Bilateral Bargaining with Durable Commitment *

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Abstract

We study an infinite horizon bilateral bargaining model in which negotiators can make strategic commitments to durable offers. Commitments decay stochastically. In the model's unique stationary Nash equilibrium, agreement occurs when the first negotiator's commitment decays. As commitments decay more quickly, disagreement is shorter and the terms of the agreement become more equal. Conversely, as the rate of decay tends to zero, the expected duration of disagreement tends to infinity and the eventual agreement gives all the surplus to one party.

KEYWORDS: Bargaining, Commitment, Disagreement

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1 Introduction

Thomas Schelling (1956, 1960, 1966) pioneered the analysis of bilateral bargaining as a game of strategic commitment. By committing to an irrevocable bargaining position, a negotiatiator aims to force concessions from the opponent. As Schelling (1960, Appendix B) emphasizes, there is no *a priori* reason to expect that the outcome of such a struggle for dominance will be symmetric, or even efficient. Instead, the outcome depends on the available commitment technologies.

Schelling's view of bargaining as a struggle for commitment has had a deep impact on the analysis of conflict and cooperation by political scientists (e.g., Snyder and Diesing, 1977; Fearon, 1998). Yet, despite notable contributions by Crawford (1982), Fershtman and Seidman (1993), Perry and Reny (1993), Muthoo (1996), and Li (2007), it is still true that formal bargaining theory has paid scant attention to strategic commitments (Binmore, Osborne, and Rubinstein, 1992, p. 200). Instead, the study of dynamic bargaining has revolved around models that admit only the minimal commitment necessary to make credible short-lived contract offers, as in Ståhl (1972) and Rubinstein (1982).[1]

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[1]There is also a literature on strategic bargaining in which players may be inherently (i.e., exogenously) committed to some bargaining stance, and where this commitment is not directly observable to the opponent; see Kambe (1999), Abreu and Gul (2000), and Compte and Jehiel (2002).

Building on the two-stage model of Crawford (1982), as modified by Ellingsen and Miettinen (2008) (henceforth E-M), we here seek to formalize Schelling's argument within an infinite horizon framework. The model yields precise and simple predictions concerning the effect of commitment possibilities and patience on the outcome of bargaining.

Bargaining proceeds as follows. Each period is split into two stages. In the first stage, players can make binding contract offers, unless they are already committed to an earlier offer. In the second stage, players can sign on to their opponent's offer, unless they have committed themselves to an incompatible offer.

Our analysis rests on two key assumptions. The first assumption is that, in each period, a commitment only sticks with some probability $q < 1$. With probability $1 - q$ the commitment decays completely. As long as the own commitment has not decayed, a player cannot accept any incompatible offer by the opponent, but after such a decay the player is free to accept or reject any offer by the opponent. After a rejection, the flexible player may either commit to a new offer or remain flexible.

The possibility of highly asymmetric equilibrium outcomes arises because players who discount the future are better off accepting a small share than by waiting a long time in order to get a large share.

The second key assumption is that, everything else equal, players prefer not to make an own offer. That is, it is better to sign on to the opponent's offer than to have the opponent sign on to an own offer of the same deal. This assumption can be defended on several grounds. The most direct interpretation is that there are (lexicographically) small costs associated with formulating and conveying a firm offer.[2] As a consequence of this assumption, there cannot be a stationary equilibrium in which players commit to exactly compatible offers with positive probability, because one player is then better off by remaining flexible and sign on to the opponent's offer.

We consider two solution concepts – stationary Nash equilibrium and subgame perfect Nash equilibrium. In alternating offer bargaining games between players who have a weak (lexicographic) preference for simple strategies, Binmore, Piccione, and Samuelson (1998) show that evolutionary stability favors stationary over non-stationary Nash equilibria.[3] The set of stationary Nash equilibria in their model is small and collapses on the (unique) subgame perfect equilibrium in the limit as the time between periods tends to 0.[4] In this limit of their model, the bargaining solution obtained by Rubinstein (1982) is thus chosen by both the solution concepts that we consider.[5]

Our infinite horizon bargaining game has a unique stationary Nash equilibrium, which "nests" the insights of previous models in a natural way. In the limit as the time between periods tends to zero, the stationary equilibrium is characterized by a simple formula: When

[2] A less direct interpretation is that it is good to retain some flexibility in case the opponent makes another offer than one had expected, although such trembles should ideally be modeled directly. Ellingsen and Miettinen (2008) and Asheim and Perea (2009) show that various equilibrium refinements, including trembling hand perfection, have much cutting power in the single-period version of our model.

[3] As they write (p. 260): "The evolutionary process contains no forces favoring backward-induction reasoning. However, the evolutionary advantages of simple strategies ensure that evolutionarily successful strategies are stationary and prescribe the same behavior no matter what role the player fills."

[4] Since Binmore, Piccione, and Samuelson (1998) do not distinguish time discounting from the time between periods, they state their result in terms of players'discount factor tending to 1.

[5] In the first paper on the evolution of bargaining behavior, Young (1993) applies evolutionary stability criteria to a one-shot Nash Demand game in which players are able to observe some past play by others.

the two players have identical payoff functions, both of them start out offering the opponent a share $k/(2k+r)$, where k is the rate at which commitments decay and r is the discount rate. (The probability that a commitment sticks for a time period of length t is $q = e^{-kt}$, and the discount factor is $\delta = e^{-rt}$.) Thus, as the rate of decay tends to infinity, the offers converge to an equal split and agreement is almost immediate, just as in Rubinstein (1982). On the other hand, as the rate of decay tends to zero, offers converge to zero and the expected duration of disagreement tends to infinity, mimicking the outcome in Ellingsen and Miettinen (2008).

The game also has efficient non-stationary subgame perfect equilibria. When commitments decay fast, these subgame perfect equilibrium payoffs are all in the vicinity of the equal split. On the other hand, as commitments decay more slowly, the set of non-stationary equilibrium payoffs grows, eventually coinciding with the feasible set – that is, the same solution set as Nash's (1953) demand game. Thus, the old sceptic view that anything may happen in bargaining in our model requires both highly durable commitments and coordination on non-stationary strategies.

2 Model

There are two negotiators, henceforth called players. Players are indexed $i = A, B$ and bargain over a fixed surplus of size 1. For simplicity, we initially assume that players have identical preferences and technologies, relegating a complete characterization of asymmetric cases to Section 4.

The size of the surplus and the rationality of the players are common knowledge. A player's utility is assumed to be linear in the player's share of the surplus. Players are impatient, with per period discount factor δ.

2.1 Timing and actions

The bargaining game, call it G^∞, has infinite horizon. In each period t, actions are taken in two stages – the proposal stage and the response stage. At the proposal stage, players can make long-lived offers; at the response stage, they accept or reject offers. If the players arrive at the response stage without any offer on the table, each player has an equal chance of being picked to make a short-lived offer that the opponent may either accept or reject. In between the two stages, no new action can be taken, but time passes and commitment to offers may decay. No time passes between the end of a period and the start of a new period.[6]

2.1.1 *Period 1*

The first period is special, since players have not been able to make previous commitments. In period 1, the available actions and payoffs are the following.

(a) The proposal stage. Each player i chooses either to make a specific proposal for how to split the surplus or to wait and remain uncommitted. Specific proposals are fully characterized either by the amount offered to the opponent or by the amount demanded by the proposer. Our formulas are shorter in the former case. Thus, we let $s_1^i \in [0, 1]$ denote an offer made by player i to player j in period 1. Denote the waiting action by w, and let the set of proposal stage actions

[6]It is straightforward to generalize the model to the case in which time passes between periods, but it complicates the analysis without generating new insight.

be denoted $S = [0, 1] \cup \{w\}$. The set of randomized offers is the set of probability distributions on S. Let σ^i denote a randomized offer of player i, and let $p^i(s)$ denote the associated probability that player i takes the action s, and let $\mathcal{P}$ denote the set of all probability distributions on S. As in E-M, we assume that it is costly to make specific proposals. Initially, we assume that any offer $s \in [0, 1]$ entails a small positive cost c, whereas w is free. Subsequently, we will simplify by assuming that c is of second order magnitude (lexicographically small).

(b) Delay. Some time passes between the making of an offer and the opportunity to respond. During this time, a player's commitment to the offer can potentially decay. Specifically, if player i makes an offer $s^i \in [0, 1]$, the offer survives until the response stage with probability $q \leq 1$. With probability $1 - q$, player i instead enters the response stage with $s^i = w$. The cost

(c) The response stage. Each player now observes the offers made at the proposal stage; if a player made a mixed offer, only the realization is observed by the opponent. If only player i is committed, player j either (i) accepts the offer, getting a share s_1^i while leaving a share $1 - s_1^i$ to player i, or (ii) rejects the offer. After a rejection, Period 1's negotiation is over, and players must wait for bargaining to resume next period. If both players are committed, the outcome depends on whether the commitments are compatible or not. If $s_1^A + s_1^B < 1$, offers are incompatible, there is no agreement in period 1, and bargaining resumes next period. If $s_1^A + s_1^B \geq 1$, one of the players is randomly picked to make an accept or reject decision. If player i is picked and accepts, player i gets the share s_1^j and player j gets the remainder. If player i is picked and rejects, bargaining resumes next period. Finally, if no player has a surviving offer, i.e., if $s^A = s^B = w$, each player has an even chance of being picked to make a short-lived take-it-or -leave-it offer. Let such short-lived offers be denoted $\widetilde{s}_1^i$. It does not matter whether it is costly to make a short-lived offer, as long as the cost is sufficiently small; for simplicity we assume that it is costless. Let randomized short-lived offers be denoted $\widetilde{\sigma}_1^i$, and let $\widetilde{\mathcal{P}}$ denote the set of all probability distributions on $[0, 1]$. If the offer is accepted, the game ends and players get the corresponding payoffs. If the offer is rejected, and it is not the last period, the game continues to the next period where both players will be initially uncommitted.[7]

Observe that the set of opponent offers that player i can choose to accept is $A(s^i) = \{s^j \in [0, 1] \mid s^i + s^j \geq 1\}$. Formally, player i's response is thus a function $z^i : A(s^i) \to \{Y, N\}$, where Y denotes acceptance and N denotes rejection. A mixed response is a function $\rho^i : A(s^i) \to [0, 1]$ yielding for each offer s^j the probability that player i accepts it.

2.1.2 *Period t*

Suppose negotiations did not end before period t. Suppose moreover that in the beginning of period $t - 1$ player i was committed to the offer $s_{t-1}^i \in [0, 1]$. Then, unless the commitment subsequently decayed, player i remains committed to s_{t-1}^i at the proposal stage of period t. That is, $s_t^i = s_{t-1}^i$.

As before, the offer decays with probability $1 - q$ before the response stage, implying that offers decay exponentially. After having observed surviving offers, any uncommitted player chooses a (possibly mixed) offer σ_t. The remainder of period t is analogous with period 1.

In the following, we refer to s_t^i as player i's offer at the end of period t's commitment stage and $s_{t+}^i \in \{s_t^i, w\}$ as player i's (surviving) offer at the beginning of period t's response stage.

[7] The latter assumption is of minor importance. In the main case of interest, infinite horizon bargaining with brief time intervals, the specification of what happens in the (w, w) case is altogether irrelevant.

2.2 Histories and strategies

For simplicity, we assume that any randomization is only privately observable, and that players condition their strategies only on the public history (if at all). The public history of the game comprises the actual offers and responses as well as the observed decays of previous commitments.

Let $\mathcal{H}$ be the set of all possible finite histories. Players have perfect recall, and subject to the constraints imposed by current commitments, they can condition their actions on all previous events. Allowing mixed strategies, a commitment strategy for player i is a function $\sigma^i : \mathcal{H} \to \mathcal{P}$. Similarly a short-run offer strategy is a function $\widetilde{\sigma}^i : \mathcal{H} \to \widetilde{\mathcal{P}}$, and a response strategy is a function $\rho^i : \mathcal{H} \to [0,1]$. Accordingly, a complete strategy for player i can be written $x_i = (\sigma^i, \widetilde{\sigma}^i, \rho^i)$.

2.3 Remarks

The model is essentially an extension of E-M to the multi-period case, but there are slight differences in exposition and technical detail. We now find it more natural to speak about "offers" than "demands". The formulation "I offer s" suggests that the proposer will get *exactly* $1-s$ if the proposal succeeds, whereas the seemingly similar formulation "I demand $1-s$" could be interpreted as saying that the proposer will get *at least* $1-s$ if the proposal succeeds. Whereas E-M assumed that any excess surplus would go to the flexible player in case less than all the surplus is demanded by a committed opponent, our new formulation gets to the same outcome more directly.

Another change is that we here explicitly model the negotiators' opportunity to reject the opponent's proposal.

Finally, we are now explicit about the delay that occurs between offers and responses within a period. If there were no delay between the two stages, it would be difficult to justify the assumption $q < 1$ in the single period case.

3 Analysis

After a brief recapitulation of the single-period analysis of E-M, we go on to characterize the unique stationary Nash equilibrium of the infinite bargaining game.[8] We then describe the set of non-stationary subgame perfect equilibria. In Section 4, we generalize all results with respect both to player asymmetries and the process of commitment decay.

3.1 One period

Let us start by analyzing a single-period bargaining game, call it G^1. Suppose the cost of making a specific proposal is $c \in (0, \delta(1-q)^2/2)$.

Proposition 1 *There is a unique subgame perfect Nash equilibrium of G^1. At the proposal stage, each player i commits to the offer $s^i = 0$. In case both commitments decay and player i is chosen to be the proposer, player i makes the offer $s^i = 0$. If only player i's commitment decays,*

[8]The case of a long but finite bargaining game is quite complicated, so we leave it aside.

or if both commitments decay and player i is chosen to be responder, player i accepts any offer $s^j \in [0,1]$. Since the commitments are incompatible, there is disagreement with probability q^2.

The result corresponds closely to part (i) of Proposition 3 of E-M, but since the model is slightly different, the Appendix provides a modified proof.

Proposition 1 says that each player offers the smallest amount that a flexible opponent is willing to accept. The existence of such a bad equilibrium is unsurprising; the Nash Demand Game (NDG) too has similarly inefficient equilibria. The surprise is that no other equilibria exist. In the NDG there is a continuum of efficient equilibria, so why are there no efficient equilibria here? As indicated above, a key difference is the cost of making offers. The efficient equilibria in the NDG have the property that players make exactly compatible demands. In our model, compatible offers cannot arise in equilibrium because a player is better off by deviating to being flexible and accepting the opponent's offer.[9]

3.2 Infinitely many periods

Let us now consider the infinite horizon game, G^∞. From now on, we assume that the commitment cost c is lexicographically small.

3.2.1 Stationary equilibria

We say that a strategy is stationary if any proposal decision σ_t^i only depends on known features of the opponent's current commitment status $s^j_{(t-1)+}$, and if any acceptance decision only depends on the opponent's current proposal s_t^j. That is, $\sigma_t^i : S \to \mathcal{P}$, and $\rho_t^i : S \to [0,1]$.

When we consider stationary equilibria, we can without loss of generality confine attention to pure strategies, $\sigma_t^i : S \to S$ and $\rho_t^i : S \to \{0,1\}$.[10] The construction of a stationary equilibrium in pure strategies involves five steps.

Step (i): In a stationary equilibrium, if player i is prepared to accept an offer $s^j = s$, then the player is also prepared to accept all better offers $s^j > s$. (A rejection of a better offer could only be rational if that rejection would induce another continuation equilibrium than the rejection of s, and by definition different continuation equilibria violate stationarity.) It follows that, in a stationary equilibrium, response strategies are represented by a constant acceptance threshold.

Step (ii): A stationary offer s^i is an equilibrium offer only if it equals j's acceptance threshold: If j were to strictly prefer accepting, it would be optimal for j to accept a slightly lower offer too, and thus i should make a lower offer. If j were to strictly prefer rejecting, then it would be better for i not to make the offer (given the assumed stationarity of j's strategy).

[9] If we introduce a cost of making demands in the NDG, together with a rule that unclaimed surplus goes to the player making no demands, then this modified NDG would also have a unique and inefficient equilibrium.

[10] There cannot be an equilibrium in which an indifferent player, j, mixes between rejecting and accepting the opponent's offer: Suppose player j is indifferent between accepting and rejecting share x_j, rejecting with probability p. Given j's indifference, the upper bound of the expected payoff of player i is $(1-p)(1-x_i) + p\delta(1-\frac{x_j}{\delta}) < (1-(x_i+\varepsilon))$ for some $\varepsilon > 0$ sufficiently small. Thus the player i can deviate and propose $x_i + \varepsilon$, which the responder strictly prefers to accept, contradicting the supposition. With players not randomizing their acceptance decisions, it also follows that there cannot be an equilibrium in which a player is indifferent between two offers on the equilibrium path, but will make the lowest offer that is accepted.

Step (iii): It cannot be an equilibrium in which $s^A + s^B \geq 1$, because in this case a player would benefit from not making an offer and sign on to the other's offer instead. (Observe that this claim is only true because stationarity of strategies implies that the continuation in the event that both players are flexible is independent of previous commitment attempts.)

Step (iv)**:** There cannot be a stationary equilibrium in which only one player ever makes offers. Since the flexible opponent is better off by accepting immediately than by rejecting and facing the same offer in the future, the only candidate stationary equilibrium with one-sided offers is $s = 0$. However, if player i offers nothing, player j can profitably make an own offer $s^j \in (\delta, 1)$, which player i accepts in case i's own commitment has a loophole.

Step (v): Suppose both make offers, and that $s^A + s^B < 1$, implying that the offers are incompatible. Let V^idenote player i's equilibrium payoff. Thus, $s^i = V^j$. Thus,

$$V^i = \delta \left[q^2 V^i + q(1-q)(1-V^j) + (1-q)qV^i + (1-q)^2 \frac{1 - V^j + V^i}{2} \right], \tag{1}$$

where the first term in the bracket corresponds to the case that both commitments stick, the second term corresponds to the case that only player i's commitment sticks, the third term corresponds to the case that only player j's commitment sticks, and the fourth term corresponds to the case that neither player's commitment sticks. The unique solution to these two equations yields the equilibrium payoffs

$$V_S^A = V_S^B = \frac{\delta\,(1-q^2)}{2(1-\delta q^2)}. \tag{2}$$

Finally, we investigate the equilibrium as the period length tends to zero. Let r be the instantaneous discount rate, and let k be the instantaneous rate of decay. That is, $\delta = e^{-rt}$ and $q = e^{-kt}$. Taking the limit of (2) as t tends to $0+$ yields, after an application of L'Hôpital's rule,

$$V_S^A = V_S^B = \frac{k}{2k+r}. \tag{3}$$

Proposition 2 *The symmetric game G^∞ has a unique stationary Nash equilibrium. In the limit as period length goes to zero, both players immediately make the offer $k/(2k+r)$, and the first player to have a decaying commitment accepts the opponent's offer.*

Since the sum of offers is smaller than 1, there is always some conflict. Since offers are accepted by the first player who has a loophole, the duration of the conflict is driven entirely by the rate of decay of commitments. To compute the expected conflict duration, note that at any time t, the probability that players have reached agreement is $1 - q^2$, which in the continuous time limit equals $1 - e^{-2kt}$. The rate at which players reach agreement is given by the derivative, $2ke^{-2kt}$. Thus, conflict duration is given by the exponential distribution.[11] The expected conflict duration is,[12]

$$D(k) = \int_0^\infty t \cdot 2ke^{-2kt} dt = \frac{1}{2k}.$$

[11] Interestingly, durations of actual labor conflicts are often claimed to be approximately exponentially distributed; see for example Kiefer (1988).

[12] Integrate by parts and use L'Hôpital's rule to get the second equality. (Of course, the equality is a well known property of the exponential distribution.)

For example, if t and k are measured on a yearly scale, and the instantaneous rate of decay corresponds to one breakdown per year ($k = 1$), then the expected duration of conflict is half a year.

3.2.2 Non-stationary subgame perfect equilibria

Let us now consider whether there are efficient non-stationary subgame perfect equilibria, with or without commitment on the equilibrium path.

In an efficient non-stationary equilibrium with commitment, one player – say player B – initially chooses to be flexible, whereas the other player, player A, initially commits. Efficiency dictates that B immediately agrees to A's offer. In case there are several such equilibria, we focus on the equilibrium that maximizes player A's payoff, call this payoff $\overline{V}$.

The offer $\underline{V} = 1 - \overline{V}$, must satisfy two conditions. First, it must be sufficiently large to keep player B from deviating and making an own commitment. Call this the *lower bound condition*: $\underline{V} \geq V_L$. Second, the offer $\underline{V}$ must be sufficiently small to induce a rejection by player B whenever player A deviates and offers $b < \underline{V}$. Call this the *upper bound condition*: $\underline{V} \leq V_U$.We have found an efficient equilibrium if these bounds are consistent, i.e., if $V_U \geq V_L$. Let us now derive the two bounds.

Step (i): Deriving the lower bound. The lower bound condition is found by computing the expected payoff, V_L, that player B can ensure herself by deviating to the smallest offer that player A is willing to accept in case A's commitment decays. In order to minimize B's incentive to deviate, continuation equilibria induced by A's rejection of an offer by B should be as favorable as possible to A, thus raising the offer that B has to make. Since the best offer that B will ever accept in equilibrium is V_L, it follows that if A rejects B's (deviating) offer, A's best policy is to renew the commitment to V_L.

Letting V_H denote the expected payoff of player A in case of rejection and renewal, as evaluated from the beginning of the next period, it follows that the optimal deviation by player B is to offer exactly V_H. Thus, V_L is given by

$$\begin{aligned} V_L &= \delta \left[q^2 V_L + q(1-q)(1-V_H) + (1-q)qV_L + (1-q)^2 \frac{V_L + (1-(1-V_L))}{2} \right] \\ &= \delta[q(1-q)(1-V_H) + (1-q(1-q))V_L]. \end{aligned} \tag{4}$$

The first term on the right hand side of the first equation is B's payoff in case both players' commitments stick. The second term is B's payoff in case only A has a loophole, the third term is B's payoff in case only B has a loophole, and the fourth term is B's expected payoff in case both players have loopholes. In the latter case, each player is proposer with probability 1/2, and the proposer offers exactly the opponent's expected continuation payoff.

Analogously, we have

$$\begin{aligned} V_H &= \delta \left[q^2 V_H + q(1-q)(1-V_L) + (1-q)qV_H + (1-q)^2 \frac{1 - V_L + (1-V_L)}{2} \right] \\ &= \delta[qV_H + (1-q)(1-V_L)]. \end{aligned} \tag{5}$$

For (δ, q) inside the unit square, the unique solution to this pair of equations is

$$\begin{aligned}\widehat{V}_L &= \frac{\delta q(1-q)}{1-\delta q^2}, \\ \widehat{V}_H &= \frac{\delta(1-q)}{1-\delta q^2}.\end{aligned}$$

Comparing $\widehat{V}_L$ to V^B reveals that $\widehat{V}_L$ is smaller. Thus, we have indeed found the lower bound.

Step (ii): Deriving the upper bound. Consider a deviation to a more aggressive commitment than $\overline{V} = 1 - V_U$ by player A. To prevent such a deviation, it must be profitable for player B to reject A's proposal, and propose the continuation equilibrium that is best for player B – namely, that in which B eventually gets $1 - \widehat{V}_L$. Player B's maximum expected payoff to rejection is

$$V_U = q^2\delta V_U + q(1-q)\delta(1-\widehat{V}_L) + (1-q)q\delta V_U + (1-q)^2\delta(1-\widehat{V}_L),$$

yielding the solution

$$\widehat{V}_U = \frac{\delta(1-q)}{1-\delta q^2}.$$

Note that $\widehat{V}_U = \widehat{V}_L/q > \widehat{V}_L$.Thus, there are generally multiple efficient equilibria.

However, we are interested in the outcome as the time between periods approaches zero, in which case q approaches 1. Recalling that $\delta = e^{-rt}$ and $q = e^{-kt}$, and taking the limit as t tends to 0+, we see that if only one player commits, there is a unique pair of equilibrium payoffs. The flexible player obtains a payoff

$$\lim_{t\to 0+} \widehat{V}_L = \lim_{t\to 0+} \widehat{V}_U = \frac{k}{2k+r}.$$

and the committed player obtains the remainder, $(k+r)/(2k+r)$.

With this result in hand, it immediately follows that all efficient outcomes that yield a more even payoff distribution can also be sustained in a subgame perfect equilibrium: Let players make exactly compatible commitments that yield each player more than $k/(2k+r)$. Because a player who deviates from this strategy by waiting may be "punished" in case the opponent's offer decays and both are flexible at the response stage (by selection of the worst continuation equilibrium from then onwards), it is not worth attempting to save the lexicographically small commitment cost.

Proposition 3 *In the continuous time limit, any efficient outcome yielding each player a payoff of at least $k/(2k+r)$ can be sustained in a (non-stationary) subgame perfect equilibrium of G^∞.*

As the rate of decay goes to infinity, there difference between the stationary and the non-stationary outcomes vanish; there is immediate agreement on the equal split in both cases. However, for sufficiently slow decay rates, non-stationary strategies admit virtually any outcome.

4 Extension: Asymmetries and correlation

To what extent do our previous results depend on our assumptions that players have identical utility functions and that commitments decay independently? As we shall now show, the results generalize naturally. (In the expressions below, superscripts are used according to the conventions $i \in \{A, B\}$ and $j \neq i$.)

Let the discount factor of player i be denoted δ^i. We retain the assumption that decay rates are constant. Let q^i be the probability that player i's commitment sticks if only player i commits. If both players are committed, let p^I be the probability that both commitments survive this period, and let p^i be the probability that only player i's commitment survives. If decay rates are independent of whether the opponent commits, we thus have $p^i = q^i(1 - q^j)$. However, here we do not assume independence.

For brevity, we only consider the stationary equilibrium of the general game.

Generalizing (1) yields the two equations

$$V^i = \delta^i \left[p^I V^i + p^i(1 - V^j) + p^j V^i + (1 - p^I - p^i - p^j) \frac{V^i + (1 - V^j)}{2} \right], \tag{6}$$

which have the solution

$$V_*^i = \frac{\delta^i \left(1 + p^i - p^j - p^I\right)\left(1 - \delta^j\right)}{2 + 2\delta^j \delta^i p^I - \left(\delta^i + \delta^j\right)\left(1 + p^I\right) - (\delta^j - \delta^i)(p^i - p^j)}. \tag{7}$$

To obtain the payoffs in the continuous time case, insert $\delta^i = e^{-r^i t}$, $p^I = e^{-(k^I + k^i + k^j)t}, p^i = e^{-(k^I + k^i)t}(1 - e^{k^j t})$. Taking the limit as t tends to $0+$ yields

$$V_*^i = \frac{r^j \left(2k^j + k^I\right)}{2r^i r^j + (r^i + r^j)k^I + 2(r^i k^j + r^j k^i)}. \tag{8}$$

Inspection reveals that player i's expected payoff, V^i, is an increasing function of the opponent's decay rate, k^j, and the opponent's discount rate r^j, while it is a decreasing function of the own rates of decay and discount, k^i and r^i. In other words, a player benefits from increases in own patience and commitment, and suffers from increases in an opponent's patience and commitment.

The impact of correlated decay is slightly more subtle. Differentiation of V^i with respect to k^I yields an expression which is positive if and only if $r^i > k^j - k^i$. Thus, a greater probability of joint decay always improves the payoff for player i if i is relatively weakly committed ($k^i \geq k^j$), but not necessarily otherwise. When the own commitment is stronger, it may be better to wait for a larger expected share than to increase the probability of settling early with a smaller expected share. The result thus suggests that it will be the weaker parties who most enthusiastically welcome the presence of mediators as well as of external pressure for flexibility in negotiations.

Note finally that when decay rates go to infinity we obtain Rubinstein's result that payoffs are determined by the relative patience $r^i/(r^i + r^j)$.

Of course, these characterization results are only of interest if a stationary equilibrium continues to exist in the general case. To derive an existence condition, we must consider players' incentive to deviate from the posited equilibrium strategies. Since we have already characterized the optimal commitment strategies, the only relevant one-step deviation is for one of the players to refrain from making a commitment, instead waiting in order to agree to the opponent's proposal.[13] For player i, making an own commitment is then strictly preferable to staying flexible if and only if

$$V^i > \delta^i \left[q^j V^i + (1 - q^j) \frac{V^i + (1 - V^j)}{2} \right].$$

Inserting for V^i from (6) and simplifying, the condition becomes

$$\left(p^i - p^j + q^j - p^I\right)(1 - V^i - V^j) > 0.$$

Thus the inefficient stationary commitment equilibrium (with $V_*^i + V_*^j < 1$) exists if $p^i - p^j + q^j - p^I > 0$ for $i = A, B$. If we assume that $p^j = q^j(1 - q^i)$ (so that decay rates are independent), then $p^I = q^i q^j$, and the condition becomes $q^j - q^j q^j > 0$, which is trivially satisfied. When $p^i = p^j$, it is only when a player can powerfully reduce the opponent's commitment power by refraining to commit (and thereby bring q^j below p^I in the formula above) that the incompatible commitment equilibrium fails to exist.

5 Related literature

As noted by several game theorists, subgame perfect equilibria of bargaining models with short-lived commitment tend to display a large sensitivity to minor changes in negotiation protocols. This is troubling. In the words of Aumann (1989, p 9): "there is a feeling that procedures are not really all that relevant; that it is the possibilities for coalition forming, promising and threatening that are decisive, rather than whose turn it is to speak. [...] even when the procedures are specified, non-cooperative analyses of a cooperative game often lead to highly non-unique results". Although Aumann originally made the statement in promotion of cooperative game theory, Perry and Reny (1993, p 51) instead see it as a call for a non-cooperative study of strategic commitments – promises and threats – in models with permissive protocols regarding who may speak when. Binmore, Piccione, and Samuelson (1998) instead suggests that we focus attention on equilibria in simple (stationary) strategies. Following the advice of both teams, we study strategic commitments and focus most of our attention on equilibria in stationary strategies.

Let us now elaborate in a little more detail on other related work. As noted in the Introduction, Crawford (1982) is the seminal formal treatment of strategic commitment in bilateral bargaining.[14] Subsequent work includes Perry and Reny (1993), Muthoo (1996), Ellingsen (1997), Güth, Ritzberger and van Damme (2004), and Ellingsen and Miettinen (2008). With the notable exception of Perry and Reny (1993), a shortcoming of this literature is that it confines attention to two-period models. By construction, negotiators have only one shot at finding an agreement. Since it is impossible to continue negotiations after an impasse, it is unclear to what extent the asymmetries and impasses predicted by these models carry over to

[13] As usual, if there is no profitable one-step deviation, there is no profitable many-step deviation.

[14] Crawford also studies cases in which there is asymmetric information about commitments. We shall neglect asymmetric information here. For subsequent contributions to this literature, see especially the analysis of dynamic bargaining games with unobservable commitments by Kambe (1999), Abreu and Gul (2000), and Compte and Jehiel (2002).

more realistic settings. For example, will a negotiator really accept to get a small fraction of the surplus if a rejection today entails the opportunity to reopen negotiation at some point in the future? Therefore, we have here studied the role of strategic commitment in infinite horizon bargaining.

Our contribution is particularly closely related to Perry and Reny (1993), who study a model in which each offer has a fixed duration, preventing the proposer from accepting any counteroffer before the own offer expires. In addition to such deterministic waiting time, Perry and Reny also allow for positive response time. When the response time is shorter than the waiting time, players may use their own waiting time for strategic advantage, with the range of equilibria narrowing towards the equal split as waiting times tend to zero. However, as waiting times increase, the set of equilibria grows and in the limit any efficient outcome is sustainable as an equilibrium.[15,16]

The two major features of our predictions concern outcome asymmetry and disagreement. Let us therefore also briefly relate our contribution to alternative models of these two phenomena.

Outcome asymmetry is a typical feature of models with alternating offers and a finite horizon. However, in models such as Rubinstein (1982) the asymmetry vanishes in the infinite horizon limit, at least if the time between offers is short. On the other hand, asymmetry can be sustained if players can delay their own moves, because there is then a "last-proposer" advantage (Ma and Manove, 1993). An objection is that the alternating move structure is exogenous and that the resulting asymmetries are artificial. Güth, Ritzberger and van Damme (2004) study a two-stage model with simultaneous moves in which the equilibrium outcomes are strongly asymmetric. They assume that the size of the surplus is initially uncertain, and that each player can choose to move either before or after the uncertainty resolves. In any strict equilibrium, one player moves before and the other moves after. Note that the outcome is efficient despite the short horizon, whereas in our model strongly asymmetric outcomes are associated with substantial average delay.[17]

Disagreement, or delayed agreement, is often ascribed to the existence of asymmetric information. In Crawford (1982, Section 5), disagreement is due to uncertain and privately known costs associated with revoking a commitment.[18] Myerson and Satterthwaite (1983) show quite generally that disagreement is bound to arise with positive probability when negotiators are uncertain about the opponent's private valuation. Abreu and Gul (2000) consider a bargaining model in which players may be irrational and where uncertainty about the opponent's rationality can be a source of inefficiency. In their model, as in that of Myerson and Satterthwaite,

[15] A natural question is whether it is possible to refine the set of equilibria in the Perry-Reny model, like Ellingsen and Miettinen (2008) and Asheim and Perea (2010) refine the set of equilibria Crawford (1982).

[16] In recent independent work, Li (2010) considers a dynamic bargaining model that is complementary to ours. His key assumption is that observable commitment attempts are easy to make, but that it is difficult to make commitments stick. When the strategy sets are the same (the simultaneous moves case), this model has a vast range of equilibria, including the perpetual disagreement outcome. Again it is possible that refinements could help to select between these equilibria.

[17] Strongly asymmetric outcomes can also occur in the model of Perry and Reny (1993), but at least with their solution concept these outcomes belong to a larger set of equilibria.

[18] Muthoo (1996) assumes that the cost of revoking depends on the demand and how much the player backs off. The marginal costs of backing off determines the bargaining outcome. (Infinite marginal costs bring the model back to the case where commitment is fully irrevocable.) With common knowledge of the cost of revoking the model predicts an efficient outcome in which the allocation depends on players' relative cost of revoking.

inefficiencies disappear as the amount of private information tends to zero; see also Kambe (1999) and Compte and Jehiel (2002).

In bilateral perfect information bargaining over a single trade, it is known that delay may occur if the negotiation is subject to a deadline. The logic is easiest to see when players take turns to make offers, but may delay their moves. If the discount factor is large, the first mover may then wait to make the offer until just before the deadline.[19] If the negotiators have imperfect control over the timing of their offers, for example due to imperfect communication channels, Ma and Manove (1993) show that deadlines may induce not only delays, but also offers that are rejected with positive probability and disagreements. In a sense, this model introduces asymmetric information about valuations through the random delay; when making an offer, the proposer does not know what the responder's valuation will be when the offer arrives.

Reference-dependent preferences may also entail disagreement. If negotiators are unwilling to accept any offer that they have previously turned down, Fershtman and Seidman (1993) show that such reference-dependence can delay agreement if there is a deadline. Li (2007) strengthens the result. If players are unwilling to accept offers that do not improve on rejected offers in net present value terms, then delay is unavoidable even without a deadline. One way to view this result is that players have a particular technology for making commitments, namely rejecting the opponent's proposal.

Despite these results, it is fair to say that the mainstream view among economic theorists has been that disagreement among rational individuals who can engage in unrestricted bargaining is typically due to asymmetric information.[20] Or as Kennan and Wilson (1993, p.101) put it: "The hypothesis that private information is an underlying source of conflict is currently the only one based on the usual test of rationality, namely relentless maximizing behavior." In other words, to the extent that commitments matter at all, disagreement is caused by exogenous and unobservable commitments. We think that our paper lends credence to Schelling's view that disagreement can also be caused by rationally chosen observable commitments.

6 Conclusion

Like Perry and Reny (1993), we seek to integrate two major strands of bargaining literature. The first strand, associated with Schelling (1956, 1960) and Crawford (1982) focuses on strategic commitments. That is, it emphasizes the role of inflexibility. The second strand, associated with Ståhl (1972) and Rubinstein (1982) focuses on the opportunity for continued negotiation in the absence og agreement. That is, it emphasizes the role of flexibility.

By admitting strategic commitment as well as opportunities for continued negotiation, our model illuminates how equilibrium outcomes vary with the available commitment technology. When commitments decay swiftly, the model's predictions closely resemble the efficient and relatively symmetric outcome derived by Rubinstein. When commitments decay slowly, predictions instead resemble the inefficient and asymmetric outcomes alluded to by Schelling.

[19] Ma and Manove (1993) credit Martin Hellwig with making this point.

[20] Observe the qualifier that bargaining is feasible. Of course, when players cannot negotiate side-transfers, dynamic games of chicken have war of attrition equilibria.

7 Appendix A

7.1 Proof of Proposition 1.

It is trivial that no player i has an incentive to deviate from the proposed equilibrium: Rejection of $s^j = 0$ yields the payoff 0. Since this is identical to the payoff under acceptance, there is nothing to gain by rejecting at the response stage. Consider the offer stage. Given $s^j = 0$, any own offer $s^i \in [0,1]$ yields an expected payoff $\delta[(1-s^i)(1-q)q + (1/2)(1-q)^2] - c$, where the first term is the payoff in case the own commitment sticks and the opponent's commitment decays, and the second term is the expected payoff in case both commitments decay (in which case the random proposer will make an offer of 0). The expression is maximized by the offer $s^i = 0$. The remaining strategy, w, yields an expected payoff $\delta(1/2)[(1-q)q + (1-q)^2]$, which is smaller than $\delta[(1-q)q + (1/2)(1-q)^2] - c$ under our condition on c.

Let us next prove that G^1 has no other subgame perfect equilibrium. We do this by showing that if player j plays optimally at the response stage, all offers $s^i \neq 0$ are iteratively strictly dominated. Observe first that player i strictly prefers w to any offer $s^i \in [1/2, 1]$: The latter commitment strategy gives player i a payoff $\delta(1-s^i) - c < 1/2$ when player j chooses w. It gives at most $\delta s_j - c < s_j$ when player j chooses a compatible commitment $s_j \in [1-s_i, 1]$. Finally it gives $-c < \delta s_j$ when player j chooses an incompatible commitment.

After these strategies are eliminated, for any offer $s^i \in (0, 1/2)$, there exists some $\epsilon > 0$ such that s^i is strictly dominated by the mixed strategy $\sigma^i = (p^i(\epsilon) = 1 - s^i, p_1(w) = s^i)$: If player j plays w, player i's payoff to the pure strategy s^i is $\delta[q(1-s^i)+(1-q)(1-s^i)/2]-c$; whereas under the mixed strategy σ^i he gets $\delta[q(1-s^i)(1-\epsilon)+((1-q)(1-s^i)+s^i)/2)]-(1-s^i)c$, which is greater for sufficiently small ϵ. If player j plays $s^j \in [0, 1/2)$, then the payoff to a pure commitment strategy $s^i \in (0, 1/2)$ is $\delta[q(1-q)(1-s^i) + (1-q)qs^j + (1-q)^2/2] - c$, whereas the payoff to the mixed strategy is $\delta[q(1-q)(1-s^i)(1-\epsilon)+ (1-q+qs^i)qs^j + (1-q+qs^i)(1-q)/2] - (1-s^i)c$, which is again greater for sufficiently small ϵ.

The only remaining proposal strategies are 0 and w. Could there be a subgame perfect equilibrium in which player j plays w with probability $p^j(w) > 0$ and offers 0 otherwise? No: In this case, player i would optimally offer 0 (or some small ϵ), and the unique best response to $s^i = 0$ is $s^j = 0$. That is, $p^j(w) = 0$, a contradiction.

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