# Matching with transfers: an economist's toolbox

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  - Interpretation: 'divorce at will'

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- 'Tractable General Equilibrium'
- Different models are better suited for some purposes than for others.

# Issues related to matching: two examples

# Example 1: Assortative matching and inequality

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- Several questions; in particular:
  - Why did correlation change? Did 'preferences for assortativeness' change?
  - How do we compare single-adult households and couples? What about intrahousehold inequality?

# Example 2: College premium and the demand for college education

- **Motivation:** remarkable increase in female education, labor supply, incomes worldwide during the last decades.

FIGURE 3: FRACTION OF 30- TO 34-YEAR-OLDS WITH COLLEGE EDUCATION, COUNTRIES ABOVE MEDIAN PER CAPITA GDP AND BELOW PER CAPITA GDP, BY SEX

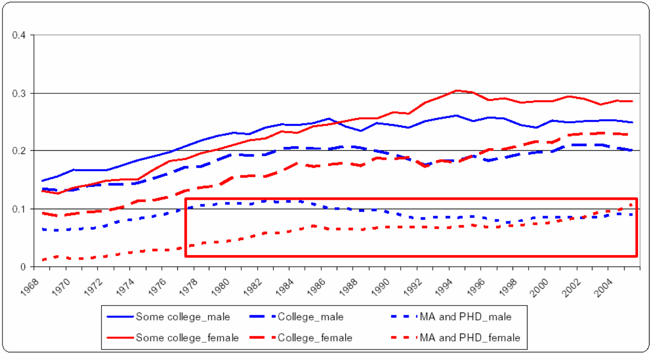

*Source:* See Figure 1.

Source: Becker-Hubbard-Murphy 2009

# Example 2: College premium and the demand for college education

- **In the US:**

Figure 13: Completed Education by Sex, Age 30-40, US 1968-2005




Source: Current Population Surveys.

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- why such different responses by gender?
- impact on intrahousehold allocation?
- impact on household behavior (expenditure, HC investment, etc.)
  $\longrightarrow$ especially relevant in developing countries!

# Roadmap

1. Matching models: general presentation
2. The case of Transferable Utility (TU)
3. Extensions:
   - Pre-investment
   - Multidimensional matching
   - Imperfectly Transferable Utility
   - Risk sharing
4. Econometric implementation

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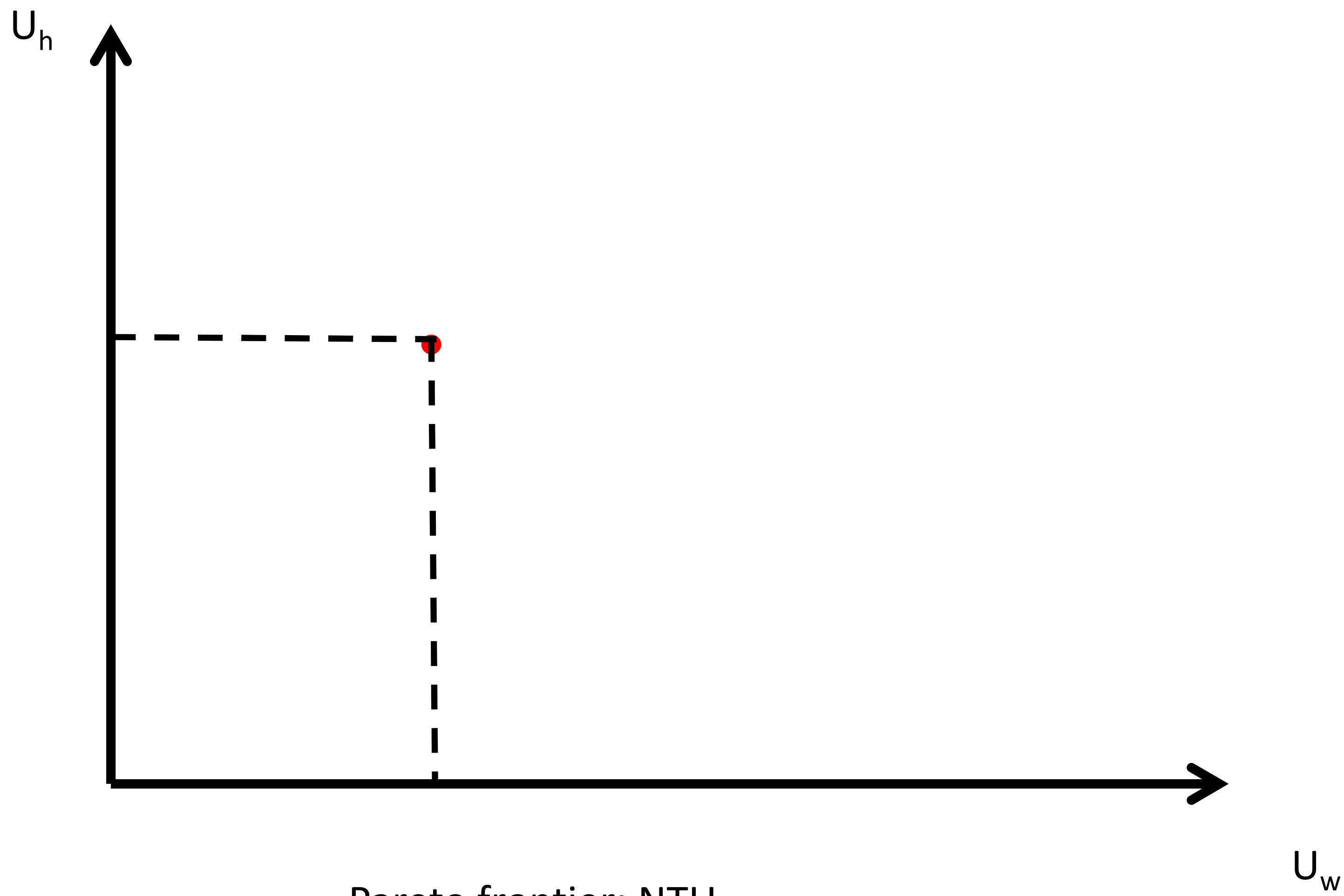
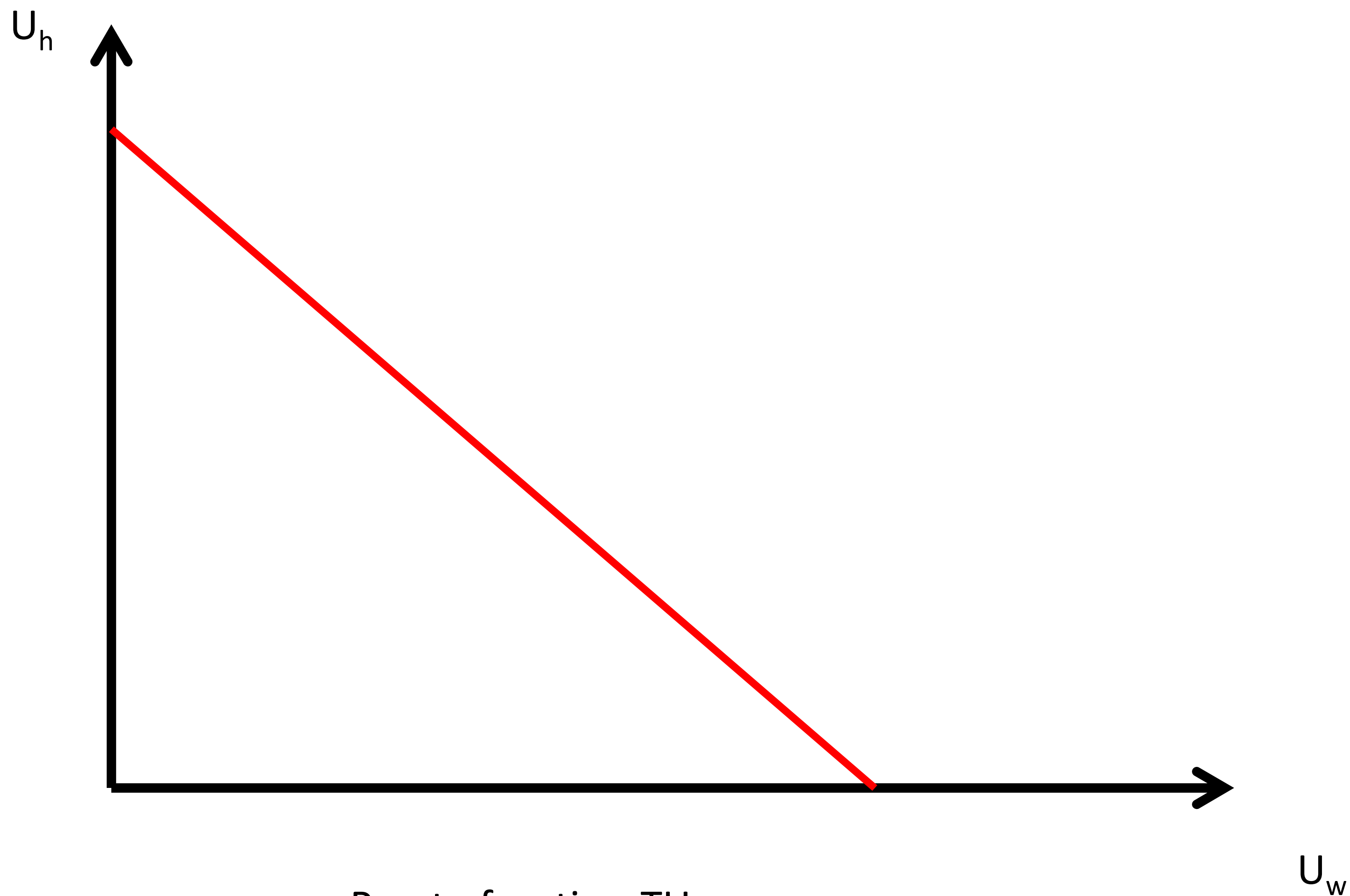
# Matching models: three main families

1. Matching under NTU (Gale-Shapley)
   Idea: no transfer *possible* between matched partners
2. Matching under TU (Becker-Shapley-Shubik)
   - Transfers possible without restrictions
   - Technology: constant 'exchange rate' between utiles
   - In particular: (strong) version of interpersonal comparison of utilities
   - $\rightarrow$ requires restrictions on preferences
3. Matching under Imperfectly TU (ITU)
   - Transfers possible
   - But no restriction on preferences
   - $\rightarrow$ technology involves variable 'exchange rate'

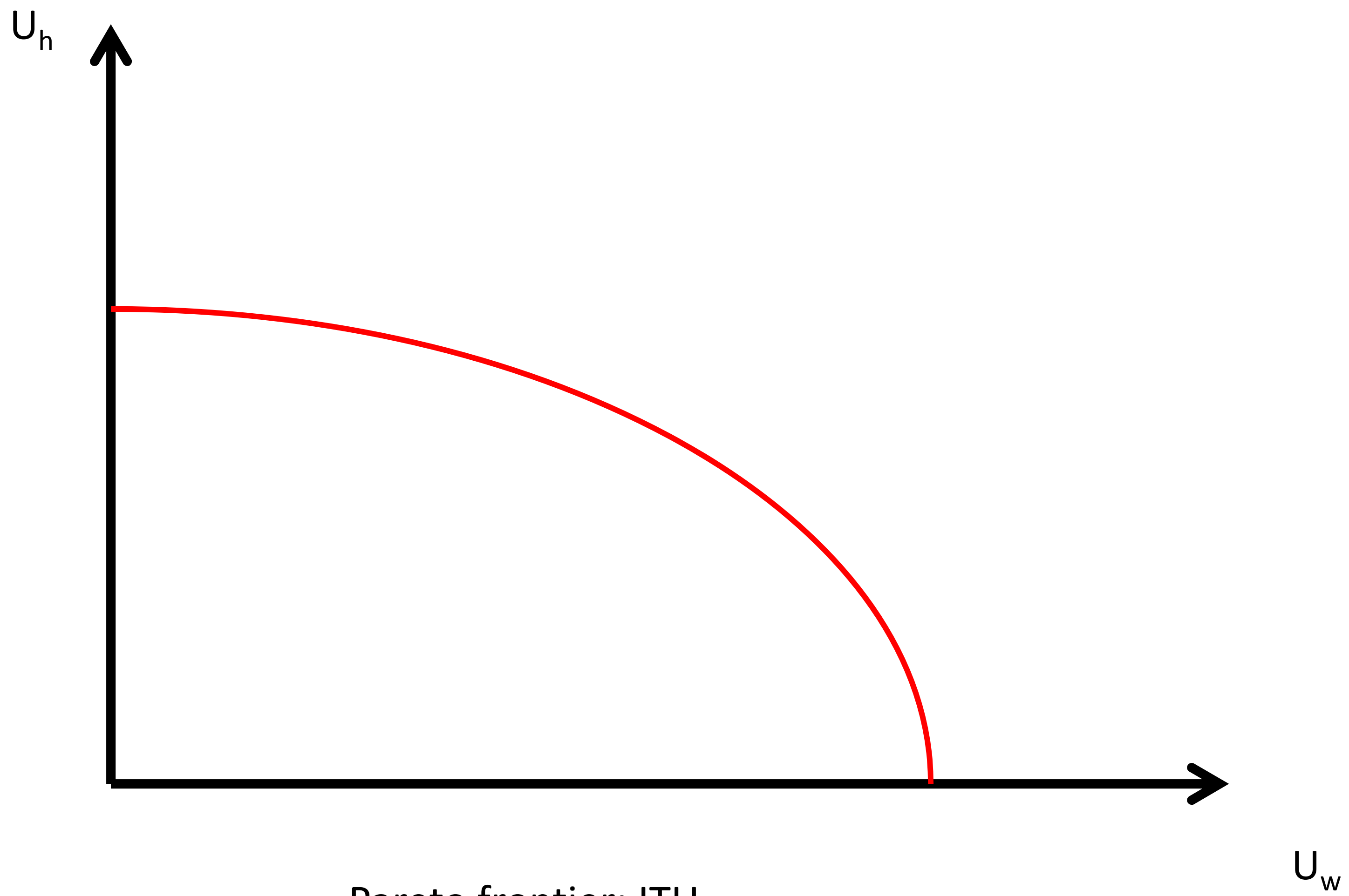
... plus 'general' approaches ('matching with contracts', from Kelso-Crawford to Milgrom-Hatfield-Kominers and friends)
... and links with: auction theory, general equilibrium.




Pareto frontier: NTU




Pareto frontier: TU




Pareto frontier: ITU

# Matching models: three main families

Similarities and differences

- All aimed at understanding who is matched with whom
- Only the last 2 address how the surplus is divided
- Only the third allows for impact on the group's aggregate behavior

# Formal structure: Common components

- Compact, separable metric spaces $X, Y$ ('women, men') with *finite* measures $F$ and $G$. Note that the spaces may be *multidimensional*
- Spaces $X, Y$ often 'completed' to allow for singles: $\bar{X} = X \cup \{\emptyset\}, \bar{Y} = Y \cup \{\emptyset\}$
- A *matching* defines of a measure $h$ on $X \times Y$ (or $\bar{X} \times \bar{Y}$) such that the marginals of $h$ are $F$ and $G$
- The matching is *pure* if the support of the measure is included in the graph of some function $\phi$
  Translation: matching is *pure* if $y = \phi(x)$ a.e.
  $\rightarrow$ no 'randomization'

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  - TU: measure $h$ *and* two functions $u(x), v(y)$ such that

    $$u(x) + v(y) = s(x,y) \text{ for } (x,y) \in \text{Supp}(h)$$

    and stability

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  - NTU: intragroup allocation *exogenously imposed*; transfers are ruled out by assumption
  - TU and ITU: intragroup allocation *endogenous*; transfers are paramount and determined (or constrained) by equilibrium conditions
  - TU: life much easier (GQL $\rightarrow$ equivalent to surplus maximization) ...
    ... but price to pay: couple's (aggregate) behavior does *not* depend on 'powers', therefore on equilibrium conditions

# Implications (crucial for empirical implementation)

- NTU: stable matchings solve

$$u(x) = \max_z \{U(x,z) | V(x,z) \geq v(z)\}$$

and

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- ITU: stable matchings solve
$$u(x)=\max_z\{F(x,z,v(z))\}\text{ and }v(y)=\max_z\{F^{-1}(z,y,u(z))\}$$
for some pair of functions $u$ and $v$.

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# Transferable Utility (TU)

## Definition

A group satisfies TU if there exists monotone transformations of individual utilities such that the Pareto frontier is an hyperplane
$u(x) + v(y) = s(x, y)$ *for all values of prices and income.*

$\rightarrow$ Marriage market: assumption on preferences?

- Model: collective (public and private consumptions, efficient decisions)
- TU if 'Generalized Quasi Linear (GQL, Bergstrom and Cornes 1981):

$$u_i(q_i, Q) = F_i\left[A_i\left(q_i^2, ..., q_i^n, Q\right) + q_i^1 b_i(Q)\right]$$

  with $b_i(Q) = b(Q)$ for all $i$ (much more general than QL)
- Then standard model: $x, y$ incomes and:

$$s(x, y) = H(x + y) = \max F_1^{-1}(u_1) + F_2^{-1}(u_2) \text{ under BC}$$

# Basic result

- If a matching is stable, the corresponding measure satisfies the *surplus maximization problem*, which is an *optimal transportation problem* (Monge-Kantorovitch):
  Find a measure $h$ on $X \times Y$ such that the marginals of $h$ are $F$ and $G$, and $h$ solves
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- Dual problem: dual functions $u(x), v(y)$ and solve
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  under the constraint
  $$u(x) + v(y) \geq s(x, y) \quad \text{for all } (x, y) \in X \times Y$$
- In particular, *the dual variables $u$ and $v$ describe an intrapair allocation compatible with a stable matching*

# Links with hedonic models

- Structure: three sets ('buyers' $X$, 'sellers' $Y$, 'products' $Z$) with measures $\mu, \nu, \sigma$. B
- Buyer $x$: quasi linear preferences $U(x,z) - P(z)$; seller $y$ maximizes profit $P(z) - c(y,z)$
- Equilibrium: price function $P(z)$ that clear markets
- Technically: function $P$ *and* measure $\alpha$ on the product set $X \times Y \times Z$ such that

(i) marginal of $\alpha$ on $X$ (resp. $Y$) coincides with $\mu$ (resp. $\nu$)

(ii) for all $(x,y,z)$ in the support of $\alpha$,

$$U(x,z) - P(z) = \max_{z' \in K} \left( U(x,z') - P(z') \right)$$
$$\text{and} \quad P(z) - c(y,z) = \max_{z' \in K} \left( P(z') - c(y,z') \right).$$

# Links with hedonic models

- Chiappori, McCann and Nesheim (2010): canonical correspondance between QL hedonic models and matching models under TU.
- Specifically:
  - Consider a hedonic model and define surplus:

    $$s(x,y) = \max_{z \in Z}(U(x,z) - c(y,z))$$

    Let $\eta$ be the marginal of $\alpha$ over $X \times Y$, $u(x)$ and $v(y)$ by

    $$u(x) = \max_{z \in K} U(x,z) - P(z) \text{ and } v(y) = \max_{z \in K} P(z) - c(y,z)$$

    Then $(\eta, u, v)$ defines a stable matching
  - Conversely, starting from a stable matching $(\eta, u, v)$,

    $$u(x) + v(y) \geq s(x,y) \geq U(x,z) - c(y,z) \quad \Rightarrow c(y,z) + v(y) \geq U(x,z$$

    For any $z$, take $P(z)$ such that

    $$\inf_{y \in J}\{c(y,z) + v(y)\} \geq P(z) \geq \sup_{x \in I}\{u(x,z) - u(x)\}$$

    then $P(z)$ is an equilibrium price for the hedonic model.

# Supermodularity and assortative matching

- Assume $X, Y$ one-dimensional. Then $s$ is supermodular if whenever $x > x'$ and $y > y'$ then

$$s(x, y) + s(x', y') > s(x, y') + s(x', y)$$

Interpretation: *single crossing* (Spence - Mirrlees)

- Consequence: matching is *assortative*
- Generalization (CMcCN ET 2010):

**Definition**

A surplus function $s : X \times Y \longrightarrow [0, \infty[$ is said to be $X-$*twisted* if there is a set $X_L \subset X_0$ of zero volume such that $\partial^x s(x_0, y_1)$ is disjoint from $\partial^x s(x_0, y_2)$ for all $x_0 \in X_0 \setminus X_L$ and $y_1 \neq y_2$ in $Y$.

Then the stable matching is unique and *pure*

# Intracouple allocation under TU

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- However, basic result:

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- Known from the outset, but ...
- ... much easier than you would think

# Pinning down intracouple allocation under TU

Assume $X, Y$ one dimensional and $s$ supermodular. Then 3 steps

- Step 1: supermodularity implies assortative matching:
  $x$ matched with $y = \psi(x)$ if *the number of women above $x$ equals the number of men above $\psi(x)$*

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- Step 2: Stability implies

$$u(x) = \max_y s(x, y) - v(y)$$

with the max being reached for $y = \psi(x)$.
Therefore

$$u'(x) = \frac{\partial s}{\partial x}(x, \psi(x)) \text{ and } v'(y) = \frac{\partial s}{\partial y}(\phi(y), y)$$

and

$$u(x) = k + \int_0^x \frac{\partial s}{\partial x}(t, \psi(t))\, dt \ , \ v(y) = k' + \int_0^y \frac{\partial s}{\partial y}(\phi(s), s)\, ds$$

$\rightarrow$ Utilities defined up to two additive constants

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  - If one gender in excess supply (say women): the 'last married' woman indifferent between marriage and singlehood
  - Note: typically, discontinuity
  - If equal number (knife-edge situation), indeterminate ...
    ... unless corner solutions

# Applications: applied theory

Various applications:

- Abortion and female empowerment (CO JPE 2006)
- Children and divorce (CW JoLE 2007)
- Male and female demand for higher education (CIW AER 2009)
- Dynamics: divorce and impact of divorce laws (CIW 10)
- Multidimensional matching:
  - general framework (Galichon Salanié 2011)
  - income/education and physical attractiveness (COQ 2011)
  - income and smoking habits (COQ 2012)
  - income and 'reproductive capital' (Low 2012)

# Roadmap

1. Matching models: general presentation
2. The case of Transferable Utility (TU)
3. *Extensions*
    - Pre-investment
    - Multidimensional matching
    - Imperfectly Transferable Utility
    - Risk sharing
4. Econometric implementation

# Extensions: roadmap

1. Pre-investment
2. Multidimensional matching
    - Theory
    - Practical Implementation
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    - General presentation
    - A specific model
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# Pre-investment

Two stage game:

1. Agents independently (non cooperatively) invest in characteristics (say in HC)
2. Agents match on these characteristics

Model solved backwards:

- For given distributions of characteristics, matching equilibrium pins down the allocation of the surplus
- This allocation defines the return from the first period investment
- 'Rational expectations': the distribution of characteristics expected by the agents when investing is realized by their investment

# Is pre-investment efficient?

Two opposite arguments:

1. ('Free rider'): My investment will increase the joint surplus, some of which goes to my (future) partner
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   Why?
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   $$u'(x) = \frac{\partial s}{\partial x}(x, \psi(x))$$
4. Application: gender unbalance: who invests more? (ACM)

# Extensions: roadmap

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Two-dimensional example:

- $X \subset \mathbb{R}^2$, $Y \subset \mathbb{R}^2$
- Surplus $S(x_1, x_2, y_1, y_2)$
- Particular case ('index'):

$$S(x_1, x_2, y_1, y_2) = \mathcal{S}(A(x_1, x_2), B(y_1, y_2))$$

Two questions:

1. Who marries whom?
2. How is the surplus shared?

# Who marries whom?

Two possible approaches:

1. 'Guess' what the matching patterns will look like; then:
   - Compute the thresholds
   - Compute the individual utilities (see below)
   - Check the stability conditions
2. Use surplus maximization
   - Always possible
   - Typically: optimal control
   - Very useful for simulations, etc.

Common caveat: matching may not be 'pure'

# Purity

- Idea: generalize the one-dimensional 'supermodularity $\Rightarrow$ assortativeness' result
- Generalization of supermodularity (CMcCN ET 2010):

**Definition**

A surplus function $S : X \times Y \longrightarrow [0, \infty[$ is said to be $X-$*twisted* if there is a set $X_L \subset X_0$ of zero volume such that $\partial^x S(x_0, y_1)$ is disjoint from $\partial^x S(x_0, y_2)$ for all $x_0 \in X_0 \setminus X_L$ and $y_1 \neq y_2$ in $Y$.

- Then the stable matching is unique and *pure*

**Definition**

The matching is pure if the measure $h$ is born by the graph of a function: for almost all $x$ there exists exactly one $y$ such that $x$ matched with $y$.

- If not: 'randomization': an open set of (say) women are indifferent between several men

# Who marries whom? (cont.)

Assume the condition is satisfied: $(y_1, y_2) = \phi(x_1, x_2)$. Then surplus maximization:

$$\max_{\phi} \int_X S(x_1, x_2, \phi(x_1, x_2))\, dF(x_1, x_2)$$

with a constraint:

The *push-forward* of $F$ through $\phi$ coincides with $G$

where the *push-forward* $\phi_{\#}F$ of $F$ through $\phi$ defined by

$$\phi_{\#}F(B) = F\left(\phi^{-1}(B)\right)$$

for any Borel $B \subset X$

$\rightarrow$ Optimal control

# Sharing the surplus

As previously, 3 steps

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with the max being reached for $y = \psi(x)$. Then 1st OC

$$\frac{\partial u}{\partial x_i} = \frac{\partial S}{\partial x_i}(x_1, x_2, \psi(x_1, x_2))$$

The PDE must be compatible:

$$\frac{\partial}{\partial x_2}\left(\frac{\partial S}{\partial x_1}(x_1, x_2, \psi(x_1, x_2))\right) = \frac{\partial}{\partial x_1}\left(\frac{\partial S}{\partial x_2}(x_1, x_2, \psi(x_1, x_2))\right)$$

If so, utilities defined up to one additive constant (and same for men)

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  If so, utilities defined up to one additive constant (and same for men)
- Step 3: pin down the constants

# Example: smoking (COQ 2011)

Setting:

- Two populations (men and women) of equal size, normalized to one.
- Socio-economic status: continuous variables $x$ and $y$, uniformly distributed over $[0, 1]$
- Smoking: dichotomic, independent of status; $k_M$ and $k_W$ proportions of smokers
- Surplus:

$$\begin{aligned}\Sigma &= s(x, y) \text{ if both spouses do not smoke} \\ \Sigma &= \lambda s(x, y) \text{ otherwise, } \lambda < 1\end{aligned}$$

- In practice

$$s(x, y) = (x + y)^2 / 2$$

# Example: smoking (COQ 2011)

Basic remark:

*The 'twisted' condition does not hold.*

Woman, index $x_0$, non smoker:

- $\partial_x \Sigma = (x_0 + y_1)$ if she marries a non smoker with index $y_1$
- $\partial_x \Sigma = \lambda (x_0 + y_2)$ if she marries a smoker with index $y_2$.

For any $y_2 \in \left[\frac{(1-\lambda)x_0}{\lambda}, 1\right]$, if $y_1 = \lambda y_2 - (1-\lambda) x_0$, then the couples $(x_0, y_1)$ and $(x_0, y_2)$ violate the twisted buyer condition; works for an open set of values $x_0$ - namely $x_0 \in \left[0, \frac{\lambda}{1-\lambda}\right]$.
Consequence:

*The stable matching may not be pure.*

# The model

Particular case: if $k_M = k_W$ then:

- All smoking women marry smoking men, and conversely
- All non smoking women marry non smoking men, and conversely

In words:

*Even if $\lambda$ very close to 1, fully discriminated submarkets*

But: in practice,

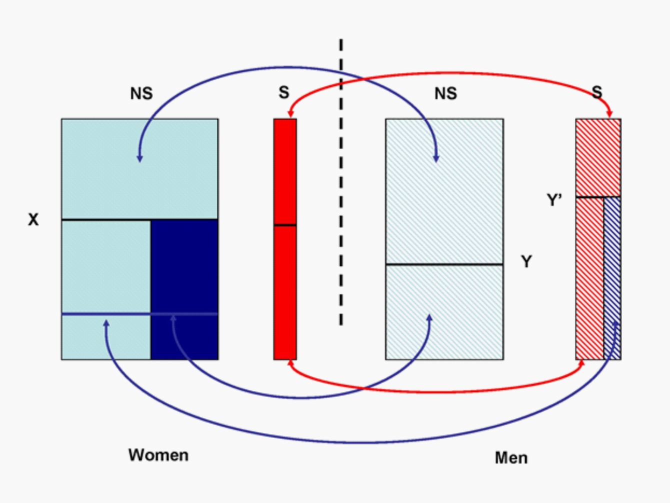
$$k_M > k_W$$

# Method 1: surplus maximization

- Four categories: $\{NW, SW, NM, SM\}$
- For each, let $P_A(t)$ denote the proba that an individual with income $t$ marries a smoker
- 'Push-forward' condition:
  - assortative matching on income within each cell
  - $\forall x \in NW$, let $\phi_{NW}(x)$ denote the income of the non smoking husband. Then
$$\int_x^1 (1 - P_{NW}(t))\, dF_{NW}(t) = \int_{\phi_{NW}(x)}^1 (1 - P_{NM}(t))\, dG_{NM}(t)$$
    which pins down $\phi_{NW}(x)$; etc.
- Finally, total surplus:
$$\begin{aligned}\Sigma &= \int_0^1 (1 - P_{NW}(t))\, S(t, \phi_{NW}(t))\, dF_{NW}(t) \\ &+ \int_0^1 P_{NW}(t)\, \lambda S(t, \phi_{NW}(t))\, dF_{NW}(t) + \ldots\end{aligned}$$

# Method 2: 'Guessing' the form of the result

Here:



# Method 2: 'Guessing' the form of the result

Then:

- Compute the utilities in each case
- Compute the thresholds (indifference conditions)
- Check stability (can be done directly using the inequality conditions)

# Particular case

Assume that

$$S(x_1, x_2, y_1, y_2) = \Sigma(A(x_1, x_2), B(y_1, y_2))$$

Then:

- one dimensional matching
- but: depends on an index that is not known

Basic intuition: two agents with the same index are equivalent for *all* potential partners; therefore they should have the same distribution of matches (i.e.: the measure $h$ only depends on $A$ and $B$).
Consequence:

- the MRS $\frac{\partial A/\partial x_1}{\partial A/\partial x_2}$ can be identified
- utility only depends on the index

# Extensions: roadmap

1. Pre-investment
2. Multidimensional matching
   - Theory
   - Practical Implementation
3. *ITU*
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# Imperfectly transferable utility: theory

Motivation

- Limitation of TU models: *all Pareto optimums correspond to the same aggregate behavior*
- Therefore, redistributing power between men and women *cannot* impact the structure of expenditures
- 'Collective' literature: important phenomenon

# Imperfectly transferable utilities

General case:

- Transfers possible...
- ... but the 'exchange rate' is not constant.
- In practice:

$$u(x) = P(x, y, v(y))$$

  with $P$ decreasing in $v$, usually increasing in $x$ and $y$.
- Stability:

$$u(x) \geq P(x, y, v(y)) \;\; \forall x \in X, y \in Y$$

- But: no longer equivalent to a maximization ('total surplus ' not defined).

# Imperfectly transferable utility: theory

- Stability

$$u(x) \geq \max_y P(x, y, v(y))$$

and equality if marriage probability positive. Hence:

$$u(x) = \max_y P(x, y, v(y))$$

1st O C:

$$\frac{\partial P}{\partial y}(x, y, v(y)) + v'(y) \frac{\partial P}{\partial v}(x, y, v(y)) = 0$$

satisfied for $x = \phi(y)$

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satisfied for $x = \phi(y)$

- Knowing $\phi$, if $\partial P / \partial y > 0$, $v$ defined up to a constant by:

$$v'(y) = -\frac{\frac{\partial P}{\partial y}(\phi(y), y, v(y))}{\frac{\partial P}{\partial v}(\phi(y), y, v(y))} > 0$$

# Imperfectly transferable utility: theory

Assortativity

- 1st OC:

$$H(y, \phi(y)) = 0 \quad \forall y$$

where

$$H(y, x) = \frac{\partial P}{\partial y}(x, y, v(y)) + v'(y)\frac{\partial P}{\partial v}(x, y, v(y)).$$

therefore

$$\frac{\partial H}{\partial y} + \frac{\partial H}{\partial x}\phi'(y) = 0 \quad \forall y,$$

# Imperfectly transferable utility: theory

Assortativity

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$$H\left(y,x\right)=\frac{\partial P}{\partial y}\left(x,y,v\left(y\right)\right)+v'\left(y\right)\frac{\partial P}{\partial v}\left(x,y,v\left(y\right)\right).$$

therefore

$$\frac{\partial H}{\partial y}+\frac{\partial H}{\partial x}\phi'\left(y\right)=0\quad\forall y,$$

- 2nd OC:

$$\frac{\partial H}{\partial y}\leq 0\quad\Leftrightarrow\quad\frac{\partial H}{\partial x}\phi'\left(y\right)\geq 0.$$

or:

$$\left(\frac{\partial^2 P}{\partial x\partial y}\left(\phi\left(y\right),y,v\left(y\right)\right)+v'\left(y\right)\frac{\partial^2 P}{\partial x\partial v}\left(\phi\left(y\right),y,v\left(y\right)\right)\right)\phi'\left(y\right)\geq 0\ \forall y \tag{1}$$

# Imperfectly transferable utility: theory

Assortative: $\phi'(y) \geq 0$ therefore

$$\frac{\partial^2 P}{\partial x \partial y}\left(\phi(y), y, v(y)\right) + v'(y) \frac{\partial^2 P}{\partial x \partial v}\left(\phi(y), y, v(y)\right) \geq 0 \quad \forall y. \tag{2}$$

or:

$$\frac{\partial^2 P}{\partial x \partial y}\left(\phi(y), y, v(y)\right) - \frac{\frac{\partial P}{\partial y}\left(\phi(y), y, v(y)\right)}{\frac{\partial P}{\partial v}\left(\phi(y), y, v(y)\right)} \frac{\partial^2 P}{\partial x \partial v}\left(\phi(y), y, v(y)\right) \geq 0 \ \forall y. \tag{3}$$

TU case: $P(x, y, v(y)) = s(x, y) - v(y)$, hence $\frac{\partial^2 P}{\partial x \partial v} = 0$ and condition

$$\frac{\partial^2 P}{\partial x \partial y} = \frac{\partial^2 s}{\partial x \partial y} \geq 0$$

# Imperfectly transferable utility: a specific model

Goal: capture two notions:

- spouses value the public good differently
- (endogenous) changes in 'powers' affect the structure of expenditures

Model:

- Continuum of men and women; $x, y$ incomes
- 1 public good, 1 private good
- Translation of distributions: matching functions (assuming assortativeness) are $\phi(y) = (y + \beta) / \alpha$ and $\psi(x) = \alpha x - \beta$.
- Male preferences:
$$u_m = c_m Q$$
- Female preferences:
$$\begin{aligned} u_f(c_f) &= -\infty \text{ if } c_f < \bar{c} \\ &= c_f + Q \text{ if } c_f \geq \bar{c} \end{aligned}$$
- In particular, efficiency implies $c_f = \bar{c}$

# Pareto frontier

- Note: $u_f \geq ((x+y)+\bar{c})/2$
- The Pareto frontier:

$$u_m = P((x+y), u_f) = (u_f - \bar{c})((x+y) - u_f),$$

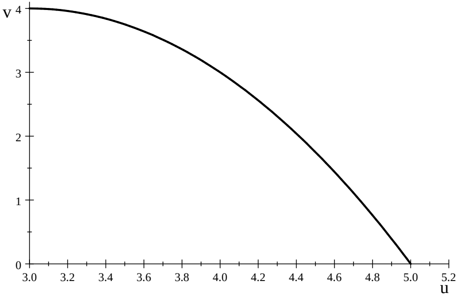


Figure: Frontière de Pareto

# Assortativeness

Here

$$\frac{\partial P\left(x+y,v\right)}{\partial\left(x+y\right)}=v-\bar{c},\frac{\partial P\left(x+y,v\right)}{\partial v}=-\left(2v-\left(\bar{c}+\left(x+y\right)\right)\right)$$

therefore

$$\frac{\partial^2 P\left(x+y,v\right)}{\partial\left(x+y\right)^2}=0 \text{ and } \frac{\partial^2 P\left(x+y,v\right)}{\partial\left(x+y\right)\partial v}=1$$

# Allocation

We have that

$$v'(y) = -\frac{\frac{\partial P}{\partial y}(\phi(y) + y, v(y))}{\frac{\partial P}{\partial v}(\phi(y), y, v(y))} = \frac{\alpha v(y) - \alpha \bar{c}}{2\alpha v(y) - (\alpha + 1) y - (\alpha \bar{c} + \beta)}.$$

Solution: let $\omega$ be the inverse of $v$, the equation becomes:

$$\omega'(v) + \frac{(\alpha+1)}{\alpha v - \alpha \bar{c}} \omega(v) = \frac{2\alpha v - (\alpha \bar{c} + \beta)}{\alpha v - \alpha \bar{c}},$$

Solution:

$$\omega(v) = K(v - \bar{c})^{-\frac{\alpha+1}{\alpha}} + \frac{2\alpha}{2\alpha+1} v - \frac{\beta + \bar{c}\alpha + 2\alpha\beta}{(\alpha+1)(2\alpha+1)},$$

# Utilities and consumptions

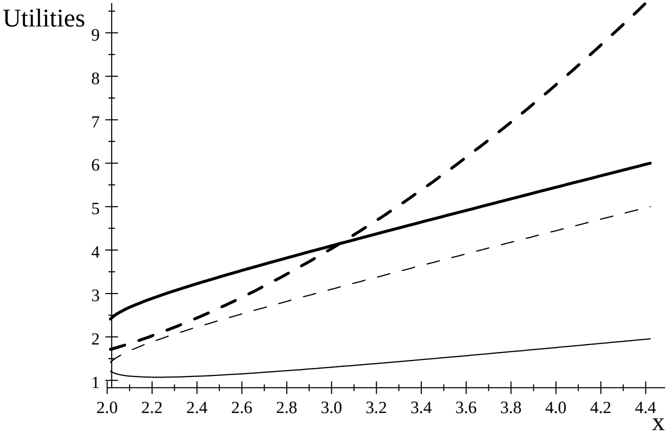


Figure: Utilities and consumptions

# Comparative statics

Start from $\lambda = .8$, and two scenarios:

1. Increase all female incomes by 25%, male unchanged
2. Increase all male incomes by 20%, female unchanged

Note that:

- 'Who marries whom' unchanged

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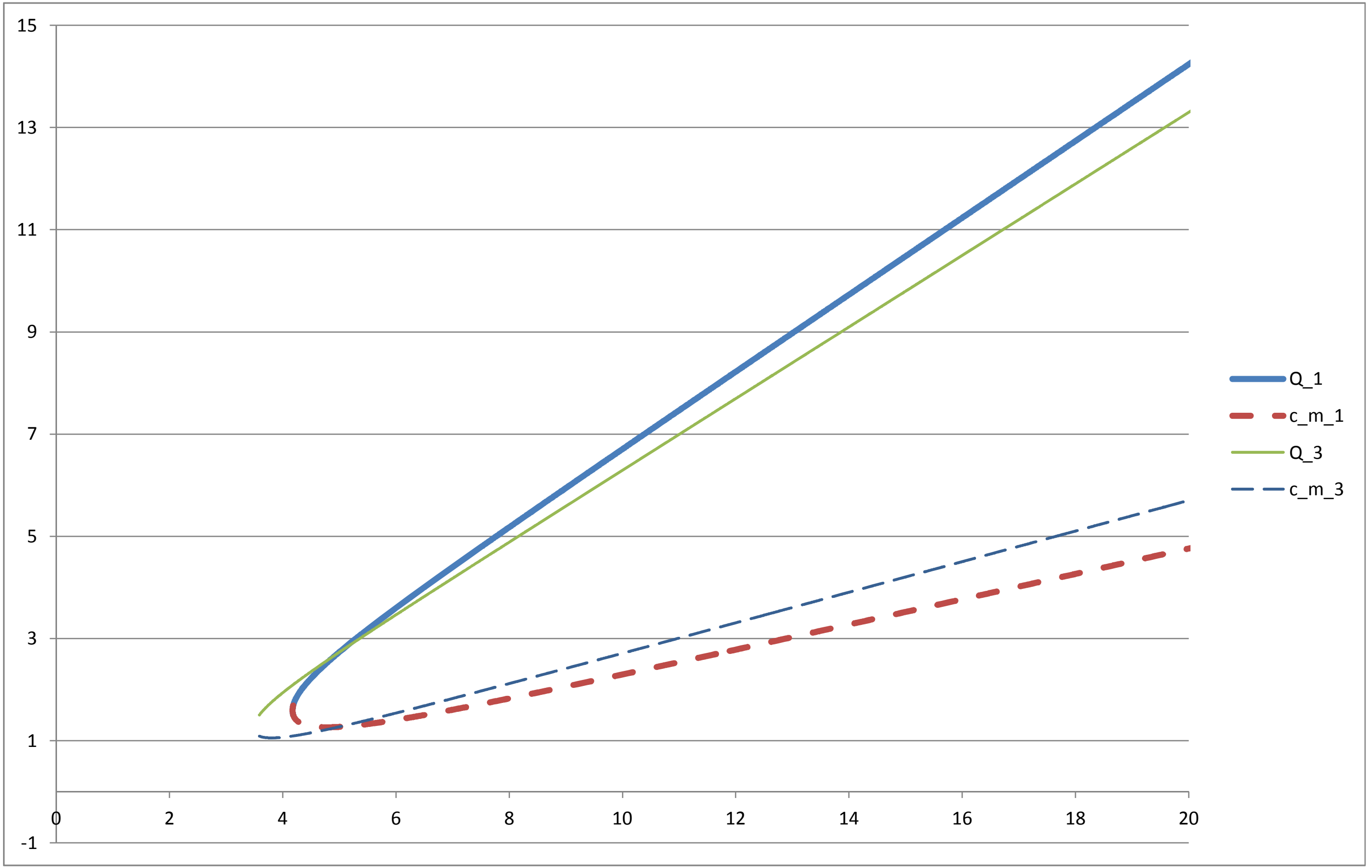
- Couples' total income unchanged
- In particular, under TU, *no impact on expenditures*
- But (presumably) here changes in powers



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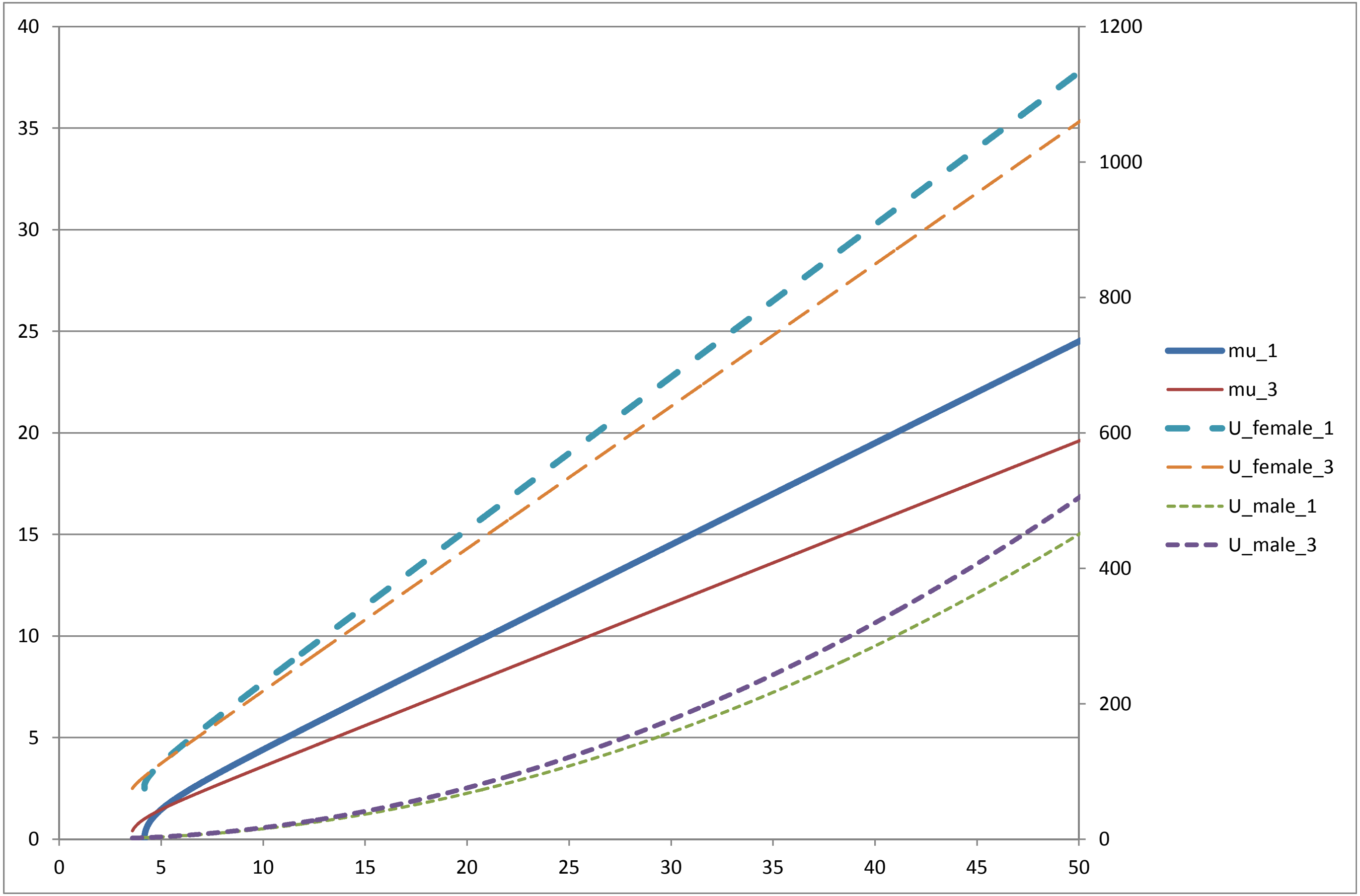

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U_male_1
U_male_3

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# Risk sharing

Two brief points:

1. Matching under TU may apply to risk sharing ...
2. ... but you still want to allow for ITU

# Risk sharing: S-W 2000

- Utilities: CRRA

$$U_m = \frac{c_m^{1-\eta}}{1-\eta}, U_f = \frac{c_f^{1-\eta}}{1-\eta}$$

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- Efficient risk sharing:

$$c_m = ky, c_f = \left(1-k\right) y$$

therefore

$$E\left(U_m\right) = \frac{k^{1-\eta}}{1-\eta} \int y^{1-\eta} dF\left(y\right), E\left(U_f\right) = \frac{\left(1-k\right)^{1-\eta}}{1-\eta} \int y^{1-\eta} dF\left(y\right)$$

and we have TU:

$$\left[\left(1-\eta\right) E\left(U_m\right)\right]^{\frac{1}{1-\eta}} + \left[\left(1-\eta\right) E\left(U_f\right)\right]^{\frac{1}{1-\eta}} = \left(\int y^{1-\eta} dF\left(y\right)\right)^{\frac{1}{1-\eta}} = S$$

# Roadmap

1. Matching models: general presentation
2. The case of Transferable Utility (TU)
3. Extensions:
    - multidimensional matching
    - Imperfectly Transferable Utility
4. *Econometric implementation*

# Econometric implementation

Basic empirical issue:

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    - about transfers ($\longrightarrow$ hedonic models)
    - about the outcome and/or the sharing ($\longrightarrow$ collective model)

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- Therefore: dual variables $(u_i, v_j)$ also random (*endogenous* distribution). Problem: nothing is known (in general) about the dual distribution.
- Stability: constrained by the inequalities

$$u_i + v_j \geq g_{ij}^{IJ} \quad \text{for any} \quad (i, j)$$

  $\rightarrow$ large number (one inequality *per potential couple*) ... of which a few are in fact equalities

# Basic issue: why did female demand for education exceed male?

Possible explanation (CIW 2009): impact of education is twofold:

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- Simple framework: *Total* college premium as the sum of these two components; CIW's story: huge discrepancies between genders regarding MCP
- Matching models adequate to distinguish, since they take singlehood as a benchmark

# Theoretical model (CIW 2009)

- Two-dimensional heterogeneity: willingness to marry and cost of acquiring education
- Two stage model:
  - Stage 1: choose education level and entry on the marriage market
  - Stage 2: matching game
- Resolution: backwards
  - solve matching for given population $\rightarrow$ dual variables: expected utility for each education level
  - then models decision to acquire education/enter the marriage market
  - Note: fixed point
- Problem 1: how to *empirically* estimate the second stage?
- Problem 2 (more ambitious): estimate the *two stage* model (ongoing work with M. Costa and C. Meghir)

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  - The sum $\alpha_i + \beta_j$ contributes to the surplus
  - Note that $E[\alpha_i \mid i \in I] = a^I \neq 0$ in general:

$$\alpha_i^J = a_i^J + \tilde{\alpha}_i^J \text{ with } E\left(\tilde{\alpha}_i^J\right) = 0$$

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- 'Second stage': match after education has been chosen but before incomes are known $\longrightarrow$ *economic* surplus if $i \in I$ marries $j \in J$:

$$S^{IJ} = E\left[s\left(x, y\right) \mid i \in I, j \in J\right]$$

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- *Total* surplus:

$$\begin{aligned} s_{ij} &= S^{IJ} + \alpha_i^J + \beta_j^I \\ &= Z^{IJ} + \tilde{\alpha}_i^J + \tilde{\beta}_j^I \end{aligned}$$

  where $I = 0$, $J = 0$ for singles, $\alpha_i^0 = \beta_j^0 = 0$ by normalization, $Z^{IJ} = S^{IJ} + a_i^J + b_j^I$ and $E\left[\tilde{\alpha}_i^J\right] = E\left[\tilde{\beta}_j^I\right] = 0$

# Econometric implementation

- The model satifies a crucial identifying assumption (Choo-Siow 2006)
  **Assumption S (separability)**: *the idiosyncratic component* $\varepsilon_{ij}$ *is additively separable:*
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- Then:

**Theorem**

*Under S, there exists $U^{IJ}$ and $V^{IJ}$ such that $U^{IJ} + V^{IJ} = Z^{IJ}$ and for any match $(i \in I, j \in J)$*

$$\begin{aligned} u_i &= U^{IJ} + \alpha_i^{IJ} \\ v_j &= V^{IJ} + \beta_j^{IJ} \end{aligned}$$

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## Theorem

*A NSC for $i \in I$ being matched with a spouse in $J$ is:*

$$\begin{aligned} U^{IJ} + \alpha_i^{IJ} &\geq U^{I0} + \alpha_i^{I0} \\ U^{IJ} + \alpha_i^{IJ} &\geq U^{IK} + \alpha_i^{IK} \quad \text{for all } K \end{aligned}$$

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  - assume the $\alpha_i^{IJ}$ are extreme value distributed
  - then logit and expected utility:

$$\bar{u}^I = E\left[\max_J \left(U^{IJ} + \alpha_i^{IJ}\right)\right] = \ln\left(\sum_J \exp U^{IJ} + 1\right) = -\ln\left(a^{I0}\right)$$

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    - assume 'some' invariance across cohorts
- Underlying question: '*did the preferences for assortative matching change*'?

# Test and identification (CSW 10)

**Idea**: structural model $\mathcal{M} = \left(Z^{IJ}, \sigma^I, \mu^J\right)$ holds for different cohorts $c = 1, ..., T$ *with varying class compositions*. Then:

$$s_{ij,c} = Z_c^{IJ} + \sigma^I \tilde{\alpha}_{i,c}^J + \mu^J \tilde{\beta}_{j,c}^I$$

where $\alpha, \beta$ extreme value distributed, with the identifying assumption:

$$Z_c^{IJ} = \zeta_c^I + \xi_c^J + Z^{IJ}$$

Interpretation: trend affecting the surplus *but not the supermodularity*

$$Z_c^{IJ} - Z_c^{IL} - Z_c^{KJ} + Z_c^{KL} = Z^{IJ} - Z^{IL} - Z^{KJ} + Z^{KL}$$

$\rightarrow$ Null: '*Preferences for assortativeness do not change*'

**Basic result:** the model is (over)identified

# Data

- American Community Survey, a representative extract of Census. The 2008 survey has info on current marriage status, number of marriages, year of current marriage (633,885 currently married couples).
- Born between 1943 and 1970 for men, 1945 and 1972
- Three education classes: HS drop out, HS graduate, College and above
- Construct 28 'cohorts'; for each cohort, matrix of marriage proportions by classes (plus singles)
- Age $\longrightarrow$ assumption: husband in cohort $c$ marries wife in cohort $c + 2$

# Results (1)

- Estimate the $Z^{IJ}$s; strongly supermodular

| Group | HSD | HSG | SC |
|---|---|---|---|
| HSD | 0.331 | 0.193 | $-0.128$ |
| HSG | 0.195 | 0.272 | 0.098 |
| SC | $-0.028$ | 0.233 | 0.468 |

Table: $Z$ values: men in rows, women in columns

- Variances:

$$\sigma_1 = .089, \sigma_2 = .06, \sigma_3 = .087, \mu_1 = .148, \mu_2 = .071, \ \mu_3 = .137$$

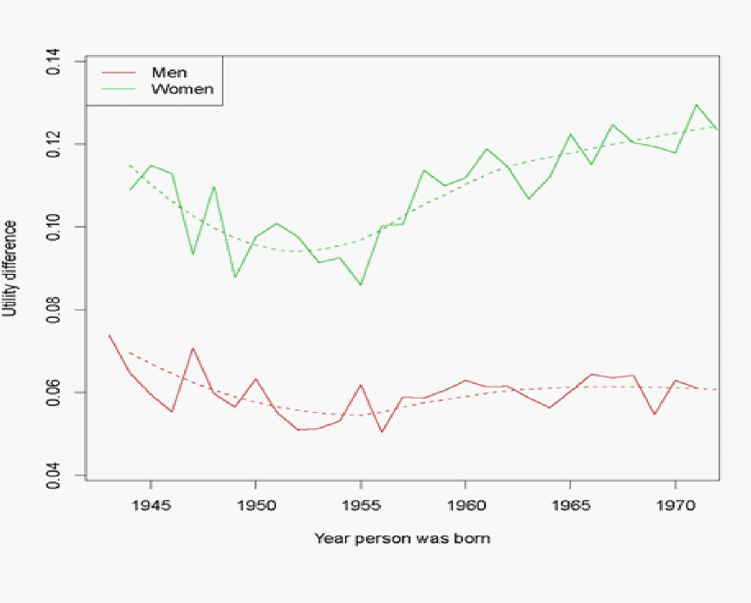
# Results: marital college premium

- In principle, marital college premium has several components:
  - Marriage probability
  - Spouse's (distribution of) education
  - Surplus generated
  - Distribution of the surplus
- Our estimates for women:

| Cohort born | 1944-46 | | 1970-72 | |
|---|---|---|---|---|
| Education | HSG | SC | HSG | SC |
| Married | 0.933 | 0.896 | 0.791 | 0.818 |
| College-educated husband | 0.380 | 0.833 | 0.376 | 0.841 |
| Marital surplus | 0.191 | 0.464 | -0.041 | 0.330 |
| Wife's share | 0.419 | 0.570 | 0.404 | 0.625 |

Table: Marital outcomes for women in early and in recent cohorts

# Results: marital college premium



# Conclusion

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2. Crucial property: intramatch allocation of surplus derived from equilibrium conditions
3. Applied theory: many applications (abortion, female education, divorce laws, children, ...)
4. Can be taken to data; structural econometric model, over identified
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