Monetary and Fiscal Policy in a Liquidity Trap with Inflation Persistence

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Abstract

This paper relies on the new Keynesian model with inflation persistence to characterize the optimal monetary and fiscal policy in a liquidity trap. It shows that, with a Phillips curve that is both forward and backward looking, the monetary policy that is implemented during a liquidity trap episode can lift the economy out of depression. The central bank does not need to commit beyond the end of the crisis to get some traction on the level of economic activity. Regarding fiscal policy, inflation persistence justifies some front-loading of government expenditures to get inflation started, which reduces the real interest rate. The magnitude of the optimal fiscal stimulus is decreasing in the degree of inflation persistence. Finally, if inflation persistence is due to adaptive expectations, rather than to price indexation, then monetary policy is useless while the optimal fiscal stimulus is large and heavily front-loaded.

Keywords: Commitment, Inflation persistence, Liquidity trap, Monetary and fiscal policy

JEL Classification: E12, E52, E62, E63

1 Introduction

The Great Recession has shown that a major constraint on the conduct of economic policy is that the nominal interest rate set by the central bank cannot be negative. The constraint is due to the simple fact that anyone would prefer to hold cash rather than a bond yielding a negative nominal return. When the zero lower bound is binding, the

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economy is said to be in the "liquidity trap".[1] This situation results in an interest rate that is excessively high and, hence, in a level of demand that is below the production capacity of the economy. As the experience of Japan over the last two decades has shown, there is no mechanism through which an economy naturally escapes a liquidity trap.

The government therefore needs to intervene in order to stimulate the level of economic activity. It can do so through expansionary fiscal policy. An increase in government spending mechanically raises the level of demand. Alternatively, Krugman (1998) and Eggertsson and Woodford (2003) have shown that, in a forward looking environment, the government can also rely on monetary policy to escape the liquidity trap. Indeed, the policy that will be implemented by the central bank after the crisis is over can influence the current level of economic activity through its effect on expectations. The key is for the central bank to credibly promise to create an output boom after the crisis.

To analyze monetary and fiscal policy in a liquidity trap, the literature has extensively relied on the new Keynesian model. However, this model is purely forward looking, which implies that inflation does not have any persistence. This is implausible in light of historical experiences such as the sharp recession caused by the Volcker disinflation episode.[2] Hence, indexation of non re-optimized prices to the last observed rate of inflation is routinely added to the new Keynesian model (see, for instance, Woodford 2003, Smets and Wouters 2003 or Christiano, Eichenbaum and Evans 2005), which introduces inflation persistence without departing from rational expectations.

However, the liquidity trap has never been analyzed in the presence of inflation persistence. The aim of this paper therefore is to characterize the optimal monetary and fiscal policy in a liquidity trap under inflation persistence. As we shall see, inflation persistence significantly enhances the efficacy of these policies and their ability to lift the economy out of the trap.

There are empirical controversies about the structural degree of inflation persistence.[3] On the one hand, reduced form estimates show that inflation persistence has declined since the onset of the Great Moderation. On the other hand, the structural degree of inflation persistence can hardly be identified over an episode of history characterized by low and stable inflation, where the policies implemented by central banks guaranteed that inflation would always revert back to target. Moreover, while in my analysis I assume a constant degree of inflation persistence, in reality persistence is likely to fluctuate over time. It might in fact rise when the economy falls into the liquidity trap. Indeed, the

[1] Once the nominal rate has fallen to zero, money and short term bonds are both zero-interest-yielding assets. Hence, an increase in the money supply through an open market operation induces agents to rely on money rather than bonds for savings, but it does not increase the quantity of money in circulation, i.e. the economy is literally "liquidity trapped".

[2] Karl Otto Pöhl, a former president of the Bundesbank, once famously remarked that "inflation is like toothpaste. Once it is out of the tube, it is hard to get it back again".

[3] See Fuhrer (2010) for a comprehensive survey.

inability of the central bank to hit its target can de-anchor inflation expectations, which would increase the structural degree of inflation persistence.[4]

For most of my analysis, I rely on the standard new Keynesian model with inflation persistence. To implement the first-best allocation with zero inflation, the nominal interest rate must be equal to the natural real interest rate, which follows an exogenous and deterministic path.[5] The source problem is that the natural real rate becomes negative for a fixed length of time, which induces the nominal rate to hit the zero lower bound. In that case, an active monetary or fiscal policy is necessary to stabilize the economy. To obtain qualitative and quantitative insights, I simulate numerically the optimal policies.

I first investigate monetary policy alone. The aforementioned well known result is that, in the absence of inflation persistence, the central bank can only get some grip on the level of economic activity by promising to create an output boom after the crisis. This requires a strong degree of commitment as this policy is not time consistent. Indeed, generating such a boom will no longer be desirable after the crisis is over. By contrast, I show that, if inflation persistence is sufficiently strong, commitment beyond the end of the crisis is not necessary to stabilize the economy. The central bank can raise inflation by committing to implement a path of positive nominal interest rates *during* the crisis.

To understand this result note that, without inflation persistence, a rise in future nominal rates reduces inflation expectations and, hence, through the forward looking behavior of agents, the current rate of inflation. However, with inflation persistence, the lower current rate of inflation further reduces future rates of inflation. This effect can be so strong as to generate a never-ending feedback loop between a fall in current inflation and in future inflation. In that case, the only rational expectation equilibrium is that a rise in future nominal rates *raises* inflation expectations which increases the current rate of inflation. This reduces the current real interest rate, which stimulates the demand for consumption.

This result shows that the interplay between the forward and backward looking components of the Phillips curve can make the monetary policy implemented *during* the liquidity trap effective. This enhances the scope of monetary policy as it is admittedly much easier for a central bank to credibly commit during the crisis than to commit beyond the end of the crisis.

I then turn to fiscal policy. Without inflation persistence, if the government cannot commit beyond the end of the crisis, then fiscal policy alone is responsible for avoiding a depression. In that case, the optimal fiscal policy is characterized by a back-loaded

[4]More precisely, if inflation expectations are de-anchored, then firms that do not re-optimize their prices in a given quarter might to choose to index them to the last observed rate of inflation rather than to the target rate of inflation. This mechanically raises the structural degree of inflation persistence.

[5]The natural real interest rate is the real interest rate that would prevail in a flexible price economy and which generates a level of demand equal to the efficient level of output.

profile of government expenditures. This is due to the purely forward looking nature of the economy: expenditures realized towards the end of the crisis stimulate the economy when they occur, but also beforehand through their effect on expectations. This optimal policy stands in sharp contrast with the common practice of governments, which is to opt for front-loaded stimulus packages. Clearly, such a policy does not make any sense in a purely forward looking environment where expenditures realized at the beginning of crisis have no persistent impact on the level of economic activity.

Inflation persistence provides a countervailing force. I show that, with inflation persistence, the government can always spend sufficiently in the first period of the crisis in order to raise inflation by a sufficient amount to guarantee that the zero lower bound will never be binding in the future. Of course, this policy of "pump priming" the economy requires a huge amount of government expenditures in the first period and is therefore unlikely to be optimal. However, this example shows that inflation persistence makes a front loading of government expenditures desirable. Simulations show that, for a strong degree of inflation persistence, the optimal fiscal stimulus is mostly front-loaded. For an intermediate degree of inflation persistence, and in the absence of support from monetary policy,[6] it is double-peaked: government expenditures are concentrated towards the very beginning and the very end of the crisis.

Simulations also show that inflation persistence substantially reduces the magnitude of the fiscal stimulus that is necessary to stabilize the economy. The interaction between the forward and the backward looking components of the Phillips curve enhance the efficacy of the fiscal stimulus. The backward looking component allows inflation to get started, while the forward looking component ensures that the expectation of future inflation raises current inflation.

While most of the paper investigates inflation persistence within a rational expectation framework, in a final section, I consider the possibility that inflation persistence results from backward looking expectations. If agents form adaptive expectations, then inflation is purely backward looking. In that case, monetary policy is useless while fiscal policy is less efficient than before. Indeed, in the absence of a forward looking component to the Phillips curve, government spending cannot raise inflation expectations. Thus, a much larger stimulus package in needed to stabilize the economy. The optimal fiscal stimulus is heavily front-loaded.

Related Literature. Keynes (1936) and Hicks (1937) were both aware of the possibility of the economy falling into a liquidity trap.[7] However, this phenomenon was seen as a

[6] In fact, when the government cannot commit beyond the end of the crisis and when persistence is not very high, it turns out to be optimal to rely on fiscal policy alone to stabilize the economy.

[7] Keynes (1936): "*There is a possibility [...] that, after the interest rate has fallen to a certain level, liquidity-preference may become virtually absolute in the sense that almost everyone prefers cash to*

purely theoretical scenario until the Japanese nominal interest rate hit the zero lower bound in the mid-1990s. This event led to the emergence of a substantial literature on the topic, starting with Krugman (1998). A considerable amount of attention has been devoted to the policy response that is needed to lift the economy out of the liquidity trap. Following Eggertsson and Woodford (2003), the literature has extensively relied on the new Keynesian framework in order to analyze the extent to which policy interventions can exploit the forward looking behavior of agents. Let us now review the candidate solutions to the liquidity trap.

A first obvious solution to the problem would be to switch to electronic money. In the absence of cash, the zero lower bound simply disappears. However, such a radical modification of the monetary system is unlikely to be adopted in the foreseeable future.[8] As an alternative, Correia, Farhi, Nicolini and Teles (2013) have shown that, in a new Keynesian model, it is possible to implement a fiscal policy that exactly replicates the allocation of resources that would result from a negative nominal interest rate. The government needs to implement a rising path of consumption taxes, which induces agents to front-load their consumption. This, however, creates a rising distortion to labor supply, which can be offset by a declining path of labor income taxes. Similarly, a rising capital subsidy also needs to be implemented. This policy is time-consistent and implements the first-best allocation of resources. However, if governments are unable or unwilling to implement such a radical change to their tax code, they then have to rely on monetary and conventional fiscal policy, i.e. government spending, to prevent a depression.

The traditional view of the liquidity trap is that monetary policy is completely useless. When the nominal interest rate is down to zero, an increase in the money supply through an open market operation induces agents to rely on money rather than bonds for savings. This does not affect the allocation of resources. Increasing the money supply is therefore like pushing on a rope.[9] Krugman (1998), Eggertsson and Woodford (2003), Jung, Teranishi and Watanabe (2005) and Werning (2012) have shown that, if agents are forward looking, then the monetary policy that will be implemented after the crisis is over can stimulate the level of demand during the crisis. However, as already mentioned, the central bank must be able to credibly commit to implement a policy that is not

holding a debt which yields so low a rate of interest. In this event the monetary authority would have lost effective control over the rate of interest. But whilst this limiting case might become practically important in future, I know of no example of it hitherto."

Hicks (1937): "*If the cost of holding money can be neglected, it will always be profitable to hold money rather than lend it out, if the rate of interest is not greater than zero. Consequently the rate of interest must always be positive. [...] If IS lies to the left [of LM], we cannot [increase employment by increasing the quantity of money]; merely monetary means will not force down the rate of interest any further.*"

[8] See Rogoff (2014) for a discussion of the costs and benefits of disposing of paper currency.

[9] Chen, Curdia and Ferrero (2012) have shown that, under segmented asset markets, a large increase in the money supply, i.e. quantitative easing, can stimulate aggregate demand through a reduction in long term interest rates.

time consistent. Eggertsson and Woodford (2003) have shown that this can be achieved through price level targeting.[10] Should the central bank be unable to adopt such a policy, the government has to resort to public spending to stimulate aggregate demand.

Christiano, Eichenbaum and Rebelo (2011), Woodford (2011) and Farhi and Werning (2013) have shown that the fiscal multiplier is much larger in a liquidity trap than in normal times.[11] The reason is simple: in normal times, an increase in government expenditures generates an inflationary pressure to which the central bank responds by sharply increasing the nominal interest rate. This raises the real interest rate, which annihilates most of the stimulative impact of the expansionary fiscal policy. By contrast, in a liquidity trap, the nominal rate is stuck against the zero lower bound while the inflationary effect of government spending reduces the real rate, which strengthens the efficacy of the stimulus.[12] My simulation results show that the magnitude of the fiscal stimulus necessary to stabilize the economy is decreasing in the degree of inflation persistence. This suggests that persistence further increases the magnitude of the fiscal multiplier in a liquidity trap.

Werning (2012) has characterized, within the new Keynesian model, the optimal time path of government expenditures during a liquidity trap episode. Interestingly he distinguishes "opportunistic" from "stimulus" spending. The former is the mechanical response to a fall in the opportunity cost of public expenditures, and is therefore always countercyclical; while the latter corresponds to the spending realized for purely stimulative purposes. Werning (2012) has shown that, under full commitment, monetary policy does most of the job of stabilizing the economy and, hence, the stimulus component can be equal to zero. By contrast, under discretionary monetary policy, a fiscal authority that can commit should implement a positive stimulus component of government expenditures. Moreover, the stimulus component should be back-loaded, which, as discussed above, is not surprising in a purely forward looking environment.

Finally, the zero lower bound has reinvigorated the debate on the optimal rate of inflation. Williams (2009) and Blanchard, Dell'Ariccia and Mauro (2010) argued that raising the target rate of inflation from 2 to 4 percent would leave more room for cutting nominal rates in crisis time, which would reduce the likelihood of falling into the liquidity trap. Billi (2011) has shown that, if the central bank cannot commit to implement a

[10] Eggertsson (2006) showed that, if taxes are distortionary, increasing public debt can be a credible way to signal that the central bank will tolerate higher inflation in the future. In a similar vein, Bhattarai, Eggertsson and Gafarov (2013) have argued that quantitative easing can credibly signal the willingness of the central bank to keep nominal rates low after the crisis.

[11] DeLong and Summers (2012) and Denes, Eggertsson and Gilbukh (2013) even argued that, in a liquidity trap, the multiplier is so large that a rise in government spending might enhance the sustainability of public debt.

[12] While most of the literature investigates the effect of an increase in public spending, Eggertsson (2010) also analyzes the stimulative impact of various tax cuts. He argues that tax cuts are expansionary if they increase demand; but, if they increase supply, they are deflationary and, hence, contractionary.

very accommodative monetary policy in the future, then the optimal rate of inflation is above 8 percent. By contrast, Coibion, Gorodnichenko and Wieland (2012) found that the optimal rate of inflation is always small, around 2 percent. The difference is partly due to their assumption of a much lower degree of inflation persistence than in Billi (2011). Ball (2013) also emphasizes inflation persistence has a major justification for raising the inflation target. These results are obtained under the assumption of a fairly low probability of hitting the zero lower bound, which is questionable given the rising concern about "secular stagnation", according to which the natural real interest rate has permanently fallen (see, for instance, Eggertsson and Mehrotra 2014). My paper does not directly contribute to the debate on the optimal inflation target since, in my analysis, I normalize the steady state rate of inflation to zero. However, I show that inflation persistence makes it easier to stabilize the economy through monetary or fiscal policy, which somewhat reduces the case for higher inflation in normal times.

2 Monetary Policy

To begin this section, I introduce the new Keynesian model with inflation persistence. I then rely on this framework to investigate the optimal monetary policy.

2.1 New Keynesian Model with Inflation Persistence

The analysis relies on the standard new Keynesian model with inflation persistence. There is a continuum of goods produced by monopolistically competitive producers. At each point in time, a representative agent chooses his labor supply and his demand for each consumption good such as to maximize his intertemporal utility. He discounts the future at rate β. His elasticity of substitution across goods is equal to ϵ, the inverse of his Frisch elasticity of labor supply is equal to η and the inverse of his intertemporal elasticity of substitution of consumption is equal to σ.

Aggregate demand has two components: private consumption and government expenditures. Let Y, C and G denote the steady state levels of output, consumption and government expenditures, respectively. Thus, $Y = C + G$. As in this section I focus on monetary policy alone, I assume that government spending is always equal to its steady state level G.

Each firm employs labor such as to produce its own variety of goods. The production function is $Y = N^{1-\alpha}$. Calvo pricing implies that, in any given period, a monopolistically competitive firm only has a probability $1 - \theta$ of being able to reset its price. Following Woodford (2003), Smets and Wouters (2003) and Christiano, Eichenbaum and Evans (2005), among others, inflation persistence is introduced by assuming that, in the absence of re-optimization, prices are indexed to inflation. More specifically, the index attaches a weight ω to the previously observed rate of inflation and the remaining weight $1-\omega$ to the trend rate of inflation. For simplicity, and without loss of generality, the trend rate of inflation is normalized to zero.

Importantly, I assume throughout that there is no aggregate uncertainty. Hence, there is perfect foresight about the future. Let c_t denote the deviation of consumption from its natural level normalized by the steady state output level. More formally:

$$c_t = \ln\left(\frac{C_t}{C_t^n}\right)\frac{C}{Y} \approx \frac{C_t - C_t^n}{Y}, \tag{1}$$

where C_t denotes the actual level of consumption at t while C_t^n denotes the natural level of consumption at t.[13] A log-linear approximation around the steady state to the optimal price setting decisions of firms yields the (hybrid) new Keynesian Phillips curve:

$$\pi_t - \omega\pi_{t-1} = \beta\left(\pi_{t+1} - \omega\pi_t\right) + \kappa c_t, \tag{2}$$

where π_t denotes the rate of inflation from $t-1$ to t and the parameter κ is equal to:

$$\kappa = \frac{\eta + \alpha + (1-\alpha)\sigma Y/C}{1+\alpha\left(\epsilon - 1\right)}\frac{1-\theta}{\theta}\left(1-\theta\beta\right). \tag{3}$$

Thus, κ is non-negative and increasing in the degree of price flexibility, i.e. κ is decreasing in θ. A key parameter of my analysis is ω which determines the degree of inflation persistence. If $\omega = 0$, there is no inflation persistence and (2) reduces to the standard new Keynesian Phillips curve. If $\omega = 1$, there is full indexation to past inflation, which implies that, in the absence of re-optimization, prices at t are automatically increased by the previously observed rate of inflation, π_{t-1}.

The other building block of the new Keynesian model is the consumption Euler equation, which, after log-linearization, is given by:

$$c_t = -\frac{1}{\sigma Y/C}\left(i_t - \pi_{t+1} - r_t^n\right) + c_{t+1}, \tag{4}$$

where i_t denotes the nominal interest rate and r_t^n the natural real interest rate, which is the real interest rate that would prevail in a flexible price economy. In other words, for the economy to produce at full capacity, the real interest rate $i_t - \pi_{t+1}$ needs to be always equal to its natural counterpart r_t^n. This natural real interest rate r_t^n is exogenous to the model. A fundamental constraint to the conduct of monetary policy is that the nominal

[13] The natural level of consumption corresponds to the consumption level that prevails in a flexible price economy in the absence of government intervention, other than a steady state level of spending maintained at G.

interest rate i_t cannot be negative:

$$i_t \geq 0. \tag{5}$$

Indeed, money is an asset that always yields a zero nominal return. Thus, agents would always prefer to hold money for savings rather than bonds yielding a negative nominal return.

The dynamics of the economy are fully characterized by the new Keynesian Phillips curve with a given initial rate of inflation equal to π_0 and by the Euler equation where the nominal interest rate i_t is set by the central bank. Monopolistic competition implies that the equilibrium of the flexible price economy is inefficient. Let us therefore assume that the government implements a proportional employment subsidy, financed by a lump sum tax, which offsets this inefficiency. We can then consider that the natural level of consumption coincides with the optimal level of consumption.

The aim of the central bank is to minimize the following loss function:

$$\sum_{t=1}^{\infty} \beta^t \left[(\pi_t - \omega \pi_{t-1})^2 + \frac{\kappa}{\epsilon} c_t^2 \right]. \tag{6}$$

The negative of this loss function corresponds to a second-order approximation to the utility function of consumers around the steady state. The inflation term captures the welfare loss from relative price distortions. Indeed, when $\omega = 0$, inflation under Calvo pricing introduces a distortion between newly reset prices and older prices. With full indexation, when $\omega = 1$, it is only changes in the rate of inflation, $\pi_t - \pi_{t-1}$, that introduce relative price distortions. Finally, with partial indexation, when $\omega \in (0, 1)$, relative price distortions occur unless $\pi_t = \omega \pi_{t-1}$ for all t, in which case firms that can reset their price choose not to deviate from the indexation rule.

The optimal monetary policy is obtained by minimizing the loss function (6) with respect to the nominal interest rates i_t for $t \geq 1$ subject to the new Keynesian Phillips curve (2) with π_0 given, to the Euler equation (4) and to the zero lower bound (5). The loss function implies that, at any time t, the first-best allocation of resources is characterized by $\pi_t = \omega \pi_{t-1}$ and $c_t = 0$. If the zero lower bound on the nominal interest rate is never binding, then this allocation can easily be implemented by setting $i_t = \pi_{t+1} + r_t^n = \omega^{t+1} \pi_0 + r_t^n$ for all $t \geq 1$.[14]

In order to investigate a liquidity trap scenario, I assume that, for exogenous reasons, the level of demand is weak from time 1 until time T, which is characterized by a natural real interest rate r_t^n that is negative from time 1 until T and positive afterwards. This implies that, starting with an inflation rate which is at or below trend, i.e. $\pi_0 \leq 0$, the

[14] In this paper, I focus on optimal allocations and do not specify the policy rules that prevent the occurrence of multiple equilibria. Typically, these rules impose a sharp rise (fall) in the nominal interest rate if inflation or consumption is higher (lower) than in the optimal allocation.

first-best allocation cannot be implemented.[15] More specifically, for numerical simulations, I will rely on the standard step function:

$$r_t^n = \begin{cases} \underline{r} & \text{if } 1 \leq t \leq T \\ \bar{r} & \text{if } T+1 \leq t \end{cases} \tag{7}$$

where $\bar{r} > 0$ and $\underline{r} < 0$. Candidate explanations for the persistence of a very low level of the natural real rate include population aging, a process of deleveraging or a rise in the concentration of wealth among individuals with a low propensity to consume. However, endogenizing the evolution of the natural real rate is beyond the scope of my analysis.

2.2 Calibration

Throughout the paper, I perform numerical simulations to investigate the main qualitative and quantitative properties of optimal monetary and fiscal policy. I therefore rely on a standard quarterly calibration of the new Keynesian model. The preference parameters are set as follows: the discount rate β is set equal to 0.99, the elasticity of substitution across goods ϵ to 6, the Frisch elasticity of labor supply $1/\eta$ to 0.5 and the intertemporal elasticity of substitution of consumption $1/\sigma$ to 1 (which corresponds to a logarithmic utility of consumption). The steady state output level Y is normalized to 1. As investment is absent from the model, steady state consumption C is set 0.8 and government expenditures G to 0.2. On the production side, the steady state labor share $1-\alpha$ is set equal to 2/3 and the Calvo parameter of price stickiness θ to 2/3, which implies an average price duration of three quarters. These parameters imply, by (3), that the Phillips curve parameter κ is approximately equal to 0.20.

I consider a scenario where the natural real interest rate r_t^n remains negative for 5 years, i.e. $T = 20$ quarters. The step function is symmetric with a natural real interest rate equal to -2% per year during the crisis, i.e. $\underline{r} = -0.005$, and equal to 2% afterwards, i.e. $\bar{r} = 0.005$. For simplicity, I assume that the initial rate of inflation π_0 is equal to 0%.

The persistence parameter ω is at the heart of my analysis. I therefore consider the two benchmark cases with no inflation persistence, $\omega = 0$, and with full indexation, $\omega = 1$. More realistically, I also consider the case where $\omega = 0.5^{1/4} \simeq 0.841$. Recall that in a first-best allocation inflation follows $\pi_t = \omega\pi_{t-1}$ or, equivalently, $\pi_{t+4} = \omega^4\pi_t$. Starting from a strictly positive rate of inflation, the indexation parameter $\omega = 0.5^{1/4}$ implies that, along the optimal path, inflation is halved every year, i.e. $\pi_{t+4} = 0.5\pi_t$. This corresponds to a plausible degree of inflation persistence. Relying on quarterly U.S. data from 1960 to

[15] If the steady state rate of inflation is not set equal to zero, then the analysis is unchanged provided that the natural real interest rate is lowered by the steady state rate of inflation. For instance, with a steady state rate equal to 2%, the natural real rate needs to fall below -2% for the zero lower bound to be binding and for the economy to fall into the liquidity trap.

2004, Milani (2007) estimates that, under rational expectations, $\omega = 0.885$. In his analysis of the optimal rate of inflation, Billi (2011) assumes $\omega = 0.9$. While the value $\omega \simeq 0.841$ seems reasonable, it must be acknowledged that there remains a considerable amount of controversy in the literature about the structural degree of inflation persistence.

2.3 Monetary Policy without Inflation Persistence

Let us begin by analyzing monetary policy in the standard new Keynesian model without inflation persistence, when $\omega = 0$. The main characteristics of the optimal monetary policy in a liquidity trap when $\omega = 0$ are already well known from the analyses of Eggertsson and Woodford (2003), Jung, Teranishi and Watanabe (2005) and Werning (2012).[16] It provides a useful benchmark against which to compare the effects of inflation persistence.

Figure 1 displays the optimal monetary policy and the corresponding allocation of resources under full commitment.[17] The thin dotted line displays the exogenous path of the natural real interest rate r_t^n, which is negative for 20 consecutive quarters. The optimal monetary policy is given by the thin solid line, which is the path of the nominal interest rate i_t to which the central bank must commit. This policy seems broadly successful at stabilizing the output gap c_t, represented by the thick solid line, which fluctuates around its natural level. It also generates some inflation π_t during the crisis, as shown by the thick dashed line. The real interest rate $r_t = i_t - \pi_{t+1}$ is given by thin dashed line. Recall that, to implement the first-best allocation $c_t = \pi_t = 0$, the real rate must always be equal to the natural real rate. However, due to the zero lower bound on the nominal interest rate, this is not possible.

To understand the logic of the optimal monetary policy with commitment, it is useful to consider first what happens in the absence of commitment. After the crisis is over, from the 21st quarter onwards, the central bank wishes to set the nominal interest rate i_t equal to the natural real interest rate $r_t^n = \bar{r} > 0$ forever such as to implement the first-best allocation, i.e. $c_t = \pi_t = 0$ for all $t \geq T + 1$. However, the anticipation of the this policy has disastrous consequences during the crisis. Indeed, if the economy produces at full capacity with no inflation at $T + 1$, then the zero lower bound on the nominal interest rate forces the real interest rate at time T, $r_T = i_T - \pi_{T+1} = 0$, to be above the natural real interest rate, $r_T^n = \underline{r} < 0$. This depresses the output level at T. Indeed, the Euler equation (4) with $i_T - \pi_{T+1} = 0$ gives $c_T = \underline{r}/(\sigma Y/C) < 0$. The depressed output level induces firms to cut their prices which generates deflation. By the Phillips curve (2), we have $\pi_T = \kappa c_T < 0$. But, deflation at T implies an even wider gap

[16] Note that Eggertsson and Woodford (2003) do not rely on a fully deterministic setup as they assume that, each period, there is a constant probability that the natural real interest rate becomes positive.

[17] The optimal monetary policy problems are formally solved in appendix A.

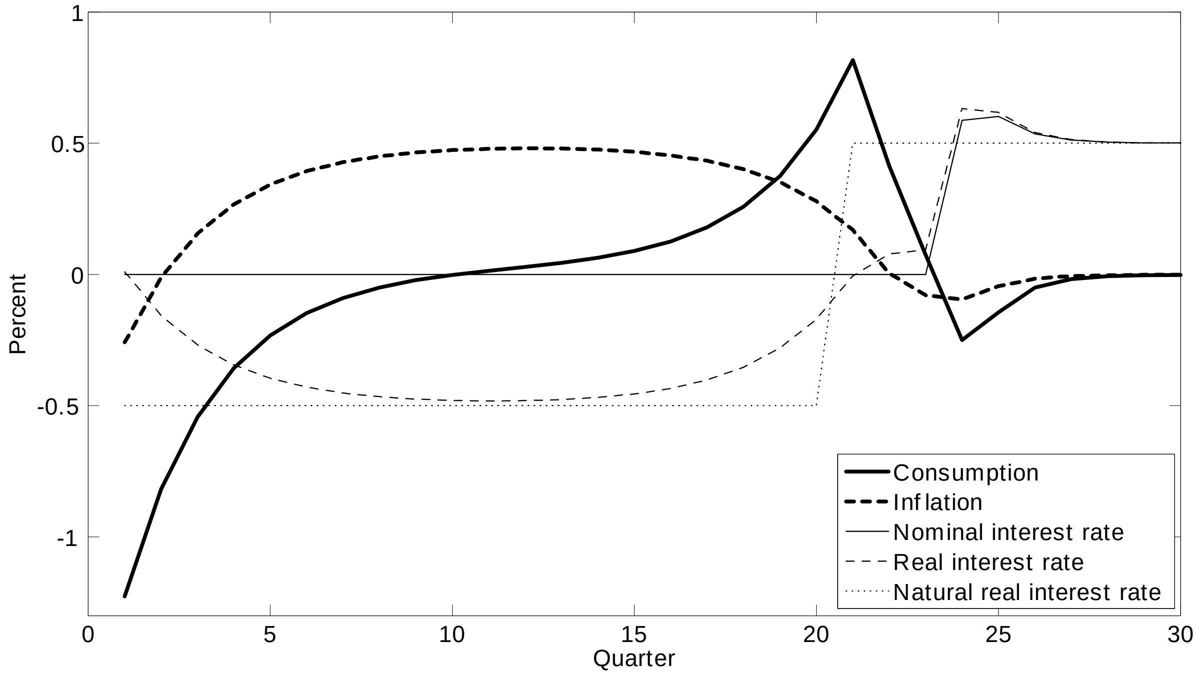

Figure 1: Optimal monetary policy with full commitment when $\omega = 0$

between the real interest rate at $T-1$, $r_{T-1} = i_{T-1} - \pi_T = -\pi_T > 0$, and the natural real rate, $r^n_{T-1} = \underline{r} < 0$. This causes an even larger depression in the output level at time $T-1$, which generates even more deflation at $T-1$. The economy is caught in a vicious deflationary spiral throughout the liquidity trap.

Figure 2 displays the path of consumption and inflation under the optimal monetary policy when the central bank cannot commit beyond time T, in which case $i_t = 0$ for $t \leq T$ and $i_t = \bar{r}$ for $t \geq T+1$. Clearly, the absence of commitment has disastrous consequences, even though the linearization of the model around the steady state does not seem appropriate for a quantitative investigation of a phenomenon of this magnitude.

As illustrated by Figure 1, the key to avoid the deflationary spiral is to commit to keeping the nominal interest rate i_t equal to 0 for some time after the crisis is over. This induces the real interest rate to be below the natural real rate, which generates an output boom after the crisis is over. As the pricing behavior of firms is forward looking, this output boom creates inflation both during the boom and beforehand, i.e. during the crisis.

The remarkable feature of the optimal policy is that keeping the nominal rate equal to zero for only three quarters after the crisis is over is sufficient to completely eliminate the vicious deflationary spiral. Indeed, as can be seen from Figure 1, the optimal policy generates just enough inflation for the real interest rate to be almost equal to the natural real rate during most of the crisis.

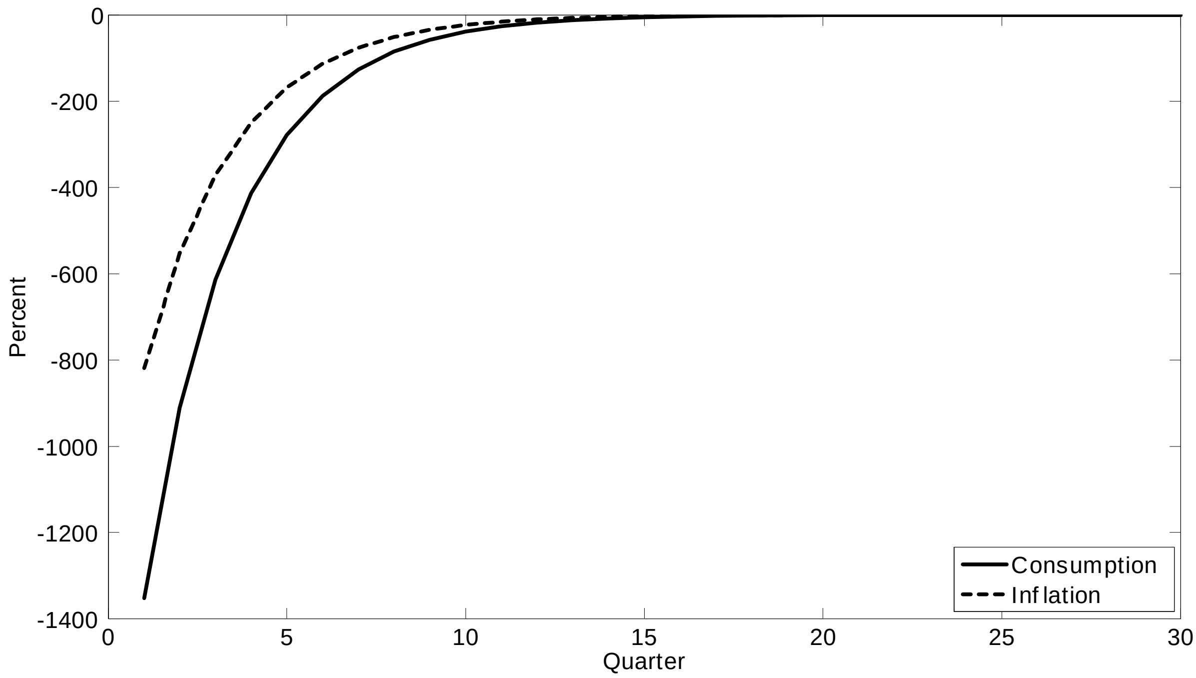

Figure 2: Optimal monetary policy with no commitment beyond T when $\omega = 0$

In sum, when $\omega = 0$, the central bank can only stabilize the level of economic activity if it can commit beyond time T. This is the key insight, initially emphasized by Krugman (1998), that the central bank needs to "credibly promise to be irresponsible".

2.4 Monetary Policy with Inflation Persistence

Let us now investigate how inflation persistence modifies the optimal conduct of monetary policy with full commitment. For simplicity, I begin by analyzing the full indexation benchmark where $\omega = 1$. Figure 3 displays the optimal monetary policy and the corresponding allocation of resources.

Recall that, when $\omega = 1$, the first-best allocation is characterized by $c_t = 0$ and $\pi_t = \pi_{t-1}$. This immediately follows from the specification of the loss function, given by (6). The first remarkable result from Figure 3 is that the optimal monetary policy implements the first-best allocation well before the end of the crisis. Indeed, from the eleventh quarter onwards, the output gap is virtually equal to zero and inflation is constant. This is due to inflation being high enough to allow the real interest to be equal to the natural real interest rate, despite the zero lower bound.

To understand the optimal policy, we therefore need to perform a detailed analysis of the dynamics of inflation. To this end, it is useful to consolidate the Euler equation and the new Keynesian Phillips curve into a single expression for inflation. For generality, I allow for any degree ω of inflation persistence. Let us consider that the economy is in

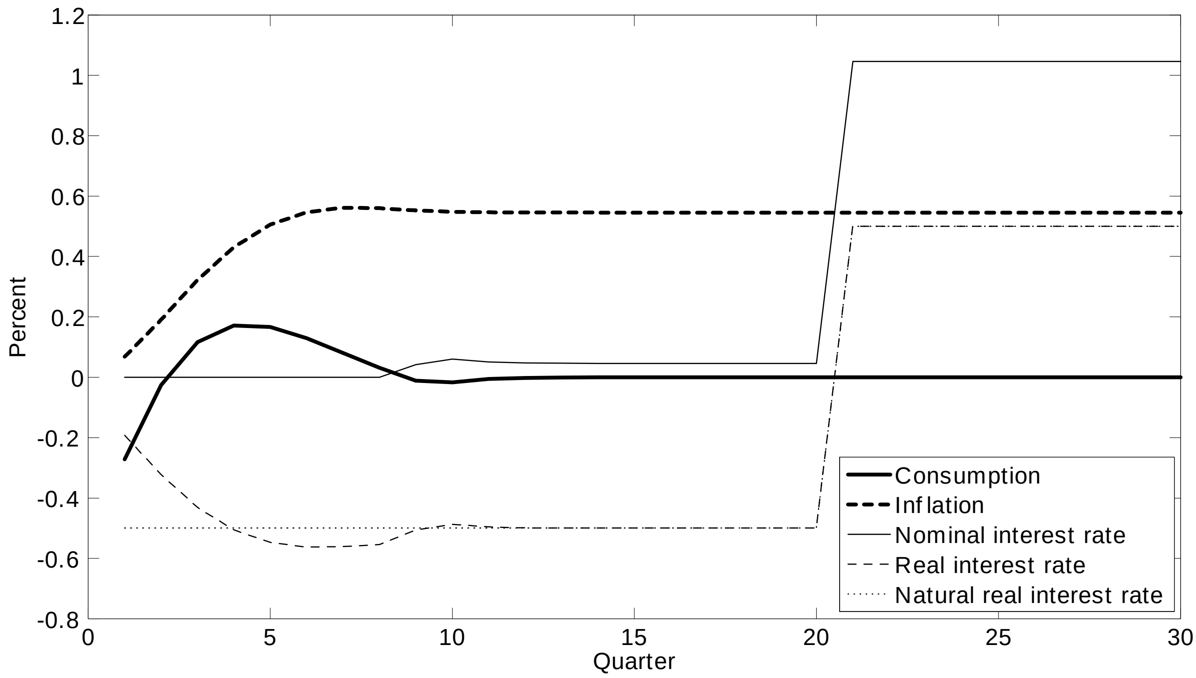

Figure 3: Optimal monetary policy with full commitment when $\omega = 1$

a first-best allocation from time $N+1$ onwards, where $c_t = 0$ and $\pi_t = \omega\pi_{t-1}$ for all $t \geq N+1$. There is no loss of generality as N can be arbitrarily large; we can even take the limit as N tends to infinity. Iterating forward on the Euler equation (4) until time N, and imposing $c_{N+1} = 0$, yields:

$$c_t = -\frac{1}{\sigma Y/C} \sum_{k=t}^{N} \left(i_k - \pi_{k+1} - r_k^n\right). \tag{8}$$

Similarly, iterating forward on the Phillips curve (2) until N, and imposing $\pi_{N+1} = \omega\pi_N$, yields:

$$\pi_t - \omega\pi_{t-1} = \kappa \sum_{k=t}^{N} \beta^{k-t} c_k. \tag{9}$$

Substituting the first equation into the second, and rearranging terms, gives:

$$\pi_t - \omega\pi_{t-1} = -\frac{\kappa}{\sigma Y/C} \sum_{k=t}^{N} \frac{1-\beta^{k+1-t}}{1-\beta} \left(i_k - \pi_{k+1} - r_k^n\right), \tag{10}$$

for any $t \in \{1, 2, ..., N\}$. This expression fully characterizes the dynamics of inflation.

The initial rise in inflation is driven by the forward looking component of the Phillips curve. In Figure 3, agents anticipate that the real interest rate will be below the natural real rate between the fourth and the ninth quarter, which, by (10), generates a rising path of inflation. Note that, as $\lim_{\beta \to 1}(1-\beta^{k+1-t})/(1-\beta) = k+1-t$, gaps between the real rate and the natural real rate have a larger impact on inflation if they occur at a distant point in the future than if they happen soon. This explains why, in Figure 3, a very small gap between the two rates more than one year ahead is sufficient to generate a sizeable increase in inflation.

We have now seen how the expectation of high inflation in the future makes it possible to implement low future real interest rates that raise the current rate of inflation. The deeper question is: How can the central bank, through its choice of future nominal interest rates, generate high inflation expectations? Before answering this question, it should be emphasized that, given a path of nominal interest rates until time N chosen by the central bank, there is a *unique* allocation of consumption and inflation that is consistent with the boundary condition that the economy must be in a first-best allocation from time $N+1$ onwards, where N is arbitrarily large.[18] It may of course seem surprising that the central bank is able to generate inflation expectations by committing to implement strictly positive nominal rates in the future. This would not be possible without inflation persistence.

To understand the intuition, note that the usual mechanism is that an increase in future nominal rates raises future real rates, which reduces inflation expectations and, through the forward looking behavior of agents, the current rate of inflation. However, with inflation persistence, a lower current rate of inflation further reduces inflation expectations. If this feedback loop is strong enough, there is no corresponding rational expectation equilibrium. In that case, the only equilibrium is that a rise in future nominal rates *raises* inflation expectations sufficiently to reduce future real rates. This, through the forward looking behavior of agents, increases current inflation, which, through inflation persistence, raises future inflation, consistently with agents' expectations.

To illustrate this mechanism, let us start by considering an extreme example where a rise in the current nominal interest rate raises current inflation. In such a case, the first-best allocation can be reached as soon as time 2, where $c_2 = 0$ and $\pi_2 = \omega\pi_1$. Equation (10) evaluated at $t = 1$ and with $N = 1$ gives:

$$\pi_1 - \omega\pi_0 = -\frac{\kappa}{\sigma Y/C}\left(i_1 - \pi_2 - r_1^n\right). \tag{11}$$

[18]Uniqueness is straightforward to prove. Select an arbitrarily large value of N and impose that the economy is in a first-best allocation from time $N+1$ onwards. Given some value of π_{N+1}, we immediately obtain $\pi_N = \pi_{N+1}/\omega$. Then, π_{N-1} can be obtained from (10) evaluated at $t = N$. We can then proceed recursively with equation (10) to obtain the whole trajectory of inflation until π_0. But, as (10) is a linear equation, the recursive substitutions yields a linear relationship between π_{N+1} and π_0. Thus, there is a unique value of π_{N+1} consistent with the preset value of π_0. This value of π_{N+1} fully characterizes the path of inflation and, through (8), of consumption. Note that for a given value of N, such an equilibrium only exists if, for all $t \geq N+1$, we have $i_t = \pi_{t+1} + r_t^n = \omega^{t-N}\pi_N + r_t^n \geq 0$; as, otherwise, it is not possible to set a nominal interest rate consistent with the economy being in a first-best allocation from $N+1$ onwards.

Using the fact that $\pi_2 = \omega\pi_1$ and rearranging terms yields:

$$\left(\frac{\kappa}{\sigma Y/C}\omega - 1\right)\pi_1 = \frac{\kappa}{\sigma Y/C}\left(i_1 - r_1^n\right) - \omega\pi_0. \tag{12}$$

Thus, if $\omega > (\sigma Y/C)/\kappa$, then π_1 is increasing in i_1. In that case, to reach the first-best allocation as soon as time 2, the central bank just needs to raise i_1 by a sufficient amount to make sure that inflation is going to be high enough in the future to guarantee that the zero lower bound will never be binding again.[19]

While the above example focuses on a single time period, the feedback loops at work are even more powerful over longer horizons. To illustrate this, consider a situation where the first-best allocation can be reached at time 3, where $c_3 = 0$ and $\pi_3 = \omega\pi_2$. Now, equation (10) with $N = 2$ evaluated at $t = 2$ and $t = 1$, respectively, gives:

$$\pi_2 - \omega\pi_1 = -\frac{\kappa}{\sigma Y/C}\left(i_2 - \pi_3 - r_2^n\right), \tag{13}$$

and:

$$\pi_1 - \omega\pi_0 = -\frac{\kappa}{\sigma Y/C}\left(i_1 - \pi_2 - r_1^n\right) - \frac{\kappa}{\sigma Y/C}(1+\beta)\left(i_2 - \pi_3 - r_2^n\right). \tag{14}$$

The usual mechanism is that an increase in i_2 raises $r_2 = i_2 - \pi_3$ which reduces both π_1, by (14), and π_2, by (13). Furthermore, the fall in π_1 is strengthened by the fall in π_2, which raises $r_1 = i_1 - \pi_2$. The crucial accelerating factor is inflation persistence which, by (13), implies that the fall in π_1 amplifies the fall in π_2. In fact, if ω is sufficiently close to 1, then, by (13), π_2 must drop by more than π_1 (for a fixed value of π_3). But, at the same time, by (14), $\kappa/\left(\sigma Y/C\right) \geq 1$ is a sufficient condition to ensure that π_1 drops by more than π_2 (for a fixed value of π_3). This clearly is inconsistent with a rational expectation equilibrium. All this occurs in addition to the previously identified feedback loop through which a fall in π_2 induces, by inflation persistence, a fall in π_3, which increases $r_2 = i_2 - \pi_3$ and further reduces π_2.

When ω is large enough, the only rational expectation equilibrium consistent with the economy reaching a first-best allocation at time 3 is that a rise in i_2 induces an even larger rise in the expected future rate of inflation π_3, which reduces $r_2 = i_2 - \pi_3$ and increases π_2. The pricing decision of firms being forward looking, the expected increase in both π_2 and π_3 generates a rise in π_1. Inflation persistence implies that the rise in π_1 will raise π_2 and the rise in π_2 will raise π_3, consistently with agents' expectations. This intuition is formally confirmed by combining the two equations above such as to

[19] It follows from footnote 18 that the desired equilibrium of the central bank is unique when we impose the boundary condition that the economy is in a first-best allocation from time $\tilde{N}+1$ onwards, for any given $\tilde{N}$ such that $\tilde{N}+1 \geq N+1 = 2$. If $\tilde{N}+1 > N+1$, the central bank needs to commit to the path of the nominal interest rate from time 2 to $\tilde{N}$ that is consistent with the first-best allocation.

eliminate π_1 and by using $\pi_3 = \omega\pi_2$. This yields:

$$\left[\frac{\kappa}{\sigma Y/C}\omega\left(2+\omega(1+\beta)\right)-1\right]\pi_2 = \frac{\kappa}{\sigma Y/C}\omega\left(i_1 - r_1^n\right) + \frac{\kappa}{\sigma Y/C}\left[1+\omega(1+\beta)\right]\left(i_2 - r_2^n\right) - \omega^2\pi_0. \tag{15}$$

Thus, π_2 is increasing in both i_1 and i_2 provided that $\omega\left(2+\omega(1+\beta)\right) > (\sigma Y/C)/\kappa$, which is a much weaker condition on ω than in the one period example (where the condition was $\omega > (\sigma Y/C)/\kappa$).

When comparing Figure 1 and 3, it is tempting to conclude that the ability of the central bank to create inflation during the liquidity trap is due to the fact that inflation remains high after the crisis is over. However, this is not the main mechanism at work. Even if we impose the boundary condition that the economy must be in a first-best allocation with zero inflation from time $N+1$ onwards, i.e. $c_t = \pi_t = 0$ for all $t \geq N+1$, it is still possible for the central bank to raise inflation by committing to a specific path of nominal rates until time N provided that inflation persistence is strong enough. Indeed, the two period example above with N set equal to 2 shows that, even if π_3 must be equal to zero, it is possible to raise π_2 by increasing i_1 or i_2. This illustrates that there are enough feedback loops over a multi-period horizon to make it possible to raise inflation by increasing nominal rates, even if the end-point is characterized by zero inflation.

Werning (2012) forcefully emphasized that the stance of monetary policy is determined by the amount of inflation it generates, not by the level of the nominal interest rate. Indeed, he shows that more accommodative monetary policies generate inflation which could eventually result in a higher nominal rate. Things are even sharper with inflation persistence where it is the rise in nominal rates that can directly generate inflation.[20]

In Figure 3, the most important consequence of the implementation of a first-best allocation before the end of the crisis is that the central bank does not need to commit beyond time T to be able to stabilize the economy. This is a direct implication of the ability of the central bank to stimulate inflation through positive nominal interest rates during the crisis. Indeed, the above examples show that no commitment is necessary when $\omega > (\sigma Y/C)/\kappa$ and commitment for one period ahead is sufficient when $\omega\left(2+\omega(1+\beta)\right) > (\sigma Y/C)/\kappa$.

This is an important result to the extent that it is presumably much easier for a central bank to credibly commit to a certain path of the nominal rate during the crisis, i.e. when the natural real rate is still negative, than after the crisis is over. Indeed, while

[20] Schmitt-Grohé and Uribe (2013) have also argued that raising nominal interest rates can boost inflation expectations in a liquidity trap. However the underlying mechanism is completely different. In their framework, there is a non-fundamental confidence shock that results in multiple equilibria. The liquidity trap is an adverse equilibrium caused by a negative confidence shock. Higher interest rates signal higher future inflation and therefore generate inflation expectations.

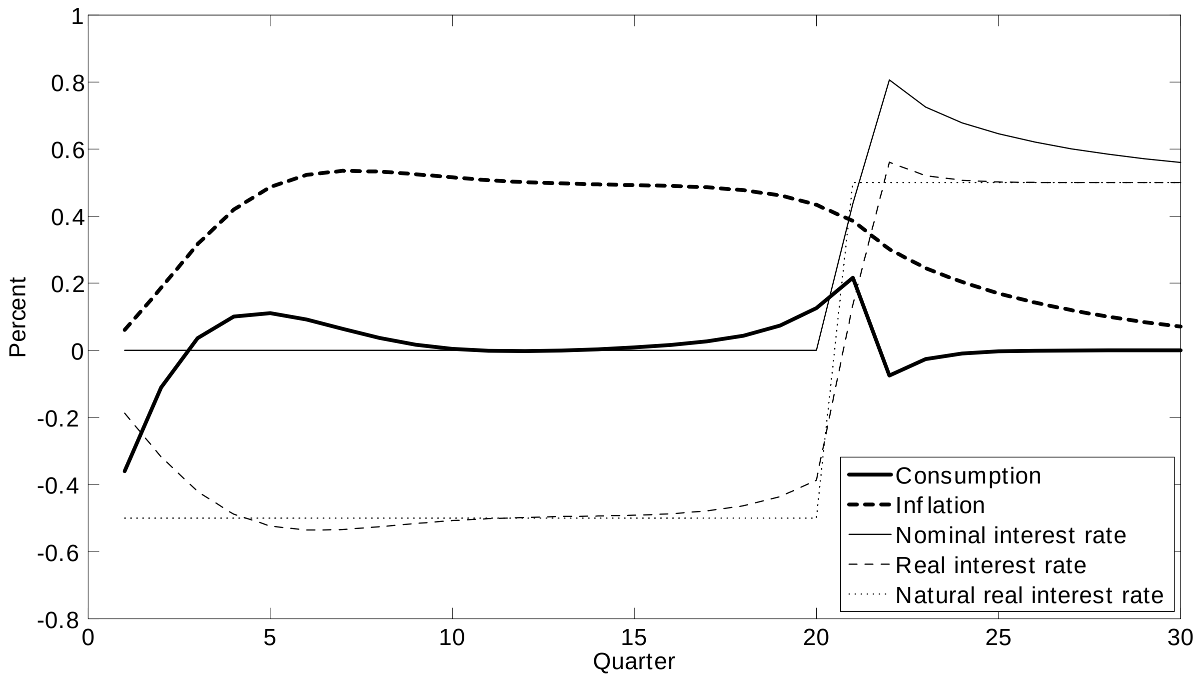

Figure 4: Optimal monetary policy with full commitment when $\omega = 0.841$

the Federal Reserve and other major central banks have engaged in forward guidance, none of them has credibly committed to behave "irresponsibly" once their economy will have recovered. Thus, inflation persistence considerably enhances the ability of central banks to steer their economies out of liquidity traps.

As already hinted, even if the central bank can only commit up to time S, with $S < T$, it is possible to reach the first-best allocation by the end of the commitment period provided that inflation is raised by a sufficient amount to ensure that the zero lower bound will not be binding in the future. More precisely, to implement a first-best allocation from $S+1$ onwards, π_S must be greater or equal to $-\underline{r}/\omega^{T+1-S} > 0$.[21] Of course, while it is interesting to know that the first-best allocation can be reached as soon as time $S+1$, this does not necessarily characterize the optimal monetary policy.

Let us now investigate quantitatively the more plausible case where prices are only partially indexed to inflation. Figure 4 displays the optimal monetary policy and the corresponding allocation under full commitment when $\omega = 0.5^{1/4} \simeq 0.841$.

Note that, in the 21st quarter, the real interest rate is below the natural real rate. The central bank therefore commits to be "irresponsible" and to create an output boom

[21] To be in a first-best allocation from $S+1$ onwards, the real interest rate must be equal to the natural real rate and the zero lower bound must never be binding. Thus, we must have $i_t = \pi_{t+1} + r_t^n \geq 0$ for all $t \geq S+1$. But, we know that $\pi_t = \omega\pi_{t-1}$ when $t \geq S+1$, which implies $\pi_t = \omega^{t-S}\pi_S$. Hence, the condition simplifies to $i_t = \omega^{t+1-S}\pi_S + r_t^n \geq 0$ for all $t \geq S+1$. As $\omega^{t+1-S}\pi_S$ is weakly decreasing in t and as $r_1^n = r_2^n = ... = r_T^n = \underline{r} < 0$, a necessary and sufficient condition is $i_T = \omega^{T+1-S}\pi_S + r_T^n \geq 0$ or, equivalently, $\pi_S \geq -\underline{r}/\omega^{T+1-S}$.

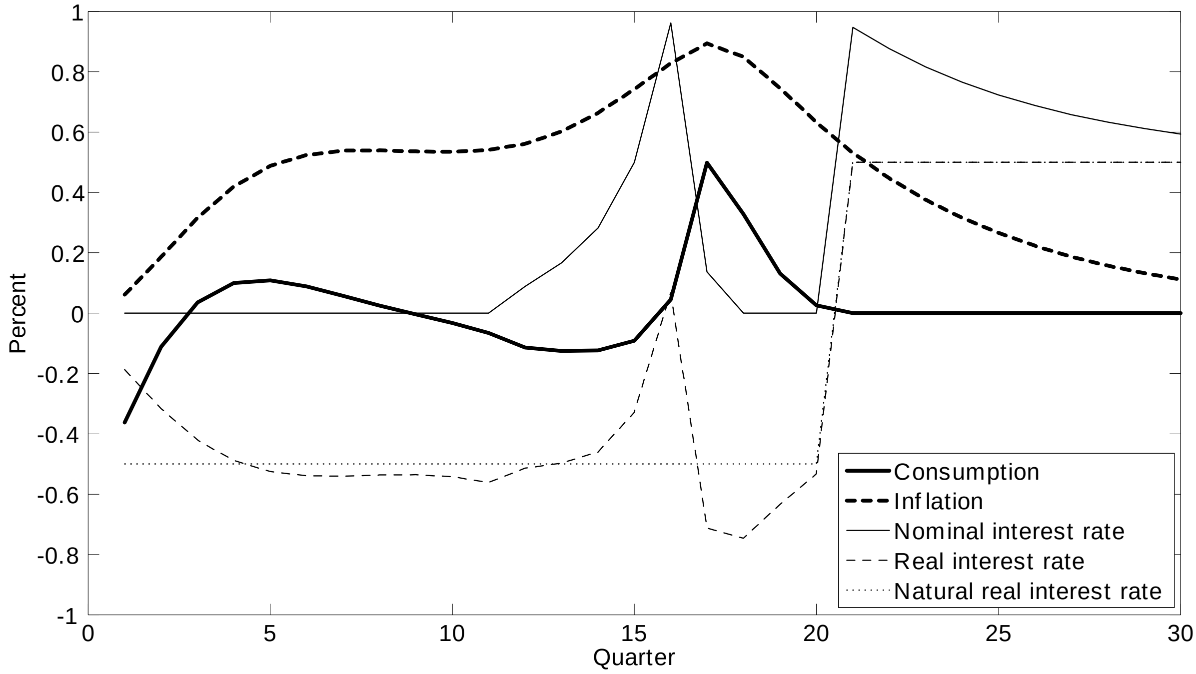

Figure 5: Optimal monetary policy with commitment up to T when $\omega = 0.841$

after the crisis is over, as in an economy without inflation persistence. However, the "irresponsible" behavior is much milder than in the standard new Keynesian model of Figure 1, where the real rate remains below the natural rate for three consecutive quarters. By comparing Figure 1 and 4, we can also observe that inflation persistence enhances the ability of the central bank to stabilize the economy. Indeed, the real interest rate tracks the natural real rate much more closely with persistence than without, which results in smaller fluctuations of the output gap.

Figure 4 shows that, when $\omega = 0.841$, the central bank uses its ability to commit beyond time T, which must therefore be valuable. It is therefore interesting to investigate the optimal monetary policy when the central bank can only credibly commit up to time T, but not beyond. It is represented in Figure 5.

As expected, the inability of the central bank to commit beyond time T results in larger fluctuations in both the output gap and inflation. It is nevertheless remarkable that the monetary policy implemented during the crisis manages, on its own, to stabilize the economy.

The central bank commits to a sequence of positive nominal interest rates during the second half of the crisis. This generates inflation expectations beforehand, i.e. in the first half of the crisis. These expectations reduce the real interest rate, which raises output and, hence, inflation, as expected by agents. Towards the very end of the crisis, the nominal interest rate is reduced to zero while inflation is high and persistent. This mechanically reduces the real rate, which creates an output boom that further increases inflation. As inflation is forward looking, this further stimulates inflation expectations in the first half of the crisis. The rise in the nominal rate around the 16th quarter is quite sharp such as to increase the real rate, which reduces output relative to its future boom level. This helps stabilize the output gap. Finally, from time $T+1$ onwards, there is no commitment and the zero lower bound on the nominal rate is no longer binding. Hence, once the crisis is over, the first-best allocation is implemented.

2.5 Welfare Analysis

To quantify the value of commitment beyond time T, let us now perform a welfare analysis for different values of the persistence parameter ω. For any allocation of consumption and inflation, we can compute the corresponding social welfare loss using (6). However, the magnitude of this loss has no clear interpretation. Thus, to obtain a meaningful measure of welfare, I define an "output gap equivalent social welfare loss" denoted by $\bar{c}$. Formally, $\bar{c}$ is the solution to following equation:

$$\sum_{t=1}^{\infty} \beta^t \left[(\pi_t - \omega \pi_{t-1})^2 + \frac{\kappa}{\epsilon} c_t^2 \right] = \sum_{t=1}^{T} \beta^t \frac{\kappa}{\epsilon} \bar{c}^2. \tag{16}$$

Hence, by definition, the welfare loss from imperfect stabilization of consumption and inflation (the left hand side) is as large as the welfare loss from a first-best allocation except for an output gap of magnitude $\bar{c}$ for the duration of the crisis (the right hand side).

The output gap equivalent social welfare losses from imperfect stabilization are reported in Table 1. For instance, without inflation persistence, i.e. $\omega = 0$, and under full commitment, fluctuations in consumption and inflation reduce welfare by as much as a 2.22% output gap throughout the crisis.[22]

	$\omega = 0$	$\omega = 0.841$	$\omega = 1$
Full commitment	2.22%	0.54%	0.32%
Commitment up to time T	1471.88%	0.71%	0.32%

Table 1: Output gap equivalent social welfare loss from imperfect stabilization

Table 1 confirms that, in the absence of inflation persistence, commitment beyond time T is essential to stabilize the economy.[23] By contrast, with full indexation of prices

[22] To be precise, given the definition of the output gap given by (1), the 2.22% output gap corresponds to a deviation of consumption from its natural level equal to 2.22% of the steady state level of output.

[23] When $\omega = 0$ and commitment beyond T is not possible, the deviations of output and inflation from the steady state are so large that the log-linearized model cannot reliably quantify the huge magnitude of the welfare loss from imperfect stabilization.

to inflation, commitment beyond time T is useless under the proposed calibration of the model. This follows from the fact that the first-best allocation is reached even before the end of the crisis, as seen in Figure 3. Finally, in the intermediary case of partial indexation, commitment beyond T is valuable, but clearly not essential to stabilize the economy during the crisis.

The other interesting result from Table 1 is that, for any degree of commitment, inflation persistence makes it considerably easier for the central bank to stabilize the level of economic activity during a liquidity trap episode. This stands in sharp contrast with the common wisdom that, in normal times, inflation persistence is a destabilizing force.

Table 2 shows that, if the initial rate of inflation π_0 is set equal to -0.025, i.e. -10% per year, instead of 0, then an intermediate degree of inflation persistence is ideal to stabilize the economy. Indeed, a very strong degree of persistence makes it harder to escape from the initial stage of strong deflation. Hence, partial indexation is preferable to full indexation. When $\omega = 0$, the model is purely forward looking and therefore independent from the initial rate of inflation π_0.

	$\omega = 0$	$\omega = 0.841$	$\omega = 1$
Full commitment	2.22%	1.41%	1.89%
Commitment up to time T	1471.88%	1.49%	1.89%

Table 2: Output gap equivalent social welfare loss when $\pi_0 = -0.025$

3 Fiscal Policy

Let us now investigate the extent to which fiscal policy can help stabilize the economy in a liquidity trap with inflation persistence.

3.1 New Keynesian Model with Government Expenditures and Inflation Persistence

Following Woodford (2011) and Werning (2012), the new Keynesian model of the previous section can easily be extended to allow for variations in government expenditures. The government relies on lump sum taxes to buy goods from each monopolistically competitive producer. In steady state, this results in an aggregate level G of government expenditures. Let g_t denote the deviation of government expenditures from its steady state level, normalized by the steady state level of output. Thus, by definition:

$$g_t = \ln\left(\frac{G_t}{G}\right)\frac{G}{Y} \approx \frac{G_t - G}{Y}, \tag{17}$$

where G_t denotes the level of government expenditures at t. The new Keynesian Phillips curve, obtained by log-linearizing the optimal price setting decision of firms around the steady state, is:

$$\pi_t - \omega\pi_{t-1} = \beta\left[\pi_{t+1} - \omega\pi_t\right] + \kappa\left[c_t + (1-\Gamma)\, g_t\right], \tag{18}$$

where κ is defined as before, by (3). The parameter Γ is equal to:

$$\Gamma = \frac{(1-\alpha)\sigma Y/C}{\eta + \alpha + (1-\alpha)\sigma Y/C}. \tag{19}$$

It corresponds to the fiscal multiplier of the flexible price economy. Under flexible prices, agents respond to higher government expenditures by working more and by consuming less. Hence, the fiscal multiplier is always between 0 and 1, i.e. $\Gamma \in (0,1)$.

If $\sigma = \infty$, agents have a zero intertemporal elasticity of substitution of consumption. In that case, the wage rate, and hence the real marginal cost of production, is determined by intertemporal substitution in consumption, since agents require an infinite increase in their current wage in order to accept working and consuming a little more at present. This results in $\kappa = \infty$. It follows that government expenditures have a negligible impact on marginal cost and on inflation, as confirmed by the fact that $\Gamma = 1$. By contrast, if $\sigma = 0$, agents have an infinite intertemporal elasticity of substitution. The wage rate, and hence the real marginal cost, is determined by the disutility of labor supply, which explains why consumption and government expenditures have an identical impact on marginal cost and on inflation, which is confirmed by $\Gamma = 0$.

The consumption Euler equation remains unchanged from the previous section:

$$c_t = -\frac{1}{\sigma Y/C}\left(i_t - \pi_{t+1} - r_t^n\right) + c_{t+1}, \tag{20}$$

where the natural real interest rate also remains specified as before, by (7).

Government expenditures are valued by consumers. More specifically, their utility function is additively separable between private consumption, government expenditures and labor supply. Let σ_G denote the inverse of the intertemporal elasticity of substitution of government expenditures. Assuming that the steady state level of government expenditures G is chosen optimally, the second-order approximation to the utility function of consumers around the steady state yields the following loss function:

$$\sum_{t=1}^{\infty}\beta^t\left[\left[\pi_t - \omega\pi_{t-1}\right]^2 + \frac{\kappa}{\epsilon}\left[c_t + (1-\Gamma)g_t\right]^2 + \frac{\kappa}{\epsilon}\gamma g_t^2\right], \tag{21}$$

where the parameter γ is defined as:

$$\gamma = \Gamma\left(1 - \Gamma + \frac{\sigma_G Y/G}{\sigma Y/C}\right). \tag{22}$$

There are two reasons why government expenditures appear in the loss function: it is valued by consumers and it affects labor supply.[24]

The equilibrium of the flexible price economy coincides with the first-best allocation of resources. Hence, the flexible price multiplier Γ gives the efficient response of output to an increase in government expenditures. But, as $C_t = Y_t - G_t$, if G_t increases by one unit, then C_t should ideally increase by $\Gamma - 1$ units, i.e. it should fall by $1 - \Gamma$ units. Thus, as can be seen from the loss function, the optimal deviation of consumption from its natural level following a fiscal shock is given by $c_t = -(1-\Gamma)g_t$.[25] The Phillips curve (18) implies that this efficient response of consumption generates an optimal rate of disinflation.

The optimal monetary policy is obtained by minimizing the loss function (21) with respect to the nominal interest rate i_t and to government spending g_t subject to the Phillips curve (18) with π_0 given, to the Euler equation (20) and to $i_t \geq 0$. The first-best allocation is characterized by $\pi_t = \omega\pi_{t-1}$ and $c_t = g_t = 0$. Without the zero lower bound, this allocation can always be implemented by setting $i_t = \pi_{t+1} + r_t^n = \omega^{t+1}\pi_0 + r_t^n$ for all $t \geq 1$. Finally, note that the model of the previous section is a special case of this model with $g_t = 0$ for all t.

3.2 Calibration

I rely on exactly the same calibration as in the previous section, which implies that $\Gamma = 0.263$. The only new deep parameter is σ_G, which is set equal to σ. Hence, $\sigma_G = 1$. This implies $\gamma = 1.247$.

3.3 Pump Priming the Economy

Before characterizing the optimal policy, I briefly consider an insightful benchmark where fiscal policy can be used in period 1 only, i.e. $g_t = 0$ for all $t \geq 2$. In particular, I show that, thanks to inflation persistence, the fiscal policy implemented at time 1 can permanently move the economy into a first-best allocation.

If the economy is in a first-best allocation from time 2 onwards, we must have: $c_t = g_t = 0$, $\pi_t = \omega\pi_{t-1} = \omega^{t-1}\pi_1$ and $i_t = \pi_{t+1} + r_t^n = \omega^t\pi_1 + r_t^n \geq 0$ for all $t \geq 2$.

[24] If $\sigma_G = 0$, consumers do not care about deviations away from the steady state level of government expenditures, which is set at the optimum of the flexible price economy. In that case, g_t only enters the loss function because of its effect on labor supply.

[25] The natural level was defined as the consumption level that prevails in a flexible price economy in the absence of government intervention. Hence, the natural level is *not* affected by fiscal shocks.

Crucially, the inflation rate π_1 must be high enough to guarantee that the zero lower bound will not be binding at 2 or thereafter. As $\omega \leq 1$, ω^t is non-increasing over time and, hence, a sufficient condition for the zero lower bound to be non-binding after time 2 is $i_T = \omega^T \pi_1 + \underline{r} \geq 0$. Thus, the smallest rate of inflation at time 1 consistent with the implementation of the first-best allocation is:

$$\pi_1 = \frac{-\underline{r}}{\omega^T}. \tag{23}$$

The equilibrium of the economy at time 1 is implicitly characterized by:

$$\pi_1 - \omega \pi_0 = \kappa \left[c_1 + (1 - \Gamma) g_1 \right], \tag{24}$$

$$c_1 = -\frac{1}{\sigma Y/C} \left(i_1 - \omega \pi_1 - \underline{r} \right). \tag{25}$$

I impose the mild condition $\omega < (\sigma Y/C)/\kappa$, which ensures that, on its own, monetary policy at time 1 cannot generate inflation. Thus, with $\pi_0 \leq 0$, the optimal monetary policy at 1 is to set $i_1 = 0$.[26] The magnitude of the fiscal stimulus g_1 necessary to escape the liquidity trap is obtained by substituting (23) and (25) with $i_1 = 0$ into (24). This yields:

$$g_1 = \frac{1}{\kappa(1-\Gamma)} \left[\frac{-\underline{r}}{\omega^T} \left(1 - \frac{\kappa \omega}{\sigma Y/C} \left(1 - \omega^{T-1} \right) \right) - \omega \pi_0 \right] > 0. \tag{26}$$

This proves by construction that fiscal policy can, within one period, allow the economy to reach a first-best allocation.

While this policy is clearly unlikely to be optimal, it shows that inflation persistence considerably enhances the ability of fiscal policy to lift the economy out of a liquidity trap. A front-loaded stimulus gets inflation started. This reduces the real interest rate, which brings it closer, or even equal, to its natural counterpart.

Thus, with inflation persistence, fiscal policy can "pump prime" the economy by generating a sufficient amount of inflation. Importantly, the efficacy of this policy does not rely on the forward looking behavior of agents.

Of course, the fiscal stimulus and the required rate of inflation are very large, unless ω is close to 1. Under the chosen calibration of the model, if $\omega = 1$, then inflation at time 1 needs to rise to 0.005, i.e. 2% per year. This requires a fiscal stimulus g_1 equal to 0.034, i.e. 3.4% of the quarterly steady state level of GDP. However, if $\omega = 0.841$, then inflation needs to rise to 0.16, i.e. 64% per year, which requires a stimulus equal to 0.935, i.e. 93.4% of quarterly GDP. The output gap equivalent social welfare loss from this policy is equal to 1.23% when $\omega = 1$ and to 36.90% when $\omega = 0.841$.[27]

[26] In fact, even if π_0 is positive, but not too high, then the optimal monetary policy is still characterized by $i_1 = 0$.

[27] The output gap equivalent social welfare loss in the presence of fiscal policy is formally defined below,

3.4 Opportunistic vs. Stimulus Spending

Following Werning (2012), I shall decompose government expenditures into two components: opportunistic spending and stimulus spending. More formally, opportunistic spending at time t is defined as:

$$g_t^* = \arg\max_{g_t} \left[c_t + (1-\Gamma)g_t\right]^2 + \gamma g_t^2. \tag{27}$$

It therefore corresponds to level of spending that the government would like to have, ignoring all dynamic general equilibrium effects. Solving (27) yields:

$$g_t^* = -\frac{1-\Gamma}{(1-\Gamma)^2+\gamma}c_t, \tag{28}$$

which, under the chosen calibration, gives $g_t^* = -0.41c_t$. In a depressed economy, the demand for consumption is low, which induces firms to reduce their demand for labor. This lowers the equilibrium wage rate, which reduces the cost of government expenditures. Hence, opportunistic spending is always countercyclical.[28] The other component, stimulus spending, is defined as the residual:

$$\hat{g}_t = g_t - g_t^*. \tag{29}$$

It corresponds to the spending induced by dynamic general equilibrium considerations, which are realized to stimulate the economy.

3.5 Fiscal Policy without Inflation Persistence

To begin the analysis of optimal monetary and fiscal policy in a liquidity trap, I consider the benchmark case without inflation persistence, where $\omega = 0$.[29] As shown by Werning (2012), under full commitment, the optimal fiscal policy is to have no stimulus spending whenever $\epsilon\sigma Y/C = 1$. In that case, the burden of stabilizing the economy exclusively relies on monetary policy, while fiscal policy only consists of countercyclical opportunistic government spending.

The proposed calibration implies that $\epsilon\sigma Y/C = 7.5 > 1$. Figure 6 displays the corresponding optimal fiscal policy when it is jointly determined with monetary policy under full commitment. Qualitatively, the other two endogenous variables of the economy, i.e. π_t and i_t, behave almost exactly as in Figure 1 and are therefore not reported. Figure 6

by equation (30). It is a straightforward generalization from the previous section.

[28] This insight would naturally extend to a model that does allow for unemployment, as the high rates of unemployment that are typical of recessions reduce the opportunity cost of government spending.

[29] The resolution of the optimal monetary and fiscal policy problem is outlined in appendix B.

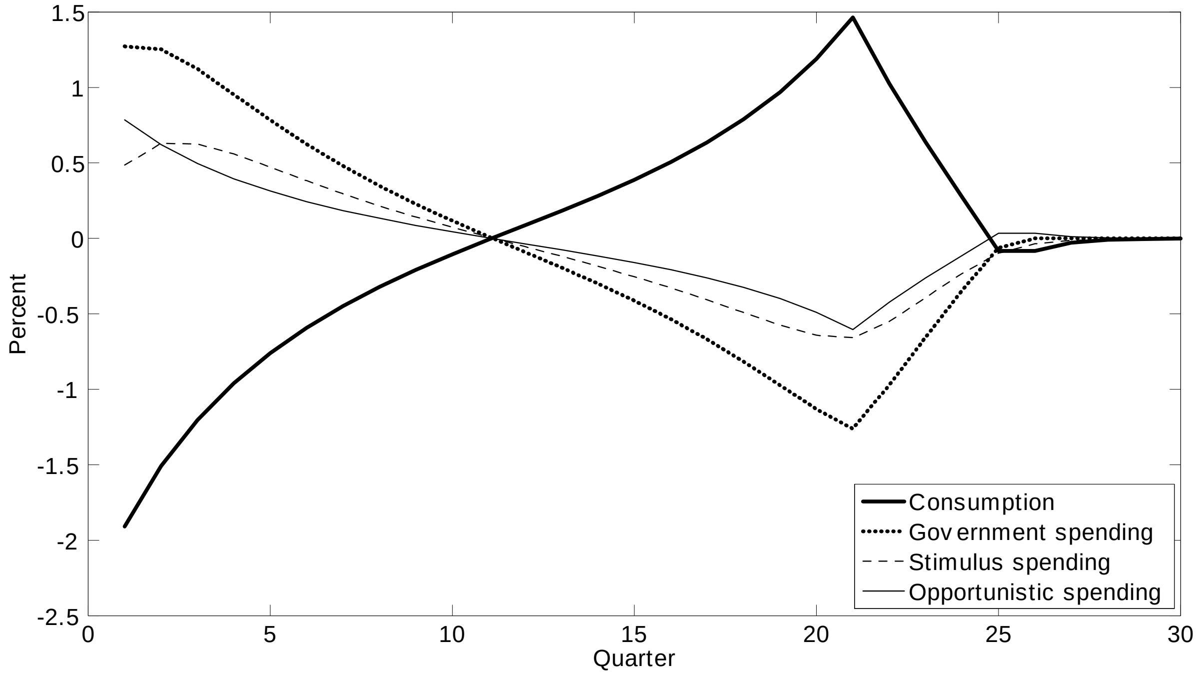

Figure 6: Optimal monetary and fiscal policy with full commitment when $\omega = 0$

shows that the stimulus component of government spending is as strongly countercyclical as the opportunistic component.

By committing to keep the nominal interest rate equal to zero for some time after the crisis is over, the central bank generates an output boom towards the end of the crisis. When the elasticity of substitution across goods ϵ is high enough to ensure that $\epsilon\sigma Y/C > 1$, then the government cares much more about price dispersion $\pi_t - \omega\pi_{t-1}$ than about the output gap $c_t + (1-\Gamma)g_t$ or about fluctuations in government spending g_t, as can be seen from the loss function (21). Then, the optimal fiscal policy is to implement a strongly countercyclical stimulus component of government spending, such as to increase the magnitude of the consumption boom. Indeed, a low level of government spending reduces the output gap $c_t + (1-\Gamma)g_t$, which allows the central bank to keep its nominal rate equal to zero for slightly longer after the crisis is over such as to enhance the magnitude of the consumption boom.[30] A larger boom implies that, in the midst of the crisis, slightly less inflation is required to stabilize the economy, i.e. the economy can cope with a slightly higher real interest rate.

Interestingly, if we discount the future at rate β, the present value of government expenditures induced by the crisis is equal to zero, i.e. $\sum_{t=1}^{\infty} \beta^t g_t = 0$. As we shall see in the next subsection, this is a general result under full commitment that also applies with

[30] In Figure 1, without fiscal policy, the nominal interest rate remains equal to zero for three quarters after the crisis is over. Under the jointly optimal monetary and fiscal policy, the nominal rate remains at the zero lower bound for four quarters and rises slightly more slowly afterwards.

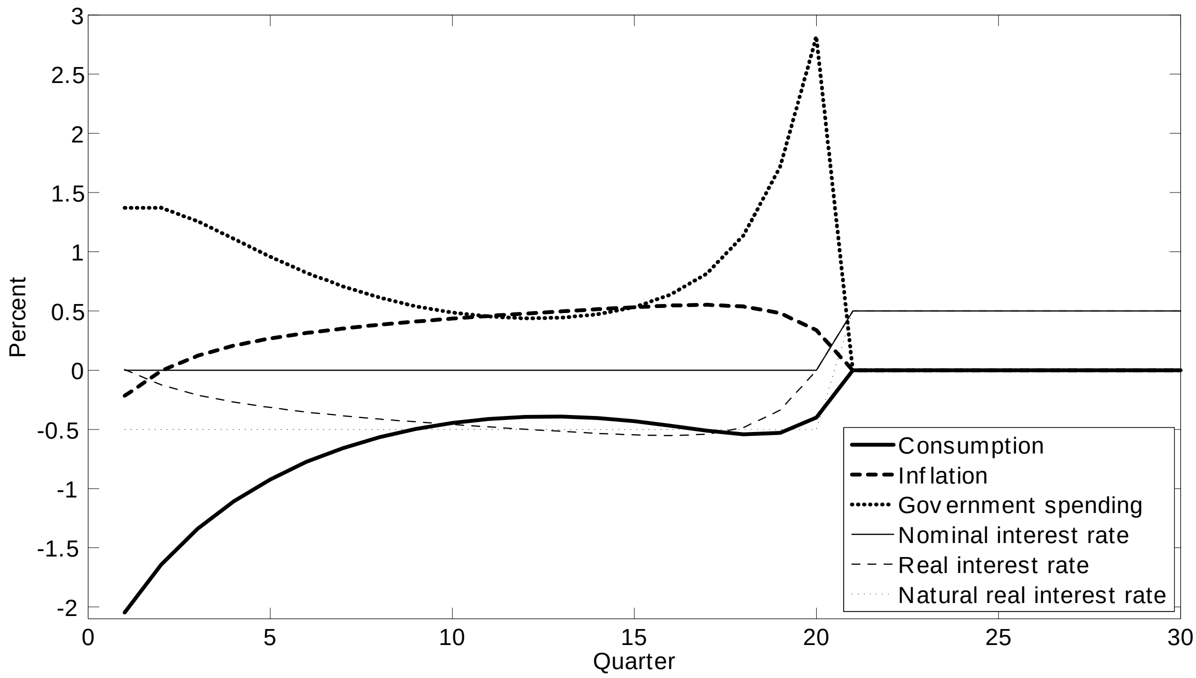

Figure 7: Optimal monetary and fiscal policy with commitment up to T when $\omega = 0$

inflation persistence.

While government expenditures are only used to fine-tune the optimal policy under full commitment, it is solely responsible for stabilizing the economy when the government cannot commit beyond time T. Figure 7 displays the equilibrium allocation under the optimal monetary and fiscal policy with commitment up to time T.

Clearly, throughout the crisis, the optimal monetary policy is to be as accommodative as possible and, hence, to set the nominal interest rate equal to zero. Without commitment beyond time T, the economy is in a first-best allocation as soon as the crisis is over, with $c_t = \pi_t = g_t = 0$ and $i_t = r_t^n = \bar{r}$ for all $t \geq T + 1$.

Note that Werning (2012) considers a different problem where fiscal and monetary policy are characterized by different degrees of commitment. In particular, he considers the case where fiscal authorities can commit up to time T, or even beyond, while monetary policy cannot commit at all. As a result, any inflation generated by fiscal policy is immediately killed off by the central bank through a rise in the nominal rate. Such a low degree of coordination between monetary and fiscal policy in crisis time seems implausible. By contrast, I assume that monetary and fiscal policy are jointly determined with commitment up to T. Hence, the central bank can commit not to raise its interest rate throughout the duration of the crisis. This enhances the scope of fiscal policy.[31]

[31] In this paper, I do not consider cases where monetary and fiscal policy are characterized by different degrees of commitment. Inflation persistence makes the model both forward and backward looking. Moreover, the deterministic length of the crisis makes the model non-stationary. Thus, solving for the

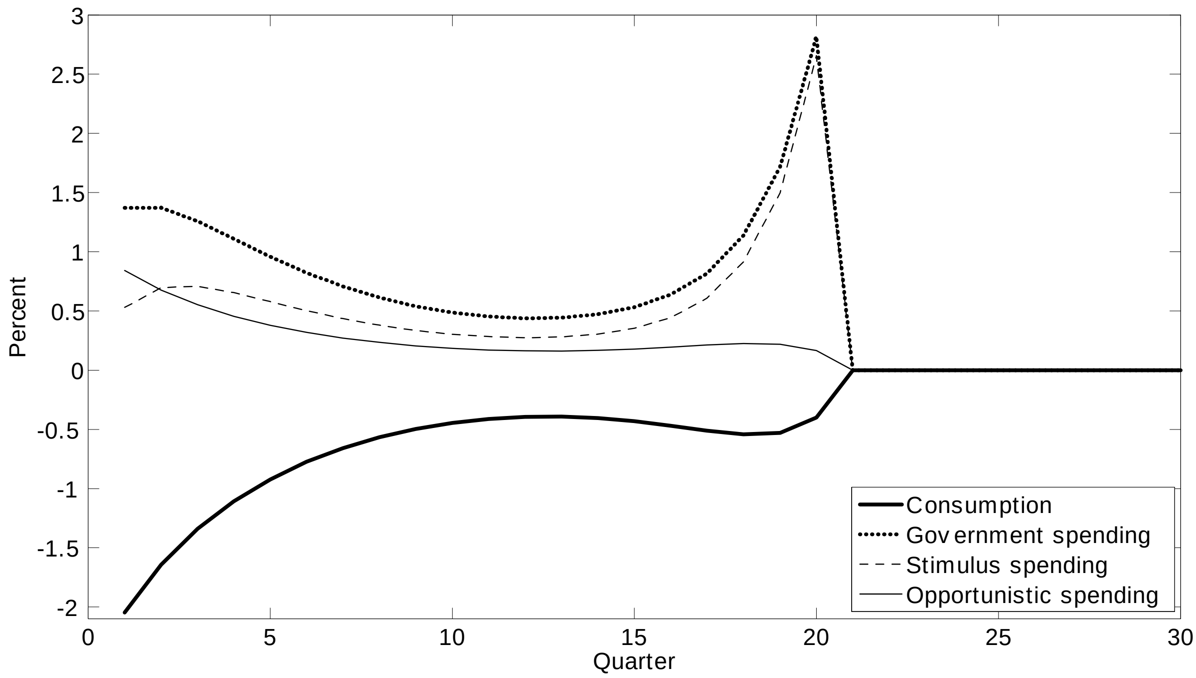

Figure 8: Optimal monetary and fiscal policy with commitment up to T when $\omega = 0$

Figure 8 displays the decomposition of the optimal path of government expenditures. It shows that stimulus spending account for nearly 70% of the optimal fiscal policy. The bulk of this stimulus is concentrated towards the very end of the crisis, with a peak at time T. This creates a boom in the output gap $c_t + (1 - \Gamma)g_t$ at the end of the crisis, driven by high government expenditures g_t rather than by high consumption c_t. The expectation of this boom generates inflation. This reduces the real interest rate, which stabilizes the demand for consumption. This fiscal policy breaks the deflationary spiral shown in Figure 2 that would occur with monetary policy alone.

If we had $\epsilon\sigma Y/C = 1$, then stimulus spending would be virtually equal to zero over the first half of the crisis. However, with $\epsilon\sigma Y/C = 7.5 > 1$, the government is willing to tolerate larger fluctuations in the output gap and in government expenditures in order to reduce fluctuations in inflations. Hence, compared to the case where $\epsilon\sigma Y/C = 1$, it implements a stimulus program which is slightly less inflationary at the end of the crisis. This needs to be compensated with more government spending throughout the crisis and especially towards the beginning when the output boom at the end of the crisis is only a distant prospect.

It is interesting to note that the optimal timing of government expenditures under

optimal monetary policy without commitment is numerically demanding. Solving for the optimal fiscal policy with commitment taking into account the discretionary response of monetary policy is therefore beyond the scope of this paper. Similarly, I do not solve for the optimal monetary policy with commitment together the optimal fiscal policy without commitment.

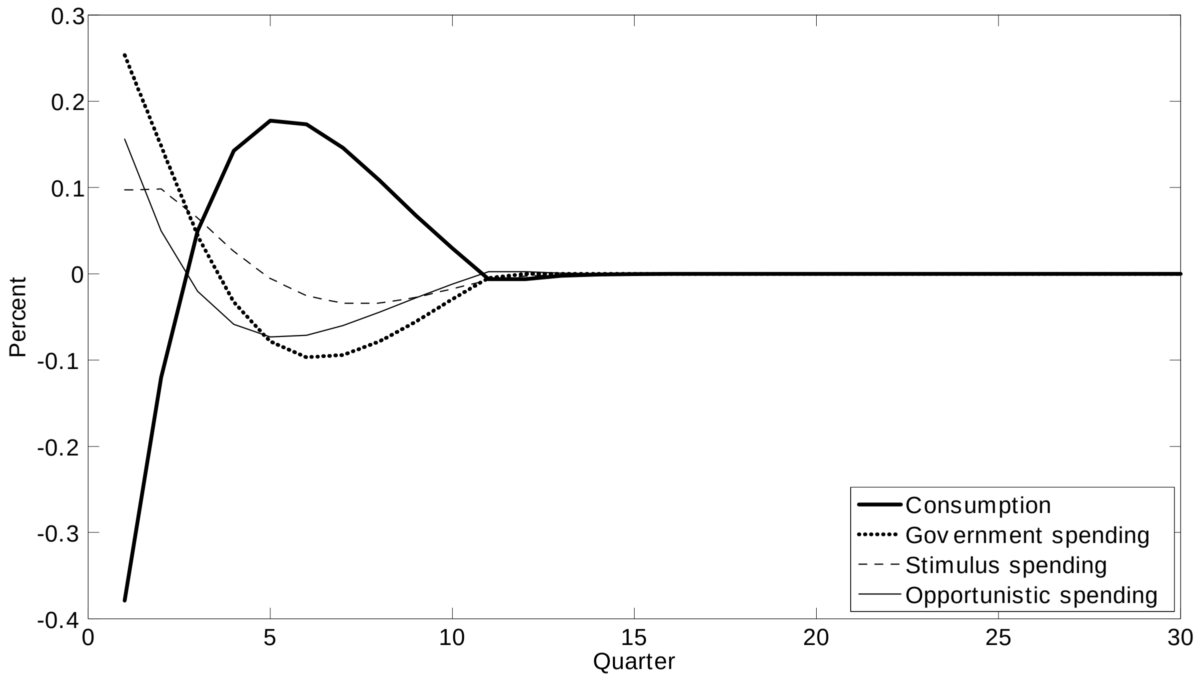

Figure 9: Optimal monetary and fiscal policy with full commitment when $\omega = 1$

rational expectations and without inflation persistence is very different from the one typically adopted by governments in crisis time. In practice, most stimulus efforts are concentrated at the beginning of crises. A typical concern is that many possible expenditures, such as infrastructure investments, cannot be realized sufficiently rapidly. They are therefore not included in stimulus packages, even though they would offer a perfect commitment device to spend in the future.

But, in a model that is purely forward looking, past stimulus spending has no impact whatsoever on the current level of economic activity. Governments must therefore believe that initial spending has a persistent stabilizing impact on the economy. To capture this, we need to add a backward looking component into the new Keynesian model; and the most natural way of doing so is to allow for inflation persistence.

3.6 Fiscal Policy with Inflation Persistence

Let us now investigate the joint determination of monetary and fiscal policy in the presence of inflation persistence. I begin by analyzing the benchmark case of full indexation, where $\omega = 1$. Figure 9 displays the optimal fiscal policy under full commitment. The other two endogenous variables of the model, i.e. π_t and i_t, behave almost exactly as in Figure 3 and are therefore not reported. The only difference is that the nominal interest rate starts rising two quarters later than in Figure 3 and by a slightly smaller amount, which is not surprising since fiscal policy now contributes to stabilizing the economy.

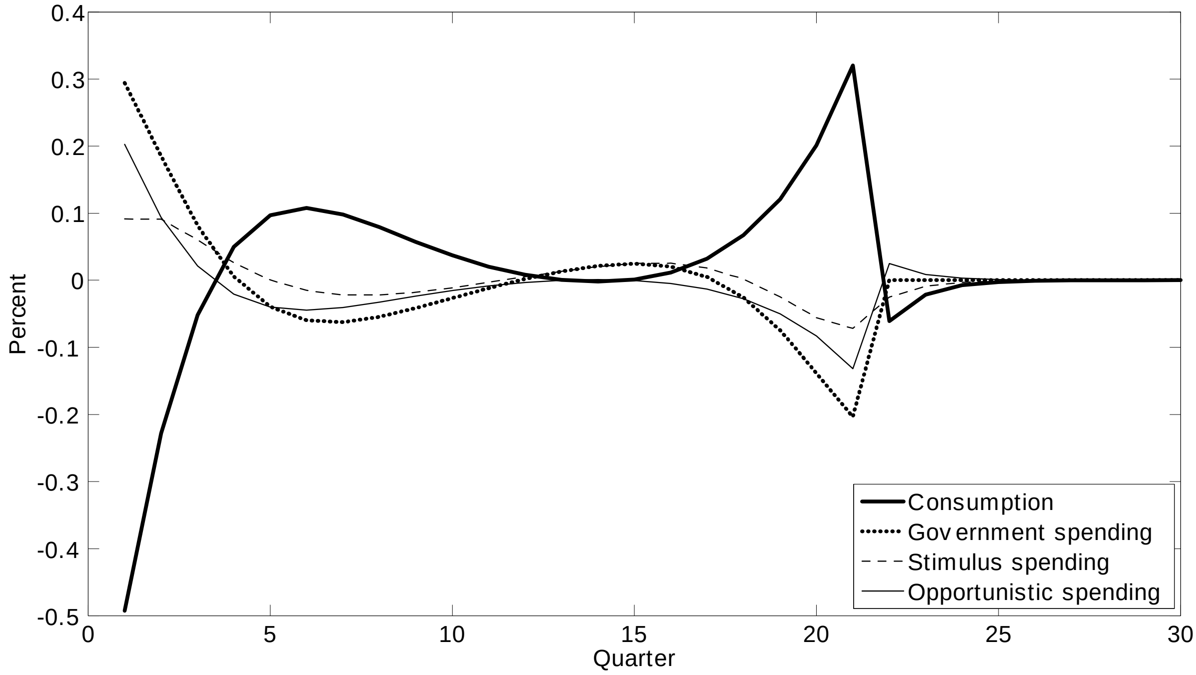

Figure 10: Optimal monetary and fiscal policy with full commitment when $\omega = 0.841$

With inflation persistence, the stimulus component of government spending is front-loaded. The aim is to create a positive output gap $c_t + (1 - \Gamma)g_t$ such as to get inflation started. After the fifth quarter, the stimulus component becomes slightly negative in order to eliminate the positive output gap, such as to stabilize inflation to its new higher level. Interestingly, if the nominal interest rate was constrained to be equal to zero throughout the crisis, the optimal fiscal policy would be virtually unchanged and social welfare would hardly decrease. This shows that governments can rely on fiscal policy alone to stabilize the economy when $\omega = 1$.

Note that the economy reaches a first-best allocation before time T. Hence, commitment beyond T is useless and the optimal monetary and fiscal policy with commitment up to T remains unchanged.

While the full indexation benchmark is insightful, it is more realistic to assume partial indexation of prices to inflation. Figure 10 displays the optimal fiscal policy when $\omega = 0.5^{1/4} \simeq 0.841$, which is jointly determined with monetary policy under full commitment. Again, the paths of inflation and of the nominal interest rate are almost unchanged from Figure 4 and are therefore not reported.

The optimal fiscal policy under partial indexation and full commitment is a combination of the policy with full indexation and with no persistence. The stimulus component is positive at the beginning of the crisis, in order to spur inflation, as under full indexation; and it is negative around the end of the crisis, in order to induce an adjustment in

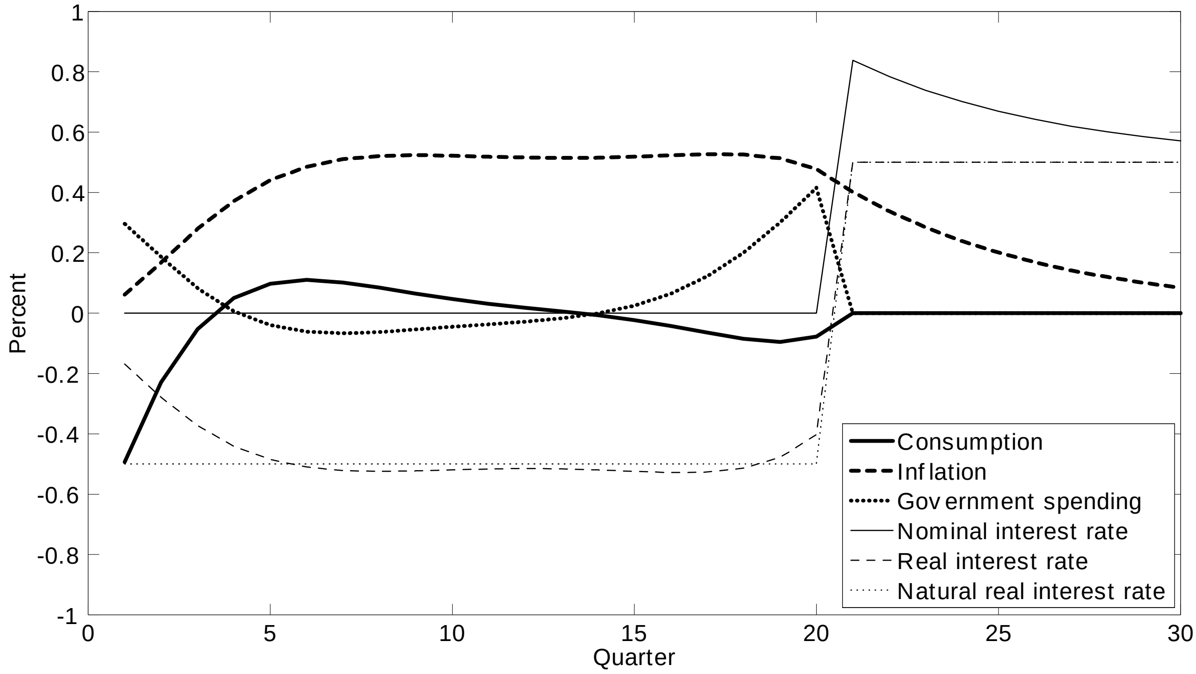

Figure 11: Optimal monetary and fiscal policy with commitment up to T when $\omega = 0.841$

monetary policy that strengthens the consumption boom,[32] as in the absence of inflation persistence.

If the fiscal authority can fully commit, then, with a discount rate equal to β, the present value of government expenditures induced by the optimal fiscal policy is always equal to zero, i.e. $\sum_{t=1}^{\infty} \beta^t g_t = 0$. This can easily be shown by combining the first-order conditions to the optimal policy problem.[33] This results holds even if the central bank commits to a path of nominal interest rates that is not optimal. It follows that, under full commitment, the fiscal policy debate should focus entirely on the timing of government expenditures, not on its average level.

Finally, Figure 11 displays the optimal allocation under partial indexation, with $\omega = 0.841$, when monetary and fiscal authorities can only commit up to time T. While monetary policy alone can stabilize the economy in that case, as shown by Figure 5, it turns out not to play any role in the presence of fiscal policy. Indeed, the optimal policy is for the central bank to commit to set the nominal interest rate equal to zero throughout the crisis.

[32] Recall that consumption is determined by the Euler equation (20) and is therefore not directly affected by the level of government expenditures.

[33] It is obtained by combining equations (B4) and (B5) from appendix B, which are the first-order conditions with respect to c_t and g_t, respectively. More precisely, equation (B4) for each time t must be multiplied by β^t. All these equations must then be added for t running from 1 to infinity. All the λ_t's for $t \geq 1$ cancel out, while we know that $\lambda_0 = 0$. Equation (B5) should then be used to replace $c_t + (1-\Gamma)g_t - \epsilon\mu_t$ by $-\gamma/(1-\Gamma)g_t$. The resulting sum can then be factorized and simplified to deliver the desired result.

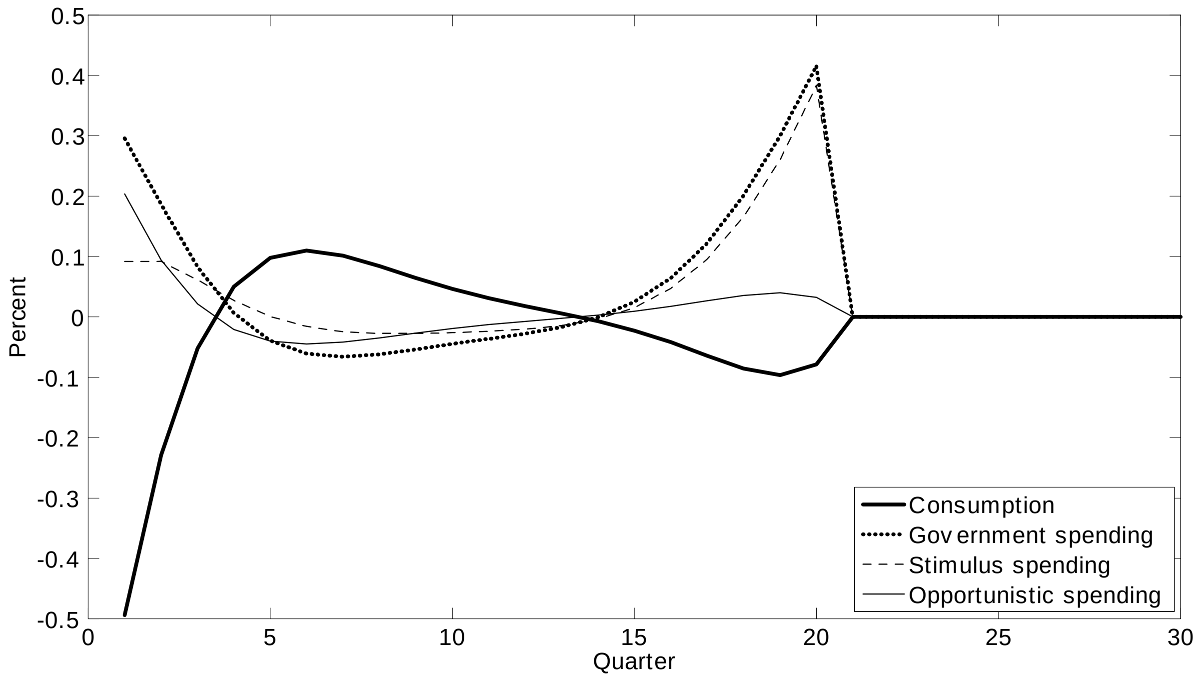

Figure 12: Optimal monetary and fiscal policy with commitment up to T when $\omega = 0.841$

The optimal fiscal policy is both front-loaded and back-loaded. Its decomposition is shown in Figure 12, which reveals that, again, the optimal policy under partial commitment is a combination of the policy with full indexation and with no persistence. The stimulus component is positive both at the beginning of the crisis, in order to get inflation going, and especially at the end, in order to create an output boom, the expectation of which is inflationary.[34] Under partial commitment, the stimulus component is by far the main driver of fiscal policy, accounting for more than two thirds of the deviation from the steady state level of government expenditures.

While the shape of the optimal fiscal policy under partial indexation (Figure 12) is broadly similar to that with no persistence (Figure 8), the magnitude of the required stimulus is much smaller in the former case. This shows that inflation persistence makes it considerably easier for fiscal policy to stabilize the economy.

3.7 Welfare Analysis

I now turn to the welfare analysis in order to quantify the effectiveness of fiscal policy. Table 3 reports the output gap equivalent social welfare loss from imperfect stabilization under an optimal monetary and fiscal policy. As in the previous section, the output gap

[34] Note that, even when $\epsilon\sigma Y/C = 1$, the stimulus component is still positive (but of a smaller magnitude) at the beginning of the crisis, which would not be the case without inflation persistence.

equivalent social welfare loss $\bar{c}$ is implicitly defined by:

$$\sum_{t=1}^{\infty}\beta^t\left[[\pi_t-\omega\pi_{t-1}]^2+\frac{\kappa}{\epsilon}[c_t+(1-\Gamma)g_t]^2+\frac{\kappa}{\epsilon}\gamma g_t^2\right]=\sum_{t=1}^{T}\beta^t\frac{\kappa}{\epsilon}\bar{c}^2. \tag{30}$$

Comparing Table 1 and 3 reveals that fiscal policy is particularly valuable when inflation persistence is low and when the government cannot commit beyond time T. The gains generated by fiscal policy are relatively small under full commitment or under partial commitment when $\omega = 1$, in which case commitment beyond time T is useless. When $\omega = 0.841$, under partial commitment, fiscal policy alone[35] yields a welfare loss of 0.56% and is therefore significantly more efficient than monetary policy alone, which yields a welfare loss of 0.71%.

	$\omega = 0$	$\omega = 0.841$	$\omega = 1$
Full commitment	1.89%	0.51%	0.30%
Commitment up to time T	2.57%	0.56%	0.30%

Table 3: Output gap equivalent social welfare loss from imperfect stabilization under optimal monetary and fiscal policy

4 Monetary and Fiscal Policy with Adaptive Expectations

So far, I have assumed that inflation persistence was due to price indexation, through the parameter ω. However, inflation persistence can instead be caused by backward looking expectations. Indeed, Milani (2007) estimates $\omega = 0.885$ under rational expectations, but $\omega = 0.032$ under adaptive learning.

In this section, I therefore set $\omega = 0$ and assume that agents form adaptive expectations throughout the duration of the crisis. The rational expectation hypothesis is indeed very demanding in crisis time. Even in the absence of uncertainty, agents need to have a high degree of sophistication in order to be able to work out the whole future trajectory of inflation. They need to think recursively from the end of the crisis, or even beyond, to the present time.[36] Thus, adaptive expectations seems to provide a sensible alternative benchmark.

[35] Recall that, under partial commitment and with $\omega = 0.841$, the optimal monetary policy is to commit to a zero nominal interest rate throughout the crisis.

[36] As a result, under rational expectations, the current level of inflation and output are very sensitive to the monetary policy implemented in a distant future. For instance, consider the optimal monetary policy with commitment up to T and with $\omega = 0.841$. Assume that the central bank commits to the path of nominal interest rates displayed in Figure 5, expect in the 18th quarter where the nominal rate is set equal to 0.1%, i.e. 0.4% per year, instead of 0%. This small change completely destabilizes the

I assume that, once the crisis is over, agents revert to rational expectations. This implies that, from time $T+1$ onwards, the economy will be in a first-best allocation with no inflation and no output gap, i.e. $\pi_t = 0$ and $c_t + (1-\Gamma)g_t = 0$ for all $t \geq T+1$. Forming rational expectations of future inflation is admittedly much simpler in normal times than in exceptional circumstances when the zero lower bound is binding. It is much easier to predict perfect stabilization after the crisis is over than to figure out the whole trajectory of inflation during the crisis. Also, the assumption of rational expectations from $T+1$ onwards simplifies the analysis as, otherwise, inflation persistence due to backward looking expectations would create a trade-off between inflation and output stabilization after the crisis is over.

For simplicity, I assume that the government cannot commit beyond time T. Hence, the fiscal authority will choose to set $g_t = 0$ for all $t \geq T+1$.

Let $\tilde{\mathbb{E}}_{t-1}$ denote the expectations formed by agents based on $t-1$ information. Agents know that inflation will be in steady state, normalized to zero, after the crisis. Thus, $\tilde{\mathbb{E}}_{t-1}[\pi_{t+k}] = 0$ for all $k \geq T+1-t$. Expectations of inflation in crisis time are adaptive and are therefore based on both the last observed rate of inflation and the steady state rate of inflation, normalized to zero. We therefore have:

$$\tilde{\mathbb{E}}_{t-1}[\pi_t] = \phi\pi_{t-1} + (1-\phi)0, \tag{31}$$

where $\phi \in [0,1]$ is a parameter capturing the influence of current inflation on future forecasts. Thus, $1-\phi$ corresponds to the extent to which inflation expectations are well anchored at the steady state rate of inflation. Iterating on adaptive expectations immediately yields:

$$\tilde{\mathbb{E}}_{t-1}[\pi_{t+k}] = \phi^{k+1}\pi_{t-1}, \tag{32}$$

for any $k \leq T-t$.

Households choose their demand for consumption such as to maximize their expected intertemporal utility. The solution to the household problem at time t is characterized by a set of Euler equations, which can be log-linearized to yield:

$$\tilde{\mathbb{E}}_{t-1}[c_{t+k}] = -\frac{1}{\sigma Y/C}\left(i_{t+k} - \tilde{\mathbb{E}}_{t-1}[\pi_{t+k+1}] - r^n_{t+k}\right) + \tilde{\mathbb{E}}_{t-1}[c_{t+k+1}], \tag{33}$$

where both the paths of the natural real interest rate r^n_t and of the nominal rate i_t until T are exogenous and publicly known. As households rely on backward looking expectations, their consumption plans made at the beginning of time t is based on the information available at the end of $t-1$. Thus, households are unable to foresee the

economy and results in a current output gap equal to -25.76%, instead of -0.36%, and a current rate of inflation of -8.82%, instead of 0.06%.

impact of their demand at time t on the aggregate price level, and on inflation, at t. This limited understanding of general equilibrium effects seems plausible in the absence of rational expectations.

Households know that, as soon as the crisis is over, the output gap will be reduced to zero and government spending will be at its steady state level. Thus, $\tilde{\mathbb{E}}_{t-1}[c_{t+k}] = 0$ for all $k \geq T + 1 - t$. Combining Euler equations for different values of k together with the fact that $\tilde{\mathbb{E}}_{t-1}[c_{T+1}] = 0$ yields:

$$\tilde{\mathbb{E}}_{t-1}[c_{t+k}] = -\frac{1}{\sigma Y/C} \sum_{l=k}^{T-t} \left(i_{t+l} - \tilde{\mathbb{E}}_{t-1}[\pi_{t+l+1}] - r^n_{t+l} \right), \tag{34}$$

for $k \leq T - t$. Note that, the actual demand c_t for consumption at t is decided at the beginning of time t based on $t - 1$ information, which implies that $c_t = \tilde{\mathbb{E}}_{t-1}[c_t]$. Hence, using the expectation rule (32) for $k \leq T - t$ and $\tilde{\mathbb{E}}_{t-1}[\pi_{T+1}] = 0$ yields, after some simplifications:

$$c_t = -\frac{1}{\sigma Y/C} \sum_{l=0}^{T-t} (i_{t+l} - \underline{r}) + \frac{\phi^2}{\sigma Y/C} \frac{1 - \phi^{T-t}}{1 - \phi} \pi_{t-1}, \tag{35}$$

where I have used the fact that $r^n_{t+l} = \underline{r} < 0$ for all $l \leq T - t$.

Inflation in the new Keynesian framework is fundamentally determined by the price setting behavior of monopolistically competitive firms. Using the definition of the aggregate price index and the link between the real marginal cost of production and the output gap, the log-linearized first-order condition to firms' optimal price setting problem can be directly expressed as a function of inflation and of the output gap. This yields:[37]

$$\pi_t = \tilde{\mathbb{E}}_{t-1} \left[\sum_{k=0}^{\infty} (\theta\beta)^k \left[\theta\kappa \left(c_{t+k} + (1 - \Gamma) g_{t+k} \right) + (1 - \theta) \pi_{t+k} \right] \right]. \tag{36}$$

When re-optimizing their own price, firms care about their future nominal marginal costs. This is why they need to form expectations of future output gaps, which determine real marginal costs, and of future inflation; both of which appear on right hand side of (36).

To determine their new price at t, firms rely on information available at the end of time $t-1$. Thus, they do not realize that the inflation rate at t is determined, through the price index, by the newly reset price that they themselves, together with symmetric firms, choose to set. By making expectations at t conditional on $t - 1$ information, I assume that neither households nor firms fully understand contemporaneous general equilibrium effects, which seems plausible in the absence of rational expectations.[38] Note that, at

[37] Taking expectations with respect to t, instead of $t - 1$, and using the law of iterated expectations, which does not hold under adaptive expectations, this expression immediately simplifies to the new Keynesian Phillips curve (18) with $\omega = 0$.

[38] This assumption can however easily be relaxed by taking expectations everywhere conditional on t,

any point in time, households and firms are forming the same expectations, which is consistent with the evidence reported by Coibion and Gorodnichenko (2014).

Substituting (34) for $k \leq T-t$ and $\tilde{\mathbb{E}}_{t-1}[c_{t+k}] = 0$ for $k \geq T+1-t$ into (36) and using the expectation rule (32) for $k \leq T-t$ and $\tilde{\mathbb{E}}_{t-1}[\pi_{t+k}] = 0$ for $k \geq T+1-t$ yields, after some simplifications:

$$\begin{aligned} \pi_t &= -\frac{\theta\kappa}{\sigma Y/C}\sum_{k=0}^{T-t}\frac{1-(\theta\beta)^{k+1}}{1-\theta\beta}\left(i_{t+k}-\underline{r}\right)+\theta\kappa(1-\Gamma)\sum_{k=0}^{T-t}(\theta\beta)^k g_{t+k} \\ &\quad + \left[(1-\theta)\phi\frac{1-(\theta\beta\phi)^{T+1-t}}{1-\theta\beta\phi}+\frac{\theta\kappa}{\sigma Y/C}\sum_{k=0}^{T-1-t}\frac{1-(\theta\beta)^{k+1}}{1-\theta\beta}\phi^{k+2}\right]\pi_{t-1}, \end{aligned} \tag{37}$$

for all $t \leq T$, where I have used the fact that $r^n_{t+k} = \underline{r} < 0$ for all $k \leq T-t$. This expression clearly shows that adaptive expectations at time t based on $t-1$ information is a source of inflation persistence provided that $\phi > 0$.

The government needs to choose the paths of nominal interest rates i_t and of government expenditures g_t that minimize the loss function:

$$\sum_{t=1}^{\infty}\beta^t\left[{\pi_t}^2+\frac{\kappa}{\epsilon}\left[c_t+(1-\Gamma)g_t\right]^2+\frac{\kappa}{\epsilon}\gamma g_t^2\right], \tag{38}$$

subject to equation (35), giving the actual consumption level at t, and to equation (37) together with a given initial rate of inflation π_0, describing the inflation dynamics. This loss function implies that the ideal rate of inflation is always equal to zero. Indeed, in the absence of price indexation, any non-zero inflation generates relative price distortions.

Let us set the initial rate of inflation equal to zero, i.e. $\pi_0 = 0$. In that case, without the zero lower bound, the optimal policy would trivially be to set the nominal interest rate i_t equal to the natural real rate r^n_t and to leave government spending constant at its steady state level. This would implement the first-best allocation. But, of course, with a negative natural real interest rate, this policy violates the zero lower bound.

Equation (37) together with the zero lower bound implies that, in the absence of fiscal policy, if $\pi_{t-1} \leq 0$, then $\pi_t < 0$. Thus, with $\pi_0 = 0$, deflation and a depressed consumption level are unavoidable throughout the crisis. In that context, the best that the central bank can do, with or without commitment, is to be as accommodative as possible by setting the nominal rate equal to zero throughout. This confirms the traditional view of the liquidity trap that, under backward looking expectations, monetary policy is useless.

instead of $t-1$. In that case, it is possible for a rise in the nominal rate to generate some inflation. The mechanism is even more straightforward than under rational expectations since, here, higher current inflation *mechanically* raises inflation expectations, as $\tilde{\mathbb{E}}_t[\pi_{t+k}] = \phi^k\pi_t$. This can reduce future real interest rate, which can raise current inflation. Note that, while potentially interesting to investigate, this case does not display any inflation persistence.

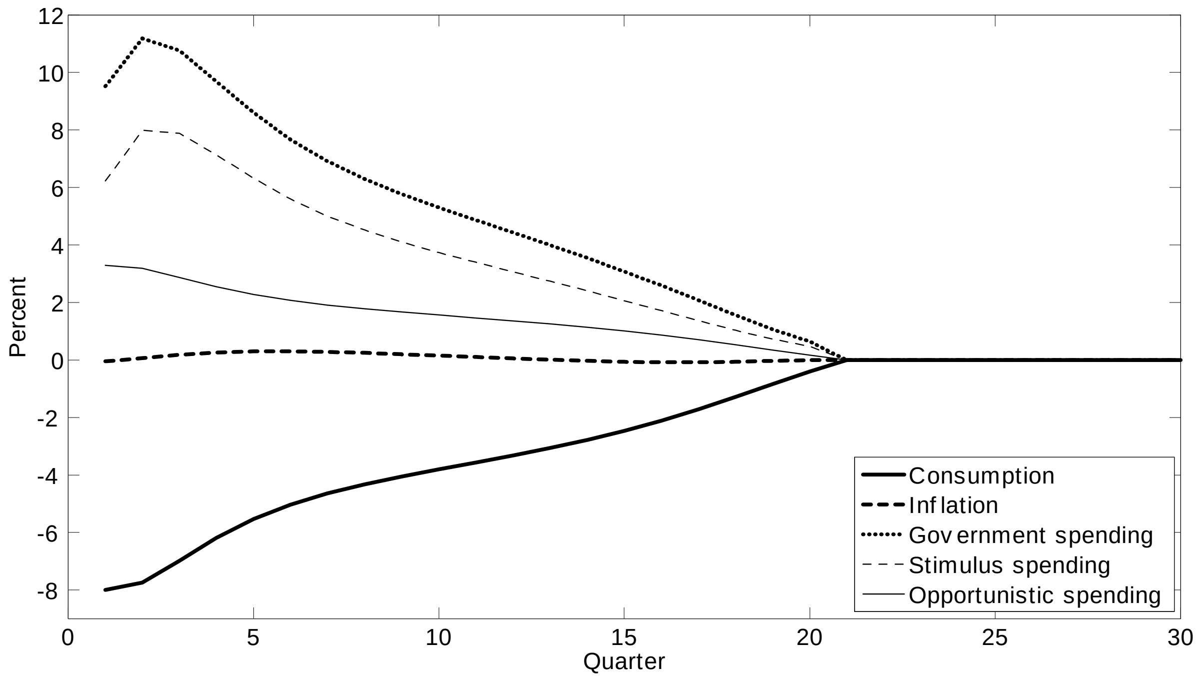

Figure 13: Optimal fiscal policy under adaptive expectations when $\phi = 0.841$

Thus, without rational forward looking behavior, inflation persistence is not sufficient to allow the central bank to stabilize the economy during a liquidity trap episode. Hence, under adaptive expectations, fiscal policy offers the only hope for economic stabilization.

The optimal fiscal policy can be characterized numerically. For the simulation, I set $\phi = 0.5^{1/4} \simeq 0.841$, which implies that the expectation at the end of time $t-1$ of inflation in one year is equal to half the current rate of inflation, i.e. $\tilde{\mathbb{E}}_{t-1}[\pi_{t+3}] = 0.5\pi_{t-1}$. Simulations confirm that it is optimal to set the nominal interest rate equal to zero throughout the crisis and to rely on fiscal policy alone to stabilize the economy. Figure 13 displays the optimal allocation. It also shows the decomposition of government spending between the opportunistic and the stimulus component, which are defined as in the previous section.

A striking result is the huge magnitude of the optimal fiscal stimulus, which in the first few quarters of the crisis exceeds 10% of quarterly steady state GDP.[39] Under rational expectations, a fiscal stimulus creates inflation expectations, which considerably enhances the efficacy of the stimulus program. Under adaptive expectations, however, fiscal policy cannot create inflation expectations, which must be compensated by a larger magnitude of government expenditures.

The fiscal stimulus serves two purposes. First, as can be seen in (37), it raises inflation, which prevents the economy from falling into deflation. Second, higher inflation raises

[39] Recall the definition of g_t given by (17).

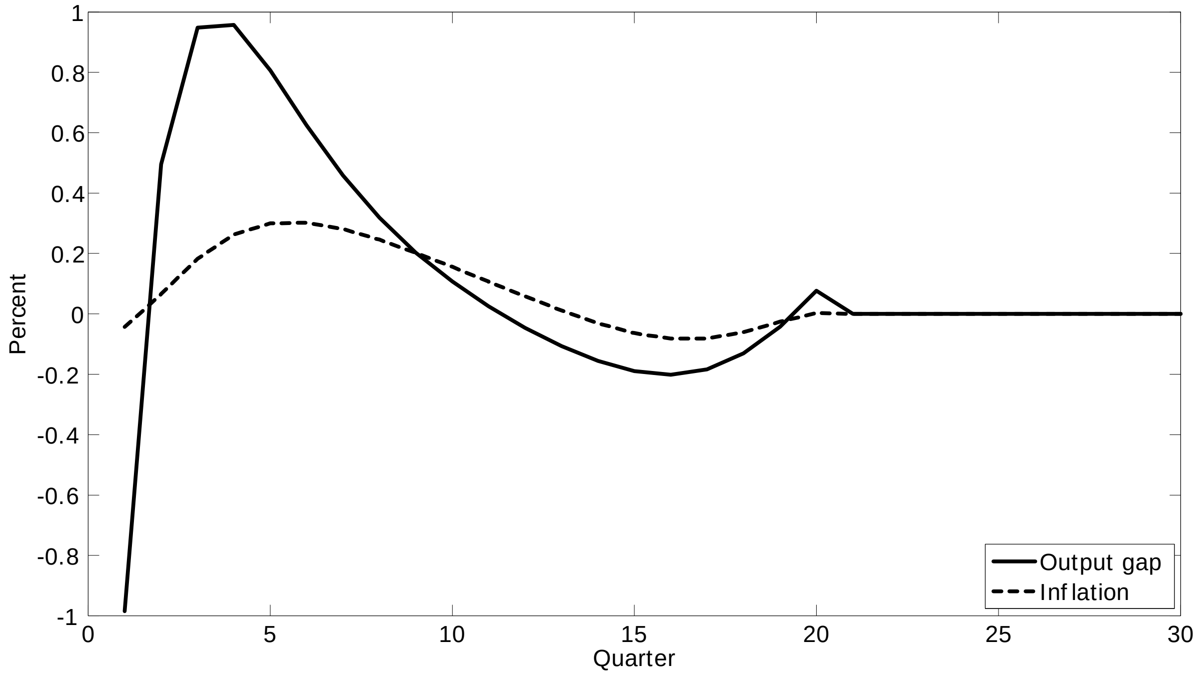

Figure 14: Output gap $c_t + (1 - \Gamma)g_t$ and inflation under the optimal policy when $\phi = 0.841$

consumption. While consumption does not seem fully stabilized, recall that what matters for social welfare is the output gap defined as $c_t + (1 - \Gamma)g_t$. Figure 14 displays both the output gap and inflation. It shows that, at the optimum, fluctuations in the output gap are rather small.

Inflation is, on average, slightly positive, but close to zero. It might therefore seem surprising that the optimal policy does not raise inflation by more. This has two explanations. First, a zero rate of inflation minimizes the relative price distortions, which weights heavily in the loss function. Second, inflation can only have a limited impact on future real interest rates. Indeed, at any time t, agents expect the real interest rate at time $t + k$, with $t + k \leq T - 1$, to be equal to $i_{t+k} - \tilde{\mathbb{E}}_{t-1}\left[\pi_{t+k+1}\right] = 0 - \phi^{k+2}\pi_{t-1}$, which is close to zero unless k is very small or ϕ is very close to 1.

Finally, Figure 13 shows that the optimal fiscal policy is heavily front-loaded, which is mostly due to the stimulus component of government spending. To understand this pattern, note that, by (34), consumption is equal to the sum of future gaps between the expected real interest rate and its natural counterpart. But, as we have just seen, the expected future real interest rates are close to zero. Thus, at the beginning of the crisis, the sum of future natural real rates is large, which results in the current and expected future levels of consumption to be very depressed. By (36), inflation is influenced by the expected level of future output gaps, defined as $c_t + (1 - \Gamma)g_t$. To avoid sharp deflation, the government needs to compensate the low levels of consumption by spending a lot.

Towards the end of the crisis, the remaining horizon is much shorter, hence agents only expect consumption to be slightly negative. This only needs to be offset by a small amount of government spending. In sum, a large and front-loaded stimulus is required to keep inflation close to zero throughout the crisis, which turns out to characterize the optimal policy when $\phi = 0.841$.

Table 4 displays the welfare implications of the optimal stabilization policy. The first line reports the welfare loss in the absence of fiscal policy and with the nominal interest rate set equal to zero throughout the crisis. Clearly, a larger value of ϕ implies a stronger degree of persistence and, hence, a stronger deflationary spiral. The effect is so strong that, for large values of ϕ, the log-linearized model does not permit a precise quantification of the welfare loss from imperfect stabilization. Fiscal policy is essential to stabilize the economy.

	$\phi = 0$	$\phi = 0.841$	$\phi = 1$
No fiscal policy	10.12%	3574.87%	$1.97 \cdot 10^9$%
Optimal fiscal policy	5.83%	7.14%	6.13%

Table 4: Output gap equivalent social welfare loss from imperfect stabilization under adaptive expectations

It might seem surprising that, under the optimal fiscal policy, the welfare loss is larger for an intermediate value of ϕ than for extreme values. When $\phi = 0$, expectations of future inflation are always equal to zero, i.e. expectations are perfectly anchored. Hence, consumption is independent of the last realization of inflation. It can therefore not be affected by fiscal policy. The real interest rate is nevertheless above the natural rate. Thus, consumption is negative, which generates deflation. The optimal fiscal policy only helps to raise inflation. However, at the optimum, inflation during the crisis does not need be raised all the way to zero. Government spending, i.e. the last term in the loss function (38), accounts for 65.8% of the welfare loss from imperfect stabilization, while deflation accounts for 25.4%.

When $\phi = 0.841$, inflation needs to be set close to zero to avoid the deflationary spiral. This requires a larger fiscal stimulus. In that case, government spending accounts for 98.0% of the welfare loss. Using equation (37), it is possible to solve for the fiscal policy that results in an inflation rate exactly equal to zero throughout the crisis. This policy generates an output gap equivalent social welfare loss of 7.24%, only 0.10% higher than under the optimal policy. This suggests that inflation targeting might be a good guide to fiscal policy in crisis time.

Finally, when $\phi = 1$, the last realization of inflation π_{t-1} does have a persistent impact on the expectations of future inflation and, hence, of future real interest rates

$i_{t+k} - \tilde{\mathbb{E}}_{t-1}[\pi_{t+k+1}] = 0 - \phi^{k+2}\pi_{t-1} = -\pi_{t-1}$. In that case, the optimal policy is to bring the inflation rate close to $-\underline{r} > 0$, such that the real rate becomes almost equal to the natural real rate. Thus, the optimal fiscal stimulus is even larger at the beginning of the crisis, reaching 14.2% of quarterly steady state GDP in the first quarter of the crisis. This generates enough inflation to "pump prime" the economy. The optimal stimulus then drops to 4.7% after a year (5th quarter) and to 1.8% after two years (9th quarter) and even less afterwards.[40] Thus, the overall size of the optimal stimulus program is smaller than when $\phi = 0.841$, which is why welfare is higher. Government expenditures are, however, even more heavily front-loaded. When $\phi = 1$, government spending account for 91.6% of the welfare loss from imperfect stabilization.

It has been assumed throughout this section that the government commits to a given path of expenditures. Thus, as can be seen from (37), future expenditures affect both the current and future rates of inflation. This effect favors back-loaded stimulus programs. An alternative would be to assume that boundedly rational agents always expect future expenditures to be at their steady state level. Thus, in (37), $\theta\kappa(1-\Gamma)\sum_{k=0}^{T-t}(\theta\beta)^k g_{t+k}$ should be replaced by $\theta\kappa(1-\Gamma)g_t$. In that case, the optimal fiscal policy is even more front loaded. Moreover, if ϕ is too small to "pump prime" the economy, i.e. to raise expectations of future inflation, then the size of the optimal stimulus package is even larger. This is not surprising since future expenditures no longer contribute to raising the current rate of inflation.

5 Conclusion

This analysis has shown that inflation persistence has major consequences for the optimal conduct of monetary and fiscal policy in a liquidity trap. If the Phillips curve is both forward and backward looking, then the monetary policy that is implemented during a liquidity trap episode can be effective and can be used to avoid a depression. The central bank does not need to be able to commit beyond the end of the crisis in order to get some traction on the economy. Regarding fiscal policy, the forward looking component of the Phillips curve makes it desirable to back-load government expenditures, while the backward looking component justifies a front-loading of expenditures. Importantly, inflation persistence considerably reduces the magnitude of the fiscal stimulus that is necessary to lift the economy out of depression. These results show that inflation persistence, which is usually perceived as a curse, is in fact a blessing in a liquidity trap.

Finally, while I have realized most of my analysis in a rational expectation framework,

[40] The main reasons why the stimulus does not drop faster is that inflation at time 1 is strongly influenced by government spending occurring shortly after, as can be seen from (37). But, from the loss function (38), the welfare cost is convex in the level of government expenditures. Thus, it is desirable to smooth the stimulus over several periods at the beginning of the crisis.

I have also considered the possibility that inflation persistence could result from adaptive expectations. In that case, the absence of forward looking behavior makes monetary policy totally ineffective. The government therefore needs to implement a fiscal stimulus that is large and heavily front-loaded.

An alternative scenario, which for conciseness has not be fully explored, is that a fraction of agents form rational expectations while the remaining fraction relies on adaptive expectations. In that case, the Phillips curve is both forward and backward looking, which should make the monetary policy implemented during the crisis potentially effective.

The steady state rate of inflation and the structural degree of inflation persistence have both been assumed to be exogenous and constant throughout this analysis. However, the extent to which inflation expectations are anchored at the official target rate of inflation is influenced by past realizations of inflation, which is a source of inflation persistence. Monetary and fiscal policy can therefore potentially be used to anchor, or to de-anchor, inflation expectations. For instance, the Japanese government has recently chosen to rely on expansionary monetary policy in order to reach a higher inflation target. Analyzing monetary and fiscal policy with endogenous inflation trend and persistence should be a promising avenue for future research.

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A Solving the Optimal Monetary Policy Problem

A.1 Full Commitment

The optimal monetary policy problem with full commitment consists in minimizing the loss function (6) subject to the new Keynesian Phillips curve (2) with π_0 given, to the Euler equation (4) and to the zero lower bound on the nominal interest rate (5). The

Lagrangian corresponding to the problem is:

$$
\begin{aligned}
\mathcal{L} \;=\; & \sum_{t=1}^{\infty} \beta^t \Big[(\pi_t - \omega\pi_{t-1})^2 + \frac{\kappa}{\epsilon} c_t^2 + \mu_t \left((1+\beta\omega)\pi_t - \omega\pi_{t-1} - \beta\pi_{t+1} - \kappa c_t \right) \\
& + \lambda_t \left(c_t + \frac{1}{\sigma Y/C} \left(i_t - \pi_{t+1} - r_t^n \right) - c_{t+1} \right) \Big],
\end{aligned} \tag{A1}
$$

where μ_t and λ_t are the Lagrange multipliers associated with the new Keynesian Phillips curve and the Euler equation, respectively. The first-order conditions with respect to i_t, π_t and c_t are respectively given by:

$$\lambda_t \geq 0 \text{ and } i_t \geq 0 \text{ with complementary slackness} \tag{A2}$$

$$\pi_t - \omega\pi_{t-1} + \mu_t - \mu_{t-1} = \beta\omega \left[\pi_t - \omega\pi_{t-1} + \mu_{t+1} - \mu_t \right] + \frac{\lambda_{t-1}}{\beta\sigma Y/C} \tag{A3}$$

$$\frac{\kappa}{\epsilon} c_t - \kappa\mu_t + \lambda_t - \frac{\lambda_{t-1}}{\beta} = 0 \tag{A4}$$

for all $t \geq 1$ and with $\mu_0 = \lambda_0 = 0$.

From $T+1$ onwards, the environment is stationary. I therefore focus on solutions such that the optimal allocation converges to a steady state where the zero lower bound is no longer binding. This implies, from (A2), that $\lambda_\infty = 0$. Hence, from (A3), $\pi_\infty(1-\omega) = 0$; from (4), $i_\infty = \pi_\infty + \bar{r}$; from (2), $c_\infty = 0$; and, from (A4), $\mu_\infty = 0$.

To solve the problem numerically, I consider a finite horizon of length N, with $N >> T$. The solution to the optimal policy problem is fully characterized by the new Keynesian Phillips curve (2) with π_0 given, the Euler equation (4), the three first-order conditions (A2), (A3) and (A4) together with two boundary conditions $\mu_{N+1} = 0$ and $\pi_{N+1} = \omega\pi_N$. These conditions guarantee that the economy is in a first-best allocation from $N+1$ onwards. This yields a system of $5N$ equations in $5N$ unknowns, which are $\{i_t, \pi_t, c_t, \mu_t, \lambda_t\}_{t=1}^N$.

One technical difficulty, due to the complementary slackness condition (A2), is that we do not know the pattern of binding zero lower bound constraints on the nominal interest rate. The solution is to guess a given pattern, to solve the problem, which yields the corresponding values $\{\lambda_t\}_{t=1}^N$, and to update the guess by considering that the zero lower bound on i_t is binding if and only if the computed value of λ_t is positive. We can then iterate until we obtain a guess that only yields non-negative values of λ_t. It is possible to check that, starting from a different initial guess, the algorithm converges to the same solution.

A.2 Partial Commitment

Let us now solve the optimal monetary policy problem when the central bank can only commit up to time T. When determining its monetary policy from time 1 to T, the central bank anticipates that it will choose to implement the first-best allocation from $T+1$ onwards. Thus, under partial commitment, the central bank takes $\pi_{T+1} = \omega\pi_T$ as an additional constraint. The Lagrangian is:

$$\begin{aligned}\mathcal{L} \;=\; \sum_{t=1}^{T} \beta^t \Big[& (\pi_t - \omega\pi_{t-1})^2 + \frac{\kappa}{\epsilon}c_t^2 + \mu_t\left((1+\beta\omega)\pi_t - \omega\pi_{t-1} - \beta\pi_{t+1} - \kappa c_t\right) \\ & + \lambda_t\left(c_t + \frac{1}{\sigma Y/C}\left(i_t - \pi_{t+1} - r_t^n\right) - c_{t+1}\right)\Big] + \beta^T\varsigma\left(\pi_{T+1} - \omega\pi_T\right),\end{aligned} \tag{A5}$$

where ς is the multiplier associated with the new constraint. The first-order conditions are still given by (A2), (A3) and (A4) for all $t \in \{1, 2, ..., T\}$, except for (A3) with $t = T$ which is replaced by:

$$\pi_T - \omega\pi_{T-1} + \mu_T - \mu_{T-1} = \frac{\omega}{\sigma Y/C}\lambda_T + \frac{\lambda_{t-1}}{\beta\sigma Y/C}. \tag{A6}$$

These first-order conditions together with the new Keynesian Phillips curve (2) and the Euler equation (4) for $t \in \{1, 2, ..., T\}$ and the boundary condition $\pi_{T+1} = \omega\pi_T$ fully characterize the solution to the optimal monetary policy problem with commitment up to time T. The pattern of binding zero lower bounds is found numerically following the procedure described in the previous subsection.[41]

B Solving the Optimal Monetary and Fiscal Policy Problem

The Lagrangian corresponding to the optimal monetary and fiscal policy problem under full commitment is:

$$\begin{aligned}\mathcal{L} \;=\; \sum_{t=1}^{\infty} \beta^t \Big[& (\pi_t - \omega\pi_{t-1})^2 + \frac{\kappa}{\epsilon}\left(c_t + (1-\Gamma)g_t\right)^2 + \frac{\kappa}{\epsilon}\gamma g_t^2 \\ & + \mu_t\left((1+\beta\omega)\pi_t - \omega\pi_{t-1} - \beta\pi_{t+1} - \kappa\left(c_t + (1-\Gamma)g_t\right)\right) \\ & + \lambda_t\left(c_t + \frac{1}{\sigma Y/C}\left(i_t - \pi_{t+1} - r_t^n\right) - c_{t+1}\right)\Big].\end{aligned} \tag{B1}$$

[41] Figure 5 provides an example where the pattern of binding zero lower bounds is non-monotone.

The first-order conditions with respect to i_t, π_t, c_t and g_t are respectively given by:

$$\lambda_t \geq 0 \text{ and } i_t \geq 0 \text{ with complementary slackness} \tag{B2}$$

$$\pi_t - \omega\pi_{t-1} + \mu_t - \mu_{t-1} = \beta\omega\left[\pi_t - \omega\pi_{t-1} + \mu_{t+1} - \mu_t\right] + \frac{\lambda_{t-1}}{\beta\sigma Y/C} \tag{B3}$$

$$\frac{\kappa}{\epsilon}\left[c_t + (1-\Gamma)g_t\right] - \kappa\mu_t + \lambda_t - \frac{\lambda_{t-1}}{\beta} = 0 \tag{B4}$$

$$(1-\Gamma)\frac{\kappa}{\epsilon}\left[c_t + (1-\Gamma)g_t\right] + \frac{\kappa}{\epsilon}\gamma g_t - (1-\Gamma)\kappa\mu_t = 0 \tag{B5}$$

for all $t \geq 1$ and with $\mu_0 = \lambda_0 = 0$.[42]

Under commitment up to time T, the first-order conditions are still given by (B2), (B3), (B4) and (B5) for all $t \in \{1, 2, ..., T\}$, except for (B3) with $t = T$ which is replaced by:

$$\pi_T - \omega\pi_{T-1} + \mu_T - \mu_{T-1} = \frac{\omega}{\sigma Y/C}\lambda_T + \frac{\lambda_{t-1}}{\beta\sigma Y/C}. \tag{B6}$$

[42] Note that, using (B4), the first-order condition with respect to g_t, (B5), can be rewritten as $(\kappa/\epsilon)\,\gamma g_t = (1-\Gamma)\left[\lambda_t - \lambda_{t-1}/\beta\right]$. This implies that fiscal policy should only be used when the zero lower bound is binding.