WHEN BAD QUALITY IS GOOD POLICY*

by

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Abstract

We investigate product quality under different market structures (monopoly vs. perfect competition) and under different risk–sharing regimes (replacement warranty vs. no warranty). Because of our particular representation of quality we can determine optimal quality and optimal risk–sharing within one single model. Quality differs between risk–sharing regimes but is independent of market structure. Optimal risk–sharing is also independent of market structure, but it can be optimal to place all risk with the risk–averse consumers instead of with the risk–neutral producer(s). The interests of consumers and producer(s) are always the same.

1. Introduction and summary

The question of how product quality depends on market structure has attracted considerable attention in the literature. It is a common belief, at least among laymen, that the quality supplied by monopolistic firms is different from that supplied by competitive firms. In particular, it is suspected that monopolistic firms take advantage of their market power to supply products of inferior quality. We take a new look at this question under the assumption that quality not only is endogeneous, as has been assumed earlier, but also that it is uncertain.

The stochastic nature of product quality is the subject of a quite separate body of literature. It asks how the risk of product failure should be optimally shared between buyers and sellers, under the assumption that quality is given exogeneously. We revisit this question also, but under the assumption that quality is endogenous as well as stochastic. This means that quality may differ depending on the risk–sharing arrangement.

Both questions, that of quality as a function of market structure and that of quality as a function of risk–sharing, are analyzed within one and the same model. Thus, we are able to combine the separate strands of literature on optimal quality and optimal risk–sharing. This approach is fruitful, since it allows us to ask if optimal risk–sharing is dependent on market structure. Another advantage of our approach in comparison with much of the earlier analysis is its simplicity. This is achieved through use of a parametrized (quadratic) utility function and a particular representation of product quality. It is quite clear – at least to us – that the assumption of parametric preferences constitutes a minor cost compared to the new insights and results that it makes possible.[1]

The main result of earlier contributions on quality and market structure is that different market structures in general give rise to different qualities because cost structures will differ, as shown by Levhari and Peles (1973), Spence (1975), Sheshinski (1976) and others, but that quality will be the same under perfect competition and monopoly if firms produce under constant returns to scale, as first demonstrated by Swan (1970) and by

Sieper and Swan (1973). We are able to show that Swan's finding remains unaffected when quality is stochastic. Abel (1983) stresses that what matters for the independence result is that the cost–minimizing quality is invariant with respect to output, and proves that this condition is fulfilled under a more general condition than constant returns to scale.

In the literature on quality and optimal risk sharing, Brown (1974) has shown that a risk–averse consumer will choose to buy insurance against product failure offered by a risk–neutral producer at an actuarially fair rate. This result is generalized in Heal (1977a) to say that risk should be shared between buyers and sellers according to their relative risk–aversion. In particular, optimal risk–sharing between a risk–neutral seller and risk–averse buyers calls for a complete warranty offered by the seller. We find this to be incorrect in general, and present conditions under which both buyers and sellers find it in their interest to place all risk with the buyers.

We also find that the level of quality in general differs between risk–sharing regimes, and establish simple conditions under which quality is higher and lower with than without a replacement warranty. Quality will be lower with than without a replacement warranty for a sufficiently low cost of replacement of malfunctioning units: a replacement warranty and high quality without a warranty constitute two alternative forms of insurance against product failure for the consumers. As for welfare, bad quality (with a warranty) may be good policy. Interestingly, the quality and risk–sharing regime that is optimal for the consumers is always optimal also for the producers, and vice versa, and is independent of the market structure, as shown by Heal (1977a) in a different model.

2. The consumer's decision problem

In order to simplify as much as possible we will consider a good that has the characteristics of a light–bulb: it either functions or fails to function. In general, one will of course find that the distribution of performance of a given good is less extreme. It would seem more "realistic" to assume that the two possible states for example are either 100 or

75 per cent performance (as when all four or only three algebraic operations work in an electronic calculator), but the nature of our representation of product quality would nevertheless be unchanged and so would the qualitative results.

When the consumer decides about how many units of the good to purchase, he does not know in advance how many of the units that will turn out to work. His decision problem is to maximize utility under uncertainty, and can be formally stated as

$$(1) \quad \max_{n} \; E[U(x, z) \mid n]$$
$$\text{s.t. } Pn + z = Y.$$

Here n denotes the decision variable of the consumer, i.e. the number of units of the good in question that he should buy. Having bought n units, x of them will eventually turn out to work. The variable x is stochastic with a known probability distribution, and the consumer is assumed to maximize his expected utility, where $E[U \mid n]$ denotes the expectation over x conditional on n units having been bought.[2] This notation serves to remind us that the probability distribution of x depends on the decision variable n. The price of the uncertain good is denoted by P while the price of the background good z is normalized at unity. The budget constraint $Pn + z = Y$ says that total spending on n units of goods with uncertain quality and on z cannot exceed the exogeneously given income Y.

If we introduce the conditional probability $p(x \mid n)$ as the probability of obtaining x functioning units out of n units bought, the objective function (1) can be equivalently written as

$$(1') \quad \max_{n} \; \sum_{x=0}^{n} p(x \mid n) U(x, z).$$

Let us denote by q the probability that one single unit will function. The number of well–functioning units x will then be a binomially distributed random variable, and the conditional probabilities in (1') are given by

$$(2) \qquad p(x|n) = \frac{n!}{x!(n-x)!} q^x (1-q)^{n-x}.$$

With this specification of the consumer's optimization problem, (1) will yield a demand function n(P, Y, q), and we can regard q as an index of quality. This very specific interpretation of quality (the probability of success of one trial, or the parameter of the binomial distribution) makes our approach different from that in most of the literature in the field, which is more general and regards q only as an (undefined) index.[3]

Before we derive the demand function two problems need to be discussed. First, we note that with the probabilities defined by (2), n is constrained to take integer values only. Thus (1) is not differentiable with respect to n, which makes the solution rather cumbersome. To be able to focus on this paper's main points, we will disregard this problem altogether and treat (1) as if it were differentiable. The "correct" solution to the optimization problem will therefore be one of the two integer values adjacent to the solution n(P, Y, q) that we have derived. Second, it turns out that even if (1) were differentiable, the solution will in general be rather complicated since changes in n will affect the probabilities p(x|n) in a complex way. Economically meaningful results are therefore hard to come by.[4]

Because of the difficulty inherent in handling probabilities of the type given by (2) we will assume that the utility function U(x, z) is quadratic:

$$U(x, z) = \alpha x - \beta x^2 + \gamma z - \delta z^2 + \varphi xz.$$

Substitution of this function together with the budget constraint into (1) enables us to write the consumer's decision problem as

$$(1'') \quad \underset{n}{\text{Max}}\ E(U|n) = \alpha qn - \beta q(1-q)n - \beta q^2n^2 + \\ + \gamma(Y - Pn) - \delta(Y - Pn)^2 + \varphi qn(Y - Pn)$$

To derive (1'') we made use of the fact that for the binomial distribution

$$E[x|n] = qn$$

and

$$E[x^2|n] = var[x|n] + (E[x|n])^2 = q(1-q)n + q^2n^2.$$

We can now derive the demand function n(P, Y, q) as:

$$(3) \qquad n = \frac{(\alpha - \beta)q + \beta^2 q - (\gamma - 2\delta Y)P + \varphi\gamma q}{2\beta q^2 + 2\delta P^2 + 2\varphi qP}$$

Expression (3) is quite simple, but to facilitate the analysis even further we will assume that the utility function is linear in the background good, i.e. that $\gamma = 1$ and $\delta = 0$. We also assume away the interaction term φxz by setting $\varphi = 0$. These assumptions make the demand function independent of income, i.e. n(P, Y, q) = n(P, q), which is critical for the results that follow.[5] Thus, in the following we will assume that demand takes the following form:

$$(3') \qquad n(P, q) = \frac{(\alpha - \beta)q + \beta q^2 - P}{2\beta q^2}$$

It is interesting to note that demand can be both increasing and decreasing in q:

$$\frac{\partial n}{\partial q} = \frac{-(\alpha - \beta)q + 2P}{2\beta q^3}$$

which implies that

$$(4) \qquad \frac{\partial n}{\partial q} \gtrless 0 \quad \text{as} \quad \frac{q}{2} \lessgtr \frac{P}{\alpha-\beta}$$

It may seem counterintuitive that a fall in quality can cause an increase in demand, but the explanation is quite natural: When quality falls the consumer runs a greater risk that he will end up with only defective units. To insure himself against this risk he buys more units. The likelihood that he will act in this way depends on the valuation that he places on the first functioning unit(s), i.e. the size of the parameter α. A large α (and a small β) tends to produce an increase in demand as quality falls.[6]

Let us end this section by pointing out a problem with a quadratic utility function. For some value of x the derivative $\partial u/\partial x$ will start to become negative. To rule out this possibility, we will assume that the parameters of the problem are such that we are always on the upward–sloping part of the utility function, i.e. $x < \alpha/2\beta$. Since this has to hold for all values of x, and since there is a possibility that all n units will turn out to function, we have the requirement that $n < \alpha/2\beta$. A necessary condition for this to hold, assuming $n \geq 1$, is of course that $\alpha > \beta$. Furthermore, by (3'), the requirement can be written as

$$(5) \qquad q(1-q) < \frac{P}{\alpha-\beta}.$$

Since (5) always has to be satisfied if the marginal utility of x is to be positive, and since $q(1-q) \leq q/2$, with equality only for $q = 1/2$, we see by (4) that for values of q close to 1/2, $\partial n/\partial q$ will always be positive.

3. Market equilibrium without a warranty

Let us now assume that producers maximize profits subject to a cost function c(n, q). Unless otherwise stated, we assume constant returns to scale,

$$c(n, q) = n \cdot c(q),$$

and that cost is a convex function of quality, c'(q) > 0 and c" > (q) . The latter assumption seems natural, considering that quality here means reliability. To take the light bulb example again, what we assume is that it costs more to produce 100 bulbs of which 90 rather than 89 function on average, and that it costs even more to make sure that 100 rather than 99 function.

It can be easily verified that when the cost function is of this form the allocation (n, q) that maximizes welfare is identical to the equilibrium allocation in a competitive market. In what follows we refer to such allocations as "competitive equilibria" but we could refer to them as "welfare maxima" as well.

Competitive market

Equilibrium in a competitive market is attained when consumer welfare U_C is maximized and when profits are zero, i.e. when P = c(q). Subscript C denotes variables in a competitive market, while subscript M denotes variables in a monopolistic market. We assume that consumers maximize over quality as well as over the number of units of goods. This seems to be a reasonable assumption for the long run. We have

$$\max_{n,q} U_C = \alpha qn - \beta q(1-q)n - \beta q^2 n^2 + Y - nc(q)$$

which yields the first–order conditions

(6) w.r.t. n: $n = \frac{\alpha-\beta}{2\beta q} + \frac{1}{2} - \frac{c(q)}{2\beta q^2}$.

(7) w.r.t. q: $n = \frac{\alpha-\beta}{2\beta q} + 1 - \frac{c'(q)}{2\beta q}$

Given that the simultaneous system (6) and (7) has a meaningful solution[7], the quality q_C resulting from a competitive market equilibrium is given by

(8) $$\epsilon(q) = 1 + \beta q^2/c(q)$$

where $\epsilon(q)$ is the elasticity of cost with respect to quality, i.e. $\epsilon(q) \equiv c'(q)q/c(q)$. The equilibrium quantity n_C can then be obtained simply by substitution of the q_C resulting from (8) into (6) or (7).

Monopolistic market

In a monopolistic market the producer solves the following problem:

$$\max_{n,q} \pi = n\cdot[P - c(q)]$$
$$\text{s.t. } P = (\alpha-\beta)q + \beta q^2 - 2\beta q^2 n$$

where the constraint is identical to the consumer's demand function (3'). This yields the first–order conditions

(9) w.r.t. n: $$n = \frac{\alpha-\beta}{4\beta q} + \frac{1}{4} - \frac{c(q)}{4\beta q^2}$$

(10) w.r.t. q: $$n = \frac{\alpha-\beta}{4\beta q} + \frac{1}{2} - \frac{c'(q)}{4\beta q}$$

We first note that this pair of equations will yield an equation in q which is identical to (8). We can therefore conclude that with a multiplicative cost function c(n, q) = $n\cdot c(q)$, the equilibrium quality q_M under monopoly will be equal to the equilibrium quality q_C under perfect competition, i.e.

$$q_M = q_C = q.$$

This result has also been derived by Spence (1974, p.422), Swan (1970), and others, but in models that differ from ours in several important respects.[8]

Second, we observe that the quantity n_M resulting from (9) and (10) must be half of the quantity n_C resulting from (6) and (7), i.e.

$$2n_M = n_C.$$

This is simply a consequence of our assumption of a quadratic utility function, which yields a demand function (3') that is linear in P. For such a demand function, the quantity under monopoly will always be half of the quantity under perfect competition – provided, of course, that the quality will be the same in the two cases.

4. Market equilibrium with a replacement warranty

Let us now assume that producers of the good with uncertain quality will undertake to replace any defective unit at "no cost" to consumers. The cost to producers of the full replacement warranty is equal to the cost of production of any number of new units that is necessary to ensure that all consumers receive the number of functioning units that they desire plus, we assume, a constant per unit transactions cost of replacement. The assumption of a replacement cost is made for two reasons. The first is simply that replacement typically entails transactions cost in the form of transportation, examination of the defective unit, etc. Second, we have formulated the consumer's decision problem without a warranty as non–sequential under the implicit assumption that transactions costs make such a purcasing strategy optimal for the consumer (cf. footnote 2 above).

Similar costs must exist for the producer when he undertakes to replace any defective unit at no cost to the consumer.

Since consumers no longer bear any risk for product failure, their decision problem reduces to ordinary utility maximization under certainty:

$$\max_{x,z} U(x, z)$$
$$\text{s.t. } Px + z = Y$$

where x now is a deterministic variable. Given the quadratic utility function $U(x, z) = \alpha x - \beta x^2 + z$ the demand function takes the form

$$(11) \quad x = \frac{\alpha - P}{2\beta}$$

which is identical to (3') for q = 1.

Monopolistic market

The decision problem of the monopolist involves maximization under uncertainty. We assume the monopolist to be risk–neutral. Consequently, he maximizes expected profits according to

$$(12) \quad \max_{x,q} E[\Pi^w|x] = Px - E[c(n, q)|x] - R \cdot E[n - x|x]$$
$$\text{s.t. the consumer's demand function (11),}$$

where R denotes the per unit cost of replacement. Here n is a stochastic variable, since the number of units the producer actually will have to produce in order to eventually supply x functioning units to the consumer is not known in advance. The superscript w denotes variables when there is a warranty; in this case profits. We note that if the cost function is linear in n, i.e. $c(n, q) = n \cdot c(q)$, then the objective function (12) can be written as

$$(12')\ \max_{x,q} \pi^W \equiv E[\Pi^W|x] = Px - c(q)E[n|x] - R\cdot E[n-x|x],$$

and thus only the expectation of n matters. For more general cost functions, however, higher moments of n could also play a role and it is therefore important to point out that the fact that the producer is risk–neutral (i.e. maximizes expected profits) does not automatically mean that he is insensitive to higher moments of the distribution of n, i.e. that in an optimum all risk should be shifted over to him.

It can be shown that the conditional expectation in (12') is as simple as:

$$E[n|x] = \frac{x}{q}.$$

Maximization of (12') subject to (11) yields the following first–order conditions:

$$(13)\quad \text{w.r.t. x:}\quad x = \frac{\alpha - c(q)/q - R(1-q)/q}{4\beta}$$

$$(14)\quad \text{w.r.t. q:}\quad qc'(q) - c(q) = R$$

This system is recursive, i.e. the solution q_M^W can be immediately derived from (14) and inserted into (13) to yield x_M^W.

Competitive market

Under perfect competition the solution (x_C^W, q_C^W) is obtained by maximizing the consumer's utility with respect to x and q and assuming that the expected profit is zero. The latter requirement implies that

$$(15)\quad P = c(q)/q + R(1-q)/q.$$

The first–order conditions for such an optimum, given (15), are

(16) w.r.t. x: $x = \frac{\alpha - c(q)/q - R(1 - q)/q}{2\beta}$

(17) w.r.t. q: $qc'(q) - c(q) = R.$

We see that (17) is identical to (14). Our result of the previous section, namely that the quality will be the same under monopoly as under perfect competition, therefore extends to the case with warranty:

$$q_M^w = q_C^w = q^w.$$

Comparing (16) to (13) we also see that $x_C^w = 2x_M^w$, which again simply depends on the linearity of the demand function.

We can summarize our results thus far in the following proposition:

Proposition 1: *If product quality is uncertain in the sense that the number of functioning units is a binomially distributed random variable, and if technology exhibits constant returns to scale, then quality will be the same in market equilibrium under perfect competition and under monopoly when there is no warranty as well as when there is a full replacement warranty.*

It remains to compare the quality established with and without a warranty to see which is higher. We can rewrite (14) or (17) as

(18) $\epsilon(q^W(R)) = 1 + R/c(q(R))$

to make the result for the warranty case comparable to that for the non–warranty case as given by (8), taking note that quality with a warranty is a function of R. Quality with and without a warranty will be the same if the second terms on the RHS of (8) and (18) have the same value. The critical value of R is

$$(19) \quad R^* = \beta q^2 \frac{c(q^W)}{c(q)},$$

where q is the solution to (8) and q^W is the solution to (18). To rank quality levels for smaller or larger values of R^* we must know the shape of the function $\epsilon(q^W(R))$. By differentiating (18) we obtain

$$\frac{d\epsilon}{dR} = \frac{c(q^W(R)) - R \quad c'(q^W(R)) \frac{1}{q \ c''(\cdot)}}{c(q^W(R))^2}.$$

The elasticity is increasing for small values of R and decreasing for large values. A simple comparison of elasticity values therefore does not suffice to determine relative quality. A higher elasticity with a warranty when the elasticity is increasing (decreasing) for the no warranty quality level indicates that the warranty quality is higher (lower), whereas a lower elasticity can indicate both a lower and higher warranty quality. We

can state our findings in the form of the following proposition:[9]

Proposition 2: *Quality is lower with than without a warranty for a small replacement cost under constant returns to quantity and decreasing returns to quality, and higher with a large replacement cost.*

The proposition has a simple economic interpretation. Consumers are risk–averse whereas producers are risk–neutral. A warranty and a high quality can be seen as alternative forms of insurance against product failure on the part of consumers (regardless of whether it is voluntary or not on the part of producers).[10] Both competitive producers and a monopolist will supply the warranty at minimum cost. When the unit cost of replacement is small the cost is minimized by a low quality and relatively many replacements. It is profitable to raise the quality as the unit cost of replacement increases. Quality with a warranty is higher than quality without a warranty for a sufficiently large unit cost of replacement.

5. Are warranties warranted?

Until now we have regarded the warranty regime as exogeneous. In this section we will determine under what conditions warranties (in the form of a full replacement warranty) are market outcomes. It will turn out that shifting the risk for product failure from risk–averse buyers to risk–neutral sellers will not necessarily raise utility and, in the case of monopoly, profits.

Let us start with the monopoly case. Profits are defined by

$$\pi_M = [P_M - c(q)]n_M$$

and

$$\pi_M^W = [(P_M^W - c(q^W)/q^W - R(1 - q^W)/q^W]x_M^W$$

where we must keep in mind that with a warranty, the profit π_M^W is actually the firms' expected profit, with expected production given by $E[n_M^W | x_M^W] = x_M^W/q^W$. It is easily shown from the demand functions (3') and (11) that the prices charged under monopoly and under perfect competition will differ according to the following relations[11]:

$$P_M = P_C + 2\beta q^2 n_M \quad \text{and} \quad P_M^w = P_C^w + 2\beta x_M^w.$$

Substituting these into the definitions of π_M and π_M^w above and taking into account that $P_C = c(q)$ and $P_C^w = c(q^w)/q^w + R(1-q^w)/q^w$ we have that

$$(20) \quad \pi_M = 2\beta q^2 n_M^2 \quad \text{and} \quad \pi_M^w = 2\beta(x_M^w)^2$$

Let us now turn to consumer utility and first take the case of no warranties. For any market form, we have by (1") that[12]

$$U_i = (\alpha-\beta)qn_i + \beta q^2 n_i - \beta q^2 n_i^2 + Y - P_i n_i. \qquad i = M, C$$

Substitution of the demand function (3') into the expression for U_i yields

$$(21) \quad U_i = \beta q^2 n_i^2 + Y = \beta(E[x_i])^2 + Y \qquad i = M, C$$

For the case with warranties, the expression for utility is very similar. We have

$$U_i^w = \alpha x_i^w - \beta(x_i^w)^2 + Y - P_i^w x_i^w. \qquad i = M, C$$

Substitution of the demand function (11) into the expression for U_i^w yields

$$(22) \quad U_i^w = \beta(x_i^w)^2 + Y \qquad i = M, C$$

Our results thus far are summarized in <u>Table 2</u>:

Table 1: Profits and consumer utility under monopoly and perfect competition

	Monopoly	Perfect competition
Warranty	$\pi_M^w = 2\beta(x_M^w)^2$ $U_M^w = \beta(x_M^w)^2 + Y$	$\pi_C^w = O$ $U_C^w = \beta(x_C^w)^2 + Y$
No warranty	$\pi_M = 2\beta(E[x_M])^2$ $U_M = \beta(E[x_M])^2 + Y$	$\pi_C = O$ $U_C = \beta(E[x_C])^2 + Y$

We see that monopoly profits are higher with warranties than without warranties if and only if $x_M^w > E[x_M]$. Similarly, by (21) and (22) consumer utility under monopoly as well as under perfect competition is higher with warranties than without warranties if and only if $x_i^w > E[x_i]$, i = M, C. We can therefore state

Proposition 3: *Given a market structure of monopoly or perfect competition, the warranty regime that is optimal for the producer(s) is also optimal for the consumers.*

One implication of this proposition is that if we take market structure as given, there is no need for legislation about warranties. The market will do the job. For example, if a monopoly finds it more profitable not to issue a warranty (i.e. $E[x_M] > x_M^w$), the consumers will attain the utility level U_M. Forcing the monopoly to issue a warranty would then change consumer utility to U_M^w, which is less than U_M. Thus, such legislation would be harmful to the consumers.

To find out which warranty regime is to be preferred and at the same time is likely to prevail in market equilibrium, we have to determine the magnitude of $E[x_M]$ relative to x_M^w as well as that of $E[x_C]$ relative to x_C^w.

Let us start with the monopoly case. By (9) we note that $E[x_M]$ can be written as

$$E[x_M] = \frac{\alpha - c(q)/q}{4\beta} - \frac{1 - q}{4}.$$

Comparing this to x_M^w, which is given by (13), we see that

$$(23) \quad x_M^w - E[x_M] = \frac{1}{4\beta}\left[\frac{c(q)}{q} - \frac{c(q^w)}{q^w}\right] + \frac{1}{4}(1-q) - \frac{1}{4}\left[(1-q^w)\frac{R}{\beta q^w}\right]$$

Let us next turn to the case of perfect competition. By (6) we have that $E[x_C] = q_C n_C$ is given by

$$E[x_C] = \frac{1}{2\beta}\left[\alpha - \frac{c(q)}{q}\right] - \frac{(1 - q)}{2}$$

while, by (16), we have that

$$x_C^w = \frac{1}{2\beta}\left[\alpha - \frac{c(q^w)}{q^w} - \frac{(1 - q^w)}{q^w}R\right]$$

We can write the difference as

$$(24) \quad x_C^w - E[x_C] = \frac{1}{2\beta}\left[\frac{c(q)}{q} - \frac{c(q^w)}{q^w}\right] + \frac{1}{2}(1-q) - \frac{1}{2}\left[(1-q^w)\frac{R}{\beta q^w}\right]$$

Condition (24) differs from condition (23) only by a scalar; the sign must be the same and depends on whether $c(q)/q$ is rising or falling, quality is larger or smaller with than without a warranty, the replacement cost is large or small, and on the magnitude of the risk aversion parameter β.

Consider first the change in the unit cost of quality. By taking the derivative of $c(q)/q$ and making use of the first order conditions (14) and (17) we obtain

$$\frac{\delta c(q)}{\delta q} = \frac{q\ c'(q) - c(q)}{q^2} > 0\ .$$

The cost function c(q) can have an intercept such that c(0) > 0 and such that the unit cost of quality is falling for low levels of q. The unit cost of quality must however be increasing in equilibrium with and without a warranty, and must, by monotonicity, be strictly increasing between the different equilibrium quality levels.

Let us investigate the sign of conditions (23) and (24) for $q = q^W$ in equilibrium. The first term on the RHS will vanish. The ratio $R/(\beta q^W)$ in the third term on the RHS must have a value of less than unity since in this case $R = \beta q^2$ by (19). The sum of the second and third terms must therefore be positive. Hence, a full replacement warranty is optimal.

Consider next the case of a vanishing replacement cost. The value of the third term on the RHS approaches zero in this case. From Proposition 2 we know that $q > q^W$ and therefore that the first term is positive. Hence, a full replacement warranty is optimal when the replacement cost becomes very small; bad quality is good policy in this case. We conclude by a continuity argument that a warranty is optimal for all replacement costs between zero and βq^2.

From Proposition 2 we also know that $q < q^W$ for a sufficiently large replacement cost. This means that the first term on the RHS becomes negative and that the value of the negative third term is high. A sufficiently large replacement cost will make no warranty optimal.

Our results are summarized in the following propositions:

Proposition 4: *It is optimal to place all risk for product failure with the risk–averse consumers when the the replacement cost is high, and with the risk–neutral producer(s) when the cost is low.*

Proposition 5: *Optimal risk–sharing is independent of market structure.*

What is the economic logic behind Proposition 4? In general terms, optimal risk–sharing is a question of balancing the gain from shifting risk from risk–averse to risk–neutral agents against the cost of replacing non–functioning units that placing the risk with the producer(s) entails. Consider condition (23) or (24) for more details on this balance of gains and costs. The first term within square brackets on the right hand side captures the influence of the unit cost of quality. A rising cost of quality calls for the regime in which quality is lower. This effect is moderated by risk aversion: the higher the risk–aversion, the weaker the effect of a low relative cost of quality. The second term captures the influence of quality without a warranty: the lower the quality without a warranty, the more attractive is a warranty. (Quality with a warranty has no similar direct effect on welfare, its effect comes via the replacement cost in the third term.) The third term captures the effect of the total cost of replacement. The total cost depends positively on the per unit cost of replacement and on the fraction of units that need to be replaced, $(1 - q^W)$. It is moderated by risk aversion: the higher the risk–aversion, the weaker the effect of the cost of replacement.

In his study of the welfare analysis of quality, risk–sharing and market structure, Heal (1977a) concluded that a risk–neutral seller should assume all risk and provide risk–averse buyers with a warranty, independent of whether the seller is a monopolist or a perfectly competitive firm. Heal proved his result about risk–sharing under the assumption that the supply of goods is inelastic. This assumption seems inappropriate, since we expect quantities, and hence costs and prices, to be different under alternative risk–sharing regimes. We have demonstrated in a different model that Heal was right provided that there are no costs of replacement. With such costs, letting the producer(s) carry all risk is not always optimal. We have shown that what counts is not only the direct effect of the replacement cost, but also its indirect effect on quality under a warranty and therefore the relative cost of quality. Heal had difficulty in explaining why risk–sharing was independent of market structure: "On an intuitive basis, it seems most unlikely that this result is dependent on the assumption of constant relative risk–aversion, though it also seems clear that a much more complex argument would be needed for the general case." One advantage of our simpler, yet more general model is that it clearly shows one set of assumptions (constant returns to quantity, decreasing returns to quality, and separation of income and demand for quality) under which optimal risk–sharing is independent of market structure.

FOOTNOTES

* We are grateful for very helpful suggestions from Peter Englund, Harald Lang, and Jörgen Weibull, and for comments by seminar participants at the Institute for International Economic Studies.

1 We also simplify by assuming a single quality, a single representative consumer, and the simplest possible risk–sharing arrangement. This means that we neglect more recent issues in the quality–warranty literature, namely the signalling function of warranties in a market with many, exogeneously given qualities (see e.g. Akerlof (1970) and Grossman (1981)), the incentive function of warranties to upgrade quality in a market without reputation (see e.g. Spence (1977)), the moral hazard problem that arises when consumers can affect the probability of product failure (Cooper and Ross (1985) and Emons (1985)), the adverse selection problem arising from different handling by different consumers (Emons (1986)), and the scope for extracting consumer surplus by self selection of consumers with different preference for quality (see e.g. Mussa and Rosen (1978)) or by giving sophisticated quality–warranty options to consumers with the same preference for quality (Braverman, Guasch and Salop (1983)).

2 We have assumed here that the consumer has to buy all his units at one and the same time. Such a strategy can be called <u>batch purchasing</u> and may be optimal in the presence of transactions costs. An example of batch purchasing is when an airforce typically buys n units of a new type of fighter on the expectation that technical problems will only make $x < n$ units operational at any given time. Another example is when one buys fruit or eggs in the market, and takes into account that a part will turn out to be bad. An alternative approach would be to assume that the consumer makes his purchases <u>sequentially</u>, i.e. that he buys one unit, examines it to see whether it functions or not (in general, to see how well

it functions), and then, conditional on the unit's performance, decides whether to buy another unit, and so on. Sequential purchasing is optimal when transactions costs are low or non–existent.

In the present paper the distinction between the consumer's different purchasing strategies is of no importance for the main argument of the analysis. Since batch purchasing leads to simpler expressions than sequential purchasing in the present context, we have chosen to deal with the former only.

3 A formally similar model but in an entirely different context – the drilling for oil – is presented in Dasgupta and Heal (1979, pp. 387f.).

4 For large n and x the binomial distribution can be approximated by the normal distribution. If so, (1) can be treated as differentiable.

5 If quality is a decision variable for the producer and if consumers differ with respect to income, then we would expect consumers to be distributed over many different qualities in equilibrium, with e.g. high–income consumers consuming high qualities and low–income consumers consuming low qualities. For a model with these equilibrium properties, see Flam and Helpman (1987).

6 A similar result can be found in Heal (1977b). His approach is different from ours, however. He models quality as the tightness of the spread of a good's performance around a constant mean. A decrease in the tightness of the distribution around the constant mean implies a fall in quality. Heal finds that the existence of a substitute for the good with uncertain quality reduces the likelihood that demand increases as quality falls. This can be shown in our model as well by letting $\varphi > 0$ so that the interaction term φxz becomes positive.

7 This is not always the case. If the cost function is quadratic, i.e. $c(q) = c_0 + c_1 q + c_2 q^2$, the solution to (8) is

$$q_C = [c_0/(c_2 - \beta)]^{1/2}$$

We see that for $c_0 = 0$, we get the meaningless solution $q_C = 0$. Note also that it must be that $c_2 - \beta > 0$.

8 In these models, quality is defined as <u>durability</u>. Durability is assumed to be deterministic, and thus there is no scope for warranties or other risk–sharing arrangments between consumers and producers. This can be easily illustrated by the following example. Assume that the cost function is quadratic. As is pointed out in footnote 7 above, the optimal quality in our model, where product performance is uncertain, will then be

$$q_M = q_C = [c_0/(c_2 - \beta)]^{1/2}$$

Spence's formulation would however yield

$$q_M = q_C = [c_0/c_2]^{1/2}$$

In that case, the risk aversion parameter β in the consumer's utility function does not appear in the expression for optimal quality.

9 With a quadratic cost function $c(q) = c_0 + c_1 q + c_2 q^2$ we have that $q = [c_0/(c_2 - \beta)]^{1/2}$ and $q^W = [(c_0 - R)/c_2]^{1/2}$, i.e. that $q > q^W$ if $R < c_0\beta/(c_2 - \beta)$, and $q < q^W$ otherwise.

10 Heal (1977b) also analyses the effects of a regulated guarantee on the profits of a monopolist to determine if the monopolist will be induced to lower quality. Heal concludes that the presumption is the opposite to ours, namely that the monopolist if anything tends to raise quality.

11 Here we have made use of the fact that, under any warranty regime i, $n_C^i = 2n_M^i$ and $q_C^i = q_M^i = q^i$.

12 For brevity, we will write U_i instead of $E[U_i | n_i]$ and $E[x_i]$ instead of $E[x_i | n_i]$.

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