Domestic or global imbalances? Rising inequality and the fall in the US current account
Job Market Paper

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Abstract

This paper shows how the rise in individual income risk in the US since the 1980s might help explain the fall in its foreign asset position. The key to this result is endogenous financial deepening in an open economy with participation-constrained domestic financial markets. More volatile income makes individuals less inclined to default on financial contracts as this triggers exclusion from future financial trade. Lower incentives to default, in turn, increase the insurability of income shocks, thus lowering the need for precautionary savings. My theoretical results show that, contrary to the case of unconstrained complete markets, individual participation-constraints guarantee a well-defined stationary equilibrium at a given world interest rate. Based on an analytical solution to the stationary consumption distribution, I show that higher income risk can lower mean consumption and aggregate asset holdings. Consumption inequality, on the other hand, is almost entirely determined by the level of world interest rates, and remains largely unaffected by changes in income risk. A quantitative exercise shows that the observed rise in individual income risk in the US since the 1980s can explain a significant fall in net foreign assets.

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1 Introduction

Over the past 25 years, the US has experienced a significant rise in both cross-sectional income inequality and the uncertainty of individual incomes. Simple economic models suggest this should have increased individual savings at the same time as consumption inequality. But instead, during the same period, US savings fell, current account deficits accumulated to about 40 percent of 2004 GDP, while consumption inequality increased only little. This paper shows how, in an open economy, a rise in individual income risk can actually lower the aggregate foreign asset position, while leaving consumption inequality largely unchanged. The crucial assumption is that individuals have access to complete domestic insurance markets, but also the option to default on contracts, at the price of permanent exclusion from financial trade. This restricts transfers under the insurance scheme to amounts that individuals find optimal to pay, rather than choose the outside option of default. Higher income risk increases individuals' incentives to remain insured and thus to honour contracts, which is equivalent to a financial deepening in the economy. Under these "debt-constraints" to complete domestic risk-sharing, I analyse the effect of changes in income risk on consumption volatility and aggregate savings in a small open economy. I analytically show that an increase in income risk can lower the mean of the stationary consumption distribution, thus decreasing the amount of stationary assets, while leaving relative consumption inequality unaffected. Also, I develop a new algorithm based on the associated planner's problem as in Marcet and Marimon (1998), to show quantitatively that the observed rise in individual income risk in the US between 1980 and 2003 can explain a significant fall in net foreign assets.

Figure 1 shows the large and, until recently, increasing US current account deficit since 1980. Understanding the reasons for the corresponding rise in foreign indebtedness is important, mainly, because different explanations have different implications for its sustainability. For example, it has been argued that the fall in US net assets is a necessarily temporary phenomenon, linked to a strong rise in US house prices, that will eventually have to unwind (see e.g. Roubini et al 2004, Roubini 2005). Other authors, however, have attributed at least a part of this fall to changes in the structure of the world economy that imply a permanently lower US net asset position. Thus, Mendoza et al (2007) have focused on the impact of capital account liberalization in countries whose domestic financial markets are less developed relative to the US. In their model, once capital markets get liberalized, higher precautionary savings and lower appetite for risk in the rest of the world result in capital flows to the US concentrated in bonds, in line with the evidence. However, the underlying comparative advantage of deeper domestic financial markets in the US is exogenous to the model. In another contribution, Fogli and Perri (2006) show how the relatively more important reduction in US macro-volatility since 1980 implies a stronger reduction in the bufferstock savings of a representative US consumer than in other countries. But importantly, while international asset trade is limited to non-contingent bonds in their model, they assume domestic trade of a set of complete state-contingent assets that warrants the focus on representative national agents. This assumption, however, has been largely rejected by the data (see for example Zeldes (1989)). Moreover, as figure 1 shows, while US debt increased, cross-sectional domestic income inequality rose strongly, partly attributable to a rise in the uncertainty of individual incomes (see Krueger and Perri (2006), and more recently Heathcote et al (2008b)). And in the absence of perfect domestic risk-sharing, these changes in income risk will affect aggregate debt dynamics.

This paper analyses net asset positions in a simple open economy model that relaxes the assumption of a representative agent, and does not assume exogenous comparative financial advantage. Instead, it makes the depth of domestic financial markets depend endogenously on the riskyness of individual income. This allows me to look at the impact of changes in idiosyncratic income and consumption risk on aggregate savings and asset positions. But importantly, it also allows me to analyse the effect of international variables, such as interest rates, on individuals' decisions and, ultimately, the domestic consumption distribution.

If non-contingent debt was the main savings vehicle of the economy, as in Fogli et al (2006), an increase in individual income risk would yield a rise, not a fall, in equilibrium savings, together with higher consumption volatility. In an economy, on the other hand, where domestic markets are complete, but individuals can default on contracts at the price of permanent exclusion from financial trade, the relationship between income risk and consumption volatility is known to be less simple. Krueger and Perri (2006) show that under this assumption of participation-constrained complete markets, a rise in income risk has two offsetting effects: first, it raises the income realizations of individuals who receive positive shocks, and thus, for a given upper limit to redistribution, increases the volatility of consumption. But higher income risk also makes the outside option of financial autarky, where it translates one-to-one into higher consumption volatility, less appealing. This second effect acts to increase the insurability of income shocks, and thus deepens financial markets and reduces consumption volatility. Krueger and Perri (2006) show that the latter, financial deepening effect becomes more important for high levels of income risk, causing consumption volatility to first rise and then fall as income risk increases. And aggregate savings mainly act as a precaution against this consumption volatility.

This paper shows analytically that the open economy setting breaks the closed economy-link between consumption risk and precautionary savings. Particularly, relaxing individual debt constraints leaves relative consumption inequality largely unchanged. Rather, it can be interpreted as an increase in the country-wide borrowing capacity that leads to an increase in stationary debt holdings, or a fall in the net asset position. To derive these results, I first show that, unlike with unconstrained complete markets, the debt-constrained small open economy has a unique stationary equilibrium that does not depend on initial conditions. So individual participation-constraints "close small open economies" (Schmidt-Grohé et al 2003). The optimality conditions of an associated planner's problem, as in Marcet and Marimon (1998), allow me to solve analytically for the stationary consumption distribution even with standard, independent Markov processes for the incomes of a large number of individuals. The stationary equilibrium has the interesting feature that consumption follows a geometric distribution whose shape depends largely on the world interest rate, while its position is determined by participation constraints. Thus, looser participation constraints increase aggregate debt holdings and decrease aggregate consumption in stationary equilibrium. However, as mentioned above, the effect of higher income risk on participation-constraints depends on the initial level of income risk, and therefore the particular economy under analysis. A second part of the paper thus looks at the US example, and evaluates the effect of the observed rise in US income volatility on its net foreign asset position and the consumption distribution quantitatively. This analysis should ideally account for changes in income heterogeneity in both the US and its main economic partners during this period. Unfortunately, comprehensive cross-country data on the evolution of income risk are as yet unavailable, and in some cases unfeasible.[1] Comparative studies of simpler inequality measures have found that, apart from the United Kingdom, other OECD countries have experienced less important increases in income inequality since 1980 than the US (see e.g. Brandolini et al 2007). To focus on the open economy effect of the relatively large changes in income heterogeneity in the US, I make the simplifying assumption of an exogenously given world interest rate.[2] To capture the change in income risk, I use the stochastic process of individual incomes in the US estimated by Krueger and Perri (2006) for the years 1980 and 2003. I develop a new algorithm based on Marcet and Marimon (1998) to compute the corresponding stationary consumption distributions and net asset positions. The results show that the increase in income risk in the US can indeed explain a significant part of the fall in the net foreign asset position.

The rest of the paper is structured as follows: Section II describes the environment of a small open economy with debt-constrained domestic financial markets. Section III derives the analytical results on the basis of the associated planner's problem, and indicates how they generalize to the closed economy case. Section IV reports the computational algorithm and quantitative results. An appendix contains most proofs.

2 A small open economy with debt-constrained domestic financial markets

This section presents a simple model of a small open economy where domestic financial markets are constrained by individual default, and defines the competitive equilibrium.

2.1 Agents, countries, time

The economy consists of a small country and a large rest of the world. The analysis focuses on the small country that takes prices of goods and assets traded with the rest of the world as given.

[1] Thus, in the UK, for example, household panel data have been collected only since the beginning of the 1990s. However, Heathcote et al (2008b) is one paper in a recent project to compare measures of individual inquality and income risk across countries. See http://www.econ.umn.edu/ fperri/Cross.html.

[2] The assumption of an exogenous interest rate has also been made in contributions concentrating entirely on the domestic consequences of increases in individual income volatility in the US. See for example Heathcote et al (2008a).

The small country is populated by a large number of individuals of unit mass. Individuals are indexed by i, located on a unit-interval $i \in \mathbb{I} = [0,1]$. Time is discrete $t \in \{0, 1, 2, ..., \infty\}$ and a unique perishable endowment good is used for consumption.

2.2 The endowment process

The consumption endowment of agent i in period t, $z_{i,t}$, takes values in a finite set Z: $z_{i,t} \in Z = \{z_1 > z_2 > ... > z_N\}, N \geq 2$. Endowments follow a stochastic process described by a Markov transition matrix F. F has strictly positive entries, is identical across agents, monotone (in the sense that the conditional expectation of an increasing function of tomorrow's income is itself an increasing function of today's income), and has a unique ergodic distribution $\Phi_Z : \mathbb{Z} \to [0,1]$, where $\mathbb{Z}$ is the power set of Z. Thus, in the long-run, aggregate income $Y = \int_{\mathbb{I}} z_i$ is constant, while individual income fluctuates.
Let s_t denote the state of the economy in period t, a vector containing individual incomes and asset holdings of all agents. Also, let $s^t = \{s_0, s_1, ...s_t\}$ denote the history of the economy up to period t, and $s^{t\prime}$ the set of possible histories following s^t.

2.3 Preferences

Agents live forever and order consumption sequences according to the utility function

$$U = E_{s_0} \sum_{t=0}^{\infty} \beta^t u(c_{i,t}) \tag{1}$$

where E_{s_0} is the mathematical expectation conditional on s_0, $0 < \beta < 1$ discounts future utility, $c_{i,t}$ is consumption by agent i in period t, and $u : R^+ \to R$ is an increasing, strictly concave, continuously differentiable function that satisfies Inada conditions and is identical for all agents in the economy. Sometimes I assume these preferences to have constant relative risk aversion, or $u = \frac{c^{1-\sigma}}{1-\sigma}$.

2.4 Asset markets

I choose a specification of the economy similar to that by Alvarez and Jermann (2000), amended for the international setting. Agents engage in sequential trade of a complete set of state-contingent bonds domestically, but international asset trade is limited to non-contingent bonds.[3]

Individual endowment realisations are verifiable and contractable, but asset contracts are not completelely enforceable: at any point, individuals can default on their contractual payments at the price of eternal exclusion from financial markets. Thus the total amount an agent can borrow today against any income state z_j tomorrow is bounded by the option to default into financial autarky. There, consumption is forever equal to income. Given the markov structure of income, the value of default as a function of the vector of current income z can be written as

$$W(z) = \sum_{t=0}^{\infty}(\beta F)^t U(z) = (I - \beta F)^{-1}U(z) \tag{2}$$

I denote holdings of bonds and Arrow-Debreu securities paying off in state s_t by b and $a(s_t)$ respectively. In any state s_t, $V(z(s_t), a(s_t), b_t)$ is the contract value as a function of income $z(s_t)$ and current asset holdings $\{a(s_t), b_t\}$.

As in Alvarez and Jermann (2000) individual i's participation constraint for any state s_{t+1} tomorrow can be written as a constraint on the claims she can issue against s_{t+1} income. This borrowing constraint is "not too tight" in the words of Alvarez and Jermann (2000) if it assures participation but does not constrain contracts otherwise

$$a_i(s_{t+1}) + Rb_{i,t+1} \geq A_i(s_{t+1}) = min\{\alpha(s_{t+1}) : V(z_i(s_{t+1}), \alpha(s_{t+1}, 0)) \geq W(z_i(s_{t+1}))\} \tag{3}$$

Note that bonds are redundant in this setting, although including them facilitates somewhat the setup of the planner's problem in open economy where aggregate bond holdings, denoted B, are potentially non-zero.

2.5 A special case: A generalisation of Kehoe and Levine's (2001) income process

I call "special case" an economy where the income process described in the previous section takes only two values $\{z_h, z_l\} = \{y_0 + \epsilon, y_0 - \epsilon\}$, $\epsilon \geq 0$, so we can write $F = [p, 1-p; 1-q, q]$. Monotonicity and absolute continuity require $0 < 1 - q < p < 1$. Also, I assume income has persistence which is not too

[3] This is non-restrictive as there is no aggregate risk and the law of large numbers holds. It requires, however, no default on foreign debt on a country level. In a previous version of this paper I show that Broner and Ventura's (2006) result applies to my setting. So perfect secondary markets prevent governments from defaulting on agents' foreign liabilities.

different in high and low income states:

$$p, q > 1/2 \tag{4}$$

$$\frac{\beta - 1}{\beta} < p - q < \frac{1 - \beta}{\beta} \tag{5}$$

I define a "**marginal rise in income risk**" as $d\epsilon > 0$.

This special case is a generalisation of an example considered, in an economy with capital, by Kehoe and Levine (2001), or more recently by Krueger and Perri (2006). With the same participation constraints, they analyse consumption risk sharing among 2 groups whose income can be high or low: $z \in \{1 - \epsilon, 1 + \epsilon\}$. In their example, the transitions for groups 1 and 2 are strictly dependent in the sense that with probability $p = 1/2$ they simply "swap" income states. This yields an equilibrium consumption distribution characterised uniquely by a difference between consumption of the high and low income group that is constant through time: unless perfect insurance is feasible, the high income group consumes some $\tilde{c}$ more than average income to meet its participation constraint. Since there are only two income histories, all low-income agents consume an equal amount, which feasibility pins downs as $\tilde{c}$ less than average income. This particular setting eliminates history dependence completely. In my special case with independent transitions, on the other hand, agents have individual income histories that translate into a non-trivial consumption distribution. Krueger and Perri (2005) consider a similar income process in a debt-constrained economy but assume independent income shocks. I allow for persistence.

2.6 Income risk and the value of default

In the above environment, individuals can default today on yesterday's contracts at the price of permanent exclusion from financial markets. The attractiveness of default is determined by the value of the outside option, individual financial autarky, which is equal to the expected utility of individual income streams. The assumption of monotonicity of both utility and transitions ensures that these autarky values are increasing in the level of current income. However, the relationship between autarky values and income risk is more difficult to characterise. Particularly, a change in risk can come via changes in transition probabilities F, via a change in the support of endowments Z, or both. In this paper, I follow Kehoe and Levine (2001) and define a rise in risk as a mean-preserving spread to the income support Z. This, however, does not imply mean-preserving spreads to the conditional income distribution for all individuals. Rather, given persistence, it raises (lowers) current and expected future income for today's high (low) income earners. So for low levels of uncertainty, higher risk increases both expected income and autarky values for the income-rich. However, although their expected income continues to rise, as a consequence of concave utility the prospect of negative shocks weighs more heavily on expected utility as higher risk decreases income, and thus consumption, in low income states. Given Inada conditions, this effect necessarily outweighs the gain in expected income at some point. Thus, autarky values of high income individuals roughly follow an inverse U-shape relation with income risk.

This concavity of autarky values in risk has been shown by Krueger and Perri (2006) for Kehoe and Levine's (2001) particular example. It can also be shown easily in the special case described above, where autarky values are

$$W_h = \frac{(1-\beta q)u(y_0+\epsilon)+\beta(1-p)u(y_0-\epsilon)}{1-\beta(q+p)-\beta^2(1-(q+p))} \tag{6}$$

$$W_l = \frac{\beta(1-q)u(y_0+\epsilon)+(1-\beta p)u(y_0-\epsilon)}{1-\beta(q+p)-\beta^2(1-(q+p))} \tag{7}$$

Given my assumptions on transition probabilities, low-income-autarky value W_l decreases in ϵ for all ϵ. However, it is easy to see that the high income-autarky value W_h is concave in ϵ with a maximum at some $\epsilon* > 0$, increases for $\epsilon < \epsilon*$, decreases for $\epsilon > \epsilon*$ and crosses the perfect insurance value at $\overline{\epsilon} > \epsilon*$.[4] Note that this result does not depend on CRRA preferences.

[4]To see this, take the first derivative of autarky values with respect to ϵ

$$\frac{dW}{d\epsilon} = (I-\beta F)^{-1}[\frac{dU(y_0+\epsilon)}{d\epsilon}, -\frac{dU(y_0-\epsilon)}{d\epsilon}] \tag{8}$$

The persistence assumptions assures that for $\epsilon = 0$ the rise in current utility dominates the fall in future expected utility. With strictly positive entries of F, however, Inada conditions on u translate to W_h, so marginal utility goes to infinity as the low income realisation goes to zero: as $\epsilon \longrightarrow y_0$, $\frac{dW_l}{d\epsilon} \longrightarrow -\infty$. By the intermediate value theorem and continuity, there exists an $\epsilon*$ with $\frac{dW_h(\epsilon*)}{d\epsilon} = 0$, and $\overline{\epsilon} > \epsilon*$ with $W_h(\overline{\epsilon}) = 0$. Also, the concavity of the utility function translates to the concavity of autarky values as a function of ϵ

$$\frac{dW^2}{d\epsilon^2} = (I-\beta F)^{-1}[\frac{dU^2(y_0+\epsilon)}{d\epsilon^2}, \frac{dU^2(y_0-\epsilon)}{d\epsilon^2}] < 0 \tag{9}$$

2.7 The household's problem

Every period, households maximise their expected utility by choosing current consumption and assets subject to budget and borrowing constraints

$$V(z(s_t), a(s_t), b_t) = max_{c_t,\{a(s_{t+1})\},b_{t+1}} \sum_{s=0}^{\infty} \beta^s u(c_{t+s})$$

$$s.t.\ c_t + \sum_{s_{t+1}} a(s_{t+1})q(s_{t+1}) + b_{t+1} \leq Rb_t + a(s_t) + z(s_t) \tag{10}$$

$$a(s_{t+1}) + Rb_{t+1} \geq A(s_{t+1}) \tag{11}$$

As shown in Alvarez and Jermann (2000) this problem has a recursive representation as

$$V(z(s), a(s), b) = max_{c,\{a(s')\},b'} \{u(c) + \beta E_s V(z', a(s'), b')\}$$

$$s.t.\ c + \sum_{s'} a(s')q(s') + b' \leq Rb + a(s) + z(s)$$

$$a(s') + Rb' \geq A(s')$$

$$A(s') = min\{\alpha(s') : V(z(s'), \alpha(s'), 0) \geq W(z(s'))\}$$

where c, b', a' are policy functions of the state variables $(z(s), a(s), b)$.

2.8 Definition of competitive equilibrium

The competitive equilibrium in this economy is a set of asset prices $q(s')$, R, a set of individual decision rules $c, b', a'(s')$ with associated value functions $V(z, a, b)$ such that

1. $V(z, a, b)$ is the households maximum value function associated to the household problem given $q(s'), R$

2. $V(z, a, b)$ is attained by $c, b', a'(s')$

3. Markets for state-contingent assets clear
   $\int_{\mathbb{I}} a_i(s_t) = 0,\ \forall s_t, t$

4. The interest rate on bonds is equal to the world interest rate R.

The competitive equilibrium is called "stationary" if prices and aggregate bond holdings are constant, and the distribution of individual consumption is stationary through time.

3 Analytical properties of the consumption distribution and aggregate savings in stationary equilibrium

In this section I show analytically how, unlike with unconstrained complete markets, individual participation constraints ensure the existence of a stationary equilibrium in a small open economy even when interest rates differ from the rate of time preference. I show how across stationary equilibria, a rise in income risk can leave consumption inequality unchanged, but decreases aggregate asset holdings if the initial level of income risk is high enough. Also, I show how market completeness does not help the most unfortunate individuals in this economy: both their current consumption and expected value from future consumption are the same as without any financial markets. Insurance, however, reduces the number of individuals in this situation significantly. To derive these results I exploit the constrained efficient nature of the economy that allows me to solve the associated planner's problem as in Marcet and Marimon (1998). I use this method, first, to derive the closed form of the consumption distribution in the special case with two income values but a continuum of agents whose incomes follow identical independent Markov processes. Contrary to previous papers by Kehoe and Levine (2001) or Krueger and Perri (2006), this allows for potentially infinite history-dependence of individual consumption. I then generalise these results to an income process with $N > 2$ and, finally, show how they translate to the closed economy equilibrium with endogenous interest rates.

3.1 The planner's problem and first order conditions

Alvarez and Jermann (2000) show that a version of the first welfare theorem applies to the closed economy version of this environment. The small open economy assumption changes aggregate feasibility constraints but, together with an appropriate No-Ponzi condition, leaves this result intact. This allows me to focus on participation-constrained efficient allocations. More particularly, I exploit the results in Marcet and Marimon (1998), and focus on the solution to the participation-constrained social planner's problem.

Marcet and Marimon (1998) show how the efficient competitive equilibrium allocation solves the following planners problem. For a given bounded measurable weighting function $\mu_{i,0} : \mathbb{I} \rightarrow R^+$ in a linear social welfare function $\Omega = \int_{\mathbb{I}} \mu_{i,0} E_0 \sum_0^\infty \beta^t u(c_{i,t})$ the problem of the planner is to distribute ressources optimally subject to individuals' participation constraints and the aggregate ressources of the economy

$$\mathbb{VV}(\Phi_{\mu_{i,0}}, B_0) = max_{\{c_{i,t}\}} \int_{\mathbb{I}} \mu_{i,0} \sum_{t=0}^{\infty} \beta^t u(c_{i,t}) \tag{12}$$

$$s.t. \int_{\mathbb{I}} c_{i,t} + B_{t+1} = \int_{\mathbb{I}} z_{i,t} + R_t B_t, \ \forall t$$

$$V_{i,t} \geq W(z_(i,t)), \ \forall t, i$$

$$B_t \geq -\frac{Y}{R-1}, \ \forall t$$

where the planner's maximum value $\mathbb{VV}$ is a function of $\Phi_{\mu_{i,0}}$, the initial distribution of multipliers induced by $\mu_{i,0}$, and aggregate bond holdings B_0. $V_{i,t}$ denotes the expected value of the consumption sequence the planner gives to agent i starting in period t, and the last line is a No-Ponzi condition on aggregate bonds B, which I assume to be 0 in period 0. Also, I assume that $\mu_{i,0}$ only takes a finite number of values.

Note that the problem in (12) is not recursive in the cross-sectional distribution of income. Intuitively, the planner optimally provides an increase in value $V_{i,t}$ to participation-constrained individual i by an increase in both current and future consumption. But this requires the planner to keep her consumption promise even if individual i receives a negative income shock tomorrow. The solution thus has potentially infinite history dependence. But Marcet and Marimon (1998) show how, based on the Lagrangian associated to the sequential planner's problem, this history-dependence can be encoded in a time varying value of individual welfare weights $\mu_{i,t}$. Particularly, the assumptions on $\Phi_{\mu_{i,0}}$, utility and transition probabilities ensure that the problem is sufficiently well-behaved to have a saddle-point representation that is recursive in a time-varying distribution of weights $\Phi_{\mu_{i,t}}$ and aggregate bond holdings[5]

$$\mathbb{VV}(\Phi_{\mu_i}, B) = \inf_{\gamma_i \geq 0} \max_{\{c_i\}} \int_{\mathbb{I}} [(\mu_i + \gamma_i)u(c_i) - \gamma_i W_i] + \beta E[\mathbb{VV}((\Phi_{\mu_i'}, B')] \quad (13)$$

$$s.t. \int_{\mathbb{I}} c_i + B' = \int_{\mathbb{I}} z_i + RB$$

$$\mu_i' = \mu_i + \gamma_i \quad (14)$$

$$B_t \geq -\frac{Y}{R-1} \ \forall t \ \Phi_{\mu_i}$$

where γ_i corresponds to the multiplier on i's participation constraint in the sequential problem (12). Note that the weights of individuals in the social welfare function are now updated every period to meet participation constraints.[6] And when γ_i is zero, so i is unconstrained, (14) ensures promise-keeping by the planner. Intuitively, by increasing multipliers the planner allocates a higher than expected consumption path to constrained individuals with positive income shocks, to keep them "happy" with the contract. The absolute weights of the remaining, unconstrained individuals are constant, but decline relative to those for individuals with positive income shocks. This leads to a gradual decline in consumption for these individuals until they either receive a positive income shock, or reach the level of constant consumption that, given prospects for future shocks, just meets the participation constraint corresponding to their income level. The solution of the planner's problem is a sharing rule $\Gamma : Z \times R^{+} \rightarrow R^{+2}$ that maps current weights μ_i and income shocks z_i into consumption c_i and new weights $\mu_i' = \mu_i + \gamma_i$.

[5]To see this, note that that the initial weighting function $\mu_{i,0}$ only takes a finite number of values, and that for every $t < \infty$ the set of possible income histories Z^t is finite and bounded. So the exogenous state space is the Euclidian Product of a countable number of compact sets, and thus, according to Tychonoff's theorem, compact. Also, given the No-Ponzi condition, aggregate bond holdings are bounded and thus lie in a convex compact set, implying that feasible consumption allocations are just a simplex, and thus a convex set, every period. With concave utility, the constraint set is therefore compact and convex, and non-empty since autarky is feasible and incentive-compatible. The Problem thus fulfills conditions A1 to A5 in Marcet and Marimon (1998), and therefore has a recursive saddle-point representation. For further detail, see the proof of uniqueness and existence in the Appendix.

[6]Again, despite the continuum of agents, the values of multipliers remain countable, since $\mu_i' = \mu_i + \gamma_i$ is a function of current income and the past value of μ_i only. So, given my assumption of a countable support of $\Phi_{\mu_{i,0}}$, the number of individual multipliers remains countable.

The first order conditions[7] for individual consumption imply

$$\frac{U'(c_{i,t})}{U'(c_{j,t})} = \frac{\mu_{j,t} + \gamma_{j,t}}{\mu_{i,t} + \gamma_{i,t}} \tag{15}$$

Thus, since $U'(c)$ is decreasing, individuals with a higher weight receive higher consumption. Also, from the first order condition for aggregate bond holdings, the interest rate is tied to the ratio of the multipliers λ, associated to the aggregate feasibility constraint in (13)

$$R = \frac{\lambda}{\beta E[\lambda']} = \frac{\beta\lambda}{\lambda'} = \frac{U'(c_i)(\mu_i)}{\beta U'(c_i')(\mu_i + \gamma_i)} \tag{16}$$

where the second equality exploits the absence of aggregate uncertainty and the law of large numbers,[8] and the third uses the intratemporal optimality conditions for consumption. Importantly, the interest rate determines the slope of marginal utility for those consumers who are unconstrained tomorrow ($\gamma_j = 0$)

$$U'(c_i) = \beta R U'(c_i') \tag{17}$$

Given monotonicity of U', this provides a law of motion for the consumption of unconstrained agents. With CRRA preferences, we can solve for c_i' as

$$c_i' = (\beta R)^{\frac{1}{\sigma}} c_i \tag{18}$$

So the lower R, the faster falls consumption of unconstrained agents. With CRRA preferences we can simplify equation (16) further by solving for c_i in terms of the multipliers, and integrating across agents, to get

$$R = \frac{1}{\beta}[\frac{C'}{C}\frac{\int_{\mathbb{I}}(\mu_i^{1/\sigma})}{\int_{\mathbb{I}}(\mu_i + \gamma')^{1/\sigma}}]^{\sigma} \tag{19}$$

[7] Note that continuously differentiable utility and a convex constraint set imply that the value function is differentiable. Also, Inada conditions and concavity of the utility function imply that the first order conditions, together with participation constraints, are sufficient to characterise the optimum.

[8] Since the state space is finite every period, the assumption of independent shocks over a continuum of agents ensures that the law of large numbers applies. Formally, $\int_{\mathbb{I}} x(i,t) = \sum_{Z\times\{\mu_{i,t}\}} \int_{\mathbb{I}} I_{\mu,z} = \sum_{Z\times\{\mu_{i,t}\}} \mathbb{III}_{\mu_t,z}$ where $I_{\mu,z}$ is the indicator function of the set $\{i : \mu_i = \mu, z_i = z\}$ and $\mathbb{III}_{\mu_t,z} \in [0,1]$ is the mass of individuals with weight μ and income z in period t. So we can replace integrals with summation over countable sets. Given the continuum of agents $i \in \mathbb{I}$, this ensures that the law of large numbers applies. So the joint distribution of income and weights μ tomorrow is known today. On the law of large numbers in economies with a continuum of agents and independent idiosyncratic risk, see Uhlig (1996).

Thus, a fall in the world interest rate either lowers aggregate consumption growth, or increases average growth in individual multipliers, or both. The first effect is standard and leads to non-existence of a stationary equilibrium in small open economies with unconstrained complete markets. The second effect comes from the participation-constrained nature of risk-sharing. It implies, for example, that unless there is perfect insurance ($\gamma_i = 0, \forall i$), the equilibrium closed economy interest rate is below the time preference rate, a result well-known from Alvarez and Jermann (2000). More generally, binding participation constraints increase the shadow value of future resources relative to today's. This is because current consumption only relaxes today's participation constraints. Future consumption relaxes all previous participation constraints, including today's, via the increase in continuation utility under the contract. So when more agents hit their participation-constraints every period, or when a given set of binding constraints becomes more binding, the planner reallocates aggregate consumption to the future. Below I show that this second effect ensures the existence of a stationary equilibrium in this economy.

Note that if $\frac{U'(z_1)}{\beta U'(z_N)} > 1$, (16) immediately yields a minimum interest rate $R_{min} > 1$ below which all individuals simply consume their endowments. This is because, whenever $1 < R < R_{min} = \frac{U'(z_1)}{\beta U'(z_N)}$, there are no participation-compatible unconstrained transitions in (17). So individual consumption is simply equal to individual income.

3.2 Existence and uniqueness of equilibrium

The closed economy version of this economy is one of the examples discussed in Marcet and Marimon (1998). An appendix proves that the planner's problem has a unique solution also in this small open economy setting. However, in both cases, we do not know if this solution is stationary in terms of the long-run behaviour of aggregate consumption and its distribution across individuals. The next section shows, by construction of the stationary consumption distribution that solves the planner's problem, that this is indeed so.

3.3 The stationary distribution of consumption

In a small open economy with complete domestic markets that are not participation-constrained, $R < 1/\beta$ implies that consumption levels are forever declining. So no stationary solution exists. With participation constraints, however, this is not an equilibrium, as the total value that the planner can distribute to individuals declines with the level of aggregate resources. A permanently downward sloping path of aggregate consumption thus necessarily violates individual participation constraints at some point in the future. Instead, in an equilibrium with participation constraints, the aggregate consumption decline slows down as participation constraints become more binding. This is because for given weights $\mu_i + \gamma_i$, individual contract values decline with aggregate resources. This requires stronger increases in relative weights of participation-constrained indivdiuals γ_i. But more binding participation constraints increase the marginal value of future resources according to equation (16). This slows the decline in aggregate consumption until it settles down at a stationary level, with a corresponding stationary distribution of individual consumption and aggregate debt holdings.

In this stationary equilibrium, consumption in all states is pinned down by participation constraints and the law of motion of unconstrained agents (17) given the exogenous interest rate R. I use this to show, by construction, the existence of the stationary consumption distribution and its characteristics. Note that this result holds for any interest rate R that satisfies $R_{min} < R < \frac{1}{\beta}$. Thus, the stationary equilibrium does not depend on initial conditions even when the world interest rate is equal to the equilibrium rate of the closed small economy. And there is no eternal growth, or decline, in aggregate consumption when they differ. So individual participation constraints provide an additional way of "closing small open economies" (Schmidt-Grohé et al 2003).

The following proposition summarises the properties of the consumption distribution in stationary equilibrium. Krueger and Perri (2005) perform a similar analysis based on the planner's recursive expenditure minimisation problem as in Atkeson and Lucas (1995). Using an equivalent to the law of motion (17), they characterise the consumption distribution in a debt-constrained economy with many agents under the assumption of i.i.d. income shocks that take two values. I generalise their results in three ways. First, in my special case, I allow for persistence of income shocks and show that consumption has a geometric distribution over declining consumption values. Second, I solve for the distribution in closed form under the assumption of CRRA preferences. And third, I generalise these results to the case of $N > 2$ income values with persistent transitions and general preferences.

Proposition 1: The stationary distribution of consumption
There exists a unique stationary equilibrium with a distribution of consumption $\Phi_C : \mathbb{C} \subseteq R^+ \longrightarrow [0,1]$.
If $1 < R < R_{min}$*, the stationary distribution of consumption is equal to that of income, so* $\Phi_C = \Phi_Z, \mathbb{C} = Z$*. If* $R_{min} < R < 1/\beta$*,* Φ_C *has* $N-1$ *overlapping subdistributions on finite supports* $\mathbb{C}_i = \{c_{1,i} > c_{2,i} > ... > c_{m_i,i}\},\ i = 1, ..., N-1$*. For each, probability mass declines geometrically from an upper bound* $c_{1,i}$ *defined by the participation-constraint for income* z_i*, across intermediate support points given by the law of motion* (17)*, to a common lower bound equal to minimum income* ($c_{m_i,i} = z_n, \forall i$)*.*

For the case $R_{min} < R < \frac{1}{\beta}$, the proof is by construction of the stationary distribution, and can be found in the appendix. For $1 < R < R_{min}$, indiviual autarky is the only allocation that fulfills optimality conditions, as shown above and well-known from e.g. Krueger and Perri (2005).

Corollary 1: The consumption distribution in the special case with $N = 2$ **and CRRA preferences**
In the special case with $N = 2$ *and CRRA preferences,* Φ_C *is*

$$\Phi(c_1) = \frac{1-q}{2-q-p} = \nu \tag{20}$$

$$\Phi(c_i|_{1<i<m}) = \nu(1-p)q^{i-1} \tag{21}$$

$$\Phi(c_m) = (1-\nu)q^{m-1} \tag{22}$$

for c_1 *the unique number solving*

$$c_1^{1-\sigma} + (1-p)[\sum_{i=1}^{\infty} \beta^i q^{i-1}\ max\{c_1^{1-\sigma}(\beta R)^{\frac{i(1-\sigma)}{\sigma}},\ (y_0-\epsilon)^{1-\sigma}\}] = \tag{23}$$

$$(1-\sigma)W_h \frac{1-\beta(p+q)-\beta^2(1-p-q)}{1-\beta q} \tag{24}$$

and

$$c_i = c_1(\beta R)^{\frac{i}{\sigma}}, 1 < i < m$$

$$c_m = y_0 - \epsilon \tag{25}$$

$$m = min\{x \in N : x > \frac{\sigma[ln(y_0-\epsilon) - ln(c_1)]}{ln(\beta R)}\} \tag{26}$$

Aggregate consumption is[9]

$$C = \nu c_1[1 + (1-p)\sum_{i=1}^{m-1}(\beta R)^{\frac{i}{\sigma}} q^{i-1}] + (1-\nu)q^{m-1}(y_0-\epsilon) \tag{29}$$

[9] If $\Phi(c_m) \approx 0$, such that truncation of the geometric distribution is negligible (which is

Proof

To obtain the support $\mathbb{C}$, define c_m as the minimum participation-compatible consumption for an individual in the low income state $z_l = y - \epsilon$. As she cannot move further down in consumption, she is necessarily participation-constrained in both income states tomorrow, receiving values W_h and W_l respectively. Thus c_m is determined from her participation constraint as

$$W_l = U(c_m) + \beta[(1-q)W_h + qW_l] \tag{30}$$

which is solved by $c_m = z_l = y_0 - \epsilon$ from the definition of W_l. So minimum consumption is equal to minimum income.

An individual in the high income state is always constrained, receiving minimum participation-compatible consumption c_1, whose value we need to determine. Tomorrow she either remains at high income, receiving value W_h. Or she gets a negative income shock and moves down in consumption according to (18). Thus, the expected value of her consumption stream under the contract can be expressed as an infinite sum of lotteries with two outcomes: either, she receives value W_h. Or, in case of a low income realisation, she receives $(\beta R)^{\frac{i-1}{\sigma}} c_1,\ i = 1$, plus participation in the next lottery for $i = 2$, and so forth. If she has not received a positive shock after m-1 periods, her consumption cannot fall further without violating her participation-constraint at low income. So the lottery remains static, yielding either value W_h, or minimum consumption $c_m = y_0 - \epsilon$ plus another draw. So c_1 is uniquely determined by her participation constraint

$$\begin{aligned} W_h = {} & \frac{c_1^{1-\sigma}}{1-\sigma} + pW_h + \\ & (1-p)\{\sum_{i=1}^{\infty} \beta^i q^{i-2}[q\ max\{(\beta R)^{\frac{i(1-\sigma)}{\sigma}} \frac{c_1^{1-\sigma}}{1-\sigma}, \frac{(y_0-\epsilon)^{1-\sigma}}{1-\sigma}\} + (1-q)W_h] \end{aligned}$$

To derive the mass function Φ, note that the stationary mass at c_1 is that at income state y_h, equal to the first entry of the normalised left eigenvector of transition matrix F associated with a unit eigenvalue $\nu = \frac{1-q}{2-q-p}$. $\Phi(c_2)$ is simply ν times transition probability to low income $(1-p)$. And $\Phi(c_i) =$

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true necessarily as $R \longrightarrow 1/\beta$), we have

$$c_1 = \{\frac{(1-\beta(p+q) - \beta^2(1-p-q))(1-\beta q(\beta R)^{\frac{1-\sigma}{\sigma}})}{(1+\beta(1-p-q)(\beta R)^{\frac{1-\sigma}{\sigma}})(1-\beta q)}(1-\sigma)W_h\}^{\frac{1}{1-\sigma}} \tag{27}$$

and aggregate consumption equals

$$C = \nu c_1[1 + \frac{(1-p)(\beta R)^{\frac{1}{\sigma}}}{1-(\beta R)^{\frac{1}{\sigma}} q}] \tag{28}$$

$\nu(1-p)q^{i-1},\ i=2,...,m-1$ declines geometrically with survival probability q, the probability of remaining at low income. Finally, the lower bound c_m has probability $\Phi(c_m) = \Phi(c_{m-1})\frac{q}{1-q} = \nu\frac{(1-p)q^{m-1}}{1-q} = (1-\nu)q^{m-1}$. Aggregate consumption is simply the sum over this distribution.

QED

Relative to previous analytical characterisations of the debt-constrained consumption distribution, note that here, there is non-trivial, in fact infinite, history dependence of individual consumption, as individuals with constant low income move down the geometric distribution. However, in the special case, $N=2$, history dependance of individual incomes reduces to the number of periods since an individual last had high income. In the general case, history dependence can be collapsed to two dimensions, the last income state where an individual was constrained, and the number of periods passed since then.

3.4 Income risk, consumption inequality and aggregate debt in stationary equilibrium

Proposition 1 allows me to derive the impact of changes in income risk on aggregate net assets and consumption inequality in a small open economy with debt-constrained domestic financial markets. This section considers first the special case with CRRA preferences, and then discusses how it generalises to the case of $N > 2$.

3.4.1 The special case $N = 2$ with CRRA preferences

We know that changes in income risk, which here widen the support of income but leave transitions unaffected, affect the consumption distribution only through their effect on autarky values W_h, W_l and thus participation constraints. Therefore, changes in income risk have a direct effect only on the minimum participation-compatible consumption levels c_1 and c_m. From Proposition 1 we know that $c_m = y_0 - \epsilon$ is strictly decreasing in ϵ. Also, we see that in the special case with CRRA preferences, the remainder of the consumption support is homogenous of degree 1 in the upper bound c_1. And c_1 is strictly increasing in autarky value at high income W_h. Thus, changes in ϵ simply shift the consumption values $c_1, ...c_{m-1}$ up and down in parallel. So we know that whenever a rise in income risk decreases W_h, it lowers stationary aggregate consumption, as it lowers the whole support of consumption. This leads to the following corollaries.

Corollary 2: Income uncertainty and foreign asset holdings in the special case with CRRA preferences
For high levels of risk ($\epsilon > \epsilon*$), a marginal rise in income risk $d\epsilon > 0$ decreases stationary asset holdings.

Proof
Note first that for $\epsilon > \epsilon*$ both W_h and W_l decline in income risk, and second that Φ is unaffected by small changes in the support $\mathbb{C}$ that leave the number of support points m unchanged. From the monotonicity of c_1 in W_h, the homogeneity of $c_i,\ i = 1, ..., m-1$ in c_1, and $c_m = y - \epsilon$ it then follows that aggregate consumption declines in ϵ for $\epsilon > \epsilon*$. For a given world interest rate stationary aggregate assets are monotonously increasing in aggregate consumption, so the result follows.

QED

Corollary 3: Variance of log-consumption in the special case with CRRA preferences
If $\Phi(c_m) \approx 0$, the variance of log-consumption is

$$Var_c = \Lambda[\frac{\log(\beta R)}{\sigma}]^2 \tag{31}$$

where $\Lambda > 0$ is a function of transition probabilities only. So higher world interest rates lower domestic consumption inequality. If there is a non-negligible mass at the truncation point, $\Phi(c_m) > 0$, this is an upper bound for the cross-sectional variance of individual consumption.

Proof
The simple algebra that leads to the result can be found in the appendix.

Thus, in a small open economy, consumption inequality can get completely decoupled from income risk in stationary equilibrium. Risk only determines the amount of aggregate consumption and assets in stationary equilibrium. And interestingly, consumption inequality is decreasing in the world interest rate.

3.4.2 General uncertainty and preferences

Corollary 2 naturally generalises to the case $N > 2$ with well-behaved, non-CRRA preferences. To see this, note that in this case, the consumption distribution can be characterised by N minimum participation-compatible consumption levels, associated to N autarky values, that provide the upper bounds for geometric sub-distributions. Within these subdistributions, the support is entirely determined by the law of motion (17), and monotonously increasing in the upper bounds. So when a rise in income risk reduces all autarky values, the whole support of consumption declines, reducing aggregate consumption and asset holdings in stationary equilibrium.

The shape of the n sub-distributions is again independent of the upper bound, with variance that decreases in R. However, changes in income risk now change relative autarky values and thus do not move the subdistributions in parallel. So the shape of the overall distribution is not independent of risk. But it is easy to show that the width of the support $\mathbb{C}$ decreases with R.

3.5 The closed economy equilibrium

This section briefly considers the equilibrium of a closed economy in analogy to the open economy case considered above. I show that the results on the shape of the consumption distribution continue to hold, while the comparative static effect of changes in income risk does not.

Without international asset trade, aggregate consumption equals aggregate income every period $C = C' = Y$. The planner's problem in this case is a simplified version of that discussed above. Particularly, it also has a unique solution. The characterisation of equilibrium is unchanged, noting that now the interest rate is endogenously determined, for the CRRA case, by

$$R_{CE} = \frac{\int_{\mathbb{I}} (\mu_i^{1/\sigma})^{\sigma}}{\beta \int_{\mathbb{I}} [(\mu_i + \gamma')^{1/\sigma}]^{\sigma}} \tag{32}$$

The closed economy interest rate R_{CE} is thus inversely related to the growth in multipliers. But despite the absence of aggregate shocks and the ergodic nature of the income process, it is not trivial to show that this equilibrium interest rate is constant. However, if we can show that there is a small open economy with zero long-run asset holdings at $\hat{R}$, we immediately know that its equilibrium allocation is that of a closed economy with constant equilibrium interest rate $\hat{R}$. Moreover, this allocation is unique, since the solution to the closed economy's planner's problem is unique. Thus, proposition 1 continues to hold conditional on the equilibrium interest rate. Particularly, inequality and R are negatively related in equilibrium, and the shape of the consumption distribution is geometric. However, a rise in income risk that reduces autarky values now raises interest rates, and reduces inequality in stationary equilibrium.[10]

[10] To see this, consider equation (32). A fall in autarky values reduces the average increase in multipliers, the numerator on the righ-hand-side, and thus raises equilibrium interest rates.

4 Individual risk and global imbalances: income uncertainty and the US net foreign asset position 1980-2003

The previous section showed that in an open, debt-constrained economy, rises in income risk can lower aggregate savings and asset positions. But importantly, this only holds for an initial level of income risk that is sufficiently high. The sign and importance of the effect of changes in income risk on asset positions thus depends on the particular economy under analysis. Also, the independence of stationary consumption inequality from income risk only holds for the special case with two income values. Thus, this section analyses a version of the model that is calibrated to match some stylised features of the US economy in the years 1980 and 2003. Particularly, I use the stochastic process for US individual incomes estimated by Krueger and Perri (2006), and compare debt holdings and consumption inequality in stationary equilibria corresponding to the two endpoints of their sample, respectively 1980 and 2003. Before turning to the results I briefly describe the calibration, and the algorithm I use to compute the stationary equilibria.

4.1 Calibration

I calibrate the income process following Krueger and Perri (2006), using their estimates for the years 1980 and 2003, the endpoints of their sample. The authors assume the log of post tax labour income plus transfers (LEA+) $log(z_t)$ to be the sum of a group specific component α_t and an idiosyncratic part y_t. The latter, in turn, is the sum of a persistent AR(1) process m_t, with persistence parameter ρ and variance σ_m^2, plus a completely transitory component ε_t which has mean zero and variance σ_ε^2.
The process for LEA+ is thus of the form

$$\begin{aligned} log(z_t) &= \alpha_t + y_t \\ y_t &= m_t + \varepsilon_t \\ m_t &= \rho m_{t-1} + \nu_t \\ \varepsilon &\sim N(0, \sigma_\varepsilon^2) \\ \nu_t &\sim N(0, \sigma_\nu^2) \end{aligned} \tag{33}$$

Using data from the Consumer Expenditure Survey (CES), the authors first partial out the group-specific component α_t as a function of education and other variables, identifying the variance of the idiosyncratic part of income y_t, as well as (from the short panel dimension of the CES) its first order autocorrelation. They then fix $\rho = 0.9989$, the value estimated by Heathcote et al (2008a), which allows them to identify σ_ν^2 and σ_ε^2.

The results show an increase in the variance of labour income of 18 percentage points between 1980 and 2003, the two periods I focus on. 11 percentage points are due to an increase in within-group inequality, out of which roughly two thirds are accounted for by an increase in the importance of persistent shocks, and one third by that of transitory shocks.

In my exercise I abstract from changes in the common wage rate and differences in the group specific component, which, in the present model as in that of Krueger and Perri, translate fully into consumption differences by construction. As a baseline calibration, I choose a CRRA utility function with coefficient of relative risk aversion of 1 (log-preferences), a discount factor of 0.96, and a constant interest rate equal to the initial closed economy equilibrium rate of 3.5 percent. I then look at the sensitivity of the results to changes in parameters, and the world interest rate. And I look at the case when agents who default are excluded from all financial transactions in the current period, but allowed to invest in non-contingent bonds in the future to smooth income shocks over time. This reduces the impact of higher income risk on the value under default.

4.2 Model Solution

To solve the model, I first approximate the persistent process for m_t with a 7-state Markov chain using Floden's (2008) amended version of the standard Tauchen and Hussey (1999) method.[11] Following Krueger and Perri (2006) I choose a binary process for the transitory shock. The computational algorithm then follows the appendix that describes the recursions to derive the stationary consumption distribution in the general case. I amend this for the fact that, with purely transitory shocks ν_t, the monotonicity condition for F does not hold. So I need to reshuffle income states occasionally in order to have decreasing minimum-participation-compatible consumption values $c_1^a > c_2^a > ... > c_N^a$ during the algorithm. The solution is facilitated by the fact that, if this monotonicity condition holds, c_i^a can be found quickly using bisections on an interval $[z_i, c_{i+1}^a]$. This yields an algorithm that is extremely efficient when solving for

[11] Note that this method accords to my assumption of widening the support Z to increase risk, but leaving the transition probabilities unchanged.

the stationary consumption distribution.[12]

4.3 Results

4.3.1 Income risk and net foreign assets

Table 1 shows the equilibrium asset positions for different specifications of the economy. In the baseline calibration (I), the rise in income risk between 1980 and 2003 leads to a fall in the stationary level of net foreign assets of more than 50 percent of annual GDP. However, this calibration features a relatively high world real interest rate, and very strong effects of income risk on the value of default, due to the assumption of permanent exclusion from all financial trade. Thus, a second calibration allows saving in non-contingent bonds starting from the period following default, and reduces the world interest rate to 2.5 percent.[13] The results are reported as calibration II in table 1. The fall in stationary assets from the observed rise in US income risk is now smaller, at less than 10 percent of GDP. This is because with saving after default, higher income risk has a smaller impact on autarky values. Calibration III in table 1 increases risk aversion in this second calibration to $\sigma = 2$. With more risk averse individuals, the income volatility under financial autarky provides stronger disincentives to default, even when agents are allowed to save in autarky. For a given level of income risk, this translates to lower stationary asset holdings. But as before, the increase in income risk between 1980 and 2003 decreases stationary assets further, by about 40 percent of GDP.

Figure 2 shows that this reduction in assets from a rise in income risk holds for all values of world interest rates in the base line calibration. But this monotonicity of stationary foreign assets in risk gets lost when agents are allowed to save under autarky, as figures 3 and 4 show. For high interest rates, the additional increase in risk now increases aggregate assets in stationary equilibrium.

4.3.2 Income and consumption risk

Figure 5 shows the consumption distributions in the baseline case, for low (1980) and high income risk (2003). The sub-distributions, of different colour in the graph, correspond to individuals that were last constrained in the same income

[12] A dual core desktop computer with two Intel 1.8 GHz processors solved for a typical equilibrium consumption distribution in less than 4 seconds.

[13] I choose the savings rate of 2.5 percent, which is close to the average ex-ante annualised real rate of 2.6 percent on 6 month US treasury bills between 1980 and 2003, deflated using University of Michigan 12 month inflation expectations.

state, and thus have a common starting value for their declining paths before the next positive shock. Importantly, these sub-distributions are geometric and their shape remains constant between 1980 and 2003 - this is because the interest rate is unchanged in the baseline case. Their positions, however, decline with the fall in autarky values caused by higher income risk. This fall is less pronounced in states that correspond to positive realisations of the binary transitory shock, such as state 1, as there, higher variance translates to an increase in current income, if not value. From table 1 we see that the corresponding change in the variance of log consumption is small, equal to 0.06 percentage points. Table 1 also shows that, with saving in autarky, the effect of higher income risk on consumption variability changes sign. This is because agents can now smooth part of the income volatility in autarky by saving. This reduces the financial deepening effect of higher income risk.
Figure 6 illustrates the relationship between interest rates and the consumption distribution. For the income process estimated for 2003, the figure shows how a lower interest rate widens the consumption distribution significantly, as analytically shown for the special case above. Figure 7 confirms this finding: the change in consumption volatility due to a change in income risk is an order of magnitude smaller than the changes caused by movements in the world interest rate.

The rise in individual income risk observed in the US since the 1980s can thus potentially explain at least part of the fall in its net foreign asset position. And interestingly, for a given interest rate this rise in income risk leaves the distribution of consumption almost unaffected. But, for given income risk, changes in world interest rates have an important effect on consumption inequality.

5 Conclusion

This paper has analysed the link between domestic income uncertainty, consumption inequality and net foreign asset positions in a small open economy. Domestic financial markets were assumed to be complete, but constrained by individuals' option to default on contracts, at the price of permanent exclusion from insurance markets. I showed that, contrary to small open economies with unconstrained complete markets, this economy has a well-defined stationary equilibrium. An analytical solution to the cross-sectional consumption distribution showed that higher income risk can indeed lower aggregate savings by making the punishment of default, financial autarky, less attractive, thus endogenously "deepening" financial markets. However, changes in income risk have only a small effect on consumption inequality, which depends mainly on the international interest rate. A calibration of the model to the US case showed that the changes in income risk observed between 1980 and 2003 might indeed explain an important part of the fall in the net foreign asset position.

Future research should generalise this analysis to the more realistic case of a 2 country economy. While conceptually and computationally unproblematic, however, this requires assumptions about the relative evolution of inequality in the US and the rest of the world, which I have shied away from. But the recent advances in comparative estimates of stochastic income processes (cf. Heathcote et al 2008b) are promising in this context.

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7 Appendix

7.1 Proof of existence and uniqueness

Result: *For every given world interest rate $R_{min} < R < \frac{1}{\beta}$, there exists a unique equilibrium allocation in the small country that is equal to the solution of the planner's problem for an appropriate weighting function μ in the social welfare function.*

Proof

I prove existence of a unique solution to the planner's problem by checking that the conditions for a simplified version of Proposition 3 in Marcet and Marimon (1998) hold in this economy.

Given the finite space of individual endowments Z we can apply a version of Tychonoff's theorem to see that the Euclidian Product Z^T is compact for countable T. So the exogenous vector of individual states lies in a compact (Borel) subset of the Euclidian Space R^T. And of course, the discrete transition function satisfies the Feller property (Assumption A1 in Marcet and Marimon (1998)). Second, given the No-Ponzi condition, for any given B_t, R, Y the set of feasible consumption allocations $c_{i,t} : \int_{mathbbI} c_i, t \leq \frac{Y}{R-1} + B_t, \forall t$ is just a simplex, so the choice vector lies in a compact and convex set (Assumption A2 in Marcet and Marimon (1998)). Third, note that our objective function is continuous, but unbounded. However, since aggregate ressources are bounded each period, so is $\int_{\mathbb{I}} U(c)$ (Assumption B1 in Marcet and Marimon (1998)). Finally, individual autarky is incentive compatible and resource feasible. So the constraint set is convex, compact, and non-empty.[14]

Given the continuous objective function, the original sequential problem (12) therefore has a solution. Also, Marcet and Marimon (1998) show that, given any initial weighting function μ, these conditions suffice to show that an allocation

[14]Strictly, we have to show that the constraint set has a non-empty interior, or that there is a real number $\varepsilon > 0$, such that $\int_{\mathbb{I}} c_i, t - Y \geq \varepsilon$ and $\int_{\mathbb{I}} [E[\sum_{t=0}^{\infty} (\mu_{i,t} + \gamma_{i,t}) U(c_{i,t}) - W(z_i)] > \varepsilon$. In fact, without knowing the solution of the problem, the existence of $\varepsilon > 0$ is not trivial to prove. However, once we have the solution, the condition is easy to check. For now, I show the existence of ε for the i.i.d. version of the special case, with $p = q = 1/2$ and $B_{t+1} = B_t = 0$. For this case it is easy to see that as long as the income uncertainty is big enough, or $\epsilon > \nu : \frac{U'(y_0+\nu)}{U'(y_0-\nu)} = \frac{2-\beta}{\beta}$, there are numbers $\xi, \widehat{\varepsilon} > 0$ such that a programme of the form $c(y_h) = y_h - \xi, c(y_l) = y_h + \xi - \widehat{\varepsilon}$ fulfills the conditions above. Intuitively, the expected discounted gain from higher consumption in future low-income states is big enough to allow a ressource-feasible reallocation of current consumption from high to low income agents. Thus the interior of the constraint set is strictly non-empty (Assumption B2 in Marcet and Marimon (1998)). But, as we will see, this history independent sharing rule is not optimal.

$\{c_{i,t}\}, i \in \mathbb{I}, t \geq 0$ solves the original problem if and only if there is a sequence of multipliers $\gamma_{i,t},\ i \in \mathbb{I}, t \geq 0$ such that $\{c_{i,t}, \gamma_{i,t}\},\ i \in \mathbb{I}, t \geq 0$ solves the saddle-point functional equation (13).
Uniqueness of the equilibrium is assured by the strict concavity of the utility function u.

QED

7.2 Proof of Proposition 1

There exists a unique stationary equilibrium with a distribution of consumption $\Phi_C : \mathbb{C} \subseteq R^+ \longrightarrow [0,1]$. *If* $R > R_{min}$, Φ_C *has* $N-1$ *overlapping subdistributions on finite supports* $\mathbb{C}_i = \{c_{1,i} > c_{2,i} > ... > c_{m_i,i}\},\ i = 1, ..., N-1$. *For each, probability mass declines geometrically from an upper bound* $c_{1,i}$ *defined by the participation-constraint for income* z_i*, across intermediate support points given by the law of motion* (17)*, to a common lower bound equal to minimum income* ($c_{m_i,i} = z_n, \forall i$).

Proof
The proof conjectures constant aggregate consumption, and then constructs a consumption distribution that fulfills the planner's (necessary and sufficient) optimality conditions and participation constraints. Its stationarity validates the conjecture and transversality conditions. In the following, I describe the recursion to construct Φ_C and $\mathbb{C}$, which uniquely maps interest rates and the structure of uncertainty into stationary distributions of consumption. First, I construct the finite number of support points $\mathbb{C}$, and then the frequency distribution Φ_C on this support.
Step 1: The support $\mathbb{C}$
The strategy is to build the support "bottom-up". We know its lower bound to be minimum income. From this I can determine the minimum participation compatible level of consumption in the second income state by the participation-constrained values, by the Law of Motion for consumption of unconstrained agents 16, and transition probabilities. These two minimum participation-compatible consumption levels allow me to determine the third in a similar fashion, etc.
To see this, denote as $c^i(c, R)$ the result of applying (17), the transition for consumption of unconstrained agents that do not receive positive income shocks, i times starting from level c at interest rate R.
1. Order income states in descending order from 1 to N and let $c_i^a,\ i = 1, 2, ..., N$

denote the minimum level of consumption today that just meets participation constraint for individuals with income z_i. Now consider a participation-constrained individual in the lowest income state z_n. She will be constrained in all income states tomorrow, thus receiving autarky value $W(i)$ for all z_i. Thus her minimum participation-compatible consumption c_n^a is determined from

$$W(N) = U(c_N^a) + \beta \sum_i p_{Ni} W(i) \tag{34}$$

From the definition of autarky value W(N) this is solved for $c_N^a = z_N$. So the minimum consumption level with participation constrained insurance is equal to minimum income.

2. Now consider minimum participation-compatible consumption in the second lowest income state $N-1$. There, individuals receive c_{N-1}^a today. They face the "danger" of moving, with probability $p_{N-1,N}$, to state N, and thus down to $c^1(c_{N-1}^a, R)$ tomorrow. With probability $p_{N-1,i}$, however, they move to states $i < N$ receiving $W(i)$. So c_{N-1}^a is uniquely determined from the participation constraint

$$W(N-1) =$$

$$U(c_{N-1}^a) + p_{N-1,N} \sum_{s=0}^{\infty} \beta^{s+1} p_{NN}^s max\{U(c^s(c_{N-1}^a, R), N), U(c_N^a)\} + \tag{35}$$

$$\beta \sum_{i=1}^{N-1} p_{N-1,i} W(i) + p_{N-1,N} \frac{\beta^2}{1 - p_{N,N}\beta} \sum_{1}^{N-1} p_{N,i} W(i) \tag{36}$$

Here, the second term on the right-hand side of the equation is the value from the declining consumption path starting at c_{N-1}^a and truncated at minimum level c_N^a, weighted by the probability to remain in income state N. The third term is the continuation value when not receiving a negative income shock tomorrow, the fourth from moving down in income tomorrow and then receiving positive income shocks at a later date. Note that the right hand side is increasing in c_{N-1}^a while the left hand side is constant. So the solution is unique.

3. Analogously, one can determine the other values c_i^a from repeated application of this algorithm.

The support of the consumption distribution $\mathbb{C}$ is simply the union of downward-sloping paths starting at minimum participation-compatible consumption c_i^a $\mathbb{C} = \cup_{i=1}^{N} \{max[c^j(c_i^a, R), c_1^a], j = 0, 1, 2, ...\}$.

Step 2: The frequency distribution on $\mathbb{C}$

I construct the frequency distribution "top-down". I know the upper bound of the distribution is the minimum participation-compatible consumption for individuals with the highest income. So its mass is equal to the stationary mass of individuals with this income value, as none of them receives less. The rest of the frequency distribution is then based on the transition probabilities of consumption for an individual in income state i at consumption c, constructed as follows:

First, denote as $z_k(c)$ the lowest income state with a minimum participation-compatible consumption greater than the first step down from c, or $k = min\{l : c^a_l > c^1(c, R)\}$. For all shocks $z_l < z_k$, an individual who consumes c today will move one step down to $c^1(c, R)$. For all shocks z_j equal to or greater than z_k she will move up to c^a_j, the minimum participation-compatible consumption corresponding to z_j. For an individual in income state i this yields transition from c as

$$p(c = c^a_j, j \geq k) = p_{i,j} \tag{37}$$

$$p(c = c^1(c, R)) = \sum_{1}^{k-1} p_{i,k} \tag{38}$$

$$p(c) = 0 \ otherwise \tag{39}$$

Thus, the probability of moving down the unconstrained path declines discretely when "passing" minimum participation-compatible levels. Thus, on the declining consumption path starting from c^a_i the distribution is geometric with $N-i$ discretely declining values of survival probability. This yields N geometric distributions truncated by the minimum level $c^a_N = z_N$. Finally, the ergodic nature of **income** ensures that all consumption states are consequents of each other. So the ergodic set associated to the transition for consumption is unique. The stationary distribution of consumption is simply determined as the unique fixed point to the consumption transition.

QED

Proof of Corollary 3: The variance of log(c) in the special case
Proof

The results follow from simple tedious algebra, considering the non-truncated geometric distribution, noting that the variance of a truncated distribution is strictly lower:

1. Denote the first entry of the left eigenvector as $\nu = \frac{(1-q)}{(2-q-p)}$, and the log of x as $\widehat{x}$.

2. The mean of $\log c$ is

$$\mu_c = \nu\{\widehat{c_h} + (1-p)[\widehat{c_h} + \sum_{i=1}^{\infty} \frac{\widehat{\beta R}}{\sigma} i q^{i-1}]\} \tag{40}$$

$$= \widehat{c_h} + \frac{1-p}{(1-q)(2-q-p)} \frac{\widehat{\beta R}}{\sigma} \tag{41}$$

3. The variance is

$$VAR_c = \nu \frac{(1-p)^2}{(1-q)^2(2-q-p)^2} [\frac{\widehat{\beta R}}{\sigma}]^2 \tag{42}$$

$$+\nu(1-p)[\frac{\widehat{\beta R}}{\sigma}]^2 \sum_{i=1}^{\infty} [i - \frac{(1-p)}{(1-q)(2-q-p)}]^2 q^{i-1} \tag{43}$$

$$= \nu[\frac{\widehat{\beta R}}{\sigma}]^2 \{\frac{(1-p)^2}{(1-q)^2(2-q-p)^2} \tag{44}$$

$$+(1-p)[\frac{(1+q)}{(1-q)^3} - 2\frac{(1-p)}{(1-q)^3(2-q-p)} + \frac{(1-p)^2}{(1-q)^3(2-q-p)^2}]\} \tag{45}$$

$$= \nu[\frac{\widehat{\beta R}}{\sigma}]^2 [-\frac{(1-p)^2}{(1-q)^3(2-q-p)} + \frac{(1-p)(1+q)}{(1-q)^3}] \tag{46}$$

$$= [\frac{\widehat{\beta R}}{\sigma}]^2 \frac{(1-p)(1+q(1-p-q))}{(1-q)^2(2-p-q)^2} \tag{47}$$

Note that for the i.i.d. case $1-p=q$ both the mean and the variance reduce to those for an ordinary geometric distribution.

QED

8 Tables and figures

Table 1:

Stationary assets and consumption inequality - different calibrations

I	**Baseline**						
	year	R	β	σ	Assets/GDP	Var(log(c))	Save in default?
	1980	1.035	0.96	1	0	0.006	No
	2003	1.035	0.96	1	-0.52	0.0054	No
II	**Save in default**						
	year	R	β	σ	Assetss/GDP	Var(log(c))	Save in default?
	1980	1.025	0.96	1	-0.42	0.0142	At 2.5%, not in t=0
	2003	1.025	0.96	1	-0.49	0.0162	At 2.5%, not in t=0
III	**Save in default,** $\sigma = 2$						
	year	R	β	σ	Assetss/GDP	Var(log(c))	Save in default?
	1980	1.025	0.96	2	-0.9635	0.0059	At 2.5%, not in t=0
	2003	1.025	0.96	2	-1.3776	0.006	At 2.5%, not in t=0

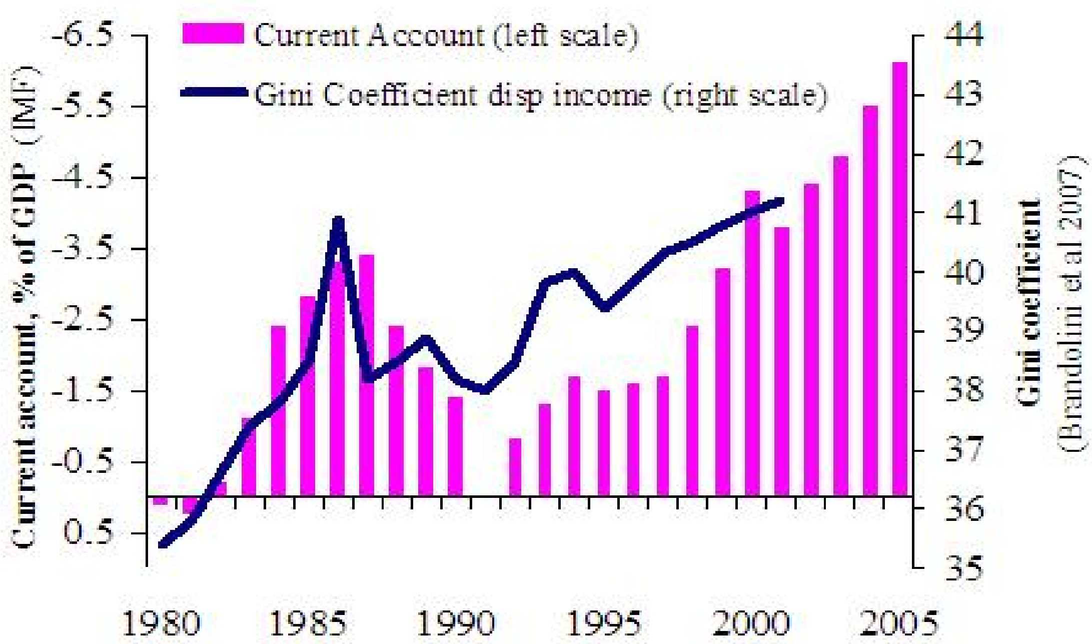

Figure 1: US current account and Gini coefficients. Source: IMF and Brandolini et al (2007)

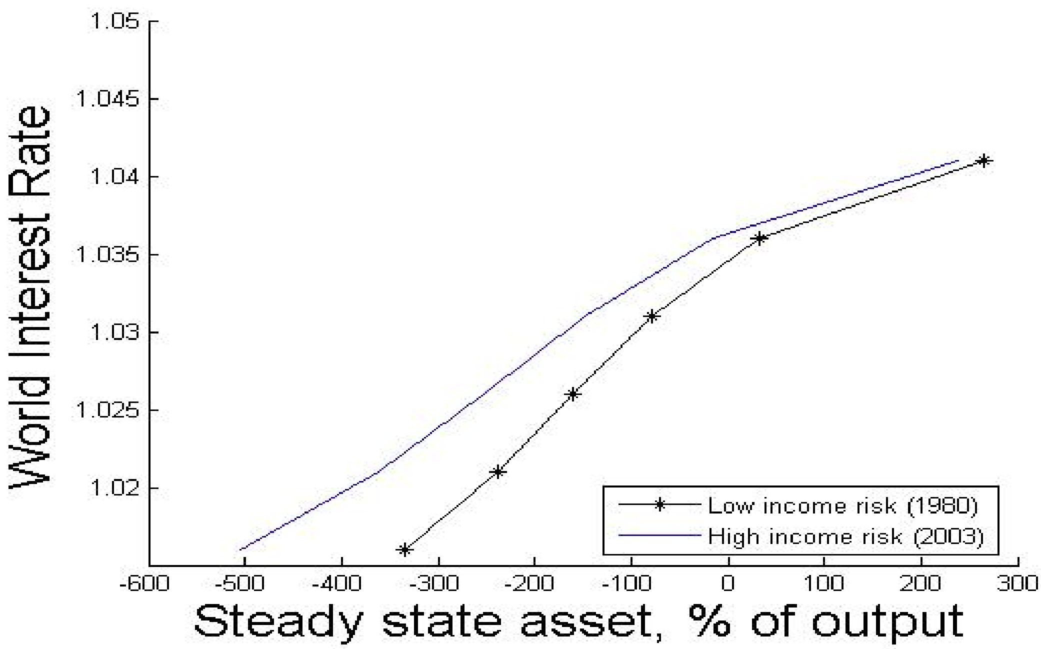

Figure 2: Asset demand function, baseline calibration.

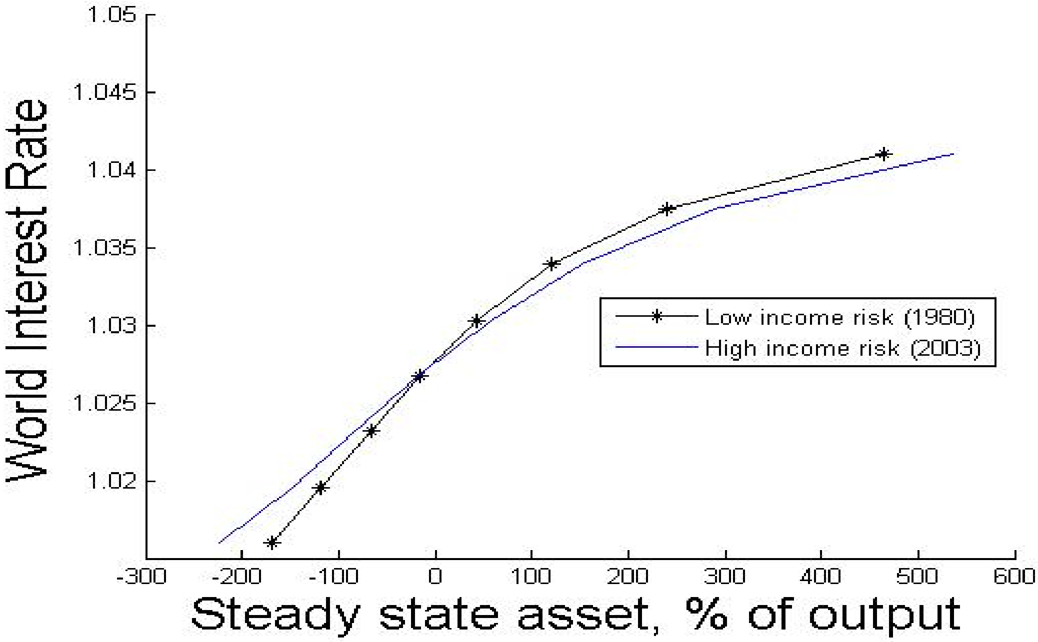

Figure 3: Asset demand function, log-preferences, saving at world interest rate but not in t=0.

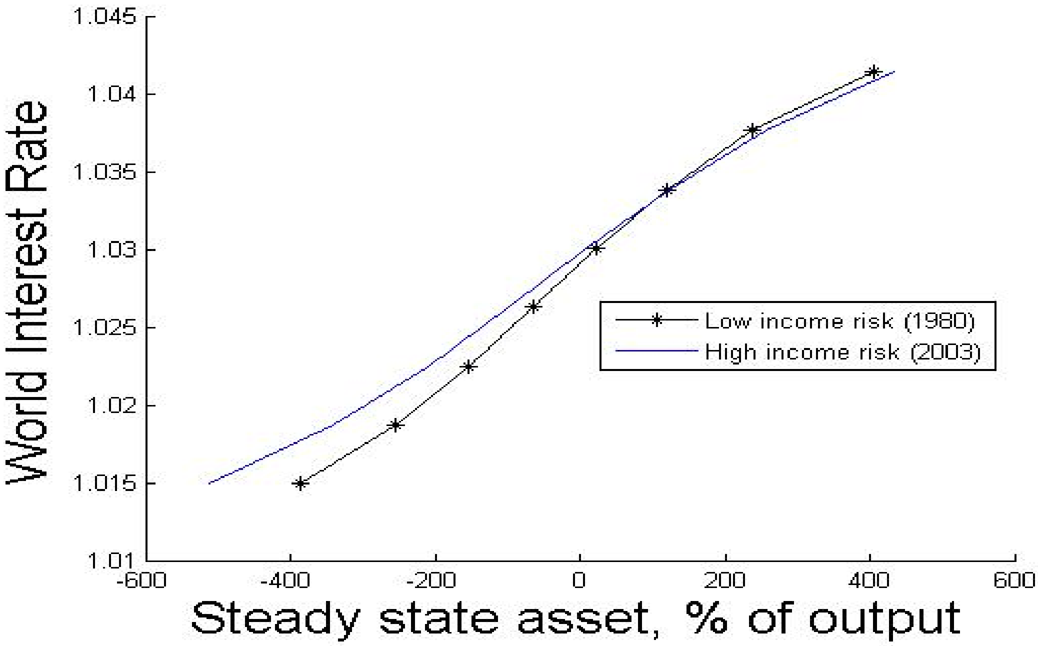

Figure 4: Asset demand function, higher risk aversion ($\sigma = 2$), saving at world interest rate but not in t=0.

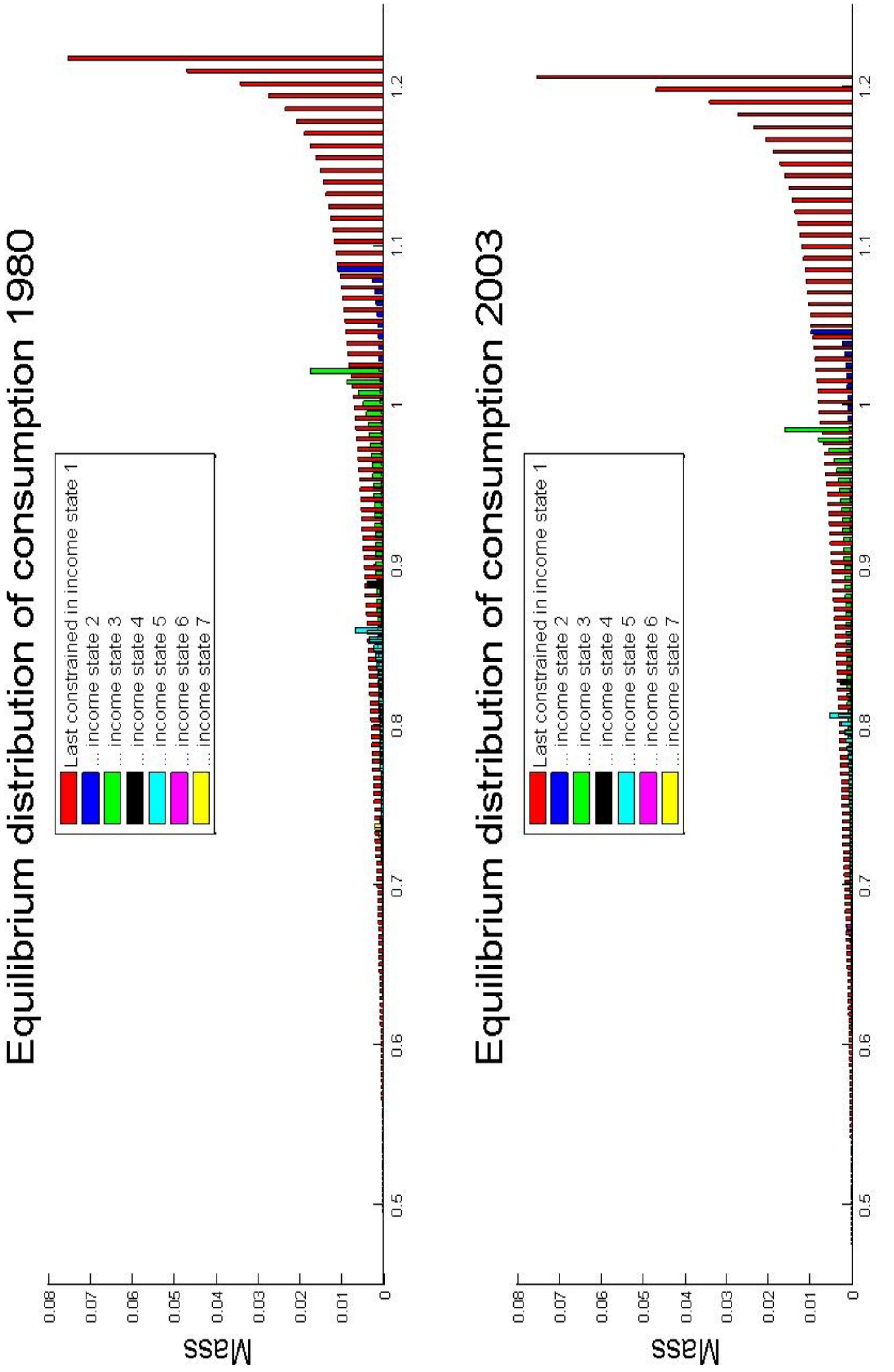

Figure 5: The consumption distribution in 1980 and 2003, baseline calibration

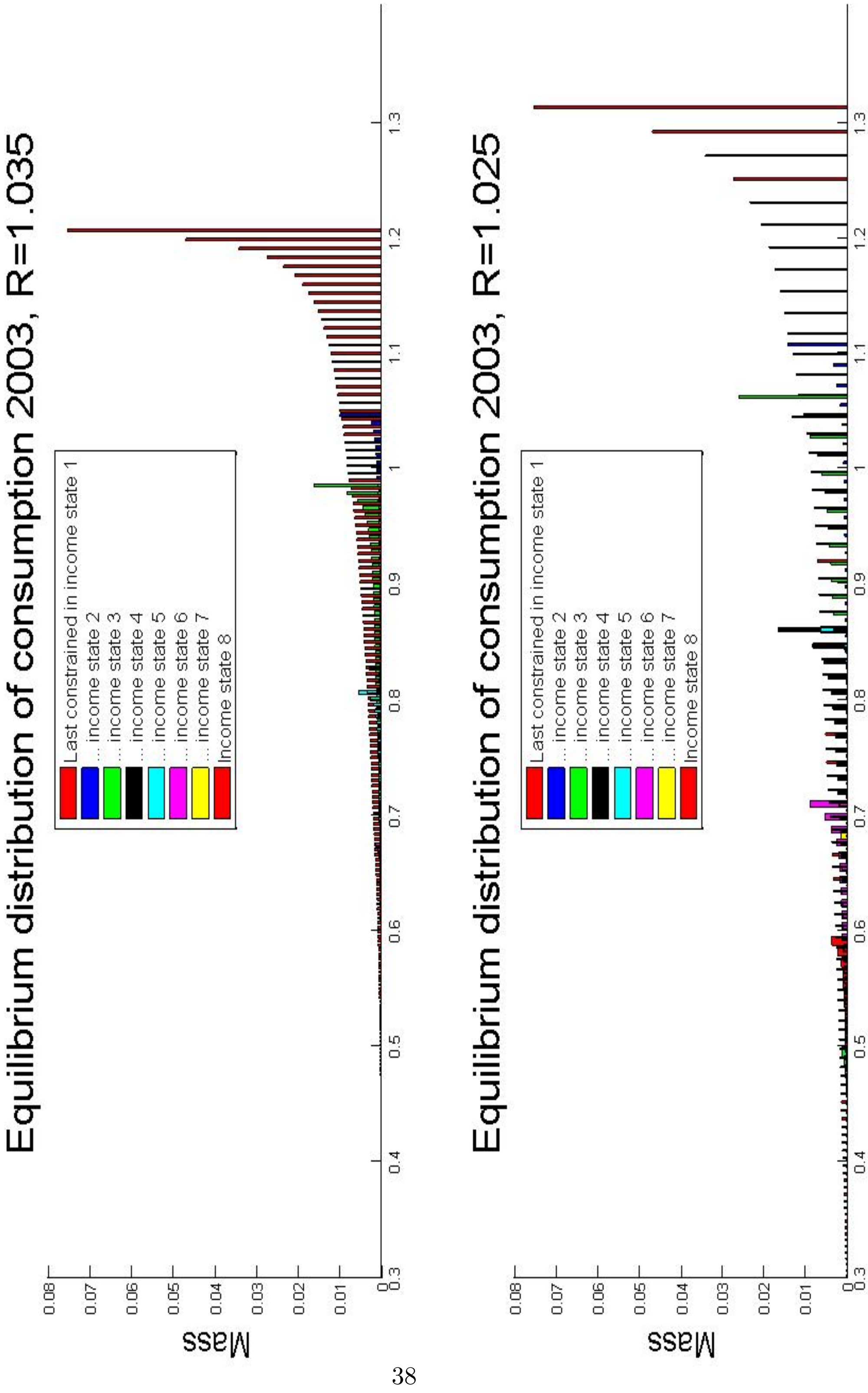

Figure 6: The consumption distribution in 2003 (high income risk), with high and

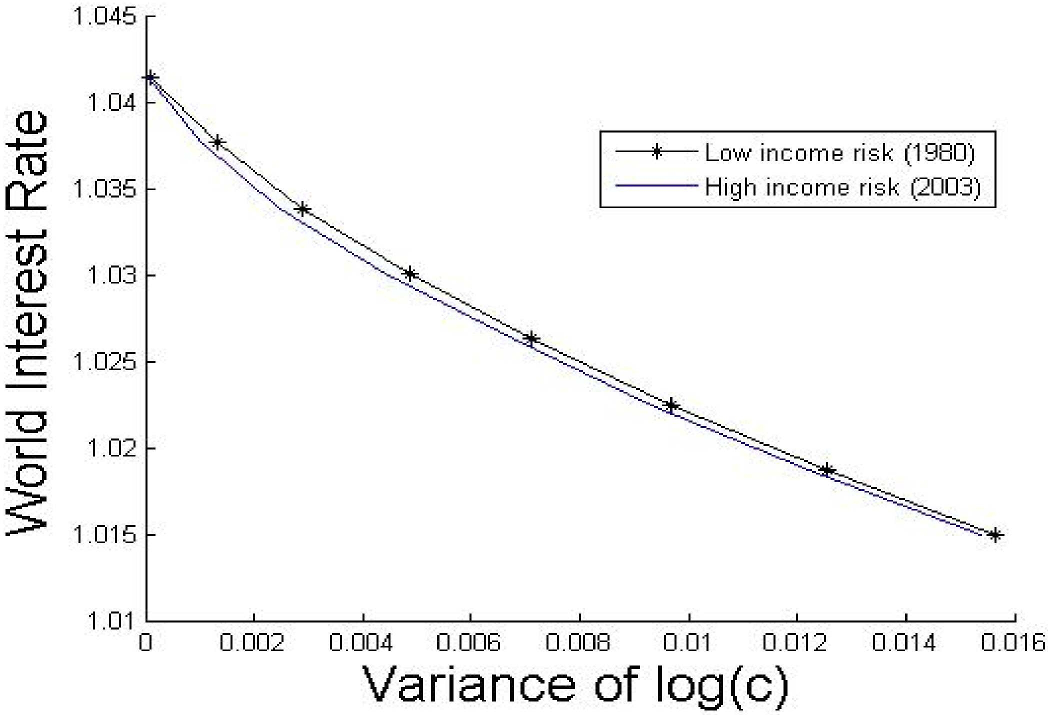

Figure 7: Variance of log(c), baseline calibration.