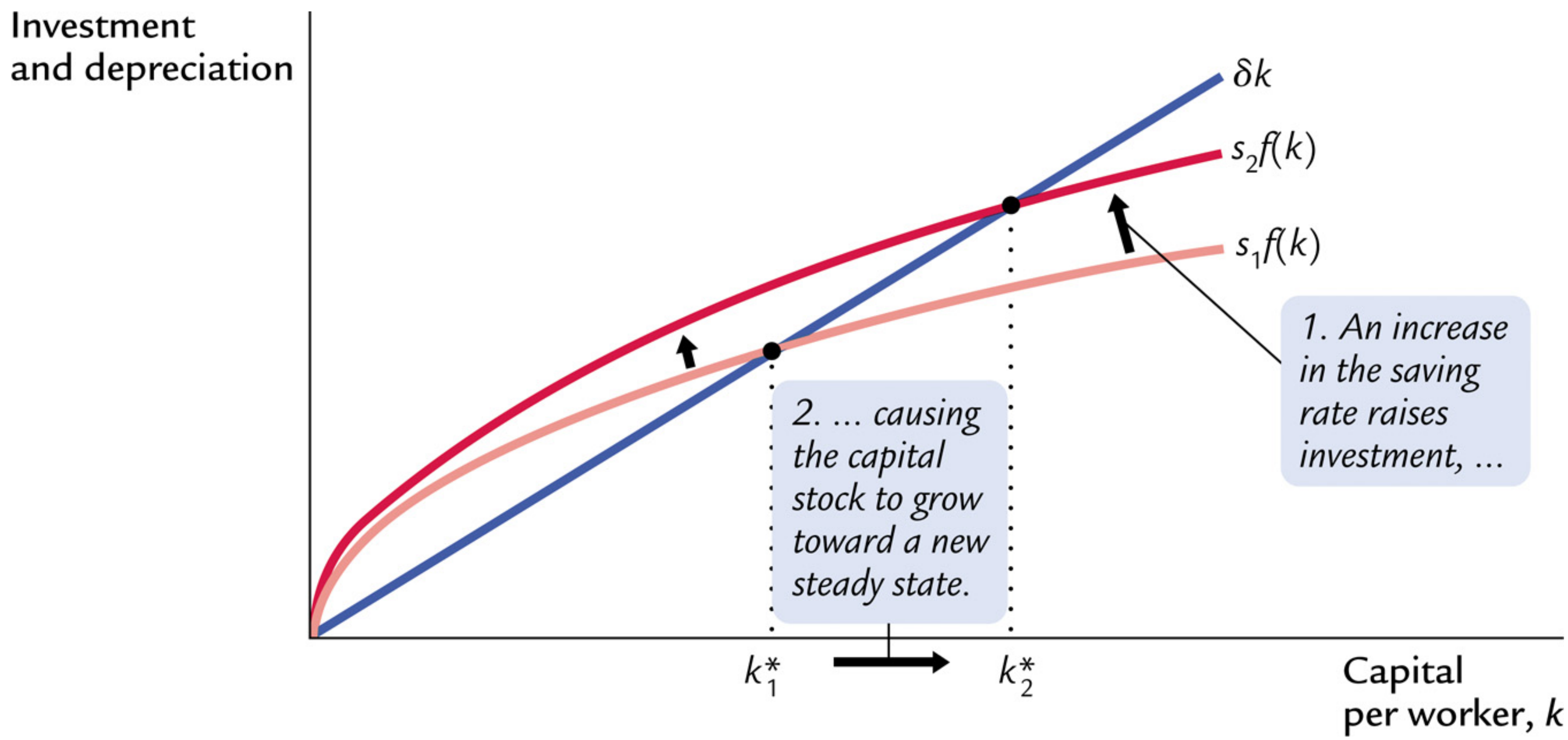


Figure 7-5: An increase in the saving rate

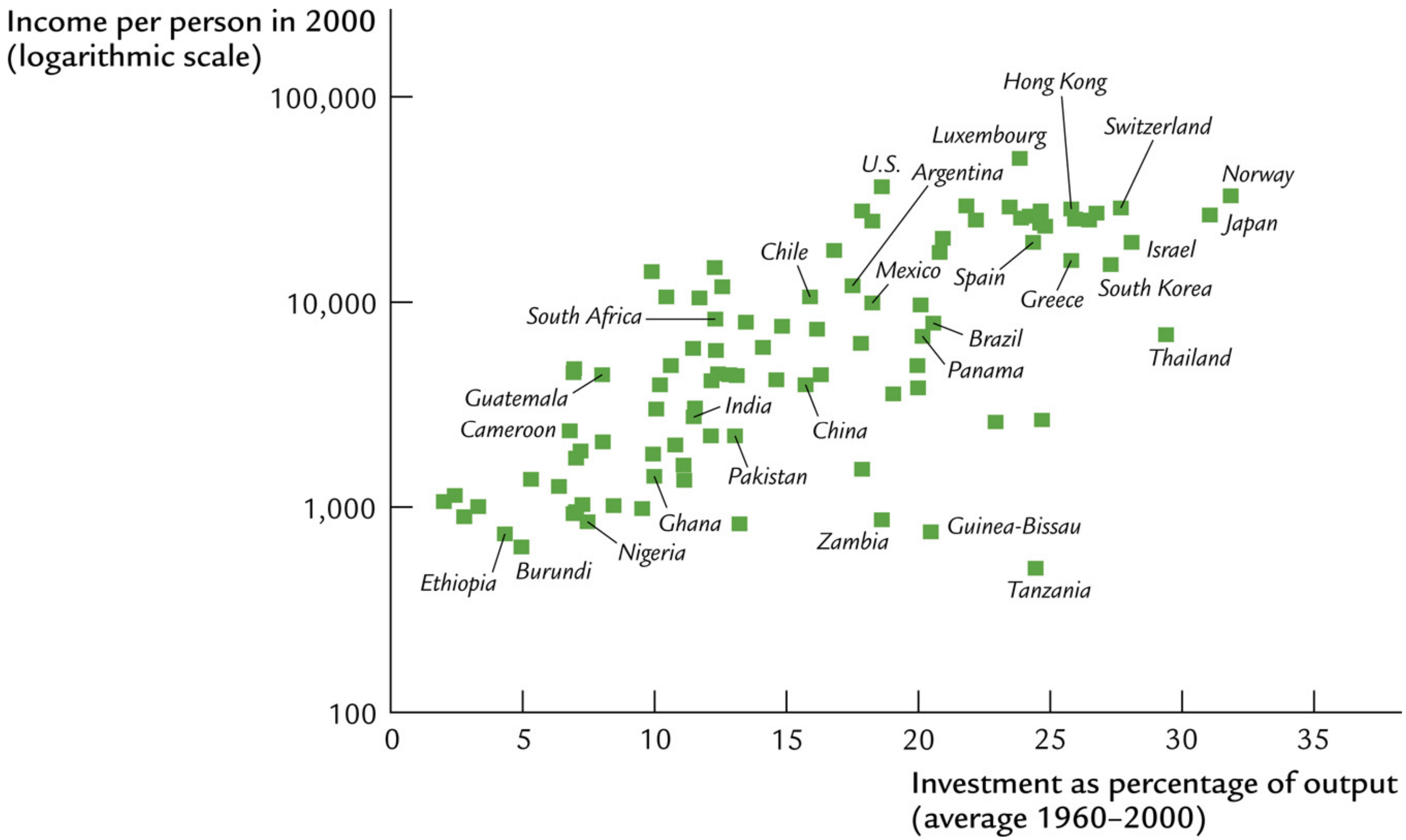


Figure 7-6: International evidence on investment rates and income per person

Golden rule of capital accumulation

Which savings rate gives the highest per capita consumption in the steady state?

$y = c + i$

$c = y - i$

In a steady state, gross investment equals depreciation:

$i = \delta k$

Hence:

$c = f(k) - \delta k$

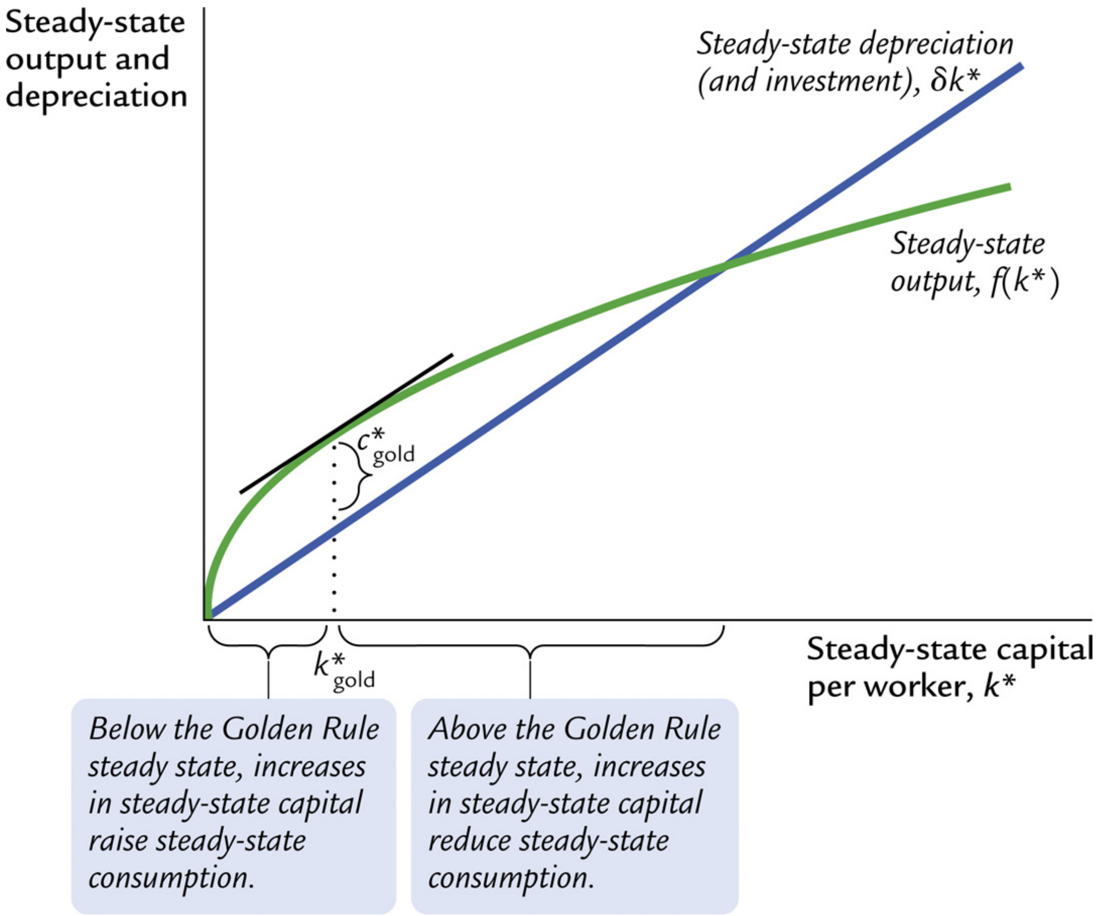
Consumption is maximised if the marginal product of capital equals the rate of depreciation, i.e. $MPC = \delta$

Mathematical derivation

Differentiate c function w.r.t. k:

$$\partial c / \partial k = f_k - \delta = 0$$

$$f_k = \delta$$



<u>Figure 7-7: Steady-state consumption</u>

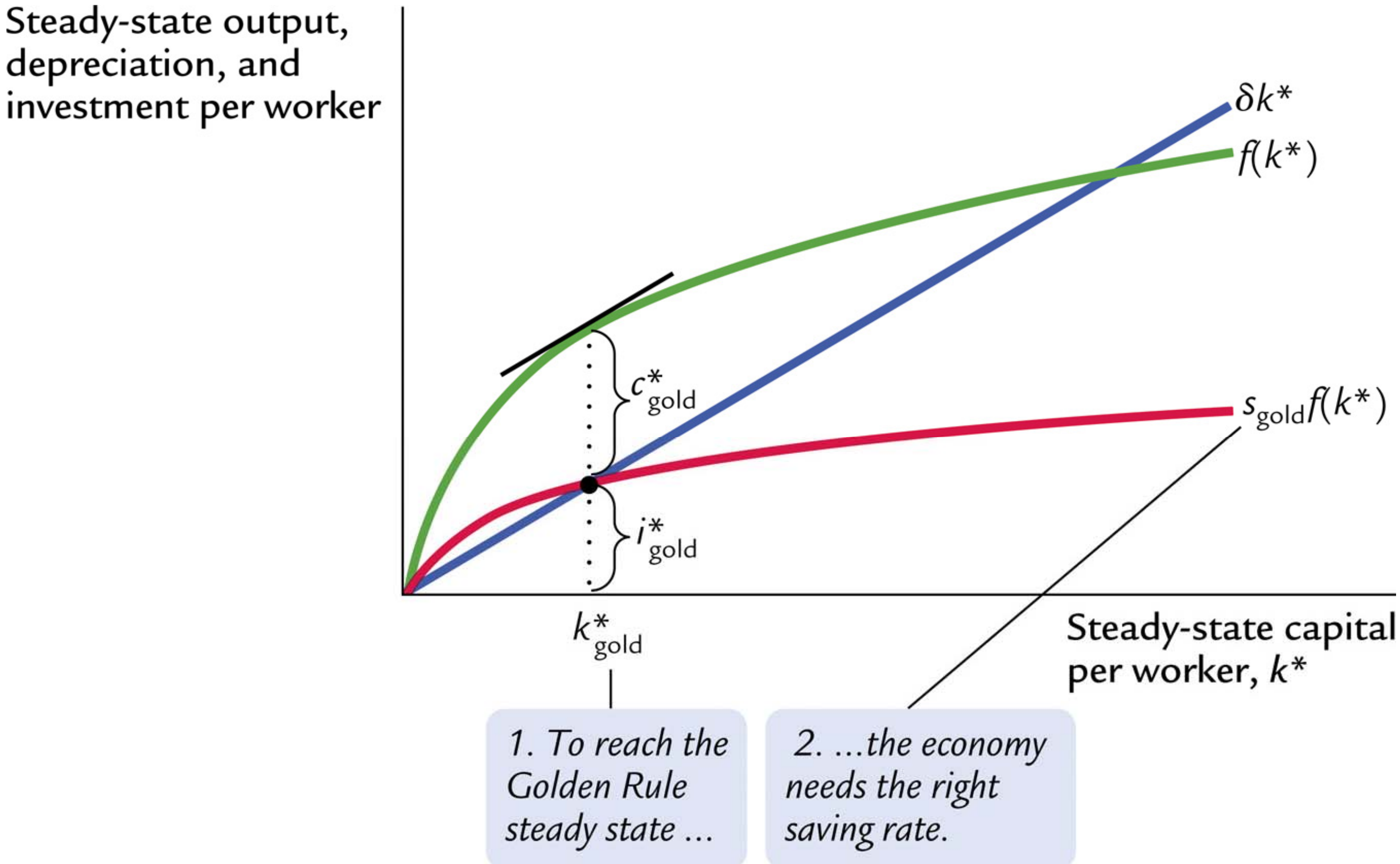


Figure 7-8: The saving rate and the golden rule

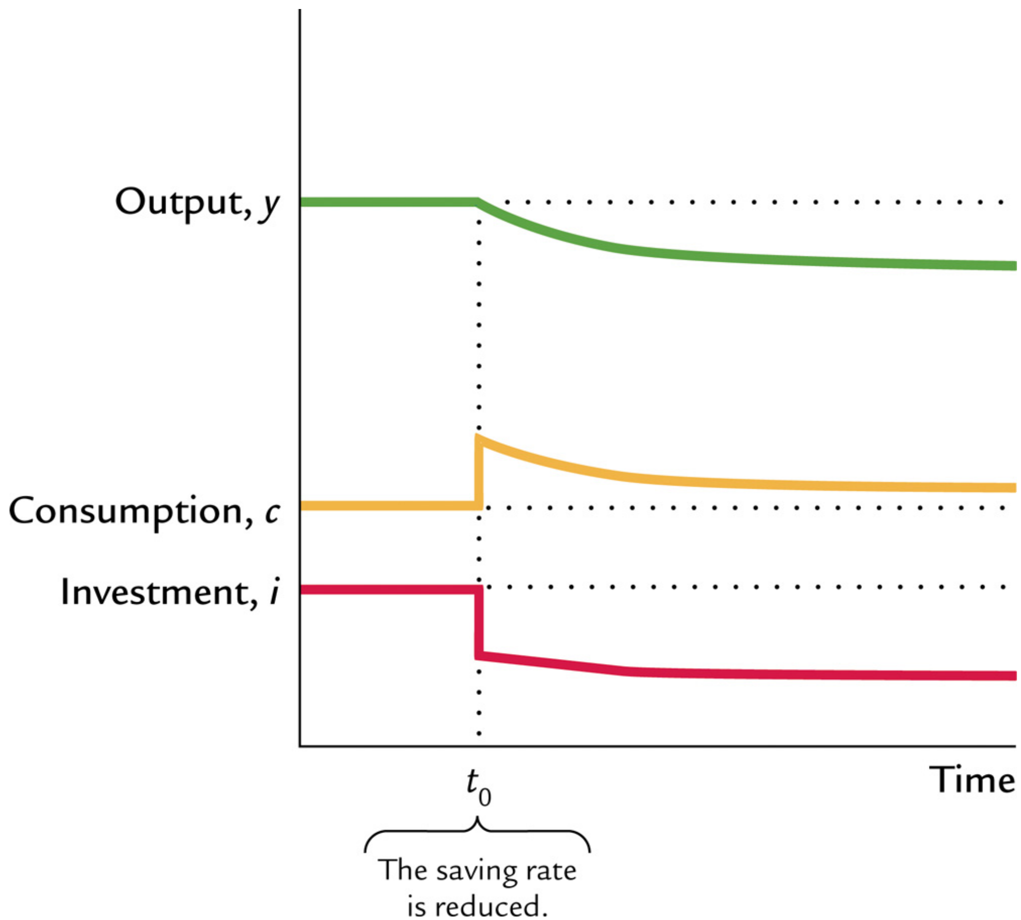


Figure 7-9: Reducing saving when starting with more capital than in the golden rule steady state

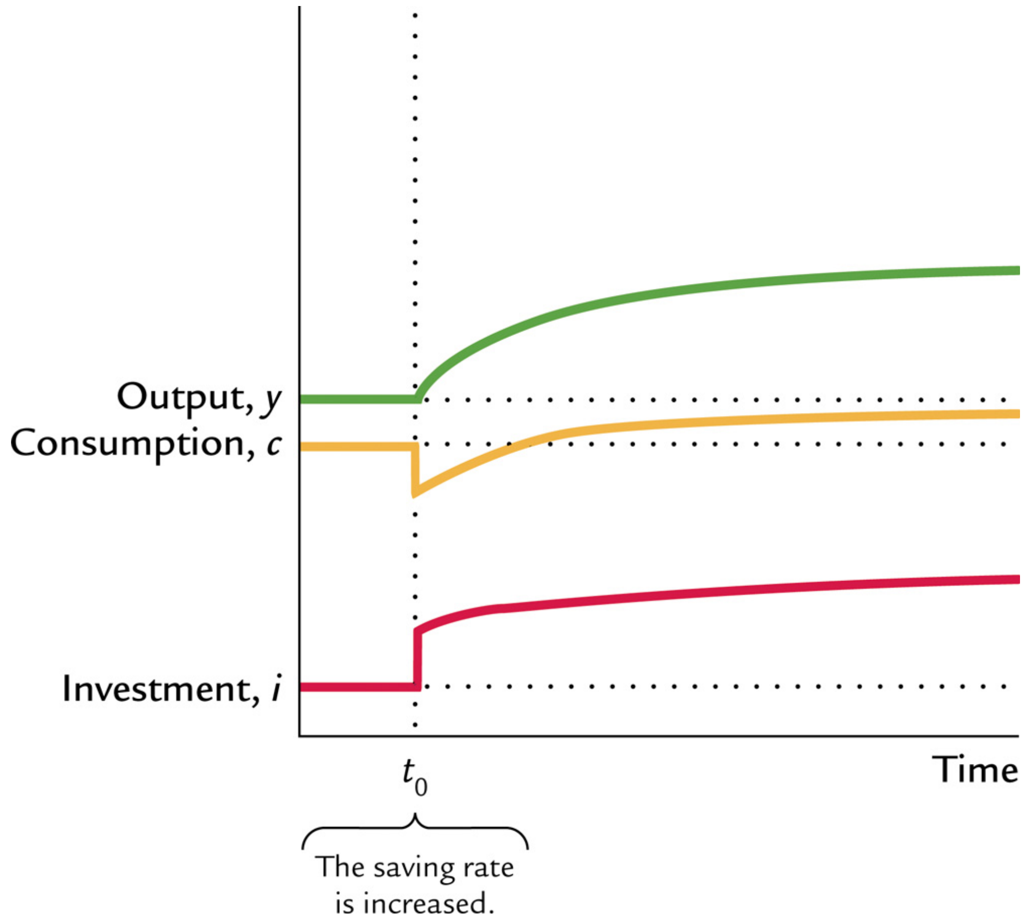


Figure 7-10: Increasing saving when starting with less capital than in the golden rule steady state

A steady state with population growth

$n = \frac{\Delta L}{L}$ = population growth

$\Delta k = i - \delta k - nk$

Change in capital intensity ($k = K/L$) = Gross investment – Depreciation – Reduction in capital intensity due to population growth

In a steady state:

$\Delta k = i - \delta k - nk = 0$, i.e. $i = (\delta + n)k = 0$

Derivation

K = capital stock, I = gross investment, L = population
$k = K/L$ = capital stock per worker (capital intensity)
$i = I/L$ = gross investment per worker

$$\Delta K = I - \delta K$$

$$\frac{\Delta K}{K} = \frac{I}{K} - \delta$$

Use that:

$$\frac{\Delta k}{k} \approx \frac{\Delta K}{K} - \frac{\Delta L}{L} \text{ and } \frac{\Delta L}{L} = n$$

$$\frac{\Delta k}{k} \approx \frac{I}{K} - \delta - n$$

Hence:

$$\frac{\Delta k}{k} \approx \frac{I}{L} \bullet \frac{L}{K} - \delta - n$$

$$\frac{\Delta k}{k} \approx \frac{i}{k} - \delta - n$$

$$\Delta k \approx i - \delta k - nk = i - (\delta + n)k$$

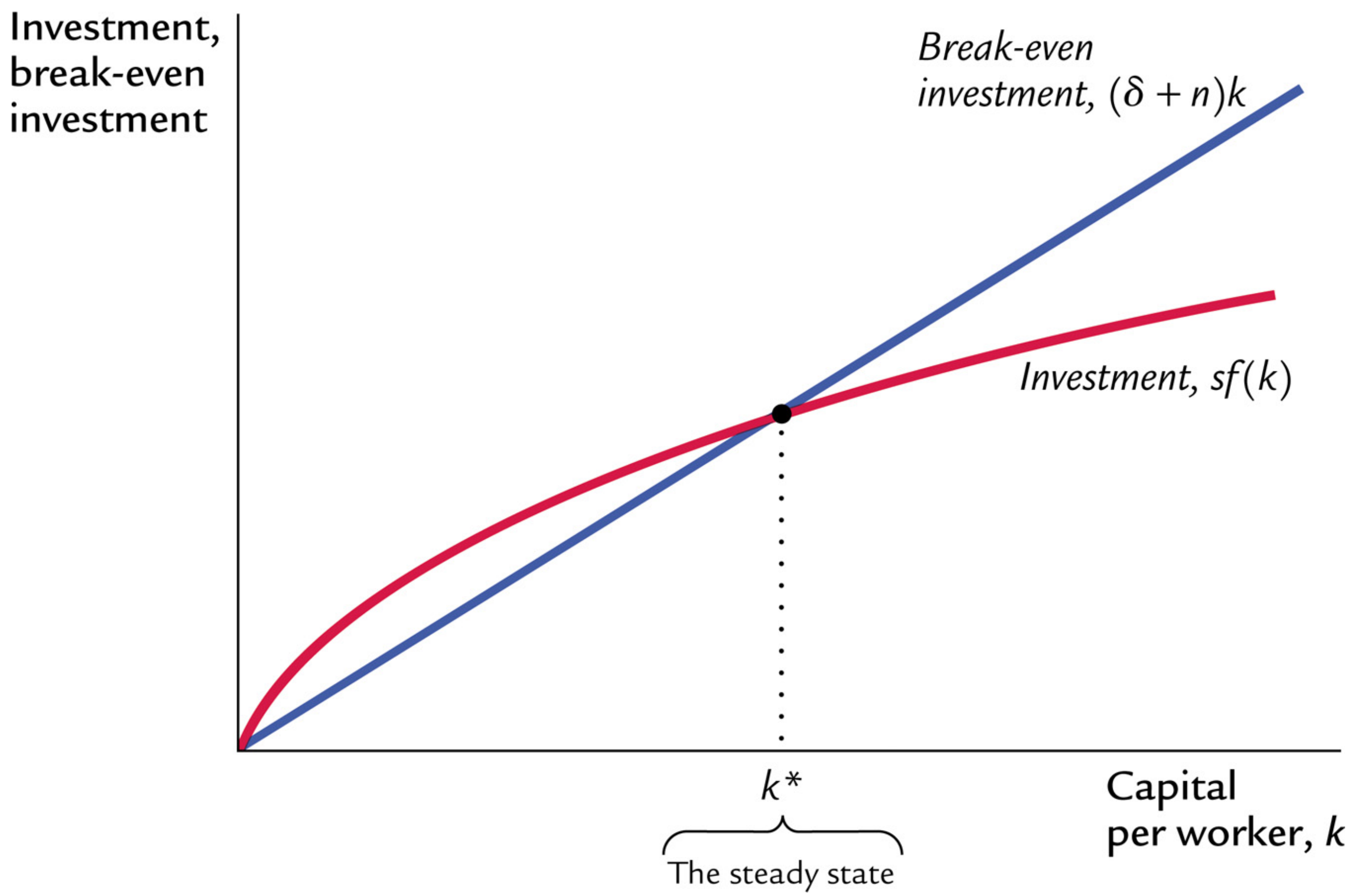
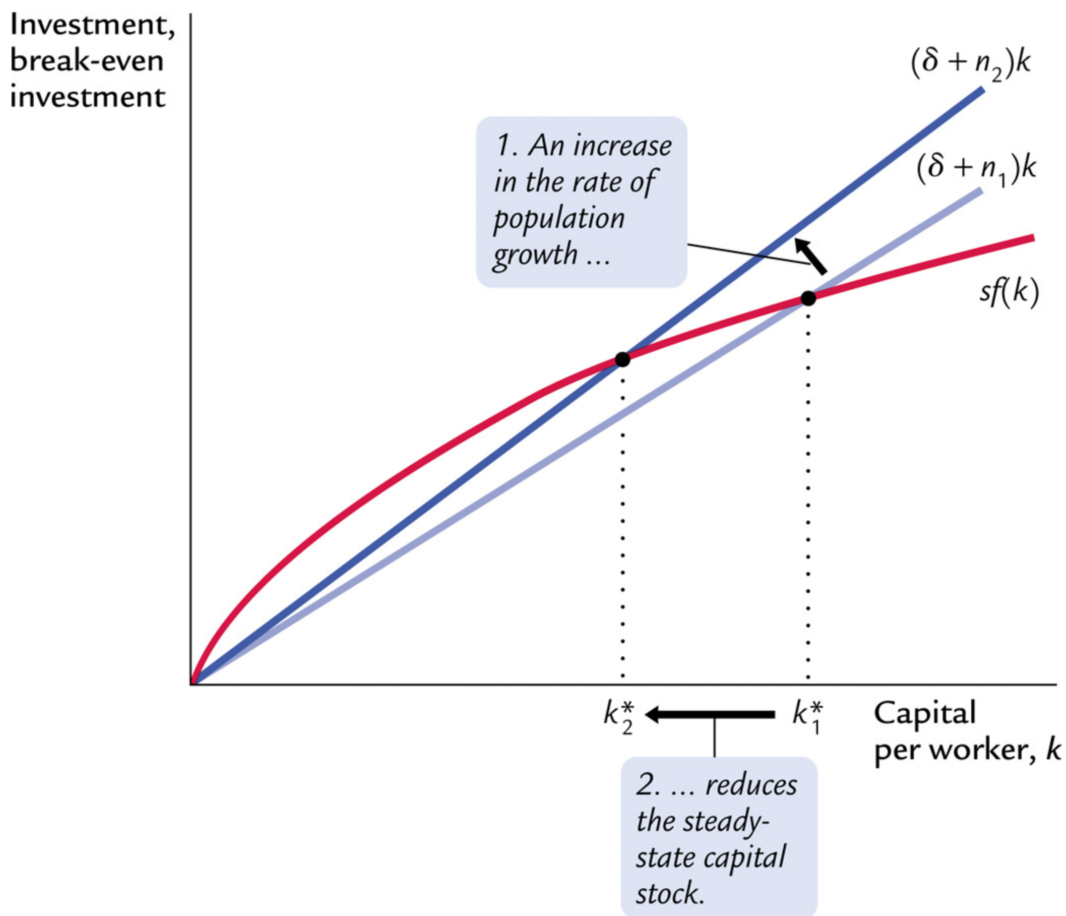


Figure 7-11: Population growth in the Solow model



<u>Figure 7-12: The impact of population growth</u>

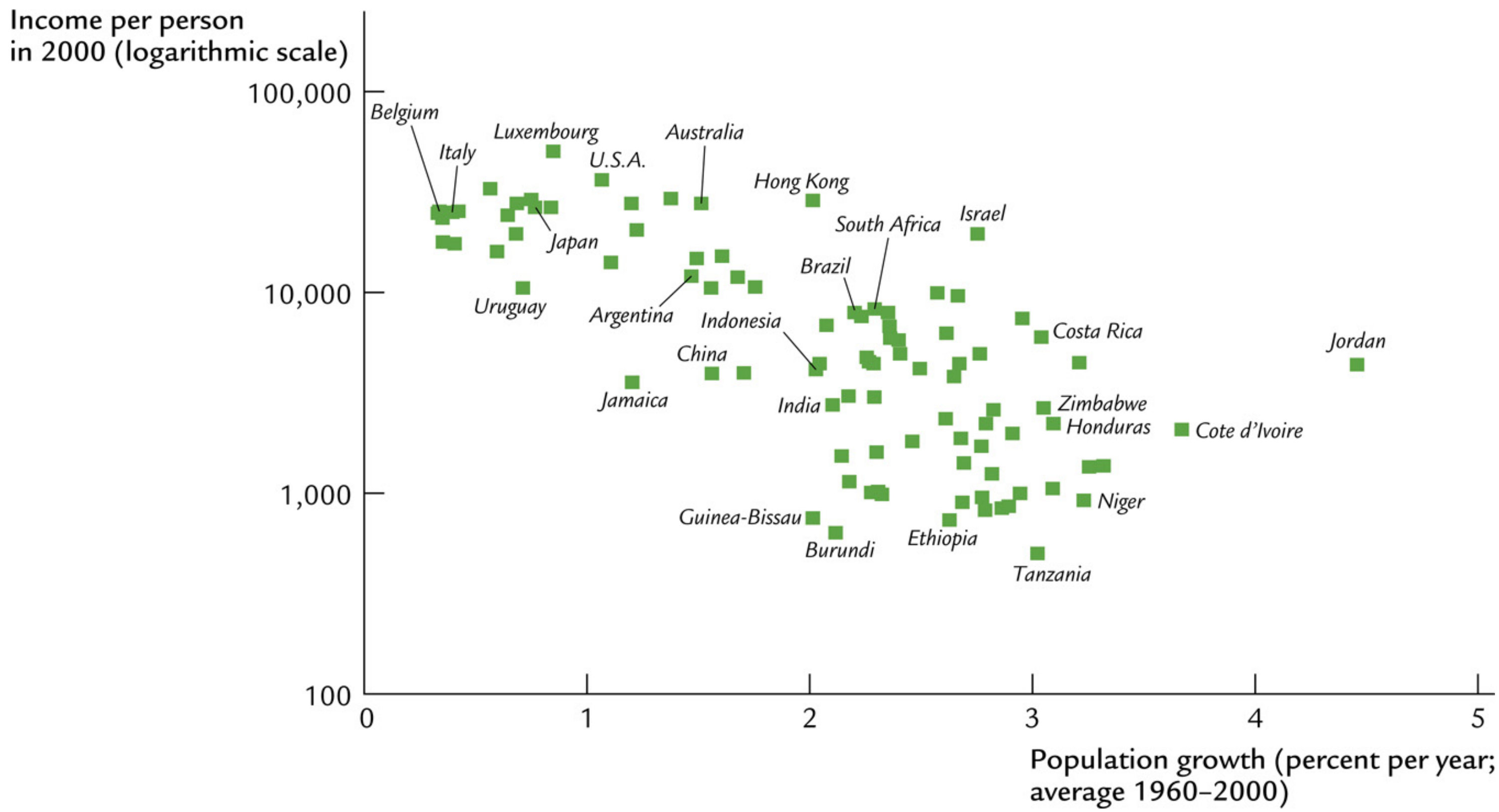


Figure 7-13: International evidence on population growth and income per person

A steady state with population growth

$$Y = F(K, L)$$

$$\frac{\Delta Y}{Y} \approx \alpha \frac{\Delta K}{K} + (1-\alpha)\frac{\Delta L}{L}$$

In a steady state, $k = K/L$ is constant. Because

$$\frac{\Delta k}{k} \approx \frac{\Delta K}{K} - \frac{\Delta L}{L} = 0,$$

We have

$$\frac{\Delta K}{K} = \frac{\Delta L}{L} = n$$

$$\because \text{ är } \frac{\Delta Y}{Y} \approx \alpha \frac{\Delta K}{K} + (1-\alpha)\frac{\Delta L}{L} = \alpha n + (1-\alpha)n = n$$

GDP growth = Population growth

<u>Golden rule with population growth</u>

$$c = y - i = f(k) - (\delta + n)k$$

Consumption per capita is maximised if *MPC* = $\delta + n$, i.e. if the marginal product of capital equals the sume of the depreciation rate and population growth

<u>Alternative formulation:</u> The net marginal product of capital after depreciation (*MPK* – δ) should equal population growth (*n*)

<u>Mathematical derivation</u>

Differentiation of c-function w.r.t *k* gives:

$$\partial c / \partial k = f_k - (\delta + n) = 0$$

$$f_k = \delta + n$$

Labour-augmenting technical progress

$Y = F(K, L \bullet E)$

E = labour efficiency

$L \bullet E$ = efficiency units of labour

$$y = \frac{Y}{LE} = F(\frac{K}{LE}, 1) = F(k, 1) = f(k)$$

$$k = \frac{K}{LE}$$

Steady state

L grows by n % per year

E grows by g % per year

$\Delta k = sf(k) - (\delta + n + g)k = 0$

Gross investment = Depreciation + Reduction in capital intensity because of population growth + Reduction in capital intensity because of technological progress

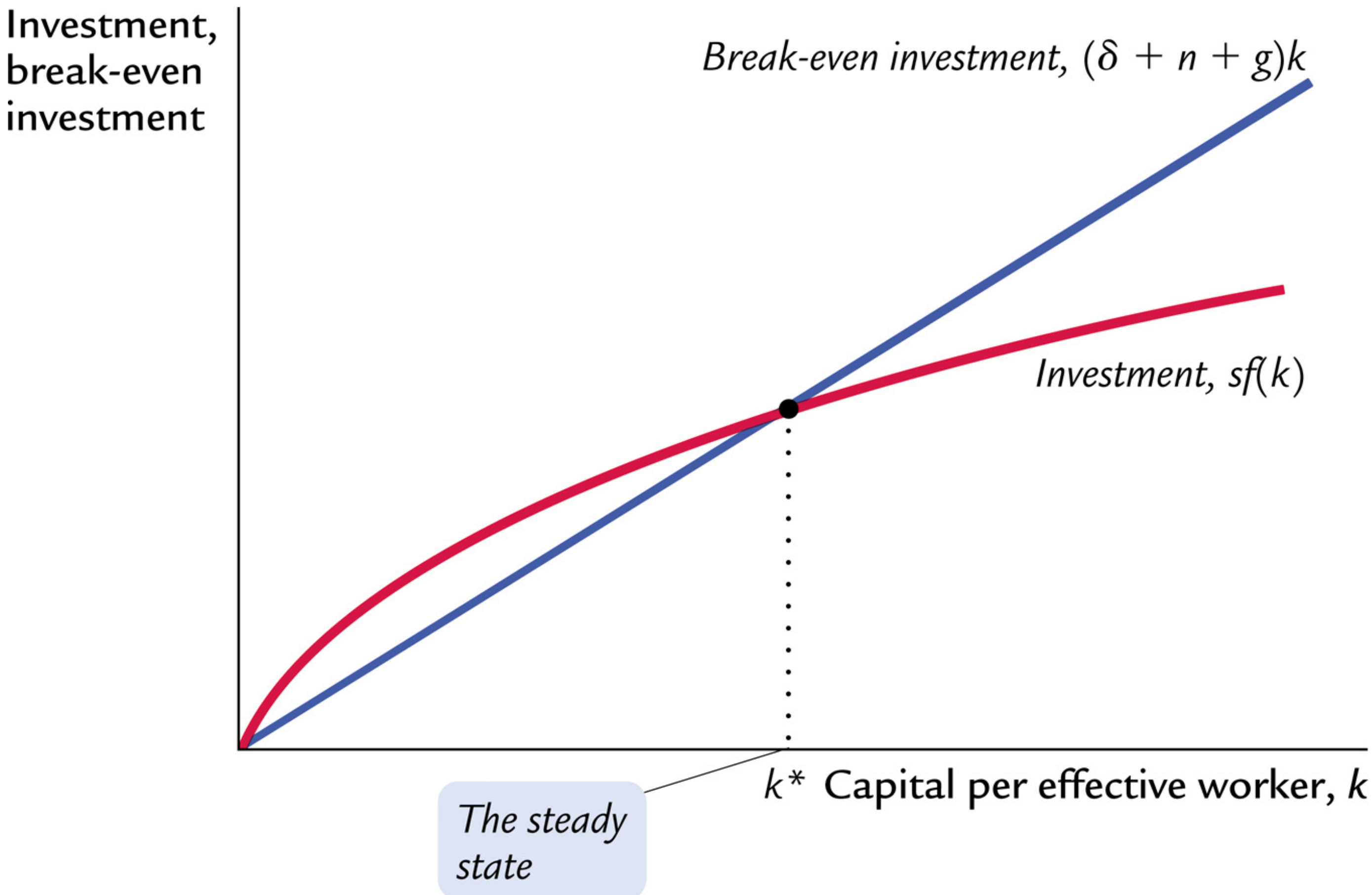


Figure 8-1: Technological progress and the Solow growth model

Growth

$$Y = K^{\alpha}(LE)^{1-\alpha}$$

$$\frac{\Delta Y}{Y} \approx \alpha \frac{\Delta K}{K} + (1-\alpha)(\frac{\Delta L}{L} + \frac{\Delta E}{E})$$

In a steady state *K/LE* is constant

$$(\Delta L / L + \Delta E / E) = n + g \Rightarrow \Delta K / K = n + g.$$

$$\frac{\Delta Y}{Y} \approx \alpha(n + g) + (1-\alpha)(n + g) = n + g$$

GDP growth = population growth+ technological progress

$$\frac{\Delta y}{y} \approx \frac{\Delta Y}{Y} - \frac{\Delta L}{L} = n + g - n = g$$

Growth in GDP per capita = rate of technological progress

Table 8-1: Steady-State growth rates in the Solow model with technological progress

Variable	Symbol	Steady-state growth rate
Capital per effective worker	$\boldsymbol{k} = \boldsymbol{K}/(\boldsymbol{L} \times \boldsymbol{E})$	0
Output per effective worker	$\boldsymbol{y} = \boldsymbol{Y}/(\boldsymbol{L} \times \boldsymbol{E})$	0
Output per worker	$(\boldsymbol{Y}/\boldsymbol{L}) = \boldsymbol{y} \times \boldsymbol{E}$	$\boldsymbol{g}$
Total output	$\boldsymbol{Y} = \boldsymbol{y} \times \boldsymbol{E} \times \boldsymbol{L}$	$\boldsymbol{n} + \boldsymbol{g}$

Golden rule with technological progress

$c = f(k) - (\delta + n + g)k$

Consumption per efficiency unit is maximised if $MPK = \delta + n + g$

The marginal product of capital should equal the sum of depreciation, population growth and technological progress

Alternative formulation: **The net marginal product (*MPK* - δ) should equal GDP growth (*n* + *g*).**

Mathematical derivation

Differentiation w.r.t. *k*:

$$\partial c / \partial k = f_k - (\delta + n + g) = 0$$

$$f_k = \delta + n + g$$

Real world capital stocks are smaller than according to the golden rule. The current generation attaches a larger weight to its own welfare than according to the golden rule.

Comparison of UK capital stock with golden rule level

$$MPK - \delta = n + g$$

$$n + g = 0.025$$

1. Capital stock is 2.5 times GDP

$$K = 2.5Y$$

2. Depreciation of capital is 10 percent of GDP

$$\delta K = 0.1Y$$

3. Capital income is 30 percent of GDP

$$MPK \times K = 0.3Y$$

Divide (2) by (1):

$$\frac{\delta K}{K} = \frac{0.1Y}{2.5Y}$$

$$\delta = 0.04$$

Divide (3) by (1)

$$\frac{MPK \times K}{K} = \frac{0.3Y}{2.5Y}$$

$$MPK = 0.12$$

$$MPK - \delta = 0.12 - 0.04 = 0.08 > n + g = 0.025$$

The capital stock is far below the golden-rule level!

Endogenous or exogenous growth

- **In the Solow model growth is exogenously determined by population growth and technological progress**
- **Recent research has focused on the role of human capital**
- **A higher savings rate or investment in human capital do not change the rate of growth in the steady state**
- **The explanation is decreasing marginal return of capital (*MPK* is decreasing in *K*)**

The AK-model

$Y = AK$

$\Delta K = sY - \delta K$

Assume *A* to be fixed!

$\Delta Y/Y = \Delta K/K$

$\Delta K/K = sAK/K - \delta K/K = sA - \delta$

$\Delta Y/Y = sA - \delta$

- **A higher savings rate *s* implies permanently higher growth**
- **Explanation: constant returns to scale for capital**
- **Complementarity between human and real capital**

<u>A two-sector growth model</u>

- **Business sector**
- **Education sector**

$Y = F[K, (1-u)EL]$ **Production function in business sector**

$\Delta E = g(u)E$ **Production function in education sector**

$\Delta K = sY - \delta K$ **Capital accumulation**

u **= share of population in education**

$\Delta E/E = g(u)$

- **A higher share of population, *u*, in education raises the growth rate permanently (cf *AK*-model – here human capital)**
- **A higher savings rate, *s*, raises growth only temporarily as in the Solow model**

Human capital in growth models

1. **Broad-based accumulation of knowledge in the system of education**

2. **Generation of ideas and innovations in research-intensive R&D sector**
3. **Learning by doing at the work place**

Policy conclusions

1. **Basic education – incentives for efficiency in the education system – incentives to choose and complete education**

2. **Put resources in top-quality R&D**

3. **Life-long learning in working life**

Technological externalities / knowledge spillovers

<u>Role of institutions</u>

- **Quality of institutions determine the allocation of scarce resources**

- **Legal systems – secure property rights**
  - “helping hand” from government (Europe)
  - “grabbing hand” from government

- **Acemoglu / Johnson /Robinson**
  - European settlers in colonies preferred moderate climates (US, Canada, NZ)
  - European-style institutions
  - Earlier institutions strongly correlated with today’s institutions