Explaining frictional wage dispersion

Tamás K Papp*

Department of Economics, Princeton University
e-mail: tpapp@princeton.edu
www.princeton.edu/~tpapp

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Abstract

Matching the magnitude of frictional wage dispersion has been difficult for most frictional search models of the labor market; only 1/30–1/4 of the frictional wage dispersion observed in the data can be accounted for using realistic calibrations. In this paper, I develop a general equilibrium model with ex ante homogeneous workers but heterogeneous firm productivities that accounts much better for the magnitude of frictional wage dispersion.

The key ingredient that makes this model different from wage posting models is the bargaining and the wage determination structure. When a firm meets an employed worker, the new firm and the incumbent firm engage in Bertrand competition in the worker's share of the match surplus; consequently the current wage of the worker depends not only on the firm's productivity, but also on the history of previous offers. I demonstrate that the latter generates about 80% of the frictional wage dispersion, and that standard wage posting models cannot achieve similar magnitudes of frictional wage dispersion with the calibration used in the paper.

In contrast to the structural estimation literature, the parametrization of the model is very parsimonious. The model is calibrated to match the worker flows, the standard deviation of log productivity and the replacement ratio, and it delivers frictional wage dispersion that matches the data very closely.

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1 Introduction

The wages of individuals are dispersed both in the cross section and over time, and explaining these differences among workers has always been one of the central questions of economics. In a competitive environment, workers would earn different wages only if their marginal products were different: wage dispersion would be the consequence of individuals having different levels of innate productive skills, education, work experience, or other forms of human capital. However, the data suggests that observable worker characteristics are not able to account for all the wage dispersion among workers: Mincerian wage regressions based on cross-sectional data on individuals use a large set of observable variables, but usually account for only 30% of the cross-sectional wage dispersion.[1]

Models of *frictional labor markets* suggest that ex ante homogeneous individuals may end up earning different wages because of the search frictions present in the market: when individuals are not able to survey all potential jobs due to search frictions, some will earn higher wages than others simply because they were more lucky when searching. However, as Hornstein et al. (2007) demonstrate, even though frictional labor market models can explain the *presence* of wage dispersion, calibrated versions of these models are unable to account for its *magnitude*.

In this paper I develop a general equilibrium model that accounts for all the frictional wage dispersion in the data when calibrated to worker flows, productivity dispersion and realistic values of the replacement ratio. Workers are ex ante homogeneous, but firms have heterogeneous productivity levels. Nevertheless, while heterogeneous productivity does generate some wage dispersion, it is not the dominant source *per se*: the key ingredient of the model is its bargaining and wage determination structure. Upon forming a match, the worker and the employer agree

[1]See Mortensen (2005) for a survey of these results.

to share the surplus value of the match over unemployment and vacancy by giving the worker a *fixed fraction of the surplus*. Either party can propose renegotiation of the contract, but if the match is separated, neither party can recall it so it is not a credible threat. When employed workers are contacted by another firm, the two firms Bertrand compete in the worker's share of the surplus. Provided that the other firm has sufficiently high productivity, outside offers to employed workers result in better terms for the worker and thus a jump in the present value of the worker's earnings. Depending on their bargaining position, workers may be willing to accept low wages with the knowledge that they will be compensated for this when outside offers make them better off in the future.

Hornstein et al. (2007) show that models without on-the-job search fail to generate enough frictional wage dispersion because increasing the dispersion of the wage offer distribution also increases the reservation wage of the worker, leaving the ratio of the average wage and the reservation wage invariant. They also conjecture that models with on-the-job search are the most likely to generate realistic frictional wage dispersion, since on-the-job search breaks the tight connection between the average and reservation wage as accepting a job offer no longer implies that the worker sacrifices the option value of searching for a better wage.

However, as I demonstrate in Section 6.3, on-the-job search *per se* is not enough to generate realistic frictional wage dispersion in wage posting models following Burdett and Mortensen (1998), even though these models are significantly closer to the data than models without on-the-job search. The most important difference between these models and the model presented in this paper is that in wage posting models, each worker draws wage offers from a *common wage offer distribution*, while in this paper, each worker is facing a *specific distribution of offers* that depend on his current bargaining position — in particular, the productivity of the firm where he is currently employed. The wage is determined implicitly, and is a function of the current firm's productivity p and the surplus share β which is the result of the bargaining process, and is dispersed in the cross-section because it is a function of the productivity levels of the last two firms which made offers to the worker. I demonstrate that for the calibrated model, the latter generates the bulk (about 80%) of the wage dispersion, and thus it is the key ingredient behind the result of the paper.

In the companion paper Papp (2008), I present a model with similar bargaining and contract structure but vintage capital and embodied technological growth, which results in the passage of time decreasing the productivity of the match, ultimately making firms exit endogenously. Despite the different productivity structure, the model's implications about wage dispersion are similar to this paper, demonstrating the robustness of the result.

Structure of the paper

The structure of the paper is as follows. Section 1.1 reviews the related literature. To provide a background motivating the present research and, in particular, the difficulty in accounting for wage dispersion, in Section 2 I briefly explain the *mean-min ratio*, a particularly useful statistic introduced by Hornstein et al. (2007), and summarize their empirical estimates of this statistic and the failure of simple models in matching it. In Section 3, I present the model and define the stationary equilibrium. Section 4 summarizes the solution method for the model while Section 5 discusses the calibration. In Section 6 I present the results, explore the source of the wage dispersion in the model and perform sensitivity analysis. In this section I also derive an approximation that characterizes the wage dispersion of a standard wage posting model in equilibrium and contrast the calibrated version of this model with the data. Section 7 concludes and discusses possible extensions. In the Appendix of the paper, I briefly explain the numerical solution technique used in solving the model, discuss the planner's problem and social efficiency, and present proofs.

1.1 Related literature

The idea that wage (or in general, price) dispersion for ex ante homogeneous agents or goods is related to imperfect information has been recognized for a long time: for example, Stigler (1961) convincingly argues that price dispersion is related to search frictions: if agents had perfect information, no price dispersion could exist. However, Diamond (1971) demonstrates that search frictions *per se* are not necessarily enough to create price dispersion: given perfect information about the buyers' options and a fixed search cost, the equilibrium will display no price dispersion or even unravel (if the initial search is costly). Burdett and Judd (1983) resolve this paradox by introducing *imperfect (and asymmetric) information*: in their model, sellers do not know how many other offers their potential buyers may have, which discourages them from demanding the highest possible price.

For labor markets, search frictions play a potentially dual role: they ensure the existence of *match rents*, and search frictions may also lead to *wage dispersion*. Note that the former feature can occur without the latter: for example in the seminal paper of Pissarides (1985) on the random matching model, there are rents for matches but no wage dispersion. Burdett and Mortensen (1998) generate frictional wage dispersion by assuming asymmetric information — similarly to Burdett and Judd (1983) — in particular, employers in their model cannot condition on the employment status and previous wage of potential employees when making wage offers, thus the wage dispersion is the result of strategic interaction between firms. This model can be generalized to incorporate heterogeneous productivity levels, for example as in Bontemps et al. (2000).

The papers mentioned above use asymmetric information and take it or leave it offers to generate wage dispersion. In this paper, I take a different route and use a bargaining mechanism that is similar to Postel-Vinay and Robin (2002) and Cahuc et al. (2006): in these two models, when an employed worker meets another firm, the incumbent and the new firm Bertrand compete in the worker's continuation value through wage offers. The model in the present paper differs from the these two papers in the following respects: (1) the object of the bargaining process is the *surplus share*, not the wage, (2) firm entry is endogenized in a general equilibrium framework, (3) vacancies have positive value, so workers cannot extract all the firms' productivity, which makes the maximum wage lower than the latter, (4) the model parametrization is parsimonious and is calibrated, not structurally estimated.

In contrast, the typical approach in the structural estimation literature (for example, Christensen et al. (2005)) is to introduce a model (which may be general or partial equilibrium), estimate some distribution (for example, the wage offer distribution) non-parametrically and then back out some other property of the model (such as the ergodic distribution of wages) and contrast it with some other aspect of the data. This paper is different in the sense that it sets up a parsimoniously parametrized general equilibrium model, calibrates it to parameters — such as worker flow rates, the real interest rate, dispersion of the log productivity and the replacement ratio — that are not directly related to wage dispersion, and then checks how well the wage dispersion that the model generates matches the data.

Another possible bargaining structure is presented in Kiyotaki and Lagos (2007). Their model also allows the displacement of an employed worker by another, making the search structure symmetrical. However, in order to make contracts self-enforcing, the bargaining situation results in spot side payments, not persistent wage differences, so bargaining and worker history *per se* are not a source of wage dispersion in the model.

2 The mean-min ratio

Since wage distributions are infinite-dimensional objects, it is useful to use some summary statistic of the wage dispersion. Commonly used summary statistics include quantile ratios (such as 90%–10% quantiles), the coefficient of variation and of course the mean-min ratio, which is *the average wage divided by the lowest wage we observe.* In this paper, I will use the latter as it has two particularly appealing features: first, it is based on the natural notion that *the lowest wage* agents are willing to accept (ie the reservation wage) *is an endogenous feature of a model with optimizing agents*; and, second, it appears to be the key summary of wage dispersion in benchmark search models.

Consider a simple model without on-the-job search, where a risk-neutral worker with discount rate r encounters wage offers with distribution $F(w)$, similarly to the sequential search model of McCall (1970). Workers receive an unemployment benefit z, and get job offers at rate λ. Exogenous separations happen at rate σ. Let $W(w)$ and U denote the present discounted value of working for wage w and unemployment, respectively. The HJB equations are

$$rW(w) = w + \sigma \underbrace{\big(U - W(w)\big)}_{\text{separation}} \tag{1}$$

$$rU = z + \lambda \underbrace{\int_{w^*} \big(W(w) - U\big) dF(w)}_{\text{finding a job}} \tag{2}$$

In (1), the worker receives a flow of w, but at rate σ he will be separated from the firm and become unemployed. In (2), unemployed workers receive a flow benefit z, and they encounter wage offers are rate λ, each drawn from the distribution $F(w)$. In the equations above I implicitly truncate the distribution F at w^* without loss of generality, as wage offers below the reservation wage would not be accepted by the worker. First, solve (1) for $W(w)$ as

$$W(w) = \frac{w + \sigma U}{r + \sigma} = \frac{w - rU}{r + \sigma} + U \tag{3}$$

From this, we know that the reservation wage is $w^* = rU$. Then substituting into (2) and using $\bar{w}$ for the average wage,

$$w^* = z + \frac{\lambda}{r + \sigma} \int_{w^*} (w - w^*) dF(w) = z + \underbrace{\frac{\lambda\big(1 - F(w^*)\big)}{r + \sigma}}_{\equiv \chi} (\bar{w} - w^*) \tag{4}$$

where I used χ to capture the job finding, separation and interest rates in a single variable. Without loss of generality, express z using the replacement ratio ρ, defined as

$$\rho = \frac{z}{\mathrm{E}[w]}$$

Then from (4),

$$\text{Mean-min ratio} = \frac{\mathrm{E}[w]}{w^*} = \frac{1 + \chi}{\rho + \chi} \tag{5}$$

In order to explore this relationship, I will use the calibration from Section 5 of this paper. The time unit is a year. The job finding rate is $\lambda_{\mathrm{UE}} = \lambda\big(1 - F(w^*)\big) = 4.845$, the separation rate is $\lambda_{\mathrm{EU}} = \sigma = 0.213$. I will choose $r = 0.04$, but the results are not sensitive to this as r is small when compared to the other rates. Then

$$\chi = 19.15$$

Hornstein et al. (2007) estimate the mean-min ratio for various data sources. Since the lowest wage is very sensitive to outliers, they also use the 5th and 10th percentile in place of the lowest wage. In this paper I will try to match the ratio of the average wage to the lowest 5th percentile, but will continue to refer to this as the mean-min ratio. They estimate this statistic to be in the range of 1.97–2.13 for the 1990 Census data,[2] and 2.08 for PSID data.[3]

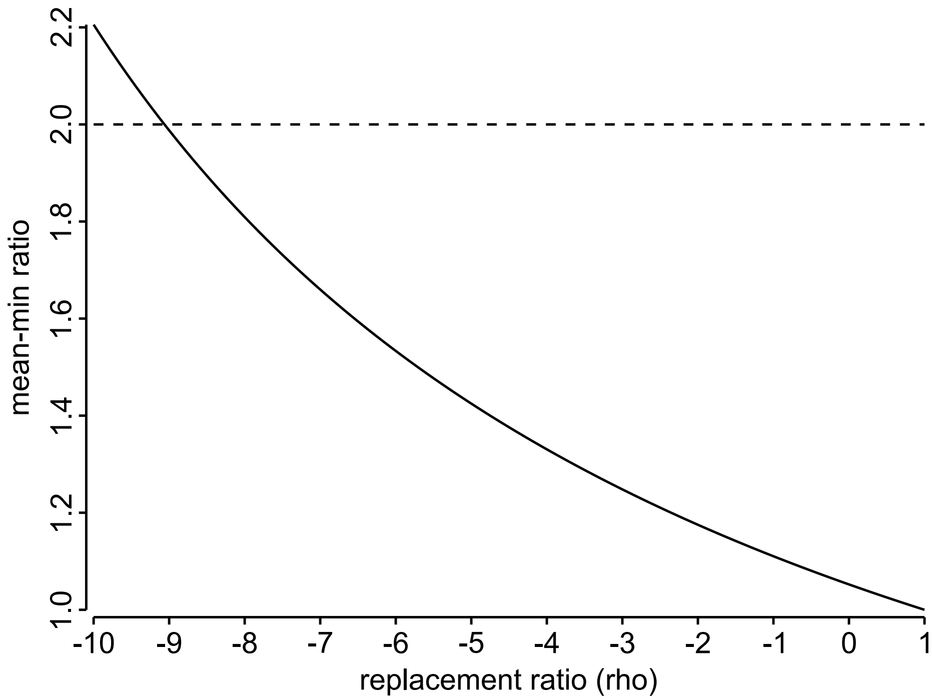

Figure 1: Mean-min ratio without on-the-job search. To match the mean-min ratio of ≈ 2 in the data (dashed line), the replacement ratio would need to be unrealistically low, in the range of -10 to -8, far outside the realistic range of 0.2–0.8.

Figure 1 shows (5), the mean-min ratio as a function of ρ. It is clear that we need an extremely low replacement ratio (around -9) to match the data, which is not a reasonable calibration.

Intuitively, the simple model presented above generates a low mean-min ratio because once workers are employed, they can no longer search, so even with a very dispersed offer distribution F whey would just use a higher reservation wage. Horn-

[2] 5% IPUMS sample, controlling for gender, race, education, potential experience; also occupation and geographical area, for the latter, restricted to cells with $N \geq 200$.

[3] PSID 1967–1996, controlling for gender, race, education, potential experience, marital status, region, occupation, occupation interacted with experience. When controlling for worker fixed effects gives 1.46, but the authors argue that this also includes match- (or firm-) specific fixed effects and should be interpreted with caution.

stein et al. (2007) discuss several extensions of the basic model above, including endogenous search effort, ability differences, risk aversion and wage shocks during unemployment, and conclude that none of them increase the mean-min ratio enough to match the data.

Because the high reservation wage is a result of sacrificing the option of searching for a job once employed, one could expect that including on the job search would lead to a sufficiently high mean-min ratio, but this is not the case: on the job search *per se* is not enough, though it is a step in the right direction. In Section 6.3 I derive an expression similar to (5) for the standard wage ladder (or wage posting) model of Mortensen (1998) embedded in the production and search framework of the present paper. Similarly to (5), the replacement ratio ρ also plays a crucial role there: while the replacement ratio that would be needed to match the mean-min ratio in the standard wage ladder model is still counterfactually low (around -0.6), the latter model is much closer to the data.

It is not surprising that the replacement ratio turns out to be a key variable for calibration: the lower the unemployment compensation, the lower the reservation wage that workers are willing to accept, which makes the mean-min ratio larger. A significant contribution of the model presented in this paper is that it generates a mean-min ratio between 2.18 and 2.07 when calibrated to a reasonable replacement ratio of $\rho \in [0.2, 0.6]$. The intuition behind this result is that the logic behind (5) does not apply in the model, because workers no longer face a common "wage offer distribution", but the potential distribution of wage offers depends on the current employment status of the worker. Wage dispersion primarily results from the heterogeneity of the latter.

3 The model

I present a general equilibrium model of a stationary economy with ex-ante homogeneous workers and heterogeneous firms, decentralized production and a frictional labor market with on the job search. There are no aggregate fluctuations in the model, and I will solve for the steady state.

Time is continuous. Production is decentralized into worker-machine pairs operating Leontief technologies. Machines are either *vacant* or *matched* with a worker. Each machine produces a constant productivity flow p when matched, and its production is normalized to 0 when vacant. There is constant in- and outflow of firms: new firms may be established for a setup cost, and firms die with firm deaths occurring as a Poisson process with constant and homogeneous rate δ.

Newly established firms start out as vacant. Firms and workers are risk-neutral and discount utility by the common discount rate r, and unemployed workers receive a flow utility z, which can be thought of as the sum of unemployment benefits and the utility derived from leisure.

When firms are established, they draw their productivity level from a distribution $F(p)$. In order to draw this productivity, the owner of the firm has to pay a fixed cost I which is a sunk cost and cannot be recovered. As we will see, some firms will never be able to hire a worker because their productivity is so low that it cannot offer a rent over when compared to unemployment. Without loss of generality, I assume that only firms with productivity above some threshold p^* are operated, the rest are immediately shut down, p^* is endogenous and will be characterized below.

The population of workers is a unit mass, with u unemployed and $1-u$ employed workers, where $1-u$ is also the number of matched firms. Workers search for vacancies both when they are unemployed and employed, but when they are working, their relative search efficiency is s compared to unemployed workers. Let

$$\hat{u} = u + (1-u)s$$

denote the effective search effort, and v the number of vacancies. A flow of *meetings*

$$m(v, \hat{u}) = Av^{\alpha}\hat{u}^{1-\alpha}$$

is generated between workers and vacancies. The decision to form a new match — and in the case of an employed worker, to break up the old one — is endogenous and will be discussed in detail below.

Matches are *separated* exogeneously, at rate σ, after which they are vacant, and as noted above, firms *die* at rate δ, which means that they shut down with a termination value of 0. Let

$$\Upsilon(p) = \text{the measure of } \textit{vacant} \text{ firms with productivity in } [p^*, p]$$
$$\Gamma(p) = \text{the measure of } \textit{matched} \text{ firms with productivity in } [p^*, p]$$

I will use $\bar{p} \in \mathbb{R} \cup \{\infty\}$ to denote the supremum of the support of p.[4] Then

$$\Gamma(\bar{p}) = 1 - u$$
$$\Upsilon(\bar{p}) = v$$

and $1 - u + v$ is the total number of firms. For algebraic convenience, I define

$$\mu = \frac{m(\hat{u}, v)}{\hat{u}v} \tag{6}$$

[4] I introduce this notation in order to avoid having to write limits when the support is infinite.

which allows us to write the following rates as

the rate	at which...
$\mu\Upsilon'(p)$	an unemployed worker meets a firm with productivity p
$\mu s\Upsilon'(p)$	an employed worker meets a firm with productivity p
μu	a firm meets an unemployed worker
$\mu s\Gamma'(p)$	a firm meets a worker employed at a firm with productivity p

3.1 Bargaining, wage determination and value functions

I will define the *(joint) surplus* as the sum of the excess value of the firm and the worker over their outside options, which are vacancy and unemployment, respectively. When a firm and a worker form a match, they agree to share the surplus according to a given ratio, by giving a share $\beta \in [0,1]$ to the worker. This contract is binding but can be unilaterally terminated by either the firm or the worker; however, in this case the match is dissolved and the worker-firm pair is separated, so it cannot be used as a credible threat *per se*. The bargaining and contract framework is similar to Postel-Vinay and Robin (2002), except that in this model, the contract state is the surplus share, not the wage.[5]

When an unemployed worker meets a firm, they engage in Nash bargaining about the surplus. I will denote the worker's bargaining power in this situation, which is an exogenous parameter in the model, as β_0.

However, when an employed worker meets another firm, the two firms Bertrand-compete for the worker in the worker's surplus. In this case, the worker also has the option of not accepting either offer and keeping his previous surplus share.

Thus the state variables of an employed worker and a matched firm are the firm's productivity p and the worker's surplus share β. This implicitly determines the wage

[5] Note however that there is an isomorphism between the surplus and the wage for a given level of productivity, so this is not a fundamental difference.

$w(p, \beta)$, which I calculate later.

Let $W(p, \beta)$, U, $J(p, \beta)$, $V(p)$ denote the value functions of an employed worker, unemployed worker, matched and vacant firm, respectively, with firm productivity p and surplus share β, where applicable. The surplus is defined as

$$S(p) = W(p, \beta) + J(p, \beta) - U - V(p) \tag{7}$$

and of course it is independent of β because the contract implies that

$$W(p, \beta) - U = \beta S(p) \tag{8}$$

$$J(p, \beta) - V(p) = (1 - \beta) S(p) \tag{9}$$

It is easy to verify that $S(p)$ is increasing in p.

Clearly, the surplus share of an unemployed worker who just found a job is $\beta = \beta_0$. In order to calculate the subsequent law of motion for the surplus share, we need to consider two cases.

Let (p, β) denote the state of a worker and firm when the worker meets another firm with productivity p'. Formally, in this game the firms submit a surplus share simultaneously. The equilibrium solution I discuss below is a Nash equilibrium, and the strategies of both firms presented below are also weakly dominant strategies.

If $p' > p$, then the new firm can always outbid the incumbent firm because $S(p') > S(p)$. In this case, the incumbent firm will offer *all* its surplus share, which is matched by an offer β' of the new firm which satisfies

$$\beta' S(p') = S(p)$$

which allows us to define

$$B(p', p) \equiv \frac{S(p)}{S(p')} \tag{10}$$

to be the new surplus share of the worker, where the domain of B is

$$\{(p', p) \mid 0 \leq p \leq p'\}$$

Here I introduce an implicit tiebreaking assumption that when receiving equally attractive offers, workers go to the firm with higher productivity, which is trilaterally efficient. Of course, this may also be motivated as the new firm willing to offer $\beta' + \varepsilon$, and having $\varepsilon \to 0$.

When $p' \leq p$, the incumbent firm is in a position to match any offer of the new firm. Then Bertrand competition ensures that the latter will offer all its surplus, and the incumbent will offer β' which solves

$$S(p') = \beta' S(p)$$

The worker will stay with the incumbent firm, but will only accept the new surplus share if dominates the current one, ie $\beta' > \beta$. Note that we can use the definition of B above here, as $B(p, p') = \beta'$ since now $p' \leq p$. Figure 2 illustrates a typical path of a worker in (p, β) space.

The HJB equation for the unemployed worker is

$$rU = z + \mu \int \big(W(p, \beta_0) - U\big) d\Upsilon(p) \tag{11}$$

which can be rewritten using (8) as

$$rU = z + \mu\beta_0 \int S(p) d\Upsilon(p) \tag{12}$$

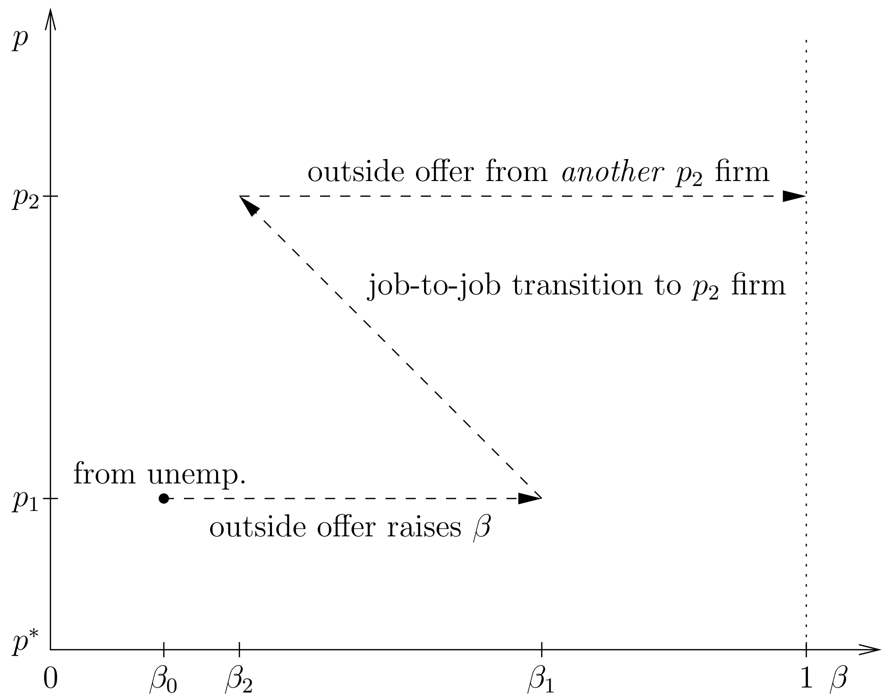

Figure 2: Typical path of worker in (p, β) space where p is productivity and β is the surplus share of the worker. Coming from unemployment, the worker finds a job with a p_1 firm, with surplus share β_0 as a result of Nash bargaining. Then an outside offer from a less productive firm raises β to β_1, but the worker stays with the same firm. Subsequently the worker meets a firm with productivity $p_2 > p_1$ and moves to the new firm, getting a lower share β_2 from the higher surplus $S(p_2)$ according to equation (10). Then the worker meets *another* p_2 firm, raising its β to 1 since it is able to extract all the surplus from two equivalent firms.

The value of the employed worker satisfies the HJB equation

$$\begin{aligned} rW(p,\beta) = w(p,\beta) + \mu s \int \Bigg(& \underbrace{[p' > p]\big(W(p', B(p',p)) - W(p,\beta)\big)}_{\text{worker is hired away}} \\ & + \underbrace{[p' \le p][B(p,p') > \beta]\big(W(p, B(p,p')) - W(p,\beta)\big)}_{\text{outside offer increases } \beta} \Bigg) d\Upsilon(p') \\ & + \underbrace{(\sigma+\delta)\big(U - W(p,\beta)\big)}_{\text{separation or firm death}} \end{aligned} \tag{13}$$

where I use the notation[6]

$$[\text{expression}] = \begin{cases} 1 & \text{if expression is true} \\ 0 & \text{otherwise} \end{cases}$$

The first term on the right hand side is the flow value of the wage. The second term is the integral over the distribution of vacancies, and covers two cases: when $p' > p$, the worker is hired away, otherwise he is not. In the latter case, the worker's surplus share will increase only if $B(p,p')$, the result of the Bertrand competition, is larger than β. Finally, if the firm dies (rate δ) or the match is exogenously separated (rate σ), the worker's value jumps to U.

The HJB equation for the value of a matched firm is

$$\begin{aligned} rJ(p,\beta) = p - w(p,\beta) + \mu s \int \Bigg(& \underbrace{[p' > p]\big(V(p) - J(p,\beta)\big)}_{\text{worker is hired away}} \\ & + \underbrace{[p' \leq p][B(p,p') > \beta]\big(J(p,B(p,p')) - J(p,\beta)\big)}_{\text{outside offer increases } \beta} \Bigg) d\Upsilon(p') \\ & + \underbrace{\sigma\big(V(p) - J(p,\beta)\big)}_{\text{separation}} + \underbrace{\delta\big(-J(p,\beta)\big)}_{\text{firm death}} \end{aligned} \tag{14}$$

and the terms mirror the ones in the worker's HJB equation (13), except that when the firm dies, its continuation value is 0.

Finally, the HJB equation for the value of a vacant firm is

$$rV(p) = \underbrace{\mu u\big(J(p,\beta_0) - V(p)\big)}_{\text{hiring from unemployment}} + \underbrace{\mu s \int [p' \leq p]\big(J(p,B(p,p')) - V(p)\big) d\Gamma(p')}_{\text{hiring from another firm}} + \underbrace{\delta\big(-V(p)\big)}_{\text{firm death}} \tag{15}$$

[6] A similar notation was introduced by Kenneth E. Iverson in the 1960s — see Knuth (1992) for a discussion.

Using (9), (15) is simplified to

$$rV(p) = \mu u(1-\beta_0)S(p) + \mu s\int_{p^*}^{p}\big(S(p)-S(p')\big)d\Gamma(p') - \delta V(p) \tag{16}$$

Also, when we add (13) and (14), the terms in the integral drop out because Bertrand competition ensures that when the worker is hired away,

$$\underbrace{W\big(p', B(p',p)\big)}_{S(p)+U \text{ from } (10)} \underbrace{-W(p,\beta)+V(p)-J(p,\beta)}_{-(S(p)+U) \text{ from } (7)} = 0$$

and since there is no change in the surplus in case the worker remains at its current employer, for outside offers that raise β we have

$$W\big(p, B(p,p')\big) - W(p,\beta) + J\big(p, B(p,p')\big) - J(p,\beta) = 0$$

After adding a (13) and (14) and subtracting (11) and (16) we arrive at

$$\begin{aligned}(r+\sigma+\delta)S(p) = p \; &\underbrace{-z-\mu\beta_0\int S(p)d\Upsilon(p)}_{\text{opportunity cost of unemployment}} \\ &\underbrace{-\mu u(1-\beta_0)S(p) - \mu s\int_{p^*}^{p}\big(S(p)-S(p')\big)d\Gamma(p')}_{\text{opportunity cost of an occupied vacancy}}\end{aligned} \tag{17}$$

Note that from (16), $V(p) > 0$ whenever $S(p) > 0$, and also $V(p) = 0$ if $S(p) = 0$. This implies that p^*, the lowest feasible productivity level, is the point where $S(p^*) = 0$. Whenever a firm draws a $p \geq p^*$ upon establishment, it will advertise the vacancy and the firm will be kept in operation (as either vacant or matched) until it dies exogenously.

Also, because $S(p^*) = 0$, it follows from (17) that

$$p^* = \underbrace{z + \mu\beta_0 \int S(p) d\Upsilon(p)}_{\text{flow value of unemployment}} = rU \tag{18}$$

3.2 The wage function

The wage is determined implicitly in (13). For notational convenience, I define $P(p, \beta)$ implicitly as the solution to

$$S\big(P(p, \beta)\big) = \beta S(p) \tag{19}$$

Intuitively, $P(p, \beta)$ is the highest productivity which would result in an outside offer raising β (and thus the wage). Since S is strictly increasing, $P(p, \beta) < p$ for $\beta < 1$, also $P(p, 0) = 0$ and $P(p, 1) = p$. Also, P is strictly increasing in p because S is.

Rearrange (13) as

$$
\begin{aligned}
w(p,\beta) =& rW(p,\beta) - \mu s \int \Bigg(\underbrace{[p' > p]\big(W(p', B(p',p)) - W(p,\beta)\big)}_{\text{worker is hired away}} \\
& - \underbrace{[p' \le p][B(p,p') > \beta]\big(W(p, B(p,p')) - W(p,\beta)\big)}_{\text{outside offer increases } \beta} \Bigg) d\Upsilon(p') \\
& + \underbrace{(\sigma+\delta)\big(U - W(p,\beta)\big)}_{\text{separation or firm death}} \\
=& r\beta S(p) + rU - \mu s \int_p^{\bar{p}} \big(S(p) - \beta S(p)\big) d\Upsilon(p') \\
& - \mu s \int_{P(p,\beta)}^{p} \big(S(p') - \beta S(p)\big) d\Upsilon(p') + (\sigma+\delta)\beta S(p) \\
=& p^* + (r+\delta+\sigma)\beta S(p) - \underbrace{\mu s S(p)(1-\beta)\big(v - \Upsilon(p)\big)}_{\text{worker is hired away}} \\
& - \underbrace{\mu s \left(\int_{P(p,\beta)}^{p} S(p') d\Upsilon(p') - \beta S(p)\big(\Upsilon(p) - \Upsilon(P(p,\beta))\big) \right)}_{\text{outside offer increases } \beta}
\end{aligned}
\tag{20}
$$

where I have used (8) and (18).

This equation demonstrates the forces that determine the wage. The first term, p^*, the the wage a worker would earn if he was employed at the least productive firm. This is because for this productivity level the joint surplus is zero, and thus this is the only wage that is possible. The second term is the share β of the flow value $(r+\delta+\sigma)S(p)$, where the separation rate is added to the interest rate. Naturally, this part is increasing in both β and p.

The third term is the negative of the expected jump in value when the worker is hired away. While $S(p)$, the basis for the bargaining position, is increasing in p, the measure of vacancies which are better than p, $v - \Upsilon(p)$ is decreasing in p, making $S(p)\big(v - \Upsilon(p)\big)$ a hump-shaped function that is 0 both at $p = p^*$ and $p = \bar{p}$.

Finally, the fourth term is the negative of the jump in value when an outside offer results in an increase in β. The larger p is, the higher fraction of outside offers will be below it, which leads to a higher expected increase in the wage.

3.3 Free entry

Firms who enter the market draw a p from the distribution F and pay the cost I. The free entry condition states that the expected value of the former should equal the latter, ie

$$I = \int [p \geq p^*] V(p) dF(p) \tag{21}$$

The inflow of new establishments (not all of which necessarily remain in operation) will be denoted by ϕ. Note that since the truncated tail of F below p^* is never observed, it is impossible to identify ϕ and F separately because we can always achieve observationally equivalent transformations (for example, more firms concentrated above p^* in F, proportionally lower ϕ, proportionally higher I). Because of this, I introduce the normalized notation

$$F^*(p) = \begin{cases} \frac{F(p)-F(p^*)}{1-F(p^*)} & \text{if } p \geq p^* \\ 0 & \text{otherwise} \end{cases} \tag{22}$$

$$I^* = I \cdot \big(1 - F(p^*)\big) \tag{23}$$

$$\phi^* = \phi \cdot \big(1 - F(p^*)\big) \tag{24}$$

Then (21) can be rewritten as

$$I^* = \int V(p) dF^*(p) \tag{25}$$

and the inflow of *new firms that are kept in operation* is ϕ^*. Intuitively, I^* is the cost of a new firm with productivity above p^* and F^* is the distribution of the inflow of firms which are kept as vacancies. Also note that by construction,

$$\phi F(p) = \phi^* F^*(p)$$

for all p.

3.4 Distributions and flows

Since the meeting of an unemployed worker and vacant firm always results in a match and workers who are searching on the job will always move to the more productive firm, the movement of workers across firms and unemployment do not depend on the value functions. Then the equilibrium of outflows and inflows from and to matches in $[p^*, p]$ is characterized by

$$\underbrace{(\sigma+\delta)\Gamma(p)}_{\text{separations and firm deaths}} + \underbrace{\mu s\Gamma(p)(v-\Upsilon(p))}_{\text{workers hired away}} = \underbrace{\mu u\Upsilon(p)}_{\text{hiring unemployed}} \tag{26}$$

We can write a similar equation for the outflows and inflows of vacancies in $[p^*, p]$,

$$\underbrace{\delta\Upsilon(p)}_{\text{firm deaths}} + \underbrace{\mu u\Upsilon(p)}_{\text{hiring unemployed}} = \underbrace{\phi^* F^*(p)}_{\text{new firms}} + \underbrace{\sigma\Gamma(p)}_{\text{separations}} + \underbrace{\mu s\Gamma(p)(v-\Upsilon(p))}_{\text{workers hired away}} \tag{27}$$

where I have used the notation introduced in Section 3.3. Adding (26) and (27) we obtain

$$\underbrace{\delta\big(\Gamma(p)+\Upsilon(p)\big)}_{\text{total firm deaths}} = \underbrace{\phi^* F^*(p)}_{\text{new firms}} \tag{28}$$

From (26),

$$\Upsilon(p) = \frac{(\sigma+\delta+\mu s v)\Gamma(p)}{\mu u + \mu s\Gamma(p)} \tag{29}$$

Also,

$$\Upsilon'(p) = \frac{(\sigma + \delta + \mu s v)\mu s}{\big(\mu u + \mu s \Gamma(p)\big)^2}\Gamma'(p) \tag{30}$$

Equation (30) implies that the density of vacancies is the density of matches multiplied by a term that is decreasing in p. Intuitively, this means that vacancies are more concentrated around the low productivity levels compared to matches, which is quite natural considering that job-to-job transitions always flow from low to high productivities.

We can express the match distribution Γ in terms of F by substituting (29) into (28), which yields

$$\Gamma(p) + \frac{(\sigma + \delta + \mu s v)\Gamma(p)}{\mu u + \mu s \Gamma(p)} = \frac{\phi^*}{\delta} F^*(p)$$

which can be rearranged as

$$\mu s \Gamma^2(p) + \left(\mu u + \sigma + \delta + \mu s v - \mu s \frac{\phi^*}{\delta} F^*(p)\right)\Gamma(p) = \mu u \frac{\phi^*}{\delta} F^*(p) \tag{31}$$

which yields a unique solution $\Gamma(p)$ at each p, given that $F(p) \geq 0$ and $\Gamma(p)$ has to be nonnegative.[7]

[7] At $p = p^*$, the only non-negative solution to (31) is $\Gamma(p^*) = 0$. For $p > p^*$, $F(p) > 0$, then using the quadratic formula, $\Gamma(p)$ solves

$$a\Gamma^2(p) + b\Gamma(p) + c = 0 \qquad \Rightarrow \qquad \Gamma(p)_{12} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

with

$$a = \mu s > 0 \qquad b = \mu u + \sigma + \delta + \mu s v - \mu s \frac{\phi^*}{\delta} F^*(p) \qquad c = -\mu u \frac{\phi^*}{\delta} F^*(p) < 0$$

so $\sqrt{b^2 - 4ac} > |b|$ which means that roots have opposite signs, and thus there is a unique $\Gamma(p) > 0$.

The fact that $\Gamma(p)$ is increasing, which makes it a proper distribution function, can be demonstrated by differentiating (28) and using (30).

At $p = \bar{p}$, (26) and (27) become

$$(\sigma + \delta)(1 - u) = \mu uv \tag{32}$$

$$\delta v + \mu uv = \phi^* + \sigma(1 - u) \tag{33}$$

which, when combined, yield

$$\underbrace{\delta(1 - u + v)}_{\text{firm deaths}} = \underbrace{\phi^*}_{\text{new firms}} \tag{34}$$

3.5 Stationary equilibrium

A stationary equilibrium of the model is a collection of unemployment and vacancy measures u and v, a productivity threshold p^*, an inflow of new firms ϕ, distribution functions for matches and vacancies $\Gamma(p)$ and $\Upsilon(p)$ and value and surplus functions U, $V(p)$, $W(p, \beta)$, $J(p, \beta)$ and $S(p)$ such that

1. Given u, v (and thus implicitly μ), p^* and the distributions $\Gamma(p)$ and $\Upsilon(p)$ the value functions satisfy the HJB equations (8), (9), (11), (13), (14), (15) and (17).

2. Given p^* and S, the smooth pasting conditions $V(p^*) = S(p^*) = 0$ hold.

3. Given $V(p)$ and p^*, the free entry condition (21) holds.

4. The flow equilibrium conditions (26) and (27) hold, also, $\Gamma(\bar{p}) = 1 - u$ and $\Upsilon(\bar{p}) = v$.

4 Model solution

While the stationary equilibrium characterized in Section 3.5 contains many objects, it can be characterized parsimoniously in just two parameters, the normalized inflow rate ϕ^* and the smallest firm productivity value p^*.

With a pair of (ϕ^*, p^*) we can solve the nonlinear system (32) and (33) for the two unknowns u and v — recall that μ is just a function of these two, as defined by (6). Given p^*, we can calculate $F^*(p)$ from $F(p)$ (see (22)), then calculate $\Gamma(p)$ and $\Upsilon(p)$ analytically at each p using (31) and (29).

Given the distributions, it is easy to solve for $S(p)$ numerically using (17) (the solution method is described in the Appendix). Thus the solution algorithm proceeds by searching for the pair (ϕ^*, p^*) so that the resulting value functions satisfy the free entry condition (21) and the boundary condition (18).

5 Calibration

Table 5 summarizes the parameters of the model. The calibration proceeds in three stages. First, I fix the value of r, α and A using standard values from the literature and a normalization. Then, in Section 5.1, I extract the transition rates between employment and unemployment states (including the job-to-job rate) using data from Fallick and Fleischman (2004) with a procedure that eliminates the time aggregation bias, and use this to calibrate of s, σ and δ. Finally, I choose the unemployment bargaining power parameter β_0 to match the replacement ratio.

The setup cost I for new firms is chosen to match the number of vacancies v implicit in the transition rates λ_{UE}, λ_{EE} and λ_{EU}. Because for the parametric form chosen for the productivity of matches Γ is the Pareto distribution, the productivity levels are scale invariant as discussed in detail in Section 5.2, so I set the unemployment benefit z to match the normalization $p^* = 1$.

The time unit of the model is a year. I set the interest rate to 4%. For the meetings function

$$m(v, \hat{u}) = Av^{\alpha}\hat{u}^{1-\alpha}$$

parameter	meaning	calibration
α	elasticity of meetings function in v	0.5, Petrongolo and Pissarides (2001)
A	scale parameter in matching function	1, normalization
r	real interest rate	0.04, Cooley (1995)
δ	firm death rate	10% from Baldwin et al. (1998)
σ	exogenous separation rate	$\sigma = \lambda_{\mathrm{EU}} - \delta$
s	relative search efficiency of employed	match λ_{EU}, λ_{UE} and λ_{EE}
$F(p)$	productivity distribution for startups	match parametrized productivity distribution of matches $\Gamma(p)$, $\mathrm{sd}(\log p) = 0.66$ from Bernard et al. (2000)
β_0	bargaining power of unemployed	avg replacement ratio of 40%
I	setup cost for new firms	match v implicit in transition rates
z	unemployment utility flow	normalization, to $p^* = 1$

Table 1: Model parameters (time unit=year).

we need to fix the value of α and A. In their survey of matching functions, Petrongolo and Pissarides (2001, Table 3) report various estimates for the elasticity of $m(v, \hat{u})$ in v that corresponds to α in this model, from which I chose the plausible mean of $\alpha = 0.5$.

Note that if, for a given A, v is an equilibrium solution of the model, then so is $\tilde{v} = Cv$ for $\tilde{A} = C^{-\alpha}A$ for any positive C — and suitable changes in $F(p)$ and I as this transformation would leave $\mu V(p)$ (and thus μv) unchanged for any $p \in [p^*, \bar{p}]$. Because of this, I will normalize A to unity:

$$A = 1 \tag{35}$$

The scale of A does affect the calibration of I and $F(p)$ because it scales the total number of vacancies as described above. However, the value of A does not make a difference in the results of the paper concerning wages, so this normalization is innocuous. Nevertheless I will not report calibrated values of I of $F(p)$ because they are dependent on the normalization.

5.1 The transition rates

The availability of micro-level panel datasets has resulted in studies (for example, Nagypál (2008)) which have refined out understanding of job-to-job transitions considerably. However, naive inference of transition rates from cross-tabulations using data of monthly or even lower frequencies is made difficult by the *time aggregation bias*: it is possible that we overlook transitions between observations. For example, if we observe a worker at one employer and then at another employer a month later, this might be a job-to-job transition, or an employment-unemployment and an unemployment-employment transition.

In order to eliminate the time aggregation bias, I use a procedure that accounts for the evolution of the conditional distributions of a Markov chain using forward equations, similarly to Shimer (2007).

Let λ_{EE}, λ_{EU}, λ_{UE}, λ_{UN}, λ_{EN}, λ_{NE}, and λ_{NU} denote the transition rates between unemployment (U), employment (E) and being outside the labor force (N). The transition rate matrix ("Q-matrix") for this process can be represented as shown below with the states ordered as E, E', U, and N and the diagonal set so that elements in a row add up to 0. The state E' means that the worker is with a different employer than he started with if he was employed at the beginning of the period. I also introduce the convention that workers from states U and N also transit to E' without loss of generality. Then, with states ordered as E, E', U, and N, the

Q-matrix looks like

$$Q = \begin{bmatrix} \cdot & \lambda_{\mathrm{EE}} & \lambda_{\mathrm{EU}} & \lambda_{\mathrm{EN}} \\ 0 & \cdot & \lambda_{\mathrm{EU}} & \lambda_{\mathrm{EN}} \\ 0 & \lambda_{\mathrm{UE}} & \cdot & \lambda_{\mathrm{UN}} \\ 0 & \lambda_{\mathrm{NE}} & \lambda_{\mathrm{NU}} & \cdot \end{bmatrix} \tag{36}$$

The evolution of transition probabilities can be described by the differential equation

$$\dot{\pi}(t) = Q^T \pi(t) \tag{37}$$

with the initial condition

$$\pi(0) = \pi_0 \tag{38}$$

where π_0 is one of the unit vectors corresponding to the initial state E, U or N. Equation (37) has the solution

$$\pi(t) = \exp\big(Q^T t\big)\pi_0$$

I solve for the λ's which match Table 2 of Fallick and Fleischman (2004) numerically, with the results shown in Table 2. Since the nonlinear equation constructed with the help of the matrix exponential is strictly increasing in its arguments, the solution is unique.

rate	value
λ_{EE}	0.278
λ_{EU}	0.213
λ_{EN}	0.314
λ_{UE}	4.845
λ_{UN}	3.963
λ_{NE}	0.542
λ_{NU}	0.417

Table 2: Calibrated transition rates — E is employment, U is unemployment, and N means that the worker is outside the labor force.

Since the model does not have workers who are outside the labor force, only the rates λ_{EE}, λ_{EU}, and λ_{UE} are used in the calibration. I will now calculate s and the values of unemployment u and v so that the calibrated model matches the data above.

The separation rate for employees is simply

$$\lambda_{\text{EU}} = \sigma + \delta \tag{39}$$

The job finding rate for unemployed is

$$\lambda_{\text{UE}} = \mu v \tag{40}$$

Then from (32), we can relate λ_{EU} and λ_{UE} to unemployment as

$$u = \frac{1}{1 + \lambda_{\text{UE}}/\lambda_{\text{EU}}} = 0.042 \tag{41}$$

The average job-to-job transition rate is

$$\lambda_{\text{EE}} = \frac{\mu s}{1-u} \int_{p^*}^{\bar{p}} \big(v - \Upsilon(p)\big) d\Gamma(p) \tag{42}$$

From (26) and (29)

$$\mu s\big(v - \Upsilon(p)\big) = \mu u \frac{\Upsilon(p)}{\Gamma(p)} - \delta - \sigma = \mu u \frac{\lambda_{\text{EU}} + \mu s v}{\mu u + \mu s \Gamma(p)} - \lambda_{\text{EU}} = \lambda_{\text{EU}} \frac{1 + \frac{1-u}{u} s}{1 + \frac{s}{u}\Gamma(p)} - \lambda_{\text{EU}}$$

where I have used (32) and (39). Then by combining the above with (42) we obtain

$$\begin{aligned}
\lambda_{\mathrm{EE}} &= -\lambda_{\mathrm{EU}} + \lambda_{\mathrm{EU}} \frac{1+\frac{1-u}{u}s}{1-u} \int_{p^*}^{\bar{p}} \frac{1}{1+\frac{s}{u}\Gamma(p)} d\Gamma(p) \\
&= -\lambda_{\mathrm{EU}} + \lambda_{\mathrm{EU}} \frac{1+\frac{1-u}{u}s}{1-u} \int_{0}^{1-u} \frac{1}{1+\frac{s}{u}x} dx \\
&= -\lambda_{\mathrm{EU}} + \lambda_{\mathrm{EU}} \frac{1+\frac{1-u}{u}s}{\frac{1-u}{u}s} \log\left(1+\frac{1-u}{u}s\right) \\
&= \lambda_{\mathrm{EU}} \left(\frac{1+y}{y}\log(1+y) - 1\right) \qquad \text{with} \quad y \equiv \frac{1-u}{u}s = \frac{\lambda_{\mathrm{UE}}}{\lambda_{\mathrm{EU}}}s
\end{aligned} \tag{43}$$

Define

$$f(y) = \left(\frac{1}{y}+1\right)\log(1+y) - 1$$

which is independent of the model parameters. It is easy to show that

$$\begin{aligned}
f'(y) &= \frac{1}{y}\left(1 - \frac{\log(1+y)}{y}\right) > 0 \qquad \text{for all } y > 0 \\
\lim_{y\to 0} f(y) &= -1 + \lim(1+y)\frac{\log(1+y)}{y} = 0 \\
\lim_{y\to\infty} f(y) &= \infty
\end{aligned}$$

which ensures that there is a unique y that satisfies (43), which can be found numerically as shown in Figure 3.

$$\frac{\lambda_{\mathrm{EE}}}{\lambda_{\mathrm{EU}}} = 1.30 = f(y) \Rightarrow y = 6.31$$

Using the value of u derived above, this implies that $s = 0.28$, that is, employed workers encounter between 1/3–1/4 of the firms that unemployed workers meet.

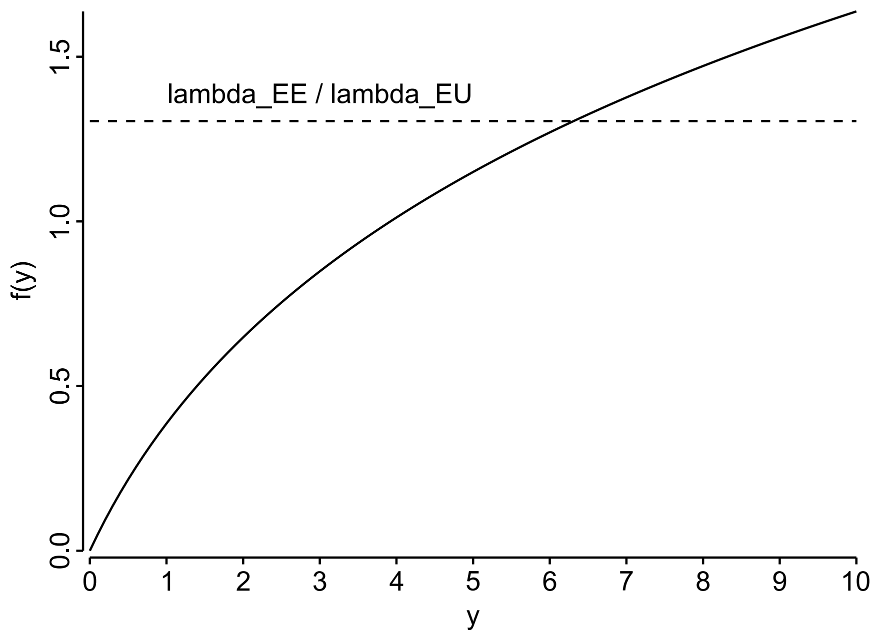

Figure 3: Numerical solution for $y = s(1-u)/u$, calibrated to match $\lambda_{\mathrm{EE}}/\lambda_{\mathrm{EU}}$ (dashed line), the ratio of the job-to-job transition rate and the separation rate.

5.2 The distribution F and bargaining power β_0

Recall that given the model parameters and the transition rates, there is a unique pointwise mapping between the entry productivity distribution F, the match distribution Γ and the vacancy distribution Υ, which is derived in Section 3.4. I will calibrate the model assuming a particular parametric form for the match distribution Γ, which then determines Υ by (29) and then F by (28).

I parametrize Γ, the distribution of matches as a Pareto(p^*, k) distribution[8] with cumulative distribution function

$$\frac{\Gamma(p; p^*, k)}{1-u} = 1 - \left(\frac{p}{p^*}\right)^{-k} \qquad \text{for } p \geq p^* \tag{44}$$

[8]Pareto distributions for firm productivity are common in the trade literature, for example in Melitz and Ottaviano (2008). Examining 11 EU countries and 18 manufacturing sectors, Del Gatto et al. (2006) find that the Pareto distribution provides a very good fit for productivity distributions.

where the normalization by $1 - u$ is necessary to make the distribution proper. It is well known that the logarithm of a random variable with Pareto distribution is exponential, in our case with parameter k. Then it follows that $\log p$ has variance $1/k$ regardless of p^*, which I calibrate to

$$\frac{1}{k} = 0.66 \qquad \Rightarrow \qquad k = 1.52$$

using the standard deviation of log labor productivity from Bernard et al. (2000, Table 1).[9]

Recall that (30) states that

$$\Upsilon'(p) = \frac{(\sigma + \delta + \mu s v)\mu s}{\big(\mu u + \mu s \Gamma(p)\big)^2} \Gamma'(p)$$

which implies that while $\Upsilon(p)$ is not a Pareto distribution, its right tail displays the same asymptotic power law behavior as the right tail of $\Gamma(p)$, with the same power coefficient because $\lim_{p\to\infty} \Gamma(p) = 1 - u$. Also, since (28) states that

$$\delta\big(\Gamma(p) + \Upsilon(p)\big) = \phi^* F^*(p)$$

this means that the tail of F also follows a power law with the same coefficient as Υ and Γ.

Since the shape of the Pareto distribution is independent of the scale parameter p^*, the model displays scale invariance in production: if value and distribution functions are the solution to the model with a particular z parameter, for another $\tilde{z} = Az$ and a rescaled $\tilde{F}$ we can obtain the new solution by rescaling the previous

[9] With industry fixed effects removed since Hornstein et al. (2007) also control for occupation when calculating the mean-min ratio for various datasets. Without the removal of industry fixed effects, the value of $1/k$ would be 0.76. Using this larger standard deviation in the calibration leads to a slightly larger wage dispersion.

one in p. A sketch of a formal proof of this property is contained in the Appendix. Therefore we can normalize the value of z (and p^*), and for algebraic simplicity I choose the normalization $p^* = 1$ so that the distribution of matches is

$$\frac{\Gamma(p)}{1-u} = 1 - p^{-k} \qquad \text{for } p \geq 1$$

Figure 4 shows the calibrated density function $\Gamma'(p)$ of matches.

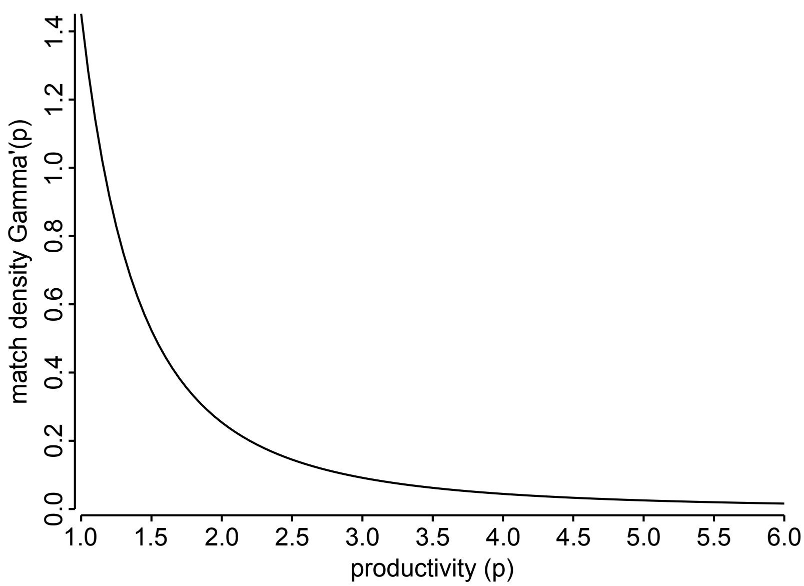

Figure 4: Density of matches $\Gamma'(p)$ in the calibrated model.

The only parameter we have left to calibrate is β_0. I solve for β_0 numerically to match the average *replacement ratio*, which is defined as

$$\rho = \frac{\text{unemployment benefit}}{\text{average wage}} = \frac{z}{\mathrm{E}[w]}$$

I will follow Shimer (2005) in calibrating to $\rho = 0.4$, and perform a sensitivity analysis in Section 6.2.

6 Results

Table 3 below displays some statistics of the calibrated model. The model appears to be successful in generating the large wage dispersion observed in the data: the mean-min ratio is very close to 2. Figure 5 shows the stationary wage distribution of

ρ	0.4
β_0	0.239
w_5	0.522
w_{25}	0.801
w_{50}	0.986
w_{75}	1.257
w_{95}	2.047
$\mathrm{E}[w]/w_5$	2.118

Table 3: Wages in the calibrated model. The mean-min ration $\mathrm{E}[w]/w_5$ is ≈ 2, matching the data.

the calibrated model. The shape of the wage distribution if unimodal and it displays fat tails, which results from the fat tails of the Pareto distribution of productivity.

In order to understand the source of the wage dispersion better, it is useful to decompose the variance of wages. Let $G(\beta|p)$ denote the stationary distribution of surplus shares conditional on the firm's productivity level. To make the distribution of matches proper, I will introduce

$$\tilde{\Gamma}(p) = \frac{\Gamma(p)}{1-u}$$

For the expected values below, I drop the arguments of w. Then

$$\mathrm{Var}(w) = \int_{p^*}^{\bar{p}} \int_0^1 \big(w(p,\beta) - E[w]\big)^2 dG(\beta|p) d\tilde{\Gamma}(p) \tag{45}$$

$$= \underbrace{\int_{p^*}^{\bar{p}} \big(E[w|p] - E[w]\big)^2 d\tilde{\Gamma}(p)}_{\text{variance from productivity dispersion}} + \underbrace{\int_{p^*}^{\bar{p}} \int_0^1 \big(w(p,\beta) - E[w|p]\big)^2 dG(\beta|p) d\tilde{\Gamma}(p)}_{\text{variance from dispersion of } \beta} \tag{46}$$

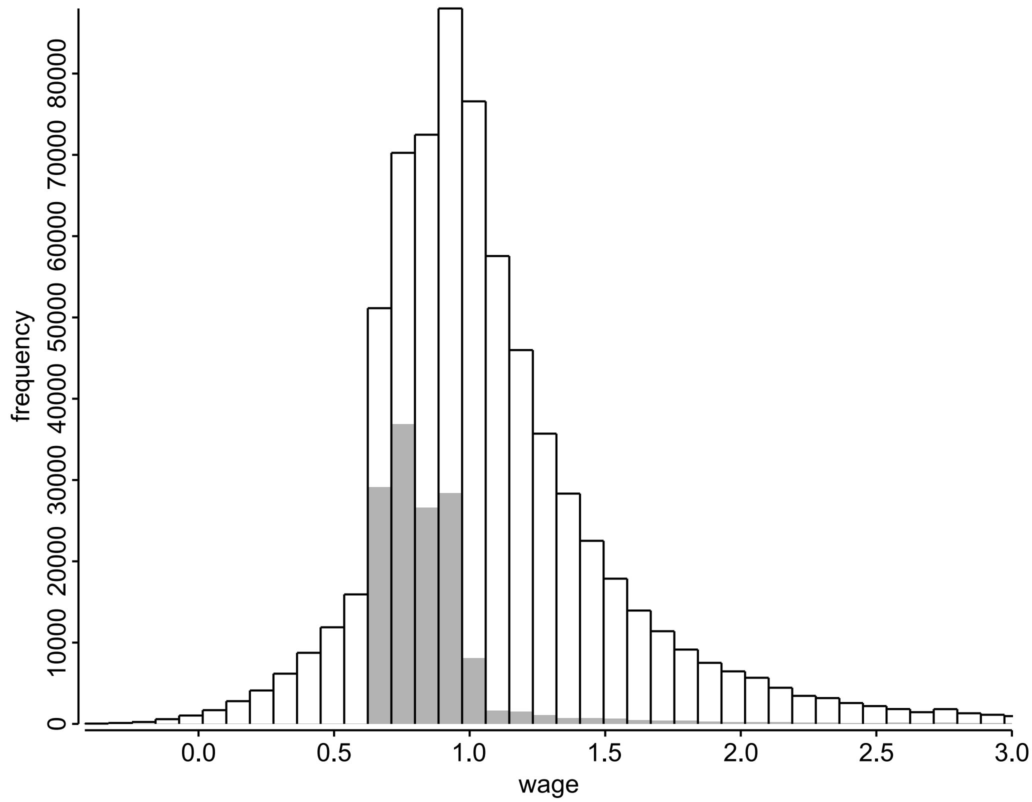

Figure 5: Wage distribution in the calibrated model. Grey columns: workers coming from unemployment.

Calculating the variance shares numerically, I find that the dispersion of βs explains 78% of the variance of wages. Figure 6 shows $\mathrm{E}[w|p]$. Clearly, it is this feature of the model which accounts for its success in explaining the large frictional wage dispersion. In order to understand this feature better, we need to look at the decomposition of $w(p, \beta)$ introduced in Section 3.2.

Figure 7 shows the wage $w(p, \beta)$ as a function of productivity and surplus share in the model. At the lowest productivity level p^*, only one wage is possible since the joint surplus is zero here so β does not matter. However, as p increases, how the surplus is shared plays a greater role in the wage.

In order to understand the cause of this dispersion, recall the wage decomposition

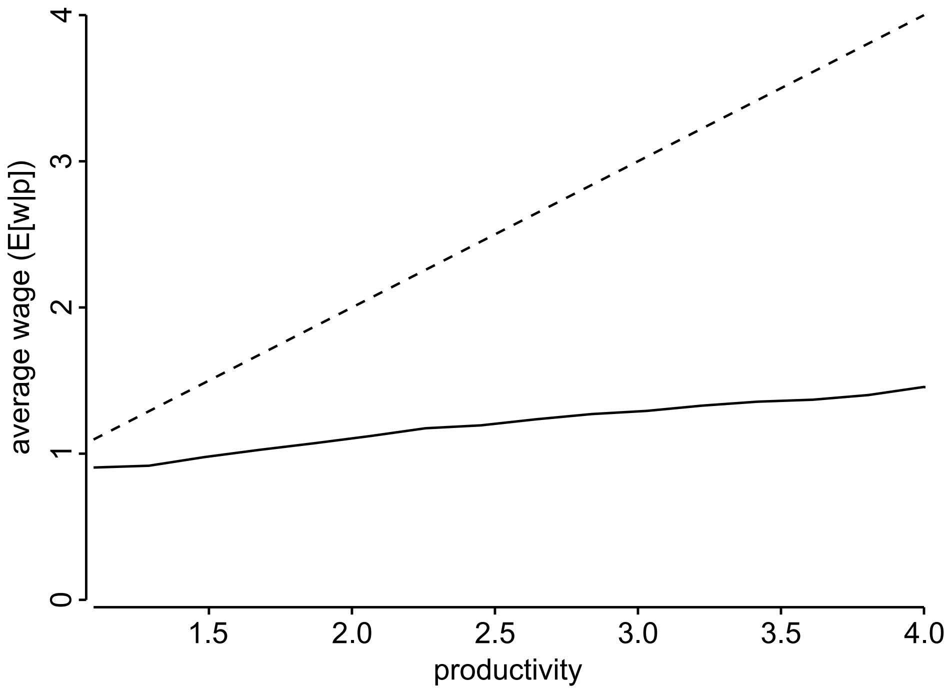

Figure 6: Expected wage given productivity E$[w|p]$. Dashed: 45° line.

equation 20 from Section 3.2:

$$w(p,\beta) = p^* + \underbrace{(r+\delta+\sigma)\beta S(p)}_{\text{flow from surplus share}} - \underbrace{\mu s S(p)(1-\beta)\big(v - \Upsilon(p)\big)}_{\text{worker is hired away}}$$
$$- \underbrace{\mu s \left(\int_{P(p,\beta)}^{p} S(p')d\Upsilon(p') - \beta S(p)\big(\Upsilon(p) - \Upsilon(P(p,\beta))\big) \right)}_{\text{outside offer increases } \beta}$$

The first term in this equation is p^*, which corresponds to the opportunity cost of unemployment as shown in (18). The second term is the flow value of surplus share of the worker, and is shown in Figure 8 — since S is increasing in p, this term is increasing in both p and β.

Figure 9 shows the contribution of the prospect of being hired away to the wage. The prospect of job-to-job transitions leads to the largest gains in value for the workers who currently have a low β, since those who have a β close to 1 already

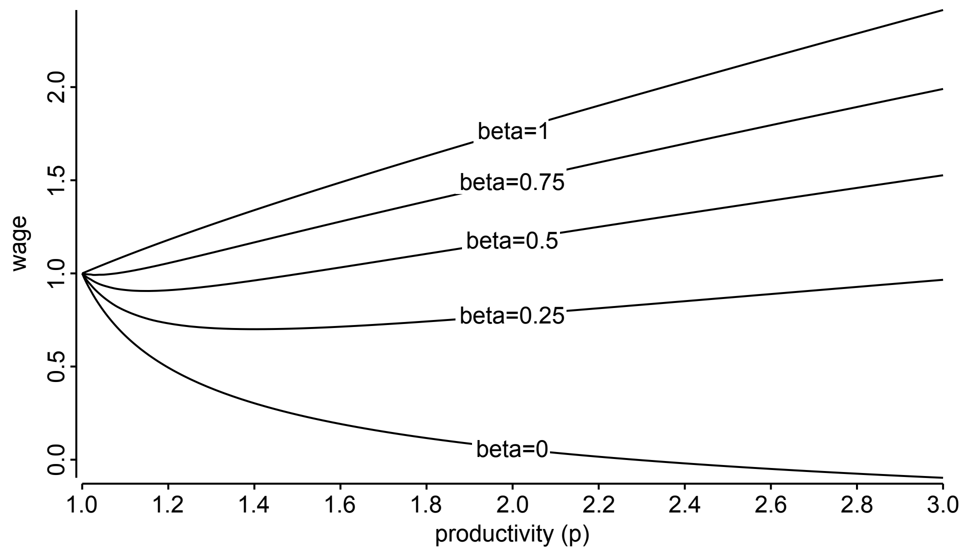

Figure 7: Wage as a function of productivity (p) and surplus share (β)

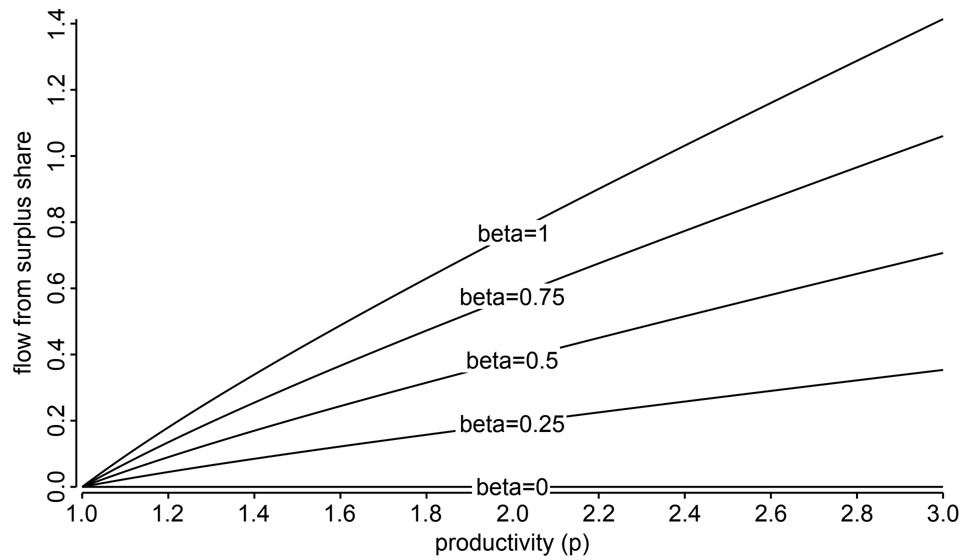

Figure 8: Wage decomposition: flow from surplus share.

obtain most of the surplus from their employer; thus the continuation surplus of $S(p)$ is only marginally better for them than their current surplus is. Since workers with more productive firm have a lower chance of meeting better firms, the contribution of this term gets smaller as p is increasing, but this effect is really slow because of the fat tails of the match distribution.

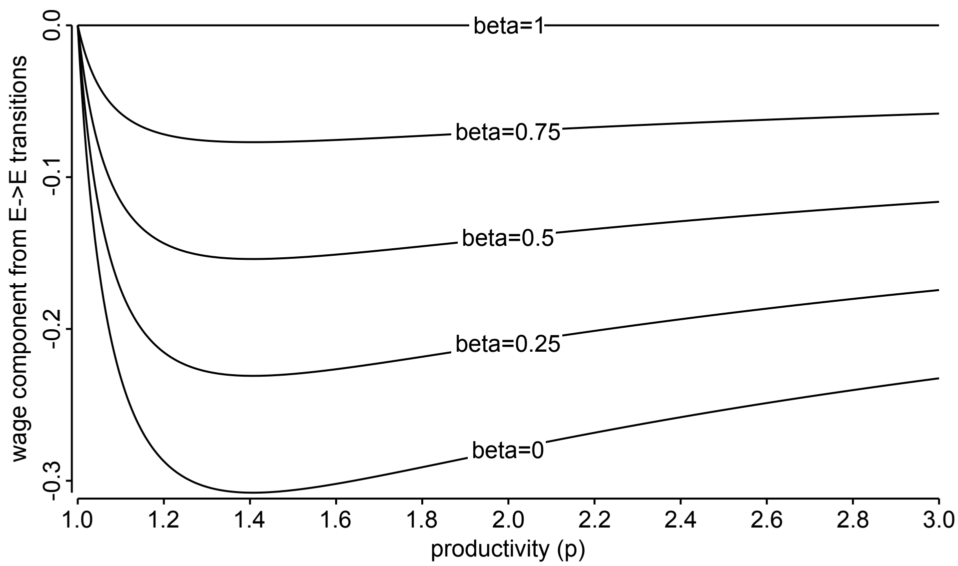

Figure 9: Wage decomposition: contribution of the prospect of being hired away by another firm.

The prospect of outside offers that increase the surplus share β, shown in Figure 10 is even more significant, as this is increasing in p. Thus workers at a firm with high p but with low β expect large increases in their surplus share, which means that they will receive a low wage for the present. When we compare the contribution of p and β to the wage, it is clear that the latter is much more significant: most of the wage dispersion is a result of the bargaining structure and the resulting dispersion in the workers' surplus share β.

Because of the latter two components (transfers and outside offers that increase β), we observe negative wages: workers at firms with high productivities are willing to accept negative wages because they expect that outside offers will either result in a transfer and then the high productivity of the incumbent firm will result in a good bargaining position, or because they know that outside offers that do not result in transfers can nevertheless raise their value significantly.

Figure 11 shows that the surplus shares of workers is evenly distributed for the

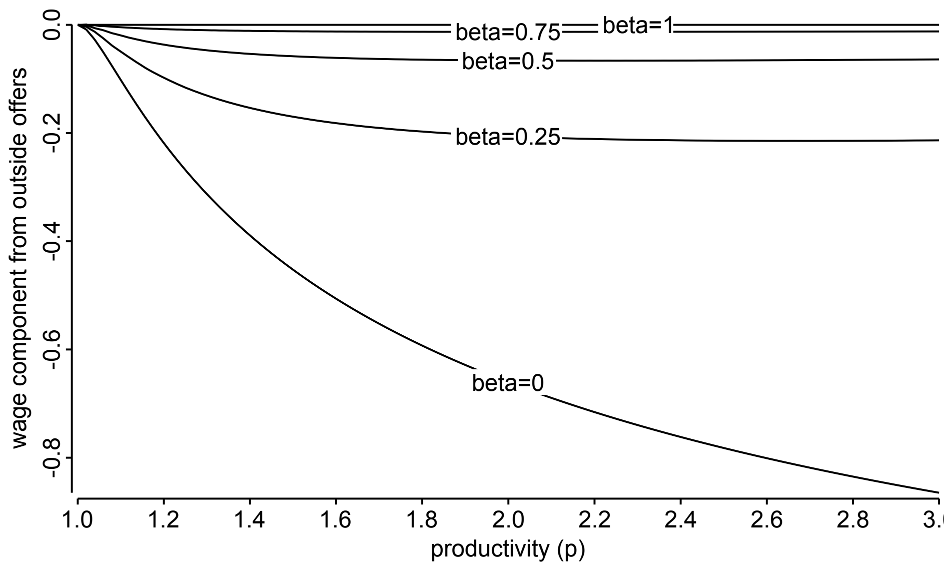

Figure 10: Wage decomposition: contribution of outside offers that raise β

workers who were working at another firm before landing at their current one. Of course, the surplus share of the workers coming from unemployment is exogenously determined by the parameter β_0.

Figure 12 shows the ergodic distribution of workers' productivities and wages. Since the distributions are concentrated around p^*, most workers end up around here, but since the tails of the Pareto distributions are quite fat, we observe quite a few workers above this. The wage-productivity pairs are fanning out to fill the envelope defined by $w(p, \beta)$ at $\beta = 0$ and $\beta = 1$, but of course workers are more concentrated at the higher βs. Also notice the line that corresponds to the mass of workers coming from unemployment who start with surplus share β_0.

Finally, Table 4 shows some regressions[10] of wage on productivity and/or tenure (time spent at the firm which currently employs the worker) as summary statistics. As we would expect from the analysis above, the explanatory power of productivity

[10]OLS regressions on artificially generated data. I do not report standard deviations or other error statistics of coefficients since with artificially generated data; these can be driven to zero asymptotically and thus have little meaning in this context.

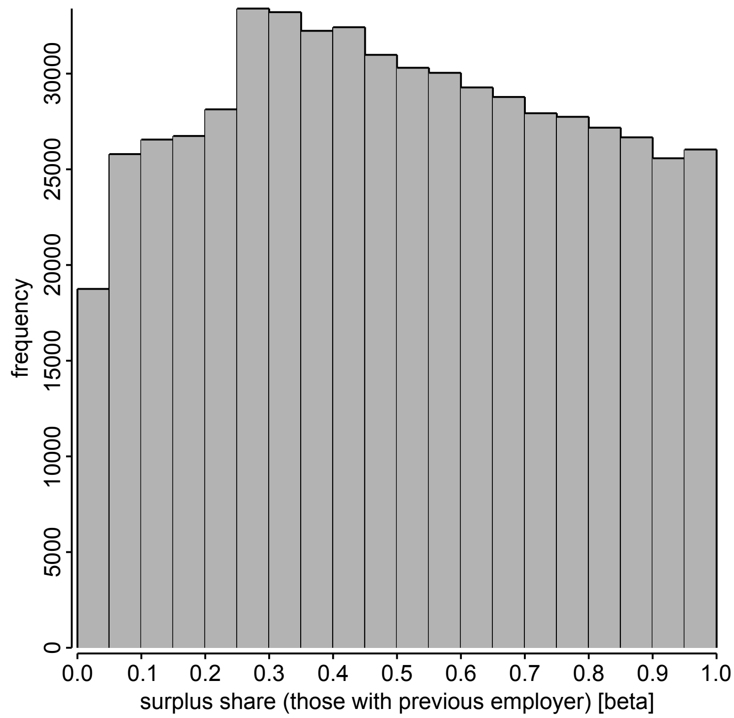

Figure 11: Ergodic distribution of surplus shares β for the workers who moved to their current firm from another one. For workers hired from unemployment, the surplus share is β_0 by construction.

for wages is quite limited because of the dispersion of surplus shares β. Because of workers whose β (and thus w) is increasing as a result of outside offers that do not result in a transfer, wage is increasing in tenure.

	(1)	(2)	(3)
p	0.052	0.073	n/a
tenure	0.052	n/a	0.062
R^2	0.283	0.142	0.217

Table 4: Regression coefficients and R^2 of regressing wage on productivity and/or tenure in calibrated model.

6.1 Hypothetical wage offer distributions

In order to facilitate the comparison of the model presented in this paper with wage posting models, it is useful to calculate the distribution of wages a worker is expected

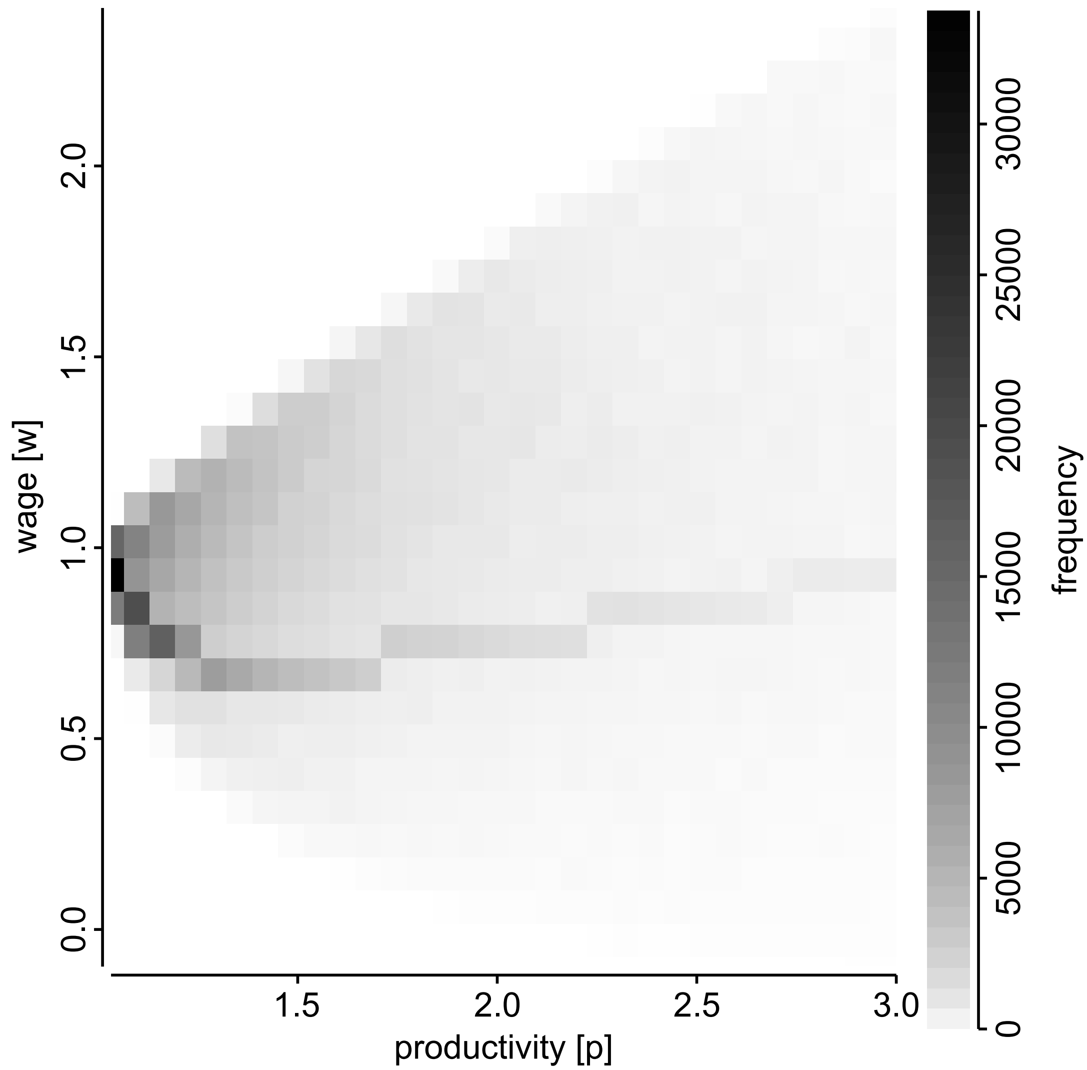

Figure 12: Endogenous distribution of (p, w). The distribution fans out to fill the envelope defined by the highest and lowest wages, shown in Figure 7. The ridge in the middle corresponds to the mass of workers hired from unemployment, whose surplus share is concentrated at β_0.

to receive after encountering an outside firm. Of course, this distribution is hypothetical in the sense that wages are not posted by firms but determined as a function of surplus shares which result from Bertrand competition.

Consider a worker working at a firm with productivity p. If he meets another firm with productivity p', Bertrand competition results in the surplus share

$$\beta' = \begin{cases} \frac{S(p)}{S(p')} & \text{if } p' \geq p \\ \frac{S(p')}{S(p)} & \text{if } p' \leq p \end{cases}$$

This will only be accepted if it is larger than the current β. Then the new wage is

$$\tilde{w}(p'; p, \beta) \equiv \begin{cases} w\Big(p', S(p)/S(p')\Big) & \text{if } p' \geq p \\ w\Big(p, \max\big(\beta, S(p')/S(p)\big)\Big) & \text{if } p' \leq p \end{cases} \tag{47}$$

For the sake of simplicity, consider the case with $\beta = 0$ so we can ignore the $\max(\beta, \cdot)$ in (47). When $p' \leq p$, the worker remains at his current firm, but the outside offer raises his surplus share. Since the surplus is increasing in p, better outside offers result in a higher wage: $\tilde{w}$ is *increasing* in p'. When the new firm is more productive than the incumbent, the new surplus share will be decreasing in p', which makes $\tilde{w}$ decreasing in p' for that region.

Having calculated $\tilde{w}$ for specific values of p, I obtain its distribution by sampling from the vacancy distribution $\Upsilon(p)$ and calculating the corresponding $\tilde{w}$. The left panels of Figure 13 show $\tilde{w}$ as a function of p' for $p = 2$ and $p = 4$ and the corresponding hypothetical wage offer distributions. The left tail of the distributions comes from the lowest wages starting at $w(p', 0)$, which is decreasing in p' as discussed in Section 3.2, so the higher p' is, the more the distribution of $\tilde{w}$ is expanded to the left. Of course with $\beta > 0$, these distributions would simply be truncated on the left at $w(p, \beta)$. The value of p also determines the maximum of $\tilde{w}$, which is at $p' = p$ when the worker meets a firm of the same productivity and Bertrand competition allows him to extract all the surplus from the match. Consequently, a higher p' expands the hypothetical wage offer distribution to the right.

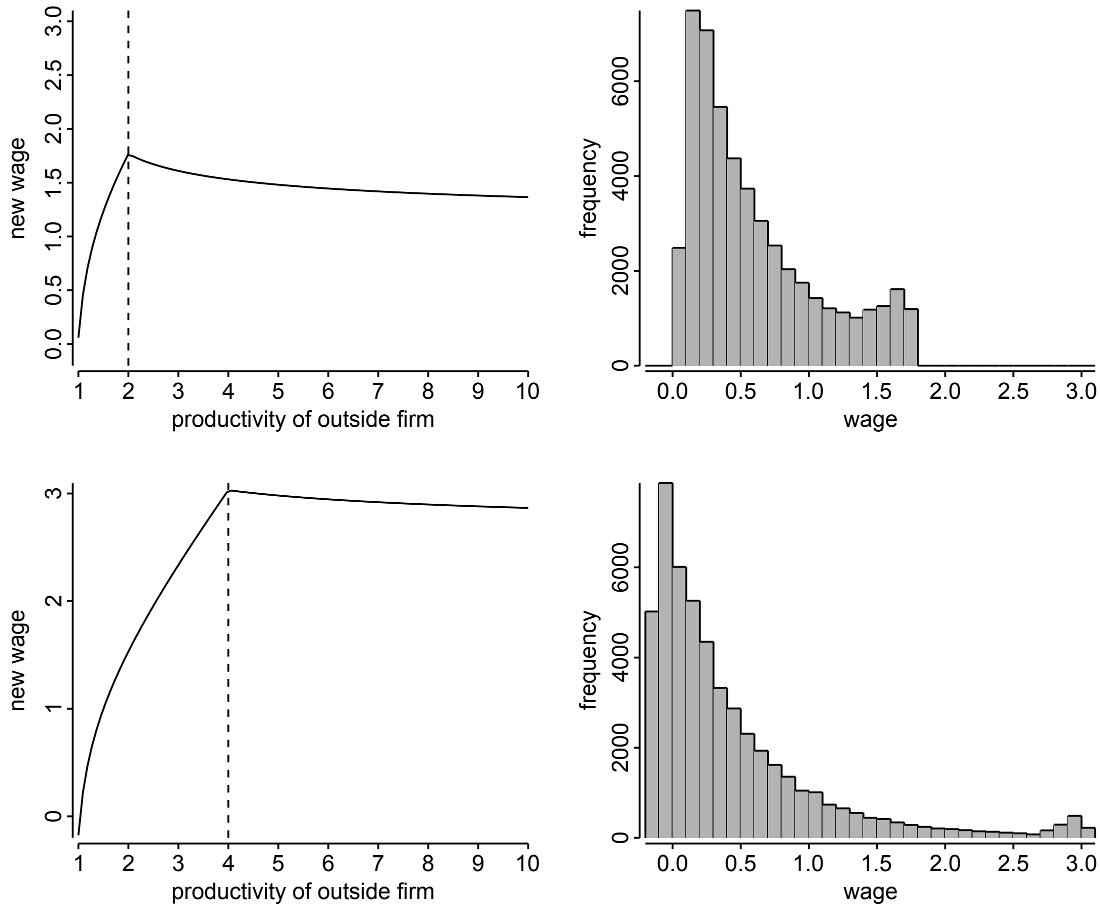

Figure 13: Hypothetical wage offer functions (left) and distributions (right). Top panels: $p = 2$, bottom panels: $p = 4$.

6.2 Sensitivity analysis

Since it plays an important role in replicating the business cycle fluctuations of the Mortensen-Pissarides type model, the calibration of the replacement ratio is a contested topic — see for example Hagedorn and Manovskii (2007). In order to check that the model is robust for various values of the replacement ratio, I also check alternative calibrations.

Figure 14 shows summary statistics of the model for a range of β_0s. Clearly the mean/min ratio is increasing in β_0, which mostly comes from the increasing average wage, with the lowest wage staying almost constant. The replacement ratio

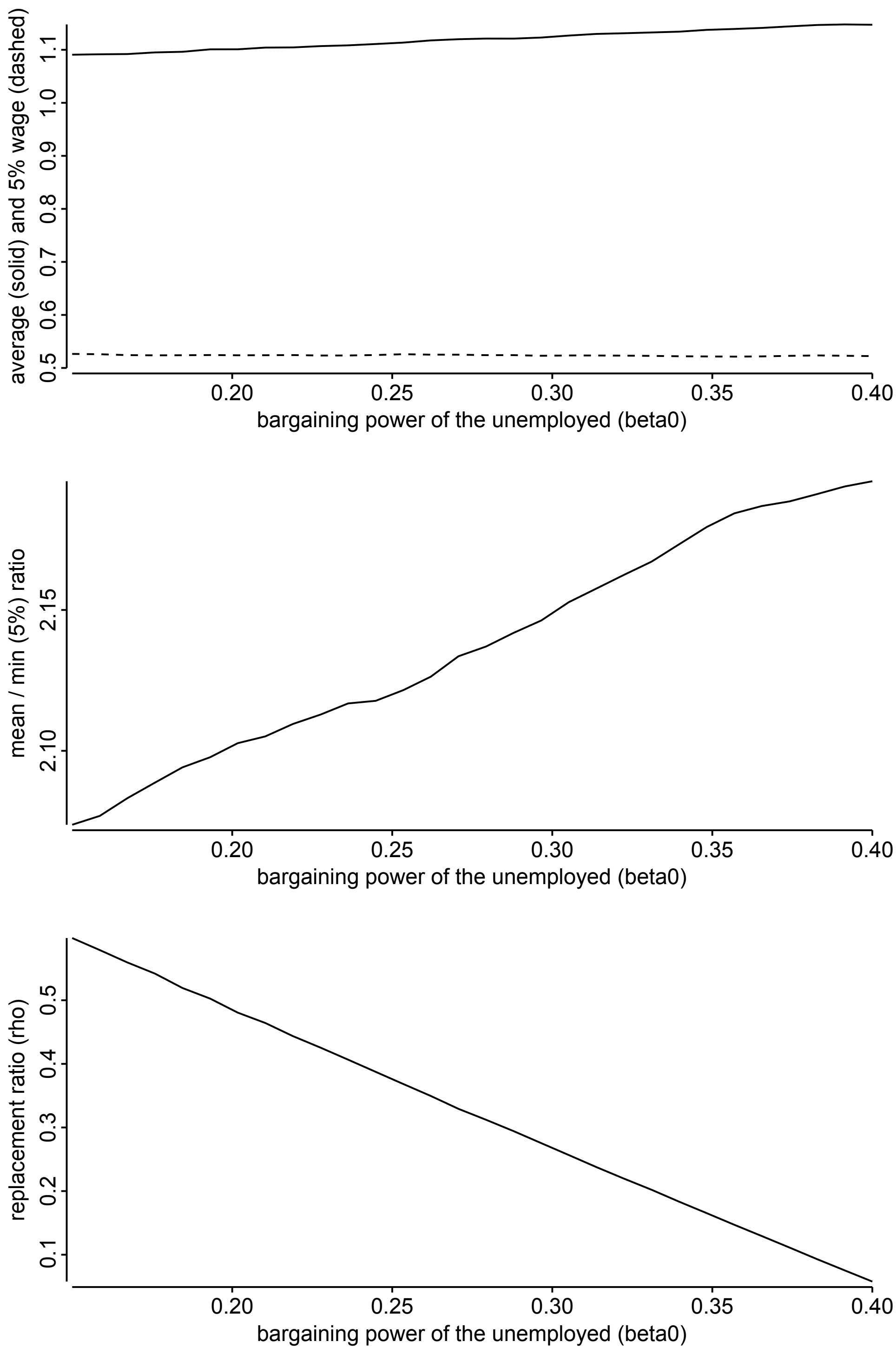

Figure 14: Sensitivity analysis of the calibration. The top panel shows the average and the 5% quantile wage. The former is increasing in β_0, the bargaining power of the unemployed, while the latter is constant. Consequently, their ratio is increasing as shown in the middle panel. The bottom panel shows the replacement ratio.

ρ generated by the model is decreasing in β_0. Table 5 displays wage statistics (and β_0) for $\rho = 0.2, 0.4, 0.6$, confirming that the mean-min ratio generated by the model is fairly robust. Also, the decomposition of variance according to (45) is virtually unaffected by the choice of ρ.

ρ	0.2	0.4	0.6
β_0	0.332	0.239	0.149
w_5	0.522	0.522	0.528
w_{25}	0.837	0.801	0.778
w_{50}	0.997	0.986	0.986
w_{75}	1.282	1.257	1.263
w_{95}	2.110	2.047	2.018
$\mathrm{E}[w]/w_5$	2.182	2.118	2.071

Table 5: Calibrations for various replacement ratios, showing that the mean-min ratio is robust to changes in the replacement ratio.

6.3 Wage dispersion in the standard wage ladder model

In order to demonstrate that a wage ladder model in the style of Burdett and Mortensen (1998) cannot match the wage dispersion in the data, it is instructive to derive the mean-min ratio it would yield if embedded in the production and matching environment of this model.

Consider a general equilibrium formulation of the wage ladder model with heterogeneous productivity as in Bontemps et al. (2000). I keep the meeting and productivity structure from Section 3, but change the wage determination mechanism: now firms post wages as a take-it-or-leave-it offer, which the employed and unemployed workers can accept upon meeting. While in wage ladder models the wage offer strategy of the firms is endogenous, it turns out that there is no need to solve for it explicitly — it is enough to assume that there is some wage offer distribution and that workers always move to a firm which is offering higher wages, and that all wages offered are above the unemployed workers' reservation wage — which must be true in equilibrium, since no firm would offer a wage which never results in hiring anyone.

I will recycle notation: in this section only, $\Upsilon(w)$ will denote the wage offer distribution and $\Gamma(w)$ the distribution of wages in the economy. Let w^* be the reservation wage, and $\bar{w}$ supremum of the support of Γ and Υ, which can possibly be infinite. The derivation below is similar to Hornstein et al. (2007, Section 7), except that this derivation results in a tighter calibration by relating the mean-min ratio to transition rates calibrated in Section 5.1 directly.

The value functions of the worker satisfy the HJB equations

$$rW(w) = w + \underbrace{\mu s \int_{w^*}^{\bar{w}} \big(W(w') - W(w)\big) d\Upsilon(w')}_{\text{transition to another firm, higher w}} + \underbrace{(\sigma + \delta)\big(U - W(w)\big)}_{\text{separation and firm death}} \tag{48}$$

$$rU = z + \underbrace{\mu \int_{w^*}^{\bar{w}} \big(W(w') - U\big) d\Upsilon(w')}_{\text{finding a job}} \tag{49}$$

At the reservation wage w^*, $U = W(w^*)$, so subtracting (48) from (49) yields

$$\begin{aligned} w^* &= z + \mu(1-s) \int_{w^*}^{\bar{w}} \big(W(w') - U\big) d\Upsilon(w') \\ &= z + \mu(1-s) \int_{w^*}^{\bar{w}} \frac{v - \Upsilon(W)}{r + \sigma + \delta + \mu s\big(v - \Upsilon(w)\big)} dw \end{aligned} \tag{50}$$

where I have used partial integration.[11] Similarly to (26), we can characterize the equilibrium match and vacancy distributions with

$$\underbrace{(\sigma + \delta)\Gamma(w)}_{\text{separations and firm deaths}} + \underbrace{\mu s \Gamma(w)(v - \Upsilon(w))}_{\text{workers hired away}} = \underbrace{\mu u \Upsilon(w)}_{\text{hiring unemployed}} \tag{51}$$

[11] Differentiating (48), we obtain

$$W'(w) = \frac{1}{r + \sigma + \delta + \mu s\big(v - \Upsilon(w)\big)}$$

which yields

$$\Gamma(w) = \frac{\mu u \Upsilon(w)}{\sigma + \delta + \mu s\big(v - \Upsilon(w)\big)} \tag{52}$$

Then consider the average wage,

$$\begin{aligned}
\mathrm{E}[w] &= \frac{1}{1-u}\int_{w^*}^{\bar{w}} w d\Gamma(w) = w^* + \int_{w^*}^{\bar{w}} \left(1 - \frac{\Gamma(w)}{1-u}\right) dw \\
&= w^* + \int_{w^*}^{\bar{w}} \frac{\left(\frac{\sigma+\delta}{v} + \mu s\right)\big(v - \Upsilon(w)\big)}{\sigma + \delta + \mu s\big(v - \Upsilon(w)\big)} dw \\
&= w^* + \mu\left(\frac{u}{1-u} + s\right)\int_{w^*}^{\bar{w}} \frac{\big(v - \Upsilon(w)\big)}{\sigma + \delta + \mu s\big(v - \Upsilon(w)\big)} dw
\end{aligned} \tag{53}$$

where I have used (52). Then I use the replacement ratio

$$\rho = \frac{z}{\mathrm{E}[w]}$$

in (50), and approximate the integral by $r \to 0$, since the real interest rate is dominated by the other rates:

$$w^* - \mathrm{E}[w] \approx \mu(1-s)\int_{w^*}^{\bar{w}} \frac{v - \Upsilon(W)}{\sigma + \delta + \mu s\big(v - \Upsilon(w)\big)} dw \tag{54}$$

Combining (54) and (53),

$$\frac{\mathrm{E}[w]}{w^*} = \frac{\lambda_{\mathrm{UE}}/\lambda_{\mathrm{EU}} + 1}{\lambda_{\mathrm{UE}}/\lambda_{\mathrm{EU}} + y + \rho(1-y)} \tag{55}$$

where I have used (41) and the definition of y from (43). Figure 15 shows (55) as a function of ρ, with y and the other rates calibrated in Section 5.1. As noted in Hornstein et al. (2007), this model comes closer to matching the wage dispersion in the data, but it would imply a replacement ratio that is still counterfactually low.

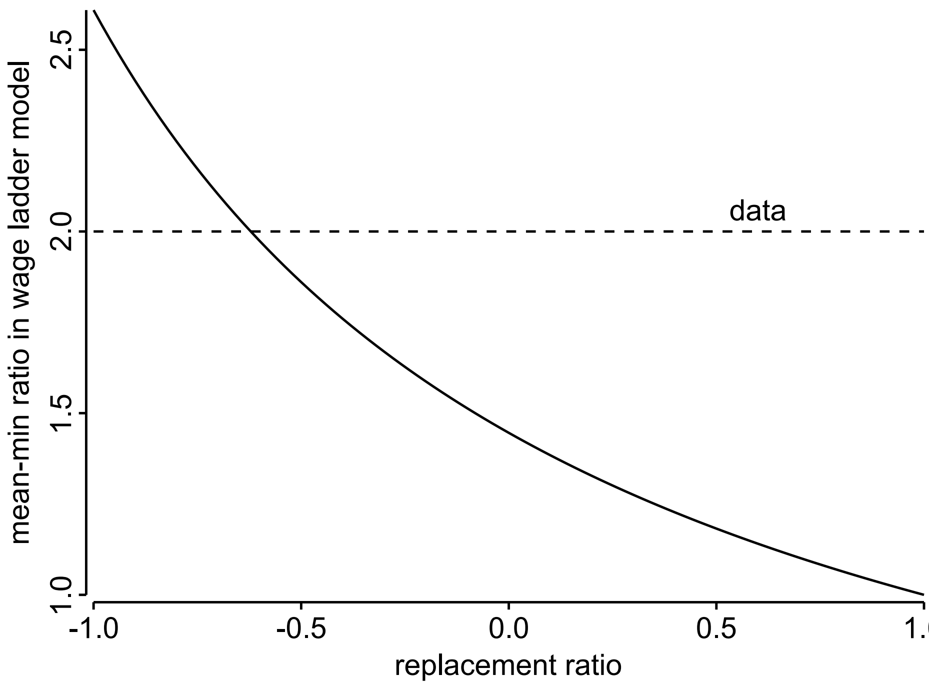

Figure 15: Wage dispersion (mean-min ratio) vs replacement ratio in a calibrated Burdett-Mortensen wage ladder model. Comparing with Figure 1, we see that on-the-job search brings the model closer to the data, but the replacement ratio required to match the mean-min ratio of ≈ 2 is still counterfactually low, indicating that wage posting models have difficulty in matching the data.

7 Conclusion

Frictional models of the labor market are able to generate *some* frictional wage dispersion, but explaining the amount of wage dispersion in the data has been a challenge to this class of models. This paper proposes a general equilibrium model with a parsimonious parameter structure, that is calibrated to worker flows, the interest rate, and the standard deviation of log productivity but no wage-distribution information *per se*. I find that the implied model matches the amount of wage dispersion very closely. The dominant source of the wage dispersion in the model is that workers do not face a common wage offer distribution: their wages depend on their bargaining position, which in turn is a function of their employment history.

While partial equilibrium estimations of a similar framework by Postel-Vinay and Robin (2002) and Cahuc et al. (2006) have yielded interesting results, I consider the general equilibrium formulation of the present model with endogenous firm entry and vacancies with positive value important because these features impose significant constraints on the model. In future work it would be beneficial to compare the general equilibrium formulation of the present paper with other frictional models of the labor market, in particular, general equilibrium wage posting models like Mortensen (1998). The most significant difference I see between the two approaches is that in the wage posting model, *workers see the same wage offer distribution*, while in the model presented in this paper *there is no common wage offer distribution* (except for the unemployed). This can potentially form a basis of an empirical comparison of the two frameworks.

Another extension of the model that would be potentially interesting for future work is the incorporation of a realistic utility function and precautionary savings. Lise (2006) demonstrates that precautionary savings has important implications for wealth levels and wage dispersion. Since in this paper the worker is comparing offers of the two employers by their value, anything that changes the curvature of the value functions could lead to important changes.

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A Appendix

A.1 Computation

Since $\Gamma(p)$ is calibrated as a Pareto distribution with an analytical form and $\Upsilon(p)$ and $F(p)$ can be calculated using (26) and (27), the only part of the model that is solved for numerically are the value and wage functions.

Recall that the surplus is characterized by (17), which I repeat below for convenience,

$$(r+\sigma+\delta)S(p) = p \underbrace{-z - \mu\beta_0 \int S(p)d\Upsilon(p)}_{\text{opportunity cost of unemployment}}$$
$$\underbrace{-\mu u(1-\beta_0)S(p) - \mu s \int_{p^*}^{p} \big(S(p) - S(p')\big) d\Gamma(p')}_{\text{opportunity cost of an occupied vacancy}}$$

Define

$$Q(p) = \int_{p^*}^{p} S(p')d\Gamma(p') \tag{56}$$
$$C = \int S(p)d\Upsilon(p) \tag{57}$$

Then we can rewrite the equation for the surplus as

$$\big(r+\sigma+\delta+\mu s\Gamma(p)+\mu u(1-\beta_0)\big)S(p) = p - z - C + Q(p) \tag{58}$$

From (56),

$$Q'(p) = S(p)\Gamma'(p) \tag{59}$$

Combining (59) with (58),

$$Q'(p) = \frac{\Gamma(p)}{r + \sigma + \delta + \mu s \Gamma(p) + \mu u(1 - \beta_0)} \big(p - z - C + Q(p)\big) \tag{60}$$

with the boundary condition $Q(p^*) = 0$. For a given p^*, I guess at C, solve for Q (and incidentally, S) by numerically solving (60), and then calculate C from (57). The solution is found when the latter equals the original guess for C, which can thus be found using univariate rootfinding methods.

Once S is available, I solve for $P(p, \beta)$ numerically using (19) and univariate rootfinding. Then it is straightforward to calculate the wage from (20).

The ergodic distribution of wages is obtained by simulation.

A.2 Efficiency

Consider the problem of a social planner who is maximizing the discounted present value of consumption (which in this model is production minus investment in new firms), and can dictate the movements of workers but is still subject to the constraints of a the meeting function in the model. Clearly, if a worker at a firm with productivity p meets another firm with productivity p', the social planner will move the worker to the new firm if and only if $p' \geq p$, so in this respect the competitive solution of the model is efficient.

However, there social planner still needs to decide the inflow of new firms ϕ and the cutoff value of productivity p^* above which firms are kept in operation.[12] In this analysis, I will concentrate on the latter.

We can analyze the problem of the social planner by calculating the discounted present value of having workers in various states. Let $\tilde{W}(p)$ and $\tilde{U}$ denote the present

[12]Assuming that if a firm with productivity level p is kept in operation, then so is $p' > p$ is without loss of generality.

value of production generated by a worker at a firm with productivity p and in unemployment, respectively. Using the same notation as in Section 3,

$$r\tilde{W}(p) = p + \mu s \int_{p}^{\bar{p}} \big(W(p') - W(p)\big) d\Upsilon(p') + (\sigma + \delta)\big(\tilde{U} - \tilde{W}(p)\big) \tag{61}$$

$$r\tilde{U} = z + \mu \int_{p^*}^{\bar{p}} (\tilde{W}(p) - \tilde{U})^+ d\Upsilon(p) \tag{62}$$

We are looking for the p^* where $\tilde{W}(p^*) = \tilde{U}$. Evaluating (61) at p^* and subtracting (62) yields

$$p^* - z = \mu(1-s) \int_{p^*}^{\bar{p}} \big(\tilde{W}(p) - \tilde{U}\big) d\Upsilon(p) = \mu(1-s) \int_{p^*}^{\bar{p}} \tilde{S}(p) d\Upsilon(p) \tag{63}$$

where I have introduced the planner's match surplus $\tilde{S}(p)$.

Intuitively, when the planner assigns a worker to a firm, he will gain a higher production flow, but also pays to opportunity cost of lower search efficiency. Below the threshold p^*, the sacrifice in search efficiency is larger than the productivity gain $p - z$.

Comparing (63) with (18), we find that the role of $1 - s$ in the former is taken by β_0 in the latter, in other words, efficiency along this dimension is achieved by the competitive equilibrium when the relative search efficiency of the employed workers is the same as the *firms'* surplus share. Since this $\beta_0 < 1 - s$ in the calibration of Section 5, firms are kept in operation even though, from the planner's point of view, operating them is inefficient because workers employed in these firms could increase social welfare by searching as unemployed.

A.3 Scale invariance of productivity

Consider a model with unemployment benefit z and initial productivity distribution $F(p)$ which leads to the equilibrium solution $\Gamma(p)$, $\Upsilon(p)$, $S(p)$, …, p^*. Then consider another model with $\tilde{z} = Az$, $\tilde{F}(p) = F(Ap)$. It is easy to show that

$$\tilde{\Gamma}(p) = \Gamma(Ap)$$
$$\tilde{\Upsilon}(p) = \Upsilon(Ap)$$

satisfy the equilibrium conditions (26) and (27) for the new model. Also,

$$\tilde{S}(p) = S(Ap)$$
$$\tilde{p}^* = Ap^*$$

since these satisfy the equilibrium conditions (17) and (18). The result follows for the other value functions by a similar rescaling, in particular, the wages in the new model are also rescaled by A.[13]

[13]From the definition (19) it is easy to show that

$$\tilde{P}(p, \beta) = AP(p, \beta)$$

then the result follows for (20).