Structural change and the Kaldor facts in a growth model with relative price effects and non-Gorman preferences[*]

Timo Boppart[†]

February 14, 2014

Abstract

U.S. data reveal three facts: (1) the share of goods in total expenditure declines at a constant rate over time, (2) the price of goods relative to services declines at a constant rate over time, and (3) poor households spend a larger fraction of their budget on goods than do rich households. I provide a macroeconomic model with non-Gorman preferences that rationalizes these facts, along with the aggregate Kaldor facts. The model is parsimonious and admits an analytical solution. Its functional form allows a decomposition of U.S. structural change into an income and substitution effect. Estimates from micro data show each of these effects to be of roughly equal importance.

Keywords: Structural change, structural transformation, relative price effect, non-Gorman preferences, Kaldor facts.
JEL classification: O14, O30, O41, D90.

1 Introduction

It is well-documented that economic growth goes hand in hand with significant shifts in the sectoral structure of output, employment, and expenditures (Kuznets, 1957). These dynamics are hard to square with balanced growth at the aggregate level, as described by the Kaldor facts – i.e., the growth rate of real per-capita output, the real interest rate, the capital-output ratio, and the labor income share are all constant over time (Kaldor, 1961).

Structural change in expenditures is commonly believed to be driven by two separate forces. First, the expenditure structure can shift because of relative price changes driven by asymmetric technologies across sectors.[1] Second, an income effect may decrease the expenditure shares of necessities (and increase the expenditure shares of luxuries) even at constant relative prices.[2] In this paper, I document three empirical facts about the post-war U.S. that speak to the underlying determinants of structural change: (i) the share of goods in total expenditure declines at a constant rate over time, (ii) the price of goods relative to services declines at a constant rate

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[1]This mechanism goes back to Baumol (1967), who stresses productivity growth differentials as a source of relative price changes. Changes in relative prices affect expenditure shares whenever the elasticity of substitution across sectors is not equal to unity.

[2]This mechanism is consistent with Engel's law, which is regarded as one of the most robust empirical regularities in economics (see Engel, 1857, Houthakker, 1957, Houthakker and Taylor, 1970, and Browning, 2008). As a consequence, many models of structural change rely on income effects. See, e.g., Matsuyama (1992), Echevarria (1997), Laitner (2000), Caselli and Coleman (2001), Kongsamut, Rebelo and Xie (2001), Gollin, Parente and Rogerson (2002), Greenwood and Seshadri (2002), and Rogerson (2008), who use quasi-homothetic intratemporal preferences or Falkinger (1990), Falkinger (1994), Zweimueller (2000), Matsuyama (2002), Foellmi and Zweimueller (2008), and Buera and Kaboski (2012a, 2012b), who generate non-homotheticity by a hierarchy of needs.

over time, and (iii) poor households spend a larger fraction of their budget on goods than do rich households. I then show that a parsimonious neoclassical growth model can rationalize not only these facts, but also the aggregate Kaldor facts. The main new element in my macroeconomic theory is to incorporate a class of preferences regularly used in applied microeconomics. These non-Gorman preferences are tractable and turn out to generate auxiliary predictions that I verify using microeconomic data.

This paper has several overlaps with the existing literature. Much of the literature focuses on a single mechanism behind structural change. Emphasizing the relative price channel, Ngai and Pissarides (2007) reconcile structural change with the Kaldor facts. Asymptotically, the same is true in Acemoglu and Guerrieri (2008).[3] Yet, by using a (homothetic) constant elasticity of substitution (CES) specification, both papers abstract from income effects. In contrast, Kongsamut, Rebelo and Xie (2001) as well as Foellmi and Zweimueller (2008) focus on income effects and specify non-homothetic preferences. However, these papers exclude relative price effects.[4] As Buera and Kaboski (2009) emphasize, no existing model with endogenous savings and balanced aggregate growth includes both forces of structural change – relative price and income effects. This gap reflects more of a theoretical challenge than a firm belief in a single driver of structural change.[5] A main contribution of my paper is to develop a parsimonious

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[3]In addition to differences in productivity growth, Acemoglu and Guerrieri (2008) stress cross-sectoral factor intensity differences together with capital deepening as a cause of relative price changes.

[4]In Kongsamut, Rebelo and Xie (2001) consistency with the Kaldor facts relies on a knife-edge condition that ties preference and technology parameters together and implies constant relative prices. Foellmi and Zweimueller (2008) have to assume that technological differences are uncorrelated with the hierarchical position of a good (and its sectoral classification).

[5]Acemoglu and Guerrieri (2008) acknowledge that income effects are an "undoubtedly important" determinant of structural change and conclude in their paper: "It

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theory which allows us to analyze both drivers of structural change.

My model relies on non-Gorman preferences. Specifically, the marginal propensity to consume goods and services differs between rich and poor households, such that income inequality affects the aggregate demand structure.[6] Although inequality matters for aggregate demand, it enters via a single sufficient statistic, which permits a tractable dynamic framework with an analytical solution. Given the model, I use micro data to estimate the preference parameters. Based on these estimates I can decompose the drivers of structural change into an income and a substitution effect. Both channels turn out to be quantitatively important, each contributing roughly 50 percent to U.S. structural change.

The structure of the paper is as follows. Section 2 summarizes the three empirical facts. Section 3 lays out the theoretical model. In Section 4, I estimate the preference parameters and quantify the two channels of structural change. Section 5 concludes.

2 Empirical regularities

In this section, I uncover three empirical regularities of the goods and service sector in post-war U.S. All price and aggregate expenditure data was obtained from the Bureau of Economic Analysis (BEA) and I follow its

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would be particularly useful to combine the mechanism proposed in this paper with non-homothetic preferences and estimate a structural version of the model with multiple sectors using data from the U.S. or the OECD." (Acemoglu and Guerrieri, 2008, p. 493.)

[6]This is similar to a hierarchy of needs approach à la Foellmi and Zweimueller (2008). At least at the product level, non-linearity of Engel curves is an empirical fact and matters since it is an explicit aim of this paper to be consistent with cross-sectional expenditure data. However, unlike in Foellmi and Zweimueller (2008), in my model inequality is allowed for, the connection to data of broadly defined sectors is straight forward and (most importantly) relative price effects can be discussed.

classification of total expenditures into 'goods' and 'services'.[7] For the micro evidence, I use data from the Consumer Expenditure Survey (CEX) of the years 1986-2011. Households with an incomplete income report, zero food expenditures, or an expenditure share of goods outside $[0, 1]$ have been excluded. For each household, I group the expenditures over a three month period into goods and services according to BEA's definition.[8] The analysis of this data shows three regularities.

Empirical Regularity 1: The share of goods in total personal consumption expenditure declines at a constant rate over time.

Figure 1 shows the share of total consumption expenditure that is spent on goods. On a logarithmic scale, the series is well approximated by a linear downward sloping trend (see dashed line). The slope of this linear fit suggests that the share of goods decreased at a constant annualized rate of 1.0 percent.

Empirical Regularity 2: The price of goods relative to services declines at a constant rate over time.

Figure 2 plots the evolution of the relative consumer price between goods and services on a logarithmic scale. The relative price trend is well approximated by a constant annualized growth rate of -1.6 percent (see dashed line). Supplementary to the (nominal) expenditure share and relative price

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[7]According to the BEA a 'good' is defined as "a tangible commodity that can be stored or inventoried" whereas a 'service' is "a commodity that cannot be stored or inventoried and that is usually consumed at the place and time of purchase".

[8]For most CEX categories this mapping is unambiguous. If a CEX category contains both some goods and services, I compare the size of these subcomponents in the BEA data and do the classification on the basis of the major subcomponents. Note that, as the BEA data, the micro data used measures expenditures, not consumption flows (see Online Appendix B.1.8 for a short discussion). Note also that the CEX interview data measures 'out-of-pocket' expenditures (i.e., net of third-party reimbursements).
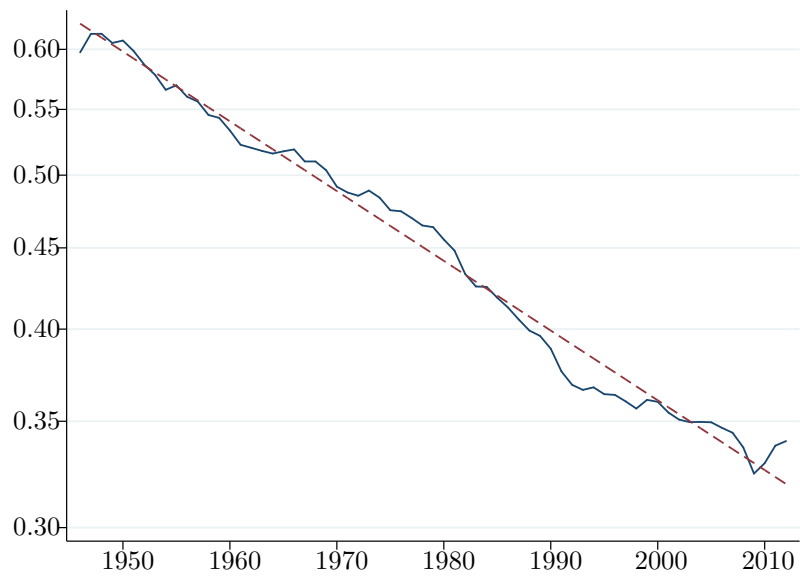
Figure 1: **Expenditure share of goods**

Notes: The figure plots the share of personal consumption expenditures devoted to goods in the U.S. on a logarithmic scale. The main (sub)categories the BEA classifies as 'goods' are: "motor vehicles and parts", "furnishings and durable household equipment", "recreational goods and vehicles", "food and beverages purchased for off-premises consumption", "clothing and footwear", "gasoline and other energy goods" and "other durable/nondurable goods". The dashed line represents the predicted values obtained by regressing the logarithmized expenditure share on time and a constant. The estimated slope coefficient and its standard error are -0.0101 and 0.00015, respectively. The regression attains an R^2 of 0.9848. Source: BEA, NIPA table 1.1.5.

of goods, the solid line in Figure 3 shows the implied dynamics for the
quantity of services relative to goods. (In the same figure, the dashed line
represents the relative quantity implied by the two linear fits in Figure 1
and 2.) The relative quantity of services is non-monotonic, increasing until
the mid-90s and then declining.
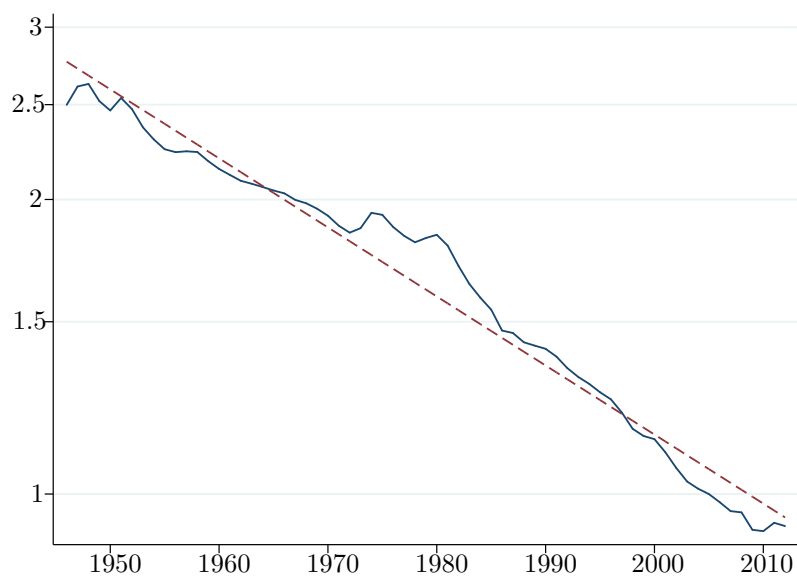
Figure 2: **Relative price between goods and services**

Notes: The figure plots the relative consumer price between goods and services on a logarithmic scale. The dashed
line represents the predicted values obtained by regressing the logarithmized relative price on a constant and time. The
estimated slope coefficient and its standard error are -0.0162 and 0.00036, respectively. The regression attains an R^2
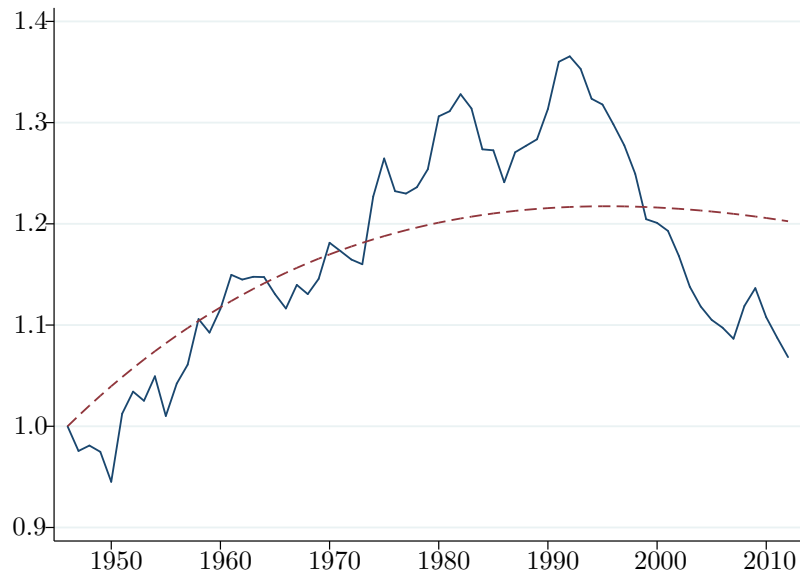of 0.9697. Source: BEA, NIPA table 1.1.4.

Figure 3: **Quantity of services relative to goods**

Notes: The figure plots the quantity of services relative to the quantity of goods on a logarithmic scale. The red dashed line represents the relative quantity implied by the linear approximations of the expenditure share and relative price in Figure 1 and 2. The relative quantity and the red dashed line are normalized to one in the year 1946. Source: BEA, NIPA table 1.1.3.

Empirical Regularity 3: Poor households spend a larger fraction of their budget on goods than do rich households.

Figure 4 plots the expenditure shares devoted to goods for the different income quintiles. At a given point in time, poorer households exhibit a larger expenditure share of goods. On the logarithmic scale, the expenditure shares of the different income quintiles decline linearly and parallel over time. This suggests that the shares of goods of rich and poor households decline at the same (constant) growth rate as the aggregate series.

Empirical Regularity 1-3 hold for other developed countries, too. Figure B.12-B.14 in Online Appendix B.1.9 show the aggregate dynamics for European OECD countries. With -0.9 and -1.4 percent, the average growth rates of the share and relative price of goods are similar to the U.S. In addition, Figure B.3 in Online Appendix B.1.9 shows Empirical Regularity

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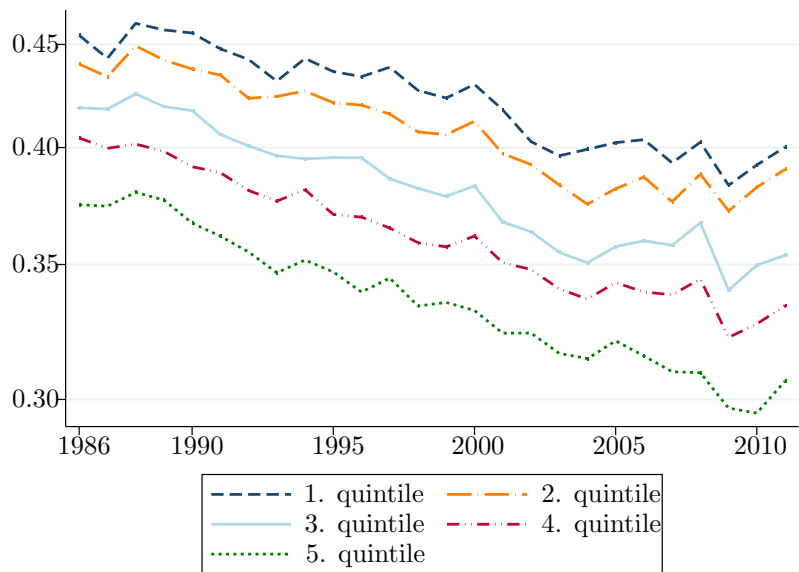
Figure 4: **Micro evidence of expenditure shares of goods**

Notes: The figure plots the expenditure share of goods for each income quintile of the U.S. on a logarithmic scale. The following expenditure categories are considered as services: food away from home; shelter; utilities, fuels and public services; other vehicle expenses; public transportation; health care; personal care; education; cash contributions; personal insurance and pensions. The remaining categories are considered as goods. The sample consists of expenditure data of 477,730 quarters (and 177,419 households). The quintiles refer to total household labor earnings after taxes plus transfers per OECD modified equivalence scale. For homeowners the imputed renting value is taken as shelter expenditures. (Figure B.1, B.11 and B.2 in the Online Appendix B.1.9 show that the picture remains qualitatively unchanged if we exclude housing expenditures, durable good expenditures or if we use total after tax income to form the quintiles. Source: CEX interview data obtained from the BLS for the year 2011 and from the ICPSR for the years 1986-2010.

3 for UK micro data.[9]

Although services became relatively more expensive over time, in Figure 3, the relative quantity of services increased over several decades.[10] This feature is inconsistent with a (homothetic) CES specification, where households should substitute towards the cheaper sector.[11] Admittedly, this rejection of a CES specification relies on the decomposition of nominal expenditures into prices and quantities, which is – especially due to the difficulty in measuring quality improvements – a controversial issue. However, the study by Boskin et al. (1996) on biases in consumer price estimates suggests that over-estimation of price growth is especially large in services. This would make the argument against a CES specification even stronger. I will not attempt to settle the debate about biases in consumer prices in this paper. Instead, using micro data, my theory offers an approach to estimating the importance of income effects, which does not rely on price data. The estimates obtained are broadly consistent with the BEA price data.

This paper aims to explain the structural change between goods and services. Empirical Regularity 2 highlights the need for a framework in which relative price effects can be discussed. Empirical Regularity 3 calls for a theory with non-homothetic preferences. For this reason, the next section provides a multi-sector growth model in which structural change is driven

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[9]All three regularities are also robust to the exclusion of durable goods. Without durable goods however, the pace of structural change is faster although the relative price growth is slower (see Figure B.8 and B.9 in Online Appendix B.1.9). This suggests that non-homotheticity of preferences is more important, which is indeed consistent with the micro data (see Figure B.3 in Online Appendix B.1.9).

[10]The same is observed for European countries and is even stronger if durable goods are excluded (see Figure B.14 and B.10 in Online Appendix B.1.9).

[11]Formally, a CES specification implies, $\frac{X_S(t)}{X_G(t)} = \left[\frac{P_G(t)}{P_S(t)}\right]^{\sigma}$, where $\sigma \geq 0$ is the elasticity of substitution and $X_j(t)$ and $P_j(t)$ are quantities and prices of the goods ($j = G$) and services ($j = S$).

by income and relative price effects. The model will fulfill Empirical Regularity 1-3 by construction. Finally, as the post-war U.S. is known as a prime example for balanced aggregate growth, the ambition of this paper is to reconcile the nonbalanced features at the sectoral level with the Kaldor facts.

3   Theoretical model

There is a unit interval of (heterogeneous) households indexed by $i \in [0,1]$. Each household consists of $N(t)$ identical members, where $N(t)$ grows at an exogenous rate $n \geq 0$. $N(0)$ is normalized to one, such that we have $N(t) = \exp[nt]$. Each member of household i is endowed with $l_i \in (\bar{l}, \infty)$, $\bar{l} > 0$, units of labor and $a_i(0) \in [0, \infty)$ units of initial wealth. The per-capita factor endowments can differ across households. Labor is supplied inelastically at every instant of time. Consequently, the aggregate labor supply $L(t) \equiv N(t) \int_0^1 l_i \, di$, grows at a constant rate n.

3.1   Preferences

All households $i \in [0, 1]$ have the following additively separable representation of intertemporal preferences

$$\mathcal{U}_i(0) = \int_0^\infty \exp\left[-(\rho - n)t\right] V\left(P_G(t), P_S(t), e_i(t)\right) dt, \qquad (1)$$

where $\rho \in (n, \infty)$ is the rate of time preference and $V\left(P_G(t), P_S(t), e_i(t)\right)$ is an indirect instantaneous utility function of each household member. This instantaneous utility function is specified over the prices of "goods" and "services", $P_G(t)$ and $P_S(t)$, and the nominal per-capita expenditure level of household i, $e_i(t)$. The indirect instantaneous utility function takes the following form

$$V\left(P_G(t), P_S(t), e_i(t)\right) = \frac{1}{\epsilon}\left[\frac{e_i(t)}{P_S(t)}\right]^\epsilon - \frac{\nu}{\gamma}\left[\frac{P_G(t)}{P_S(t)}\right]^\gamma - \frac{1}{\epsilon} + \frac{\nu}{\gamma}, \qquad (2)$$

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where $0 \leq \epsilon \leq \gamma < 1$ and $\nu > 0$.[12] I will show below that these preferences imply a household behavior which is consistent with the facts emphasized in Section 2. The specified intratemporal utility function represents a subclass of "price independent generalized linearity" (PIGL) preferences defined by Muellbauer (1975) and Muellbauer (1976). The PIGL class of preferences is more general than the Gorman class.[13] Still, PIGL preferences avoid an aggregation problem. Expenditure shares of the aggregate economy coincide with those of a household with a "representative" expenditure level (the representative household in Muellbauer's sense). Moreover, PIGL preferences ensure that this representative expenditure level is independent of prices. PIGL preferences have an explicit empirical justification and are widely used in demand system estimations (see, e.g., the "Quadratic Expenditure System" (QES) by Howe, Pollak and Wales, 1979 or the PIGLOG case in the "Almost Ideal Demand System" (AIDS) by Deaton and Muellbauer, 1980).

The specified class of instantaneous utility functions includes familiar homothetic preferences as special cases. For $\epsilon = 0$ we get the limit case with $V\left(\cdot\right) = \log\left[\frac{e_i(t)}{P_S(t)}\right] - \frac{\nu}{\gamma}\left[\frac{P_G(t)}{P_S(t)}\right]^\gamma + \frac{\nu}{\gamma}$ and with $\gamma = \epsilon = 0$ we obtain Cobb-Douglas preferences with $V\left(\cdot\right) = \log\left[\frac{e_i(t)}{P_G(t)^\nu P_S(t)^{1-\nu}}\right]$. Finally, with $\nu = 0$,

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[12]Online Appendix B.1.3 discusses further generalizations like adding a third sector. In general, with $\epsilon \neq 0$, a closed form representation of the direct utility function does not exist. An exception is if $\gamma = \epsilon$ and the direct form of (2) can be written as:

$$U(x_G^i(t), x_S^i(t)) = \frac{1}{\epsilon}\left[x_S^i(t)\right]^\epsilon \frac{\left[\frac{x_G^i(t)}{\nu}\right]^{\frac{\epsilon}{1-\epsilon}} - \nu}{\left[\left[\frac{x_G^i(t)}{\nu}\right]^{\frac{1}{1-\epsilon}} - x_G^i(t)\right]^\epsilon} - \frac{1-\nu}{\epsilon},$$

where $x_G^i(t)$ and $x_S^i(t)$ are the consumed quantities of goods and services.

[13]The PIGL class of preferences can be written as $V\left(\mathbf{P}, e_i\right) = \frac{1}{\vartheta}\left[\frac{e_i}{a(\mathbf{P})}\right]^\vartheta - b(\mathbf{P})$ (see Muellbauer, 1975). e_i is the expenditure level, $\mathbf{P}$ is the price vector, $a(\mathbf{P})$ is a linearly homogeneous function and $b(\mathbf{P})$ is homogeneous of degree zero. The Gorman form restricts $\vartheta = 1$ (see Online Appendix B.1.2 for a longer discussion).

we would have only one consumption sector and CRRA preferences. Lemma 1 shows a restriction under which function (2) satisfies the standard properties of a utility function.

Lemma 1. *Function* (2) *is a valid indirect utility specification if and only if*

$$e_i(t)^\epsilon \geq \left[\frac{1-\epsilon}{1-\gamma}\right]\nu P_G(t)^\gamma P_S(t)^{\epsilon-\gamma}. \tag{3}$$

Proof. See Online Appendix B.1.1.1. □

Henceforth, I assume that condition (3) is fulfilled. Later, two conditions in terms of exogenous parameters are stated, which jointly ensure condition (3) for all individuals at each date.

Households maximize (1) with respect to $\{e_i(t), a_i(t)\}_{t=0}^\infty$, subject to the budget constraint

$$\dot{a}_i(t) = [r(t) - n]\, a_i(t) + w(t)l_i - e_i(t), \tag{4}$$

and a standard transversality condition, which can be expressed as

$$\lim_{t\to\infty} e_i(t)^{\epsilon-1}P_S(t)^{-\epsilon}a_i(t)\exp\left[-(\rho - n)t\right] = 0. \tag{5}$$

$r(t)$ and $w(t)$ is the (nominal) interest and wage rate and $a_i(t)$ denotes per-capita wealth of household i at date t. $a_i(0)$ is exogenously given. In each instant of time each household i takes $P_G(t)$ and $P_S(t)$ as given and chooses the per-capita consumption of goods, $x_G^i(t)$ and services, $x_S^i(t)$, such that the instantaneous utility is maximized. This intratemporal optimization is subject to the budget constraint $e_i(t) = P_G(t)x_G^i(t) + P_S(t)x_S^i(t)$.

3.2 Technology

There are three output goods: the output of the two consumption sectors $Y_G(t)$ and $Y_S(t)$ and an "investment good", $Y_I(t)$, which can be transformed

one-to-one into capital, $K(t)$. Capital depreciates at constant rate $\delta \geq 0$. This implies for the law of motion of capital

$$\dot{K}(t) = X_I(t) - \delta K(t), \tag{6}$$

where $X_I(t)$ is aggregate gross investment (in terms of investment goods) at date t. The consumption sectors produce under perfect competition according to the following technologies

$$Y_j(t) = \exp\left[g_j t\right] L_j(t)^\alpha K_j(t)^{1-\alpha}, \ j = G, S, \tag{7}$$

where $L_j(t)$ and $K_j(t)$ denote labor and capital, respectively, allocated to sector j at date t. Both production factors are fully mobile and wage rate $w(t)$ and rental rate $R(t)$ equalize across sectors. $\alpha \in (0,1)$ is the output elasticity of labor, which is identical across sectors. Total factor productivity (TFP) expands at a constant, exogenous and sector-specific rate $g_j \geq 0$. The investment good is produced by a linear technology

$$Y_I(t) = A K_I(t), \tag{8}$$

with $A > \delta$.[14] The market of investment goods is competitive too. Henceforth, I normalize the price of the investment good at each date to one, i.e., $P_I(t) = 1, \ \forall t$.

The AK structure prevents transitional dynamics with the aim to focus the discussion on the phenomenon of interest: The coexistence of structural change and balanced growth on the aggregate level. Alternatively, one could assume that the investment sector $j = I$ produces according to a neoclassical production technology with Harrod-neutral technical change, $F\left[K_j(t), \exp\left(\tilde{g} t\right) L_j(t)\right]$ and the consumption sectors $j = G, S$ use – apart

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[14]The specified production side is similar to Rebelo (1991). Endogenous growth is feasible because $K(t)$ is a "core" capital good, whose production does not involve any non-reproducible factor. If $g_j \neq 0$, for some $j = G, S$, the economy has an exogenous driver of growth too.

from a time-varying Hicks-neutral term – the same technology. As illustrated in Online Appendix B.1.4 such a model is consistent with a globally stable *steady state* which features identical dynamics as the equilibrium with the AK specification.

Finally, it is worth noting that the entire model is specified in terms of final output as opposed to value-added. This means that in order to derive theoretical implications for sectoral value-added shares, the exact production processes with intermediate inputs have to be specified (see Herrendorf, Rogerson and Valentinyi, 2013 for the empirical differences of these two perspectives).[15] Online Appendix B.1.5 shows how the input-output structure of the economy can be modeled and used to make predictions on the sectoral value-added level.

3.3 Resource constraints and market clearing

In equilibrium, capital and labor markets have to clear, i.e.,

$$L(t) = L_G(t) + L_S(t), \text{ and } K(t) = K_G(t) + K_S(t) + K_I(t), \ \forall t. \quad (9)$$

Let us write the aggregate demands as $X_j(t) \equiv N(t) \int_0^1 x_j^i(t) di, \ j = G, S$. Then, market clearing in the goods, service and investment good markets requires

$$Y_j(t) = X_j(t), \ j = G, S, I, \ \forall t. \quad (10)$$

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[15]For instance if we would want to calibrate α to the sectoral labor's income share the entire input-output structure of the economy needs to be taken into account. Valentinyi and Herrendorf (2008) do this for the U.S. for the year 1997 and estimate labor's income shares for gross manufacturing output, gross service output, overall consumption and total gross output that are all between 0.65 and 0.67. Online Appendix B.1.6 illustrates the equilibrium dynamics with sectoral factor intensity differences. In this case the model is similar to the one by Acemoglu and Guerrieri (2008) and the Kaldor facts hold only asymptotically. Still, structural change is also determined by an income effect.

Since the price of the investment good is chosen as a numéraire, asset market clearing implies

$$N(t) \int_0^1 a_i(t)di = K(t), \ \forall t. \tag{11}$$

Finally, the market rate of return of capital has to equalize the rental rate net of depreciation, i.e., $r(t) = R(t) - \delta$, $\forall t$.

3.4 Definition of an equilibrium

In this economy, an equilibrium is defined as follows:

Definition 1. *A dynamic competitive equilibrium is a time path of households' per-capita expenditure levels, wealth stocks and consumption quantities* $\left\{e_i(t), a_i(t), x_j^i(t)\right\}_{t=0}^{\infty}$, $j = G, S$, $\forall i$; *an evolution of prices, wage, interest and rental rate,* $\{P_j(t), w(t), r(t), R(t)\}_{t=0}^{\infty}$, $j = G, S$ *and a time path of factor allocations* $\{L_G(t), L_S(t), K_G(t), K_S(t), K_I(t)\}_{t=0}^{\infty}$, *which is consistent with household and firm optimization, perfect competition, resource constraints and market clearing conditions* (9)-(11).

In the following, I characterize the equilibrium as the outcome of decentralized markets. However, since all markets are complete and competitive, the Welfare Theorems apply and the dynamic competitive equilibrium coincides with the solution to a social planner's problem.

3.5 Solving the model

3.5.1 Firm behavior

The equilibrium in production is characterized by the following lemma.

Lemma 2. *Firm optimization implies at each date t,*

$$r(t) = A - \delta, \tag{12}$$

$$w(t) = A \frac{\alpha}{1-\alpha} \frac{K_G(t) + K_S(t)}{L(t)}, \tag{13}$$

$$P_j(t) = \exp\left[-g_j t\right] \left[\frac{A}{1-\alpha}\right] \left[\frac{K_G(t) + K_S(t)}{L(t)}\right]^\alpha, \ j = G, S, \tag{14}$$

$$Y_j(t) = \exp\left[g_j t\right] \left[\frac{L(t)}{K_G(t) + K_S(t)}\right]^\alpha K_j(t), \ j = G, S, \tag{15}$$

and

$$\frac{K_G(t)}{L_G(t)} = \frac{K_S(t)}{L_S(t)} = \frac{K_G(t) + K_S(t)}{L(t)}. \tag{16}$$

Proof. See Online Appendix B.1.1.2. □

Because of the AK production function the real, investment good denominated interest rate is constant. Moreover, since the output elasticity of labor is the same in the two consumption sectors, capital intensities equalize (see (16)). This also implies that changes in the relative price between goods and services are fully determined by the relative TFP growth rates g_j, $j = G, S$.

3.5.2 Household behavior

Applying Roy's identity to the indirect utility function (2) gives the Marshallian demand functions. The resulting expenditure system is summarized in the next lemma.

Lemma 3. *(i) At each point in time, intratemporal preferences imply the following expenditure system*

$$x_G^i(t) = \nu \frac{e_i(t)}{P_G(t)} \left[\frac{P_S(t)}{e_i(t)}\right]^\epsilon \left[\frac{P_G(t)}{P_S(t)}\right]^\gamma, \tag{17}$$

and

$$x_S^i(t) = \frac{e_i(t)}{P_S(t)} \left[1 - \nu \left[\frac{P_S(t)}{e_i(t)}\right]^\epsilon \left[\frac{P_G(t)}{P_S(t)}\right]^\gamma\right], \tag{18}$$

where $x_j^i(t)$, $j = G, S$, is household i's per-capita consumption of goods/services at date t.

(ii) *With $\epsilon > 0$, the expenditure elasticity of demand is positive, but strictly smaller than unity for goods and larger than unity for services. With $\epsilon = 0$, we have homothetic preferences (expenditure elasticities of both sectors are equal to unity).*

(iii) *The elasticity of substitution between goods and services,*

$$\sigma_i(t) = 1 - \gamma - \frac{\nu \left[\frac{P_G(t)}{P_S(t)}\right]^{\gamma}}{\left[\frac{e_i(t)}{P_S(t)}\right]^{\epsilon} - \nu \left[\frac{P_G(t)}{P_S(t)}\right]^{\gamma}} [\gamma - \epsilon], \qquad (19)$$

is less than or equal to unity (for all households at each date).

Proof. See Online Appendix B.1.1.3. □

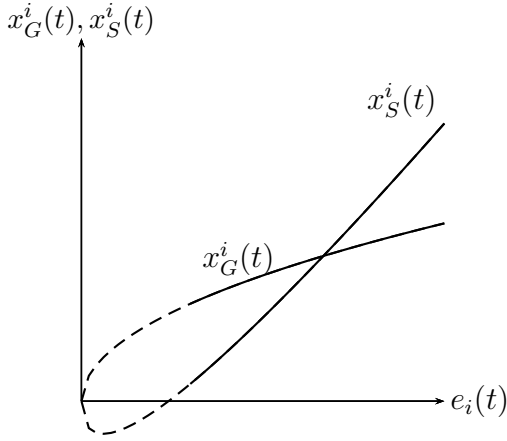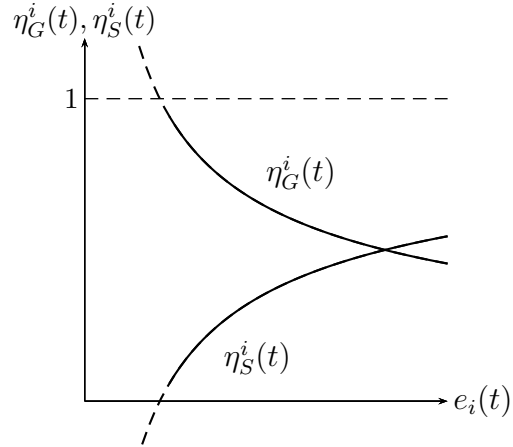
Figure 5: **Engel curves**   Figure 6: **Expenditure shares**

Notes: $\epsilon > 0$ is assumed. As indicated by the dashed sections, preferences are only well defined if condition (3) holds (i.e., $e_i(t)$ exceeds a certain threshold).

In general, the Marshallian demands $x_G^i(t)$ and $x_S^i(t)$ are not linearly homogenous in the per-capita expenditure level. The expenditure shares de-

voted to the two consumption sectors, $\eta_j^i(t)$; $j = G, S$, are[16]

$$\eta_G^i(t) = \nu \left[\frac{P_S(t)}{e_i(t)}\right]^{\epsilon} \left[\frac{P_G(t)}{P_S(t)}\right]^{\gamma} \text{ and } \eta_S^i(t) = 1 - \nu \left[\frac{P_S(t)}{e_i(t)}\right]^{\epsilon} \left[\frac{P_G(t)}{P_S(t)}\right]^{\gamma}. \quad (20)$$

For $\epsilon > 0$, Figure 5 and 6 plot the sectoral Engel curves and expenditure shares as functions of the per-capita expenditure level. As the non-linear Engel curves reveal, preferences are in general non-homothetic and do not even fall into the Gorman class. The expenditure elasticity of demand for goods is $1 - \epsilon$. As long as $\epsilon > 0$, goods are necessities whereas service is a luxury with an expenditure elasticity of demand that exceeds unity. Consequently, in the cross-section, richer households spend a smaller fraction of their budget on goods. This is consistent with the Empirical Regularity 3. In the time series, the implication is that – even if relative prices do not change – growing expenditure levels lead to a declining expenditure share devoted to goods. In contrast, if $\epsilon = 0$, preferences are homothetic and expenditure shares are independent of the expenditure level.

Sign and magnitude of relative price changes on the expenditure shares are controlled by the elasticity of substitution across sectors. In general, the elasticity of substitution is non-constant, but the assumption $0 \leq \epsilon \leq \gamma < 1$ ensure that it is less than unity (see part (iii) of Lemma 3).[17] In the literature there seems to be a consensus that this is the empirically relevant case.[18] This notion is also confirmed in Section 4. An elasticity of

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[16]Note that (2) (and the resulting demand system) is asymmetric. This is because the theory aims to replicate an equilibrium path along which the share of goods decreases at a constant rate (see Empirical Regularity 1). The symmetric counterpart of (2) is discussed in Online Appendix B.1.3.1.

[17]With $\epsilon > \gamma$ or $\epsilon < 0$, the specified utility function could also generate cases where the expenditure elasticity of demand for goods or the elasticity of substitution exceeds unity.

[18]Acemoglu and Guerrieri (2008), Ngai and Pissarides (2008) and Buera and Kaboski (2009) calibrate the elasticity of substitution to 0.76, 0.1 and an asymptotic value of 0.5, respectively. Herrendorf, Rogerson and Valentinyi (2013) estimate an asymptotic elas-

substitution below unity implies that the sector that experiences a relative price increase, grows in terms of expenditure shares (and Baumol's cost disease applies). If the elasticity of substitution were larger than one, the structural change would run in the opposite direction.

Next, we turn to the household's intertemporal optimization problem, whose solution is characterized in the following lemma.

Lemma 4. *Intertemporal optimization yields the Euler equation*

$$(1 - \epsilon)g_{e_i}(t) + \epsilon g_{P_S}(t) = r(t) - \rho, \qquad (21)$$

where $g_{e_i}(t)$ is the growth rate of per-capita consumption expenditures of household i and $g_{P_S}(t)$ is the growth rate of the price of services at date t.

Proof. Since instantaneous utility, $V(\cdot)$, is increasing and strictly concave in $e_i(t)$ we can use the current value Hamiltonian to solve the intertemporal maximization problem. The Hamiltonian is given by

$$\mathcal{H} = V(\cdot) + \lambda_i(t)\left[a_i(t)\left[r(t) - n\right] + w(t)l_i - e_i(t)\right].$$

Then, the first-order conditions are $\dot{\lambda}_i(t) = \lambda_i(t)\left[\rho - r(t)\right]$ and $e_i(t)^{\epsilon-1}P_S(t)^{-\epsilon} = \lambda_i(t)$, which can be rewritten as (21). $\qquad\square$

The Euler equation takes the same functional form as in the standard neoclassical growth model with CRRA preferences. Additionally, since $g_{e_i}(t)$ is the only term that involves a household index i, the Euler equation implies that the growth rate of per-capita expenditure levels is the same for all households at a given point in time, or formally,

$$g_{e_i}(t) = g_e(t), \ \forall i. \qquad (22)$$

ticity of substitution of 0.85 for final consumption expenditure (which is the perspective taken in this paper) and 0.002 if consumption value-added is considered. The elasticity of substitution is related to the price elasticity of demand of the service sector for which estimates are below unity (see Ngai and Pissarides, 2008, for a discussion). Finally, the elasticity of substitution between goods and services has been estimated in international macroeconomics (see, e.g., Stockman and Tesar, 1995 who obtain a value of 0.44).

3.5.3 Aggregation

Since preferences do not fall into the Gorman class, aggregation is non-trivial. Nonetheless, the next proposition shows that the demand side of the economy can be summarized in a tractable way.

Proposition 1. *Under household optimization,*

(i) *the aggregate expenditure share of goods, $\eta_G(t) \equiv \frac{P_G(t)X_G(t)}{E(t)}$, is*

$$\eta_G(t) = \nu \left[\frac{P_S(t)}{\frac{E(t)}{N(t)}} \right]^{\epsilon} \left[\frac{P_G(t)}{P_S(t)} \right]^{\gamma} \phi, \tag{23}$$

where $\phi \equiv \int_0^1 \left[\frac{e_i(0)N(0)}{E(0)} \right]^{1-\epsilon} di$ is a scale invariant (inverse) measurement of inequality of per-capita consumption expenditures across households and $E(t) \equiv N(t) \int_0^1 e_i(t)di$ is aggregate expenditure. Furthermore, we have

$$E(t) = P_G(t)X_G(t) + P_S(t)X_S(t). \tag{24}$$

(ii) *a household with $e_i(t) = \frac{E(t)}{N(t)}\phi^{-\frac{1}{\epsilon}} \equiv e^{RA}(t)$ is the representative agent in Muellbauer's sense.[19]*

(iii) *the intertemporal behavior of the demand side is fully characterized by the following Euler equation, budget constraints and transversality conditions:*

$$(1 - \epsilon)\left[g_E(t) - n\right] + \epsilon g_{P_S}(t) = r(t) - \rho, \; \forall t, \tag{25}$$

where $g_E(t)$ is the growth rate of $E(t)$,

$$\dot{a}_i(t) = \left[r(t) - n\right] a_i(t) + w(t)l_i - e_i(0) \exp\left[\int_0^t g_E(\varsigma) - n \, d\varsigma\right], \; \forall i, t, \tag{26}$$

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[19]For $\epsilon = 0$, we have – according to Muellbauer's definition – the limit case with $e^{RA}(t) = \frac{E(t)}{N(t)}$.

and

$$\lim_{t \to \infty} a_i(t) \exp\left[-\int_0^t r(\varsigma) - n \, d\varsigma\right] = 0, \ \forall i, \tag{27}$$

where $a_i(0)$, $\forall i$, is exogenously given.

Proof. Aggregation of individual demands gives

$$X_G(t) = \nu P_G(t)^{-1} P_S(t)^\epsilon \left[\frac{P_G(t)}{P_S(t)}\right]^\gamma \left[\frac{E(t)}{N(t)}\right]^{-\epsilon} E(t)\phi(t),$$

$$X_S(t) = \frac{E(t)}{P_S(t)} - \nu P_S(t)^{\epsilon-1} \left[\frac{P_G(t)}{P_S(t)}\right]^\gamma \left[\frac{E(t)}{N(t)}\right]^{-\epsilon} E(t)\phi(t),$$

where $\phi(t) = \int_0^1 \left[\frac{e_i(t)N(t)}{E(t)}\right]^{1-\epsilon} di$. These two equations imply (23) and (24), where $\phi(t)$ is constant over time because of (22) and because it is scale invariant in all $e_i(t)$. (20) and (23) show that a household exhibits the same expenditure shares as the aggregate economy if $e_i(t) = \frac{E(t)}{N(t)}\phi^{-\frac{1}{\epsilon}}$. For part (iii): (22) implies $g_{e_i}(t) = g_E(t) - n$, $\forall i$, allowing us to rewrite (21) as (25). Substituting $e_i(t)$ in (4) by $e_i(0) \exp\left[\int_0^t g_E(\varsigma) - n \, d\varsigma\right]$ yields (26). Using (21) in (5) and ignoring the positive constant $e_i(0)$ gives (27). □

For a given path of prices, $\{r(t), w(t), P_G(t), P_S(t)\}_{t=0}^\infty$, this proposition fully characterizes the demand side of the economy. Since preferences are part of the PIGL class a representative agent in Muellbauer's sense exists (see Muellbauer, 1975 and Muellbauer, 1976). A household with the (price independent) representative expenditure level, $e^{RA}(t)$, exhibits the same expenditure shares as the aggregate economy. In order to compute the aggregate demand structure $\eta_G(t)$, it is sufficient to know the average per-capita expenditure level and one additional scale invariant inequality measure of per-capita expenditure levels $\phi(t) = \int_0^1 \left[\frac{e_i(t)N(t)}{E(t)}\right]^{1-\epsilon} di$.[20]

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[20]With $\epsilon > 0$, a high dispersion of per-capita expenditure levels is associated with a low value of ϕ. ϕ is a (negative) monotonic transformation of an Atkinson index (Atkinson, 1970) with relative inequality aversion ϵ (see Online Appendix B.1.7 for an illustration). In the homothetic case, we have a representative agent economy in the narrower sense, where inequality does not matter (i.e., $\phi = 1$).

In addition, intertemporal optimization implies the same per-capita expenditure growth rate for all households at any given point in time (see (22)). Consequently, all individual expenditure dynamics can be described as a function of average expenditure growth and ϕ, which is constant over time. This property renders the model tractable, and allows to solve it analytically, despite household heterogeneity, non-Gorman intratemporal preferences and intertemporal optimization.[21]

3.5.4 Equilibrium path

The dynamic competitive equilibrium is fully characterized by the equations (6), (8)-(15) and (23)-(27). The endogenous variables are: $X_j(t)$ and $Y_j(t)$, $j = G, S, I$; $a_i(t)$ and $e_i(0)$, $\forall i$; $E(t)$, $P_j(t)$, $j = G, S$; $w(t)$, $r(t)$, $L_j(t)$, $j = G, S$; $K(t)$ and $K_j(t)$, $j = G, S, I$. $a_i(0)$, $\forall i$, is exogenously given. Solving for the dynamic competitive equilibrium, we obtain the following proposition.

Proposition 2. *Suppose that the exogenous parameters satisfy the following constraints:*

$$A - \delta - \rho + \epsilon g_S > 0, \tag{28}$$

$$\rho > (1-\alpha)\epsilon\left[A - \delta - n\right] + n + \epsilon g_S, \tag{29}$$

$$\alpha^\epsilon \bar{l}^\epsilon \geq \frac{1-\epsilon}{1-\gamma}\nu\left[\frac{L(0)}{K(0)}\frac{A\left(1 - (1-\alpha)\epsilon\right)}{\rho - n - \epsilon g_S - \epsilon(1-\alpha)\left(A - \delta - n\right)}\right]^{\epsilon(1-\alpha)}, \tag{30}$$

and

$$\gamma\left[g_S - g_G\right] - \epsilon\left[\frac{g_S + (1-\alpha)\left[A - \delta - \rho\right]}{1 - (1-\alpha)\epsilon}\right] \leq 0. \tag{31}$$

Then, there exists a unique dynamic competitive equilibrium path along which

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[21]In contrast to models with 0/1 preferences (see, e.g., Foellmi and Zweimueller, 2006 and Foellmi, Wuergler and Zweimueller, 2009) this model focuses on the intensive margin of consumption. Moreover, the model at hand allows us to study any – possibly continuous – income distribution with a lower bound such that condition (3) is fulfilled.

(i) per-capita consumption expenditures, wages, aggregate capital and capital allocated to the consumption sectors grow at constant rates

$$g_E^* - n = g_w^* = \frac{A - \delta - \rho + \epsilon g_S}{1 - (1 - \alpha)\epsilon} > 0, \tag{32}$$

$$g_K^* = g_{K_G + K_S}^* = g_E^*. \tag{33}$$

The saving rate is constant and the real, investment-good denominated interest rate is given by $A - \delta$. The prices of goods and services change at constant rates

$$g_{P_j}^* = -g_j + \alpha \left[g_E^* - n \right], \ j = G, S. \tag{34}$$

(ii) the expenditure share devoted to goods changes at constant rate

$$g_{\eta_G}^* = -\gamma \left[g_G - g_S \right] - \epsilon \left[g_S + (1 - \alpha) \left[g_E^* - n \right] \right] \leq 0. \tag{35}$$

Capital and labor allocated to the goods sector grow at constant rates

$$g_{K_G}^* = g_K^* + g_{\eta_G}^* \leq g_K^* \leq g_{K_S}^*(t), \ and \ g_{L_G}^* = n + g_{\eta_G}^* \leq n \leq g_{L_S}^*(t), \ \forall t. \tag{36}$$

The relative price between consumption goods and services changes at a constant rate

$$g_{P_G}^* - g_{P_S}^* = g_S - g_G. \tag{37}$$

Proof. See Appendix A.1. □

Proposition 2 demonstrates that the model reconciles structural change and changing relative prices at a sectoral level with balanced growth on the aggregate. Part (i) illustrates that the model features on the aggregate the standard properties of neoclassical growth theory (i.e., the Kaldor facts hold). The per-capita output growth rate, the capital-output ratio, the saving rate and the labor income share are constant over time. Moreover, the real, investment good denominated interest rate is equal to $A - \delta$. Since

relative prices change at constant rates (see (34)), any price index with constant sectoral weights grows at a constant rate too. Hence, deflated by any constant-weights-price-index, the real per-capita expenditure growth rate and real interest rate would be constant. However, in an economy with structural change, the sectoral weights of the true cost of living price index adjust over time. This implies that, deflated by the true cost of living price index, the growth rate is not constant. Consequently, a Baumol's cost disease can arise, where the low productivity growth sector constitutes an increasing fraction of total expenditures. However, changes in the growth rate of the price index due to weight adjustments are typically relatively small (see Ngai and Pissarides, 2004).[22]

The model exhibits no transitional dynamic and can be solved analytically. As in Rebelo (1991), the equilibrium growth rate is increasing in the marginal product of capital, A, and decreasing in the rate of time preference, ρ, and the depreciation rate, δ. A specific model characteristic is that the intertemporal substitution elasticity of expenditure, $\frac{1}{1-\epsilon}$, is tied together with the expenditure elasticity of demand for goods, ϵ.[23]

Part (ii) of Proposition 2 emphasizes the equilibrium's non-balanced features on the sectoral level. Although the Kaldor facts hold, expenditure shares as well as relative prices change over time. The functional forms of these changes are notable too. Consistent with Empirical Regularity 1 and

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[22]The growth rate of the partial true cost of living price index of household i is defined as $g_P^{TCL}(t) = g_{P_S}(t) + \eta_G^i(t)\left[g_{P_G}(t) - g_{P_S}(t)\right]$ (see Pollak, 1975). In the data, the relative price growth rate is -1.6 percent and in 2011 the aggregate expenditure share of goods was 0.34, whereas its asymptotic value is zero. Hence, measured by the true cost of living price index of the representative household, the model predicts the real interest rate in 2011 to be 0.005 higher than its asymptotic value.

[23]With $\epsilon = 0$, this interdependence reflects the result obtained by Ngai and Pissarides (2007): If preferences are homothetic, reconciliation of structural change with the Kaldor facts requires that the intertemporal substitution elasticity of expenditures is equal to unity.

2, the model predicts that both the expenditure share of goods and the price of goods relative to services decrease at constant rates. The change in the aggregate demand structure translates to the production side (see (36)). Capital allocated to the goods sector grows at a lower rate than the aggregate capital stock, which itself grows at a lower rate than capital allocated to the service sector. The same applies to the allocation of labor. If n is small relative to $g^*_{\eta_G}$, the absolute quantity of labor allocated to the goods sector can even decrease.

The required parametric restrictions (28)-(31) are innocuous. In particular, reconciliation of the non-balanced features of growth with the Kaldor facts does not depend on any knife-edge condition. Positive capital accumulation and growth in per-capita terms is ensured by (28). Condition (29) is necessary and sufficient for the transversality condition to hold. Furthermore, it is also sufficient to ensure finite utility. Condition (30) makes sure that condition (3) is met for all households at $t = 0$. Moreover, together with condition (31), it ensures condition (3) along the entire equilibrium path.

The expenditure structure changes along the equilibrium path because per-capita expenditure levels grow and because the relative price between goods and services changes. Given preferences, any change in the consumption bundle can be decomposed in a substitution effect (along Hicksian demands) and an income effect (residual effect). This decomposition implies that the expenditure share for goods of household i changes along the equilibrium path at rate $-\left[g_G - g_S\right]\left(\gamma - \epsilon\eta^i_G(t)\right)$ due to the substitu-

tion effect.[24] The remaining decline at rate $-\epsilon\left[g_S + (1-\alpha)\left[g_E^* - n\right]\right] - \epsilon\left[g_G - g_S\right]\eta_G^i(t) \leq 0$ is associated with the income effect. Since service is a luxury, the income effect decreases the goods share.[25] Depending on the direction of the relative price change, the substitution effect can go in either direction. If the TFP growth rate is larger in the service sector (i.e., $g_S > g_G$) the two effects go in opposite directions. But, as will be shown in Section 4, in the empirically relevant case the income and substitution effect run in the same direction and Baumol's cost disease is reinforced by the income effect.

The decomposition of the decline in the goods share in a substitution and income effect is individual specific. For richer households (with a lower $\eta_G^i(t)$), the substitution effect is relatively more important. Consequently, as all $\eta_G^i(t)$ decline, the relative importance of the income effect as a determinant of the aggregate structural change decreases over time.[26] However, the sum of substitution and income effect is not individual specific. According to the model, for all individuals, the expenditure share of goods decreases at the identical constant rate $g_{\eta_G}^*$. This is consistent with the linear and parallel decline of the logarithmized expenditure shares of different income quintiles (see Figure 4).

The model further makes a testable auxiliary prediction on the cross-sectional relation of expenditure shares and total expenditure levels. Log-

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[24]Using Hicksian demand, $\eta_G^i(t)$ can be written as $\eta_G^i(t) = \nu\left[\epsilon\left[V_i(t) + \frac{\nu}{\gamma}\left[\frac{P_G(t)}{P_S(t)}\right]^\gamma + \frac{1}{\epsilon} - \frac{\nu}{\gamma}\right]\right]^{-1}\left[\frac{P_G(t)}{P_S(t)}\right]^\gamma$. For the elasticity of $\eta_G^i(t)$ with respect to $\frac{P_G(t)}{P_S(t)}$ this implies $\gamma - \epsilon\eta_G^i(t)$. The change of the relative price along the equilibrium path is given by (37). If the relative price does not change (i.e., if $g_S = g_G$) there is no substitution effect.

[25]With homothetic preferences (i.e., $\epsilon = 0$) there is no income effect.

[26]Since preferences allow for a representative agent in Muellbauer's sense, the substitution effect of the aggregate economy is the same as the substitution effect for the representative agent.

arithmizing both sides of (20) gives

$$\log \eta_G^i(t) = b(t) - \epsilon \log e_i(t), \tag{38}$$

where $b(t) \equiv \log[\nu P_S(t)^{\epsilon-\gamma} P_G(t)^\gamma]$. Hence, allowing for a time dependent intercept $b(t)$, the model predicts an iso-elastic relation between the expenditure share of goods and the per-capita expenditure level of different households. Figure 7 depicts the conditional scatter plot between the logarithm of these two variables for the income quintiles of different years. It is striking how well a linear approximation fits the data.
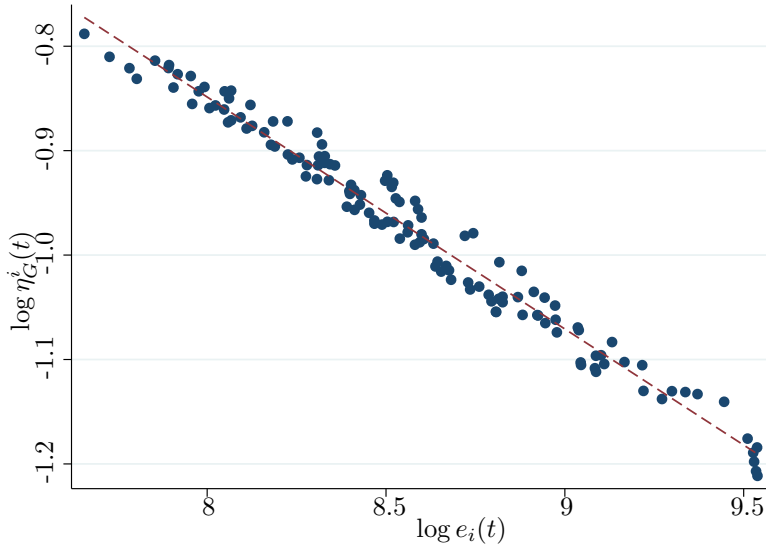

Figure 7: **Scatter plot of cross-sectional variation**

Notes: For the years 1986-2011, the figure depicts the conditional scatter-plot between the logarithmized expenditure level per-equivalent scale and the logarithmized expenditure share of goods of each income quintile, where we allowed in each year for a separate (distinct) intercept. The slope of the fitted line is -0.2222. This slope is the same as if we regressed the logarithmized expenditure share on the logarithmized expenditure level per equivalent scale and time dummies. The R^2 of this underlying regression is 0.9776 and the standard error of the slope coefficient is 0.0042. Source: CEX interview data obtained from the BLS for the year 2011 and from the ICPSR for the years 1986-2010.

Next, I characterize the equilibrium toward which the economy converges as time goes to infinity. To do so, we define:

Definition 2. *The asymptotic equilibrium is the dynamic competitive equilibrium path toward which the economy converges as time goes to infinity.*

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We have the following proposition (asymptotic equilibrium values are denoted by a superscript A).

Proposition 3. *Suppose condition* (31) *holds with strict inequality (i.e., there is structural change). Then, in the asymptotic equilibrium,*

  (i) *the expenditure share devoted to goods is equal to zero, i.e., $\eta_G^A = 0$,*

  (ii) *the expenditure elasticity of demand is $1 - \epsilon$ for goods and unity for services,*

  (iii) *the elasticity of substitution between goods and services, σ_i^A, is equal to $1 - \gamma$ for all households i.*

Proof. Since (31) holds with strict inequality, η_G converges to 0 (see (35)) and the elasticities of Lemma 3 converge to the corresponding values. □

Part (i) of Proposition 3 shows that the service sector is the asymptotically dominant consumption sector. The existence of an asymptotically dominant sector is a common feature of the models by Ngai and Pissarides (2007), Acemoglu and Guerrieri (2008) and Foellmi and Zweimueller (2008). The asymptotic dominance of the service sector is not a result of a trivial disappearance of the good sector. In absolute terms, the asymptotically consumed quantity of goods goes to infinity – even in per-capita terms.

Part (ii) and (iii) of Proposition 3 illustrate that the model is relatively parsimonious. The expenditure elasticity of demand and the elasticity of substitution across sectors control the sign and magnitude of relative price and income effects on the demand structure. The model has two parameters, ϵ and γ, which control separately the asymptotic values of these two elasticities. In general, both income and substitution effects are even asymptotically present (note that all the properties stated in Proposition 2 hold asymptotically too). The possibility to calibrate freely a sector's

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asymptotic expenditure elasticity of demand is a constrast to other non-homothetic preferences used in the literature. In Foellmi and Zweimueller (2008), the asymptotic expenditure elasticity of demand of the dominated sector(s) is zero, whereas it is unity for all sectors in Kongsamut, Rebelo and Xie (2001).[27]

So far, it has been shown that the model is consistent with a unique dynamic competitive equilibrium path, along which the Kaldor facts hold and changes in expenditure shares and relative prices occur. The model has been constructed to match a decline of the expenditure share of goods and the relative price of goods at a constant rate. Furthermore, the theory is consistent with the functional form of non-homotheticity observed in micro data. An additional test of the model is to examine its quantitative prediction. According to the theory, the following relationship holds (see (23))

$$g^*_{\eta_G} = -\epsilon \left(g^*_E - g^*_{P_S} - n \right) + \gamma \left(g^*_{P_G} - g^*_{P_S} \right), \tag{39}$$

where we made use of the constancy of the involved growth rates along the equilibrium path. The data suggests $g^*_{\eta_G} = -0.010$, $g^*_E - g^*_{P_S} - n = 0.016$ and $g^*_{P_G} - g^*_{P_S} = -0.016$.[28] Plugging these values into (39) we conclude that

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[27]A Stone-Geary specification's inexistence of asymptotic income effects leads to a suboptimal fit of the data. Buera and Kaboski (2009) show in their calibration: "The model fails to match the sharper increase in services and decline in manufacturing after 1960. [...] Explaining this would require a large, delayed income effect toward services. This is not possible with the Stone-Geary preferences, where the endowments and subsistence requirements are most important at low levels of income." (Buera and Kaboski, 2009, p. 473-474.) Moreover, with quasi-homothetic preferences, income effects are one-to-one connected to the subsistence level(s), often leading to binding subsistence levels in empirical estimations. Contrary to this, in the presented theory, ϵ controls the magnitude of the income effect for any given expenditure and price path (as well as for any given initial expenditure shares).

[28]See Figure 1 and 2 as well as Figure B.5 in the Online Appendix B.1.9, which illustrate that the model approximates the three series well.

the model is quantitatively consistent with the observed structural change, growth and relative price dynamics as long as the (ϵ, γ)-combination fulfills

$$\epsilon + \gamma = 0.625. \tag{40}$$

For such a calibration, the model's equilibrium path also generates a hump-shaped evolution of the relative quantity of services.[29]

The relative size of the two parameters ϵ and γ control the elasticity of substitution and the expenditure elasticity of demand. Consequently, restriction (40) is uninformative about the magnitude of the income and substitution effect on the structural change. It is the aim of the next subsection to quantify these two forces.

4 Quantification of income and substitution effects

The theory's prediction for a household's expenditure structure is given by (38). This suggests that ϵ can be identified as the slope coefficient in a regression of a household's logarithmized expenditure share of goods on the logarithmized expenditure level, while controlling for time fixed effects. In this regression, infrequently bought items, like durable goods or clothing, lead to endogeneity of the expenditure level. Following the literature (see, e.g., Blundell, Pashardes and Weber, 1993), I hence instrument expenditure levels by household income.[30] Column (1) in Table 1 reports the

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[29]We have $\frac{X_S(t)}{X_G(t)} = \frac{P_G(t)}{P_S(t)} \frac{1 - \eta_G(t)}{\eta_G(t)}$ or in terms of growth rate along the equilibrium path $g_{X_S}(t) - g_{X_G}(t) = g^*_{P_G} - g^*_{P_S} - \frac{1}{1 - \eta_G(t)} g^*_{\eta_G}$. With the observed growth rates of the relative price and expenditure share, this implies that $\frac{X_S(t)}{X_G(t)}$ reaches its maximum at $\eta_G(t) = 0.375$. The dashed line in Figure 3 shows the relative quantity implied by $g^*_{\eta_G} = -0.010$ and $g^*_{P_G} - g^*_{P_S} = -0.016$.

[30]An alternative approach to solve the simultaneity problem is to group households according to their income level. As can be inferred from Figure 7, or Figure B.4 in the

results of such a time fixed effects instrumental-variable (IV) regression using quarterly CEX data. The estimated ϵ is 0.18 and homotheticity of preferences (i.e., $\epsilon = 0$) is clearly rejected. In column (2), I additionally control for a large set of household characteristics (sex, race, skill level, place of residence and household size and age composition). The estimate for ϵ slightly increases to 0.22.[31] Column (3) presents results when using Panel Study of Income Dynamics (PSID) data. I find an estimate in a very similar ballpark as before. The results are hence robust to using a different data source.[32]

In column (4) I exclude durable goods. This leads to a larger estimated degree of non-homotheticity of 0.29.[33] Column (5) reports the results of a corresponding ordinary least-squares (OLS) regression. Compared to the OLS estimate, the IV estimate is significantly smaller. This suggests that there still exists an endogeneity bias due to infrequently bought items.

Overall, we conclude, that the cross-sectional data allows to estimate ϵ precisely and suggests a value of about 0.22. This value implies an expenditure elasticity of demand for goods of 0.78. An alternative way to interpret the parameter value is to look at the elasticity of substitution implied by condition (40). With $\epsilon = 0.220$, a replication of the aggregate structural change requires $\gamma = 0.405$. This means that the elasticity of substitution converges (from below) to an asymptotic value of $1 - \gamma = 0.596$ (see Proposition 3). This value of the elasticity of substitution is in the range of other estimates

Online Appendix B.1.9 for the UK, this leads to very similar estimates for ϵ.

[31] Figure B.6 in the Online Appendix B.1.9 shows the estimates for ϵ if we run the regression of column (2) for each year separately. $\hat{\epsilon}$ is stable over time and most of the estimates lie between 0.20 and 0.25.

[32] Moreover, Table B.2 in the Online Appendix B.1.9 shows that similar coefficients are obtained if we use CEX diary data.

[33] This is consistent with the fact that without durable good, the pace of structural change is faster although relative prices change at a slower rate (see Figure B.8 and B.9 in the Online Appendix B.1.9).

Dependent variable: $\log \eta_G^i(t)$

	baseline			excluding durables	
	(1)	(2)	(3)	(4)	(5)
$-\log e_i(t)$	0.181***	0.224***	0.168***	0.293***	0.181***
	(0.002)	(0.003)	(0.024)	(0.003)	(0.001)
Children share		0.124***	0.049	0.203***	0.211***
		(0.004)	(0.051)	(0.004)	(0.004)
Elderly share		-0.052***	0.014	-0.060***	-0.025***
		(0.003)	(0.145)	(0.003)	(0.003)
Residence indicators	No	Yes	Yes	Yes	Yes
Family size indicators	No	Yes	Yes	Yes	Yes
Ref. person controls	No	Yes	Yes	Yes	Yes
Data	CEX	CEX	PSID	CEX	CEX
Sample years	86-11	86-11	2009	86-11	86-11
Method	IV	IV	IV	IV	OLS
Observations	477,730	425,402	4,770	425,402	425,402
R^2	0.013	0.041	0.152	0.143	0.156

Table 1: **Cross-sectional estimation of ϵ**

Notes: Standard errors in parentheses. *** significant at 1 percent, ** significant at 5 percent, * significant at 10 percent. All regressions with the CEX data include quarter fixed effects (104 groups). The logarithmized expenditure level per equivalent scale is instrumented by the logarithmized after tax labor earnings plus transfers per equivalent scale in the CEX data and by the logarithmized "Total Labor Income" plus "Total Transfer Income" plus "Social Security Income" per OECD equivalent scale in the PSID data. "Children share" and "Elderly share" measure the share of household members with age < 18 and ≥ 65, respectively. "Residence indicators" consists of regional indicators (4 groups in the CEX data and 5 groups in the PSID data), a rural/urban dummy in the case of the CEX data, as well as indicators of different population sizes of the city of residence (5 groups in the CEX data and 6 groups in the PSID data). "Family size indicators" consists of 11 groups. "Ref. person controls" consists of the age, the sex, skill level indicators (7 groups) and race indicators (4 groups in the CEX data and 7 groups in the PSID data) of the reference person. For a detailed data description see Online Appendix B.1.8. Using total income (instead of labor income) as an instrument leads to very similar results (see Table B.1 in the Online Appendix B.1.9).

and calibrations (see footnote 18).

These baseline estimates for ϵ and γ highlight that both channels of structural change are of empirical importance. The model could generate the observed structural change with an income effect alone (and an asymptotic elasticity of substitution equal to unity). However, this would require an ϵ of 0.625 (see 40), implying an expenditure elasticity of demand for goods of $1 - \epsilon = 0.375$. Such a strong income effect is clearly at odds with cross-sectional data.[34] Conversely, the homothetic case with $\epsilon = 0$ is also clearly rejected by the data.

For given preference parameters, we can decompose the aggregate structural change into a substitution and an income effect. With $\epsilon = 0.220$ and $\gamma = 0.405$, the model suggests that in the year 1946, 44 percent of the observed structural change is attributable to a substitution effect, whereas the remaining 56 percent is attributable to the income effect.[35] In 2011, the corresponding numbers are 53 percent and 47 percent respectively. Furthermore, the model predicts that the relative contribution of the substitution effect will asymptotically converge to 65 percent.

The time fixed effects regressions of Table 1 exploit solely cross-sectional variation in nominal expenditure levels and shares.[36] Hence, the estimates for ϵ do not rely on sector specific price data. Even if the price index of goods or services is mismeasured, the estimated degree of non-homotheticity is unaffected and a statement about the importance of in-

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[34]See Figure B.7 in the Online Appendix B.1.9 for a graphical illustration of this fact.

[35]In 1946, the goods sector accounted for 60 percent of total personal consumption expenditures. Then, the change in expenditure share attributed to the substitution effect is equal to an annualized rate of $(-0.405 + 0.220 \cdot 0.6) \cdot 1.6 = -0.435$ (see footnote 26).

[36]This is a contrast to the recent empirical work by Buera and Kaboski (2009) and Herrendorf, Rogerson and Valentinyi (2013), who also estimate the relative contribution of income and substitution effects on structural change in aggregate U.S. data.

come effects can still be made. The estimated (residual) γ however would be affected by price mismeasurements (and consequently also the estimated asymptotic elasticity of substitution).

As an alternative, using price data, ϵ and γ can jointly be estimated from the micro panel data. The functional form (20) implies

$$\log \eta_G^i(t) = \log \nu + \gamma \log \left[\frac{P_G(t)}{P_S(t)} \right] - \epsilon \log \left[\frac{e_i(t)}{P_S(t)} \right], \qquad (41)$$

and suggests regressing the logarithmized expenditure share on a constant, the logarithmized relative price and the (instrumented) logarithmized expenditure level in terms of services. Comparing these estimates to the ones of the baseline approach tests the consistency of framework and data in two ways: First, it checks whether the variation, which is in the baseline specification absorbed by the time fixed effects, is consistent with the (measured) price data. And second, it tests whether the micro panel estimate for γ is broadly consistent with the structural change in the aggregate data.

Using the price data, an IV regression as suggested by (41) yields $\hat{\epsilon} = 0.182$ and $\hat{\gamma} = 0.410$ with standard errors of 0.002 and 0.004. Compared to the time fixed effects regression in column (1) of Table 1, the estimated degree of non-homotheticity is basically unchanged. Finally, the estimated ϵ and γ closely match restriction (40). This exercise suggests micro estimates which are surprisingly consistent with the observed structural change in the aggregate (post-war) data.[37]

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[37]The BEA provides aggregate data back to 1929. Figure B.15 and B.16 in Online Appendix B.1.9 show the expenditure structure predicted by the aggregate price and expenditure data as well as the micro panel estimates $\hat{\epsilon} = 0.182$ and $\hat{\gamma} = 0.410$. In contrast to a generalized Stone-Geary specification, the model does account for the late rise of the service economy emphasized by Buera and Kaboski (2012a, 2012b).

5   Conclusion

This paper presents a parsimonious growth theory that is consistent with structural change, relative price dynamics and the Kaldor facts. The model allows us to analyze both explanations of structural change – income and substitution effects – simultaneously. To the best of my knowledge, no such theory was previously available.

The virtues of the theory are twofold. First, the model's functional form fits the data very well and the framework can replicate the observed structural change quantitatively. Moreover, not only the model's aggregate predictions but also the implied cross-sectional variation in the expenditure structure are confirmed by the data. The paper also shows how this cross-sectional variation can be exploited to estimate the model's key parameters and quantify the two driving forces of structural change. They are of roughly equal importance.

The second virtue is given by the exact replication of the Kaldor facts, which is clearly desirable from an empirical point of view. In the data, we see a fast and persistent structural change. Reconciling this with a relatively stable interest, saving, and aggregate growth rate is challenging. Although some calibrations of models of structural change are approximately consistent with the Kaldor facts, others are clearly not. This paper suggests that this shortcoming is mainly a consequence of the functional form of the specified intratemporal utility function.

The exact replication of the Kaldor facts is appealing from a theoretical perspective too. Structural change is interrelated to many important aspects in macroeconomics like demographics, labor supply, income inequality and convergence, international trade, environmental economics or biased technical change. These phenomena are often outlined in standard one-sector neoclassical growth models (with balanced growth). To analyze them in a

multi-sector model, a theory of structural change which is at the same time analytically tractable and empirically exact is a prerequisite.[38] I hope the presented framework proves to be useful in order to study these important questions.

References

Acemoglu, D. and Guerrieri, V. (2008). "Capital Deepening and Non-balanced Economic Growth", *Journal of Political Economy*, **116**(3), 467-497.

Acemoglu, D. (2009). *"Modern Economic Growth"*, Princeton, New Jersey: Princeton University Press.

Atkinson, A. B. (1970). "On the measurement of inequality", *Journal of Economic Theory*, **2**(3), 244-263.

Baumol, W. (1967). "Macroeconomics of Unbalanced Growth: The Anatomy of Urban Crisis", *American Economic Review*, **57**(3), 415-426.

Blundell, R., Pashardes, P. and Weber, G. (1993). "What do we Learn About Consumer Demand Patterns from Micro Data?", *American Economic Review*, **83**(3), 570-597.

Boskin, M., Dulberger, E., Gordon, R., Griliches, Z., and Jorgenson, D. (1996) "Toward a more accurate measure of the cost of living", Final Report to the Senate Finance Committee from the advisory commission to study the Consumer Price Index.

Browning, M. (2008). "Engel's Law", The New Palgrave Dictionary of Economics, Second Edition, Palgrave Macmillan.

Buera, F. and Kaboski, J. P. (2009). "Can Traditional Theories of Structural Change fit the Data?", *Journal of the European Economic Association*, **7**(2-3), 469-477.

Buera, F. and Kaboski, J. P. (2012a). "Scale and the origins of structural change", *Journal of Economic Theory*, **147**(2), 684-712.

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[38]For example, it would be interesting to augment the utility function of this paper to allow for home production (as considered by Ngai and Pissarides, 2008, Rogerson, 2008, Buera and Kaboski, 2012a, 2012b or Ngai and Petrongolo, 2013).

Buera, F. and Kaboski, J. P. (2012b). "The Rise of the Service Economy", *American Economic Review*, **102**(6), 2540-69.

Caselli, F. and Coleman, J. (2001). "The U.S. Structural Transformation and Regional Convergence: A Reinterpretation", *Journal of Political Economy*, **109**(3), 584-617.

Deaton, A. and Muellbauer, J. (1980). "An Almost Ideal Demand System", *American Economic Review*, **70**(3), 312-326.

Echevarria, C. (1997). "Changes in Sectoral Composition Associated with Economic Growth", *International Economic Review*, **38**(2), 431-452.

Engel, E. (1857). "Die Productions- und Consumptionsverhaeltnisse des Koenigsreichs Sachsen", *Zeitschrift des Statistischen Bureaus des Königlich Sächsischen Ministeriums des Inneren*, No. 8 and 9.

Falkinger, J. (1990). "On Growth Along a Hierarchy of Wants", *Metroeconomica*, **41**(3), 209-223.

Falkinger, J. (1994). "An Engelican Model of Growth and Innovation With Hierarchic Demand and Unequal Incomes", *Ricerche Economiche*, **48**(2), 123-139.

Foellmi, R. and Zweimueller, J. (2006). "Income Distribution and Demand-Induced Innovations", *Review of Economic Studies*, **73**(4), 941-960.

Foellmi, R. and Zweimueller, J. (2008). "Structural Change, Engel's Consumption Cycles and Kaldor's Facts of Economic Growth", *Journal of Monetary Economics*, **55**(7), 1317-1328.

Foellmi, R., Wuergler, T. and Zweimüller, J. (2009). "The Macroeconomics of Model T", *IEW Working Paper*, No. 459.

Gollin, D., Parente, S. and Rogerson, R. (2002). "The Role of Agriculture in Development", *American Economic Review*, **92**(2), 160-164.

Greenwood, J. and Seshadri, A. (2002). "The U.S. Demographic Transition", *American Economic Review*, **92**(2), 153-159.

Herrendorf, B., Rogerson, R. and Valentinyi, A. (2013). "Two Perspectives on Preferences and Structural Transformation", *American Economic Review*, **103**(7), 2752-2789.

Howe, H., Pollak, R. and Wales, T. (1979). "Theory and Time Series Estimations of the Quadratic Expenditure System", *Econometrica*, **47**(5), 1231-1246.

Houthakker, H. S. (1957). "An International Comparison of Household Patterns, Commemorating the Century of Engel's Law", *Econometrica*, **25**(4), 532-551.

Kaldor, N. (1961). *"Capital Accumulation and Economic Growth"*, In The Theory of Capital: Proceedings of a Conference of the International Economic Association, edited by Friedrich A. Lutz and Douglas C. Hague. London: Macmillan.

Kongsamut, P., Rebelo, S. and Xie, D. (2001). "Beyond Balanced Growth", *Review of Economic Studies*, **68**(237), 869-882.

Kuznets, S. (1957). "Quantitative Aspects of the Economic Growth of Nations II: Industrial Distribution of National Product and Labor Force", *Economic Development and Cultural Change*, **5**(4), 1-111.

Laitner, J. (2000). "Structural Change and Economic Growth", *Review of Economic Studies*, **67**(232), 545-561.

Matsuyama, K. (1992). "Agricultural Productivity, Comparative Advantage, and Economic Growth", *Journal of Economic Theory*, **58**(2), 317-334.

Matsuyama, K. (2002). "The Rise of Mass Consumption Societies", *Journal of Political Economy*, **110**(5), 1053-1069.

Muellbauer, J. (1975). "Aggregation, Income Distribution and Consumer Demand", *Review of Economic Studies*, **62**(4), 526-543.

Muellbauer, J. (1976). "Community Preferences and the Representative Consumer", *Econometrica*, **44**(5), 979-999.

Ngai, R. and Petrongolo, C. (2013). "Gender Gaps and the Rise of the Service Economy", *CEPR Discussion Paper*, No. 1204.

Ngai, R. and Pissarides, C. (2004). "Structural Change in a Multi-Sector Model of Growth", *CEPR Discussion Paper*, No. 627.

Ngai, R. and Pissarides, C. (2007). "Structural Change in a Multi-Sector Model of Growth", *American Economic Review*, **97**(1), 429-443.

Ngai, R. and Pissarides, C. (2008). "Trends in hours and economic growth", *Review of Economic Dynamics*, **11**(2), 239-256.

Pollak, R. A. (1975). "The intertemporal cost of living index", *Annals of Economic and Social Measurement*, **4**(1), 179-198.

Rebelo, S. (1991). "Long-Run Policy Analysis and Long-Run Growth", *Journal of Political Economy*, **99**(3), 500-521.

Rogerson, R. (2008). "Structural Transformation and the Deterioration of European Labor Market Outcomes", *Journal of Political Economy*, **116**(2), 235-259.

Stockman, A. C. and Tesar, L. L. (1995). "Tastes and Technology in a Two-Country Model of the Business Cycle: Explaining International Comovements", *American Economic Review*, **85**(1), 168-185.

Valentinyi, A. and Herrendorf, B. (2008). "Measuring factor income shares at the sectoral level", *Review of Economic Dynamics*, **11**(4), 820-835.

Zweimueller, J. (2000). "Schumpeterian Entrepreneurs Meet Engel's Law: The Impact of Inequality on Innovation-Driven Growth", *Journal of Economic Growth*, **5**(2), 185-206.

A.1 Appendix: Proof of Proposition 2

Proof. First, I show that there exists a unique equilibrium in which $g_e(t)$ grows at a constant rate. (24), (10), (14) and (15) imply $E(t) = \frac{A}{1-\alpha}[K_G(t) + K_S(t)]$. Hence, we have $g_E(t) = g_e(t) + n = g_{K_G+K_S}(t)$. Using this in (14) yields (34). Plugging (12) and (34) into (25) we get $[1 - (1-\alpha)\epsilon]\, g_e(t) = A - \delta - \rho + \epsilon g_S$. This proves that we have $g_e(t) = g_e^*$, $\forall t$ in equilibrium. Next, we show that – given $g_e(t) = g_e^*$ – the transversality condition holds if and only if per-capita wealth grows at rate g_e^* too. With (12), the transversality condition, (27), can be rewritten as

$$\lim_{t \to \infty} a_i(t) \exp\left[-(A - n - \delta)t\right] = 0, \ \forall i. \qquad (A.1)$$

(13), $g_E^* = g_{K_G+K_S}^*$ and $g_E(t) = g_e^* + n$ yield $g_w = g_e^*$. Then, with (12), the flow budget constraint, (26), simplifies to $\dot{a}_i(t) = [A - \delta - n]\, a_i(t) - [e_i(0) - w(0)l_i]\exp[g_e^* t]$. This linear differential equation has the following solution (see, e.g., Acemoglu, 2009, Section B.4)

$$a_i(t) = \mathcal{A}_i \exp\left[(A - \delta - n)\, t\right] + \frac{e_i(0) - w(0)l_i}{A - \delta - n - g_e^*}\exp\left[g_e^* t\right], \qquad (A.2)$$

where $\mathcal{A}_i$ is a constant which is to be determined. Using this expression in (A.1) we get

$$\lim_{t \to \infty} \mathcal{A}_i + \frac{e_i(0) - w(0)l_i}{A - \delta - n - g_e^*} \exp\left[- (A - \delta - n - g_e^*)\, t\right] = 0.$$

Then, the transversality condition is fulfilled if and only if $\mathcal{A}_i = 0$ (note that (29) ensures that $A - \delta - n - g_e^* > 0$). $\mathcal{A}_i = 0$ implies that $a_i(t)$ grows at constant rate g_e^*. Since this is the case for all households $i \in [0,1]$, this proves uniqueness of the equilibrium path with $g_E^* = g_K^*$.

Next, we show that (30) and (31) jointly ensure condition (3) for all individuals at each date. The poorest household has no wealth and a labor endowment of $\bar{l}$. Consequently, she consumes her entire income (see (A.2)), i.e., $e_i(t) = w(t)\bar{l}$, $\forall t$. Then, in the view of (14), at $t = 0$, condition (3) can be rewritten as

$$w(0)^\epsilon \bar{l}^\epsilon \geq \nu \left[\frac{1 - \epsilon}{1 - \gamma}\right] \left[\frac{A}{1 - \alpha}\right]^\epsilon \left[\frac{K_G(0) + K_S(0)}{L(0)}\right]^{\alpha\epsilon}. \qquad (A.3)$$

Note that (6), (8), (9) and (10) yield $\frac{K_G(t) + K_S(t)}{K(t)} = \frac{A - \delta - g_K^*}{A}$ and we have $\frac{K_G(0) + K_S(0)}{L(0)} = \frac{w(0)}{A} \frac{1 - \alpha}{\alpha}$ (see (13)). Then, (A.3) can be written as

$$\alpha^\epsilon \bar{l}^\epsilon \geq \nu \left[\frac{1 - \epsilon}{1 - \gamma}\right] \left[\frac{L(0)}{K(0)} \frac{A}{A - \delta - g_K^*}\right]^{\epsilon(1 - \alpha)}.$$

Plugging in the expression for g_K^*, we see that this condition coincides with (30). The nominal expenditure levels and all prices grow at constant rates in equilibrium. Hence, given condition (3) holds at date $t = 0$, it also holds for $t > 0$ if $\epsilon(g_E^* - n) \geq \gamma g_{P_G}^* + (\epsilon - \gamma)g_{P_S}^*$. This is guaranteed by condition (31) and completes the proof of part (i). For part (ii): (35) is the growth rate version of (23), where we used the equilibrium growth rates of prices and expenditures. Additionally, we have $g_{\eta_G}(t) = g_{P_G}(t) + g_{X_G}(t) - g_E^* \leq 0$. With (34), $g_{X_G}(t) = g_G + \alpha g_{L_G}(t) + (1 - \alpha)g_{K_G}(t)$ and $g_{K_G}(t) - g_{L_G} = g_K^* - n$ (see (16)) this implies (36). Finally, (34) follows immediately from (37). $\quad\square$