Expectations-Based Reference-Dependent Life-Cycle Consumption*

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Job Market Paper

Abstract

This paper incorporates a new preference specification of expectations-based loss aversion, which has been broadly applied in microeconomics, into a classic macro model to offer a unified explanation for three empirical observations about life-cycle consumption. First, loss aversion rationalizes excess smoothness and sensitivity, the empirical observation that consumption responds to income shocks with a lag. Intuitively, such lagged responses allow the agent to delay painful losses in consumption until his expectations have adjusted. Second, the preferences generate a hump-shaped consumption profile. Early in life, consumption is low due to a first-order precautionary-savings motive. But, as uncertainty resolves over time, this motive becomes dominated by time-inconsistent overconsumption that eventually leads to declining consumption toward the end of life. Third, consumption drops at retirement. Prior to retirement, the agent wants to overconsume his uncertain income before his expectations catch up. Post retirement, however, income is no longer uncertain, so that overconsumption is associated with a certain loss in future consumption. As an empirical contribution, I structurally estimate the preference parameters using life-cycle consumption data. My estimates match those obtained in experiments and other micro studies and generate the degree of excess smoothness observed in macro consumption data.

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I Introduction

In the last thirty years, the consumption literature has debated numerous explanations for three major empirical observations about life-cycle consumption: excess smoothness and sensitivity in consumption, a hump-shaped consumption profile, and a drop in consumption at retirement.[1] This paper offers a unified explanation based on expectations-based reference-dependent preferences, which have been developed by Koszegi and Rabin (2006, 2007, 2009) to discipline and broadly apply the insights of prospect theory.[2] The preferences formalize the idea that changes in expectations about consumption generate instantaneous utility and that losses in expectations about consumption hurt more than gains please. While these preferences have been shown to explain evidence in various micro domains, this paper validates the preferences in a classic macro domain. My explanation for the three consumption facts relies on intuitions that are reminiscent of the micro evidence that the preferences were developed to explain and may provide new foundations for prominent ideas in the macro consumption literature. Moreover, I show that the preferences generate new behavior and welfare predictions. These welfare predictions are important, because whether or not consumption, which represents two-thirds of GDP, should be excessively smooth matters for labor market reforms and countercyclical policies.

I first explain the preferences in greater detail. In each period, the agent's instantaneous utility consists of two components. "Consumption utility" is determined by his level of consumption and corresponds to the standard model of utility. "Gain-loss utility" is determined by his expectations about consumption relative to his reference point and corresponds to a prospect-theory model of utility. The agent's reference point is determined by his previous beliefs about both his present consumption and his entire stream of future consumption. The agent experiences "contemporaneous" gain-loss utility when he compares his actual present consumption with his probabilistic beliefs about present consumption. In this comparison, he encounters a sensation of gain or loss relative to each consumption outcome that he had previously expected. Additionally, the agent experiences "prospective" gain-loss utility when he compares his updated beliefs about future consumption with his previous beliefs, encountering gain-loss utility over what he has learned about future consumption. Thus, gain-loss utility can be interpreted as utility over good and bad "news" about consumption.

I analyze an agent with such "news-utility" preferences in a life-cycle consumption model. The agent lives for a finite number of periods; at the beginning of each period, he observes the realization of a permanent and a transitory income shock and then decides how much

[1]Refer to Attanasio and Weber (2010) for a comprehensive survey of the life-cycle consumption literature.

[2]Prospect theory (Kahneman and Tversky (1979)) states that people care about gains and losses relative to a reference point, where small losses hurt more than small gains give pleasure, i.e., people are loss averse.

to consume and save. I first assume that the agent's consumption utility is an exponential CARA function. This assumption produces a closed-form solution, which allows a precise understanding of the intuitions behind the preferences' implications. I then show that all of the implications hold if I instead assume a power-utility CRRA function.[3]

As the first key implication, these preferences generate excess smoothness and sensitivity in consumption, which refer to the empirical observations that consumption initially under-responds to income shocks and then adjusts with a delay.[4] Such consumption responses are puzzling from the perspective of the standard model, in which consumption fully adjusts immediately, but can be explained by expectations-based loss aversion. A simplified intuition is that, in the event of an adverse income shock, unexpected losses in consumption today are more painful than expected reductions in the future. Accordingly, the agent delays unexpected losses in consumption until his expectations will have adjusted in the future. Losses in present consumption are more painful than losses in future consumption because losses in future consumption depend on future income shocks and are thus, to some extent, uncertain.

Beyond resolving these puzzles, the preferences are consistent with another stylized fact, namely a hump-shaped life-cycle consumption profile. A hump-shaped profile is characterized by increasing consumption at the beginning but decreasing consumption toward the end of life.[5] This hump results from the interaction of two features of the preferences, a first-order precautionary-savings motive and a beliefs-based time inconsistency. First, the preferences motivate precautionary savings because loss aversion makes fluctuations in consumption painful in expectation. These fluctuations hurt relatively less, however, higher on the concave utility curve, which brings about an additional motive to save. This savings motive depends on concavity and is a first-order consideration, as opposed to the precautionary-savings motive under standard preferences.[6] Second, the preferences are subject to a beliefs-based time

[3]I assume a standard model environment, as proposed by Carroll (2001) and Gourinchas and Parker (2002), but my results are robust to many alternative environmental assumptions such as liquidity constraints, different income profiles, different savings devices, portfolio choice, endogenous labor supply, mortality risk, an endogenous retirement date, different pension designs, and income and expenditure risk after retirement.

[4]This inherently related interpretation of excess smoothness and sensitivity is first put forward by Deaton (1986) and Campbell and Deaton (1989), in response to the seminal paper by Flavin (1981) (more recent empirical evidence for excess smoothness using micro data is provided by Attanasio and Pavoni (2011)). I focus on this interpretation because my basic model features only contemporaneous income shocks. A delayed response to last period's permanent income shock can be interpreted as a response to expectedly high income. The empirical evidence for consumption responses to expected income shocks is surveyed in Jappelli and Pistaferri (2010). In a model that features contemporaneous shocks to future income, excess sensitivity would be explained by news utility, if the contemporaneous shock to future income concerns income in the next period. Other interpretations of excess sensitivity and the model's explanatory power with regards to them as well as alternative theoretical explanations are reviewed in the next section.

[5]Empirical evidence for a hump-shaped consumption profile is provided by Fernandez-Villaverde and Krueger (2007) and Gourinchas and Parker (2002).

[6]In a first-order approximation of savings, the effect of uncertainty depends on the second derivative of

inconsistency. The agent behaves inconsistently because he takes his expectations as given when increasing present consumption, but takes his expectations into account when increasing future consumption. However, once the future rolls around, he will take his expectations as given and only consider the pleasure of increasing consumption above expectations rather than increasing consumption and expectations. As a result, the agent overconsumes relative to the optimal pre-committed consumption path that maximizes his expected utility. To summarize, the precautionary-savings motive keeps consumption low at the beginning of life. However, the need for precautionary savings decreases when uncertainty resolves over time. At some point, precautionary savings are dominated by the beliefs-based time inconsistency causing overconsumption. Such overconsumption in mid-life will force the agent to choose a declining consumption path by the end of life.

Finally, the preferences predict a drop in consumption at retirement.[7] In a standard life-cycle model, income uncertainty is absent during retirement, eliminating both the precautionary-savings motive and the beliefs-based time inconsistency. The inconsistency is eliminated because the agent no longer allocates uncertain labor income but allocates certain income. Certainty implies that time-inconsistent overconsumption today is associated with a certain loss in future consumption. Because the certain loss hurts more than overconsumption gives pleasure, the agent suddenly controls his time-inconsistent desire to overconsume and his consumption drops at retirement. This result is robust to assuming small uncertainty, for instance, inflation or pension risk, or discrete uncertainty, for instance, health shocks.[8]

Beyond these three implications, the preferences generate several new and testable predictions about consumption and savings. For instance, excess smoothness is increasing in the agent's horizon and prevalent for temporary shocks only if the agent does not face permanent shocks additionally. Moreover, I draw potentially important welfare conclusions by looking at the optimal pre-committed consumption path. Excess smoothness increases welfare and is even more pronounced on the pre-committed path. In contrast, the hump-shaped consumption profile and the drop in consumption at retirement decrease welfare and are absent

the utility function and the precautionary-savings motive does not go to zero when uncertainty becomes small. This result was obtained by Koszegi and Rabin (2009) in a two-period, two-outcome model.

[7] The empirical evidence regarding the prevalence of a drop in consumption at retirement is debated. A series of papers, e.g., Banks et al. (1998), Bernheim et al. (2001), Battistin et al. (2009), Haider and Stephens (2007), Schwerdt (2005) find that consumption drops at retirement taking work-related expenses into account, and my data display such a drop. Moreover, Ameriks et al. (2007) and Hurd and Rohwedder (2003) provide evidence that the drop in consumption is anticipated. However, Aguiar and Hurst (2005) find that the drop is absent when properly controlling for health shocks and home production.

[8] When the consumption function is discrete, overconsumption results in a discrete gain that is compared with a discrete loss in future consumption, which allows the agent to credibly plan to overconsume less than if consumption is continuously distributed. The agent chooses discrete consumption, if uncertainty is discrete or if uncertainty is very small in which case the decrease in expected utility from experiencing gain-loss utility outweighs the increase in expected utility from smoothing consumption perfectly.

on the pre-committed path.

By way of providing alternative explanations, I explore habit-formation, hyperbolic-discounting, temptation-disutility, and standard preferences. I find that only news utility provides such a unified explanation independent of assumptions other explanations rely on, e.g., a power-utility function, hump-shaped income profiles, liquidity constraints, preference shifters, or non-separabilities of consumption and leisure. However, I believe that a more significant contribution is that the explanations' intuitions connect the micro evidence that reference-dependent preferences were developed to reconcile with several compelling concepts in the macro consumption literature. For instance, loss aversion explains the two macro consumption puzzles and is an experimentally robust risk preference, which has been used to explain major behavioral phenomena, such as the endowment and disposition effects, and used to explain other macro phenomena, such as stock market non-participation and the equity-premium puzzle.[9] The explanation of the equity-premium puzzle is nicely related to the explanation of the excess-smoothness puzzle: both rely on loss aversion smoothing consumption relative to movements in asset prices or permanent income.[10] To explain the other life-cycle facts, the preferences intuitively unify precautionary savings, which have been studied extensively in the standard consumption literature, and a beliefs-based time inconsistency, which is reminiscent of hyperbolic discounting.

To quantitatively evaluate the model, I structurally estimate the preference parameters using life-cycle consumption data.[11] I can identify all preference parameters because each parameter generates specific variation in consumption growth over the life-cycle. I then compare my estimates to those found in the microeconomic literature by exploiting the fact that all the preference parameters have narrow ranges determined by existing behavioral evidence and common sense. I then show that my estimates are not only in line with the micro literature and generate reasonable attitudes towards small and large wealth bets but also match the empirical evidence for excess smoothness and sensitivity in aggregate data.[12]

The paper is organized as follows. After a literature review, I explain the model environ-

[9]The endowment effect refers to the phenomenon that people become less willing to give up an item once they own it. If the item is in people's possession, foregoing it feels like a loss. The most famous study is Kahneman et al. (1990), in which students are given a mug and then offered the chance to sell it. The authors find that the payment that students' ask for once they own the mug is twice the payment they are offering to purchase it. More recently, Ericson and Fuster (2010) demonstrate that subjects' expectations to keep rather than possess an item accounts for the gap between the asking and purchase price. The disposition effect (Odean (1998)) is an anomaly related to the tendency of investors to sell winners (stocks that have gone up in value) but keep losers (stocks that have gone down in value) to avoid the realization of losses.

[10]Because consumption is too smooth relative to movements in asset prices, a high equity premium in the canonical asset-pricing economy requires unreasonably high second-order risk aversion.

[11]I follow the two-stage method-of-simulated-moments approach of Gourinchas and Parker (2002) and use pseudo-panel data from the Consumer Expenditure Survey as provided by the NBER.

[12]I use NIPA consumption and income data following Ludvigson and Michaelides (2001).

ment, preferences, and equilibrium concept in Section III. Then, I derive the model's main predictions in closed form under the assumption of exponential utility in Section IV. After briefly outlining the power-utility model, I then calibrate both models to assess whether the quantitative predictions match the empirical evidence and structurally estimate the model's parameters in Section V. Section VI outlines several extensions. Section VII concludes.

II Literature Review

The static model of reference-dependent preferences, Koszegi and Rabin (2006, 2007), has been used to explain experimental and other microeconomic evidence in various contexts.[13] In a fully dynamic and stochastic model, I obtain predictions that modify and extend the results about consumption and savings obtained by Koszegi and Rabin (2009) in a two-period, two-outcome model.[14] In particular, I generalize the implications for precautionary savings and time-inconsistent overconsumption and intuitively combine them to explain the hump-shaped consumption profile and the drop in consumption at retirement. In contrast, the excess-smoothness result is new. It is different from the result by Koszegi and Rabin (2009) that the news-utility agent consumes windfall gains but delays windfall losses, which depend on the windfall gains and losses coming as a surprise, i.e., an initially certain consumption path.[15] It is similarly different from the result by Bowman et al. (1999) that the loss averse

[13]Heidhues and Koszegi (2008, 2010), Herweg and Mierendorff (2012), and Rosato (2012) explore the implications for consumer pricing, which are tested by Karle et al. (2011), Herweg et al. (2010) do so for principal-agent contracts, and Eisenhuth (2012) does so for mechanism design. An incomplete list of papers providing direct evidence for Koszegi and Rabin (2006, 2007) preferences is Sprenger (2010) on the implications of stochastic reference points, Abeler et al. (2012) on labor supply, Gill and Prowse (2012) on real-effort tournaments, Meng (2010) on the disposition effect, and Ericson and Fuster (2010) on the endowment effect (not confirmed by Heffetz and List (2011)). Barseghyan et al. (2010) structurally estimate a model of insurance-deductible choice. Suggestive evidence is provided by Crawford and Meng (2009) on labor supply, Pope and Schweitzer (2011) on golf players' performance, and Sydnor (2010) on deductible choice. Moreover, the numerous conflicting papers on the endowment effect can be reconciled with the notion of expectations determining the reference point. All of these papers consider the static preferences, but as the dynamic preferences of Koszegi and Rabin (2009) are a straightforward extension, the evidence is equally valid for the dynamic preferences. Moreover, the notion that agents are loss averse with respect to news about future consumption is indirectly supported by all experiments, which use monetary payoffs because these concern future consumption.

[14]Koszegi and Rabin (2009) develop the dynamic preferences from the static model of Koszegi and Rabin (2006, 2007) by introducing contemporaneous and prospective gain-loss utility in the instantaneous utility function. In so doing, the authors generalize the static "outcome-wise" gain-loss comparison to a "percentile-wise" comparison. I generalize the static comparison slightly differently by assuming that the agent experiences outcome-wise gain-loss utility only over uncertainty that has been realized. Because this comparison preserves an outcome-wise structure and is a linear operator, it is considerably more tractable. Moreover, because the psychological intuition of the separated comparison is also reasonable, I see this modification as a minor contribution to exploring the preferences.

[15]This result carries over to environments characterized by discrete income uncertainty. To elaborate on this result and relax the assumption concerning a period's horizon, which constitutes a calibrational degree

agent delays losses only to remain at his deterministic reference point.[16] In contrast, the reference point is stochastic in my model because consumption is continuously distributed. The stochastic reference point induces delayed consumption adjustments to both good and bad income shocks because today's reference point is more sensitive to today's consumption and savings plan than tomorrow's reference point, which will adjust to today's plan.

Excess smoothness in consumption cannot be generated by a time-separable utility function as made clear by Ludvigson and Michaelides (2001) among others. To obtain excess smoothness, a predominant additional assumption is borrowing constraints as analyzed by Deaton (1991) among others. However, this assumption faces two problems. Theoretically, the agent expects these constraints and ensures that they are not binding for most income realizations. Empirically, the implied asymmetry in excess smoothness could not ultimately be confirmed. Borrowing constraints are binding more often in a model that features a time-inconsistency problem, as analyzed by Angeletos et al. (2001) and Laibson et al. (2012). In these models, sophisticated hyperbolic-discounting preferences imply that the agent restricts his consumption opportunities with illiquid savings against which he can borrow only up to some constraint.[17] To the extent that his borrowing constraint binds or his liquid asset holdings bunch at zero, his consumption is excessively smooth.[18]

By explaining excess smoothness and sensitivity with preferences, I resume a literature pioneered by Caballero (1995), who assumes that agents consume near-rationally, and Fuhrer (2000) and Michaelides (2002), who assume internal multiplicative habit formation. The

of freedom in a model with first-order risk aversion, I outline a model extension that assumes that the agent receives large income shocks every couple periods but merely discrete income shocks in in-between periods. Discrete uncertainty allows the agent to make a credible plan to overconsume less, but implies that he will consume entire small gains and delay entire small losses. There exists evidence that people consume small windfall gains almost entirely (as surveyed in Jappelli and Pistaferri (2010)), which has been related to excess sensitivity in consumption as the permanent income model would predict a marginal propensity to consume out of transitory shocks that is close to zero.

[16]These models of loss aversion predict a purely asymmetric response. The empirical evidence on potentially asymmetric responses to income innovations, which would also be predicted by liquidity constraints, is surveyed in Jappelli and Pistaferri (2010). The evidence is very mixed; a famous paper by Shea (1995) finds that consumption is more excessively sensitive to expected income declines than increases, which is inconsistent with liquidity constraints or myopia but consistent with loss aversion. However, Krueger and Perri (2010) find the opposite result.

[17]Demand for commitment is also generated by temptation-disutility preferences, as specified in Gul and Pesendorfer (2004) and analyzed by Bucciol (2012) in a life-cycle context.

[18]Laibson et al. (2012) and Laibson (1997) put forward an interpretation of excess sensitivity that focuses on a high marginal propensity to consume out of transitory income shocks, as the permanent income model would predict this propensity to consume to be close to zero. A high marginal propensity to consume out of transitory income shocks is prevalent in this model if the agent's time-inconsistent overconsumption dominates precautionary savings or if the model is extended such that the agent has access to an illiquid asset. In such a model, all preference specifications that feature a time-inconsistency problem, i.e., news utility, hyperbolic discounting, or temptation disutility, will predict a high propensity to consume as the agent makes wealth inaccessible to his overconsuming future selves using the illiquid asset.

basic concept of news utility appears similar to habit formation. However, the life-cycle implications are very different; most importantly, I confirm the conclusion of Michaelides (2002) that habit formation generates excess smoothness at the cost of unreasonably high wealth accumulation due to high effective risk aversion. Furthermore, Flavin and Nakagawa (2008) define a utility function over two consumption goods, one representing non-durable consumption and one representing housing, which is characterized by adjustment costs. As the utility function depends non-separably on the two goods, non-durable consumption is excessively smooth and sensitive. A similar utility function is assumed by Chetty and Szeidl (2010), however, this function is separable in the two goods, which implies that consumption is excessively smooth and sensitive with respect to the durable good only. Moreover, Reis (2006) assumes that agents face costs when processing information and thus optimally decide to update their consumption plans sporadically, Tutoni (2010) assumes that consumers are rationally inattentive as in Sims (2003), and Attanasio and Pavoni (2011) show that excessively smooth consumption results from consumption insurance being incomplete due to a moral hazard problem with hidden savings.

Several papers show that the standard and hyperbolic agents' consumption profiles are hump shaped under the assumption of power utility, sufficient impatience, and a hump-shaped income profile, such as Carroll (1997), Gourinchas and Parker (2002), and Laibson et al. (2012).[19] Other papers that generate a hump-shaped consumption profile are Caliendo and Huang (2008) with overconfidence, Attanasio (1999) with family size effects, Deaton (1991) with borrowing constraints, Feigenbaum (2008) and Hansen and Imrohoroglu (2006) with mortality risk, Bullard and Feigenbaum (2007) and Heckman (1974) with consumption-leisure choice, and Fernandez-Villaverde and Krueger (2007) with consumer durables. Caliendo and Huang (2007) and Park (2011) show that a hump-shaped consumption profile can be generated by assuming that the agent has a shorter planning horizon, i.e., five to twenty-six years, rather than his true horizon. In partial equilibrium, matching the hump shape is trivial, as preference and environmental parameters are jointly calibrated; thus, Park (2011) shows that short-term planning can still generate the hump in a well-calibrated general-equilibrium model.[20] Finally, Caliendo and Huang (2011) show that a hump-shaped consumption profile and an anticipated drop in consumption can be generated by assuming implementation costs of saving.

[19]The hump-shaped profile constitutes a puzzle to the extent that the life-cycle profile of consumption must be monotonic if utility is an additively separable function of consumption, discounting is geometric, and markets are complete. Nevertheless, I argue that the news-utility hump is more robust to alternative assumptions about the discount factor, interest rates, income profile, and more in line with the empirical hump of income and consumption.

[20]A simple real-business-cycle model with expectations-based reference dependence generates realistic moments with the preference parameters that I estimate in this paper as shown by Pagel (2012a).

III The Life-Cycle Consumption Model

I first define a general life-cycle model environment to formally introduce the preferences and equilibrium concepts.

III.1 The model environment

The agent lives for a total of T discrete periods indexed by $t \in \{1, ..., T\}$. At the beginning of each period, a vector $S_t \sim F_{S_t}$ realizes that consists of random shocks, which are independent of each other and over time. The realization of S_t is denoted s_t. The model's exogenous state variables are represented by the vector Z_t, which evolves according to the following law of motion

$$Z_t = f^Z(Z_{t-1}, S_t). \tag{1}$$

After observing s_t and Z_t, the agent decides how much to consume C_t.[21] The model's endogenous state variable is cash-on-hand X_{t+1} and is determined by the following budget constraint

$$X_{t+1} = f^X(X_t - C_t, Z_t, S_{t+1}). \tag{2}$$

All of the model's variables that are indexed by t realize in period t. Because the agent's preferences are defined over both outcomes and beliefs, I explicitly define his probabilistic "beliefs" about each of the model's period t variables from the perspective of any prior period as follows.

Definition 1. Let $I_t = \{X_t, Z_t, s_t\}$ denote the agent's information set in some period $t \leq t + \tau$. Then, the agent's probabilistic beliefs about any model variable $V_{t+\tau}$ conditional on period t information is denoted by $F^t_{V_{t+\tau}}(v) = Pr(V_{t+\tau} < v | I_t)$, and $F^{t+\tau}_{V_{t+\tau}}$ is degenerate.

Throughout the paper, I assume rational expectations, i.e., the agent's beliefs about any of the model's variables equal the objective probabilities determined by the economic environment.

III.2 Expectations-based reference-dependent preferences

Having outlined the model environment, I now introduce the agent's preferences. To facilitate the exposition, I first explain the static model of expectations-based reference dependence,

[21] Throughout most of the paper, I consider a standard life-cycle environment in which the agent's stochastic labor income is $Y_t = f^Y(P_{t-1}, S^p_t, S^T_t)$, which depends on his permanent income P_{t-1}, a permanent shock $S^P_t \sim F_{S^P_t}$, and a transitory shock $S^T_t \sim F_{S^T_t}$. He decides how much to consume C_t and how much to save in a risk-free asset that pays a net return r such that his cash-on-hand X_{t+1} is determined by $X_{t+1} = (X_t - C_t)(1 + r) + Y_{t+1}$.

as specified in Koszegi and Rabin (2006, 2007), and then introduce the dynamic model of Koszegi and Rabin (2009).

The static preferences. The agent's utility function consists of two components. First, he experiences "consumption utility" $u(c)$, which corresponds to the standard model of utility and is solely determined by consumption c. Second, he experiences "gain-loss utility" $\mu(u(c) - u(r))$. The gain-loss utility function $\mu(\cdot)$ corresponds to the prospect-theory model of utility determined by consumption c relative to the reference point r. $\mu(\cdot)$ is piecewise linear with slope η and a coefficient of loss aversion λ, i.e., $\mu(x) = \eta x$ for $x > 0$ and $\mu(x) = \eta\lambda x$ for $x \leq 0$. The parameter $\eta > 0$ weights the gain-loss utility component relative to the consumption utility component and $\lambda > 1$ implies that losses are weighed more heavily than gains, i.e., the agent is loss averse. Koszegi and Rabin (2006, 2007) allow for stochastic consumption, distributed according to $F_c(c)$, and a stochastic reference point, distributed according to $F_r(r)$. Then, the agent experiences gain-loss utility by evaluating each possible outcome relative to all other possible outcomes

$$\int_{-\infty}^{\infty} (\eta \int_{-\infty}^{c} (u(c) - u(r)) dF_r(r) + \eta\lambda \int_{c}^{\infty} (u(c) - u(r)) dF_r(r)) dF_c(c). \tag{3}$$

Additionally, the authors make the central assumption that the distribution of the reference point F_r equals the agent's fully probabilistic rational beliefs about consumption c.

The dynamic preferences. In the dynamic model of Koszegi and Rabin (2009), the utility function consists of consumption utility, "contemporaneous" gain-loss utility about current consumption, and "prospective" gain-loss utility about the entire stream of future consumption. Thus, total instantaneous utility in period t is given by

$$U_t = u(C_t) + n(C_t, F_{C_t}^{t-1}) + \gamma \sum_{\tau=1}^{\infty} \beta^{\tau} \boldsymbol{n}(F_{C_{t+\tau}}^{t,t-1}). \tag{4}$$

The first term on the left-hand side of equation (4), $u(C_t)$, corresponds to consumption utility in period t. Before turning to the subsequent terms in equation (4), which consider consumption and beliefs, I define an "admissible consumption function". This function allows me to explicitly describe the probabilistic structure of the agent's beliefs about any of the model's variables at any future date. Because the agent fully updates his beliefs in each period and because the shocks are independent over time, I consider a stationary function that depends only on this period's cash-on-hand X_t, the vector of exogenous state variables Z_t, the realization of the vector of shocks s_t, and calendar time t.

Definition 2. The consumption function in any period t is admissible if it can be written as a function $C_t = g_t(X_t, Z_t, s_t)$ that is strictly increasing in the realization of each shock $\frac{\partial g_t(X_t, Z_t, s_t)}{\partial s_t} > 0$. Repeated substitution of the law of motion, equation (1), and the budget constraint, equation (2), allows me to rewrite $C_{t+\tau} = g_{t+\tau}(X_{t+\tau}, Z_{t+\tau}, S_{t+\tau}) = h^t_{t+\tau}(X_t, Z_t, s_t, S_{t+1}, ..., S_{t+\tau})$.

I now return to the two remaining terms on the left-hand side of equation (4). The first term, $n(C_t, F^{t-1}_{C_t})$, corresponds to gain-loss utility over contemporaneous consumption; here, the agent compares his present consumption C_t with his beliefs $F^{t-1}_{C_t}$. According to Definition 1, the agent's beliefs $F^{t-1}_{C_t}$ correspond to the conditional distribution of consumption in period t given the information available in period $t-1$. The agent experiences gain-loss utility over "news" about contemporaneous consumption as follows

$$n(C_t, F^{t-1}_{C_t}) = \eta \int_{-\infty}^{C_t} (u(C_t) - u(c)) dF^{t-1}_{C_t}(c) + \eta\lambda \int_{C_t}^{\infty} (u(C_t) - u(c)) dF^{t-1}_{C_t}(c) \tag{5}$$

where C_t and $F^{t-1}_{C_t}(c)$ are explicitly described via the admissible consumption function, i.e., $C_t = g_t(X_t, Z_t, s_t) = h^{t-1}_t(X_{t-1}, Z_{t-1}, s_{t-1}, s_t)$ and $F^{t-1}_{C_t}(c) = Pr(h^{t-1}_t(X_{t-1}, Z_{t-1}, s_{t-1}, S_t) < c)$.

The third term in equation (4), $\gamma \sum_{\tau=1}^{\infty} \beta^\tau \boldsymbol{n}(F^{t,t-1}_{C_{t+\tau}})$, corresponds to gain-loss utility, experienced in period t, over the entire stream of future consumption. Prospective gain-loss utility about period $t+\tau$ consumption depends on $F^{t-1}_{C_{t+\tau}}$, the agent's beliefs he entered the period with, and on $F^t_{C_{t+\tau}}$, the agent's updated beliefs about consumption in period $t+\tau$. Again the probabilistic structure of these beliefs can be explicitly described via the admissible consumption function, i.e., $h^t_{t+\tau}(X_t, Z_t, s_t, S_{t+1}, ..., S_{t+\tau})$. Importantly, the prior and updated beliefs about $C_{t+\tau}$, $F^{t-1}_{C_{t+\tau}}$ and $F^t_{C_{t+\tau}}$, are not independent distribution functions because future shocks $S_{t+1}, ..., S_{t+\tau}$ are contained in both. Thus, there exists a joint distribution, which I denote by $F^{t,t-1}_{C_{t+\tau}} \neq F^t_{C_{t+\tau}} F^{t-1}_{C_{t+\tau}}$.[22] Because the agent compares his newly formed beliefs with his prior beliefs, he experiences gain-loss utility over "news" about future

[22] I calculate prospective gain-loss utility $\boldsymbol{n}(F^{t,t-1}_{C_{t+\tau}})$ by generalizing the "outcome-wise" comparison, specified in Koszegi and Rabin (2006, 2007) and reported in equation (15), to account for the potential dependence of F_r and F_c, i.e.,

$$\boldsymbol{n}(F_{c,r}) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mu(u(c) - u(r)) dF_{c,r}(c, r). \tag{6}$$

If F_r and F_c are independent, equation (6) reduces to equation (15). However, if F_r and F_c are non-independent, equation (6) and equation (15) yield different values. Suppose that F_r and F_c are perfectly correlated, as though no update in information occurs. Equation (15) would yield a negative value because the agent experiences gain-loss disutility over his previously expected uncertainty, which seems unrealistic. In contrast, equation (6) would yield zero because the agent considers the dependence of prior and updated beliefs, which captures future uncertainty, thereby separating uncertainty that has been realized from uncertainty that has not been realized. Thus, I call this gain-loss formulation the separated comparison. Koszegi and Rabin (2009) generalize the outcome-wise comparison to a "percentile-wise" ordered comparison. The separated and ordered comparisons are equivalent for contemporaneous gain-loss utility. However, for

consumption as follows

$$\boldsymbol{n}(F^{t,t-1}_{C_{t+\tau}}) = \int_{-\infty}^{\infty}(\eta\int_{-\infty}^{c}(u(c)-u(r)) + \eta\lambda\int_{c}^{\infty}(u(c)-u(r)))dF^{t,t-1}_{C_{t+\tau}}(c,r) \quad (7)$$

with $F^{t,t-1}_{C_{t+.\tau}}(c,r)$ given by $F^{t,t-1}_{C_{t+\tau}}(c,r) = Pr(h^{t}_{t+\tau}(X_{t-1}, Z_{t-1}, s_{t-1}, s_t, S_{t+1}, ..., S_{t+\tau}) < c$, $h^{t-1}_{t+\tau}(X_{t-1}, Z_{t-1}, s_{t-1}, S_t, S_{t+1}, ..., S_{t+\tau}) < r)$.

The agent exponentially discounts prospective gain-loss utility by $\beta \in [0,1]$. Moreover, he discounts prospective gain-loss utility relative to contemporaneous gain-loss utility by a factor $\gamma \in [0,1]$. Thus, he puts the weight $\gamma\beta^{\tau} < 1$ on prospective gain-loss utility regarding consumption in period $t+\tau$. Because both contemporaneous and prospective gain-loss utility are experienced over news, the preferences can be referred to as "news utility".

III.3 The model solution

The news-utility agent's lifetime utility in each period $t = \{1, ..., T\}$ is

$$u(C_t) + n(C_t, F^{t-1}_{C_t}) + \gamma\sum_{\tau=1}^{T-t}\beta^{\tau}\boldsymbol{n}(F^{t,t-1}_{C_{t+\tau}}) + E_t[\sum_{\tau=1}^{T-t}\beta^{\tau}U_{t+\tau}], \quad (8)$$

where $\beta \in [0,1)$, $u(\cdot)$ is a HARA[23] utility function, $\eta \in [0,\infty)$, $\lambda \in [1,\infty)$, and $\gamma \in [0,1]$. I also consider hyperbolic-discounting or $\beta\delta-$preferences, as developed by Laibson (1997); the $\beta\delta-$agent's lifetime utility is given by $u(C^b_t) + bE_t[\sum_{\tau=1}^{T-t}\beta^{\tau}u(C^b_{t+\tau})]$ where $b \in [0,1]$ is the hyperbolic-discount factor. Needless to say, standard preferences, as analyzed by Carroll (2001), Gourinchas and Parker (2002), or Deaton (1991), are a special case of the above models for either $\eta = 0$ or $b = 1$. I now define two equilibrium concepts: the monotone-personal equilibrium and monotone-pre-committed equilibrium.

The monotone-personal equilibrium. I define the model's "monotone-personal" equilibrium in the spirit of the preferred-personal equilibrium solution concept, as defined by Koszegi and Rabin (2009), but within the outlined environment and admissible consumption function as follows.

Definition 3. The family of admissible consumption functions $C_t = g_t(X_t, Z_t, s_t)$ is a monotone-personal equilibrium for the news-utility agent if, in any contingency, $C_t = g_t(X_t, Z_t, s_t)$

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prospective gain-loss utility, they are qualitatively similar but quantitatively slightly different. As a linear operator, the separated comparison is more tractable. Moreover, it simplifies the equilibrium-finding process because it preserves the outcome-wise nature of contemporaneous gain-loss utility.

[23] A utility function $u(c)$ exhibits hyperbolic absolute risk aversion (HARA) if the risk tolerance $-\frac{u''(c)}{u'(c)}$ is a linear function of c.

maximizes (8) subject to (2) and (1) under the assumption that all future consumption corresponds to $C_{t+\tau} = g_{t+\tau}(X_{t+\tau}, Z_{t+\tau}, s_{t+\tau})$. In each period t, the agent takes his beliefs about consumption $\{F^{t-1}_{C_{t+\tau}}\}^{T-t}_{\tau=0}$ as given in the maximization problem.

The monotone-personal equilibrium can be obtained by simple backward induction; thus, it is time consistent in the sense that beliefs map into correct behavior and vice versa. In other words, I derive the equilibrium consumption function under the premise that the agent enters period t, takes his beliefs as given, optimizes over consumption, and rationally expects to behave in this manner in the future. If I obtain a consumption function by backward induction that is admissible, then the monote-personal equilibrium corresponds to the preferred-personal equilibrium as defined by Koszegi and Rabin (2009). For the hyperbolic-discounting agent, the monotone-personal equilibrium corresponds to the solution of Laibson (1997).

The monotone-pre-committed equilibrium. The monotone-personal equilibrium maximizes the agent's utility in each period t when he takes his beliefs as given. However, if the agent could pre-commit to his consumption in each possible contingency, he would choose a different consumption path. I define this path as the model's "monotone-pre-committed" equilibrium in the spirit of the choice-acclimating equilibrium concept, as defined by Koszegi and Rabin (2007), but within the outlined environment and admissible consumption function as follows.

Definition 4. The family of admissible consumption functions, $C_t = g_t(X_t, Z_t, s_t)$ for each period t, is a monotone-pre-committed equilibrium for the news-utility agent, if, in any contingency, $C_t = g_t(X_t, Z_t, s_t)$ maximizes (8) subject to (2) and (1) under the assumption that all future consumption corresponds to $C_{t+\tau} = g_{t+\tau}(X_{t+\tau}, Z_{t+\tau}, s_{t+\tau})$. In each period t, the agent's maximization problem determines both his beliefs $\{F^{t-1}_{C_{t+\tau}}\}^{T-t}_{\tau=0}$ and consumption $\{C_{t+\tau}\}^{T-t}_{\tau=0}$.

I derive the equilibrium consumption function under the premise that the agent can pre-commit to an optimal, history-dependent consumption path for each possible future contingency and thus jointly optimizes over consumption and beliefs. This equilibrium is not time consistent because the agent would deviate if he were to take his beliefs as given and optimize over consumption alone.

Equilibrium existence and uniqueness. I demonstrate the existence and uniqueness of the monotone-personal and monotone-pre-committed equilibria for special environments,

such as exponential utility and permanent and transitory normal shocks, under one parameter condition. The condition is that F_{S_t} must be sufficiently dispersed such that the equilibrium consumption functions fall into the class of admissible consumption functions.[24] For the monotone-pre-committed equilibrium, an additional parameter constraint $\eta(\lambda-1) < 1$ is required to ensure global concavity of the agent's maximization problem. For other environments, such as power utility and permanent and transitory log-normal shocks, simulations using numerical backward induction suggest that the monotone-personal and monotone-pre-committed equilibria exist and are unique for most reasonable calibrations.[25]

IV Theoretical Predictions about Consumption

In the following, I explain the closed-form solution of the exponential-utility model in detail to illustrate the model's predictions formally and intuitively. After briefly outlining the model's monotone-personal equilibrium in Proposition 1, I flesh out the second-to-last period's decision problem to explain the model's theoretical predictions. Proposition 2 and Corollary 1 formalize excess smoothness and sensitivity in consumption. Lemma 1 discusses how the precautionary-savings motive competes with the prospective gain-loss discount factor; the net of these forces leads to a hump-shaped consumption profile, which is formalized in Proposition 3. Proposition 4 determines consumption during retirement, and Proposition 5 characterizes when consumption drops at retirement. After these main predictions, I discuss several more subtle consumption implications and new comparative statics. Finally, Proposition 6 characterizes the implications of the monotone-pre-committed equilibrium.

I begin by briefly explaining the model's environment and stating the equilibrium consumption function to convey a general impression of the model's solution. The agent's utility function is exponential $u(C) = -\frac{1}{\theta}e^{-\theta C}$, where $\theta \in (0, \infty)$. His additive income process $Y_t = P_{t-1} + s_t^P + s_t^T$ is characterized by a permanent $S_t^P \sim N(\mu_{Pt}, \sigma_{Pt}^2)$ and transitory $S_t^T \sim N(\mu_{Tt}, \sigma_{Tt}^2)$ normal shocks, and his permanent income is $P_t = P_{t-1} + s_{t+1}^P$. His

[24] Moreover, in Section IV.4 and Appendix B.4, I argue that the model's equilibrium is not affected qualitatively or quantitatively, if this condition does not hold. If the consumption function is decreasing over some range, the agent would simply chose a flat function over this range and the admissible consumption function requirement would be weakly satisfied.

[25] Carroll (2011) and Harris and Laibson (2002) demonstrate the existence and uniqueness of equilibria for the standard and sophisticated hyperbolic-discounting agent in similar environments. In these models, the equilibrium consumption functions fall in the class of admissible consumption functions. For the standard agent, the monotone-personal equilibrium corresponds to the pre-committed equilibrium. For the sophisticated hyperbolic-discounting agent, the monotone-personal equilibrium does not correspond to the monotone-pre-committed equilibrium, which instead corresponds to the standard agent's equilibrium.

end-of-period asset holdings are denoted $A_t = X_t - C_t$ and his budget constraint is given by

$$X_{t+1} = (X_t - C_t)(1+r) + Y_{t+1} \Rightarrow A_{t+1} = A_t R + Y_{t+1} - C_{t+1}. \tag{9}$$

In Appendix B.2.1, I show that the agent's optimal consumption function is

$$C_t = (1 - a(T-t))(1+r)A_{t-1} + P_{t-1} + s_t^P + (1 - a(T-t))s_t^T - a(T-t)\Lambda_t. \tag{10}$$

His consumption depends on his assets, income, horizon, and interest rate; the latter two are captured in the annuitization factor $a(T-t) = \frac{\sum_{j=0}^{T-t-1}(1+r)^j}{\sum_{j=0}^{T-t}(1+r)^j}$. Moreover,

$$\Lambda_t = \frac{1}{\theta} log(\frac{1 - a(T-t)}{a(T-t)} \frac{\psi_t + \gamma Q_t(\eta F(s_t^P + (1 - a(T-t))s_t^T) + \eta\lambda(1 - F(s_t^P + (1 - a(T-t))s_t^T))}{1 + \eta F(s_t^P + (1 - a(T-t))s_t^T) + \eta\lambda(1 - F(s_t^P + (1 - a(T-t))s_t^T))}), \tag{11}$$

where $F(\cdot) = F_{S_t^P + (1-a(T-t))S_t^T}(\cdot)$ and ψ_t and Q_t are constant. Thus, Λ_t varies with the shock realizations but is independent of permanent income or assets. The standard and hyperbolic-discounting agents' monotone-personal equilibria have the same structure except that Λ_t^s and Λ_t^b only vary with the agent's horizon.

Proposition 1. *There exists a unique monotone-personal equilibrium in the finite-horizon exponential-utility model if* $\sqrt{\sigma_{Pt}^2 + (1 - a(i))^2 \sigma_{Tt}^2} \geq \sigma_t^*$ *for all* $t \in \{1, ..., T\}$.

This proposition's proof and the proofs of the following propositions can be found in Appendix B.4. All of the following propositions are derived within this model environment and hold in any monotone-personal equilibrium if one exists.

IV.1 Excess smoothness and sensitivity in consumption

Excess smoothness and sensitivity in consumption are two robust empirical observations, which emerged from tests of the permanent income hypothesis. The permanent income hypothesis postulates that the marginal propensity to consume out of permanent income shocks is one and that future consumption growth is not predictable using past variables. However, numerous studies find that the marginal propensity to consume is less than one because consumption underresponds to permanent income shocks; thus, consumption is excessively smooth according to Deaton (1986). Moreover, numerous studies find that past changes in income have predictive power for future consumption growth because consumption adjusts with a delay; thus, consumption is excessively sensitive according to Flavin (1985). Campbell and Deaton (1989) explain how these observations are intrinsically related; consumption

underresponds to permanent income shocks and thus adjusts with a delay. In this spirit, I define excess smoothness and sensitivity for the exponential-utility model as follows.

Definition 5. Consumption is excessively smooth if $\frac{\partial C_t}{\partial s_t^P} < 1$ everywhere and excessively sensitive if $\frac{\partial \Delta C_{t+1}}{\partial s_t^P} > 0$ everywhere.

This definition has an empirical analogue: an ordinary least squares (OLS) regression of period $t+1$ consumption growth on the realization of the permanent shock in periods $t+1$ and t; for the two OLS coefficients β_1 and β_2, the above definition implies that consumption is excessively smooth if $\beta_1 = \frac{\partial C_t}{\partial s_t^P}|_{s_t^P=\mu_{Pt}} < 1$ and excessively sensitive if $\beta_2 = \frac{\partial \Delta C_{t+1}}{\partial s_t^P}|_{s_t^P=\mu_{Pt}} > 0$.

Proposition 2. *The news-utility agent's consumption is excessively smooth and sensitive.*

I briefly present a simplified intuition for this result to then explain the agent's first-order condition in greater detail and provide the full intuition.[26] The agent's marginal gain-loss utility today is more sensitive to his savings than his marginal gain-loss utility tomorrow, as his reference point today is invariable while his reference point tomorrow will have adjusted to his savings plan today. As a result, in the event of an adverse shock, the agent prefers to delay the reduction in consumption until his reference point has decreased. Additionally, in the event of a good shock, the agent prefers to delay the increase in consumption until his reference point has increased.

To explain this result in greater detail, I flesh out the agent's decision-making problem in the second-to-last period assuming that transitory shocks are absent, $A_{T-2} = P_{T-2} = 0$, and the permanent income shock is independent and identically distributed (i.i.d.) normal $S_{T-1}^P, S_T^P \sim F_P = N(\mu_P, \sigma_P)$. In period $T-1$, the agent chooses how much to consume C_{T-1} and save $s_{T-1}^P - C_{T-1}$. His optimal consumption growth is given by

$$\Delta C_T = s_T^P + \frac{1}{\theta} log((1+r)\frac{\psi_{T-1} + \gamma Q_{T-1}(\eta F_P(s_{T-1}^P) + \eta\lambda(1 - F_P(s_{T-1}^P)))}{1 + \eta F_P(s_{T-1}^P) + \eta\lambda(1 - F_P(s_{T-1}^P))}). \quad (12)$$

I explain each component of the fraction in equation (12) in detail. The denominator is marginal consumption and contemporaneous gain-loss utility in period $T-1$; the latter consists of two terms. First, the agent compares his actual consumption to all consumption outcomes that would have been less favorable and experiences a gain weighted by η, i.e., $\eta \int_{-\infty}^{C_{T-1}} (u(C_{T-1}) - u(c)) F_{C_T}^{T-1}(c)$. Second, the agent compares his actual consumption to all outcomes that would have been more favorable and experiences a loss weighted by $\eta\lambda$, i.e., $\eta\lambda \int_{C_{T-1}}^{\infty} (u(C_{T-1}) - u(c)) F_{C_{T-1}}^{T-2}(c)$. Because the agent takes his beliefs as given in the

[26] This result can be generalized to a HARA utility function, arbitrary horizons, and arbitrary income uncertainty.

monotone-personal equilibrium, his marginal consumption and marginal contemporaneous gain-loss utility equals

$$u'(C_{T-1}) + u'(C_{T-1})(\eta F^{T-2}_{C_{T-1}}(C_{T-1}) + \eta\lambda(1 - F^{T-2}_{C_{T-1}}(C_{T-1})))). \tag{13}$$

This expression can be simplified by replacing $F^{T-2}_{C_{T-1}}(C_{T-1})$ with $F_P(s^P_{T-1})$ because any admissible consumption function is increasing in the shock realization.

The second term of the numerator in equation (12) is marginal prospective gain-loss utility over future consumption $C_T = (s^P_{T-1} - C_{T-1})(1 + r) + s^P_{T-1} + S^P_T$. I denote the expected marginal utility of the last period's income shock $Q_{T-1} = \beta E_{T-1}[u'(S^P_T)]$.[27] As the agent's admissible consumption is increasing in the shock realization and he takes his beliefs as given, his marginal prospective gain-loss utility corresponds to the same weighted sum of $F_P(s^P_{T-1})$

$$(1 + r)u'((s^P_{T-1} - C_{T-1})(1 + r) + s^P_{T-1})\gamma Q_{T-1}(\eta F_P(s^P_{T-1}) + \eta\lambda(1 - F_P(s^P_{T-1}))). \tag{14}$$

The first term of the numerator in equation (12) is marginal future consumption and gain-loss utility. I denote the expected marginal consumption and gain-loss utility of the last period's income shock ψ_{T-1}, which equals Q_{T-1} plus $\beta E_{T-1}[\eta(\lambda-1)\int_{S^P_T}^{\infty}(u'(S^P_T)-u'(s))dF_P(s)]$. Consequently, marginal expected consumption and gain-loss utility are given by $(1+r)u'((s^P_{T-1} - C_{T-1})(1 + r) + s^P_{T-1})\psi_{T-1}$.

The fraction in equation (12) is increasing in s^P_{T-1} for any γ iff $\psi_{T-1} > Q_{T-1}$. The difference between ψ_{T-1} and Q_{T-1} corresponds to expected marginal gain-loss utility that is constant because the future reference point adjusts to today's savings plan. Thus, a positive share of tomorrow's marginal utility is inelastic to today's savings, which implies that tomorrow's marginal utility is less sensitive to changes in savings than today's marginal utility. Today's marginal contemporaneous and prospective gain-loss utility is relatively high or low in the event of an adverse or positive shock. In contrast, expected marginal gain-loss utility is constant because tomorrow's reference point will have adjusted to today's plan. Thus, the agent will consume relatively more in the event of an adverse shock and relatively less in the event of a positive shock. According to Definition 5, consumption is excessively smooth $\frac{\partial C_{T-1}}{\partial s^P_{T-1}} < 1$ and excessively sensitive $\frac{\partial \Delta C_T}{\partial s^P_{T-1}} > 0$.

In contrast, the standard agent's consumption growth is $\Delta C^s_T = s^P_T + \frac{1}{\theta}log((1+r)Q_{T-1})$, and the hyperbolic-discounting agent's consumption growth is $\Delta C^b_T = s^P_T + \frac{1}{\theta}log((1 + r)bQ_{T-1})$. Thus, the consumption of these agents is neither excessively smooth nor ex-

[27]Exponential utility implies that $u'(* + \cdot) = u'(*)u'(\cdot)$ and thus works well with additive risk.

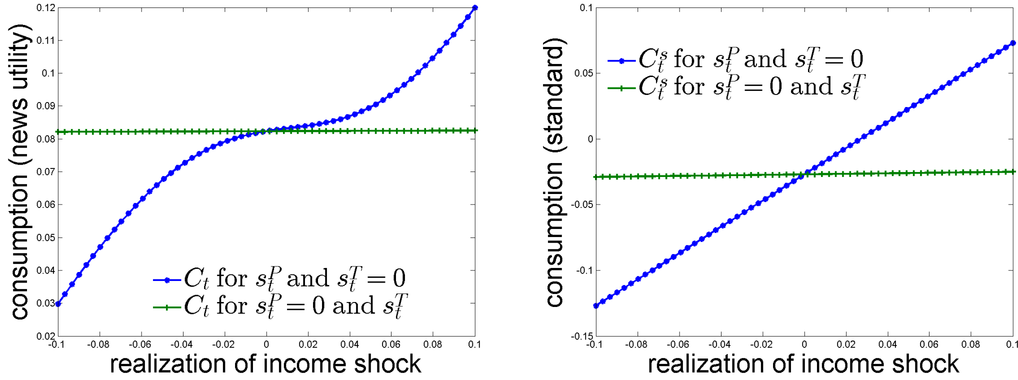

Figure I: Exponential-utility consumption functions of news-utility and standard agents.

cessively sensitive.

To illustrate the quantitative implications of excess smoothness and sensitivity, I run the data-free linear regression of consumption growth on income

$$\Delta C_{t+1} = \alpha + \beta_1 \Delta Y_{t+1} + \beta_2 \Delta Y_t + \varepsilon_{t+1}.$$

For the news-utility model, I obtain $\beta_1 \approx 0.22$ and $\beta_2 \approx 0.18$ and a marginal propensity to consume out of permanent shocks of approximately 71%.[28] In contrast, for the standard model the marginal propensity is one.[29]

For illustration, Figure I displays the news-utility and standard agents' consumption functions for realizations within two standard deviations of each shock, while the other is held constant. The flatter part of the news-utility consumption function generates excess smoothness and sensitivity.

[28] I retain the normal income process outlined in Section IV assuming that permanent and transitory shocks are i.i.d., i.e., $Y_t = P_{t-1} + s_t^P + s_t^T \sim N(P_{t-1} + \mu_P + \mu_T, \sigma_P^2 + \sigma_T^2)$. I choose the environment parameters so as to roughly generate the volatility of the log-normal income process that is typically used in the life-cycle consumption literature. I choose the agent's horizon T, his retirement period R, his initial wealth A_0 and P_0, and the interest rate r in accordance with the life-cycle literature. Additionally, I choose the preference parameters in line with the microeconomic literature and experimental evidence, which is explained in detail in Section V.3. The parameters are $\mu_P = 0$, $\sigma_P = 5\%$, $\mu_T = 0$, $\sigma_T = 7\%$, $\beta = 0.978$, $r = 2\%$, $\theta = 2$, $\eta = 1$, $\lambda = 2$, $\gamma = 0.75$, $A_0 = 0$, and $P_0 = 0.1$.

[29] Running the regression

$$\Delta C_{t+1} = \alpha + \beta_1 (s_{t+1}^P - \mu_P) + \beta_2 \Delta (s_t^P - \mu_P) + \varepsilon_{t+1}$$

yields $\beta_1^s \approx 1$ and $\beta_2^s \approx 0$ in the standard model and $\beta_1 \approx 0.71$ and $\beta_2 \approx 0.31$ in the news-utility model. The transitory shock introduces a spurious negative correlation between ΔC_{t+1} and ΔY_t because $\Delta Y_{t+1} = s_{t+1}^P + s_{t+1}^T - s_t^T$ and $\Delta Y_t = s_t^P + s_t^T - s_{t-1}^T$.

IV.2 The hump-shaped consumption profile

Fernandez-Villaverde and Krueger (2007), among others, show that lifetime consumption profiles are hump-shaped, even when controlling for cohort, family size, number of earners, and time effects.[30] In the following, I demonstrate that the preferences generate a hump-shaped consumption profile as a result of the net of two competing features – an additional first-order precautionary-savings motive and the agent's discount factor on prospective gain-loss utility γ.

Precautionary savings and prospective news discounting. Income uncertainty has a first-order effect on savings in the news-utility model. This "first-order precautionary-savings motive" is added to the precautionary savings motive of the standard agent, which is a second-order motive.[31] This result is highlighted by Koszegi and Rabin (2009) in a two-period, two-outcome model.

Definition 6. There exists a first-order precautionary-savings motive iff $\frac{\partial(s_{T-1}^P - C_{T-1})}{\partial \sigma_P}|_{\sigma_P=0} > 0$.

However, the agent wishes to increase his consumption and decrease his savings if he discounts prospective gain-loss utility relative to contemporaneous gain-loss utility, i.e., $\gamma < 1$. This discounting is reminiscent of $\beta\delta-$preferences. The following lemma formalizes these two opposing forces.[32]

Lemma 1.
1. Precautionary savings: News utility introduces a first-order precautionary-savings motive.
2. Implications for consumption growth: There exists a $\bar{\gamma}^s < 1$, implicitly determined by $\Delta C_T = \Delta C_T^s$, such that, iff $\bar{\gamma}^s < \gamma$, the news-utility agent's consumption growth in period T is higher than the standard agent's consumption growth for any realization of s_{T-1}^P and s_T^P, and $\frac{\partial \bar{\gamma}^s}{\partial \sigma_P} < 0$.

The intuition for the first part of the lemma is as follows. The agent anticipates being exposed to gain-loss fluctuations in period T, which are painful in expectation because losses hurt more than gains give pleasure. Additionally, the painfulness of these fluctuations is proportional to marginal consumption utility, which is lower higher on the utility curve. Thus, the agent has an additional incentive to increase savings. The intuition for the second part

[30] Moreover, Fernandez-Villaverde and Krueger (2007) find suggestive evidence that non-separability between consumption and leisure, which was promoted by Attanasio (1999) and previous papers, cannot explain more than 20% of the hump in consumption.

[31] Refer to Gollier (2001).

[32] This result and those following can be generalized to any HARA utility function, arbitrary horizons, and labor income uncertainty.

of the lemma is straightforward. If $\gamma < 1$, the agent is more concerned about contemporaneous than prospective gain-loss utility; thus, he wishes to increase his consumption and decrease his savings. Consequently, the presence of news utility might increase or decrease consumption relative to the standard model depending on the net of two parameters $\sigma_P > 0$ and $\gamma < 1$.

In the following, I develop a more formal intuition for the standard and additional precautionary-savings motive and demonstrate that the assumption $\psi_{T-1} > Q_{T-1}$, which I made previously, always holds. As shown above, the marginal value of savings is $(1 + r)u'((s^P_{T-1} - C_{T-1})(1 + r) + s^P_{T-1})\psi_{T-1}$, where ψ_{T-1} equals the shock's expected marginal consumption plus expected marginal gain-loss utility

$$\beta E_{T-1}[u'(S^P_T)] + \beta E_{T-1}[\eta(\lambda - 1)\int_{S^P_T}^{\infty}(u'(S^P_T) - u'(s))dF_P(s)]. \tag{15}$$

The integral in equation (15) reflects the expected marginal utility of all gains and losses, which partly cancel, such that only the overweighted component of the losses remains, i.e., $\eta(\lambda-1)(\cdot)$. The key point is that this integral is always positive if $u'' < 0$ and thus captures the additional precautionary-savings motive, implies that $\psi_{T-1} > Q_{T-1}$, and is increasing in η, λ, and σ_P. Because $\frac{\partial(s^P_{T-1}-C_{T-1})}{\partial\sigma_P}|_{\sigma_P=0} > 0$, this motive is first order, as the news-utility agent is first-order risk averse. In contrast, the standard precautionary-savings motive is captured by $Q_{T-1} = \beta E_{T-1}[u'(S^P_T)]$, which is larger than $\beta u'(E_{T-1}[S_T])$ if $u''' > 0$, according to Jensen's inequality. This standard precautionary-savings motive is second order, i.e., $\frac{\partial(s^P_{T-1}-C_{T-1})}{\partial\sigma_P}|_{\sigma_P=0} = 0$, as the standard agent is second-order risk averse.[33]

The resulting hump-shaped consumption profile. The two competing news-utility features – the additional precautionary-savings motive and $\gamma < 1$ – make it likely that the life-cycle consumption profile is hump shaped.

Definition 7. I say that the agent's consumption profile is hump shaped if consumption is increasing at the beginning of his life $\Delta C_1 > 0$ and decreasing $\Delta C_T < 0$ at the end of his life.

Proposition 3. *Suppose $\sigma_{Pt} = \sigma_P$ for all t and T is large; then, there exists a σ_P in $[\underline{\sigma_P}, \overline{\sigma_P}]$ such that, if $\gamma < 1$, $log((1+r)\beta) \in [-\triangle, \triangle]$, and $\triangle$ is small, the news-utility agent's lifetime*

[33] As shown by Benartzi and Thaler (1995) and Barberis et al. (2001), first-order risk aversion resolves the equity premium puzzle, which highlights that agents must have implausibly high second-order risk aversion to reconcile the historical equity premium because aggregate consumption is smooth compared with asset prices. The excess-smoothness puzzle highlights that aggregate consumption is too smooth compared to labor income, and again, first-order instead of second-order risk aversion is a necessary ingredient for resolving the puzzle.

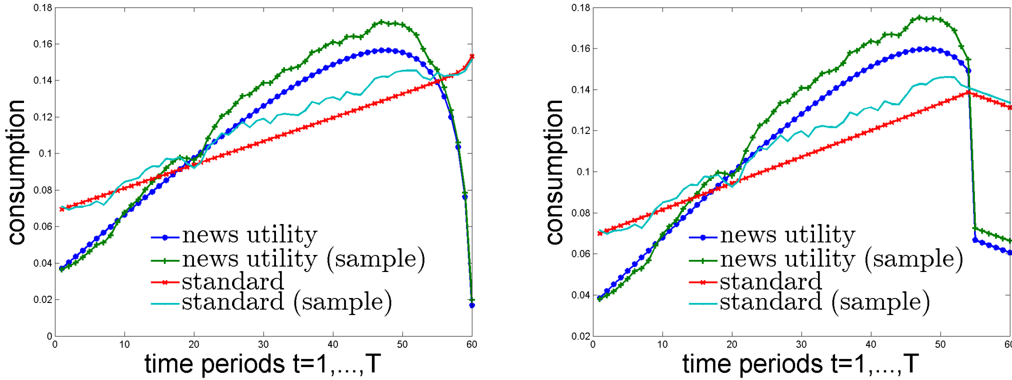

Figure II: Exponential-utility life-cycle profiles of news-utility and standard agents.

consumption path is hump shaped.

The basic intuition is illustrated in Lemma 1. The relative strengths of the additional precautionary-savings motive and $\gamma < 1$ determine whether the presence of gain-loss utility increases or decreases the news-utility agent's consumption relative to the standard model. When the agent's horizon increases, the precautionary-savings motive accumulates because uncertainty accumulates. Accordingly, at the beginning of life, the presence of gain-loss utility is likely to reduce consumption and increase consumption growth unless γ is small. Toward the end of life, however, the additional precautionary-savings motive is relatively small, and $\gamma < 1$ is likely to decrease consumption growth. More formally, the two conditions $\Delta C_{t+1} \lesseqgtr 0$ reduce to $\Lambda_t \lesseqgtr 0$ as $T-t$ becomes large or $T-t$ becomes small. The sign of Λ_t is determined by the relative values of $\frac{\psi_t}{Q_t} > 1$ and $\gamma < 1$. As $T-t$ increases, $\frac{\psi_t}{Q_t}$ increases such that $\gamma < 1$ loses relative importance and Λ_t is more likely to be positive. In contrast, $\frac{\psi_{T-1}}{Q_{T-1}}$ is small such that $\gamma < 1$ is likely to cause Λ_{T-1} to be negative.

Figure II displays the news-utility and standard agents' life-cycle consumption profiles.[34] The figure displays the average consumption profile of 300 identical agents who encounter different realizations of s_t^P and s_t^T and the consumption profile if $s_t^P = 0$ and $s_t^P = 0$ for all t. As can be observed from the figure, the news-utility agent's consumption profile is hump shaped. This hump is very robust to different parameter choices, which I discuss in Section V.3. In contrast, the standard agent's profile is V-shaped, which demonstrates that exponential utility and a random-walk income process do not promote the desired hump. Moreover, the figure displays the hump in the presence of a retirement period, which I explain next.

[34] I use the same calibration as in the quantitative exercise in Section IV.1.

IV.3 News-utility consumption during and at retirement

IV.3.1 News-utility consumption during retirement

I now add a retirement period at the end of life. I assume that in periods $t \in \{T-R, T\}$, the agent earns his permanent income without uncertainty. I first formalize a general prediction of the news-utility agent's consumption during retirement, in which I generalize a result obtained by Koszegi and Rabin (2009) in a two-period model.[35]

Proposition 4. *If uncertainty is absent, both the monotone-personal equilibrium and monotone-pre-committed equilibrium of the news-utility agent correspond to the standard agent's equilibria iff $\gamma \geq \frac{1}{\lambda}$. Iff $\gamma < \frac{1}{\lambda}$ then the monotone-pre-committed equilibrium of the news-utility agent corresponds to the standard agent's equilibrium and the monotone-personal equilibrium of the news-utility agent corresponds to a $\beta\delta-$agent's monotone-personal equilibrium with the hyperbolic-discount factor given by $b = \frac{1+\gamma\eta\lambda}{1+\eta}$.*

The news-utility agent is likely to follow the standard agent's path if uncertainty is absent. The basic intuition is that the agent associates a certain loss in future consumption, which is very painful, with an increase in present consumption. Thus, unless the agent discounts prospective gain-loss utility significantly, he follows the utility-maximizing standard agent's path. More formally, suppose that the agent allocates his deterministic cash-on-hand between consumption today C_{T-1} and tomorrow C_T. Under rational expectations, he cannot fool himself; hence, he cannot experience actual gain-loss utility in equilibrium in a deterministic model. Accordingly, his expected-utility maximization problem corresponds to the standard agent's maximization problem, and his monotone-pre-committed equilibrium thus corresponds to the standard agent's problem determined by $u'(C_{T-1}) = \beta(1+r)u'(C_T)$. Suppose that the agent's beliefs about consumption in both periods correspond to this pre-committed equilibrium path. Taking his beliefs as given, the agent will deviate if the gain from consuming more today exceeds the discounted loss from consuming less tomorrow, i.e.,

$$u'(C_{T-1})(1+\eta) > \beta(1+r)u'(C_T)(1+\gamma\eta\lambda).$$

Thus, he follows the standard agent's path iff $\gamma \geq \frac{1}{\lambda}$ because the pain of the certain loss in future consumption is greater than the pleasure gained from present consumption. However, if $\gamma < \frac{1}{\lambda}$, the agent chooses a consumption path that just meets the consistency constraint and behaves as a $\beta\delta-$ or hyperbolic-discounting agent with hyperbolic discount factor $b = \frac{1+\gamma\eta\lambda}{1+\eta} < 1$.

[35] This result can be generalized to a HARA utility function.

IV.3.2 News-utility consumption at retirement

During retirement, the implications of the agent's prospective gain-loss discount factor γ are simple: it needs to be sufficiently low to overcome the certain loss in future consumption. I now examine the pre-retirement period to derive two additional implications of $\gamma < 1$. The first concerns a drop in consumption at retirement, and the second shows how excess sensitivity in consumption arises in the absence of future uncertainty.

The drop in consumption at retirement. The empirical evidence on the prevalence of a drop in consumption at retirement is debated. While a series of papers (see Attanasio and Weber (2010) for a survey) have found that consumption drops at retirement, Aguiar and Hurst (2005) cannot confirm this finding when controlling for the sudden reduction of work-related expenses, the substitution of home production for market-purchased goods and services, and health shocks. In my data, I find such a drop in consumption at retirement even for non-work-related expenditures. Moreover, I consider the evidence provided by Schwerdt (2005) compelling because the author explicitly controls for home production and focuses on German retirees, who receive large state-provided pensions, which require little self organization, and for whom health is a complement to consumption thanks to proper insurance coverage. Moreover, Ameriks et al. (2007) and Hurd and Rohwedder (2003) provide evidence that the drop in consumption is anticipated. I first define a drop in consumption as follows.

Definition 8. There occurs a drop in consumption at retirement if consumption growth at retirement ΔC_{T-R} is negative and smaller than consumption growth after retirement ΔC_{T-R+1}.

As an example, if $\gamma \geq \frac{1}{\lambda}$, the news-utility agent's post-retirement consumption growth equals that of the standard agent's, i.e., $\frac{1}{\theta} log(\beta(1+r)) \approx 0$, whereas consumption growth at retirement is $\frac{1}{\theta} log(\beta(1+r)) + \frac{1}{\theta} g^s$ with $g^s \in \{log(\frac{1+\gamma\eta\lambda}{1+\eta\lambda}), log(\frac{1+\gamma\eta}{1+\eta})\} < 0$ for the news-utility agent and remains zero for the standard agent.[36]

Proposition 5. *If $\gamma < 1$, $log((1+r)\beta) \in [-\triangle, \triangle]$, and $\triangle$ is small, the news-utility agent's monotone-personal consumption path is characterized by a drop at retirement.*

After the beginning of retirement the agent is less inclined to overconsume than before. The basic intuition for overconsumption in the pre-retirement period is that the agent allocates house money, i.e., labor income that he was not certain that he would receive, and thus

[36]This and the following results can be generalized to a HARA utility function, arbitrary horizons, and arbitrary income uncertainty.

wants to consume before his expectations catch up iff $\gamma < 1$. During retirement, the agent associates a certain loss in future consumption with a surprise in present consumption. In contrast, in the pre-retirement period, the agent finds the loss in future consumption merely as painful as a slightly less favorable realization of his labor income, i.e., the agent trades off being somewhere in the gain domain today versus being somewhere in the gain domain tomorrow instead of a sure gain today with a sure loss tomorrow. The agent's first-order condition in period $T-1$ absent uncertainty in period T is given by

$$u'(C_{T-1}) = \beta(1+r)u'(C_T)\frac{1+\gamma(\eta F_P(s^P_{T-1}) + \eta\lambda(1-F_P(s^P_{T-1})))}{1+\eta F_P(s^P_{T-1}) + \eta\lambda(1-F_P(s^P_{T-1}))}. \quad (16)$$

In equation (16), it can immediately be seen that iff $\gamma = 1$, contemporaneous and prospective marginal gain-loss utility cancel. However, iff $\gamma < 1$, the agent reduces the weight on future utility relative to present utility by a factor between $\frac{1+\gamma\eta\lambda}{1+\eta\lambda}$ and $\frac{1+\gamma\eta}{1+\eta} < 1$. During retirement, the news-utility agent follows the standard agent's consumption path if γ is sufficiently high and a $\beta\delta-$agent's consumption path with discount factor $b = \frac{1+\gamma\eta\lambda}{1+\eta}$ otherwise. Because $\frac{1+\gamma\eta}{1+\eta} < min\{\frac{1+\gamma\eta\lambda}{1+\eta}, 1\}$ iff $\gamma < 1$, the agent's factor that reduces the weight on future utility is necessarily lower in the pre-retirement period than after retirement, which implies that consumption drops at retirement.[37] The other agents' consumption paths do not exhibit a drop in consumption at retirement. Quantitatively, Figure II displays a substantial drop in consumption at retirement.

The assumption of no uncertainty during retirement is made in all standard life-cycle consumption models, as these abstract from portfolio choice; thus, the drop in consumption at retirement is a necessary artifact of news-utility preferences in the standard environment. However, the drop is robust to three alternative assumptions: small income uncertainty during retirement, due to inflation risk for instance, potentially large discrete consumption uncertainty, due to health shocks for instance, or mortality risk.[38] Furthermore, if I were to

[37]What happens if uncertainty in the pre-retirement period becomes small? The drop in consumption depends on the support of uncertainty. First, suppose the agent expects a continuous shock, the variance of which becomes small. So long as a monotone-personal equilibrium exists, there occurs a drop at retirement. However, if the variance of the shock becomes very small, the agent will follow a flat consumption path at some point. Nevertheless, the agent will not be able to follow his deterministic consumption path, but reduces the weight on future marginal value by a factor in the range of $\{\frac{1+\gamma\eta}{1+\eta}, \frac{1+\gamma\eta\lambda}{1+\eta}\}$. Thus, if $\frac{1}{\lambda} \leq \gamma \leq 1$, there occurs a drop for good realizations, and if $\gamma < \frac{1}{\lambda}$, there occurs a drop for all realizations. Second, suppose the agent expects a shock with some probability. If the probability of a shock occurring becomes small, the agent's consumption in the pre-retirement period approaches his deterministic consumption path; this eliminates the drop because the agent's first-order condition is no longer subject to a change in the weighting of future versus present marginal value.

[38]The drop in consumption is due to the fact that the agent overconsumes before retirement but consumes efficiently after retirement. If income uncertainty is very small, the agent is able to credibly plan a flat consumption level independent of the realization of his income shock because the benefits of smoothing

observe a consumption path that is much flatter during retirement than before retirement and interpret this observation from the perspective of the standard model, I may conclude that the agent does not decumulate assets sufficiently rapidly after retirement compared to his pre-retirement asset decumulation. Such a lack of asset decumulation during retirement constitutes another life-cycle consumption puzzle that is observed by Hurd (1989), Disney (1996), and Bucciol (2012) and explained by the model.[39]

Excess sensitivity in the pre-retirement period. In the following, I outline an additional result regarding excess sensitivity in the pre-retirement period.[40] This result is related to Proposition 7 in Koszegi and Rabin (2009), in which the authors find that if $\frac{1}{\lambda} < \gamma < 1$, then the news-utility agent might entirely consume small gains but entirely delay small losses when he is surprised by them.[41]

Corollary 1. *Iff $\gamma < 1$, the news-utility agent's monotone-personal equilibrium consumption is excessively smooth and sensitive in the pre-retirement period.*

The basic intuition is that the agent can effectively reduce his sense of loss by delaying the cut in consumption. Iff $\gamma < 1$, the agent cares more about contemporaneous than prospective gain-loss utility and thus overconsumes in the presence of uncertainty, as explained above. Moreover, he overconsumes even more when experiencing a relatively bad realization because losses are overweighted. Because the agent overconsumes relatively more in the event of a bad

consumption perfectly do not warrant the decrease in expected utility from experiencing gain-loss utility. In such a small-uncertainty situation, the agent is able to commit to a flat consumption level that induces less overconsumption after retirement than before retirement such that consumption drops. I formally explain this result about overconsumption in Section IV.4. Moreover, discrete uncertainty after retirement induces less overconsumption than before retirement for the same reason that no uncertainty causes less overconsumption. If uncertainty is discrete, overconsumption is associated with a discrete gain in present consumption and a discrete loss in future consumption. Because the discrete loss hurts more than the discrete gain, the agent may credibly plan a consumption level that induces less overconsumption than the baseline continuous-outcome equilibrium. Finally, mortality risk does not affect the result because the agent would not experience gain-loss utility relative to being dead.

[39]This puzzle can also be explained by bequest motives (Hurd (1989)) and large medical expenditures shocks (Nardi et al. (2011)).

[40]This result can be generalized to a HARA utility function, arbitrary horizons, and arbitrary income uncertainty.

[41]In this example, the agent's consumption is excessively smooth and sensitive for surprise losses, but the opposite is true for surprise gains. In the same setup, the agent's consumption would also be excessively smooth and sensitive, according to Definition 5, for gains if they are sufficiently large and thus not entirely consumed or if they are expected. The agent might entirely consume an unexpected gain because it brings about a change in the weighting of future versus present marginal value in the agent's first-order condition. More formally, absent uncertainty, the agent follows the standard agent's path, as $\frac{1}{\lambda} < \gamma$, whereas in the event of a surprise gain, he puts a weight of $\frac{1+\gamma\eta\lambda}{1+\eta} < 1$ on future consumption. Thus, if the gain is small, the change in the weight the agent places on future marginal consumption utility induces the agent to consume the entire gain.

shock and relatively less in the event of a good shock, he delays his adjustment to consumption. Mathematically, the agent behaves like a $\beta\delta-$agent, weighting future consumption by a factor of $b \in \{\frac{1+\gamma\eta\lambda}{1+\eta\lambda}, \frac{1+\gamma\eta}{1+\eta}\}$. Thus, the agent's weight on future consumption is particularly low when the income realization is relatively bad, i.e., $F_P(s^P_{T-1}) \approx 0$. In turn, variation in $F_P(s^P_{T-1})$ leads to variation in $\Delta C_T = \frac{1}{\theta}log(\beta(1+r)) + \frac{1}{\theta}log(\frac{1+\gamma(\eta F_P(s^P_{T-1})+\eta\lambda(1-F_P(s^P_{T-1})))}{1+\eta F_P(s^P_{T-1})+\eta\lambda(1-F_P(s^P_{T-1}))})$ and consumption is excessively smooth and sensitive because an increase in the permanent shock increases the fraction determining ΔC_T. Moreover, for any given η and λ consumption is more excessively smooth and sensitive if γ is low.

IV.4 New predictions about news-utility consumption

In the following, I highlight several additional news-utility predictions for consumption that are new and testable comparative statics. I first explain the agent's consumption function, equation (10), in detail to highlight some subtle predictions about how the marginal propensity to consume varies with the realization of the permanent and transitory shocks and the agent's horizon. To explain the consumption function, I assume that $T-t$ is large such that $a(T-t) \approx \frac{1}{1+r}$. Then, in each period t, the agent consumes the interest payments of his last period's asset holdings rA_{t-1}, his entire permanent income $P_{t-1} + s^P_t$, and the per-period value of his temporary shock $\frac{r}{1+r}s^T_t$. Λ_t captures the agent's patience compared to the market, his precautionary savings, and his marginal gain-loss utility. In the event of a negative shock, Λ_t is low and the agent consumes more out of his end-of-period asset holdings and thus spreads the consumption adjustment to his entire future. Λ_t varies more with the permanent shock than with the transitory shock because marginal gain-loss utility varies with $F^{t-1}_{C_t}(C_t)$, which varies little with the transitory shock as the agent only consumes the per-period value of the transitory shock $\frac{r}{1+r}s^T_t$ and $\frac{r}{1+r}$ is small. This observation constitutes the first novel prediction of the news-utility model: consumption is more excessively sensitive for permanent than for transitory shocks in an environment with permanent shocks. This prediction can be seen in Figure I. In an environment with transitory shocks alone, however, news-utility consumption is excessively sensitive with respect to transitory shocks.

A second prediction is that the degree of excess smoothness and sensitivity is decreasing in income uncertainty σ_P. If σ_P is small, the agent's beliefs change more rapidly relative to the change in the realization of the shock; hence, the consumption function is more flat for realizations around μ_P. A third prediction is that any bell-shaped shock distribution induces bounded variation in Λ_t and thus the agent's excess sensitivity. If the agent is affected by a tail realization, the actual value of the low-probability shock matters less because neighboring states have very low probability; thus, the variation in Λ_t is bounded. A fourth prediction

is that consumption is more excessively smooth when the agent's horizon increases for two reasons: first the marginal propensity to consume out of permanent shocks declines when the precautionary-savings motive accumulates and second $a(T-t)$ is increasing in $T-t$. However, consumption is relatively less excessively sensitive when the agent's horizon increases because excess smoothness is proportional to $a(T-t)$ while excess sensitivity is not.

In the following, I explain how the second prediction can be taken to the extreme: the consumption function may be completely flat if σ_p is small.[42] To formally discuss this result about flat consumption, I return to the two-period, one-shock model. Suppose that the absolute level of the shock increases; then, holding C_{T-1} constant, the marginal value of savings declines and the agent's first-order condition implies that consumption should increase. However, $F_P(s^P_{T-1})$ also increases, and marginal gain-loss utility is lower, such that the agent's optimal consumption should decrease. Suppose that s^P_{T-1} increases marginally but $F_P(s^P_{T-1})$ increases sharply, which could occur if F_P is a very narrow distribution. In this case, the lower marginal gain-loss utility that decreases consumption dominates such that the first-order condition predicts decreasing consumption over some range in the neighborhood of the expected value μ_P where F_P increases most sharply if F_P is bell shaped. However, a decreasing consumption function cannot be an equilibrium because the agent would unnecessarily experience gain-loss utility over the decreasing part of consumption, which decreases expected utility unnecessarily. In the decreasing-consumption function region, the agent could choose a flat consumption function instead. In such a situation, he does not respond to shocks at all, i.e., his consumption is perfectly excessively smooth and sensitive, which resembles liquidity constraints or adjustment costs to consumption. Moreover, the agent may choose a credible consumption plan with a flat section, which induces less overconsumption than the original plan. Suppose the agent chooses a flat consumption level for realizations of s^P_{T-1} in $\underline{s}$ and $\overline{s}$. Then, $\overline{s}$ is chosen where the original consumption function just stops decreasing, which corresponds to the lowest possible level of the flat section of consumption $\overline{C}_{T-1}$, which I explicitly describe in Appendix 1. Moreover, in Appendix 1, I show that the agent's consistency constraint for not increasing consumption beyond $\overline{C}_{T-1}$ for any $s^P_{T-1} \in [\underline{s}, \overline{s}]$ always holds. Thus, I can conclude that flat consumption results in less overconsumption than the baseline equilibrium.

[42]This prediction about flat consumption is also highlighted by Heidhues and Koszegi (2008).

IV.5 Comparison to the agent's pre-committed equilibrium and welfare implications

In order to assess the preferences' welfare implications, I briefly explain the consumption implications of the monotone-pre-committed equilibrium that maximizes expected utility by jointly optimizing over consumption and beliefs. The pre-committed equilibrium is not credible without an appropriate commitment device because the agent overconsumes once he wakes up and takes his beliefs as given. I call this overconsumption phenomenon beliefs-based present bias because the agent prefers to enjoy the pleasant surprise of increasing consumption above expectations today instead of increasing both his consumption and expectations tomorrow.[43] Empirically, there is abundant laboratory and field evidence for time-inconsistent overconsumption, preference reversals, and demand for commitment devices.[44] Theoretically, the hyperbolic-discounting model of Laibson et al. (2012) is very successful in explaining life-cycle consumption. In the next proposition, I formalize how the consumption implications differ in the monotone-pre-committed equilibrium if one exists. Then, I explain beliefs-based present bias in detail and show how it differs from hyperbolic discounting.

Proposition 6. *Comparison to the monotone-pre-committed equilibrium.*
1. If $\sigma_{Pt} > 0$ for any t, then the monotone-pre-committed consumption path does not correspond to the monotone-personal equilibrium consumption path.
2. The news-utility agent's monotone-pre-committed consumption is excessively smooth and sensitive.
3. News-utility preferences introduce a first-order precautionary-savings motive in the monotone-pre-committed equilibrium, monotone-pre-committed consumption is lower $C^c_{T-1} < C_{T-1}$, and the gap increases in the event of good income realizations $\frac{\partial(C^c_{T-1}-C_{T-1})}{\partial s^P_{T-1}} > 0$.
5. The news-utility agent's monotone-pre-committed consumption path is not necessarily characterized by a hump-shaped consumption profile and consumption does not drop at retirement.

Suppose that the agent can pre-commit to an optimal, history-dependent consumption path for each possible future contingency. Then, the agent's marginal gain-loss utility is no longer solely composed of the sensation of increasing consumption in one particular contingency; additionally, the agent considers that he will experience fewer sensations of

[43] Koszegi and Rabin (2009) argue in Proposition 6 that the agent overconsumes relative to the optimal pre-committed path in the presence of uncertainty.

[44] See, e.g., DellaVigna (2009), Frederick et al. (2002), or Angeletos et al. (2001) for a survey of the theory and empirical evidence.

gains and more feelings of loss in all other contingencies. Thus, marginal gain-loss utility has a second component, $-u'(C_{T-1})(\eta(1-F^{t-1}_{C_t}(C_t))+\eta\lambda F^{t-1}_{C_t}(C_t))$, which is negative such that the pre-committed agent consumes less. Moreover, this negative component dominates if the realization is above the median, i.e., $F^{t-1}_{C_t}(C_t) > 0.5$. Thus, in the event of good income realizations, pre-committed marginal gain-loss utility is negative. In contrast, non-pre-committed marginal gain-loss utility is always positive because the agent enjoys the sensation of increasing consumption in any contingency. Therefore, the degree of present bias is reference dependent and less strong in the event of bad income realizations, when increasing consumption is the optimal response even on the pre-committed path. Moreover, this negative component implies additional variation in marginal gain-loss utility such that pre-committed consumption is more excessively smooth and sensitive.

The analysis of the pre-committed equilibrium allows me to draw potentially important welfare conclusions. The result that excess smoothness is an optimal response and even more pronounced on the pre-committed path stands in contrast to the welfare implications of liquidity constraints, the potentially most popular alternative explanation for excess smoothness. In contrast, the result that the life-cycle consumption profile is not necessarily hump shaped and that consumption does not drop at retirement in the pre-committed equilibrium appears to be in line with alternative explanations such as hyperbolic discounting and illiquid savings as proposed by Laibson et al. (2012), inattention as proposed by Reis (2006), incomplete consumption insurance as proposed by Attanasio and Pavoni (2011), an incomplete planning horizon as proposed by Park (2011), or overconfidence as proposed by Caliendo and Huang (2008).

Beyond the observation that the news-utility agent is unable to follow his expected-utility maximizing path, the news-utility implications for welfare and the costs of business cycle fluctuations differ from those of the standard model. In Section IV, I demonstrate that income uncertainty has a first-order effect on savings and thus welfare in the news-utility model; i.e., the news-utility agent dislikes fluctuations in consumption much more than the standard agent. In the spirit of Lucas (1978), I compute the share λ_W of initial wealth A_1 that the agent would be willing to give up for a risk-free consumption path. In the power-utility model for the calibration given in Table I, I obtain a share of approximately 47.83% for the news-utility agent, whereas the standard agent's share is 8.65%.

Beliefs-based present bias is both conceptually different from hyperbolic-discounting preferences and observationally distinguishable. The four main differences are the following. First, news utility introduces an additional precautionary-savings motive that is absent in the hyperbolic-discounting model. Second, because of this precautionary-savings motive the news-utility agent does not have a universal desire to pre-commit himself to the standard

agent's consumption path, in contrast to hyperbolic-discounting preferences. Third, news utility predicts that the agent's degree of present bias is reference dependent and lower in bad times; hence, he behaves better in bad times. Fourth, the news-utility agent's degree of present bias depends on the uncertainty he faces. In the absence of uncertainty, the agent's present bias is absent so long as $\gamma > \frac{1}{\lambda}$. In the presence of small or discrete uncertainty, the agent's degree of present bias is less than in the presence of large and dispersed uncertainty, as shown in Section IV.4.

V Quantitative Predictions about Consumption

In the following, I assess whether the model's quantitative predictions match the empirical evidence. Because it is commonly argued that exponential utility is unrealistic, I present the numerical implications of a power-utility model, i.e., $u(C) = \frac{C^{1-\theta}}{1-\theta}$, to demonstrate that all of the predictions hold in model environments that are commonly assumed in the life-cycle consumption literature.[45] In Section V.1, I first outline the power-utility model. In Section V.2, I structurally estimate the power-utility model's parameters. In Section V.3, I compare my estimates with those in the microeconomic literature and explain each in detail.

V.1 The power-utility model

The income processes and model environment. I follow Carroll (1997) and Gourinchas and Parker (2002), who specify income Y_t to be log-normal and characterized by a deterministic permanent income growth G_t, permanent shocks, and transitory shocks, which allow for a low probability of unemployment or illness

$$Y_t = P_t N_t^T = P_{t-1} G_t N_t^P N_t^T$$

$$N_t^T = \left\{ \begin{array}{cc} e^{s_t^T} & \text{with probability } 1-p \text{ and } s_t^T \sim N(\mu_T, \sigma_T^2) \\ 0 & \text{with probability } p \end{array} \right\} \; N_t^P = e^{s_t^P} \;\; s_t^P \sim N(\mu_P, \sigma_P^2).$$

The life-cycle literature suggests fairly tight ranges for the parameters of the log-normal income process, which are approximately $\mu_T = \mu_P = 0$, $\sigma_T = \sigma_P = 0.1$, and $p = 0.01$. G_t typically implies a hump-shaped income profile. Nevertheless, I initially assume that $G_t = 1$ for all t to highlight the model's predictions in an environment that does not simply generate a hump-shaped consumption profile via a hump-shaped income profile. In addition to the

[45] The power-utility model cannot be solved analytically, but it can be solved by numerical backward induction, as shown by Gourinchas and Parker (2002) or Carroll (2001), among others. The numerical solution is illustrated in greater depth in Appendix B.5.4.

standard and hyperbolic agents, I display results for internal, multiplicative habit-formation preferences, as assumed in Michaelides (2002), and temptation-disutility preferences, as developed by Gul and Pesendorfer (2004), following the specification of Bucciol (2012). The utility specifications can be found in Appendix B.1. For the habit-formation agent, I roughly follow Michaelides (2002) and choose $h = 0.45$, which matches the excess-smoothness evidence. The tempted agent's additional preference parameter $\tau = \frac{\lambda^{td}}{1+\lambda^{td}} = 0.1$ is chosen according to the estimates of Bucciol (2012).[46]

Comparison of life-cycle consumption profiles. Figure III contrasts the five agents' consumption paths with the average CEX consumption and income data, which I explain in Section V.2. The habit-formation agent's consumption profile is shown only in part because he engages in extremely high wealth accumulation due to his high effective risk aversion, even if I choose a lower value for h than the one that fits the excess-sensitivity evidence.[47] Hyperbolic-discounting preferences tilt the consumption profile upward at the beginning and downward at the end of life. Temptation disutility causes severe overconsumption at the beginning of life, which then dies out when alternative consumption opportunities diminish. All of the preference specifications except habit formation generate a hump-shaped consumption profile. The consumption path of all agents is increasing at the beginning of life because power utility renders them unwilling to borrow; however, all agents are sufficiently impatient such that consumption eventually decreases.[48] Nevertheless, at first glance, the news-utility agent's hump looks more similar to the empirical consumption profile with slowly increasing consumption at the beginning of life and decreasing consumption shortly before retirement.

Moreover, Figure III shows a substantial drop in consumption at retirement for both the news-utility consumption profile and the CEX consumption data. Thus, I conclude that the news-utility agent's lifetime consumption profile looks very similar to the average consumption profile from the CEX data, which I explain in greater detail in the next section.

[46]For the news-utility parameters, I use the same calibration as in the quantitative exercise in Section IV.1.

[47]This result about wealth accumulation confirms a finding by Michaelides (2002).

[48]Because power utility eliminates the possibility of negative or zero consumption and because of the small possibility of zero income in all future periods, the agents will never find it optimal to borrow. Moreover, power utility implies prudence such that all agents have a standard precautionary-savings motive. However, this motive is rather weak because the standard agent's consumption begins to decrease rather early in life. Moreover, in a model with only transitory shocks and no zero-income state, the precautionary savings motive is so weak that the standard agent's consumption is flat throughout. Attanasio (1999) criticizes this weak motive as lacking realism. In contrast, even if permanent shocks and unemployment are absent, the news-utility model generates a hump-shaped consumption profile.

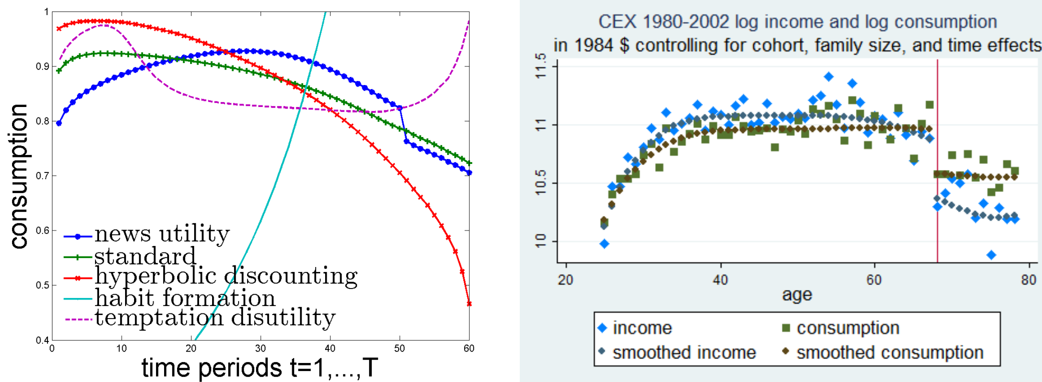

Figure III: Power-utility life-cycle profiles and CEX consumption and income data.

V.2 Structural estimation

Method of simulated moments procedure. I structurally estimate the news-utility parameters using a methods-of-simulated-moments procedure following Gourinchas and Parker (2002), Laibson et al. (2012), and Bucciol (2012). The procedure has two stages. In the first stage, I estimate the structural parameters governing the environment $\Xi = (\mu_P, \sigma_P, \mu_T, \sigma_T, p, G, r, a_0, R, T)$ using standard techniques and obtain results perfectly in line with the literature. Given the first-stage estimates $\hat{\Xi}$ and their associated variances $\hat{\Omega}_\Xi$, in the second stage, I estimate the preference parameters $\boldsymbol{\theta} = (\eta, \lambda, \gamma, \beta, \theta)$ by matching the simulated and empirical average life-cycle consumption profiles. The empirical life-cycle consumption profile is the average consumption at each age $a \in [1, T]$ across all household observations i. More precisely, it is $ln\bar{C}_a = \frac{1}{n_a}\sum_{i=1}^{n_a} ln(\bar{C}_{i,a})$ with $ln(\bar{C}_{i,a})$ being the household i's log consumption at age a of which n_a are observed. The theoretical population analogue to $ln\bar{C}_a$ is denoted by $lnC_a(\theta, \Xi)$ and the simulated approximation is denoted by $ln\hat{C}_a(\boldsymbol{\theta}, \Xi)$. Moreover, I define $g(\boldsymbol{\theta}, \Xi) = lnC_a(\boldsymbol{\theta}, \Xi) - ln\bar{C}_a$ and

$$\hat{g}(\boldsymbol{\theta}, \Xi) = ln\hat{C}_a(\boldsymbol{\theta}, \Xi) - ln\bar{C}_a.$$

In turn, if $\boldsymbol{\theta}_0$ and Ξ_0 are the true parameter vectors, the procedure's moment conditions imply that $E[g(\theta_0, \Xi_0)] = E[lnC_a(\theta, \Xi) - ln\bar{C}_a] = 0$. In turn, let W denote a positive definite weighting matrix then

$$q(\boldsymbol{\theta}, \Xi) = \hat{g}(\boldsymbol{\theta}, \Xi) W^{-1} \hat{g}(\boldsymbol{\theta}, \Xi)'$$

is the weighted sum of squared deviations of the simulated from their corresponding empirical moments. I assume that W is a robust weighting matrix rather than the optimal weighting

matrix to avoid small-sample bias. More precisely, I assume that W corresponds to the inverse of the variance-covariance matrix of each point of $ln\bar{C}_a$, which I denote by Ω_g^{-1} and consistently estimate from the sample data. Taking $\hat{\Xi}$ as given, I minimize $q(\boldsymbol{\theta}, \hat{\Xi})$ with respect to $\boldsymbol{\theta}$ to obtain $\hat{\boldsymbol{\theta}}$ the consistent estimator of $\boldsymbol{\theta}$ that is asymptotically normally distributed with standard errors

$$\Omega_\theta = (G_\theta' W G_\theta)^{-1} G_\theta' W [\Omega_g + \Omega_g^s + G_\Xi \Omega_\Xi G_\Xi'] W G_\theta (G_\theta' W G_\theta)^{-1}.$$

Here, G_θ and G_Ξ denote the derivatives of the moment functions $\frac{\partial g(\boldsymbol{\theta}_0, \Xi_0)}{\partial \boldsymbol{\theta}}$ and $\frac{\partial g(\boldsymbol{\theta}_0, \Xi_0)}{\partial \Xi}$, Ω_g denotes the variance-covariance matrix of the second-stage moments as above that corresponds to $E[g(\boldsymbol{\theta}_0, \Xi_0) g(\boldsymbol{\theta}_0, \Xi_0)']$, and $\Omega_g^s = \frac{n_a}{n_s} \Omega_g$ denotes the sample correction with n_s being the number of simulated observations at each age a. As Ω_g, I can estimate Ω_Ξ directly and consistently from sample data. For the minimization, I employ a Nelder-Mead algorithm. For the standard errors, I numerically estimate the gradient of the moment function at its optimum. If I omit the first-stage correction and simulation correction the expression becomes $\Omega_\theta = (G_\theta' \Omega_g^{-1} G_\theta)^{-1}$. Finally, I can test for overidentification by comparing $\hat{g}(\hat{\theta}, \hat{\Xi})$ to a chi-squared distribution with $T - 5$ degrees of freedom.

Data. I use data from the Consumer Expenditure Survey (CEX) for the years 1980 to 2002 as provided by the NBER.[49] The CEX is conducted by the Bureau of Labor Statistics and surveys a large sample of the US population to collect data on consumption expenditures, demographics, income, and assets. As suggested by Harris and Sabelhaus (2001), consumption expenditures consists of food, tobacco, alcohol, amusement, clothing, personal care, housing, house operations such as furniture and housesupplies, personal business, transportation such as autos and gas, recreational activities such as books and recreational sports, and charity expenditures; alternatively, I could consider non-durable consumption only. Income consists of wages, business income, farm income, rents, dividends, interest, pension, social security, supplemental security, unemployment benefits, worker's compensation, public assistance, foodstamps, and scholarships. The data is deflated to 1984 dollars.

Because the CEX does not survey households consecutively, I generate a pseudo panel that averages each household's consumption and income at each age. I only consider non-student households that meet the BLS complete income reporter requirement and complete all four quarterly interviews. Furthermore, I only consider households that are older than

[49] This data set extraction effort is initiated by John Sabelhaus and continued by Ed Harris, both of the Congressional Budget Office. The data set links the four quarterly interviews for each respondent household and collapses all the spending, income, and wealth categories into a consistent set of categories across all years under consideration.

25 years; that are retired after age 68, the average retirement age in the US according to the OECD; and that are younger than 78, the average life expectancy in the US according to the UN list.

To control for cohort, family size, and time effects, I employ average cohort techniques (see, e.g., Verbeek (2007), Attanasio (1998), and Deaton (1985)). More precisely, as I lack access to the micro consumption data for each household i at each age a, I pool all observations and estimate $log(C_{i,a}) = \xi_0 + \alpha_a + \gamma_c + f_s + X_{i,a}^{'}\beta^{ia} + \epsilon_{i,a}$. Here, ξ_0 is a constant, α_a is a full set of age dummies, γ_c is a full set of cohort dummies, and f_s is a full set of family size and number of earners dummies. Essentially, these sets of dummies allow me to consider the sample means of my repeated cross-section $C_{c,a}^{*} = E[log(C_{i,a})|c, a]$ and $X_{c,a}^{*} = E[X_{i,a}^{*}|c, a]$ for each cohort c at age a. Using the sample means brings about an errors-in-variables problem, which, however, does not appear to make a difference in practice as the sample size of each cohort-age cell is large. Because age α_a and cohort γ_c effects are not separately identifiable from time effects, I proxy time effects by including the regional unemployment rate as an additional variable in $X_{i,a}^{*}$ beyond a dummy for retirement following Gourinchas and Parker (2002). As an alternative to a full set of dummies, I use fifth-order polynomials in order to obtain a smooth consumption profile. After running the pooled regression, I back out the consumption data uncontaminated by cohort, time, family size, and number of earners effects and construct the average empirical life-cycle profile by averaging the data across households at each age. The same exercise is done for income. Figure III displays the average empirical income and average empirical consumption profiles.

Identification. Theoretically, the functional form of Λ_t, or the agent's first-order condition in the power-utility model, imply that news utility introduces such specific variation in consumption growth that all preference parameters are identified in the finite-horizon model, i.e., η, λ, γ, β, and θ, because the Jacobian has full rank.[50] Roughly speaking, the shape of the consumption profile identifies β and θ. Because consumption tracks income too closely and peaks too early in the standard model, $\eta > 0$ and $\lambda > 1$ can be identified. Finally, the drop in consumption at retirement identifies $\gamma < 1$. More precisely, I am interested in β, θ, η, λ, γ. As explained in Appendix B.5.4, the agent's consumption is determined by the following first-order condition $u'(c_{T-i}) = \frac{\Psi_{T-i}^{'}+\gamma\Phi_{T-i}^{'}(\eta F_{c_{T-i}}^{T-i-1}(c_{T-i})+\eta\lambda(1-F_{c_{T-i}}^{T-i-1}(c_{T-i})))}{1+\eta F_{a_{T-i}}^{T-i-1}(a_{T-i})+\eta\lambda(1-F_{a_{T-i}}^{T-i-1}(a_{T-i}))}$ of which I observe the inverse and log average of all households. $\Phi_{T-i}^{'}$ represents future marginal consumption utility, as in the standard model, and is determined by β and θ, which can be

[50]Numerically, I confirm this result in a Monte Carlo simulation and estimation exercise. Moreover, because previous studies cannot separately identify η and λ, I confirm that I obtain similar estimates when I assume $\eta = 1$ and only estimate the other parameters.

separately identified in a finite-horizon model. Ψ'_{T-i} represents future marginal consumption and news utility and is thus determined by something akin of $\eta(\lambda-1)$. $\eta F^{T-i-1}_{a_{T-i}}(a_{T-i}) + \eta\lambda(1-F^{T-i-1}_{a_{T-i}}(a_{T-i}))$ and $\eta F^{T-i-1}_{c_{T-i}}(c_{T-i})+\eta\lambda(1-F^{T-i-1}_{c_{T-i}}(c_{T-i}))$ represents the weighted sum of the cumulative distribution function of savings, a_{T-i}, and consumption, c_{T-i}, of which merely the average determined by $\eta 0.5(1+\lambda)$ is observed. Thus, I have two equations in two unknowns and can separately identify η and λ. Finally, γ enters the first-order condition distinctly from all other parameters.

Structural estimation results. I employ a two-stage method-of-simulated-moments procedure. In the first stage, I estimate all of the structural parameters governing the environment $\hat{\mu}_P$, $\hat{\sigma}_P$, $\hat{\mu}_T$, $\hat{\sigma}_T$, $\hat{p}$, $\hat{G}$, $\hat{r}$, $\hat{a}_0$, $\hat{R}$, and $\hat{T}$, i.e., $\hat{\mu}_P = -0.002$, $\hat{\mu}_T = -0.0031$, $\hat{\sigma}_P = 0.18$, $\hat{\sigma}_T = 0.16$, and $\hat{p} = 0.0031$, which are in accordance with the literature. The mean of Moody's municipal bond index is $r = 3.1\%$. Moreover, because 25 is chosen as the beginning of life by Gourinchas and Parker (2002), I choose $\hat{R} = 11$ and $\hat{T} = 54$. At age 25, I estimate the mean ratio of liquid wealth to income as 0.0096 under the assumption that $P_0 = 1$.

I estimate the preference parameters β, θ, η, λ, and γ and obtain $\hat{\theta} = 0.79$, $\hat{\beta} = 0.97$, $\hat{\eta} = 1.1$, $\hat{\lambda} = 2.4$, and $\hat{\gamma} = 0.53$. I display all first- and second-stage structural parameter estimates as well as their standard errors in Table I.[51] The second-stage standard errors are adjusted for first-stage uncertainty and the sampling correction; while the former increases the standard errors considerably the latter has very little effect as noted by Laibson et al. (2012). The preference parameters are estimated very tightly, and I cannot reject the overidentification test, which is a surprisingly positive result given the number of moments T and the number of parameters, which is only five. In contrast, for the standard model, the standard errors are considerably larger and I reject the overidentification test, as do Gourinchas and Parker (2002). Finally, I obtain suggestive evidence for one of the new comparative statics generated by news utility; the excess-smoothness ratio in the CEX data increases from 0.68 at age 25 to 0.82 at the start of retirement.[52]

[51] Alternatively, I use a more complex set of moments to estimate the preference parameters, namely the degree of excess smoothness in consumption, the extent of the drop in consumption at retirement, and four other points of the life-cycle consumption profile. The resulting estimates and their standard errors are quantitatively very similar to the original ones.

[52] For comparison, Figure IV in Appendix A displays the consumption and income data of Gourinchas and Parker (2002) as well as the authors' fitted consumption profile (i.e. the standard model) and the fitted consumption of the news-utility model using the authors' baseline estimation results, which are displayed in Table IV in Appendix A, as well as $\eta = 1$, $\lambda = 2$, and $\gamma = 0.85$ for the news-utility model. As noted by Gourinchas and Parker (2002), the standard agent's consumption peaks somewhat too early and increases too steeply with income growth. News utility causes consumption to peak later and to increase less steeply at the beginning of life. In the paper Gourinchas and Parker (2002) display the average empirical consumption profile for the average empirical household size profile, whereas I display the profile for a single household, which emphasizes the differences and thus facilitates the comparison.

TABLE I:
FIRST-STAGE PARAMETER ESTIMATION RESULTS

	$\hat{\mu}_P$	$\hat{\sigma}_P$	$\hat{\mu}_T$	$\hat{\sigma}_T$	$\hat{p}$	$\hat{G}_t$	$\hat{r}$	P_0	$\frac{\hat{A}_0}{\hat{P}_0}$	$\hat{R}$	$\hat{T}$
estimate	0	0.19	0	0.15	0.0031	$e^{Y_{t+1}-Y_t}$	3.1%	1	0.0096	11	54
standard error		(0.004)		(0.006)	(0.001)	$(\hat{\Omega}_{\hat{G}})$	(0.003)		(0.005)		

SECOND-STAGE PARAMETER ESTIMATION RESULTS

	news-utility model					standard model	
	$\hat{\beta}$	$\hat{\theta}$	$\hat{\eta}$	$\hat{\lambda}$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\theta}$
estimate	0.97	0.77	0.97	2.33	0.59	0.9	2.01
standard error	(0.001)	(0.011)	(0.068)	(0.018)	(0.021)	(0.029)	(0.091)
$\chi(\cdot)$			43.3			101.2	

The overidentification test's critical value at 5% is 67.5.

V.3 Discussion of the estimated preference parameters

I now show that my estimates are perfectly in line with the micro literature, generate reasonable attitudes towards small and large wealth bets, and match the empirical evidence for excess smoothness and sensitivity in aggregate data.

Comparison to the microeconomic and experimental literature. I refer to the literature for the standard preference parameter estimates $\beta \approx 1$ and $\theta \approx 1$ but discuss the news-utility parameter estimates, i.e., η, λ, and γ, in greater detail. In particular, I demonstrate that my estimates are consistent with existing micro evidence on risk and time preferences. In Table III in Appendix A, I illustrate the risk preferences over gambles with various stakes of the news-utility, standard, and habit-formation agents. In particular, I calculate the required gain G for a range of losses L to make each agent indifferent between accepting or rejecting a 50-50 win G or lose L gamble at a wealth level of 300,000 in the spirit of Rabin (2001) and Chetty and Szeidl (2007).[53]

First, I want to demonstrate that my estimates match risk attitudes towards bets regarding immediate consumption, which are determined solely by η and λ because it can be reasonably assumed that utility over immediate consumption is linear. In Table III, it can be seen that the news-utility agent's contemporaneous gain-loss utility generates reasonable attitudes towards small and large gambles over immediate consumption. Moreover, $\eta \approx 1$ and $\lambda \approx 2.5$ are consistent with the laboratory evidence on loss aversion over immediate

[53]In a canonical asset-pricing model, Pagel (2012b) demonstrates that news-utility preferences constitute an additional step towards resolving the equity-premium puzzle, as they match the historical level and the variation of the equity premium while simultaneously implying plausible attitudes towards small and large wealth bets.

consumption, i.e., the endowment effect literature.[54] In contrast, since I assume linear utility over immediate consumption, the standard and habit-formation agents are risk neutral. Second, I elicit the agents' risk attitudes by assuming that each of them is presented the gamble after all consumption in the current period has taken place. The news-utility agent will only experience prospective gain-loss utility over the gamble's outcome, which is determined by γ. Empirical estimates for the quasi-hyperbolic parameter β in the $\beta\delta-$model typically range between 0.7 and 0.8 (e.g., Laibson et al. (2012)). Thus, the experimental and field evidence on peoples' attitudes towards intertemporal consumption trade-offs dictates a choice of $b = \gamma \approx 0.7$ when $\beta \approx 1$, which is roughly in line with my estimate. In Table III, it can be seen that the news-utility agent's risk attitudes take reasonable values for small, medium, and large stakes. The habit-formation agent is risk neutral for small and medium stakes and somewhat more risk averse for large stakes than the standard agent, who only exhibits reasonable risk attitudes for very large stakes.

Excess smoothness and excess sensitivity in aggregate data. I now go on to demonstrate that my estimates are not only consistent with those found in the micro literature but generate the degree of excess smoothness found in macro data. I simulate 200 consumption and income data points of 1000 individuals to then aggregate their consumption and income and run the regression

$$\Delta log(\bar{C}_{t+1}) = \alpha + \beta_1 \Delta log(\bar{Y}_{t+1}) + \beta_2 \Delta log(\bar{Y}_t) + \varepsilon_{t+1}$$

following Campbell and Deaton (1989).[55] The results are displayed in Table II. In the news-utility model, I obtain a coefficient $\beta_2 \approx 0.27$ and the excess smoothness ratio, i.e., $\frac{\sigma(\Delta log(\bar{C}_t))}{\sigma(\Delta log(\bar{Y}_t))}$ as defined in Deaton (1986), is 0.74, whereas in the standard model, I obtain $\beta_2^s \approx 0.01$

[54]For illustration, I borrow a concrete example from Kahneman et al. (1990), in which the authors distribute a good (mugs or pens) to half of their subjects and ask those who received the good about their willingness to accept (WTA) and those who did not receive it about their willingness to pay (WTP) if they traded the good. The median WTA is \$5.25, whereas the median WTP is \$2.75. Accordingly, I infer $(1+\eta)u(mug) = (1+\eta\lambda)2.25$ and $(1+\eta\lambda)u(mug) = (1+\eta)5.25$, which implies that $\lambda \approx 3$ when $\eta \approx 1$. I obtain a similar result for the pen experiment. Unfortunately, thus far, I can only jointly identify η and λ. If the news-utility agent were only to exhibit gain-loss utility, I would obtain $\eta\lambda 2.25 \approx 5.25$ and $\eta 2.25 \approx 2.25$, i.e., $\lambda \approx 2.3$ and $\eta \approx 1$ both identified. Alternatively, if I assume that the market price for mugs (or pens), which is \$6 in the experiment (or \$3.75), equals $(1+\eta)u(mug)$ (or $(1+\eta)u(pen)$), then I can estimate $\eta = 0.74$ and $\lambda = 2.03$ for the mug experiment and $\eta = 1.09$ and $\lambda = 2.1$ for the pen experiment. These latter assumptions are reasonable given the induced-market experiments of Kahneman et al. (1990). $\eta \approx 1$ and $\lambda \approx 2.5$ thus appear to be reasonable estimates that are typically used in the literature concerning the static preferences.

[55]I simulate data for each individual at normalized wealth level $\frac{A_0}{P_0} = 1$ and date $t = 50$. The regression results are similar for different wealth levels and time horizons.

Table II:
Excess-Smoothness and Sensitivity Regression Results

Model	news-utility		habit		standard		hyperbolic		tempted	
	β_1	β_2	β_1^h	β_2^h	β_1^s	β_2^s	β_1^b	β_2^b	β_1^{td}	β_2^{td}
coefficient	0.67	0.27	0.69	0.38	0.93	0.01	0.94	0.01	1.01	-0.01
t-statistic	86.7	34.2	141	76.3	187	1.32	205	1.33	135	-1.76
e-s ratio	0.74		0.80		0.95		0.96		1.04	

Aggregate regression results of 1000 individuals with $N = 200$ simulated data points.

and 0.95.[56] Regressing consumption growth on lagged labor income growth in aggregate data, I obtain an OLS estimate for β_2 of approximately 0.23 and an excess-smoothness ratio of approximately 0.68.[57] Unsurprisingly, temptation disutility does not generate excess smoothness and sensitivity, while habit formation does. However, habit formation appears to generate too little excess smoothness and too much excess sensitivity and has unrealistic implications for the life-cycle consumption profile, which I explored in the previous section. I conclude that the estimates obtained from CEX consumption data simultaneously match the degree of excess smoothness and sensitivity found in aggregate data.

VI Extensions

In the following, I briefly outline four extensions of the basic life-cycle model that I have developed separately.

Extensions. As a first extension, I introduce both illiquid savings and credit-card borrowing to demonstrate that the beliefs-based time inconsistency generates simultaneous demand for illiquid retirement savings and excessive credit-card borrowing. I assume that the agent can borrow against his illiquid savings up to his natural borrowing constraint, which is determined by the discounted value of his accumulated illiquid savings.[58] I again find that only the news-utility model is able to robustly generate the collection of life-cycle consumption facts. My findings differ from those of Laibson et al. (2012) because I do not assume the ex-

[56] All consumption adjustment takes place after a single period because the agent's preferences are characterized by full belief updating. However, the variation in consumption adjusts via end-of-period asset holdings and is thereby spread out over the entire future. Empirically, Fuhrer (2000) and Reis (2006) find that consumption peaks one year after the shock and that the consumption response dies out briefly after the first year.

[57] I follow Ludvigson and Michaelides (2001) and use NIPA deflated total, nondurable, or services consumption and total disposable labor income for the years 1947 to 2011.

[58] I call this borrowing constraint natural, following Carroll (2001), because power utility and the possibility of zero income in all future periods induce the agent to never want to borrow beyond this constraint.

istence of non-natural borrowing constraints. Absent such constraints, only those hyperbolic agents at the margin of zero liquid asset holdings would delay consumption adjustments to shocks or tolerate a drop in consumption at retirement. In a model with illiquid savings, any agent with a time-inconsistency problem, i.e., the news-utility, hyperbolic, or tempted agent, will use illiquid savings to make wealth unavailable in the future and thus reduce future consumption. This unavailability of wealth implies that the future agent will exhibit a high marginal propensity to consume out of permanent as well as transitory income shocks. In contrast, the marginal propensity to consume out of transitory income shocks is close to zero in a model without illiquid savings or absent a time-inconsistency problem. Thus, a high marginal propensity to consume out of transitory income shocks can be interpreted as excess sensitivity in consumption rather than a delayed response to income shocks (Laibson (1997) and Laibson et al. (2012) among others).

As a second extension, I let the agent endogenously determine his work hours in response to fluctuations in wages. In the event of an adverse shock, he can maintain high consumption by working more instead of consuming his savings. Thus, if the agent's labor supply is relatively elastic, his consumption becomes more excessively smooth and less excessively sensitive.

As a third extension, I allow the agent to invest in a risky asset in addition to his risk-free asset. I obtain four main implications for portfolio choice. First, the agent chooses a low portfolio share or does not participate in the stock market, as he is first-order risk averse even in the presence of labor income.[59] Second, his optimal portfolio share decreases in the return realization. In the event of a good return realization, the agent chooses a lower portfolio share to realize the good news about future consumption and play safe. Third, the agent exhibits a time-inconsistency for risk. Taking his beliefs as given, the agent is inclined to opt for a higher portfolio share to enjoy the prospect of high future consumption, as he resides on a low-risk path. Fourth, the agent can diversify across time because the expected loss of his investment increases with the square root of his investment horizon whereas his expected return increases linearly. All of these predictions smooth the agent's risky asset holdings relative to the standard model. Thus, I obtain a novel prediction of stickiness in

[59]The result about first-order risk aversion in the presence of background risk stands in contrast to earlier analyzes, such as Barberis et al. (2006) and Koszegi and Rabin (2007, 2009). Barberis et al. (2006) consider utility specifications that exhibit first-order risk aversion at one point. Background risk takes the agent away from this point and he becomes second-order risk averse with respect to additional risk. However, the reference point is stochastic in this paper's model, so that it exhibits first-order risk aversion over the entire support of background risk. Koszegi and Rabin (2007, 2009) consider situations in which background risk is large and utility potentially linear and find that, in the limit, the agent becomes second-order risk averse. However, labor income risk is not large relative to stock market risk in a life-cycle portfolio framework and the agent's utility function is unlikely to be linear in a model that is calibrated to realistic labor income and stock-market risk at an annual horizon.

portfolio choice, which has been observed in household portfolio data by Calvet et al. (2009) or Brunnermeier and Nagel (2008).

As a fourth extension, I assume that the agent receives a large income shocks every couple periods but is subject to merely discrete or small income uncertainty in in-between periods. As I have shown in Section IV.4, in the presence of sufficiently small income uncertainty, the agent will choose a flat consumption level independent of the realization of the income shock. And whenever he is able to discretize his consumption, he overconsumes less than in periods in which he is subject to large income uncertainty that makes flat consumption non-credible. This model extension allows to relax an important calibrational degree of freedom associated with the preferences, that is, the period's length. Moreover, in a setting with merely discrete uncertainty, I reobtain a result first emphasized by Koszegi and Rabin (2009): the agent may consume entire small windfall gains but delay entire small windfall losses.

VII Conclusion

This paper demonstrates that expectations-based reference-dependent preferences can not only explain micro evidence, such as the endowment effect or cab-driver labor supply, but also offer a unified explanation for major life-cycle consumption facts. Excess smoothness and sensitivity in consumption, two widely analyzed macro consumption puzzles, are explained by loss aversion, a robust risk preference analyzed in experimental research and a popular explanation for the equity premium puzzle. Intuitively, the agent wants to allow his expectations-based reference point to decrease or increase prior to adjusting consumption. Moreover, a hump-shaped consumption profile and a drop in consumption at retirement are explained by the interplay of news-utility risk and time preferences. A hump-shaped consumption profile results from the net of two preference features. The news-utility agent's consumption path is steeper at the beginning of life because loss aversion generates an additional precautionary-savings motive, which accumulates more rapidly than the standard precautionary-savings motive in the agent's horizon. However, the news-utility agent's consumption path declines toward the end of life because the expectations-based reference point introduces a time-inconsistency problem: expected utility is higher in an optimal pre-committed equilibrium in which the agent simultaneously optimizes over consumption and beliefs. The pre-committed equilibrium is non-credible, however, because the agent overconsumes once he wakes up and takes his beliefs as given. Once the agent retires, however, time-inconsistent overconsumption is associated with a certain loss in future consumption. Thus, the agent is suddenly able to behave himself, and his consumption drops at retirement. I explore the intuition for the model's results in depth by solving an exponential-utility model

in closed form. Moreover, assuming power-utility as standard in the literature, I structurally estimate the preference parameters and obtain estimates that are in line with the existing micro evidence and generate the degree of excess smoothness found in aggregate data.

In the future, I wish to further explore expectations-based reference dependence as a potential micro foundation for behavioral biases that have been widely documented. For instance, all of my life-cycle results support the notion that fluctuations in beliefs about consumption are painful. If people have some discretion in choosing how much information to gather, they might choose to "stick their head into the sand" occasionally to avoid fluctuations in beliefs that are painful on average; i.e., people are rationally inattentive. For instance, a long-term investor might choose to not check on his portfolio, particularly when he suspects that it might have decreased in value; this behavior has been termed the Ostrich effect. Similarly, a CEO might choose to not evaluate a project when he suspects that it is performing poorly. An outsider, who acquires all information he does not have a stake in, will perceive the investor's or CEO's behavior as overconfident and extrapolative because their expectations are based on an overly favorable and outdated information set whenever they have received adverse but only incomplete information.

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A More Figures and Tables

Table III:
Risk Attitudes over Small and Large Wealth Bets

Loss (L)	standard	news-utility contemp.	news-utility prospective	habit-formation
10	10	15	22	10
200	200	300	435	200
1000	1000	1500	2166	1000
5000	5000	7500	10719	5000
50000	50291	75000	105487	52502
100000	100406	150000	2066770	112040

For each loss L, the table's entries show the required gain G to make each agent indifferent between accepting and rejecting a 50-50 gamble win G or lose L at a wealth level of 300,000 and a permanent income of 100,000 (power-utility model).

Table IV:
Baseline Estimation Results of Gourinchas and Parker (2002)

μ_n	σ_n	μ_u	σ_u	p	r	β	θ	γ_0	γ_1	P_0	A_0	T
0	$\sqrt{0.044}$	0	$\sqrt{0.0212}$	0.00302	0.0344	0.9598	0.514	0.0701	0.071	1	0.3	40

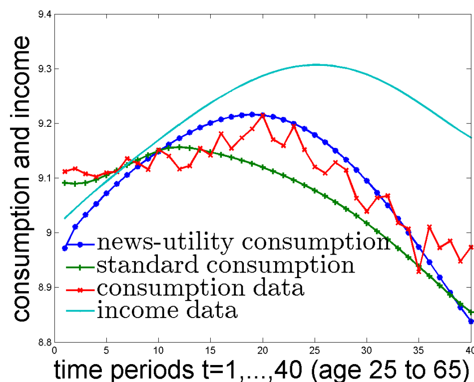

Figure IV: Consumption and income profiles and the fitted model's consumption from Gourinchas and Parker (2002).
The news-utility consumption follows the same specification except for the choice of news-utility parameters $\eta = 1$, $\lambda = 2$, and $\gamma = 0.85$.

B Derivations and Proofs

B.1 Summary of utility functions under consideration

I briefly summarize the lifetime utility of all preference specifications that I consider. I define the "news-utility" agent's lifetime utility in each period $t = \{0, ..., T\}$ as

$$u(C_t) + n(C_t, F_{C_t}^{t-1}) + \gamma \sum_{\tau=1}^{T-t} \beta^{\tau} \boldsymbol{n}(F_{C_{t+\tau}}^{t,t-1}) + E_t[\sum_{\tau=1}^{T-t} \beta^{\tau} U_{t+\tau}]$$

with $\beta \in [0, 1]$, $u(\cdot)$ a HARA[60] utility function, $\eta \in (0, \infty)$, $\lambda \in (1, \infty)$, and $\gamma \in [0, 1]$. Additionally, I first consider standard preferences as analyzed by Carroll (2001), Gourinchas and Parker (2002), and Deaton (1991), among many others. The "standard" agent's lifetime utility is given by

$$u(C_t^s) + E_t[\sum_{\tau=1}^{T-t} \beta^{\tau} u(C_{t+\tau}^s)].$$

Second, I consider internal, multiplicative habit-formation preferences as assumed in Michaelides (2002). The "habit-forming" agent's lifetime utility is given by

$$u(C_t^h) - hu(C_{t-1}^h) + E_t[\sum_{\tau=1}^{T-t} \beta^{\tau} (u(C_{t+\tau}^h) - hu(C_{t+\tau-1}^h))]$$

with $h \in [0, 1]$.

Third, I consider $\beta\delta-$ or hyperbolic-discounting preferences as developed by Laibson (1997). The "$\beta\delta-$" or "hyperbolic-discounting" agent's lifetime utility is given by

$$u(C_t^b) + bE_t[\sum_{\tau=1}^{T-t} \beta^{\tau} u(C_{t+\tau}^b)]$$

with $b \in [0, 1]$ corresponding to the $\beta\delta-$agent's β.

Fourth, I consider temptation-disutility preferences as developed by Gul and Pesendorfer (2004) following the specification of Bucciol (2012). The "tempted" agent's lifetime utility is given by

$$u(C_t^{td}) - \lambda^{td}(u(\tilde{C}_t^{td}) - u(C_t^{td})) + E_t[\sum_{\tau=1}^{T-t} \beta^{\tau} (u(C_{t+\tau}^{td}) - \lambda^{td}(u(\tilde{C}_{t+\tau}^{td}) - u(C_{t+\tau}^{td})))]$$

with $\tilde{C}_t^{td}$ being the most tempting alternative consumption level and $\lambda^{td} \in [0, \infty)$.

[60] A utility function $u(c)$ is said to exhibit hyperbolic absolute risk aversion (HARA) if the level of risk tolerance, $-\frac{u''(c)}{u'(c)}$ is a linear function of c.

B.2 Derivation of the exponential-utility model

B.2.1 The finite-horizon model

A simple derivation of the second-to-last period can be found in the text. The exponential-utility model can be solved through backward induction. In the following, I outline the model's solution for period $T-i$ in which the agent chooses how much to consume C_{T-i} and how much to invest in the risk-free asset A_{T-i}. I guess and verify the model's consumption function

$$C_{T-i} = \frac{(1+r)^i}{f(i)}(1+r)A_{T-i-1} + P_{T-i-1} + s^P_{T-i} + (1 - \frac{f(i-1)}{f(i)})s^T_{T-i} - \frac{f(i-1)}{f(i)}\Lambda_{T-i}$$

with

$$\Lambda_{T-i} = \frac{1}{\theta}log(\frac{(1+r)^i}{f(i-1)}\frac{\psi_{T-i} + \gamma Q_{T-i}(\eta F(s^P_{T-i} + \frac{(1+r)^i}{f(i)}s^T_{T-i}) + \eta\lambda(1 - F(s^P_{T-i} + \frac{(1+r)^i}{f(i)}s^T_{T-i}))}{1 + \eta F(s^P_{T-i} + (1 - \frac{f(i-1)}{f(i)})s^T_{T-i}) + \eta\lambda(1 - F(s^P_{T-i} + (1 - \frac{f(i-1)}{f(i)})s^T_{T-i}))}).$$

and $f(i) = \sum_{j=0}^{i}(1+r)^j = (1+r)^i\frac{1+r-(\frac{1}{1+r})^i}{r}$ (in the text $a(i) = \frac{f(i-1)}{f(i)}$). Then, the budget constraint $A_{T-i} = (1+r)A_{T-i-1} + Y_{T-i} - C_{T-i}$ determines end-of-period asset holdings

$$A_{T-i} = \frac{f(i-1)}{f(i)}(1+r)A_{T-i-1} + \frac{f(i-1)}{f(i)}s^T_{T-i} + \frac{f(i-1)}{f(i)}\Lambda_{T-i}.$$

Λ_{T-i} is a function independent of A_{T-i-1} and P_{T-i-1} but dependent on s^P_{T-i} and s^T_{T-i}. In the last period the agent consumes everything such that $\Lambda_T = 0$. As a first step to verify the solution guess, I sum up the expectation of the discounted consumption function utilities from period $T-i$ to T

$$\beta E_{T-i-1}[\sum_{\tau=0}^{i}\beta^\tau u(C_{T-i+\tau})] = u(P_{T-i-1} + \frac{(1+r)^i}{f(i)}(1+r)A_{T-i-1})Q_{T-i-1}$$

$$= -\frac{1}{\theta}exp\{-\theta(P_{T-i-1} + \frac{(1+r)^i}{f(i)}(1+r)A_{T-i-1})\}Q_{T-i-1},$$

with Q_{T-i-1} given by

$$Q_{T-i-1} = \beta E_{T-i-1}[exp\{-\theta(s^P_{T-i} + (1 - \frac{f(i-1)}{f(i)})s^T_{T-i} - \frac{f(i-1)}{f(i)}\Lambda_{T-i})\} + exp\{-\theta(s^P_{T-i} + \frac{(1+r)^i}{f(i)}s^T_{T-i} + \frac{(1+r)^i}{f(i)}\Lambda_{T-i})\}Q_{T-i}]$$

Q_{T-i-1} is a constant if Λ_{T-i} depends only on s^P_{T-i} and s^T_{T-i}. To derive the above sum, I simply plug in the asset-holding function into each future consumption function. For instance, C_{T-i+1} is given by

$$C_{T-i+1} = \frac{(1+r)^{i-1}}{f(i-1)}(1+r)A_{T-i} + P_{T-i} + s^P_{T-i+1} + (1 - \frac{f(i-2)}{f(i-1)})s^T_{T-i+1} - \frac{f(i-2)}{f(i-1)}\Lambda_{T-i+1}$$

$$= \frac{(1+r)^i}{f(i)}(1+r)A_{T-i-1}+\frac{(1+r)^i}{f(i)}s^T_{T-i}+\frac{(1+r)^i}{f(i)}\Lambda_{T-i}+P_{T-i-1}+s^P_{T-i}+s^P_{T-i+1}+(1-\frac{f(i-2)}{f(i-1)})s^T_{T-i+1}-\frac{f(i-2)}{f(i-1)}\Lambda_{T-i+1}.$$

The consumption function and it's sum allows me to write down the agent's continuation utility in period $T-i-1$ as follows

$$u(P_{T-i-1}+\frac{(1+r)^i}{f(i)}(1+r)A_{T-i-1})\psi_{T-i-1} = -\frac{1}{\theta}exp\{-\theta(P_{T-i-1}+\frac{(1+r)^i}{f(i)}(1+r)A_{T-i-1})\}\psi_{T-i-1}$$

with ψ_{T-i-1} given by

$$\psi_{T-i-1} = \beta E_{T-i-1}[exp\{-\theta(s^P_{T-i}+(1-\frac{f(i-1)}{f(i)})s^T_{T-i}-\frac{f(i-1)}{f(i)}\Lambda_{T-i})\}+\omega(exp\{-\theta(s^P_{T-i}+(1-\frac{f(i-1)}{f(i)})s^T_{T-i}-\frac{f(i-1)}{f(i)}\Lambda_{T-i})\})$$

$$+\gamma Q_{T-i}\omega(exp\{-\theta(s^P_{T-i}+\frac{(1+r)^i}{f(i)}s^T_{T-i}+\frac{(1+r)^i}{f(i)}\Lambda_{T-i})\})+exp\{-\theta(s^P_{T-i}+\frac{(1+r)^i}{f(i)}s^T_{T-i}+\frac{(1+r)^i}{f(i)}\Lambda_{T-i})\}]\psi_{T-i}$$

and $\omega(x)$ for any random variable $X \sim F_X$, where the realization is denoted by x, is

$$\omega(x) = \eta\int_{-\infty}^{x}(x-y)dF_X(y) + \eta\lambda\int_{x}^{\infty}(x-y)dF_X(y).$$

The above expression for ψ_{T-i-1} can be easily inferred from the agent's utility function. The first component in ψ_{T-i-1} corresponds to the expectation of consumption utility in period $T-i$, the second to contemporaneous gain-loss in period $T-i$, the third to prospective gain-loss in period $T-i$ that depends on the sum of future consumption utilities Q_{T-i}, and the last to the agent's continuation value. Moreover, for any random variable $Y \sim F_Y = F_X$ note that

$$\int_{-\infty}^{\infty}\omega(g(x))dF_X(x) = \int_{-\infty}^{\infty}\{\eta\int_{-\infty}^{x}\underbrace{(g(x)-g(y))}_{<0 \text{ if } g'(\cdot)<0}dF_Y(y)+\eta\lambda\int_{x}^{\infty}\underbrace{(g(x)-g(y))}_{>0 \text{ if } g'(\cdot)<0}dF_Y(y)\}dF_X(x) > 0$$

$$\int_{-\infty}^{\infty}\{\eta\int_{-\infty}^{x}\underbrace{(g(x)-g(y))}_{<0 \text{ if } g'(\cdot)<0}dF_Y(y)+\eta\int_{x}^{\infty}\underbrace{(g(x)-g(y))}_{>0 \text{ if } g'(\cdot)<0}dF_Y(y)+\eta(\lambda-1)\int_{x}^{\infty}\underbrace{(g(x)-g(y))}_{>0 \text{ if } g'(\cdot)<0}dF_Y(y)\}dF_X(x) > 0$$

$$= \int_{-\infty}^{\infty}\{\eta(\lambda-1)\int_{x}^{\infty}\underbrace{(g(x)-g(y))}_{>0 \text{ if } g'(\cdot)<0}dF_Y(y)\}dF_X(x) > 0$$

if $\lambda > 1$, $\eta > 0$, and $g'(\cdot) < 0$. The above consideration implies that $\psi_{T-i-1} > Q_{T-i-1}$ necessarily if $\theta > 0$ such that $u(\cdot)$ is concave. Now, I turn to the agent's maximization problem in period $T-i$, which is given by

$$u(C_{T-i}) + n(C_{T-i}, F^{T-i-1}_{C_{T-i}}) + \gamma\sum_{\tau=1}^{i}\beta^{\tau}\boldsymbol{n}(F^{T-i,T-i-1}_{C_{T-i+\tau}}) + u(P_{T-i} + \frac{(1+r)^{i-1}}{f(i-1)}A_{T-i})\psi_{T-i}.$$

I want to find the agent's first-order condition. I begin by explaining the first derivative of contemporaneous gain-loss utility $n(C_{T-i}, F^{T-i-1}_{C_{T-i}})$. The agent takes his beliefs about period

$T-i$ consumption $F^{T-i-1}_{C_{T-i}}$ as given such that

$$\frac{\partial n(C_{T-i}, F^{T-i-1}_{C_{T-i}})}{\partial C_{T-i}} = \frac{\partial(\eta \int_{-\infty}^{C_{T-i}}(u(C_{T-i}) - u(c))dF^{T-i-1}_{C_{T-i}}(c)) + \eta\lambda \int_{C_{T-i}}^{\infty}(u(C_{T-i}) - u(c))dF^{T-i-1}_{C_{T-i}}(c))}{\partial C_{T-i}}$$

$$= u'(C_{T-i})(\eta F^{T-i-1}_{C_{T-i}}(C_{T-i})+\eta\lambda(1-F^{T-i-1}_{C_{T-i}}(C_{T-i}))) = u'(C_{T-i})(\eta F(s^P_{T-i}+\frac{(1+r)^i}{f(i)}s^T_{T-i})+\eta\lambda(1-F(s^P_{T-i}+\frac{(1+r)^i}{f(i)}s^T_{T-i})))$$

the last step results from the guessed consumption function and the assumption that admissible consumption functions are increasing in both shocks. Here, I abuse notation somewhat by writing $F(\cdot) = F_{S^P_{T-i}+\frac{(1+r)^i}{f(i)}S^T_{T-i}}(\cdot)$. The first derivative of the agent's prospective gain-loss utility $\sum_{\tau=1}^{i}\beta^\tau \boldsymbol{n}(F^{T-i,T-i-1}_{C_{T-i+\tau}})$ over the entire stream of future consumption utilities $u(P_{T-i}+\frac{(1+r)^i}{f(i-1)}A_{T-i})Q_{T-i}$ can be inferred in a similar manner. Recall that Q_{T-i} is a constant under the guessed consumption function; thus, the agent only experiences gain-loss utility over the realized uncertainty in period $T-i$, i.e.,

$$\frac{\partial \sum_{\tau=1}^{\infty}\beta^\tau \boldsymbol{n}(F^{T-i-1,T-i}_{C_{T-i+\tau}})}{\partial A_{T-i}} = \sum_{\tau=1}^{\infty}\beta^\tau \frac{\partial}{\partial A_{T-i}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mu(u(c)-u(r))dF^{T-i-1,T-i}_{C_{T-i+\tau}}(c,r)$$

$$= \frac{\partial}{\partial A_{T-i}}\int_{-\infty}^{\infty}\mu(u(P_{T-i}+\frac{(1+r)^i}{f(i-1)}A_{T-i})Q_{T-i}-u(x)Q_{T-i})dF^{T-i-1}_{P_{T-i}+\frac{(1+r)^i}{f(i-1)}A_{T-i}}(x)$$

$$= \frac{(1+r)^i}{f(i-1)}exp\{-\theta(P_{T-i}+\frac{(1+r)^i}{f(i-1)}A_{T-i})\}Q_{T-i}(\eta F(s^P_{T-i}+\frac{(1+r)^i}{f(i)}s^T_{T-i})+\eta\lambda(1-F(s^P_{T-i}+\frac{(1+r)^i}{f(i)}s^T_{T-i})))$$

and again, $F(s^P_{T-i}+\frac{(1+r)^i}{f(i)}s^T_{T-i})$ results from the solution guess for A_{T-i} times $\frac{(1+r)^i}{f(i-1)}$ and the fact that future consumption is increasing in both shocks. The derivative of the agent's continuation utility with respect to A_{T-i} is simply given by

$$\frac{(1+r)^i}{f(i-1)}exp\{-\theta\frac{(1+r)^i}{f(i-1)}A_{T-i}\}\psi_{T-i}.$$

In turn, in any period $T-i$ the news-utility agent's first-order condition (normalized by P_{T-i}) is given by

$$exp\{\underbrace{-\theta((1+r)A_{T-i-1}+s^T_{T-i}-A_{T-i})}_{=-\theta(C_{T-i}-P_{T-i})\text{ budget constraint}}\}(1+\eta F(s^P_{T-i}+\frac{(1+r)^i}{f(i)}s^T_{T-i})+\eta\lambda(1-F(s^P_{T-i}+\frac{(1+r)^i}{f(i)}s^T_{T-i})))$$

$$= \frac{(1+r)^i}{f(i-1)}exp\{-\theta\frac{(1+r)^i}{f(i-1)}A_{T-i}\}(\psi_{T-i}+\gamma Q_{T-i}(\eta F(s^P_{T-i}+\frac{(1+r)^i}{f(i)}s^T_{T-i})+\eta\lambda(1-F(s^P_{T-i}+\frac{(1+r)^i}{f(i)}s^T_{T-i}))).$$

The first-order condition can be rewritten to obtain the optimal consumption and end-of-

period asset holdings functions and the function Λ_{T-i}

$$\Lambda_{T-i} = \frac{1}{\theta} log(\frac{(1+r)^i}{f(i-1)} \frac{\psi_{T-i} + \gamma Q_{T-i}(\eta F(s^P_{T-i} + \frac{(1+r)^i}{f(i)} s^T_{T-i}) + \eta\lambda(1 - F(s^P_{T-i} + \frac{(1+r)^i}{f(i)} s^T_{T-i}))}{1 + \eta F(s^P_{T-i} + (1 - \frac{f(i-1)}{f(i)}) s^T_{T-i}) + \eta\lambda(1 - F(s^P_{T-i} + (1 - \frac{f(i-1)}{f(i)}) s^T_{T-i}))})$$

and the guessed consumption function can be verified.

B.2.2 The infinite-horizon model

Suppose $\sigma_{Pt} = \sigma_P$ and $\sigma_{Tt} = \sigma_T$ for all t and $T, i \to \infty$. I use a simple guess and verify procedure to find the infinite-horizon recursive equilibrium; alternatively, the solution can be obtained by simple backward induction taking T and i to infinity. The infinite-horizon model consumption and asset-holding functions are given by

$$C_t = Y_t + rA_{t-1} - \frac{1}{1+r} s^T_t - \Lambda_t = P_{t-1} + s^P_t + rA_{t-1} + \frac{r}{1+r} s^T_t - \Lambda_t \text{ and } A_t = A_{t-1} + \frac{1}{1+r} s^T_t + \Lambda_t.$$

The first-order condition normalized by P_t is given by

$$exp\{-\theta(1+r)A_{t-1} - \theta s^T_t + \theta A_t\}(1 + \eta F(s^P_t + \frac{r}{1+r} s^T_t) + \eta\lambda(1 - F(s^P_t + \frac{r}{1+r} s^T_t)))$$

$$= rexp\{-\theta r A_t\}(\psi + \gamma Q(\eta F(s^P_t + \frac{r}{1+r} s^T_t) + \eta\lambda(1 - F(s^P_t + \frac{r}{1+r} s^T_t))).$$

Solving for optimal end-of-period asset holdings yields

$$A_t = A_{t-1} + \frac{1}{1+r} s^T_t + \underbrace{\frac{1}{\theta(1+r)} log(r \frac{\psi + \gamma Q(\eta F(s^P_t + \frac{r}{1+r} s^T_t) + \eta\lambda(1 - F(s^P_t + \frac{r}{1+r} s^T_t)))}{1 + \eta F(s^P_t + \frac{r}{1+r} s^T_t) + \eta\lambda(1 - F(s^P_t + \frac{r}{1+r} s^T_t))})}_{=\Lambda_t}.$$

Consumption is then determined by the budget constraint

$$C_t = Y_t + rA_{t-1} - \frac{1}{1+r} s^T_t - \Lambda_t = P_{t-1} + s^P_t + rA_{t-1} + \frac{r}{1+r} s^T_t - \Lambda_t.$$

Q and ψ are constant in an i.i.d. environment and given by

$$Q = \frac{\beta E_t[exp\{-\theta(s^P_{t+1} + \frac{r}{1+r} s^T_{t+1} - \Lambda_{t+1})\}]}{1 - \beta E_t[exp\{-\theta(s^P_{t+1} + \frac{r}{1+r} s^T_{t+1} + r\Lambda_{t+1})\}]}$$

$$\psi = \frac{\beta E_t[exp\{-\theta(s^P_{t+1} + \frac{r}{1+r} s^T_{t+1} - \Lambda_{t+1})\} + \omega(exp\{-\theta(s^P_{t+1} + \frac{r}{1+r} s^T_{t+1} - \Lambda_{t+1})\}) + \gamma Q\omega(exp\{-\theta(s^P_{t+1} + \frac{r}{1+r} s^T_{t+1} + r\Lambda_{t+1})\})]}{1 - \beta E_t[exp\{-\theta(s^P_{t+1} + \frac{r}{1+r} s^T_{t+1} + r\Lambda_{t+1})\}]}.$$

B.2.3 The optimal pre-committed equilibrium

Suppose the agent has the ability to pick an optimal history-dependent consumption path

for each possible future contingency in period zero when he does not experience any gain-loss utility. Thus, in period zero the agent chooses optimal consumption in period t in each possible contingency jointly with his beliefs, which of course coincide with the agent's optimal state-contingent plan. For instance, consider the joint optimization over consumption and beliefs for $C(Y^*)$ when income Y^* has been realized

$$\frac{\partial}{\partial C(Y^*)}\{\int\int \mu(u(C(Y))-u(C(Y')))dF_Y(Y')dF_Y(Y)\}$$

$$= \frac{\partial}{\partial C(Y^*)}\int \eta \int_{-\infty}^{Y}\{(u(C(Y))-u(C(Y')))dF_Y(Y')+\eta\lambda\int_{Y}^{\infty}(u(C(Y))-u(C(Y')))dF_Y(Y')\}dF_Y(Y)$$

$$= u'(C(Y^*))(\eta F_Y(Y^*)+\eta\lambda(1-F_Y(Y^*))) - u'(C(Y^*))(\eta(1-F_Y(Y^*))+\eta\lambda F_Y(Y^*))$$

$$= u'(C(Y^*))\eta(\lambda-1)(1-2F_Y(Y^*)) \text{ with } \eta(\lambda-1)(1-2F_Y(Y^*))>0 \text{ for } F_Y(Y^*)<0.5.$$

Consider the difference to the term in the initial first-order condition $u'(C_t)(\eta F_{C_t}^{t-1}(C_t)+\eta\lambda(1-F_{C_t}^{t-1}(C_t)))$: when choosing the pre-committed plan the additional utility of increasing consumption a little bit is no longer only made up of the additional step in the probability distribution; instead the two additional negative terms consider that in all other states of the world the agent experiences less gain feelings and more loss feelings because of increasing consumption in that contingency. The equation says that the marginal utility of state Y^* will be increased by news utility if the realization is below the median. For realizations above the median the marginal utility will be decreased and the agent will consume relatively less.

Unfortunately there is a problem arising in the pre-commitment optimization problem that has been absent in the non-pre-committed one: When beliefs are taken as given the agent optimizes over two concave functions consumption utility and the first part of gain-loss utility, accordingly the first-order condition pins down a maximum. In contrast, when the agent chooses his beliefs simultaneously to his consumption additionally the second convex part of gain-loss utility is optimized over. The additional part determining marginal utility $-u'(C_t)(\eta(1-F_{C_t}^{t-1}(C_t)))+\eta\lambda F_{C_t}^{t-1}(C_t)))$ is largest for particular good income realizations, since increasing consumption in these states implies additional loss feelings in almost all other states of the world. It can be easily shown that the sufficient condition of the optimization problem holds if the parameters satisfy following simple condition: $\eta(\lambda-1)(2F_{C_t}^{t-1}(C_t)-1)<1$. Accordingly, for $\eta(\lambda-1)<1$, which is true for a range of commonly used parameter combinations, the first-order condition pins down the optimum.

From the above consideration it can be easily inferred that the optimal pre-committed consumption function in the exponential-utility model is thus given by

$$\Lambda_{T-i}^c = \frac{1}{\theta}log(\frac{(1+r)^i}{f(i-1)}\frac{\psi_{T-i}^c+\gamma Q_{T-i}^c\eta(\lambda-1)(1-2F(s_{T-i}^P+\frac{(1+r)^i}{f(i)}s_{T-i}^T))}{1+\eta(\lambda-1)(1-2F(s_{T-i}^P+\frac{(1+r)^i}{f(i)}s_{T-i}^T))})$$

with

$$Q_{T-i-1}^c = E_{T-i-1}[\beta exp\{-\theta(s_{T-i}^P+(1-\frac{f(i-1)}{f(i)})s_{T-i}^T-\frac{f(i-1)}{f(i)}\Lambda_{T-i}^c)\}+\beta exp\{-\theta(s_{T-i}^P+\frac{(1+r)^i}{f(i)}s_{T-i}^T+\frac{(1+r)^i}{f(i)}\Lambda_{T-i}^c)\}Q_{T-i}^c]$$

and

$$\psi^c_{T-i-1} = \beta E_{T-i-1}[exp\{-\theta(s^P_{T-i}+(1-\frac{f(i-1)}{f(i)})s^T_{T-i}-\frac{f(i-1)}{f(i)}\Lambda^c_{T-i})\}+\omega(exp(-\theta(s^P_{T-i}+(1-\frac{f(i-1)}{f(i)})s^T_{T-i}-\frac{f(i-1)}{f(i)}\Lambda^c_{T-i})\})$$

$$+\gamma Q^c_{T-i}\omega(exp\{-\theta(s^P_{T-i}+\frac{(1+r)^i}{f(i)}s^T_{T-i}+\frac{(1+r)^i}{f(i)}\Lambda^c_{T-i})\})+\frac{1}{\theta}exp\{-\theta(s^P_{T-i}+\frac{(1+r)^i}{f(i)}s^T_{T-i}+\frac{(1+r)^i}{f(i)}\Lambda^c_{T-i})\}\psi^c_{T-i}.$$

B.3 The other agent's exponential-utility consumption functions

By the same arguments as for the derivation of the news-utility model, the "standard" agent's consumption function in period $T-i$ is

$$A^s_{T-i} = \frac{f(i-1)}{f(i)}(1+r)A^s_{T-i-1} + \frac{f(i-1)}{f(i)}s^T_{T-i} + \frac{f(i-1)}{f(i)}\Lambda^s_{T-i}$$

$$C^s_{T-i} = \frac{(1+r)^i}{f(i)}(1+r)A^s_{T-i-1} + P_{T-i-1} + s^P_{T-i} + (1-\frac{f(i-1)}{f(i)})s^T_{T-i} - \frac{f(i-1)}{f(i)}\Lambda^s_{T-i}$$

$$\Lambda^s_{T-i} = \frac{1}{\theta}log(\frac{(1+r)^i}{f(i-1)}Q^s_{T-i})$$

$$Q^s_{T-i-1} = \beta E_{T-i-1}[exp\{-\theta(s^P_{T-i}+(1-\frac{f(i-1)}{f(i)})s^T_{T-i}-\frac{f(i-1)}{f(i)}\Lambda^s_{T-i})\}+\beta exp\{-\theta(s^P_{T-i}+\frac{(1+r)^i}{f(i)}s^T_{T-i}+\frac{(1+r)^i}{f(i)}\Lambda^s_{T-i})\}Q^s_{T-i}].$$

In the infinite-horizon equilibrium $Q^s = \psi^s$ in an i.i.d. environment with $\sigma_{Pt} = \sigma_P$ and $\sigma_{Tt} = \sigma_T$ for all t

$$A^s_t = A^s_{t-1} + \frac{1}{1+r}s^T_t + \underbrace{\frac{1}{\theta(1+r)}log(r\psi^s)}_{=\Lambda^s}$$

$$\psi^s = Q^s = \frac{\beta E_t[exp\{-\theta(s^P_{t+1} + \frac{r}{1+r}s^T_{t+1} - \Lambda^s)\}]}{1-\beta E_t[exp\{-\theta(s^P_{t+1} + \frac{r}{1+r}s^T_{t+1} + r\Lambda^s)\}]}$$

$$C^s_t = Y_t + rA^s_{t-1} - \frac{1}{1+r}s^T_t - \Lambda^s = P_t + s^P_t + rA^s_{t-1} + \frac{r}{1+r}s^T_t - \Lambda^s.$$

The "tempted" agent's maximization problem is given by

$$max_{C^{td}_t}\{u(C^{td}_t) - \lambda^{td}(u(\tilde{C}^{td}_t) - u(C^{td}_t)) + E_t[\sum_{\tau=1}^{T-t}\beta^\tau(u(C^{td}_{t+\tau}) - \lambda^{td}(u(\tilde{C}^{td}_{t+\tau}) - u(C^{td}_{t+\tau})))]\}$$

with $\tilde{C}^{td}_{t+\tau}$ being the most tempting alternative. In period T as the agent cannot die in debt the most tempting alternative is $\tilde{C}^{td}_T = X^{td}_T$ but the agent will consume X_T anyway thus temptation disutility is zero and $Q^{td}_{T-1} = Q^s_{T-1}$. In period $T-1$ the agent's consumption is then given by

$$A^{td}_{T-1} = \frac{1+r}{2+r}A^{td}_{T-2} + \frac{1}{2+r}s^T_{T-1} + \frac{1}{2+r}\Lambda^{td}_{T-1}$$

$$C^{td}_{T-1} = \frac{1+r}{2+r}(1+r)A^{td}_{T-2} + P_{T-2} + s^P_{T-1} + \frac{1+r}{2+r}s^T_{T-1} - \frac{1}{2+r}\Lambda^{td}_{T-1}$$

with $\Lambda^{td}_{T-1} = \frac{1}{\theta}log((1+r)\frac{1}{1+\lambda^{td}}Q^{td}_{T-1})$ and $Q^{td}_{T-1} = \beta E_{T-1}[exp\{-\theta(s^P_T + s^T_T)\}]$.

What's the agent's most tempting alternative in period $T-1$? The value of cash-on-hand is X^{td}_{T-1} but the most tempting alternative is $\tilde{C}^{td}_T \to \infty$ as consumption could be negative in the last period $C_T \to -\infty$, which would yield $lim_{C^{td}_T \to -\infty} u(C^{td}_T) = lim_{C^{td}_T \to -\infty} - \frac{1}{\theta}e^{-\theta C^{td}_T} \to -\infty$. Accordingly, Q^{td}_{T-2} enters $lim_{\tilde{C}^{td}_T \to -\infty} u'(\tilde{C}^{td}_T) = lim_{\tilde{C}^{td}_T \to -\infty} e^{-\theta \tilde{C}^{td}_T} \to 0$

$$Q^{td}_{T-2} = \beta E_{T-2}[exp\{-\theta(s^P_{T-1} + \frac{1+r}{2+r}s^T_{T-1} - \frac{1}{2+r}\Lambda^{td}_{T-1})\} - \lambda^{td}(exp\{-\theta(s^P_{T-1} + \frac{1+r}{2+r}s^T_{T-1} - \frac{1}{2+r}\Lambda^{td}_{T-1})\}$$

$$\underbrace{- exp\{\theta\frac{1+r}{2+r}(1+r)A^{td}_{T-2} - \theta\tilde{C}^{td}_T\})}_{\to 0} + exp\{-\theta(s^P_{T-1} + \frac{1+r}{2+r}s^T_{T-1} + \frac{1+r}{2+r}\Lambda^{td}_{T-1})\}Q^{td}_{T-1}]$$

$$Q^{td}_{T-2} = E_{T-2}[\beta exp\{-\theta(s^P_{T-1} + \frac{1+r}{2+r}s^T_{T-1} - \frac{1}{2+r}\Lambda^{td}_{T-1})\}(1-\lambda^{td}) + \beta exp\{-\theta(s^P_{T-1} + \frac{1+r}{2+r}s^T_{T-1} + \frac{1+r}{2+r}\Lambda^{td}_{T-1})\}Q^{td}_{T-1}].$$

And in period $T-i$

$$A^{td}_{T-i} = \frac{f(i-1)}{f(i)}(1+r)A^{td}_{T-i-1} + \frac{f(i-1)}{f(i)}s^T_{T-i} + \frac{f(i-1)}{f(i)}\Lambda^{td}_{T-i}$$

$$C^{td}_{T-i} = \frac{(1+r)^i}{f(i)}(1+r)A^{td}_{T-i-1} + P_{T-i-1} + s^P_{T-1} + (1 - \frac{f(i-1)}{f(i)})s^T_{T-i} - \frac{f(i-1)}{f(i)}\Lambda^{td}_{T-i}$$

$$\Lambda^{td}_{T-i} = \frac{1}{\theta}log(\frac{(1+r)^i}{f(i-1)}\frac{1}{1+\lambda^{td}}Q^{td}_{T-i})$$

$$Q^{td}_{T-i-1} = \beta E_{T-i-1}[exp\{-\theta(s^P_{T-1} + (1 - \frac{f(i-1)}{f(i)})s^T_{T-i} - \frac{f(i-1)}{f(i)}\Lambda^{td}_{T-i})\}(1-\lambda^{td}) + \beta exp\{-\theta(s^P_{T-1} + \frac{(1+r)^i}{f(i)}s^T_{T-i} + \frac{(1+r)^i}{f(i)}\Lambda^{td}_{T-i})\}Q^{td}_{T-i}].$$

And for $T \to \infty$

$$A^{td}_t = A^{td}_{t-1} + \frac{1}{1+r}n_t + \underbrace{\frac{1}{\theta(1+r)}log(r\frac{1}{1+\lambda^{td}}Q^{td})}_{=\Lambda^{td}}$$

$$C^{td}_t = Y_t + rA^{td}_{t-1} - \frac{1}{1+r}n_t - \Lambda^{td} = P_{t-1} + s^P_t + rA^{td}_{t-1} + \frac{r}{1+r}s^T_t - \Lambda^{td}$$

$$Q^{td} = \frac{\beta E_t[exp\{-\theta(s^P_{t+1} + \frac{r}{1+r}s^T_{t+1} - \Lambda^{td})\}(1-\lambda^{td})]}{1 - \beta E_t[exp\{-\theta(s^P_{t+1} + \frac{r}{1+r}s^T_{t+1} + r\Lambda^{td})\}]}.$$

The "hyperbolic-discounting" agent's consumption in period $T-i$ is

$$A^b_{T-i} = \frac{f(i-1)}{f(i)}(1+r)A^b_{T-i-1} + \frac{f(i-1)}{f(i)}s^T_{T-i} + \frac{f(i-1)}{f(i)}\Lambda^b_{T-i}$$

$$C^b_{T-i} = \frac{(1+r)^i}{f(i)}(1+r)A^b_{T-i-1} + P_{T-i-1} + s^P_{T-i} + (1 - \frac{f(i-1)}{f(i)})s^T_{T-i} - \frac{f(i-1)}{f(i)}\Lambda^b_{T-i}$$

$$\Lambda^b_{T-i} = \frac{1}{\theta} log(\frac{(1+r)^i}{f(i-1)} bQ^b_{T-i})$$

$$Q^b_{T-i-1} = \beta E_{T-i-1}[exp\{-\theta(s^P_{T-i}+(1-\frac{f(i-1)}{f(i)})s^T_{T-i}-\frac{f(i-1)}{f(i)}\Lambda^b_{T-i})\}+\beta exp\{-\theta(s^P_{T-i}+\frac{(1+r)^i}{f(i)}s^T_{T-i}+\frac{(1+r)^i}{f(i)}\Lambda^b_{T-i})\}Q^b_{T-i}].$$

and for $T \to \infty$

$$A^b_t = A^b_{t-1} + \frac{1}{1+r}s^T_t + \underbrace{\frac{1}{\theta(1+r)} log(rbQ^b)}_{=\Lambda^b}$$

$$C^b_t = Y_t + rA^b_{t-1} - \frac{1}{1+r}s^T_t - \Lambda^b = P_{t-1} + s^P_t + rA^b_{t-1} + \frac{r}{1+r}s^T_t - \Lambda^b$$

$$Q^b = \frac{\beta E_t[exp\{-\theta(s^P_{t+1} + \frac{r}{1+r}s^T_{t+1} - \Lambda^b)\}]}{1 - \beta E_t[exp\{-\theta(s^P_{t+1} + \frac{r}{1+r}s^T_{t+1} + r\Lambda^b)\}]}.$$

B.4 Proofs of Section IV:

B.4.1 Proof of Proposition 1

If the consumption function derived in Section B.2.1 belongs to the class of admissible consumption functions then the equilibrium exists and is unique as the equilibrium solution is obtained by maximizing the agent's objective function, which is globally concave, and there is a finite period that uniquely determines the equilibrium. Please refer to Section B.2.1 for the derivation of the consumption function. σ^*_t is implicitly defined by the two admissible consumption function restrictions $\frac{\partial C_{T-i}}{\partial s^P_{T-i}} > 0$ and $\frac{\partial C_{T-i}}{\partial s^T_{T-i}} > 0$ as

$$C_{T-i} = \frac{(1+r)^i}{f(i)}(1+r)A_{T-i-1} + P_{T-i-1} + s^P_{T-i} + (1-a(i))s^T_{T-i} - a(i)\Lambda_{T-i}$$

the restrictions are equivalent to $\frac{\partial a(i)\Lambda_{T-i}}{\partial s^P_{T-i}} < 1$ and $\frac{\partial a(i)\Lambda_{T-i}}{\partial s^T_{T-i}} < 1 - a(i)$ as $\frac{\partial \Lambda_{T-i}}{\partial s^P_{T-i}}, \frac{\partial \Lambda_{T-i}}{\partial s^T_{T-i}} > 0$ (since $\psi_{T-i} > \gamma Q_{T-i}$ (for any concave utility function which I have shown in Section B.2)). Recall that $a(i) = 1 - \frac{(1+r)^i}{f(i)} = \frac{f(i-1)}{f(i)}$. Then, σ^*_{T-i} is implicitly defined by the two restrictions

$$\frac{\partial a(i)\Lambda_{T-i}}{\partial s^P_{T-i}} = \frac{a(i)}{\theta(\frac{1-a(i)}{a(i)})} \frac{\frac{(\psi_{T-i}-\gamma Q_{T-i})\eta f_{s^P_{T-i}+\frac{(1+r)^i}{f(i)}s^T_{T-i}}(s^P_{T-i}+\frac{(1+r)^i}{f(i)}s^T_{T-i})(\lambda-1)}{1+\eta F(s^P_{T-i}+\frac{(1+r)^i}{f(i)}s^T_{T-i})+\eta\lambda(1-F(s^P_{T-i}+\frac{(1+r)^i}{f(i)}s^T_{T-i}))}}{\psi_{T-i} + \gamma Q_{T-i}(\eta F(s^P_{T-i} + \frac{(1+r)^i}{f(i)}s^T_{T-i}) + \eta\lambda(1 - F(s^P_{T-i} + \frac{(1+r)^i}{f(i)}s^T_{T-i})))} < 1$$

and

$$\frac{\partial a(i)\Lambda_{T-i}}{\partial s^T_{T-i}} = \frac{a(i)}{\theta(\frac{1-a(i)}{a(i)})} \frac{\frac{(\psi_{T-i}-\gamma Q_{T-i})\eta f_{s^P_{T-i}+\frac{(1+r)^i}{f(i)}s^T_{T-i}}(s^P_{T-i}+\frac{(1+r)^i}{f(i)}s^T_{T-i})(\lambda-1)}{1+\eta F(s^P_{T-i}+\frac{(1+r)^i}{f(i)}s^T_{T-i})+\eta\lambda(1-F(s^P_{T-i}+\frac{(1+r)^i}{f(i)}s^T_{T-i}))}}{\psi_{T-i} + \gamma Q_{T-i}(\eta F(s^P_{T-i} + \frac{(1+r)^i}{f(i)}s^T_{T-i}) + \eta\lambda(1 - F(s^P_{T-i} + \frac{(1+r)^i}{f(i)}s^T_{T-i})))} < 1-a(i).$$

Here, the normal pdf of any random variably X is denoted by f_X. Increasing σ_{Pt} and

σ_{Tt} unambiguously decreases $f_{S^P_{T-i}+\frac{(1+r)^i}{f(i)}S^T_{T-i}}(s^P_{T-i}+\frac{(1+r)^i}{f(i)}s^T_{T-i})$ and thereby $\frac{\partial a(i)\Lambda_{T-i}}{\partial s^P_{T-i}}$ and $\frac{\partial a(i)\Lambda_{T-i}}{\partial s^T_{T-i}}$. Thus, there exists a condition $\sigma^2_{Pt}+(\frac{(1+r)^i}{f(i)})^2\sigma^2_{Tt}\geq\sigma^*_t$ for all t which ensures that an admissible consumption function exists that uniquely determines the equilibrium (given the admissible consumption functions in each future period until the final period) because the optimization problem is globally concave.

If uncertainty is small then the consumption function may be decreasing over some range, i.e., $\frac{\partial C_{T-i}}{\partial s^P_{T-i}}<0$ or $\frac{\partial C_{T-i}}{\partial s^T_{T-i}}<0$. I now show that the agent would pick a consumption function that is instead of decreasing flat and thus weakly increasing in the shock realizations, i.e., $\frac{\partial C_{T-i}}{\partial s^P_{T-i}}\geq 0$ or $\frac{\partial C_{T-i}}{\partial s^T_{T-i}}\geq 0$. To discuss this result in a simple framework, I return to the two-period, one-shock model. Suppose that the absolute level of the shock increases; then, holding C_{T-1} constant, the marginal value of savings declines and the agent's first-order condition implies that consumption should increase. However, $F_P(s^P_{T-1})$ also increases, and marginal gain-loss utility is lower, such that the agent's optimal consumption should decrease. Suppose that s^P_{T-1} increases marginally but $F_P(s^P_{T-1})$ increases sharply, which could occur if F_P is a very narrow distribution. In this case, the lower marginal gain-loss utility that decreases consumption dominates such that the first-order condition predicts decreasing consumption over some range in the neighborhood of the expected value μ_P where F_P increases most sharply if F_P is bell shaped. However, a decreasing consumption function cannot be an equilibrium because the agent would unnecessarily experience gain-loss utility over the decreasing part of consumption, which decreases expected utility unnecessarily. In the decreasing-consumption function region, the agent could choose a flat consumption function instead. In the following I show that the agent may choose a credible consumption plan with a flat section. Suppose the agent chooses a flat consumption level for realizations of s^P_{T-1} in $\underline{s}$ and $\overline{s}$. Then, $\overline{s}$ is chosen where the original consumption function just stops decreasing, which corresponds to the lowest possible level of the flat section of consumption $\overline{C}_{T-1}$. In that is then determined by

$$u'(\overline{C}_{T-1})=(1+r)u'((\overline{s}-\overline{C}_{T-1})(1+r)+\overline{s})\frac{\psi_{T-1}+\gamma Q_{T-1}(\eta F_P(\overline{s})+\eta\lambda(1-F_P(\overline{s})))}{1+\eta F_P(\overline{s})+\eta\lambda(1-F_P(\overline{s}))}$$

in which case $\underline{s}$ is determined by

$$u'(\overline{C}_{T-1})=(1+r)u'((\underline{s}-\overline{C}_{T-1})(1+r)+\underline{s})\frac{\psi_{T-1}+\gamma Q_{T-1}(\eta F_P(\underline{s})+\eta\lambda(1-F_P(\underline{s})))}{1+\eta F_P(\underline{s})+\eta\lambda(1-F_P(\underline{s}))}.$$

The agent's consistency constraint for not increasing consumption beyond $\overline{C}_{T-1}$ for any

$s^P_{T-1} \in [\underline{s}, \overline{s}]$ is given by

$$u'(\overline{C}_{T-1}) < (1+r)u'((s^P_{T-1}-\overline{C}_{T-1})(1+r)+s^P_{T-1})\frac{\psi_{T-1}+\gamma Q_{T-1}(\eta F_P(s^P_{T-1})+\eta\lambda(1-F_P(s^P_{T-1})))}{1+\eta F_P(\overline{s})+\eta\lambda(1-F_P(\overline{s}))}$$

and always holds as can be easily inferred. This result can be easily generalized to any horizon, thus, if the consumption function is decreasing over some range, the agent can credibly replace the decreasing part with a flat section as described above.

B.4.2 Proof of Proposition 2

Please refer to the derivation of the exponential-utility model Section B.2 for a detailed derivation of Λ_{T-i}. According to Definition 5 consumption is excessively smooth if $\frac{\partial C_t}{\partial s^P_t} < 1$ and excessively sensitive if $\frac{\partial \Delta C_{t+1}}{\partial s^P_t} > 0$. Consumption growth is

$$\Delta C_{T-i} = s^P_{T-i} + (1-a(i))s^T_{T-i} - a(i)\Lambda_{T-i} + \Lambda_{T-i-1}$$

so that $\frac{\partial C_{T-i}}{\partial s^P_{T-i}} < 1$ iff $\frac{\partial \Lambda_{T-i}}{\partial s^P_{T-i}} > 0$ and $\frac{\partial \Delta C_{T-i}}{\partial s^P_{T-i-1}} > 0$ iff $\frac{\partial \Lambda_{T-i-1}}{\partial s^P_{T-i-1}} > 0$. Since $\psi_{T-i} > \gamma Q_{T-i}$ (for any concave utility function which I have shown in Section B.2) it can be easily seen that $\frac{\partial \Lambda_{T-i}}{\partial s^P_{T-i}} > 0$, i.e.

$$\frac{\partial \Lambda_{T-i}}{\partial s^P_{T-i}} = \frac{1}{\theta(\frac{1-a(i)}{a(i)})}\frac{\frac{(\psi_{T-i}-\gamma Q_{T-i})\eta f_{s^P_{T-i}+\frac{(1+r)^i}{f(i)}s^T_{T-i}}(s^P_{T-i}+\frac{(1+r)^i}{f(i)}s^T_{T-i})(\lambda-1)}{1+\eta F(s^P_{T-i}+\frac{(1+r)^i}{f(i)}s^T_{T-i})+\eta\lambda(1-F(s^P_{T-i}+\frac{(1+r)^i}{f(i)}s^T_{T-i}))}}{\psi_{T-i}+\gamma Q_{T-i}(\eta F(s^P_{T-i}+\frac{(1+r)^i}{f(i)}s^T_{T-i})+\eta\lambda(1-F(s^P_{T-i}+\frac{(1+r)^i}{f(i)}s^T_{T-i})))} > 0.$$

The same holds true for the infinite-horizon model

$$\Delta C_t = s^P_t + \frac{r}{1+r}s^T_t - \Lambda_t + (1+r)\Lambda_{t-1}$$

as Λ_t is increasing in the permanent shock

$$\frac{\partial \Lambda_t}{\partial s^P_t} = \frac{1}{\theta(1+r)r}\frac{\frac{(\psi-\gamma Q)\eta f_{s^P_t+\frac{r}{1+r}s^T_t}(s^P_t+\frac{r}{1+r}s^T_t)(\lambda-1)}{1+\eta F(s^P_t+\frac{r}{1+r}s^T_t)+\eta\lambda(1-F(s^P_t+\frac{r}{1+r}s^T_t))}}{\psi+\gamma Q(\eta F(s^P_t+\frac{r}{1+r}s^T_t)+\eta\lambda(1-F(s^P_t+\frac{r}{1+r}s^T_t)))} > 0.$$

Accordingly, $\frac{\partial \Lambda_t}{\partial s^P_t} > 0$ as $\psi > \gamma Q$. Thus, if $s^P_t \uparrow$ then $\Lambda_t \uparrow$ and the shock induced change in consumption is less than one and the period t shock induced change in one-period ahead consumption ΔC_{t+1} is larger than zero.

The difference to the standard, tempted, and quasi-hyperbolic discounting agents is that $\frac{\partial \Lambda^{s,t,b}_t}{\partial s^P_t} = 0$ for all t such that consumption is neither excessively sensitive nor excessively smooth.

B.4.3 Proof of Lemma 1

I start with the first part of the lemma, the precautionary-savings motive. In the second-

to-last period of the simple model outlined in the text, the first-order condition is given by

$$u'(C_{T-1}) + u'(C_{T-1})(\eta F_P(s^P_{T-1}) + \eta\lambda(1 - F_P(s^P_{T-1}))))$$
$$= (1+r)u'((s^P_{T-1} - C_{T-1})(1+r) + s^P_{T-1})\gamma \underbrace{\beta E_{T-1}[u'(S^P_T)]}_{Q_{T-1}}(\eta F_P(s^P_{T-1}) + \eta\lambda(1 - F_P(s^P_{T-1})))$$

$$+(1+r)u'((s^P_{T-1}-C_{T-1})(1+r)+s^P_{T-1})\underbrace{\beta E_{T-1}[u'(S^P_T) + \eta(\lambda-1)\int_{S^P_T}^{\infty}(u'(S^P_T) - u'(y))dF_P(y)]}_{\psi_{T-1}}.$$

From Section B.2 I know that $\psi_{T-1} > Q_{T-1}$ because for any two random variables $X \sim F_X$ and $Y \sim F_Y$ with $F_X = F_Y$ in equilibrium

$$\int_{-\infty}^{\infty}\{\eta\int_{-\infty}^{x}\underbrace{(g(x) - g(y))}_{<0 \text{ if } g'(\cdot)<0}dF_Y(y) + \eta\lambda\int_{x}^{\infty}\underbrace{(g(x) - g(y))}_{>0 \text{ if } g'(\cdot)<0}dF_Y(y)\}dF_X(x) > 0$$

if $\lambda > 1$, $\eta > 0$, and $g'(\cdot) < 0$. Thus, $\beta E_{T-1}[\eta(\lambda-1)\int_{S^P_T}^{\infty}(u'(S^P_T) - u'(y))dF_P(y)] > 0$ if $u''(\cdot) > 0$ the agent is risk averse or $u(\cdot)$ is concave. Moreover, it can be easily seen that $\frac{\partial\beta E_{T-1}[\eta(\lambda-1)\int_{S^P_T}^{\infty}(u'(S^P_T)-u'(y))dF_P(y)]}{\partial\eta} > 0$ and $\frac{\partial\beta E_{T-1}[\eta(\lambda-1)\int_{S^P_T}^{\infty}(u'(S^P_T)-u'(y))dF_P(y)]}{\partial\lambda} > 0$. Then, for any value of savings $A_{T-1} = s^P_{T-1} - C_{T-1}$ the right hand side of the first-order condition is increased by the presence of expected gain-loss disutility if $\sigma_P > 0$ whereas if $\sigma_P = 0$ then $\psi_{T-1} = Q_{T-1}$. The increase of the agent's marginal value of savings by the presence of expected gain-loss disutility depends on $\sigma_P > 0$, but does not go to zero as $\sigma_P \to 0$ so that the additional precautionary savings motive is first-order $\frac{\partial(s^P_{T-1}-C_{T-1})}{\partial\sigma_P}|_{\sigma_P=0} > 0$ as can be easily shown for any normally distribution random variable $X \sim F_X = N(\mu, \sigma^2)$

$$E_{T-1}[\eta(\lambda-1)\int_{X}^{\infty}(u'(X) - u'(y))dF_X(y)]$$

$$= e^{-\theta\mu}\int_{-\infty}^{\infty}(\eta\int_{-\infty}^{z}(e^{-\theta\sigma z} - e^{-\theta\sigma\varepsilon})dF_{01}(\varepsilon) + \eta\lambda\int_{z}^{\infty}(e^{-\theta\sigma z} - e^{-\theta\sigma\varepsilon})dF_{01}(\varepsilon))dF_{01}(z) \text{ with } z, \varepsilon \sim F_{01} = N(0,1)$$

$$= e^{-\theta\mu}\eta(\lambda-1)\int_{z}^{\infty}(e^{-\theta\sigma z} - e^{-\theta\sigma\varepsilon})dF_{01}(\varepsilon))dF_{01}(z)$$

$$= e^{-\theta\mu}\eta(\lambda-1)\int_{-\infty}^{\infty}\{(1 - F_{01}(z))e^{-\theta\sigma z} - e^{\frac{1}{2}\theta^2\sigma^2}F_{01}(-\theta\sigma - z)\}dF_{01}(z)$$

$$\frac{\partial(\cdot)}{\partial\sigma}|_{\sigma=0} = e^{-\theta\mu}\eta(\lambda-1)\int_{-\infty}^{\infty}\{-\theta z(1-F_{01}(z))e^{-\theta\sigma z} - \theta\sigma e^{\frac{1}{2}\theta^2\sigma^2}F_{01}(-\theta\sigma-z) + \theta e^{\frac{1}{2}\theta^2\sigma^2}F_{01}(-\theta\sigma-z)\}dF_{01}(z)|_{\sigma=0}$$

$$= e^{-\theta\mu}\theta\eta(\lambda-1)\int_{-\infty}^{\infty}\{-z + zF_{01}(z) + F_{01}(-z)\}dF_{01}(z) \approx e^{-\theta\mu}\theta\eta(\lambda-1)0.7832 > 0.$$

Thus, news-utility introduces a first-order precautionary-savings motive.

In the second part of the lemma the implications for consumption can be immediately seen by comparing the agents' first-order conditions. The standard agent's first-order condition in period $T-1$ is given by

$$u'(C_{T-1}) = Ru'((s^P_{T-1} - C_{T-1})R + s^P_{T-1})Q_{T-1}.$$

The difference to the news-utility model can be seen easily: First, $\frac{\psi_{T-1}}{Q_{T-1}} > 1$ implies that

$$\frac{\frac{\psi_{T-1}}{Q_{T-1}} + \gamma(\eta F_P(s^P_{T-1}) + \eta\lambda(1 - F_P(s^P_{T-1})))}{1 + \eta F_P(s^P_{T-1}) + \eta\lambda(1 - F_P(s^P_{T-1}))} > 1$$

for γ high enough such that the news-utility agent consumes less than the standard agent if he does not discount prospective gain-loss utility very highly. Moreover, as $\frac{\psi_{T-1}}{Q_{T-1}}$ is increasing in σ_P the threshold value for γ, i.e., $\bar{\gamma}$, in each comparison is decreasing in σ_P.

B.4.4 Proof of Proposition 3

The agent optimally chooses consumption and asset holdings in periods $T-i = 1, ..., T$ for any horizon T. I defined a hump-shaped consumption profile as characterized by increasing consumption and asset holdings in the beginning of life $C_1 < C_2$ and decreasing consumption in the end of life $C_T < C_{T-1}$ (note that, I derive the thresholds $\underline{\sigma_P}$ and $\overline{\sigma_P}$ for $s^P_t = 0$ and $s^T_t = 0$ in all periods, since Λ_{T-i} is skewed this is not exactly the average consumption path but the difference is minor). The first characteristic requires $C_1 < C_2$ which implies that

$$\frac{(1+r)^{T-1}}{f(T-1)}(1+r)A_0 + P_0 - \frac{f(T-2)}{f(T-1)}\Lambda_1 < \frac{(1+r)^{T-2}}{f(T-2)}(1+r)A_1 + P_0 - \frac{f(T-3)}{f(T-2)}\Lambda_2$$

so that $\Lambda_1 > \frac{f(T-3)}{f(T-2)}\Lambda_2$ and since $\frac{f(T-3)}{f(T-2)} < 1$ this holds always if $\Lambda_1 > 0$ as T becomes large since in the limit $\Lambda_1 = \Lambda_2$. Recall that if $\lambda > 1$ and $\eta > 0$ then $\psi_{T-i} > Q_{T-i}$, $\psi_{T-i} > \psi_{T-i+1}$ and $Q_{T-i} > Q_{T-i+1}$ and $\psi_{T-i} - Q_{T-i} > \psi_{T-i+1} - Q_{T-i+1}$ for all i and $\frac{\psi_{T-i}}{Q_{T-i}}$ approaches it's limit $\frac{\psi}{Q}$ as i and T become large. $\underline{\sigma_P}$ is then implicitly defined by the requirement $\Lambda_1 > 0$ which is equivalent to

$$\frac{(1+r)^{T-1}}{f(T-2)}\frac{\psi_1 + \gamma Q_1 \eta\frac{1}{2}(1+\lambda)}{1 + \eta\frac{1}{2}(1+\lambda)} = \frac{r}{1 - (\frac{1}{1+r})^{T-1}}\frac{\psi_1 + \gamma Q_1 \eta\frac{1}{2}(1+\lambda)}{1 + \eta\frac{1}{2}(1+\lambda)} > 1.$$

Accordingly, if $\frac{\psi_1}{Q_1}$ (which is determined by expected marginal gain-loss utility) is large enough relative to γ the agent chooses an increasing consumption path. For $T \to \infty$ the condition boils down to

$$r\frac{\psi + \gamma Q \eta\frac{1}{2}(1+\lambda)}{1 + \eta\frac{1}{2}(1+\lambda)} > 1 \Rightarrow r(\psi + \gamma Q \eta\frac{1}{2}(1+\lambda)) > 1 + \eta\frac{1}{2}(1+\lambda)$$

for which a sufficient condition is $\gamma Q > \frac{1}{r}$.

The second characteristic requires $C_T < C_{T-1}$ which implies that

$$(1+r)A_{T-1} < \frac{1+r}{2+r}(1+r)A_{T-2} - \frac{1}{2+r}\Lambda_{T-1}$$

and is equivalent to $\Lambda_{T-1} < 0$. Thus, $\overline{\sigma_P}$ is implicitly defined by $\Lambda_{T-1} < 0$

$$\frac{1}{\theta}log((1+r)\frac{\psi_{T-1} + \gamma Q_{T-1}\eta\frac{1}{2}(1+\lambda)}{1+\eta\frac{1}{2}(1+\lambda)}) < 0 \Rightarrow (1+r)\frac{\psi_{T-1} + \gamma Q_{T-1}\eta\frac{1}{2}(1+\lambda)}{1+\eta\frac{1}{2}(1+\lambda)} < 1.$$

Note that, because $\beta(1+r) \approx 1$ the standard agent will choose an almost flat consumption path such that $(1+r)Q_{T-1} \approx 1$. Thus, the news-utility agent chooses a mean falling consumption path in the end of life as long as $\frac{\psi_{T-1}}{Q_{T-1}}$ is not too large or γ is not too close to one.

B.4.5 Proof of Proposition 4

In the deterministic setting, $s_t^P = s_t^T = 0$ for all t such that the news-utility agent will not experience actual news utility in a subgame-perfect equilibrium because he cannot fool himself and thus $\psi_t = Q_t$ for all t. Thus, the expected-utility maximizing path corresponds to the standard agent's one which is determined in any period $T-i$ by the following first-order condition

$$exp\{-\theta(1+r)A_{T-i-1} + \theta A_{T-i}\} = \frac{(1+r)^i}{f(i-1)}exp\{-\theta\frac{(1+r)^i}{f(i-1)}A_{T-i}\}Q_{T-i}^s.$$

If the agent believes he follows the above path then the consistency constraint (increasing consumption is not preferred) has to hold

$$exp\{-\theta(1+r)A_{T-i-1} + \theta A_{T-i}\}(1+\eta) < \frac{(1+r)^i}{f(i-1)}exp\{-\theta\frac{(1+r)^i}{f(i-1)}A_{T-i}\}Q_{T-i}^s(1+\gamma\eta\lambda).$$

Thus, if $\eta < \gamma\eta\lambda \Rightarrow \gamma > \frac{1}{\lambda}$ the agent follows the expected-utility maximizing path. Whereas for $\gamma \leq \frac{1}{\lambda}$ news-utility consumption is characterized by equality of the consistency constraint, because the agent will choose the lowest consumption level that just satisfies it. Then, the first-order condition becomes equivalent to a $\beta\delta-$agent's first-order condition with $b = \frac{1+\gamma\eta\lambda}{1+\eta} < 1$.

In the infinite-horizon model, a simple perturbation argument gives the following consistency constraint

$$exp(-\theta(1+r)A_{t-1} + \theta A_t)(1+\eta) < rexp(-\theta r A_t)Q(1+\gamma\eta\lambda),$$

because $\psi = Q$. However, if $\gamma > \frac{1}{\lambda}$ the news-utility agent finds it optimal to follow the expected-utility-maximizing standard agent's path

$$exp(-\theta(1+r)A_{t-1} + \theta A_t) = rexp(-\theta r A_t)Q^s \Rightarrow A_t = A_{t-1} + \Lambda^s \Rightarrow C_t = rA_{t-1} + Y_t - \Lambda^s$$

$$\Lambda^s = \frac{1}{\theta(1+r)} log(rQ^s) \text{ with } Q^s = \frac{\beta exp(-\theta(-\Lambda^s))}{1-\beta exp(-\theta r\Lambda^s)}.$$

Whereas for $\gamma \leq \frac{1}{\lambda}$ news-utility consumption will choose the lowest consumption level that just satisfies his consistency constraint. Then, the first-order condition becomes equivalent to a $\beta\delta-$agent's first-order condition with $b = \frac{1+\gamma\eta\lambda}{1+\eta} < 1$.

B.4.6 Proof of Proposition 5

I say that the news-utility agent's consumption path features a drop in consumption at retirement, if the change in consumption at retirement is negative and smaller than it is after the start of retirement, i.e., ΔC_{T-R} is negative and smaller than ΔC_{T-R+1}. In general, after the start of retirement the news-utility agent's consumption growth follows the standard model or the hyperbolic-discounting model. Thus, the news-utility agent's implied hyperbolic-discount factor after retirement is $b^R \in \{\frac{1+\gamma\eta\lambda}{1+\eta}, 1\}$, which is larger than the news-utility agent's implied hyperbolic-discount factor at retirement. In the at-retirement period the weight on future marginal value versus current marginal consumption is between $\{\frac{1+\gamma\eta\lambda}{1+\eta\lambda}, \frac{1+\gamma\eta}{1+\eta}\}$ and since $\frac{1+\gamma\eta\lambda}{1+\eta\lambda} < \frac{1+\gamma\eta}{1+\eta} < \frac{1+\gamma\eta\lambda}{1+\eta} < 1$ the hyperbolic-discount factor implied by at-retirement consumption growth is necessarily lower than the hyperbolic-discount factor implied by post-retirement consumption growth. Thus, consumption growth at retirement will necessarily be less than consumption growth after retirement. Moreover, if consumption growth after retirement is approximately zero, because $log((1+r)\beta) \in [-\triangle, \triangle]$ with $\triangle$ small then consumption growth at retirement will be negative.

Let me formalize the agent's consumption growth at and after retirement. After retirement the news-utility agent's consumption growth is $\Delta C_{T-R+1} = C_{T-R+1} - C_{T-R} = -a(R-1)\Lambda_{T-R+1} + \Lambda_{T-R}$ and will correspond to a hyperbolic-discounting agent's consumption with $b \in \{\frac{1+\eta\gamma\lambda}{1+\eta}, 1\}$ such that the agent's continuation utilities in period $T-R$ and $T-R+1$ (which determine Λ_{T-R} and Λ_{T-R+1}) correspond to

$$Q_{T-R} = \psi_{T-R} = Q^b_{T-R} \text{ and } Q_{T-R+1} = \psi_{T-R+1} = Q^b_{T-R+1}$$

such that

$$\Lambda_{T-R} = \frac{1}{\theta} log(\frac{(1+r)^R}{f(R-1)} bQ^b_{T-R}) \text{ and } \Lambda_{T-R+1} = \frac{1}{\theta} log(\frac{(1+r)^{R-1}}{f(R-2)} bQ^b_{T-R+1})$$

thus if $log((1+r)\beta) \in [-\triangle, \triangle]$ with $\triangle$ small then consumption growth after retirement will be approximately zero (if $b = 1$ and the news-utility agent follows the standard agent's path) or negative if $b < 1$ (if the news-utility agent follows a hyperbolic-discounting path but $log((1+r)\beta) \approx 0$)). Consumption growth at retirement is $\Delta C_{T-R} = C_{T-R} - C_{T-R-1} = -a(R)\Lambda_{T-R} + \Lambda_{T-R-1}$. Λ_{T-R} will correspond to a hyperbolic-discounting agent's value with $b \in \{\frac{1+\eta\gamma\lambda}{1+\eta}, 1\}$ as above. But, Λ_{T-R-1} will correspond to a hyperbolic-discounting agent's value with $b \in \{\frac{1+\gamma\eta\lambda}{1+\eta\lambda}, \frac{1+\gamma\eta}{1+\eta}\}$ and it can be easily seen that $\frac{1+\gamma\eta\lambda}{1+\eta\lambda} < \frac{1+\gamma\eta}{1+\eta} < \frac{1+\gamma\eta\lambda}{1+\eta} < 1$. Thus, from above

$$\Lambda_{T-R} = \frac{1}{\theta} log(\frac{(1+r)^R}{f(R-1)} bQ^b_{T-R})$$

and if the news-utility agent would continue this hyperbolic path implied by the past retire-

ment $b \in \{\frac{1+\eta\gamma\lambda}{1+\eta}, 1\}$ then

$$\Lambda^b_{T-R-1} = \frac{1}{\theta} log(\frac{(1+r)^{R+1}}{f(R)} b Q^b_{T-R-1})$$

whereas in fact his Λ_{T-R-1} is given by

$$\Lambda_{T-R-1} = \frac{1}{\theta} log(\frac{(1+r)^{R+1}}{f(R)} \frac{\psi_{T-R-1} + \gamma Q_{T-R-1}(\eta F(s^P_{T-R-1} + \frac{(1+r)^{R+1}}{f(R+1)} s^T_{T-R-1}) + \eta\lambda(1 - F(s^P_{T-R-1} + \frac{(1+r)^{R+1}}{f(R+1)} s^T_{T-R-1}))}{1 + \eta F(s^P_{T-R-1} + \frac{(1+r)^{R+1}}{f(R+1)} s^T_{T-R-1}) + \eta\lambda(1 - F(s^P_{T-R-1} + \frac{(1+r)^{R+1}}{f(R+1)} s^T_{T-R-1}))})$$

with $\psi_{T-R-1} = Q_{T-R-1} = Q^b_{T-R-1}$ because there is no uncertainty from period $T - R$ on. As can be easily seen iff $\gamma < 1$ then $\Lambda_{T-R-1} < \Lambda^b_{T-R-1}$ because

$$\frac{Q^b_{T-R-1} + \gamma Q^b_{T-R-1}(\eta F(s^P_{T-R-1} + \frac{(1+r)^{R+1}}{f(R+1)} s^T_{T-R-1}) + \eta\lambda(1 - F(s^P_{T-R-1} + \frac{(1+r)^{R+1}}{f(R+1)} s^T_{T-R-1}))}{1 + \eta F(s^P_{T-R-1} + \frac{(1+r)^{R+1}}{f(R+1)} s^T_{T-R-1}) + \eta\lambda(1 - F(s^P_{T-R-1} + \frac{(1+r)^{R+1}}{f(R+1)} s^T_{T-R-1}))} < Q^b_{T-R-1}$$

for instance, if $F(\cdot) = 0.5$ then $\frac{1+\gamma\frac{1}{2}\eta(1+\lambda)}{1+\frac{1}{2}\eta(1+\lambda)} < 1$ iff $\gamma < 1$. Thus, news-utility consumption growth is smaller at retirement than after retirement. Moreover, it is negative because it is either approximately zero after retirement (if $b^R = 1$) or negative after retirement (if $b^R = \frac{1+\gamma\eta\lambda}{1+\eta} < 1$).

B.4.7 Proof of Corollary 1

After retirement the news-utility agent's consumption from period $T - R$ on will correspond to a hyperbolic-discounting agent's consumption with $b \in \{\frac{1+\eta\gamma\lambda}{1+\eta}, 1\}$ such that the agent's continuation utilities correspond to

$$Q_{T-R-1} = \psi_{T-R-1} = Q^b_{T-R-1}$$

thus Λ_{T-R-1} is given by

$$\Lambda_{T-R-1} = \frac{1}{\theta} log(\frac{(1+r)^{R+1}}{f(R)} \frac{\psi_{T-R-1} + \gamma Q_{T-R-1}(\eta F(s^P_{T-R-1} + \frac{(1+r)^{R+1}}{f(R+1)} s^T_{T-R-1}) + \eta\lambda(1 - F(s^P_{T-R-1} + \frac{(1+r)^{R+1}}{f(R+1)} s^T_{T-R-1}))}{1 + \eta F(s^P_{T-R-1} + \frac{(1+r)^{R+1}}{f(R+1)} s^T_{T-R-1}) + \eta\lambda(1 - F(s^P_{T-R-1} + \frac{(1+r)^{R+1}}{f(R+1)} s^T_{T-R-1}))})$$

as can be easily seen iff $\gamma < 1$ then $\frac{\partial \Lambda_{T-R-1}}{\partial s^P_{T-R-1}} > 0$ and consumption is excessively smooth and sensitive in the pre-retirement period.

B.4.8 Proofs of the new predictions about consumption (Section IV.4)

The new predictions can be easily inferred from the above considerations.

1. Consumption is more excessively sensitive for permanent than for transitory shocks in an environment with permanent shocks. In an environment with transitory shocks only, however, news-utility consumption is excessively sensitive with respect to transitory shocks. Λ_t varies more with the permanent shock than with the transitory

shock, because the agent is consuming only the per-period value $\frac{r}{1+r}s_t^T$ of the period t transitory shock, such that $F_{C_t}^{t-1}(C_t)$ varies little with s_t^T. However, in the absence of permanent shocks Λ_t would vary with $F_{S_{T-i}^T}(s_{T-i}^T)$ which fully determines $F_{C_t}^{t-1}(C_t)$ even though consumption itself will increase only by the per-period value $\frac{r}{1+r}s_t^T$ of the transitory shock. Thus, consumption is excessively sensitive for transitory shocks when permanent shocks are absent. With permanent shocks, however, consumption is excessively sensitive for transitory shocks only when the horizon is very short or the permanent shock has very little variance such that the transitory shock actually moves $F_{S_{T-i}^P+\frac{(1+r)^i}{f(i)}S_{T-i}^T}(s_{T-i}^P+\frac{(1+r)^i}{f(i)}s_{T-i}^T)$ despite the fact that it is discounted by $\frac{(1+r)^i}{f(i)}$.

2. The degree of excess smoothness and sensitivity is decreasing in the amount of economic uncertainty σ_P. If σ_P is small, the agent's beliefs change more quickly relative to the change in the realization of the shock; hence, the consumption function is more flat for realizations around μ_P. The consumption function C_{T-i} is less increasing in the realizations of the shocks $s_{T-i}^P+\frac{(1+r)^i}{f(i)}s_{T-i}^T$ if $\frac{\partial\Lambda_{T-i}}{\partial s_{T-i}^P}$ is relatively high. As can be seen easily, $\frac{\partial\Lambda_{T-i}}{\partial s_{T-i}^P}$ is increasing in $f_{S_{T-i}^P+\frac{(1+r)^i}{f(i)}S_{T-i}^T}(s_{T-i}^P+\frac{(1+r)^i}{f(i)}s_{T-i}^T)$ which is high if $f_{S_{T-i}^P+\frac{(1+r)^i}{f(i)}S_{T-i}^T}$ is very high at $s_{T-i}^P=\mu_P$ which happens if $f_{S_{T-i}^P+\frac{(1+r)^i}{f(i)}S_{T-i}^T}$ is a very narrow distribution, i.e., σ_P is small.

3. Any bell-shaped shock distribution induces the variation in Λ_{T-i} and thereby the amount of excess sensitivity to be bounded. If the agent is hit by an extreme shock, the actual value of the low probability realization matters less because neighboring states have very low probability. The expression $\eta F(s_{T-i}^P+\frac{(1+r)^i}{f(i)}s_{T-i}^T)+\eta\lambda(1-F(s_{T-i}^P+\frac{(1+r)^i}{f(i)}s_{T-i}^T))$ is bounded if the two shocks' distributions are bell shaped. Thus, the variation in Λ_{T-i} is bounded.

4. Consumption is more excessively sensitive and excessively smooth when the agent's horizon increases, because the marginal propensity to consume out of permanent shocks declines when the additional precautionary-savings motive accumulates. $\frac{\psi_{T-i}}{Q_{T-i}}$ is increasing in i and approaches a constant $\frac{\psi}{Q}$ when $T\to\infty$ and $i\to\infty$. Then, the variation in Λ_{T-i} is increasing in i. And since consumption growth ΔC_{T-i} is determined by $-a(i)\Lambda_{T-i}+\Lambda_{T-i-1}$ on average the larger variation in Λ_{T-i} translates into a higher coefficient in the OLS regression. This can be seen by looking at $\frac{\partial\Lambda_{T-i}}{\partial s_{T-i}^P}$, i.e.,

$$\frac{\partial a(i)\Lambda_{T-i}}{\partial s_{T-i}^P}=\frac{a(i)}{\theta(\frac{1-a(i)}{a(i)})}\frac{\frac{(\psi_{T-i}-Q_{T-i})\eta f_{S_{T-i}^P+\frac{(1+r)^i}{f(i)}S_{T-i}^T}(s_{T-i}^P+\frac{(1+r)^i}{f(i)}s_{T-i}^T)(\lambda-1)}{1+\eta F(s_{T-i}^P+\frac{(1+r)^i}{f(i)}s_{T-i}^T)+\eta\lambda(1-F(s_{T-i}^P+\frac{(1+r)^i}{f(i)}s_{T-i}^T))}}{\psi_{T-i}+Q_{T-i}\gamma(\eta F(s_{T-i}^P+\frac{(1+r)^i}{f(i)}s_{T-i}^T)+\eta\lambda(1-F(s_{T-i}^P+\frac{(1+r)^i}{f(i)}s_{T-i}^T)))}>0.$$

As $a(i)=\frac{f(i-1)}{f(i)}$ is increasing in i because $f(i)=\sum_{j=0}^i(1+r)^j$ and thus $\frac{(a(i))^2}{1-a(i)}$ is increasing in i and approaching a constant and $\frac{\psi_{T-i}}{Q_{T-i}}$ is increasing in i and approaching a constant it can be easily seen that $\frac{\partial a(i)\Lambda_{T-i}}{\partial s_{T-i}^P}$ is increasing in i which means that con-

sumption becomes more excessively smooth as the agent's horizon increases. Moreover, as $\frac{a(i)}{1-a(i)}$ is increasing in i too $\frac{\partial \Lambda_{T-i}}{\partial s^P_{T-i}}$ is increasing in i which means that consumption becomes more excessively sensitive as the agent's horizon increases.

B.4.9 Proof of Proposition 6

In the following, I assume that the following parameter condition (which ensures that the agent's maximization problem is globally concave) holds $\eta(\lambda-1)<1$. All of the following proofs are direct applications of the prior proofs for the monotone-personal equilibrium just using Λ^c_{T-i} instead of Λ_{T-i}. Thus, I make the exposition somewhat shorter.

1. The personal and pre-committed consumption functions are different in each period as can be seen in Section B.2. But, if there's no uncertainty and $\gamma > \frac{1}{\lambda}$ then the personal and pre-committed consumption functions both correspond to the standard agent's consumption function as shown in the proof of Proposition 4.

2. Please refer to the derivation of the exponential-utility pre-committed model in Section B.2 for a detailed derivation of Λ^c_{T-i}. According to Definition 5 consumption is excessively smooth if $\frac{\partial C^c_{t+1}}{\partial s^P_{t+1}} < 1$ and excessively sensitive if $\frac{\partial \Delta C^c_{t+1}}{\partial s^P_t} > 0$. Consumption growth is
$$\Delta C^c_{T-i} = s^P_{T-i} + (1-a(i))s^T_{T-i} - a(i)\Lambda^c_{T-i} + \Lambda^c_{T-i-1}$$
so that $\frac{\partial C^c_{T-i}}{\partial s^P_{T-i}} < 1$ iff $\frac{\partial \Lambda^c_{T-i}}{\partial s^P_{T-i}} > 0$ and $\frac{\partial \Delta C^c_{T-i}}{\partial s^P_{T-i-1}} > 0$ iff $\frac{\partial \Lambda^c_{T-i-1}}{\partial s^P_{T-i-1}} > 0$. Since $\psi^c_{T-i} > \gamma Q^c_{T-i}$ it can be easily seen that $\frac{\partial \Lambda^c_{T-i}}{\partial s^P_{T-i}} > 0$, i.e.
$$\frac{\partial \Lambda^c_{T-i}}{\partial s^P_{T-i}} = \frac{1}{\theta(\frac{1-a(i)}{a(i)})} \frac{\frac{(\psi_{T-i}-\gamma Q_{T-i})\eta(\lambda-1)2f_{s^P_{T-i}+\frac{(1+r)^i}{f(i)}s^T_{T-i}}(s^P_{T-i}+\frac{(1+r)^i}{f(i)}s^T_{T-i})}{1+\eta(\lambda-1)(1-2F(s^P_{T-i}+\frac{(1+r)^i}{f(i)}s^T_{T-i}))}}{\psi^c_{T-i}+\gamma Q^c_{T-i}\eta(\lambda-1)(1-2F(s^P_{T-i}+\frac{(1+r)^i}{f(i)}s^T_{T-i}))} > 0.$$
Thus, optimal pre-committed consumption is excessively smooth and sensitive.

3. The first-order condition of the second-to-last period in the exemplified model of the text is
$$u'(C^c_{T-1}) = (1+r)u'((s^P_{T-1}-C^c_{T-1})(1+r)+s^P_{T-1})\frac{\psi_{T-1}+\gamma Q_{T-1}\eta(\lambda-1)(1-2F_P(s^P_{T-1}))}{1+\eta(\lambda-1)(1-2F_P(s^P_{T-1})))}.$$
By the exact same argument as above $\psi_{T-1} > Q_{T-1}$ such that news utility introduces a first-order precautionary-savings motive in the pre-committed equilibrium. Compare the above first-order condition with the one for personal-monotone consumption C_{T-1},

i.e.,

$$u'(C_{T-1}) = (1+r)u'((s^P_{T-1} - C_{T-1})(1+r) + s^P_{T-1}) \frac{\psi_{T-1} + \gamma Q_{T-1}(\eta F_P(s^P_{T-1}) + \eta\lambda(1 - F_P(s^P_{T-1})))}{1 + \eta F_P(s^P_{T-1}) + \eta\lambda(1 - F_P(s^P_{T-1}))))}.$$

Because $\eta F_P(s^P_{T-1}) + \eta\lambda(1 - F_P(s^P_{T-1})) > \eta(\lambda - 1)(1 - 2F_P(s^P_{T-1}))$ for all s^P_{T-1} and $\psi_{T-1} > \gamma Q_{T-1}$ monotone personal consumption is higher $C_{T-1} > C^c_{T-1}$ than pre-committed consumption. Moreover, the difference $\eta F_P(s^P_{T-1}) + \eta\lambda(1 - F_P(s^P_{T-1})) - \eta(\lambda - 1)(1 - 2F_P(s^P_{T-1})) = \eta(1 - F_P(s^P_{T-1})) + \eta\lambda F_P(s^P_{T-1})$ is increasing in s^P_{T-1} such that the difference in consumption $C_{T-1} - C^c_{T-1}$ is increasing in s^P_{T-1}.

4. Consider the pre-committed first-order conditions before and after retirement. After retirement the news-utility agent's consumption from period $T - R$ on will correspond to the standard agent's one, i.e.,

$$Q_{T-R-1} = \psi_{T-R-1} = Q^s_{T-R-1}$$

thus Λ_{T-R-1} is given by

$$\Lambda_{T-R-1} = \frac{1}{\theta} log(\frac{(1+r)^{R+1}}{f(R)} \frac{\psi_{T-R-1} + \gamma Q_{T-R-1}\eta(\lambda - 1)(1 - 2F(s^P_{T-R-1} + \frac{(1+r)^{R+1}}{f(R+1)} s^T_{T-R-1}))}{1 + \eta(\lambda - 1)(1 - F(s^P_{T-R-1} + \frac{(1+r)^{R+1}}{f(R+1)} s^T_{T-R-1}))})$$

it can be easily seen that

$$\frac{\partial \Lambda_{T-R-1}}{\partial \gamma} < 0 \text{ only if } F(s^P_{T-R-1} + \frac{(1+r)^{R+1}}{f(R+1)} s^T_{T-R-1})) < 0.5.$$

Thus, $\gamma < 1$ does not necessarily increase or decrease Λ_{T-R-1} and thereby consumption growth at retirement is not necessarily negative and smaller than consumption growth after retirement. There is no systematic underweighting of marginal utility before or after retirement and there does not occur a drop in consumption at retirement for $\gamma < 1$. The same argument that $\gamma < 1$ does not necessarily lead to a reduction in consumption growth even in the end of life when ψ_{T-i} and Q_{T-i} are small implies that the pre-committed consumption path is not necessarily hump shaped.

B.5 Derivation of the power-utility model

In the following, I outline the numerical derivation of the model with a power-utility specification $u(C_t) = \frac{C_t^{1-\theta}}{1-\theta}$. I start with the standard model to then explain the news-utility model in detail.

B.5.1 The standard model

The standard agent's maximization problem in any period $T-i$ is

$$max\{u(C_{T-i})+\sum_{\tau=1}^{i}\beta^{\tau}E_{T-i}[u(C_{T-i+\tau})]\}$$

$$\text{subject to } X_t=(X_{t-1}-C_{t-1})R+Y_t \text{ and } Y_t=P_{t-1}G_t e^{s_t^P}e^{s_t^T}=P_t e^{s_t^T}.$$

The maximization problem can be normalized by $P_{T-i}^{1-\theta}$ and then becomes in normalized terms ($X_t=P_t x_t$ for instance)

$$max\{u(c_{T-i})+\sum_{\tau=1}^{i}\beta^{\tau}E_{T-i}[\prod_{j=1}^{\tau}(G_{T-i+j}e^{s_{T-i+j}^P})^{1-\theta}u(c_{T-i+\tau})]\}$$

$$\text{subject to } x_t=(x_{t-1}-c_{t-1})\frac{R}{G_t e^{s_t^P}}+y_t \text{ and } y_t=e^{s_t^T}.$$

The model can be solved by numerical backward induction as done by Gourinchas and Parker (2002) and Carroll (2001). The standard agent's first-order condition is

$$u'(c_{T-i})=\Phi_{T-i}^{'}=\beta RE_{T-i}[\frac{\partial c_{T-i+\tau}}{\partial x_{T-i+1}}(G_{T-i+1}e^{s_{T-i+1}^P})^{-\theta}u'(c_{T-i+\tau})+(1-\frac{\partial c_{T-i+1}}{\partial x_{T-i+1}})(G_{T-i+1}e^{s_{T-i+1}^P})^{-\theta}\Phi_{T-i-1}^{'}]$$

with his continuation value

$$\Phi_{T-i-1}^{'}=\beta RE_{T-i-1}[\frac{\partial c_{T-i}}{\partial x_{T-i}}(G_{T-i}e^{S_{T-i}^P})^{-\theta}u'(c_{T-i})+(1-\frac{\partial c_{T-i}}{\partial x_{T-i}})(G_{T-i}e^{S_{T-i}^P})^{-\theta}\Phi_{T-i}^{'}]$$

where it can be easily noted that

$$P_{T-i}\Phi_{T-i}^{'}=E_{T-i}[\frac{\partial X_{T-i+1}}{\partial A_{T-i}}\frac{\partial\sum_{\tau=1}^{i}\beta^{\tau}u(C_{T-i+\tau})}{\partial X_{T-i+1}}]=E_{T-i}[\frac{\partial X_{T-i+1}}{\partial A_{T-i}}(\frac{\partial\beta u(C_{T-i+\tau})}{\partial X_{T-i+1}}+\frac{\partial\sum_{\tau=1}^{i}\beta^{\tau+1}u(C_{T-i+1+\tau})}{\partial X_{T-i+1}})]$$

$$=E_{T-i}[\frac{\partial X_{T-i+1}}{\partial A_{T-i}}\frac{\partial\beta u(C_{T-i+\tau})}{\partial X_{T-i+1}}+\frac{\partial X_{T-i+1}}{\partial A_{T-i}}\frac{\partial X_{T-i+2}}{\partial X_{T-i+1}}\frac{\partial X_{T-i+1}}{\partial X_{T-i+2}}\frac{\partial\sum_{\tau=1}^{i}\beta^{\tau+1}u(C_{T-i+1+\tau})}{\partial X_{T-i+1}}]$$

$$=E_{T-i}[\frac{\partial X_{T-i+1}}{\partial A_{T-i}}\frac{\partial\beta u(C_{T-i+\tau})}{\partial X_{T-i+1}}+\frac{\partial X_{T-i+1}}{\partial A_{T-i}}\frac{\partial X_{T-i+2}}{\partial X_{T-i+1}}\frac{\partial\sum_{\tau=1}^{i}\beta^{\tau+1}u(C_{T-i+1+\tau})}{\partial X_{T-i+2}}]$$

$$=\beta RE_{T-i}[\frac{\partial u(C_{T-i+\tau})}{\partial X_{T-i+1}}+\frac{\partial X_{T-i+2}}{\partial X_{T-i+1}}\frac{\partial\sum_{\tau=1}^{i}\beta^{\tau}u(C_{T-i+1+\tau})}{\partial X_{T-i+2}}]$$

$$=\beta RE_{T-i}[\frac{\partial u(C_{T-i+\tau})}{\partial X_{T-i+1}}+\frac{\partial A_{T-i+1}}{\partial X_{T-i+1}}\underbrace{E_{T-i+1}[\frac{\partial X_{T-i+2}}{\partial A_{T-i+1}}\frac{\partial\sum_{\tau=1}^{i}\beta^{\tau}u(C_{T-i+1+\tau})}{\partial X_{T-i+2}}]}_{P_{T-i+1}\Phi_{T-i-1}^{'}}]$$

$$= \beta R E_{T-i}[\frac{\partial u(C_{T-i+\tau})}{\partial X_{T-i+1}} + \frac{\partial(X_{T-i+1} - C_{T-i+1})}{\partial X_{T-i+1}} P_{T-i+1}\Phi'_{T-i-1}] = \beta R E_{T-i}[\frac{\partial u(C_{T-i+\tau})}{\partial X_{T-i+1}} + (1 - \frac{\partial C_{T-i+1}}{\partial X_{T-i+1}}) P_{T-i+1}\Phi'_{T-i-1}].$$

${\Phi'}_{T-i}$ is a function of savings a_{T-i} thus I can solve for each optimal consumption level c^*_{T-i} on a grid of savings a_{T-i} as $c^*_{T-i} = (\Phi'_{T-i-1})^{-\frac{1}{\theta}} = (f^{\Phi'}(a_{T-i}))^{-\frac{1}{\theta}}$ to then find each optimal level of consumption for each value of the normalized cash-on-hand grid x_{T-i} by interpolation. This endogenous-grid method has been developed by Carroll (2001). Alternatively, I could use the Euler equation instead of the agent's continuation value but this solution illustrates the upcoming solution of the news-utility model of which it is a simple case.

B.5.2 The monotone-personal and pre-committed equilibrium in the second-to-last period

Before starting with the fully-fledged problem, I outline the second-to-last period for the case of power utility. In the second-to-last period the agent allocates his cash-on-hand X_{T-1} between contemporaneous consumption C_{T-1} and future consumption C_T, knowing that in the last period he will consume whatever he saved in addition to last period's income shock $C_T = X_T = (X_{T-1} - C_{T-1})R + Y_T$. According to the monotone-personal equilibrium solution concept, in period $T-1$ the agent takes the beliefs about contemporaneous and future consumption he entered the period with $\{F^{T-2}_{C_{T-1}}, F^{T-2}_{C_T}\}$ as given and maximizes

$$u(C_{T-1}) + n(C_{T-1}, F^{T-2}_{C_{T-1}}) + \gamma\beta \boldsymbol{n}(F^{T-1,T-2}_{C_T}) + \beta E_{T-1}[u(C_T) + n(C_T, F^{T-1}_{C_T})]$$

which can be rewritten as

$$u(C_{T-1}) + \eta \int_{-\infty}^{C_{T-1}} (u(C_{T-1}) - u(c)) F^{T-2}_{C_{T-1}}(c) + \eta\lambda \int_{C_{T-1}}^{\infty} (u(C_{T-1}) - u(c)) F^{T-2}_{C_{T-1}}(c)$$

$$+\gamma\beta \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (u(c) - u(r)) F^{T-1,T-2}_{C_T}(c, r) + \beta E_{T-1}[u(C_T) + \eta(\lambda - 1) \int_{C_T}^{\infty} (u(C_T) - u(c)) F^{T-1}_{C_T}(c)].$$

To gain intuition for the model's predictions, I explain the derivation of the first-order condition

$$u'(C_{T-1})(1 + \eta F^{T-2}_{C_{T-1}}(C_{T-1}) + \eta\lambda(1 - F^{T-2}_{C_{T-1}}(C_{T-1}))) = \gamma\beta R E_{T-1}[u'(C_T)](\eta F^{T-2}_{A_{T-1}}(A_{T-1}) + \eta\lambda(1 - F^{T-2}_{A_{T-1}}(A_{T-1}))))$$

$$+\beta R E_{T-1}[u'(C_T) + \eta(\lambda - 1) \int_{C_T}^{\infty} (u'(C_T) - u'(c)) F^{T-1}_{C_T}(c)].$$

The first two terms in the first-order condition represent marginal consumption utility and gain-loss utility over contemporaneous consumption in period $T-1$. As the agent takes his beliefs $\{F^{T-2}_{C_{T-1}}, F^{T-2}_{C_T}\}$ as given in the optimization, I apply Leibniz's rule for differentiation under the integral sign. This results in marginal gain-loss utility being the sum of states that would have promised less consumption $F^{T-2}_{C_{T-1}}(C_{T-1})$, weighted by η, or more consumption $1 - F^{T-2}_{C_{T-1}}(C_{T-1})$, weighted by $\eta\lambda$,

$$\frac{\partial n(C_{T-1}, F^{T-2}_{C_{T-1}})}{\partial C_{T-1}} = u'(C_{T-1})(\eta F^{T-2}_{C_{T-1}}(C_{T-1}) + \eta\lambda(1 - F^{T-2}_{C_{T-1}}(C_{T-1}))).$$

Note that, if contemporaneous consumption is increasing in the realization of cash-on-hand then I can simplify $F^{T-2}_{C_{T-1}}(C_{T-1}) = F^{T-2}_{X_{T-1}}(X_{T-1})$. Returning to the maximization problem the third term represents prospective gain-loss utility over future consumption C_T experienced in $T-1$. As before, marginal gain-loss utility is given by the weighted sum of states $u'(C_T)(\eta F^{T-2}_{A_{T-1}}(A_{T-1})+\eta\lambda(1-F^{T-2}_{A_{T-1}}(A_{T-1})))$. Note that $F^{T-2}_{C_T}(c)$ is defined as the probability $Pr(C_T < c|I_{T-2})$. Applying a logic similar to the law of iterated expectation

$$Pr(C_T < c|I_{T-2}) = Pr(A_{T-1}R + Y_T < c|I_{T-2}) = Pr(A_{T-1} < \frac{c - Y_T}{R}|I_{T-2})$$

thus if savings are increasing in the realization of cash-on-hand then I can simplify $F^{T-2}_{A_{T-1}}(A_{T-1}) = F^{T-2}_{X_{T-1}}(X_{T-1})$.

The last term in the maximization problem represents consumption and gain-loss utility over future consumption C_T in the last period T, i.e., the first derivative of the agent's continuation value with respect to consumption or the marginal value of savings. Expected marginal gain-loss utility $\eta(\lambda-1)\int_{C_T}^{\infty}(u'(C_T) - u'(c))F^{T-1}_{C_T}(c)$ is positive for any concave utility function such that

$$\Psi'_{T-1} = \beta R E_{T-1}[u'(C_T)+\eta(\lambda-1)\int_{C_T}^{\infty}(u'(C_T)-u'(c))F^{T-1}_{C_T}(c)] > \beta R E_{T-1}[u'(C_T)] = \Phi'_{T-1}.$$

As expected marginal gain-loss disutility is positive, increasing in σ_Y, absent if $\sigma_Y = 0$, and increases the marginal value of savings, I say that news-utility introduces an "additional precautionary-savings motive". The first-order condition can now be rewritten as

$$u'(C_{T-1}) = \frac{\Psi'_{T-1} + \gamma\Phi'_{T-1}(\eta F^{T-2}_{X_{T-1}}(X_{T-1}) + \eta\lambda(1 - F^{T-2}_{X_{T-1}}(X_{T-1})))}{1 + \eta F^{T-2}_{X_{T-1}}(X_{T-1}) + \eta\lambda(1 - F^{T-2}_{X_{T-1}}(X_{T-1}))}.$$

Beyond the additional precautionary-savings motive $\Psi'_{T-1} > \Phi'_{T-1}$ implies that an increase in $F^{T-2}_{X_{T-1}}(X_{T-1})$ decreases

$$\frac{\frac{\Psi'_{T-1}}{\Phi'_{T-1}} + \gamma(\eta F^{T-2}_{X_{T-1}}(X_{T-1}) + \eta\lambda(1 - F^{T-2}_{X_{T-1}}(X_{T-1})))}{1 + \eta F^{T-2}_{X_{T-1}}(X_{T-1}) + \eta\lambda(1 - F^{T-2}_{X_{T-1}}(X_{T-1}))},$$

i.e., the terms in the first-order condition vary with the income realization X_{T-1} so that consumption is excessively smooth and sensitive.

B.5.3 The monotone-pre-committed equilibrium in the second-to-last-period

The first-order condition for pre-committed consumption in period $T-1$ is

$$u'(C^c_{T-1}) = \frac{\Psi'_{T-1} + \gamma\Phi'_{T-1}\eta(\lambda-1)(1 - 2F^{T-2}_{X_{T-1}}(X_{T-1}))}{1 + \eta(\lambda-1)(1 - 2F^{T-2}_{X_{T-1}}(X_{T-1}))}$$

by the same arguments as in the exponential-utility model derivation of the pre-committed

equilibrium.

B.5.4 The monotone-personal equilibrium path in all prior periods

The news-utility agent's maximization problem in any period $T-i$ is given by

$$u(C_{T-i}) + n(C_{T-i}, F_{C_{T-i}}^{T-i-1}) + \gamma \sum_{\tau=1}^{i} \beta^{\tau} \boldsymbol{n}(F_{C_{T-i+\tau}}^{T-i,T-i-1}) + \sum_{\tau=1}^{i} \beta^{\tau} E_{T-i}[U(C_{T-i+\tau})]$$

again, the maximization problem can be normalized by $P_{T-i}^{1-\theta}$ as all terms are proportional to consumption utility $u(\cdot)$. In normalized terms, the news-utility agent's first-order condition in any period $T-i$ is given by

$$u'(c_{T-i}) = \frac{\Psi'_{T-i} + \gamma\Phi'_{T-i}(\eta F_{c_{T-i}}^{T-i-1}(c_{T-i}) + \eta\lambda(1 - F_{c_{T-i}}^{T-i-1}(c_{T-i})))}{1 + \eta F_{a_{T-i}}^{T-i-1}(a_{T-i}) + \eta\lambda(1 - F_{a_{T-i}}^{T-i-1}(a_{T-i}))}$$

I solve for each optimal value of c_{T-i}^{*} for a grid of savings a_{T-i}, as Ψ'_{T-i} and Φ'_{T-i} are functions of a_{T-i} until I find a fixed point of c_{T-i}^{*}, a_{T-i}, $F_{a_{T-i}}^{T-i-1}(a_{T-i})$, and $F_{c_{T-i}}^{T-i-1}(c_{T-i})$. The latter two can be inferred from the observation that each $c_{T-i} + a_{T-i} = x_{T-i}$ has a certain probability given the value of savings a_{T-i-1} I am currently iterating on. However, this probability varies with the realization of permanent income $e^{s_{T-i}^{P}}$ thus I cannot fully normalize the problem but have to find the right consumption grid for each value of $e^{s_{T-i}^{P}}$ rather than just one. The first-order condition can be slightly modified as follows

$$u'(e^{s_{T-i}^{P}}c_{T-i}) = \frac{e^{s_{T-i}^{P}}\Psi'_{T-i} + \gamma e^{s_{T-i}^{P}}\Phi'_{T-i}(\eta F_{c_{T-i}}^{T-i-1}(c_{T-i}) + \eta\lambda(1 - F_{c_{T-i}}^{T-i-1}(c_{T-i})))}{1 + \eta F_{a_{T-i}}^{T-i-1}(a_{T-i}) + \eta\lambda(1 - F_{a_{T-i}}^{T-i-1}(a_{T-i}))}$$

to find each corresponding grid value. Note that, the resulting two-dimensional grid for c_{T-i} will be the normalized grid for each realization of s_t^T and s_t^P, because I multiply both sides of the first-order conditions with $e^{s_{T-i}^{P}}$. Thus, the agent's consumption utility continuation value is

$$\Phi'_{T-i-1} = \beta R E_{T-i-1}[\frac{\partial c_{T-i}}{\partial x_{T-i}}(G_{T-i}e^{S_{T-i}^{P}})^{-\theta}u'(c_{T-i}) + (1 - \frac{\partial c_{T-i}}{\partial x_{T-i}})(G_{T-i}e^{S_{T-i}^{P}})^{-\theta}\Phi'_{T-i}].$$

The agent's news-utility continuation value is given by

$$P_{T-i-1}^{-\theta}\Psi'_{T-i-1} = \beta R E_{T-i-1}[\frac{dC_{T-i}}{dX_{T-i}}u'(C_{T-i}) + \eta(\lambda-1)\int_{C_{T-i}<C_{T-i}^{T-i-1}}(\frac{dC_{T-i}}{dX_{T-i}}u'(C_{T-i}) - x)dF_{\frac{dC_{T-i}}{dX_{T-i}}u'(C_{T-i})}^{T-i-1}(x)$$

$$+\gamma\eta(\lambda-1)\int_{C_{T-i}<C_{T-i}^{T-i-1}}(\frac{dA_{T-i}}{dX_{T-i}}P_{T-i}^{-\theta}\Phi'_{T-i} - x)dF_{\frac{dA_{T-i}}{dX_{T-i}}P_{T-i}^{-\theta}\Phi'_{T-i}}^{T-i-1}(x) + (1 - \frac{dC_{T-i}}{dX_{T-i}})P_{T-i}^{-\theta}\Psi'_{T-i}]$$

(here, $\int\limits_{c_{T-i}<c_{T-i}^{T-i-1}}$ means the integral over the loss domain) or in normalized terms

$$\Psi'_{T-i-1} = \beta R E_{T-i-1}[\frac{dc_{T-i}}{dx_{T-i}} u'(c_{T-i})(G_{T-i}e^{s^P_{T-i}})^{-\theta}$$

$$+\eta(\lambda-1) \int\limits_{c_{T-i}<c_{T-i}^{T-i-1}} (\frac{dc_{T-i}}{dx_{T-i}} u'(c_{T-i})(G_{T-i}e^{S^P_{T-i}})^{-\theta} - x) dF^{T-i-1}_{\frac{dc_{T-i}}{dx_{T-i}} u'(c_{T-i})(G_{T-i}e^{S^P_{T-i}})^{-\theta}}(x)$$

$$+\gamma\eta(\lambda-1) \int\limits_{c_{T-i}<c_{T-i}^{T-i-1}} (\frac{da_{T-i}}{dx_{T-i}} \Phi'_{T-i}(G_{T-i}e^{S^P_{T-i}})^{-\theta} - x) dF^{T-i-1}_{\frac{da_{T-i}}{dx_{T-i}} \Phi'_{T-i}(G_{T-i}e^{S^P_{T-i}})^{-\theta}}(x) + (1-\frac{dc_{T-i}}{dx_{T-i}})(G_{T-i}e^{S^P_{T-i}})^{-\theta}\Psi'_{T-i}].$$

B.5.5 Risk attitudes over small and large stakes

First, I suppose the agent is offered a gamble about immediate consumption in period t after that period's uncertainty has resolved and that period's original consumption has taken place. I assume that utility over immediate consumption is linear. Then, the agent is indifferent between accepting or rejecting a 50-50 win G or lose L gamble if$0.5G - 0.5L + 0.5\eta G - 0.5\eta\lambda L = 0$. Second, I suppose the agent is offered a monetary gamble or wealth bet that concerns future consumption. Suppose $T \to \infty$. I assume that his initial wealth level is $A_t = 100,000$ and $P_t = 300,000$. Let $f^{\Psi}(A)$ and $f^{\Phi}(A)$ be the agent's continuation value as a function of the agent's savings A_t. Then, the agent is indifferent between accepting or rejecting a 50-50 win G or lose L gamble if

$$0.5\eta(f^{\Phi}(A_t+G)-f^{\Phi}(A_t))+0.5\eta\lambda(f^{\Phi}(A_t-L)-f^{\Phi}(A_t))+0.5f^{\Psi}(A_t+G)+0.5f^{\Psi}(A_t-L) = f^{\Psi}(A_t-L).$$

B.6 Habit formation

Consider an agent with internal, multiplicative habit formation preferences $u(C_t, H_t) = \frac{(\frac{C_t}{H_t^h})^{1-\theta}}{1-\theta}$ with $H_t = H_{t-1} + \vartheta(C_{t-1} - H_{t-1})$ and $\vartheta \in [0,1]$ (Michaelides (2002)). Assume $\vartheta = 1$ such that $H_t = C_{t-1}$. For illustration, in the second-to-last period his maximization problem is

$$u(C_{T-1}, H_{T-1}) + \beta E_{T-1}[u(R(X_{T-1}-C_{T-1})+Y_T, H_T)] = \frac{(\frac{C_{T-1}}{H_{T-1}^h})^{1-\theta}}{1-\theta} + \beta E_{T-1}[\frac{1}{1-\theta}(\frac{R(X_{T-1}-C_{T-1})+Y_T}{H_T^h})^{1-\theta}]$$

which can be normalized by $P_T^{(1-\theta)(1-h)}$ (then $C_T = P_T c_T$ for instance) and the maximization problem becomes

$$\frac{P_{T-1}^{(1-\theta)(1-h)}(\frac{c_{T-1}}{h_{T-1}^h})^{1-\theta}}{1-\theta} + \beta P_{T-1}^{(1-\theta)(1-h)} E_{T-1}[\frac{1}{1-\theta}(G_T e^{s_T^P})^{(1-\theta)(1-h)}(\frac{(x_{T-1}-c_{T-1})\frac{R}{G_T e^{s_T^P}} + y_T}{h_T^h})^{1-\theta}]$$

which results in the following first-order condition

$$c_{T-1}^{-\theta} = h_{T-1}^{-\theta h+h} \beta E_{T-1}[(G_T e^{s_T^P})^{-\theta(1-h)} (\frac{c_T}{h_T^h})^{-\theta} (R + \frac{c_T}{h_T} h)] = \Phi'_{T-1}$$

with Φ'_{T-1} being a function of savings $x_{T-1} - c_{T-1}$ and habit h_T. The first-order condition can be solved very robustly by iterating on a grid of savings a_{T-1} assuming $c^*_{T-1} = (\Phi'_{T-1})^{-\frac{1}{\theta}} = (f^{\Phi'}(a_{T-1}, h_T))^{-\frac{1}{\theta}}$ and $h_T = c^*_{T-1} \frac{1}{G_T e^{s_T^P}}$ until a fixed point of consumption and habit has been found. The normalized habit-forming agent's first-order condition in any period $T - i$ is given by

$$c_{T-i}^{-\theta} = h_{T-i}^{-\theta h+h} \Phi'_{T-i} = h_{T-i}^{-\theta h+h} \beta E_{T-i}[(G_{T-i+1} e^{S_{T-i+1}^P})^{-\theta(1-h)} (\frac{c_{T-i+1}}{h_{T-i+1}^h})^{-\theta} (R \frac{dc_{T-i+1}}{dx_{T-i+1}} + \frac{c_{T-i+1}}{h_{T-i+1}} h)$$

$$+(1 - \frac{dc_{T-i+1}}{dx_{T-i+1}})(G_{T-i+1} e^{S_{T-i+1}^P})^{-\theta(1-h)} \Phi'_{T-1}].$$