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Essays on Macroeconomics and Political Economy

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Abstract

This thesis consists of three self-contained essays dealing with different aspects of macroeconomics and political.

The Relative Price of Investment Goods and Sectoral Contract Dependence: We develop a quantitative model to explain the relationship between TFPs on the aggregate and sector levels and contracting institutions across countries. First, we document the empirical fact that investment goods sector is more contract dependent than the consumption goods sector. The model shows that incomplete contract enforcement induces distortions in the production process which come from the "hold up" problem between a final goods firm and its suppliers. The poorer is the contract enforcement institution, the larger is the distortion. Because investment goods sector is more contract dependent, its productivity suffers more from the distortion. In turn, countries endowed with weaker contract enforcement institutions face higher relative prices of investment goods because of the relative inefficiency of producing investment goods, invest a lower fraction of their income, and end up being poor. We find that the proposed mechanism is quantitatively relevant.

A Ricardian Model of Labor Market with Directed Search In this paper, I analyze how search frictions will affect the allocation in a Ricardian model of the labor market. The equilibrium shows that the matching pattern is partially mixed: Some tasks are only performed by skilled workers; some are only performed by unskilled workers; the remaining tasks are performed by both skilled and unskilled workers. The mixed matching pattern implies a mismatch in equilibrium. It turns out that the reason for the mismatch has its roots in search friction. I calibrate the model and show that the magnitude of the mismatch

is quantitatively small. The model also generates wage inequality among identical workers, as well as between-skill inequality. In addition, I analyze the effect of labor market institutions such as unemployment benefits and labor income taxation. These labor market institutions have interesting implications for the unemployment rate and mismatch.

A Dynamic Analysis of the Free Rider Problem: How Distorted Policy Help Special Interest Group Organized In this paper, I argue that special interest groups overcome their free-rider problem thanks to distorted government policy. As public policy confers monopoly privileges on a special interest group, it can also preserve and promote group organization. The key in my story is a dynamic incentive: when distorted policy generates rents for an interest group, each member of the group wish to make contributions not just to raise their rents today; they want to sustain their cooperation so that they will be able to influence policy in the future. Our theory predicts that overcoming of free-rider problem also lead to inefficient policy persistence.

To Yanhong Yu

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Contents

Chapter 1

Introduction

This thesis consists of three self-contained essays on issues related to the macroeconomics of climate change. The first two chapters are relatively similar in terms of the questions asked and the models used to answer them. They both use neoclassical growth models where the world is treated as one large economy. They both consider issues of intertemporal incentives for fossil-fuel use and the use of taxation to correct for the externalities through climate change caused by the burning of fossil fuels. One might say that these two chapters deal with allocation over time. The third chapter, instead, uses models with many countries and considers how effects of climate change propagate between countries through market mechanisms. That is, it considers allocation across countries.

Climate change has become a topic of intense public debate in recent years. One contributing factor to this was the publication of the Stern Review (Stern, 2007). The basic mechanisms that are driving climate change have been known for a long time. More than a hundred years ago the increase in the global temperature following an increased concentration of greenhouse gases in the atmosphere was calculated fairly accurately. At that time, however, this was not necessarily considered a threat (for instance, the Swedish chemist and physicist Svante Arrhenius who was one of the pioneers thought, understandably, that a warmer climate might well be beneficial). Over time, the problems and risks associated with climate change have become more and more apparent.

The Intergovernmental Panel on Climate Change was created in 1988.[1] It has since then published four assessment reports and a fifth is scheduled to be published in 2013 and 2014. The fourth assessment report, published in 2007, received much attention and the organization, together with Al Gore, was awarded the 2007 Nobel Peace Prize.

Climate change is not a new topic within economic research either. Perhaps the best known economist working with these issues is William Nordhaus; he has studied the interaction between the climate and the economy since the 1970s. Nordhaus has also developed one of the most widely used family of tools that jointly model the economy and the climate: the DICE/RICE models.[2] While his importance for bringing together models of the climate and the economy is difficult to overestimate, these models (and most other so called IAMs, i.e., integrated assessment models) have a problem: they are highly complex and difficult to use for qualitative interpretation. One reason for this is that they consist of a large set of equations that can only be solved numerically.

The Mistra SWEdish research programme on Climate, Impacts and Adaptation (Mistra-SWECIA), which I have been a part of, was started in 2008. One of the main purposes of the macroeconomic modeling part of the programme was to approach the problem somewhat differently. The economic part of the models should be based on modern macroeconomic theory, making the models accessible to mainstream macroeconomists. The models should also be more transparent.

The work in this thesis very much reflects this aim for transparency. Rather than using large complex models, the chapters in this thesis explores qualitative issues using tractable models. I also think that the results derived in the thesis point to the value of this approach. When setting up an integrated assessment model, a number of assumptions

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[1] See www.ipcc.ch.

[2] DICE and RICE stands for "Dynamic Integrated model of Climate and the Economy" and "Regional dynamic Integrated model of Climate and the Economy", respectively. See, e.g., Nordhaus & Boyer (2000) for a description of the models.

must be explicitly or implicitly made. These assumptions can completely change the qualitative behavior of the model.

One important part of any climate-economy model is the supply of fossil fuels. Fossil-fuel resources are finite (at relevant time scales) and there is a cost of extracting them. An important question is which of these aspects of the resource is more important for extraction decisions. If the finiteness, or scarcity, of the resources is more important, comparing the value of fossil-fuel use at different points in time will be an important driver behind the intertemporal pattern of extraction. If, instead, the costs of extraction are more important, the extraction decisions will be more about weighing current extraction costs against the current value of fossil-fuel use at each point in time.

Another important issue is alternative-energy generation. Large reductions in fossil-fuel use without large reductions in material well-being will require a rapid increase in the use of energy generated by alternative sources. The way that the alternative-energy generation is modeled can have significant consequences for the behavior of IAMs. For example, if the production function is assumed to have some degree of complementarity between energy and other inputs the alternative-energy assumption becomes important. If the model abstracts from alternative energy, or if alternative energy is exogenously given, the complementarity between energy and other inputs translates into complementarity between fossil fuel and other inputs. If, instead, the capacity for generating alternative energy comes from use of inputs such as installed capital for energy generation, this implies something very different regarding the complementarity between fossil fuel and other inputs.

Furthermore, the functional forms for the production and utility functions must be specified. A relatively common assumption regarding the production function is that energy is combined with other inputs, such as labor and capital, according to a Cobb-Douglas production function. In both chapters 2 and 3, it can be seen that this assumption, especially if combined with the assumption that the utility function is logarithmic, significantly simplifies the analysis. It can, however, also be seen that these assumptions take away some mechanisms that would be present if more realistic assumptions were made.

It may not be a very surprising conclusion that the assumptions made when building a model affects the results that the model delivers. I would, however, argue that the different possibilities that I consider in this thesis lie within the span of model assumptions used and that conclusions are sometimes drawn that rely on the particular assumptions made. At the same time, the quantitative basis for making these assumptions is sometimes weak. Thus, while the analysis in this thesis often stops at the point where the consequences of making the different possible assumptions have been determined, this points to fruitful avenues for future research. Quantitative analysis of these possible choices is needed to find out what the "right" assumptions are and the quantitative consequences for model output such as optimal taxes on fossil-fuel use must be determined.

Chapter 2, Technological Trends and the Intertemporal Incentives for Fossil-Fuel Use, analyzes how (the expectations about) the future developments of different kinds of technology affect the intertemporal incentives for fossil-fuel use. Given that fossil-fuel resources are finite, the decision of when to extract should be based on the value of fossil-fuel use at different points in time. This means that the expectations about the future value of fossil-fuel use matters also for the extraction decisions made today. The future development of technology is an important determinant of this future value. The literature on the Green Paradox (see van der Werf and di Maria, 2011, for a survey of this literature) has recognized the importance of this aspect of the fossil-fuel supply for the effects of policies aimed at reducing the emissions of CO_2 from the burning of fossil fuels. What is found in this literature is that announced policies that reduce the future value of fossil-fuel use will tend to increase the current amount of fossil-fuel use and thereby potentially exacerbate the problem of climate change. The commonly discussed policies are announcements of higher future taxes on fossil-fuel use or investments that will increase the future supply of alternative energy.

The topic of this chapter is to consider how (the expectations about) the future developments of a wider range of technologies affect the intertemporal incentives for fossil-fuel use. The technology trends that I consider are technology for: alternative-energy generation, fossil-fuel based energy generation, energy savings, productivity of other (complementary) inputs, i.e., labor and sometimes capital, and total factor productivity (TFP). The analysis in this chapter is carried out using neoclassical models. I use these models to determine the effect of a future change in the state of each of the technologies on the path of fossil-fuel use. The general conclusion is that improvements in (the expectations about) the future state of technologies for alternative-energy generation, energy efficiency and TFP all increase fossil-fuel use before the change takes place. The effect of changes in the efficiency of non-energy inputs is the reverse, while the effect of changes in fossil-fuel based energy technology is ambiguous. These conclusions are robust to a number of different possible assumptions. Thus, the effects of changes in the future technology for alternative-energy generation and energy efficiency confirm the findings in the Green Paradox literature.

The analysis indicates that the joint effects of all technology trends should be considered rather than looking at one type of technology in isolation. In reality technology trends are the results of research. Increasing spending on one type of research will typically have effects also on the amount of research on other types technologies, e.g., through crowding out.

Throughout this chapter, I emphasize the scarcity aspect of the fossil-fuel supply. This seems to be the crucial assumption. If fossil-fuel supply is, instead, mostly driven by extraction costs, some results may be reversed.

Chapter 3, The Role of the Nature of Damages, considers different ways in which climate change can be assumed to affect the economy (e.g. through various damages) and to what extent the choice of how to model these climate effects matters. The most common way to introduce the effects of climate change into economic models is to assume that it affects productivity or utility. Some of the expected effects of climate change, e.g., storms and floods, rather destroy the capital stock. Modeling the effects of a climate change as increased depreciation of capital therefore seems plausible. In this chapter I consider to what extent it matters whether climate is assumed to affect productivity, utility or the depreciation of capital.

I carry out my analysis in two different ways. Firstly, under some simplifying assumptions, I derive a formula for the optimal tax on fossil-fuel use. The optimal tax at each point in time can be written as a constant times current production, where the constant adds up the three different effects that climate change has on the economy. Golosov et al. (2011) derive a similar formula for the optimal tax when considering only climate change effects on productivity. The formula derived in chapter 3 can therefore be seen as a generalization of that formula. The assumptions I make in order to derive the formula are also similar.

Secondly, I use a two-period model with exogenous climate to analyze how the allocation of fossil-fuel use over time is affected by the effects of climate change. I consider two different cases for the fossil-fuel supply: an oil case, where the resources are finite but I abstract from extraction costs, and a coal case, where I abstract from the finiteness of the resource but extraction requires the use of inputs. I find that, for both the oil and coal cases, a decrease in second-period productivity and a worsening of the second-period climate state have the same qualitative effects on the allocation of fossil-fuel use while an increase in the depreciation of capital has the opposite effect. The effects are also very different in the coal case compared to the oil case.

In the second part of this chapter, I treat climate as exogenous. The derived effects are still indicative of how a decentralized equilibrium would respond to expected climate change. I then ask whether these reactions to climate change will amplify or dampen climate change. Answering this question requires assumptions about how to best represent the effects of climate change in a two-period model and what constitutes amplification of climate change in the oil and coal cases, respectively. Under the interpretation I choose, climate effects on productivity or utility will dampen climate change in the oil case and amplify it in the coal case. Conversely, climate effects on the depreciation of capital will amplify climate change in the oil case, at least if the supply of alternative energy is exogenously given, but dampen it in the coal case.

Chapter 4, Indirect Effects of Climate Change, investigates how direct effects of climate change in some countries have indirect effects on other countries going through changing world market prices of goods and financial instruments. The direct effects of climate change are expected to differ a great deal across different countries. However, since the economies of countries are interconnected in various ways the direct effects will be propagated between countries through market mechanisms. This means that when calculating the total effects of climate change these indirect effects must also be taken into account.

In this chapter I consider two such channels: trade in goods and trade in financial instruments. For both of these channels the indirect effects go through changing world-market prices of goods and financial instruments. If climate change decreases the productivity of a country that is a net exporter of a good, the world market price will go up, decreasing the welfare in countries that are net importers of that good. Weather events cause uncertainty. Financial instruments can be used to decrease this uncertainty by offering insurance against bad outcomes. The probability distribution of weather events is expected to change due to climate change. This means that the world market prices of financial instruments will change as the probability distribution of weather events changes. The indirect effects going through the price changes of assets will benefit or hurt countries depending on whether they are net buyers or net sellers of the assets.

Climate change depends primarily on total global emissions of greenhouse gases while the geographical source of the emissions are largely irrelevant. This means that cost-efficient mitigation of climate change (reduction of emissions of greenhouse gases) requires reductions in all countries. The uneven distribution of the effects of climate change poses a problem for efficient mitigation since countries willingness to participate in mitigation efforts can be expected to be closely related to the costs from climate change they are expected to suffer. This is made worse by the fact that there seems to be a negative correlation between emissions of greenhouse gases and the vulnerability to climate change. Since the indirect effects of climate change will give a different distribution of the total effects compared to the distribution of direct effects, these indirect effects can make it easier or more difficult to reach agreements about mitigation efforts depending on whether the indirect effects make the countries' interests more or less aligned. The net effects will depend on the relation between the direct effects and the trade patterns. I argue, based on a stylized two country example, that trade in goods will tend to make the countries' interests more aligned while trade in financial instruments will tend to make the countries' interests less aligned.

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Chapter 2

The Relative Price of Investment Goods and Sectoral Contract Dependence

2.1 Introduction

This paper is about understanding why GDP per worker co-varies positively with the PPP-adjusted investment rate and negatively with the relative price of investment goods. In this paper, we find cross-country differences in the quality of contract enforcement institutions to be an important factor for such a pattern.

Heston and Summers (1988, 1996) first emphasized the rate of investment at international prices versus that at domestic prices. When investment goods are valued using international prices, investment rates are strongly positive correlated with the income level. But when domestic prices are used, the positive association becomes much weaker; see figure 2.1 and figure 2.2 which illustrate such a pattern for 189 countries in 2005 and are constructed using data from Penn World Table version 6.3. Whereas the correlation between the purchasing power parity (PPP) investment rate and PPP GDP per worker is 0.504, that between the domestic price investment rate and PPP GDP per worker is only 0.137. At domestic prices, poor countries do not invest much less than do rich countries. This evidence suggests that the domestic relative price of investment accounts for the difference between investment rates at domestic prices versus those at international prices and is much higher in poor countries. The fact that the relative price of investment goods to consumption goods in poor countries is much higher is reported by Delong and Summers (1991), Easterly (1993), and Jones (1994) and documented in Figure 2.3.

There are two closely related rationalizations for the above evidence. Chari, Kehoe and McGrattan (1996) and Restuccia and Urrutia (2001) emphasize the cross-country distortion of the investment, i.e., the variation in investment distortion is responsible for this. Instead, Hsieh and Klenow (2007) argue that poor countries have a higher relative price of investment goods simply because they are relatively less efficient in the production of these goods. This would make investment goods relatively more expensive, thus lowering the PPP-adjusted investment rates. Recently, Herrendorf and Valentinyi (2010) show that in the equipment and construction sectors (investment goods sector), the sectoral TFP differences between developing countries and the United States are much larger than in the aggregate. However, in manufactured consumption, the sectoral TFP differences are about equal to the aggregate TFP differences, and they are much smaller in services. Their research echoes the point made by Hsieh and Klenow (2007). The next challenge is to understand the origin of either form of cross-country heterogeneity (in relative TFPs or in wedges). In this paper, we provide a micro-foundation for the variation in the difference in sectoral production efficiency and evaluate its economic significance.

Our model strategy follows the framework developed by Acemoglu, Antras and Helpman (2007) which analyzes the relationship between contractual incompleteness, technological complementarity, and technology adoption. They find that greater contractual incompleteness leads to the adoption of less advanced technologies and differences in contractual institutions generate endogenous comparative advantages across countries. Like Acemoglu, Antras and Helpman (2007), our model combines several well-established approaches. The first is the representation of technology as the range of intermediate inputs used by firms; a greater range of intermediate inputs increases productivity by allowing greater specialization and thus corresponds to more "advanced" technology. The second is that when investments are relation-specific, underinvestment will occur if contracts are incomplete *à la* Williamson (1975, 1985), Grossman and Hart (1986), Hart and Moore (1990), and Caballero and Hammour (1998). Specificity in a relationship reduces the flexibility of a separation decision, which induces a reluctance in the investment decision. To avoid this "hold up problem", it needs prior protection through a comprehensive and enforceable contract. This combination enables us to investigate how the contractual incompleteness exerts an unbalanced effect on the production efficiency of different sectors with a different range of intermediate inputs.

We embed the incomplete contract and relation-specific investment into a fairly standard two-sector growth model with capital accumulation. Two sectors produce investment goods and consumption goods, respectively. All activities undertaken by suppliers are relation-specific, and a fraction of those are *ex ante* contractible, while the rest are nonverifiable and noncontractible. The range of contractible activities depends on the contracting institutional quality. Countries have access to the same technologies. However, they differ in the quality of contract enforcement institutions. Our key assumption is that firms producing investment goods need more (or more complex) intermediate inputs which need relation-specific investments, i.e. the investment goods sector need to negotiate contracts with more suppliers. Data drawn from the US Input-Output provides strong support for this hypothesis. The fraction of contractible activities is our measure of the quality of contracting institutions. Suppliers are contractually obligated to their duties in the contractible activities, but they are free to choose their investments in noncontractible activities and withdraw their services. This combination of noncontractible investments and relationship-specificity leads to an *ex post* multilateral bargaining problem. Here we solve this multilateral bargaining problem by an exit game *à la* Krishna and Serrano (1996) to determine the division of the *ex post* surplus between final goods producer and its suppliers. we derive an explicit solution for this division of surplus, which enables us to have a simple characterization of the equilibrium.

A supplier's expected payoff in the bargaining game determines her willingness to invest in noncontractible activities. Since she is not the full residual claimant of the output gains derived from her investment, she tends to underinvest. Greater contractual incompleteness thus reduces supplier investments, making production less efficient. Since the proportion of each supplier's *ex post* bargaining revenue is decreasing with the number of suppliers, a more complex production process will discourage the suppliers' investment. The distortion generated by incomplete contracts reduces the producing efficiency, more subtly the distortion is heterogeneous and more damaging to the sector with more intermediate inputs. Hence, this unequal efficiency loss leads to co-variation between the quality of the contracting institution and relative price of investment goods which is determined by sectoral relative productity. The result that countries with contracting institutions with worse quality display a higher relative price of investment is consistent with the pattern shown in Figure 2.4. This figure relates the relative price of investment to an index of the quality of the contracting institution for a cross-section of countries. As our primary measure of contracting institution quality, we use the "rule of law" index from Kaufmann, Kraay, and Mastruzzi (2003). This is a weighted average of a number of variables that measure individuals' perceptions of the effectiveness and predictability of the judiciary and the enforcement of contracts in each country in 2005.

We use our model to quantitatively analyze the cross-country patterns in aggregate and sector-level TFP. We discipline our analysis by requiring that the calibration matches the US data on the complexity of intermediate inputs across sectors, i.e. the share of intermediate inputs. We quantify the relationship between the quality of the contracting institution and aggregate economy performence (output per worker, aggregate/sector-level TFPs, and investment rate, etc). We find that the quality of the contracting institution has sizable effects on output per worker, aggregate and sector-level TFPs, and investment rate.

Our paper is closely related to those of Acemoglu, Antras and Helpman (2007) and Costinot (2009) who consider the effect of countries' contracting environments on comparative advantage. Levchenko (2007) and Nunn (2009) use a different classification of industries into more and less contract dependent groups. They find that countries with better contracting institutions specialize in exports of goods which are more contract dependent. Moreover, the impact of contractual institutions on exports is quantitatively large.[1]

Our paper is also closely related to recent contributions by Caselli and Gennaioli (2003), Restuccia and Rogerson (2008), Castroet al (2004, 2009), Esosa and Cabrillana (2008), Buera et al (2009) and Guner, Ventura and Xu (2008). In common with these authors, we examine the implications of allocative inefficiencies for economics development. Our paper is most closely related and complementary to two papers in the literature that emphasize the effect of financial frictions on the relative price of investment. Castro et al (2009) start from the premise that the investment goods sector is characterized by more volatile underlying idiosyncratic productivity shocks. They show that in economies with less risk-sharing, sectors with more volatile shocks (investment goods) are particularly unproductive. Buera et al (2009) begin with the observations that cross-sector differences in the scale of establishments which is defined as workers per establishment, then sectors (e.g., manufacturing) with larger scales of operation have more financing needs, and are hence disproportionately vulnerable to financial frictions. Therefore, they show

[1] In this paper, I also adopt two classifications used by Levchenko (2007) and Nunn (2009). Interestingly, I find investment goods to be more contract dependent than consumption goods.

that financial frictions can account for a substantial sector-level relative productivity.

The rest of the paper is organized as follows. In section 2.2, we provide evidence in support of our assumption on the cross sectoral variation in contract dependence. We introduce the model in section 2.3, and characterize the partial equilibrium. In section 2.4, we define and characterize the dynamic general equilibrium allocation. In section 2.5, we describe our calibration procedure and the quantitative comparative statics assessment. Finally, in section 2.6, we conclude the paper. Proofs of the main results are provided in the Appendix.

2.2 Empirical Evidence

In this section, we provide evidences supporting our premise that investment goods sectors are more contract dependent, i.e. the investment goods sectors need to establish more specific-relations with their intermediate inputs suppliers.

Here we use two measures of sector-level contract dependence. The first index is a measure of production complexity. Here we use the Herfindahl index of intermediate input use computed from the US input-output use table for 1997. The Herfindahl index has been used as a measure of production complexity and institutional dependence before: for example, Blanchard and Kremer (1997) use this index to measure the complexity of intermediate inputs and Levchenko (2006) uses this index as a measure of institutional dependency. we define the index of complexity (i.e., The Herfindahl index) for sector i, called H_i as

$$H_i = \sum_j (\phi_{ij})^2$$

where ϕ_{ij} is the share of intermediate input j in the production of final good i. The reason for using it rather than simply using the number of intermediates employed in production is the following. If intermediate input use is dominated by one or two inputs (high concentration), and all other intermediates are used to a very small extent, then what really matters to the final good producer is its relationship with the largest one or two suppliers. The scope for and importance of expropriation by suppliers of minor inputs is probably much smaller than that by important suppliers. Thus, simply taking the number of intermediates may give excessive weight to insignificant input suppliers and overestimate the effective reliance on institutions.

The second index is a measure of the importance of relation-specific investments across industries. we construct a index that directly measures the relation-specificity of intermediate inputs used in the production process. This variable is used in Nunn (2007). This variable using data from Rauch (1999) identifies which intermediate inputs that require relationship-specific investments. As an indicator of whether an intermediate input is relationship specific, he uses whether it is sold on an organized exchange and whether it is reference priced in a trade publication. If an input is sold on an organized exchange or reference priced in a trade publication, the market for this good is thick, with many alternative buyers. If many buyers for an input exist, then the scope for the hold-up problem is limited. If a buyer attempts to renegotiate a lower price, the seller can simply take the input and sell it to another buyer. The variable measures the proportion of intermediate inputs in each I-O category that are neither bought and sold, nor reference priced, i.e.

$$Z_i = \sum_j \phi_{ij} R_j$$

where ϕ_{ij} is the share of intermediate input j in the production of i and R_j is the proportion of inputs j that are neither sold on an organized exchange nor reference priced.

Our next task is to assign each of the sectors to different categories. We rely on the 1997 benchmark input-output use table for the US of the Bureau of Economic Analysis. The use table tells us the fraction of output that flows from each six-digit sector to any of the other six-digit sectors and to final demand, respectively. we first group the usage of a product into three categories, intermediate (T), consumption (C) and investment (I). This is done by aggregating personal, federal, and state consumption expenditures into a single consumption category and similarly for investment expenditures. As for the intermediate category, we aggregate intermediate expenditures of 483 private sectors and federal and state enterprises enterprises. Since the use table does not provide a breakdown of imports, exports, and changes in inventories into consumption and investment, we choose to ignore these final demand items. For each six-digit sector j, we compute the share of output destined to intermediate uses, $\frac{Y_T(j)}{(Y_T(j)+Y_C(j)+Y_I(j))}$, then we rule out the sectors with a contribution to final demand of less than 50% of their output. For each remaining sector j, we compute the share of final demand destined for consumption, $\frac{Y_C(j)}{(Y_C(j)+Y_I(j))}$. We assign all remaining sectors with a share of no less than 50% to consumption good sectors and those with a share of more than 50% to the investment good sectors. Our primary empirical results are essentially the same when we change the threshold.

We show that two sectoral measures of contract dependence in the investment good sector are higher than those in consumption sectors. The Herfindahl indexes of different consumption good and investment sectors are summarized in table 2.1. For the Herfindahl index, we report the average and weighted average Herfindahl index for two sectors. The weighted average Herfindahl is calculated by

$$H_h = \sum_j \left(\frac{M_h(j)}{M_h} \right) H_h(j) \ \ , \ h = I, C$$

where $M_h(j)$ is the output of sector j which is defined either as consumption goods or investment goods and M_h is the aggregate output of consumption goods or investment goods. Two measures of the Herfindahl index in two sectors indicate that the investment goods sector has a higher Herfindahl index than the consumption goods sector. To give you an intuitive impression of the relation between a Herfindahl index and the complexity of an intermediate input, we assume that all inputs are symmetric and list the number of intermediate inputs and the corresponding Herfindahl index value. As for the specificity measure, we can clearly see that this measure is on average higher in the investment goods sector. In turn, it implies that the investment goods sector is the more contract intensive sector.

Table 2.1: Sectoral contract dependence

Sector	Average H_i	Weighted Average H_i	Average Z_i
Final goods	0.1013		0.60
Consumption goods	0.1239	0.1532	0.52
Investment goods	0.0618	0.0568	0.71

Table 2.2: Herfindahl index and the number of intermediate inputs

N	7	8	9	10	11	12	13	14	15	16	17
H_i	0.14	0.13	0.11	0.1	0.09	0.08	0.08	0.07	0.07	0.063	0.06

To address whether empirical regularity still holds in the developing country, we also use the China Input-output use table in 2007 to calculate the Herfindahl index of intermediate input use. The result shown in table 2.3 confirms the conclusion that the investment goods sector uses more complex intermediate inputs.

Table 2.3: China's sectoral Herfindahl index

Sector	Herfindahl index H_i
Final goods	0.1182
Consumption goods	0.1275
Investment goods	0.0910

One caveat should be noted, however. The Herfindahl index computed from China's data should not be quantitatively compared with values from US data. The reason is that the US input-output table is defined by the six-digit sector (483 sectors), while the Chinese input-output matrix uses a three-digit sector (135 sectors) definition. But we can still get the qualitative implication that investment goods sectors use more complicated intermediate inputs than consumption sectors.

We conducted a series of robustness checks. Our results do not change in any appreciable way when we adjust the sample selection criteria.

2.3 Model

First, we will present the simple one-period model given the capital stock K and labor endowment L. In the next section, we will embed this one-period model into an infinite horizon two-sector growth model.

2.3.1 Technology

We consider a simple standard two-sector model. The production technologies in the consumption and investment sectors are

$$I = A_I[\sum_{i=1}^{N_I} q_I(i)^\alpha di]_I^{\frac{m}{\alpha}}[{K_I}^\gamma {L_I}^{1-\gamma}]^{1-m} \quad , \quad 0 < \alpha < 1 \tag{2.1}$$

$$C = A_C[\sum_{i=1}^{N_C} q_C(i)^\alpha di]_I^{\frac{m}{\alpha}}[{K_C}^\gamma {L_C}^{1-\gamma}]^{1-m} \quad , \quad 0 < \alpha < 1. \tag{2.2}$$

Variables A_I and A_C are two sector's productivities. $q\,(i)$ is the quantity of intermediate input i. N_I and N_C are the varieties of intermediate input and thus, the degree of specialization in two sectors, respectively. α determines the degrees of complementarity between inputs; since $\alpha \in (0, 1)$, the elasticity of substitution between them $1/\,(1-\alpha)$ is always greater than one. There is a large number of profit-maximizing suppliers that can produce the necessary intermediate goods. We assume that each intermediate input needs to be produced by a different supplier with whom the firm needs to have a contract. A supplier assigned to the production of an intermediate input needs to undertake relationship-specific investments in a unit measure of (symmetric) activities, each activity $q(i,j)$ is produced by technology $K(i,j)^{\gamma}L(i,j)^{1-\gamma}$. Here we make one key assumption: $N_I > N_C$, which means that the production of investment goods needs more varieties of intermediate inputs than the production of consumption goods and the production of intermediate inputs involves the relationship-specific investment. The production function of intermediate inputs is Cobb-Douglas and it is symmetric in the activities:

$$q_I(i) = \exp[\int_0^1 \ln(K_{qI}(i,j)^{\gamma} L_{qI}(i,j)^{1-\gamma} dj] \tag{2.3}$$

$$q_C(i) = \exp[\int_0^1 \ln(K_{qC}(i,j)^{\gamma} L_{qC}(i,j)^{1-\gamma} dj]. \tag{2.4}$$

This formulation allows a tractable derivation of contractual incompleteness case in section 2.4, where a subset of the investments necessary for production are noncontractible.

2.3.2 Complete contract

Let the prices of capital goods and consumption goods be P_I and P_C, respectively. We choose the consumption good as the numeraire which means there is no loss of generality in setting the absolute prices to $P_C \equiv 1$ and $P_I \equiv P$. The interest rate and the wage rate are R and W, respectively. Given the price of investment goods P and the factor prices R and W, we obtain the following maximization problem of the investment goods firm:

$$\max_{K_I, L_I} A_I[\sum_{i=1}^{N_I} q_I(i)^{\alpha} di]^{\frac{m}{\alpha}} [{K_I}^{\gamma} {L_I}^{1-\gamma}]^{1-m} P - \sum_{i=1}^{N_I} (\tau_I(i) + S_{qI}(i)) - RPK_I - WL_I$$

$$s.t \;\; (\tau_I(i) + S_{qI}(i)) - RPK_{qI}(i) - WL_{qI}(i) = 0.$$

The payment to supplier i consists of two parts: an *ex ante* payment $\tau_I(i) \in \mathbb{R}$ before the investment level $\{K_{qI}(i,j), L_{qI}(i,j)\}$ is reached, and a payment $S_{qI}(i)$ after the investments. The second equation is the suppliers' participation constraint and we assume that the value of the suppliers' outside option is zero.

Similarly, we obtain the consumption goods producer's problem

$$\max_{K_C,L_C} A_C[\sum_{i=1}^{N_C} q_C(i)^\alpha di]^{\frac{m}{\alpha}}[K_C{}^\gamma L_C{}^{1-\gamma}]^{1-m} - \sum_{i=1}^{N_C}(\tau_C(i) + S_{qC}(i)) - RPK_C - WL_C$$

$$s.t \ \ (\tau_C(i) + S_{qC}(i)) - RPK_{qC}(i) - WL_{qC}(i) = 0.$$

Proposition 2.1. *With complete contracts, there exists a unique equilibrium with the relative price of investment goods to consumption goods*

$$P = \frac{A_C}{A_I}\left(\frac{N_C^{\frac{m}{\alpha}-m}}{N_I^{\frac{m}{\alpha}-m}}\right).$$

Proof. See the Appendix. □

Proposition 2.1 implies that the relative price of investment goods to consumption goods is inversely related to A_C/A_I and the number of intermediate firms. The most important difference from Hsieh and Klenow (2007 AER) is the term $N_C^{m/\alpha-m}/N_I^{m/\alpha-m}$; here the number of intermediate inputs is also presented as a part of sectoral TFPs. It reflects the idea that a greater range of intermediate inputs increases productivity by allowing greater specialization and thus corresponds to more "advanced" technology as discussed by Romer (1990) and Grossman and Helpman (1991).

2.3.3 Incomplete contract

We now consider the same environment under incomplete contracts. We model the imperfection of the contracting institutions by assuming that there exists a $\mu \in [0, 1]$ such that, for every intermediate input j, investments in activities $0 \leq j \leq \mu$ are contractible, while investments in activities $\mu < j \leq 1$ are not contractible. Here, μ can be considered as a shortcut measure of the quality of the contracting institution. The better is the contract enforcement institution, the greater proportion of activities can be contracted, i.e. larger μ. Consequently, a contract stipulates investment levels $\{K_{qh}(i, j), L_{qh}(i, j)\}_{h=I,C}$ for the μ contractible activities, but does not specify the investment levels in the remaining

$1-\mu$ noncontractible activities. $\mu = 1$ corresponds to the case under the complete contracts. Instead, suppliers choose their investments in noncontractible activities in anticipation of the *ex post* distribution of revenue, and may decide to withhold their services in these activities from the firm. We assume the *ex post* distribution of revenue to be governed by multilateral bargaining. In this paper, we adopt the strategic bargaining setting of Jun (1987), Chae and Yang (1994) and Krishna and Serrano (1996) as the solution concept for this multilateral bargaining game.

Timing of events

- The final goods firms offer contracts $[\{K_{qh}(i,j), L_{qh}(i,j)\}_{j=0}^{\mu}, \tau_h(i)]_{h=I,C}$ and make the investment $\{K_h, L_h\}_{h=I,C}$ for every intermediates input $i \in [0, N_h]$, where the investment level in a contractible activity is $\{K_{qh}(i,j), L_{qh}(i,j)\}_{j=0}^{\mu}$ and $\tau_h(i)$ is an up-front payment to supplier i. The payment $\tau_h(i)$ can be positive or negative.

- Potential suppliers decide whether to apply for the contracts. Then, the firm chooses N_h suppliers, one for each intermediate input i.

- All suppliers $i \in [0, N_h]$ simultaneously choose investment levels $\{K_{qh}(i,j), L_{qh}(i,j)\}$ for all $j \in [0,1]$. In contractible activities $j \in [0, \mu]$, they invest $\{K_{qh,A}, L_{qh,A}\}$ as the contract, in noncontractible activities $j \in [\mu, 1]$ they invest $\{K_{qh,B}, L_{qh,B}\}$.

- The suppliers and the firm bargain over the division of revenue and, at this stage, suppliers can withhold their services in noncontractible activities.

- Output is produced and sold, and the revenue RE_h is distributed according to the bargaining agreement.

We characterize the symmetric subgame perfect equilibrium (SSPE) where bargaining outcomes are determined by the strategic bargaining game afterwards.

Definition of Equilibrium

Behavior along the symmetric subgame perfect equilibrium can be described by a tuple $\{K_{qh,A}, L_{qh,A}, K_{qh,B}, L_{qh,B}, K_h, L_h\}_{h=I,C}$, where $\{K_h, L_h\}_{h=I,C}$ represents the level of investment made by final goods firms, $\{K_{qh,A}, L_{qh,A}\}_{h=I,C}$ the investment in contractible activities, $\{K_{qh,B}, L_{qh,B}\}_{h=I,C}$ the investment in noncontractible activities, and $\tau_h(i)_{h=I,C}$ the up-front payment to each supplier in two sectors, respectively. The SSPE can be characterized by backward induction. First, consider the penultimate stage of the game, with all investments being sunk. Given these investments, suppliers and final goods firms engage in multilateral bargaining. Denote the bargaining revenue of supplier i under these circumstances by $\{S_{qh}(i)\}_{h=I,C}$. We will derive an explicit formula for this value in the following subsection. The noncontractible investment in activities j by supplier i $\{K_{qh,B}(i), L_{qh,B}(i)\}_{h=I,C}$ are chosen to maximize the *ex post* revenue $S_{qh}[K_{qh,A}, L_{qh,A}, K_{qh,B}, L_{qh,B}, K_{qh,B}(i), L_{qh,B}(i), K_h, L_h]$ minus the cost of production of noncontractible investment activities, $(1-\mu)(RPK_{qh,B}(i) + WL_{qh,B})$, given the investment in contractible activities and final goods firm. In a symmetric equilibrium, we need $K_{qh,B}(i) = K_{qh,B}$, $L_{qh,B}(i) = L_{qh,B}$; in other words, it must be a fixed-point given by:

$$\{K_{qh,B}, L_{qh,B}\} = \arg \max_{K_{qh,B}(i), L_{qh,B}(i)} S_{qh}[K_{qh,A}, L_{qh,A}, K_{qh,B}(i), L_{qh,B}(i), \\ K_{qh,B}, L_{qh,B}, K_h, L_h] - (1-\mu)\left(RPK_{qh,B}(i) + WL_{qh,B}(i)\right).$$

These equations can be considered as an "incentive compatibility constraint", with the additional symmetry requirement. In a symmetric equilibrium with final goods firms' investment $\{K_h, L_h\}_{h=I,C}$, with investment in a contractible investment given by $\{K_{qh,A}, L_{qh,A}\}_{h=I,C}$ and with a noncontractible investment equal to $\{K_{qh,B}, L_{qh,B}\}_{h=I,C}$, the aggregate revenue is given by $A_h P_h N_h^{\frac{m}{\alpha}}[(K_{qh,A}^{\gamma} L_{qh,A}^{1-\gamma})^{\mu}(K_{qh,B}^{\gamma} L_{qh,B}^{1-\gamma})^{1-\mu}]^m [K_h^{\gamma} L_h^{1-\gamma}]^{1-m}$, $h = I, C$. Moreover, by symmetric condition, the bargaining revenue of the final goods firm is obtained as a residual:

$$S_h[K_{qh,A}, L_{qh,A}, K_{qh,B}, L_{qh,B}, K_h, L_h] = A_h P_h N_h^{\frac{m}{\alpha}}[(K_{qh,A}^{\gamma} L_{qh,A}^{1-\gamma})^{\mu}(K_{qh,B}^{\gamma} L_{qh,B}^{1-\gamma})^{1-\mu}]^m$$
$$[K_h^{\gamma} L_h^{1-\gamma}]^{1-m} - N_h S_{qh}[K_{qh,A}, L_{qh,A}, K_{qh,B}, L_{qh,B}, K_h, L_h].$$

Now consider the stage in which the firm chooses suppliers. Suppliers expect to receive no less than their outside option which we assume as zero. Therefore, for production to take place, the final good producer has to offer a contract that satisfies the participation constraint of suppliers under incomplete contracts, i.e.,

$$S_{qh}(i)+\tau_h(i) \geq \mu(RP_h K_{qh,A}(i)+WL_{qh,A}(i))+(1-\mu)(RPK_{qh,B}(i)+WL_{qh,B}(i)).$$

In other words, each supplier should expect her bargaining revenue *ex post* plus the up-front payment to cover the cost of contractible and noncontractible investment.

The maximization problem of the final good firms can then be written as:

$$\max_{K_h, L_h, \tau_h} S_h[K_{qh,A}, L_{qh,A}, K_{qh,B}, L_{qh,B}, K_h, L_h] - N_I \tau_h - (RPK_h + WL_h)$$

subject to suppliers' "incentive compatibility constraint" and participation constraint.

With no restrictions on τ_h, the participation constraint will be satisfied with equality; otherwise the firm could reduce τ_h without violating the participation constraint and increase its profit. Therefore, we can solve τ_h from this constraint, substitute the solution into the firm's problem and obtain

$$\max_{K_{qh,A}, L_{qh,A}, K_{qh,B}, L_{qh,B}, K_h, L_h} S_h[K_{qh,A}, L_{qh,A}, K_{qh,B}, L_{qh,B}, K_h, L_h] + N_h[S_{qh} -$$
$$\mu(RPK_{qh,A} + WL_{qh,A}) - (1-\mu)(RPK_{qh,B} + WL_{qh,B})] - (RPK_h + WL_h)$$

subject to the "incentive compatibility constraint" of suppliers.

The SSPE $\{K_{qh,A}, L_{qh,A}, K_{qh,B}, L_{qh,B}, K_h, L_h\}_{h=I,C}$ solves this problem, and the corresponding up-front payment satisfies

$$\tau_h = \mu(RPK_{qh,A} + WL_{qh,A}) + (1-\mu)(RPK_{qh,B} + WL_{qh,B})$$
$$- S_{qh}[K_{qh,A}, L_{qh,A}, K_{qh,B}, L_{qh,B}, K_h, L_h].$$

Bargaining

We now derive the bargaining solution in this strategic game. Here, we adopt the strategic multilateral bargaining game *à la* Jun (1987), Chae and Yang (1994) and Krishna and Serrano (1996). As an illustration, let us describe the negotiation rules of the Krishna-Serrano exit game. The bargaining players are one final goods firm and N suppliers. First, the final goods firm will make a public proposal, a division of the revenue and the others must respond to it by accepting or rejecting the proposal. The responses are made simultaneously. Those who accept it leave the game with the shares awarded by the proposer, while the rejecters continue to bargain with the proposer over the part of the surplus that has not been committed to any player. A new proposal comes from one of the rejecters, and so on. Some features of this game are worth drawing attention to. First, in any subgame, the person who makes the offer will receive a payoff if and only if all other players accept her offer. Second, while the players who accept the offer receive the share immediately, this can happen in two different ways which are formally equivalent. Players could receive the amount from an existing "pie". But in this game, it is possible that no "pie" exists unless all players agree. The model can then be interpreted as one where the proposing player purchases the rights to represent players who accept the offer and pays the amount by borrowing at no cost outside the game. So, here we can understand the extensive game in this way. In this game, there is a unique solution, and the solution inherits the properties of Rubinstein's bilateral bargaining game, including its immediate agreement and the first proposer's advantage (the equilibrium shares are $1/\left[1+(N-1)\,\delta\right]$ for the first proposer and $\delta/\left[1+(N-1)\,\delta\right]$ for each responder). This is a relatively simple rule for the division of revenue between the firm and its suppliers. The formal derivation of the equilibrium can be found in the appendix.

Lemma 2.1. *Suppose that supplier i invests $\{K_{qh,A}(i), L_{qh,A}(i), K_{qh,B}(i), L_{qh,B}(i)\}$, and the level of final goods firms' investments is K_h. Then, the bargain-*

ing value of supplier i is

$$S_{qh}[K_{qh,A}, L_{qh,A}, K_{qh,B}, L_{qh,B}, K_h, L_h] = \frac{\delta}{1+N_h\delta} RE_h.$$

The bargaining value for final goods firms is

$$S_h[K_{qh,A}, L_{qh,A}, K_{qh,B}, L_{qh,B}, K_h, L_h] = \frac{1}{1+N_h\delta} RE_h.$$

where RE_h is the total revenue of the final good firms

$$RE_h = A_h P_h N_h^{\frac{m}{\alpha}} [K_{qh,A}^{\gamma} L_{qh,A}^{1-\gamma}]^{\mu m} [K_{qh,B}^{\gamma} L_{qh,B}^{1-\gamma}]^{(1-\mu)m} [K_h^{\gamma} L_h^{1-\gamma}]^{1-m}.$$

Proof. See the Appendix. □

The derived parameter $\frac{1}{1+N_h\delta}$ represents the bargaining power of final goods firm; it is decreasing in N_h and δ. More intermediate inputs, i.e., a higher N_h, decreases the final goods firm's bargaining power, because with more players, she gets a smaller share of the revenue. The parameter $\delta \in [0, 1]$ measures the patience of the players. Since the final goods firm is the first proposer, suppliers should be given more to induce them to accept her proposal If players are more patient.

Equilibrium

To characterize a SSPE, we first derive the optimal investment level of noncontractible activities by an intermediate goods producer:

$$\{K_{qh,B}, L_{qh,B}\} = \arg \max_{K_{qh,B}(i), L_{qh,B}(i)} \frac{\delta A_h P_h Q\left(K_{qh,A}, L_{qh,A}, K_{qh,B}, L_{qh,B}, K_{qh,B}\left(i\right), L_{qh,B}\left(i\right)\right)}{1+N_h\delta}$$
$$[K_h^{\gamma} L_h^{1-\gamma}]^{1-m} - (1-\mu)[RPK_{qh,B}(i) + WL_{qh,B}(i)],$$

where

$$Q\left(K_{qh,A}, L_{qh,A}, K_{qh,B}, L_{qh,B}, K_{qh,B}\left(i\right), L_{qh,B}\left(i\right)\right)$$
$$= \left\{\left(N_h - 1\right)[K_{qh,A}^{\gamma} L_{qh,A}^{1-\gamma}]^{\mu\alpha} [K_{qh,B}^{\gamma} L_{qh,B}^{1-\gamma}]^{(1-\mu)\alpha} + [K_{qh,A}^{\gamma} L_{qh,A}^{1-\gamma}]^{\mu\alpha} [K_{qh,B}^{\gamma}(i) L_{qh,B}^{1-\gamma}(i)]^{(1-\mu)\alpha}\right\}^{\frac{m}{\alpha}}.$$

Combining the first-order conditions of these problems and imposing the symmetric condition, we obtain

$$\frac{\delta m\gamma A_h P_h N_h^{\frac{m}{\alpha}-1}\left[\left(K_{qh,A}^{\gamma} L_{qh,A}^{1-\gamma}\right)^{\mu}\left(K_{qh,B}^{\gamma} L_{qh,B}^{1-\gamma}\right)^{(1-\mu)}\right]^{m}[K_h^{\gamma} L_h^{1-\gamma}]^{1-m}}{\left(1+N_h\delta\right) K_{qh,B}} = RP \tag{2.5}$$

$$\frac{\delta m\left(1-\gamma\right)A_{h}P_{h}N_{h}^{\frac{m}{\alpha}-1}\left[\left(K_{qh,A}^{\gamma}L_{qh,A}^{1-\gamma}\right)^{\mu}\left(K_{qh,B}^{\gamma}L_{qh,B}^{1-\gamma}\right)^{(1-\mu)}\right]^{m}[K_{h}^{\gamma}L_{h}^{1-\gamma}]^{1-m}}{\left(1+N_{h}\delta\right)L_{qh,B}}=W. \tag{2.6}$$

From FOCs, we obtain each supplier's cost of noncontractible activities

$$\begin{aligned}&(1-\mu)[RPK_{qh,B}+WL_{qh,B}]\\&=\frac{\left(1-\mu\right)\delta m}{1+N_{h}\delta}A_{h}P_{h}N_{h}^{\frac{m}{\alpha}-1}[K_{qh,A}^{\gamma}L_{qh,A}^{1-\gamma}]^{\mu m}[K_{qh,B}^{\gamma}L_{qh,B}^{1-\gamma}]^{(1-\mu)m}[K_{h}^{\gamma}L_{h}^{1-\gamma}]^{1-m}\\&=\frac{\left(1-\mu\right)m}{N_{h}}S_{qh}\end{aligned}$$

and solving for the fixed point by substituting $K_{qh,B}(i)=K_{qh,B}$ and $L_{qh,B}(i)=L_{qh,B}$ yields a unique $K_{qh,B}$ and $L_{qh,B}$:

$$K_{qh,B}=\frac{\gamma}{1-\gamma}\frac{W}{RP}L_{qh,B} \tag{2.7}$$

$$K_{qh,B}=\{\frac{\gamma\delta mA_{h}N_{h}^{\frac{m}{\alpha}-1}P_{h}}{\left(1+N_{h}\delta\right)RP}(\frac{\left(1-\gamma\right)RP}{\gamma W})^{(1-\gamma)(1-\mu)m}\}^{\frac{1}{1-(1-\mu)m}} \tag{2.8}$$
$$[K_{qh,A}^{\gamma}L_{qh,A}^{1-\gamma}]^{\frac{\mu m}{1-(1-\mu)m}}[K_{h}^{\gamma}L_{h}^{1-\gamma}]^{\frac{1-m}{1-(1-\mu)m}}$$

$$[K_{qh,B}^{\gamma}L_{qh,B}^{1-\gamma}]^{(1-\mu)m}=\left[\frac{\gamma\delta mA_{h}N_{h}^{\frac{m}{\alpha}-1}P_{h}}{\left(1+N_{h}\delta\right)RP}(\frac{\left(1-\gamma\right)RP}{\gamma W})^{(1-\gamma)}\right]^{\frac{(1-\mu)m}{1-(1-\mu)m}} \tag{2.9}$$
$$[K_{qh,A}^{\gamma}L_{qh,A}^{1-\gamma}]^{\frac{\mu m}{1-(1-\mu)m}(1-\mu)m}[K_{h}^{\gamma}L_{h}^{1-\gamma}]^{\frac{1-m}{1-(1-\mu)m}(1-\mu)m}.$$

Note that $K_{qh,B}$ and $L_{qh,B}$ are increasing in $\{K_{qh,A},L_{qh,A}\}$ and $\{K_{h},L_{h}\}$, since the marginal productivity of noncontractible activities rises with investment in other activities; investments in noncontractible and contractible activities are complementary. Another implication is that investment in noncontractible activities is decreasing in N_h. Mathematically, this follows from the fact that $\frac{N_{h}^{\frac{m}{\alpha}-1}}{1+N_{h}\delta}$ is decreasing in N_h. The economics of this relationship is the outcome of bargaining. The share of every supplier in revenue $\frac{\delta}{1+N_{h}\delta}$ is decreasing in N_h; hence, the marginal incentive for investment decreases with the number of suppliers.

In turn, we characterize the final goods firms' problem by plugging in the participation constraint

$$\tau_{qh} + S_{qh} - \mu(RPK_{qh,A} + WL_{qh,A}) - (1-\mu)(RPK_{qh,B} + WL_{qh,B}) = 0.$$

Thus, we obtain the final goods firms' problem

$$\max_{K_{qh,A}, L_{qh,A}, K_{qh,B}, L_{qh,B}, K_h, L_h} S_h[K_{qh,A}, L_{qh,A}, K_{qh,B}, L_{qh,B}, K_h, L_h]$$
$$+ N_h[S_{qh} - \mu(RPK_{qh,A} + WL_{qh,A}) - (1-\mu)(RPK_{qh,B} + WL_{qh,B})] - (RPK_h + WL_h).$$

Now combining the cost function of noncontractible activities and the expression of noncontractible investment, we can derive the following proposition.

Proposition 2.2. *The relative price of investment to consumption goods in an incomplete contract scenario is delivered as*

$$P = \frac{A_C N_C^{m/\alpha - m}}{A_I N_I^{m/\alpha - m}} \left\{ \frac{1 + N_I\delta}{1 + N_C\delta} \left(\frac{1 + N_C\delta - (1-\mu)\, m\delta}{1 + N_I\delta - (1-\mu)\, m\delta} \right)^{1-(1-\mu)m} \right\}. \tag{2.10}$$

Proof. See the Appendix. □

Since $N_I > N_C$, we can easily see that in the incomplete contract scenario, the relative price of investment goods is a decreasing function of the contract completeness measure μ, i.e the worse the contract enforcement institution, the higher the relative price of capital goods.

Implications of incomplete contracts

Now, we provide several comparative static results on the SSPE under incomplete contracts, and compare the incomplete contract equilibrium investment levels to the equilibrium under complete contracts. The purpose of the analysis is to provide some intuitions for the result that the relative price decreases with the contract enforcement institution measure μ. The main results are provided in the next proposition.

Proposition 2.3. *If $\mu < 1$, the unique SSPE satisfies that for $h = I, C$*

$$\frac{\partial \left(K_{qh,A}/K_h\right)}{\partial \mu} = 0, \frac{\partial \left(L_{qh,A}/L_h\right)}{\partial \mu} = 0,$$

$$K_{qh,B} < K_{qh,A}, \ L_{qh,B} < L_{qh,A},$$

$$\frac{\partial \left(K_{qh,B}/K_{qh,A}\right)}{\partial \mu} < 0, \ \frac{\partial \left(L_{qh,B}/L_{qh,A}\right)}{\partial \mu} < 0,$$

$$\frac{\partial \left(K_{qh,B}/K_h\right)}{\partial \mu} < 0, \ \frac{\partial \left(L_{qh,B}/L_h\right)}{\partial \mu} < 0,$$

$$\frac{\partial \left(\frac{K_{qI,B}/K_{qI,A}}{K_{qC,B}/K_{qC,A}}\right)}{\partial \mu} > 0,$$

$$\frac{\partial R}{\partial \mu} < 0, \ \frac{\partial W}{\partial \mu} < 0.$$

Proof. See the Appendix. □

The main results in this proposition are intuitive. Suppliers invest less in contractible activities than in contractible activities, in particular

$$\frac{K_{qh,B}}{K_{qh,A}} = \frac{\delta \left(1 - (1 - \mu) m\right)}{\left(1 + N_h \delta - (1 - \mu) m \delta\right)} < 1$$

which is derived in the appendix. Intuitively, the firm is the full residual claimant of the return to investments in contractible activities and it dictates these investments in the contract such that $K_{qh,A}/K_h$ and $L_{qh,A}/L_h$ is invariant with μ. In contrast, investments in noncontractible activities are decided by the suppliers, who are not full residual claimants of the return generated by these investment and thus, underinvest in these activities as compared to the contractible activities. In addition, the investment level of final goods firms and investments in both contractible and noncontractible activities are increasing in the fraction of contractible activities μ (the quality of contracting institutions). Better contracting institutions imply that a greater fraction of activities receive the higher investment level $\{K_{qh,A}, L_{qh,A}\}_{h=I,C}$ rather than $\{K_{qh,B}, L_{qh,B}\}_{h=I,C}$. It makes the choice of a higher investment level for final goods firms more profitable. A higher investment $\{K_{qh}, L_{qh}\}_{h=I,C}$, in turn, increases the profitability of further investments in contractible and noncontractible activities. A better contracting institution also compresses the gap between $\{K_{qh,A}, L_{qh,A}\}_{h=I,C}$ and $\{K_{qh,B}, L_{qh,B}\}_{h=I,C}$ because with a higher fraction of contractible activities, the marginal return to investment in noncontractible activities is also higher. Given the aggregate capital stock and labor supply, the worse is the contracting institution, the lower is the factor price. This result is due to the fact that when contracting institutions become worse, firms' demand for capital and labor is lower and diminishing demand drives the factor prices downwards.

In the end, the investment of noncontractible activities in the investment goods sector will increase more than that in the consumption goods sector in response to an improvement in contracting institutions. The intuition is that contract incompleteness is more damaging to technologies with greater complex intermediate inputs, because there are more significant investment distortions in this sector. The last result implies that sectors with more complex intermediate inputs are more contract dependent. The asymmetric effect of contracting institutions across sector leads to the relative price co-varying with the quality of the contract institution.

From the proof in the appendix, we can see that given the investment level of final goods firms $\{K_h, L_h\}_{h=I,C}$, the implied level of investment in contractible activities under an incomplete contract is identical to the investment level in contractible activities under a complete contract. This highlights the fact that differences in the investment in contractible activities between these economics environments result from differences in noncontractible activities. We can see that the distortions created by bargaining between final goods firms and their suppliers decrease the investment level of noncontractible activities. Intuitively, the investment level of final goods firms is distorted because incomplete contracts reduce the investment in noncontractible activities below the level of investment in contractible activities and this "underinvestment" reduces the profitability of technologies with a high N. As $\mu \to 1$ (contractual imperfections disappear), the bracketed term on the right-hand side of

(2.10) goes to 1 and the relative price coincides with that in a complete contract scenario.

To better understand why the relative price of investment co-varies with the contracting institution, we do a development accounting exercise for two sectors. By this simple exercise, we show the unbalanced effect of contracting institutions on sectoral TFPs. Now assume the simple aggregate production function for two sectors

$$Y_h = TFP_h \bar{K}_h^{\gamma} \bar{L}_h^{1-\gamma} \ , \ h = I, C,$$

where Y_h represents real output in sector h, TFP_h is the residual TFP, $\bar{K}_h$ is real physical capital and $\bar{L}_h$ is labor input (here $\bar{K}_h$ and $\bar{L}_h$ are aggregate capital and labor which include capital and labor input by both suppliers and final goods firms). The outcome of this development accounting exercise is laid out as Lemma 2.1.

Lemma 2.2. *The TFP in both sectors can be expressed as*

$$TFP_h = \frac{A_h N_h^{\frac{m}{\alpha}-m} m^m (1-m)^{(1-m)} \delta}{(1+N_h\delta)} \left(\frac{1+N_h\delta - (1-\mu) m\delta}{\delta - (1-\mu) m\delta} \right)^{1-(1-\mu)m},$$

and the relative price of investment is the ratio of the TFP of two sectors, i.e.

$$P = \frac{TFP_C}{TFP_I}.$$

Proof. See the Appendix. □

We can seperate sectoral TFP as two parts: one is raw TFP which is represented by the term

$$A_h N_h^{\frac{m}{\alpha}-m} m^m (1-m)^{(1-m)},$$

and its value are fully determined by technology; another is wedge which is represented by the term

$$\frac{\delta}{1+N_h\delta} \left(\frac{1+N_h\delta - (1-\mu) m\delta}{\delta - (1-\mu) m\delta} \right)^{1-(1-\mu)m},$$

and its value hinges on technology and contracting institution and measures the disortion generated by imperfect contract. TFPs in both sectors increase with μ, i.e. the better is the contracting institution, the higher are the TFPs. But imperfect contract generates larger distortion in contract dependent sector. It can be seen from the wedge term. Therefore, when the contracting institution deteriorates, TFP in the investment goods sector declines more than that in the consumption goods sector. The driving force behind the changing TFP is the misallocation of investment between contractible and noncontractible activities. This misallocation leads to resource misallocation along the production chain. Since the overall production efficiency depends on the allocation efficiency along the production chain, the resource misallocation along the production chain generates a huge efficiency loss. The two figures at the bottom of Figure 2.5 trace the effect of the contracting institution on the measured TFPs of the capital goods sector (left panel) and the consumption goods sector (right panel). While TFP declines by around 48 percent in the consumption goods sector, the capital goods sector's TFP declines by 68 percent with incomplete contract enforcement[2]. This result is consistent with the empirical observations that productivity differences across countries are sharpest for the capital goods sectors. The pattern of relative productivity between the two sectors leads to the price of investment goods relative to that of consumption goods being higher in countries with a worse contracting institution.

2.4 General Equilibrium

2.4.1 The Competitive Equilibrium of a Closed Economy

We embed a simple one-period model into an infinite horizon growth model. The production technologies are the same as those in the preceding section. We also assume that the aggregate endowment of labor

[2]The parameters value are calibrated in section 2.5.

is given by 1 and the aggregate capital stock is given by K. Specifically, we assume that each worker supplies one unit of labor inelastically and there are one mass of workers. The representative agent's problem is standard:

$$\sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}-1}{1-\sigma}$$

subject to the constraints

$$K_{t+1} = I_t + (1-\lambda)K_t,$$

$$P_t I\left(t\right) + C_t = W_t L + R_t K_t.$$

Here, we normalize the price of consumption goods as one and define the depreciation rate as λ .

Definition 2.1. *Given an initial aggregate capital stock $K_0 > 0$, a competitive equilibrium is a sequence of allocation consumption $\{C_t\}_{t=0}^{\infty}$, aggregate capital and capital allocation across different sectors $\{K_t, K_{h,t}, K_{qh,A,t}, K_{qh,B,t}\}_{t=1,h=I,C}^{\infty}$, labor supply allocation across different sectors $\{L_t, L_{h,t}, L_{qh,A,t}, L_{qh,B,t}\}_{t=0,h=I,C}^{\infty}$, and a sequence of prices $\{R_t, W_t, P_t\}_{t=0}^{\infty}$ such that*

1. *Given the interest rate and the wage rate, investment goods firms and their suppliers solve SSPE for $P_{I,t} = p_t$, and consumption goods firms and their suppliers solve SSPE.*

2. *Market clearing conditions*

   $$K_{I,t}+K_{C,t}+\mu\left(N_I K_{qI,A,t} + N_C K_{qC,A,t}\right)+(1-\mu)\left(N_I K_{qI,B,t} + N_I K_{qC,B,t}\right) = K$$

   and

   $$L_{I,t}+L_{C,t}+\mu\left(N_I L_{qI,A,t} + N_C L_{qC,A,t}\right)+(1-\mu)\left(N_I L_{qI,B,t} + N_I L_{qC,B,t}\right) = 1,$$

   and the interest rate and the wage rate are the same across sectors.

3. *Gross investment equals the production of investment goods: $I_t = Y_{I,t}$ and law of motion of capital stock is $K_{t+1} = I_t + (1-\lambda)K_t$.*

4. *Goods market clear: $C_t = A_C[\sum_{i=1}^{N_C} q_{C,t}(i)^{\alpha} di]^{\frac{m}{\alpha}}[{K_{C,t}}^{\gamma} {L_{C,t}}^{1-\gamma}]^{1-m}$.*

Suppose that A_C and A_I grow at a constant rate, g_A. Then, in the balanced growth path, we can derive the following proposition.

Proposition 2.4. *Along the balanced-growth path, (i) investment rates at an international price and a domestic price and the relative price of investment are invariant, the relative price is given by proposition 2.2 and (ii) investment rates at domestic prices are given by*

$$\frac{I}{Y} = \frac{1 - \beta\gamma\left(g + \lambda\right)}{\beta\gamma\left(g + \lambda\right)}$$

where $(1 + g) = (1 + g_A)^{1/(1-\gamma)}$.

Proof. See the Appendix. □

The first claim of proposition 2.4 is easy to prove. In the previous section, we have known that given any capital stock and labor supply, the relative price of investment is constant if the quality of contractual institutions remains invariant. Then, we can easily derive the conclusion that the relative price is kept constant along the balanced growth path. Assume the international relative price to be p_w, then $(\frac{I}{Y})_{ppp} = \frac{p_w I}{C + p_w I} = \frac{p_w}{C/I + p_w}$. Since C and I grow at the same rate and p_w is kept constant along the balanced-growth path, it is easily shown that $(\frac{I}{Y})_{ppp}$ is constant. By the similar reasoning, we can reach the conclusion that the investment rate at the domestic price is constant. The proof of a second claim can be found in the appendix. Notice that the investment rate does not hinge upon the contractual institution.

2.5 Quantitative Analysis

The purpose of this section is to develop quantitative implications for the co-variation between the quality of contracting institutions and other variables of interest, for example GDP, TFP and the investment rate measured in both domestic and international prices. To do this comparative statics excersice, we compare the steady states of different countries with the same technology but different contracting institutions. Therefore, in this section we assume no technology progress, i.e. $g_A = 0$.

2.5.1 Calibration

To make our model more amenable to quantitative analysis, we calibrate preference and technology parameters such that a perfect contract enforcement economy matches key aspects of the US, a relatively undistorted economy. In particular, our target moments include standard macroeconomic aggregates and features of the complexity of intermediate inputs across sectors.

We need to specify values for nine parameters: four technological parameters, N_I, N_C, α, m, and the depreciation rate λ, two parameters describing the subjective discount rate β and δ (here β and δ refer to the discount rate of one year and one month respectively) and the relative risk aversion coefficient σ.

The relative risk aversion coefficient σ is set to 1.5, a standard value in quantitative analysis. The one year depreciation rate is set at $\lambda = 0.06$, and we pick $\gamma = 1/3$ to match the empirical evidence on capital shares. Finally, the model requires a discount factor of $\beta = 0.92$ to match the annual interest rate of four percent. The one month discount rate δ is then 0.993.

Now we are thus left with the four parameters that are more specific to our study. The first parameter is the degree of complementarity of intermediate inputs within the production function of the final goods firm. Estimating production functions is notoriously difficult, and we know of no good estimates of this parameter. Nevertheless, since we have no direct way of relating α to aggregate data, we will show the implications of changes in the quality of contracting institutions for different values of α. Fortunately, analysis in the previous sections demonstrate that the result of comparative statics does depend on the value of α. Hence, we will pick $\alpha = 0.25$. There are three crucial parameters for explaining the large differences in the relative price of capital goods across countries: the intermediate goods share σ, and the number of intermediate inputs of two sectors, N_I and N_C. There is detailed empirical evidence on the magnitude of the intermediate goods share σ. Basu (1995) recommends a value of 0.5 based on the numbers from Jorgenson, Gollop and Fraumeni (1987) for the US economy between 1947 and 1979. Here we will take $m = 0.5$. Next, we need to assign values to the complexity parameters N_I and N_C. In section 2.2, we provide simple average and weighted average Herfindahl indexes for two sectors in the US. Here we use the weighted average Herfindahl index to calibrate the complexity level across sectors. Now we assume that all intermediate inputs are symmetric, then the value of the weighted average Herfindahl index implies that N_I and N_C are 18 and 6, respectively. However, there is a potential problem as such measures will depend on the US sectoral composition, while the sectoral composition may change with the level of development. This issue will be disregarded in our benchmark calibration.

Table 2.4: Parameter values

β	δ	σ	λ	γ	α	m	NI	NC
0.92	0.993	1.5	0.06	1/3	0.25	0.5	18	6

2.5.2 Analysis of Results

In this section, we quantify the effect of imperfect contracting institution on economic development. We first show that incomplete contracts have a substantial adverse impact on output per worker. In our exercises, the lower per capita income in economies with an incomplete contract enforcement is primarily explained by their low aggregate TFPs, with particularly low productivities in investment goods sectors. These results are consistent with the empirical findings in Section 2.2. For the parameter values listed in table 2.4, figures 6 and 7 depict the steady-state values implied by all $\mu \in [0, 1]$. The figures are expressed relative to the outcome under perfect contracting institution ($\mu = 1$).

Considering Figure 2.6 and Figure 2.7, we can confirm that the relative price of investment goods is monotonely decreasing in μ. From the previous sector, we know that this relative price change comes from the asymmetric drop of TFPs when the contracting institution deteriorates. This huge drop of TFP leads to a sizable decrease of investment in a steady state which results in a fall in the capital stock at the same magnitude. The declining TFP and capital stock caused by imperfect contracting institutions will generate a great reduction in GDP which is measured by both the international price (the price under a perfect contracting institution) and the domestic price. But the investment rate measured by the domestic price is invariant with the quality of the contractual institution. This is not the case for the PPP-adjusted investment rate which is decreasing with the contractual incompleteness. Since the investment rate at the domestic price does not change, the only channel affecting the PPP-adjusted investment rate is via the changing relative price of investment. Notice that the comparative statics of a PPP-adjusted investment rate does not depend on the particular value of the international price. What matters is that the international price does not vary with the contracting institution. Finally, we can see that the huge impacts of the contracting institution are exerted on the wage rate and consumption and it generates the sizable welfare loss.

The main message of this simple quantitative comparative statics exercise is that cross-country differences in contracting institutions can potentially generate a divergence in the relative price of investment, investment rates, TFP, and GDP, which are quantitatively significant and in accordance with the empirical evidence.

2.6 Conclusions

This paper contributes to a very active line of research which investigates the sources of the sizeable cross-country sectoral heterogeneity. It does so by introducing a quantitative theory that links the quality of contracting institutions to output per worker, aggregate TFP, sector-level relative productivity, and relative prices. Because of the hold-up problem, an incomplete contract friction distorts the investment of final goods firms and their intermediate suppliers, and has sizable adverse effects on a country's output per worker and aggregate productivity. The investment goods sector needs more complex intermediate inputs than the consumption goods sector, and hence has more contracting needs. For this reason, investment goods producers are more vulnerable to contract imperfection. We have shown that this mechanism is quantitatively relevant. The quantitative analysis manifests that our mechanism can account for a large fraction of the observed variation in relative prices and investment rates. Our analysis shows how micro-level technological differences across sectors interact with the imperfect contracting institution. And it helps us better understand macroeconomics issues such as aggregate productivity and sector-level relative productivity.

A version of our model that allowed for international trade would generate trade patterns that are qualitatively consistent with the empirical evidence in the international trade literature. Especially, in line with what was documented by Eaton and Kortum (2001), developed countries specialize in the production of equipment goods, which need more complex intermediate inputs. Therefore, our paper provides a microfounded theory for why developed countries specialize in the investment goods sector.

Our last thought concerns the possibility that institutions may affect long-run growth. Empirical evidence shows that the capital goods sector is R&D intensive and R&D activities along the production chain are complementary. If this is the case, our theory suggests that innovative activities should be concentrated in countries with a high contractual institutional quality.

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2.A Proofs of the Propositions and Lemmas

This appendix contains outlines of the proofs of the propositions and lemmas reported in the paper.

2.A.1 Proof of Proposition 2.1

Solving the problem of the consumption goods firm, we obtain the following first-order conditions

$$m\gamma A_I P N_I^{\frac{m}{\alpha}} [{K_{qI}}^{\gamma} {L_{qI}}^{1-\gamma}]^m [{K_I}^{\gamma} {L_I}^{1-\gamma}]^{1-m} K_{qI}^{-1} = N_I R P,$$
$$m(1-\gamma) A_I P N_I^{\frac{m}{\alpha}} [{K_{qI}}^{\gamma} {L_{qI}}^{1-\gamma}]^m [{K_I}^{\gamma} {L_I}^{1-\gamma}]^{1-m} L_{qI}^{-1} = N_I W,$$
$$(1-m)\gamma A_I P N_I^{\frac{m}{\alpha}} [{K_{qI}}^{\gamma} {L_{qI}}^{1-\gamma}]^m [{K_I}^{\gamma} {L_I}^{1-\gamma}]^{1-m} K_I^{-1} = RP,$$
$$(1-m)(1-\gamma) A_I P N_I^{\frac{m}{\alpha}} [{K_{qI}}^{\gamma} {L_{qI}}^{1-\gamma}]^m [{K_I}^{\gamma} {L_I}^{1-\gamma}]^{1-m} L_I^{-1} = W.$$

Similarly, solving the problem of the investment goods firm, we obtain

$$m\gamma A_C N_C^{\frac{m}{\alpha}} [{K_{qC}}^{\gamma} {L_{qC}}^{1-\gamma}]^m [{K_C}^{\gamma} {L_C}^{1-\gamma}]^{1-m} K_{qC}^{-1} = N_C R P,$$
$$m(1-\gamma) A_C N_C^{\frac{m}{\alpha}} [{K_{qC}}^{\gamma} {L_{qC}}^{1-\gamma}]^m [{K_C}^{\gamma} {L_C}^{1-\gamma}]^{1-m} L_{qC}^{-1} = N_C W,$$
$$(1-m)\gamma A_C N_C^{\frac{m}{\alpha}} [{K_{qC}}^{\gamma} {L_{qC}}^{1-\gamma}]^m [{K_C}^{\gamma} {L_C}^{1-\gamma}]^{1-m} K_C^{-1} = RP,$$
$$(1-m)(1-\gamma) A_C N_C^{\frac{m}{\alpha}} [{K_{qC}}^{\gamma} {L_{qC}}^{1-\gamma}]^m [{K_C}^{\gamma} {L_C}^{1-\gamma}]^{1-m} L_C^{-1} = W.$$

From these first-order conditions, we can derive

$$\frac{L_I}{K_I} = \frac{L_{qI}}{K_{qI}} = \frac{L_C}{K_C} = \frac{L_{qC}}{K_{qC}} = \frac{1-\gamma}{\gamma} \frac{RP}{W}, \tag{2.11}$$

$$\frac{K_{qI}}{K_I} = \frac{L_{qI}}{L_I} = \frac{m}{1-m} \frac{1}{N_I}, \tag{2.12}$$

$$\frac{K_{qC}}{K_C} = \frac{L_{qC}}{L_C} = \frac{m}{1-m} \frac{1}{N_C}. \tag{2.13}$$

Furthermore, we can also obtain the following equation

$$\frac{A_I P N_I^{\frac{m}{\alpha}} [{K_{qI}}^{\gamma} {L_{qI}}^{1-\gamma}]^m [{K_I}^{\gamma} {L_I}^{1-\gamma}]^{1-m} K_I^{-1}}{A_C N_C^{\frac{m}{\alpha}} [{K_{qC}}^{\gamma} {L_{qC}}^{1-\gamma}]^m [{K_C}^{\gamma} {L_C}^{1-\gamma}]^{1-m} K_C^{-1}} = 1.$$

Substituting for (4.10), (4.11) and (4.12), we obtain

$$P = \frac{A_C}{A_I} \left(\frac{N_C^{\frac{m}{\alpha}-m}}{N_I^{\frac{m}{\alpha}-m}} \right).$$

2.A.2 Proof of Lemma 2.1

First, we will state the main result of the bargaining game which is proved as the main theorem in Krishna and Serrano (1996).

Theorem 1. Suppose that players in N are bargaining over a pie of size q in the game $G(N;q)$. Let (x^*, α^*) be the solution to the characteristic equations for $(N;q)$:

$$u_i(x_i) = \delta u_i(x_i + \alpha),\ i \in N;$$

$$x(N) + \alpha = q.$$

The unique perfect equilibrium agreement in $G(N;q)$ is $\left(x_1^* + \alpha^*, x_{-1}^*\right)$ and is achieved in period 1.

The proof of this theorem can be found in Krishna and Serrano (1996).

Now let us use this theorem to prove Lemma 2.1. Since the firms maximize profit, they have a linear utility function. In our extensive bargaining game, we assume that the final goods firm proposes first, hence by the theorem we know that the proposer gets the share $x^* + \alpha^*$ and the suppliers obtain the share x^*. Following the above theorem, we can obtain

$$x^* = \delta(x^* + \alpha^*)$$

$$Nx^* + \alpha^* = 1$$

where we assume that $q = 1$. It is easily seen that $x^* = \frac{\delta}{1+(N-1)\delta}$, $\alpha^* = \frac{1-\delta}{1+(N-1)\delta}$. It means that the final goods firm gets the share $\frac{1}{1+(N-1)\delta}$, and that suppliers obtain the share $\frac{\delta}{1+(N-1)\delta}$. Then, the claim in Lemma 2.1 can be proved.

2.A.3 Proof of Proposition 2.2

Now substituting (4.9) and the cost function of noncontractible activities, we obtain the final goods firm's problem

$$\max_{\substack{K_{qh,A},L_{qh,A}\\,K_h,L_h}} \left(1-\frac{(1-\mu)\,m\delta}{1+N_h\delta}\right) A_h P_h N_h^{\frac{m}{\alpha}} \left[\frac{\gamma\delta m A_h N_h^{\frac{m}{\alpha}-1} P_h}{(1+N_h\delta)\,RP}(\frac{(1-\gamma)\,RP}{\gamma W})^{(1-\gamma)}\right]^{\frac{(1-\mu)m}{1-(1-\mu)m}}$$
$$[K_{qh,A}^{\gamma}L_{qh,A}^{1-\gamma}]^{\frac{\mu m}{1-(1-\mu)m}}[K_h^{\gamma}L_h^{1-\gamma}]^{\frac{1-m}{1-(1-\mu)m}} - \mu N_h[RPK_{qh,A}+WL_{qh,A}] - [RPK_h+WL_h]$$

Then, we obtain the first-order conditions for the investment goods sector

$$\left(1-\frac{(1-\mu)\,m\delta}{1+N_I\delta}\right)\frac{\gamma m[K_{qI,A}^{\gamma}L_{qI,A}^{1-\gamma}]^{\frac{\mu m}{1-(1-\mu)m}}[K_I^{\gamma}L_I^{1-\gamma}]^{\frac{1-m}{1-(1-\mu)m}}K_{qI,A}^{-1}}{1-(1-\mu)m}Q_I = N_I RP$$

$$\left(1-\frac{(1-\mu)\,m\delta}{1+N_I\delta}\right)\frac{\gamma\,(1-m)\,[K_{qI,A}^{\gamma}L_{qI,A}^{1-\gamma}]^{\frac{\mu m}{1-(1-\mu)m}}[K_I^{\gamma}L_I^{1-\gamma}]^{\frac{1-m}{1-(1-\mu)m}}K_I^{-1}}{1-(1-\mu)m}Q_I = RP$$

$$\left(1-\frac{(1-\mu)\,m\delta}{1+N_I\delta}\right)\frac{(1-\gamma)\,m[K_{qI,A}^{\gamma}L_{qI,A}^{1-\gamma}]^{\frac{\mu m}{1-(1-\mu)m}}[K_I^{\gamma}L_I^{1-\gamma}]^{\frac{1-m}{1-(1-\mu)m}}L_{qI,A}^{-1}}{1-(1-\mu)m}Q_I = N_I W$$

$$\left(1-\frac{(1-\mu)\,m\delta}{1+N_I\delta}\right)\frac{(1-\gamma)\,(1-m)\,[K_{qI,A}^{\gamma}L_{qI,A}^{1-\gamma}]^{\frac{\mu m}{1-(1-\mu)m}}[K_I^{\gamma}L_I^{1-\gamma}]^{\frac{1-m}{1-(1-\mu)m}}L_I^{-1}}{1-(1-\mu)m}Q_I = W$$

where $Q_I = A_I P N_I^{\frac{m}{\alpha}}\left[\frac{\gamma\delta m}{1+N_I\delta}A_I N_I^{\frac{m}{\alpha}-1}\left(\frac{1}{R}\right)(\frac{(1-\gamma)RP}{\gamma W})^{(1-\gamma)}\right]^{\frac{(1-\mu)m}{1-(1-\mu)m}}$. Combine these first-order conditions with the first-order conditions in the consumption goods sector

$$\left(1-\frac{(1-\mu)\,m\delta}{1+N_C\delta}\right)\frac{\gamma m[K_{qC,A}^{\gamma}L_{qC,A}^{1-\gamma}]^{\frac{\mu m}{1-(1-\mu)m}}[K_C^{\gamma}L_C^{1-\gamma}]^{\frac{1-m}{1-(1-\mu)m}}K_{qC,A}^{-1}}{1-(1-\mu)m}Q_C = N_C RP$$

$$\left(1-\frac{(1-\mu)\,m\delta}{1+N_C\delta}\right)\frac{\gamma\,(1-m)\,[K_{qC,A}^{\gamma}L_{qC,A}^{1-\gamma}]^{\frac{\mu m}{1-(1-\mu)m}}[K_C^{\gamma}L_C^{1-\gamma}]^{\frac{1-m}{1-(1-\mu)m}}K_C^{-1}}{1-(1-\mu)m}Q_C = RP$$

$$\left(1-\frac{(1-\mu)\,m\delta}{1+N_C\delta}\right)\frac{(1-\gamma)\,m[K_{qC,A}^{\gamma}L_{qC,A}^{1-\gamma}]^{\frac{\mu m}{1-(1-\mu)m}}[K_C^{\gamma}L_C^{1-\gamma}]^{\frac{1-m}{1-(1-\mu)m}}L_{qC,A}^{-1}}{1-(1-\mu)m}Q_C = N_C W$$

$$\left(1-\frac{(1-\mu)\,m\delta}{1+N_C\delta}\right)\frac{(1-\gamma)\,(1-m)\,[K_{qC,A}^{\gamma}L_{qC,A}^{1-\gamma}]^{\frac{\mu m}{1-(1-\mu)m}}[K_C^{\gamma}L_C^{1-\gamma}]^{\frac{1-m}{1-(1-\mu)m}}L_C^{-1}}{1-(1-\mu)m}Q_C = W$$

where $Q_C = A_C N_C^{\frac{m}{\alpha}} \left[\frac{\gamma\delta m}{1+N_C\delta} A_C N_C^{\frac{m}{\alpha}-1} \left(\frac{1}{RP}\right) \left(\frac{(1-\gamma)RP}{\gamma W}\right)^{(1-\gamma)} \right]^{\frac{(1-\mu)m}{1-(1-\mu)m}}$, we can derive

$$\frac{L_I}{K_I} = \frac{L_{qI,A}}{K_{qI,A}} = \frac{L_C}{K_C} = \frac{L_{qC,A}}{K_{qC,A}} = \frac{1-\gamma}{\gamma}\frac{RP}{W} \tag{2.14}$$

$$\frac{K_{qI,A}}{K_I} = \frac{L_{qI,A}}{L_I} = \frac{m}{1-m}\frac{1}{N_I} \tag{2.15}$$

$$\frac{K_{qC,A}}{K_C} = \frac{L_{qC,A}}{L_C} = \frac{m}{1-m}\frac{1}{N_C}. \tag{2.16}$$

Furthermore, we also obtain the following equation

$$\frac{\left(1-\frac{(1-\mu)m\delta}{1+N_I\delta}\right) Q_I [K_{qI,A}^{\gamma} L_{qI,A}^{1-\gamma}]^{\frac{\mu m}{1-(1-\mu)m}} [K_I^{\gamma} L_I^{1-\gamma}]^{\frac{1-m}{1-(1-\mu)m}} \left[\frac{\gamma(1-m)}{1-(1-\mu)m} K_I^{-1}\right]}{\left(1-\frac{(1-\mu)m\delta}{1+N_C\delta}\right) Q_C [K_{qC,A}^{\gamma} L_{qC,A}^{1-\gamma}]^{\frac{\mu m}{1-(1-\mu)m}} [K_C^{\gamma} L_C^{1-\gamma}]^{\frac{1-m}{1-(1-\mu)m}} \left[\frac{\gamma(1-m)}{1-(1-\mu)m} K_C^{-1}\right]} = \frac{RP}{RP} = 1 \tag{2.17}$$

substituting (4.13), (4.14) and (2.16) into (2.17), we obtain

$$P = \frac{A_C N_C^{m/\alpha - m}}{A_I N_I^{m/\alpha - m}} \left\{ \frac{1+N_I\delta}{1+N_C\delta} \left(\frac{1+N_C\delta - (1-\mu)\, m\delta}{1+N_I\delta - (1-\mu)\, m\delta} \right)^{1-(1-\mu)m} \right\}.$$

In the following, we proceed to prove the relative price of investment goods is a decreasing function of the contracting institution μ. To prove this claim, we only need to prove that $\left(\frac{1+N_C\delta-(1-\mu)m\delta}{1+N_I\delta-(1-\mu)m\delta}\right)^{1-(1-\mu)m}$ is a decreasing function of μ. Now define $f(\mu) = \left(\frac{1+N_C\delta-(1-\mu)m\delta}{1+N_I\delta-(1-\mu)m\delta}\right)^{1-(1-\mu)m}$ and prove equivalently that $\ln f(\mu)$ is a decreasing function. The derivative of $\ln f(\mu)$ with respect to μ is

$$m\left[\ln(1+N_C\delta-(1-\mu)\, m\delta) - \ln(1+N_I\delta-(1-\mu)\, m\delta)\right] +$$
$$(1-(1-\mu)\, m)\left[\frac{m\delta}{1+N_C\delta-(1-\mu)\, m\delta} - \frac{m\delta}{1+N_I\delta-(1-\mu)\, m\delta}\right].$$

Now we need to prove that it is negative in the domain $\mu \in [0,1]$. Since

$$\ln(1+N_C\delta-(1-\mu)\, m\delta) - \ln(1+N_I\delta-(1-\mu)\, m\delta) < 0$$
$$\frac{\delta}{1+N_C\delta-(1-\mu)\, m\delta} - \frac{\delta}{1+N_I\delta-(1-\mu)\, m\delta} > 0$$

then

$$m\left[\ln(1+N_C\delta-(1-\mu)\, m\delta) - \ln(1+N_I\delta-(1-\mu)\, m\delta)\right] +$$
$$(1-(1-\mu)\, m)\left[\frac{m\delta}{1+N_C\delta-(1-\mu)\, m\delta} - \frac{m\delta}{1+N_I\delta-(1-\mu)\, m\delta}\right]$$

$$< m \left\{ \begin{array}{c} \left[\ln\left(1+N_C\delta-(1-\mu)\,m\delta\right)+\frac{1}{1+N_C\delta-(1-\mu)m\delta}\right]- \\ \left[\ln\left(1+N_I\delta-(1-\mu)\,m\delta\right)+\frac{1}{1+N_I\delta-(1-\mu)m\delta}\right] \end{array} \right\}.$$

The function $g(x)=\ln x+\frac{1}{x}$ is a decreasing function if $x>1$, therefore

$$\left\{ \begin{array}{c} \left[\ln\left(1+N_C\delta-(1-\mu)\,m\delta\right)+\frac{1}{1+N_C\delta-(1-\mu)m\delta}\right]- \\ \left[\ln\left(1+N_I\delta-(1-\mu)\,m\delta\right)+\frac{1}{1+N_I\delta-(1-\mu)m\delta}\right] \end{array} \right\} < 0.$$

Given that $1+N_I\delta-(1-\mu)\,m\delta > \left(1+N_C\delta-(1-\mu)\,m\delta\right) > 1$, the function $f(\mu)$ is a decreasing function of μ.

2.A.4 Proof of Proposition 2.3

Proof: From the first-order condition for the intermediate goods firm and the final goods firm, we obtain

$$\frac{L_I}{K_I}=\frac{L_C}{K_C}=\frac{L_{qI,A}}{K_{qI,A}}=\frac{L_{qC,A}}{K_{qC,A}}=\frac{L_{qI,B}}{K_{qI,B}}=\frac{L_{qC,B}}{K_{qC,B}}=\frac{1-\gamma}{\gamma}\frac{RP}{W}, \tag{2.18}$$

$$\frac{K_{qh,A}}{K_h}=\frac{L_{qh,A}}{L_h}=\frac{m}{1-m}\frac{1}{N_h}. \tag{2.19}$$

From (2.19) we can easily get the first claim proved.

Since the capital labor ratio is the same as implied by (2.18) in all firms, the aggregate capital labor ratio satisfies

$$\frac{L}{K}=\frac{1-\gamma}{\gamma}\frac{RP}{W}. \tag{2.20}$$

Combining with (4.8) and (2.19), we obtain

$$K_{qh,B}=\left\{\frac{\gamma\delta m}{1+N_h\delta}A_h(P_h/P)N_h^{\frac{m}{\alpha}-1}R^{-1}\right\}^{\frac{1}{1-(1-\mu)m}}\left[\frac{m}{(1-m)\,N_h}\right]^{\frac{\mu m}{1-(1-\mu)m}}\left(\frac{K}{L}\right)^{\gamma-1}K_h.$$

Now we need to solve factor prices R and W. Combining (2.20) and the following two first-order conditions

$$\left(1-\frac{(1-\mu)\,m\delta}{1+N_I\delta}\right)Q_I[K_{qI,A}^{\gamma}L_{qI,A}^{1-\gamma}]^{\frac{\mu m}{1-(1-\mu)m}}[K_I^{\gamma}L_I^{1-\gamma}]^{\frac{1-m}{1-(1-\mu)m}}\left[\frac{\gamma\,(1-m)}{1-(1-\mu)m}K_I^{-1}\right]=RP,$$

$$\left(1-\frac{(1-\mu)\,m\delta}{1+N_I\delta}\right)Q_I[K_{qI,A}^{\gamma}L_{qI,A}^{1-\gamma}]^{\frac{\mu m}{1-(1-\mu)m}}[K_I^{\gamma}L_I^{1-\gamma}]^{\frac{1-m}{1-(1-\mu)m}}\left[\frac{(1-\gamma)\,(1-m)}{1-(1-\mu)m}L_I^{-1}\right]=W,$$

we obtain

$$R = \frac{\gamma m^m (1-m)^{1-m} \delta^{(1-\mu)m} A_I N_I^{\frac{m}{\alpha}-m}}{1+N_I\delta} \left(\frac{1+N_I\delta-(1-\mu)m\delta}{1-(1-\mu)m}\right)^{1-(1-\mu)m} \left(\frac{K}{L}\right)^{\gamma-1}, \tag{2.21}$$

$$W = \frac{(1-\gamma) m^m (1-m)^{1-m} \delta^{(1-\mu)m} A_C N_C^{\frac{m}{\alpha}-m}}{1+N_C\delta} \left(\frac{1+N_C\delta-(1-\mu)m\delta}{1-(1-\mu)m}\right)^{1-(1-\mu)m} \left(\frac{K}{L}\right)^{\gamma}. \tag{2.22}$$

And we can easily prove that

$$\frac{\partial R}{\partial \mu} > 0, \frac{\partial W}{\partial \mu} > 0.$$

Substituting for the factor price and the relative price of investment, we can obtain

$$K_{qI,B} = \frac{m}{(1-m)N_I} \frac{\delta(1-(1-\mu)m)}{(1+N_I\delta-(1-\mu)m\delta)} K_I,$$

$$K_{qC,B} = \frac{m}{(1-m)N_C} \frac{\delta(1-(1-\mu)m)}{(1+N_C\delta-(1-\mu)m\delta)} K_C.$$

Then

$$K_{qI,B} = \frac{\delta(1-(1-\mu)m)}{(1+N_I\delta-(1-\mu)m\delta)} K_{qI,A},$$

$$K_{qC,B} = \frac{\delta(1-(1-\mu)m)}{(1+N_C\delta-(1-\mu)m\delta)} K_{qC,A}.$$

Therefore, we can derive that

$$K_{qh,B} < K_{qh,A},\ L_{qh,B} < L_{qh,A}$$

$$\frac{\partial(K_{qh,B}/K_{qh,A})}{\partial \mu} < 0,\ \frac{\partial(L_{qh,B}/L_{qh,A})}{\partial \mu} < 0.$$

Furthermore, we obtain

$$\frac{K_{qI,B}/K_{qI,A}}{K_{qC,B}/K_{qC,A}} = \frac{1+N_C\delta-(1-\mu)m\delta}{1+N_I\delta-(1-\mu)m\delta},$$

then

$$\frac{\partial\left(\frac{K_{qI,B}/K_{qI,A}}{K_{qC,B}/K_{qC,A}}\right)}{\partial \mu} > 0.$$

2.A.5 Proof of Lemma 2.2

The technologies in two sectors can be rewritten as

$$Y_h = A_h N_h^{\frac{m}{\alpha}} \left(K_{qh,A}^{\gamma} L_{qh,A}^{1-\gamma}\right)^{\mu m} \left(K_{qh,B}^{\gamma} L_{qh,B}^{1-\gamma}\right)^{(1-\mu)m} \left(K_h^{\gamma} L_h^{1-\gamma}\right)^{(1-m)}.$$

Given the aggregate factor inputs in two sectors as $\bar{K}_h$ and $\bar{L}_h$, the following conditions must be satisfied

$$N_h(\mu K_{qh,A} + (1-\mu) K_{qh,B}) + K_h = \bar{K}_h,$$

$$N_h(\mu L_{qh,A} + (1-\mu) L_{qh,B}) + L_h = \bar{L}_h.$$

From the proof of proposition 2.3 we obtain

$$K_{qh,A} = \phi_{h,A} K_h, \;\; L_{qh,A} = \phi_{h,A} L_h,$$

$$K_{qh,B} = \phi_{h,B} K_h, \;\; L_{qh,B} = \phi_{h,B} L_h,$$

where $\phi_{h,A} = \frac{m}{(1-m)N_h}$ and $\phi_{h,B} = \frac{m}{(1-m)N_h} \frac{\delta(1-(1-\mu)m)}{(1+N_h\delta-(1-\mu)m\delta)}$. Then, we can derive that

$$K_{qh,A}^{\gamma} L_{qh,A}^{1-\gamma} = \frac{\phi_{h,A}}{1 + N_h\left(\mu\phi_{h,A} + (1-\mu)\phi_{h,B}\right)} \bar{K}_h^{\gamma} \bar{L}_h^{1-\gamma},$$

$$K_{qh,B}^{\gamma} L_{qh,B}^{1-\gamma} = \frac{\phi_{h,B}}{1 + N_h\left(\mu\phi_{h,A} + (1-\mu)\phi_{h,B}\right)} \bar{K}_h^{\gamma} \bar{L}_h^{1-\gamma},$$

$$K_h^{\gamma} L_h^{1-\gamma} = \frac{1}{1 + N_h\left(\mu\phi_{h,A} + (1-\mu)\phi_{h,B}\right)} \bar{K}_h^{\gamma} \bar{L}_h^{1-\gamma},$$

and then we can rewrite the production as

$$Y_h = A_h N_h^{\frac{m}{\alpha}} \left\{ \frac{\phi_{h,A}^{\mu m} \phi_{h,B}^{(1-\mu)m}}{1 + N_h\left(\mu\phi_{h,A} + (1-\mu)\phi_{h,B}\right)} \right\} \bar{K}_h^{\gamma} \bar{L}_h^{1-\gamma}.$$

Then, TFP can be expressed as

$$TFP_h = \frac{A_h N_h^{\frac{m}{\alpha}-m} m^m (1-m)^{(1-m)} \delta}{(1+N_h\delta)} \left(\frac{1 + N_h\delta - (1-\mu) m\delta}{\delta - (1-\mu) m\delta}\right)^{1-(1-\mu)m}.$$

Using the similar derivation as the proof of proposition 2.2, we can see that TFPs in two sectors are a strictly increasing function of μ.

2.A.6 Proof of Proposition 2.4

Given the exogenous TFP growth rate g_A, the Euler equation is

$$C_t^{-\sigma} = \beta R_{t+1} C_{t+1}^{-\sigma},$$

then

$$R = \frac{C_{t+1}^{\sigma}}{\beta C_t^{\sigma}} = \frac{(1+g)^{\sigma}}{\beta}, \tag{2.23}$$

where $(1+g) = (1+g_A)^{1/(1-\gamma)}$. Combining (2.21) and (2.23), we can solve the capital stock

$$K = \left[\frac{\gamma m^m (1-m)^{1-m} A_I N_I^{\frac{m}{\alpha}-m} \left(\frac{\delta}{1+N_I\delta}\right)^{(1-\mu)m} \left(\frac{1+N_I\delta-(1-\mu)m\delta}{(1+N_I\delta)(1-(1-\mu)m)}\right)^{1-(1-\mu)m}}{(1+g)^{\sigma}/\beta - 1 + \lambda} \right]^{\frac{1}{1-\gamma}}. \tag{2.24}$$

Then, the capital stock grows at the rate $(1+g_A)^{1/(1-\gamma)}$. In the following we can solve for the share of capital (and labor) devoted to the investment goods sector by law of motion

$$I = TFP_I \bar{K}_I^{\gamma} \bar{L}_I^{1-\gamma} = TFP_I \phi_I K^{\gamma}, \tag{2.25}$$

$$I = (g+\lambda) K, \tag{2.26}$$

where $\phi_I = \frac{\bar{K}_I}{K} = \frac{\bar{L}_I}{L}$. Then with (2.24), (2.25) and(2.26), we obtain

$$\phi_I = \frac{\gamma (g+\lambda)}{(1+g)^{\sigma}/\beta - 1 + \lambda}.$$

Then, the ratio of consumption to investment is

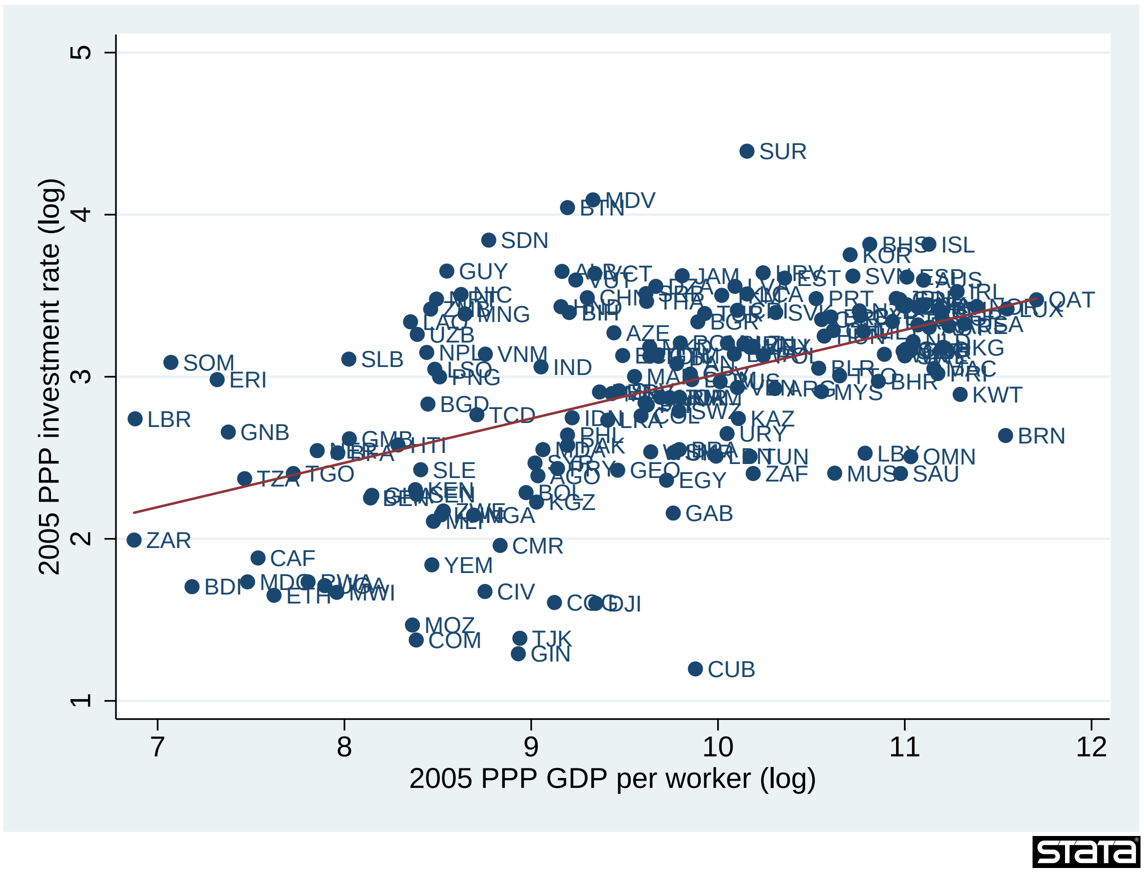
$$\frac{C}{I} = \frac{TFP_C (1-\phi_I)}{TFP_I \phi_I} = \frac{1-\phi_I}{\phi_I} P,$$

which is constant along the balanced growth path. Finally, we reach the conclusion

$$\frac{I}{Y} = \frac{PI}{PC+PI} = \frac{1}{1+\phi_I} = \frac{(1+g)^{\sigma}/\beta - 1 + \lambda}{(1+g)^{\sigma}/\beta - 1 + \lambda + \gamma (g+\lambda)}.$$

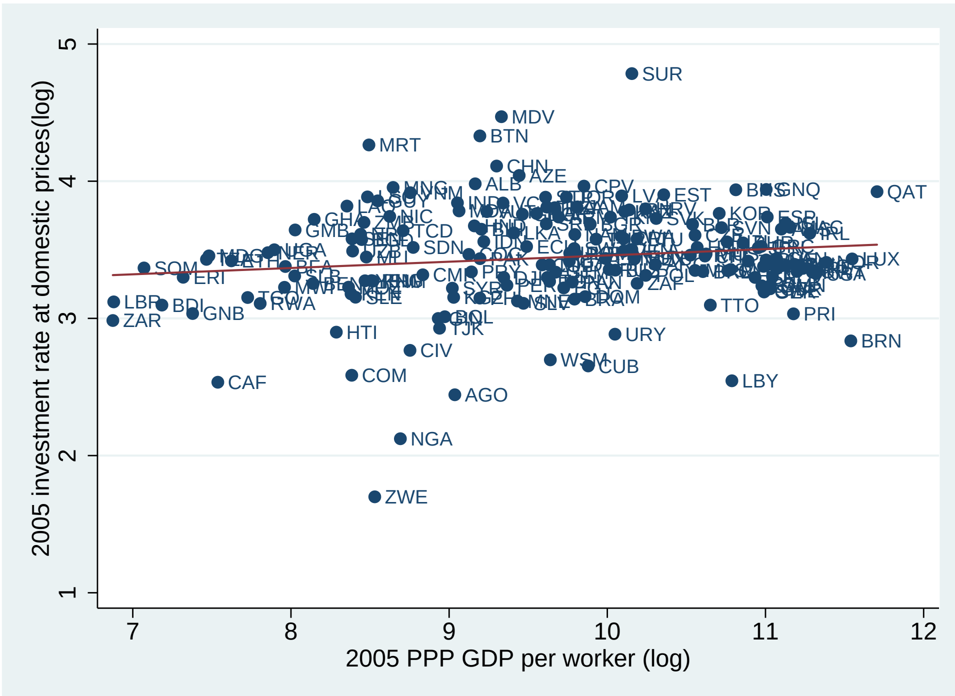
2.B Figures

Figure 2.1: PPP investment rates and income levels



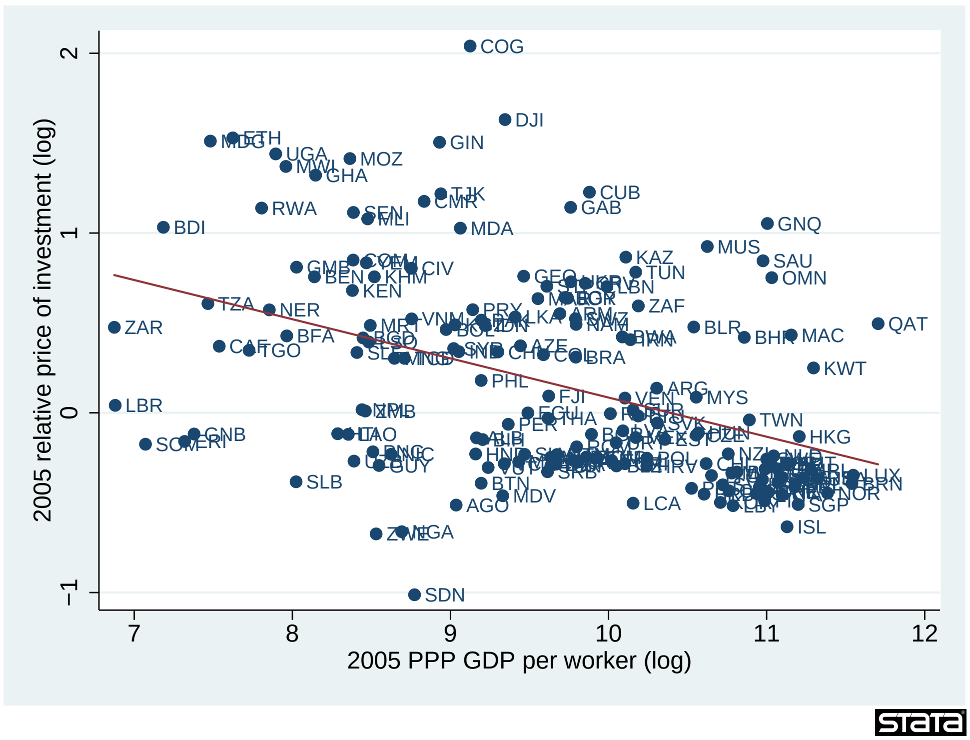
Raw correlation: 0.504. This figure displays the relationship between PPP GDP per worker and the PPP investment rates, in natural logs. Source: Penn World Tables version 6.3.

Figure 2.2: Investment rates at domestic prices and income levels



Raw correlation: 0.137. This figure displays the relationship between PPP GDP per worker and the investment rates at domestic prices, in natural logs. Source: Penn World Tables version 6.3.

Figure 2.3: Relative prices of investment and income levels



Raw correlation: -0.428. This figure displays the relationship between PPP GDP per worker and the investment rates at domestic prices, in natural logs. Source: Penn World Tables version 6.3.

Figure 2.4: Relative prices of investment and rule of law

Raw correlation: -0.394. This figure displays the relationship between relative prices of investment and index of rule of law. Source: Penn World Tables version 6.3 and Worldwide Governance Indicators.

Figure 2.5: Comparative Statics

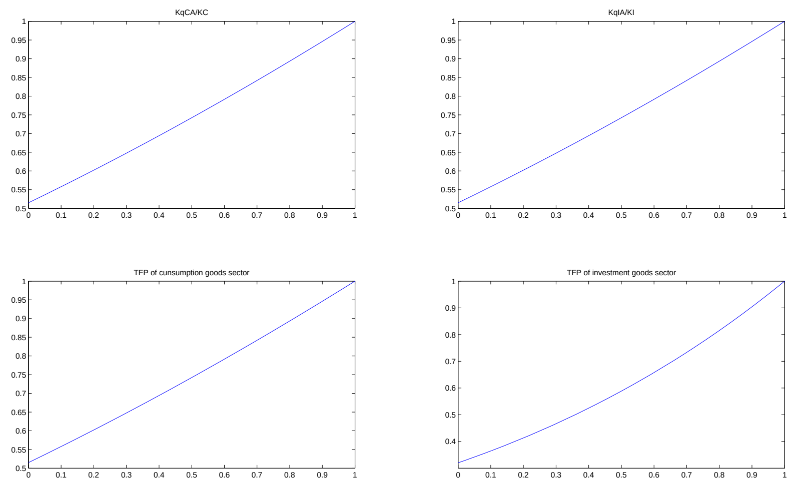


Figure 2.6: Steady State I

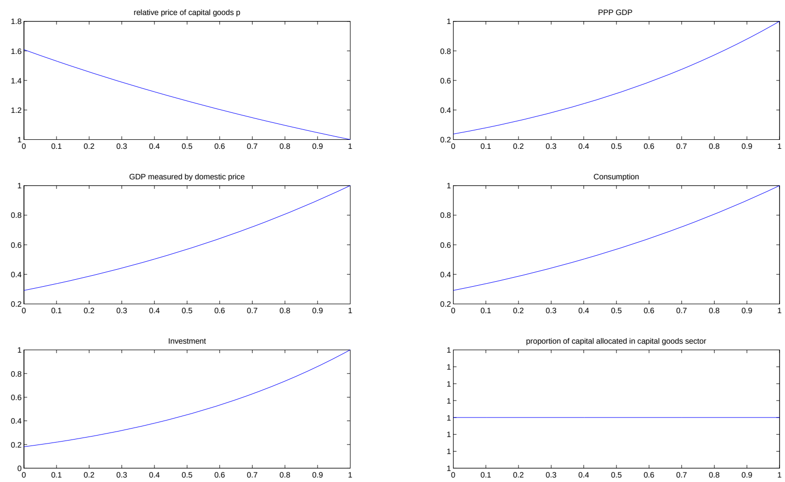
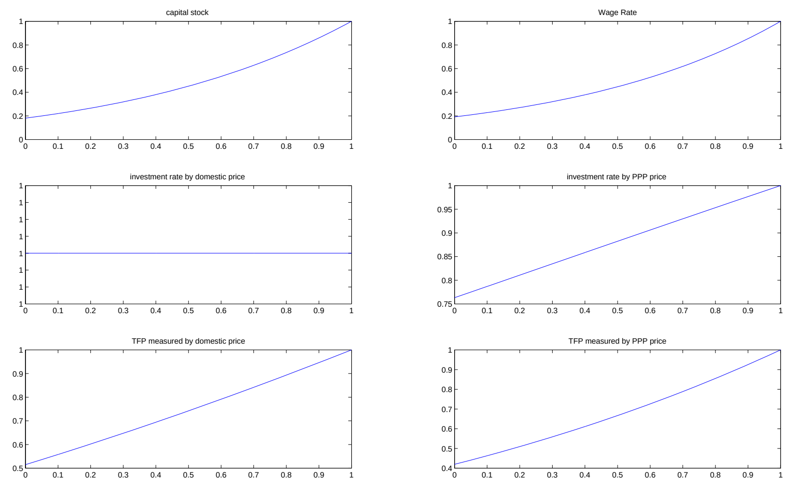


Figure 2.7: Steady State II



Chapter 3

A Ricardian Model of Labor Market with Directed Search

3.1 Introduction

In the economy, we always observe that different workers with different levels of education are allocated to different occupations and earn different wages. Recently, the Ricardian model of the labor market developed by Autor, Levy and Murnane (2003), Acemoglu and Autor (2011) and Costinot and Vogel (2011) provides a natural benchmark to analyze this phenomenon. In their terminology, a task is a unit of work activity that produces output. A skill is a worker's endowment of capabilities for performing various tasks. Different workers with different skill levels have distinct comparative advantage for performing different tasks. Therefore, this task-based approach emphasizes that skills are applied to tasks to produce output—skills do not directly produce output. The distinction between skills and tasks makes the assignment issue relevant. In this paper, I will answer the question: how will the labor market friction, in particular the search/matching friction, affect the assignment, thereby affecting the income inequality and the unemployment rate? To address this question, I embed a directed search model *à la* Shi (2002) and Shimer (2005) following the approach of "competitive search" (Moen, 1997) into a simple Ricardian model of the labor market. The reason why I adopt a directed search framework rather than a Diamond-Mortensen-Pissarides-style random searching framework is the following: directed search is a more reasonable assumption when the workers' trait determining heterogeneity is observable (e.g., education), whereas random matching fits better when the source of heterogeneity is not directly observable (e.g., ability or vintage-specific skills). Here, I distinguish between the skilled and unskilled worker by their education levels which is observable by employer. To make a comparison with the case with search friction, I first lay down a Ricardian model of the labor market assuming competitive market. The equilibrium features a perfect assortative matching: unskilled and skilled workers will perform different tasks. The technology and endowment of different workers will determine a threshold which separates the whole tasks into two parts which are performed by two kinds of workers, respectively. I also show that this Ricardian model of the labor market is isomorphic to a canonical model[1]. This means that I provide a micro foundation for the canonical model and we can exploit this relation to calibrate the Ricardian model of the labor market later on.

Despite the increased complexity of introducing the directed search model into a Ricardian model of the labor market, my analysis shows the main features of the equilibrium by using the numerical method. In equilibrium, the matching pattern is partially mixed. Some tasks can be performed by both skilled and unskilled applicants but favor skilled workers, while some tasks are only performed by either skilled workers or unskilled workers. Therefore, a search friction generates a mismatch. The mechanism of generating this mismatch has its roots in the search friction which is modeled as a coordination failure in this paper. The search friction does not only generate the mismatch but also the within group wage difference. In the competitive labor market, the same workers receive the same wage, but with a search friction, the

[1] The canonical model has two skills, high and low. The aggregate production function is $Y = [(A_L L)^\rho + (A_H H)^\rho]^{\frac{1}{\rho}}$. It makes no distinction between skills and tasks (occupations).

same workers may receive different wages. The same workers have the same expected wage rate, i.e. the workers are willing to accept the lower wage with a higher job finding rate. In this model, the same workers applying for the same job in a task receive the same wage rate and job finding rate, the within group difference in the wage rate only exists for the same workers who apply for the jobs in different tasks.

The existence of mismatch means that the economy could not reach the frontier of production efficiency. A reasonable question then follows: how much output loss is generated by mismatch? To answer this question, we need to calibrate this model and do a quantitative analysis. The model must be extended to a dynamic setting if it is to be taken quantitatively seriously. Therefore, I extend it into a dynamic setting such that I can do the quantitative exercise. From the analysis, I found that the quantitative effect of mismatch is small in terms of output loss generated by mismatch. At the same time, the within group income inequality which is also caused by a search friction is also small.

Closely related to search friction, a perspective that emphasizes the importance of the assignment of skills to tasks also calls for an additional study of the role of labor market institutions. I introduce several policy variables that feature prominently in the actual labor market: unemployment benefits and labor income taxes. For example, I find that higher unemployment benefits and labor income taxes will increase the level of mismatch and the unemployment rate especially for unskilled workers.

My model combines three main elements: (1) a Ricardian model of the labor market, (2) a search/matching friction in the labor market and (3) labor market regulation. As a result, it is related to a large and rapidly growing literature.

The Ricardian model of the labor market incorporates a clear distinction between workers' skills and job tasks (or occupations) and allow the assignment of workers with different skills to tasks to be determined in equilibrium by labor supplies, technologies, and task demands, as suggested by Autor, Levy and Murnane (2003), Acemoglu and Autor (2010) and Costinot and Vogel (2011). Costinot and Vogel (2011) consider a model with a continuum of skills as well as a continuum of tasks. Under a comparative advantage (log super modularity) assumption, Costinot and Vogel (2011) characterize the labor market equilibrium in terms of two ordinary differential equations, one determining the match between skills and tasks and the other determining the wage as a function of assignment. But this is achieved under the assumption of a competitive labor market. To the best of my knowledge, all analyses of the Ricardian model of the labor market are based on a competitive labor market. In this paper, I incorporate search friction into a model with a continuum of tasks as well as two kinds of skills. It is a first step towards exploring the effect of market imperfection on the assignment in a Ricardian model of the labor market.

To incorporate search friction into the Ricardian model of the labor market, I formulate a directed search model à la Shi (2002) and Shimer (2005) following the approach of "competitive search" (Moen, 1997). Shi (2002) and Shimer (2005) are both pioneering works on a directed search model when heterogeneity is present on both sides, i.e. firms and workers. Shi (2002) analyzes a directed search model with two types of workers (skilled and unskilled) and two types of firms (high-tech and low-tech). In this environment, skilled workers only apply for high-tech jobs, while unskilled workers apply for both types of jobs. Compared with the two types of workers and the two types of firms' environment, the Ricardian model of the labor market is a more general framework since it contains an infinite number of tasks where different workers perform with different comparative advantages. I think that it is important to understand what is the effect of a search/matching friction in this more generalized framework.

My work is also related to the line of literature on the quantitative effect of the labor market institutional features that have an impact on the propagation of different macroeconomic shocks by using a search/matching framework. Prominent examples include Bertola and Ichino (1995), Hornstein, Krusell and Violante (2007), Ljungqvist and Sargent (1998, 2008), Marimon and Zilibotti (1999), Mortensen and Pissarides (1999) and Pissarides (2007). But all these works are motivated by the salient increase in unemployment in Europe in recent decades and focus on the unemployment rate. These papers pay less attention to the effect of labor market institutions on the issue of allocating skills to tasks. I think that my work is a also contribution to this line of literature.

The rest of the paper is organized as follows. In section 3.2, I will introduce a simple task model based on Acemoglu and Autor (2011) and Costinot and Vogel (2011). I show that this simple task model is isomorphic to the canonical model and provide a natural benchmark to incorporate a search friction. In section 3.3, I will incorporate the directed search à la Shi (2002) and Shimer (2005) into the task model. I will first characterize the one-stage equilibrium and discuss the qualitative property. To do the quantitative analysis, I extend the model to the infinite horizon and calibrate it in section 3.4. In section 3.5 I will discuss the effect of the different labor market regulations such as unemployment insurance and labor income taxation. Finally, in section 3.6, I conclude the paper. Proofs of the main results are provided in the Appendix.

3.2 A Ricardian Model with Competitive Labor Market

Here I set up a simple Ricardian model of a labor market with a competitive labor market and characterize the equilibrium. At the end of this section, I establish the relation with the canonical model.

3.2.1 Economic environment and competitive equilibrium

The unique final good is produced by combining a continuum of tasks represented by the unit interval, $[0,1]$. In particular,

$$Y = \exp\left[\int_0^1 \ln y\left(i\right) di\right] \tag{3.1}$$

where Y denotes the output of a unique final good and $y\left(i\right)$ is referred to as the production of task i. I assume all markets to be competitive. Throughout, I choose the price of the final good as the numeraire.

Each task is produced by the following technology function

$$y\left(i\right) = A_L \alpha_L\left(i\right) l\left(i\right) + A_H \alpha_H\left(i\right) h\left(i\right) \tag{3.2}$$

where A terms represent a factor-augmenting technology, and $\alpha_L\left(i\right)$, $\alpha_H\left(i\right)$ are the task productivity schedules, designating the productivity of low and high skill workers in different tasks. It is important to see that this production function for task services implies that each task can be performed by low or high skill workers, but the comparative advantage of skill groups differs across tasks which is captured by the α terms. The differences in comparative advantage play a central role in a Ricardian model of the labor market. I impose the following assumption on the structure of comparative advantage throughout this paper.

Assumption 3.1. *$\alpha_L\left(i\right) = \left(1-i\right)^{\eta}$ and $\alpha_H\left(i\right) = i^{\eta}$, $\eta > 0$.*

It is easily seen that $\alpha_L\left(i\right)/\alpha_H\left(i\right)$ is continuously differentiable and strictly decreasing. This assumption specifies the structure of comparative advantage. It can be interpreted as higher indices corresponding to more complex tasks in which high skill workers perform better than low skill workers.

The economy is endowed with N workers who are distinguished by an observable skill level (e.g. education attainment). A fraction s of workers are skilled worker and the remaining workers are unskilled, i.e. $s = \frac{H}{N}$. I use a subscript $k \in \{H, L\}$ to indicate two types of workers, high and low skilled workers.

Assumption 3.2. *$A_H > A_L$ and $H < L$ where H and L are the aggregate endowments of high and low skill workers in this economy.*

This assumption is the sufficient condition such that the high skill worker has a higher wage than the low skill worker.

I will characterize the competitive equilibrium in the following. In particular, there will exist an I such that all tasks $i < I$ will be performed by low skill workers, and all tasks $i > I$ will be performed by high skill workers.

Lemma 3.1. *In any equilibrium, there exists I such that $0 < I < 1$ and for any $i < I$, $h(i) = 0$, for any $I < i < 1$, $l(i) = 0$.*

The competitive equilibrium can be characterized by the following conditions:

1. Law of one price for skills

$$W_L = p(i) A_L \alpha_L(i) \quad \text{for any } i < I,$$

$$W_H = p(i) A_H \alpha_H(i) \quad \text{for any } i > I.$$

   Then, the wage difference is determined by the productivity difference at the threshold

$$\frac{W_H}{W_L} = \frac{A_H \alpha_H(I)}{A_L \alpha_L(I)} .$$

2. Goods market (or task market) clearing condition:

$$p(i) y(i) = p(i') y(i') = Y.$$

   Then, in the interval $i < I$, we obtain

$$p(i) A_L \alpha_L(i) l(i) = p(i') A_L \alpha_L(i') l(i') .$$

   Combining with the condition for the one price law for skills, we obtain

$$l(i) = l(i') \text{ for any } i < I, \tag{3.3}$$

similarly

$$h(i) = h(i') \text{ for any } i > I. \tag{3.4}$$

3. No arbitrage condition across skills: the threshold task I must be such that it can be profitably produced using either high skilled or unskilled workers:

$$A_L \alpha_L(I)\, l(I) = A_H \alpha_H(I)\, h(I). \tag{3.5}$$

4. Labor market clearing condition:

$$\int_0^I l(i)\, di = L, \tag{3.6}$$

$$\int_I^1 h(i)\, di = H. \tag{3.7}$$

By (3.3), (3.4), (3.6), and (3.7), we obtain that

$$l(I) = L/I, \tag{3.8}$$

$$h(I) = H/(1-I). \tag{3.9}$$

Plugging (3.8), and (3.9) into (3.5), we can obtain

$$I = 1/\left(1 + \left(\frac{A_H H}{A_L L}\right)^{\frac{1}{1+\eta}}\right).$$

Given I, we can solve equilibrium allocation $l(i)$ and $h(i)$. Given the allocation, it is easy to solve the price and wage and obtain the following proposition.

Proposition 3.1. *The competitive equilibrium is characterized by the threshold condition I*

$$I = 1/\left(1 + \left(\frac{A_H H}{A_L L}\right)^{\frac{1}{1+\eta}}\right).$$

In the domain $i \in [0, I]$, the L/I measure of unskilled workers is allocated to each task. In the domain $i \in [I, 1]$, $H/(1-I)$ the measure of skilled workers is allocated to each task.

The wage premium is given by

$$\frac{w_H}{w_L} = \left(\frac{A_H}{A_L}\right)^{1/(1+\eta)} \left(\frac{H}{L}\right)^{-\eta/(1+\eta)}.$$

Lemma 3.1 and proposition 3.1 show that the matching pattern of a Ricardian model of the labor market is assortative, i.e. the whole range of tasks is separated into two domains by a threshold I which is determined by the technology and endowment of different factors: in one domain, skilled workers have a comparative advantage, in another domain unskilled workers show a comparative advantage. Skilled and unskilled workers are allocated to the tasks in which they have a comparative advantage. The competitive equilibrium features assortative matching with a positive and perfect correlation between matched workers' and firms' type.

3.2.2 The relation with the canonical model

Most economics analysis of changes in wage structure and skill differentials builds on the canonical model which is proposed in Tinbergen (1974, 1975) and developed in Welch (1973) and Katz and Murphy (1992), among many others. The effects of relative demand and supply of skills are typically modeled in an environment with just two types of workers (skilled and unskilled) and a constant elasticity of substitution aggregate production function. The technology function is

$$Y = [(A_L L)^\rho + (A_H H)^\rho]^{\frac{1}{\rho}},$$

where $\frac{1}{1-\rho} \in [0, +\infty)$ is the elasticity of substitution between high skill and low skill labor, and A_L and A_H are factor-augmenting technology terms..

With the competitive labor market, we have that

$$w_H = \partial Y / \partial H = [(A_L L)^\rho + (A_H H)^\rho]^{\frac{1}{\rho}-1} A_H^\rho H^{\rho-1},$$

$$w_L = \partial Y/\partial L = [(A_L L)^\rho + (A_H H)^\rho]^{\frac{1}{\rho}-1} A_L^\rho L^{\rho-1},$$

then

$$\frac{w_H}{w_L} = \frac{\partial Y/\partial H}{\partial Y/\partial L} = \left(\frac{A_H}{A_L}\right)^\rho \left(\frac{H}{L}\right)^{\rho-1}.$$

A relation between the assignment model and the canonical model clearly emerges. There is a one to one correspondence between ρ and η: $\rho = \frac{1}{1+\eta}$, i.e. $\eta = \frac{1-\rho}{\rho}$. If we define that $\rho = \frac{1}{1+\eta}$, we will have a Ricardian model of the labor market which is isomorphic to the canonical model. We can treat the Ricardian model as a micro foundation of the canonical model. Acemoglu and Autor (2011) have provided a special case that $\eta = 1$ which is isomorphic to the canonical model with $\rho = 1/2$. To the best of my knowledge, the establishment of a general isomorphic relation between the canonical model and the Ricardian model of the labor market has not previously been discussed.

The previous analysis assumes a frictionless labor market, but is it well known that market imperfection plays an important role in the labor market? Then, what is the equilibrium if we consider the case with search friction? In the next section, I will thus try to address the question: when I incorporate the search friction, what is the matching pattern?

3.3 A one shot model with directed search

The directed search model which I adopted here is based on the work by Shouyong Shi (2002) and Shimer (2005). It takes time to match workers with jobs. I model the search friction by assuming that each worker can apply to at most one job in a period. Unmatched jobs and workers produce nothing and get 0 payoffs. The time horizon is one period (in the next section I will extend this model to a dynamic setting). The interaction between firms and workers can be represented as a three-stage game. First, the firm enters and all firms simultaneously post and commit to wages and selection criteria of workers, knowing that their decisions will affect workers' application decisions. Second, after observing all posted wages and selection criteria, workers choose which firm to apply to. Finally, firms select workers according to the announced criteria, pay the promised wage, and production immediately follows. Workers who are not hired are unemployed, and jobs that are unfilled are vacant. There is an $M(i)$ measure of firms to recruit for each task. The number of vacancies is endogenously determined in equilibrium. Denote the aggregate measure of firms as $M = \int_0^1 M(i)\, di$. I focus on the case where the market is large and neither side of the market is infinitely larger than the other side. Each firm only recruits one employee. Each firm obtains the zero expected profit from creating different jobs. All jobs cost c to set up. Given the large numbers of workers and firms and the coordination problem across firms and workers, it is natural to focus on a symmetric, mixed strategy equilibrium, where ex ante identical workers and firms use the same strategy and workers randomize over a set of preferable jobs.

A type k worker's strategy is a set probability $\{p_k(i,j)\}_{k=H,L}$, $i \in [0,1]$ and j indexes the different firms, where $p_k(i,j)$ is the probability with which the type k worker applies firm j in a type-i task. Here, I assume that the probability of applying a specific type-i task is the same. Since each worker's application probabilities add up to one, I obtain

$$\int_0^1 p_k(i,j)\, dj di = 1.$$

A type i firm's strategy consists of the wages $w_k(i)$, $i \in [0,1]$ and a selection rule $\chi(i) \in [0,1]$. Here, I assume that all firms in task i choose the same strategy. The selection rule is only applied when the firm receives both types of applicants. In this case, the firm prefers a skilled worker if $\chi(i) = 1$, prefers an unskilled worker if $\chi(i) = 0$ and it is indifferent between the two types of workers if $\chi(i) \in (0,1)$. If the firm received z identical applicants, each applicant gets the job with probability $1/z$. These selection rules are announced before workers apply, and like the announced wages, they are committed to by the firm. In the following analysis, I also show that the firm prefers the same kind of worker *ex ante* and *ex post* that workers apply to firms.

Each worker maximizes her expected utility (in the one shot case, maximizing expected utility is equal to maximizing expect wage), making a trade-off between a wage and the probability of obtaining the job. When M, N approach infinity, the application probability approaches zero and is not convenient for the analysis. An alternative is queue length, defined as the expected number of workers applying to a firm. Let $x_k(i,j)$ be the queue length of the type-k worker who applies to a type-i firm j. Each worker's application probability for the same task is the same, therefore, $x_H(i) = x_H(i,j) = p_H(i,j)\,H$, $x_L(i) = x_L(i,j) = p_L(i,j)\,L$. These queue lengths are finite. I will treat the queue length as a worker's strategy $\{x_k(i)\}_{k=H,L}$. Since each worker's application probabilities add up to one, the following restriction must hold

$$\int_0^1 \int_0^{M(i)} x_H(i,j)\,dvdi = \int_0^1 M(i)\,x_H(i)\,di = H, \tag{3.10}$$

$$\int_0^1 \int_0^{M(i)} x_L(i,j)\,dvdi = \int_0^1 M(i)\,x_L(i)\,di = L. \tag{3.11}$$

Let $q_k(i)$ be the probability with which a type-k worker gets a type-i job. The probabilities are given by the following equations

$$q_H(i) = \left[\chi(i) + (1-\chi(i))\,e^{-x_L(i)}\right] g(x_H(i))\,; \tag{3.12}$$

$$q_L(i) = \left[1-\chi(i) + \chi(i)\,e^{-x_H(i)}\right] g(x_L(i))\,, \tag{3.13}$$

where

$$g(x) = \frac{1-e^{-x}}{x}. \tag{3.14}$$

Since the explanations for equations 3.12 and 3.13 are similar, I will only discuss equation 3.12. A type-i firm chooses a skilled worker either when no unskilled worker has applied to the firm, or when one or more unskilled workers have applied but the type-i firm favors a skilled applicant. The first case happens with probability $(1-p_L(i))^{(1-s)N}$, the second with $\chi(i)\left[1-(1-p_L(i))^{(1-s)N}\right]$ and the sum of these probabilities is $\chi(i) + (1-\chi(i))\exp(-x_L(i))$ in the limit. Conditional on choosing a skilled applicant, the firm chooses a particular skilled applicant with probability $\left[1-\left(1-p_H(i)^{sN}\right)\right]/(p_H(i)\,sN)$, which is the probability that the firm receives one or more skilled applicants divided by the expected number of skilled applicants to that firm. The limit of this probability is $g(x_H(i))$. The function $g(x)$ is continuous and a strictly decreasing and strictly convex function, with $g(0)=1$ and $g(\infty)=0$. Therefore, the probability of getting a type-i job $q_k(i)$ strictly decreases in $x_k(i)$. The intuition is simple: A type-k worker who applies to a type-i firm is less likely to be chosen if more type-k workers apply to the firm. The probability of filling a type-i vacancy with a skilled worker or an unskilled worker is $\left[\chi(i)+(1-\chi(i))\,e^{-x_L(i)}\right]\left(1-e^{-x_H(i)}\right)$ and $\left[1-\chi(i)+\chi(i)\,e^{-x_H(i)}\right]\left(1-e^{-x_L(i)}\right)$, respectively. The function $g(x)$ is continuous and a strictly increasing and strictly convex function, with $g(0)=1$ and $g(\infty)=0$. Therefore, the probability of filling a type-i vacancy strictly increases in $x_k(i)$. The intuition is also straightforward: A type-i vacancy with a larger number of expected applicants is more likely to be filled.

To describe a worker's decision, let U_k be a type-k worker's expected utility (here the expected wage in the one shot case) in equilibrium, which is taken as given by the individual agent when $M, N \to \infty$. A type-k worker applies to a task-i firm if and only if the expected wage from the firm is no less than U_k. However, it cannot be the case that $q_k(i)\,w_k(i) > U_k$ in equilibrium. If a firm's offer yields $q_k(i)\,w_k(i) > U_k$, all type-k workers will apply to that firm with probability 1, yielding $x_k(i) \to \infty$, then $q_k(i) = 0$, which contradicts $q_k(i)\,w_k(i) > U_k$. Thus, a type-k worker's strategy is

$$x_k(i)\begin{cases} \in (0,\infty)\,, \text{if } q_k(i)\,w_k(i) = U_k, \\ \quad = 0, \quad \text{if } q_k(i)\,w_k(i) < U_k, \end{cases} \tag{3.15}$$

which shows a worker's trade-off between a wage and the matching probability: a low wage job must be compensated by a high employment probability, otherwise the type-k worker will not apply, i.e. the expected queue length is zero.

Now I turn to the firms' decisions. A type-i firm's expected profit is

$$\begin{aligned}\pi(i) =& \left[\chi(i) + (1-\chi(i))\, e^{-x_L(i)}\right]\left(1-e^{-x_H(i)}\right)\left(p(i)\, A_H \alpha_H(i) - w_H(i)\right) \\ &+ \left[1-\chi(i) + \chi(i)\, e^{-x_H(i)}\right]\left(1-e^{-x_L(i)}\right)\left(p(i)\, A_L \alpha_L(i) - w_L(i)\right).\end{aligned} \tag{3.16}$$

$\left[\chi(i) + (1-\chi(i))\, e^{-x_L(i)}\right]$ is the probability with which a type-i firm chooses a skilled worker. The firm gets one or more skilled applicants with probability $\left(1-e^{-x_H(i)}\right)$ and a skilled worker yields the profit $\left(p(i)\, A_H \alpha_H(i) - w_H(i)\right)$. The first term on the right-hand side is the expected profit from getting one skilled worker, and the second term is the expected profit from getting an unskilled worker.

A firm maximizes the expected profit, taking the expected utilities (U_H, U_L) and the other firms' strategy as given. That is, a task-i firm chooses $(w_H(i), w_L(i), \chi(i))$ to solve:

$$(P1) \quad \max \pi(i) \text{ s.t. } (3.15).$$

A firm does not take the queue lengths as given; rather, it takes (3.15) as a constraint. Given (U_H, U_L), the firm effectively chooses queue lengths by adjusting the wage. In particular, a firm can offer a high wage to increase its matching probability. In contrast to the Bertrand competition, an individual firm's wage offer affects the queue length smoothly rather than discontinuously, because the x_k smoothly depends on the wage offer. That is, a marginal wage increase can only attract a marginal increase in the expected number of applicants.

3.3.1 Equilibrium definition and Characterization

Definition 3.1. *An equilibrium consists of the measure of type-i firm $M(i)$, workers' expected utilities $(U_L; U_H)$, firms' strategies $(w_k(i), \chi(i))_{k=H,L}$, workers' strategies (queue length) $x = (x_L(i); x_H(i))$, goods (tasks) price $p(i)$, task output $y(i)$ and aggregate output Y such that*

1. *Given workers' expected utilities $(U_L; U_H)$ and other firms' strategies, each type-i firm's strategies solve the firm's problem (P1)*

2. *Observing firms' decisions, each worker's decision obeys* $q_k(i)\, w_k(i) = U_k$.

3. *The mass of firms* $M(i)$ *is pinned down such that firms earn zero net profit* $\pi(i) = c$.

4. *The aggregate applicants equal the endowment of the economy:* $\int_0^1 x_L(i)\, M(i)\, di = L$ *and* $\int_0^1 x_H(i)\, M(i)\, di = H$.

5. *The measure of different workers employed in task* i *is* $l(i) = e^{-x_H(i)}\left(1 - e^{-x_L(i)}\right) M(i)$ *and* $h(i) = \left(1 - e^{-x_H(i)}\right) M(i)$.

6. *The output of task* i *satisfies (3.2).*

7. *Goods market clearing demand* $p(i)\, y(i) = p(i')\, y(i') = PY = Y$.

For task i firms, there are three possibilities of $(x_L(i), x_L(i))$. They are $x_L(i) = 0, x_H(i) > 0; x_L(i) > 0, x_H(i) = 0$; and $x_L(i) > 0, x_H(i) > 0$. Under Assumptions 3.1 and 3.2, the following lemma greatly simplifies the analysis.

Lemma 3.2. *In equilibrium, all three possibilities will appear. If* $x_L(i) > 0, x_H(i) > 0$, $\chi(i) = 1$, *i.e. the firm always prefers a skilled worker if there are both kinds of applicants. Moreover, I obtain that* $U_H > U_L$, $x_L(i) = \bar{x}_L$ *if* $x_H(i) = 0$, *and* $x_H(i) = \bar{x}_H$ *if* $x_L(i) = 0$.

From the analysis in the previous section, we know that with the frictionless labor market, the equilibrium allocation is characterized by a threshold condition which separates the tasks into two domains. Skilled and unskilled workers are separately allocated to two domains. What is the difference in the allocation between the competitive labor market and the frictional labor market? From the following proposition, we know that the allocation with search friction is different from the frictionless case. Then, the main difference is that there exist domains where skilled and unskilled workers are both allocated.

Proposition 3.2. *There is a unique equilibrium. The equilibrium allocation can be divided into three domains. Two threshold conditions*

$$\frac{A_L \alpha_L (I_1)}{A_H \alpha_H (I_1)} = \frac{e^{\bar{x}_L} U_L}{U_H - (1 - e^{\bar{x}_L}) U_L} \tag{3.17}$$

and

$$\frac{A_L \alpha_L (I_2)}{A_H \alpha_H (I_2)} = \frac{U_L}{U_H - (1 - 1) U_L} = \frac{U_L}{U_H} \tag{3.18}$$

divide the domain $[0, 1]$ *into three parts: In the domain* $[0, I_1]$*, only unskilled workers apply; in the domain* (I_1, I_2)*, both skilled and unskilled workers apply; in the domain* $[I_2, 1]$*, only skilled workers apply. In the first domain, the queue length of unskilled applicants* $x_L (i) = \bar{x}_L$ *is the same and given by*

$$(e^{\bar{x}_L} - \bar{x}_L - 1) U_L = c. \tag{3.19}$$

Similarly, in the third domain, the queue length of skilled applicants $x_H (i) = \bar{x}_H$ *is the same and given by*

$$(e^{\bar{x}_H} - \bar{x}_H - 1) U_H = c. \tag{3.20}$$

In the second domain, the queue lengths of different applicants are given by the following two equations

$$\left[e^{x_H(i)} - x_H (i) - 1\right] U_H + \left[e^{x_H(i)} \left(e^{x_L(i)} - 1\right) - x_L (i)\right] U_L = c, \tag{3.21}$$

$$\frac{A_L \alpha_L (i)}{A_H \alpha_H (i)} = \frac{e^{x_L(i)} U_L}{U_H - (1 - e^{x_L(i)}) U_L}. \tag{3.22}$$

From these two equations, we know that in the second domain, $x_L (i)$ *is a decreasing function of* i*, and instead* $x_H (i)$ *is an increasing function of* i*. The selection rule is* $\chi (i) = 1$ *in this domain.*

The equilibrium is defined as the distribution of queue length, firms and wage rate, so there is no analytical solution to this problem. Fortunately, I can use the numerical method to analyze the features of the equilibrium. The algorithm for computing a one shot equilibrium is presented in the Appendix.

From figures 3.1-3.4, we can clearly see the equilibrium allocation. I will describe the equilibrium allocation by figures 3.1-3.4 one by one. Figure 3.1 characterizes the queue lengths of different workers in the three domains. The queue length of unskilled workers in the first domain and that of skilled workers in the third domain are constant. In particular, in the second domain, the queue lengths of the skilled and unskilled worker are both positive. Moreover, based on (3.21) and (3.22), we know that the queue length of skilled workers increases, while the queue length of unskilled worker decreases. Figure 3.2 show the wage rate of different workers. In the first domain, the wage rate of unskilled workers is the same. This is due to the fact that with the same queue length, the unskilled worker faces the same probability of matching rate. From equation (3.15), we then know that firms pay them the same wage rate. A similar argument applies for skilled applicants who receive the same wage rate in the third domain. In the second domain, we will see that the wage rates of an unskilled worker and a skilled worker are increasing functions of task index i. The reason for this pattern also lies in the workers' tradeoff between the wage rate and the possibility of getting a job. The matching rate of the unskilled worker in the second domain is $e^{-x_H(i)}\left[1-e^{-x_L(i)}\right]/x_L(i)$. Then, with a decreasing $x_L(i)$ and an increasing $x_H(i)$, the probabilities of getting a type-i job for the unskilled worker decrease in the second domain which leads to an increasing wage rate of unskilled workers by equation (3.15). The logic can be applied for the skilled worker also in this domain, but there is a difference in the matching rate of two kinds of workers, the matching rate of the skilled worker is $\left[1-e^{-x_H(i)}\right]/x_H(i)$ which is a decreasing function of i in the second domain. Therefore, the wage rate of the skilled worker in the second domain increases. From the previous description, we will find a subtle difference in the matching rate of two kinds of workers in the second domain: the matching rate of skilled workers is not affected by the strategies of the unskilled worker, instead the matching rate of unskilled workers is affected by the skilled workers' application. This means that a skilled worker's application will generate an externality for the unskilled worker, but not vice versa. Figure 3.3 describe the measure of type-i entried firms in the three domains, in the first and third domains the measure of the entried firms is constant across different tasks, in the second domain the measure of type-i firms is a decreasing function of i. Figure 3.4 show the distribution of employed workers across different tasks. Skilled workers are only employed in the second and third domain and the measure of the employed skilled worker increases in the second domain and remains constant in the third domain. In contrast, unskilled workers are only employed in the first and second domain and the measure of employed unskilled workers decreases in the second domain and is constant in the first domain. For the employment rate of different workers, we know the key determined factor to be the matching rate of the different workers. Since the matching rate is not the same across different tasks for different workers, the aggregate difference of the matching rate for different workers is not obvious. But by using numerical experiments, we know that the unemployment rate of unskilled workers is higher than that of skilled workers.

3.3.2 Analysis of results

From proposition 3.2, we know that these three domains will exist in equilibrium. Compared with the competitive labor market, we can see a salient difference: first, after incorporating the search/matching friction, unemployment is inevitable. The unemployment rate will be discussed in more detail later on. Second, there will be a domain where skilled and unskilled workers both apply. In the case of a competitive labor market, the matching pattern is perfect assortative matching. But with a search friction, we see that the matching pattern is not perfectly assortative, i.e. two kinds of workers are both allocated to a subset of tasks. The imperfect assortative matching implies a mismatch in the second mixed domain. Here, mismatch means that it is possible to exchange the jobs that two skilled and unskilled workers perform in the second domain such that it will generate an output gain for both workers. Such a mismatch appears because of coordination failure (here I model coordination failure as the reason for the search/matching friction) and the continuous changing comparative advantage profile $\alpha_L(i)/\alpha_H(i)$. Because of coordination failure, workers face a tradeoff between the wage rate and the expected possibility of getting this job. Therefore, in the second domain, both kinds of workers are willing to continuously adjust their application strategies. This pattern of application strategies leads to a smooth change in queue length in the second domain. But in the case of the competitive labor market, coordination failure will not exist and thus, the worker does not face the uncertainty of getting a job and will get a job with certainty. Therefore, the difference in the wage rate and the comparative advantage totally determine the allocation of different workers. I have show there to be a mismatch when there is search friction; moreover, the severity of mismatch which is represented by the largeness of the second domain is determined by the degree of search friction. Here the degree of search friction is represented by parameter c. If c is larger, firms should pay more to generate a vacancy. If c goes to zero, the measure of the entried firms will approach infinity. With an infinite number of vacancies, the queue length will converge to zero, i.e. the matching rate of workers goes to 1. Therefore, the unemployment rate will approach 0. Meanwhile, from equations (3.17) and (3.18) we know that I_1 will converge to I_2 which leads to the disappearance of the second domain and the mismatch. I have qualitatively shown the existence of mismatch, but the severity of the mismatch is a quantitative question. To address this question, I extend the model to a dynamic setting such that I can calibrate it and do a quantitative analysis.

The search friction also generates the within group wage differential. Without search friction, there is no wage difference in the case of the competitive labor market. The reason why the same worker accepts different wage rates is that with search friction, the same worker cares about the expected wage instead of the wage rate per se. Then, the lower wage is compensated by a higher possibility of getting a job. If the search friction becomes smaller, the within group wage differential will decrease.

From figure 3.1, we see that different workers feature distinctive queue lengths, therefore, different workers should have different matching rates and unemployment rates. Since the matching rate is not the same for the identical workers who apply for different types of jobs, we cannot get an analytical expression of the average matching rate for different workers. But with a numerical experiment, I find that the unemployment rate of the skilled worker is lower than that of the unskilled worker.

3.4 Dynamic Model and Auantitative Analysis

Now I extend the time horizon to infinity. This extension is interesting because one would like to know whether the results are robust to firms' and workers' dynamic concerns and it is important to do the quantitative analysis to gauge the quantitative effect of mismatch. The description here is kept sketchy and the details are in the Appendix.

Two elements are added for the dynamic analysis: a recruiting cost $c > 0$, which is incurred by a vacancy *in* each period, and an exogenous job separation rate $\sigma > 0$. Except for lost production and wages, there is no other cost of job separation. The sequence of events is as follows. At the beginning of each period, new firms enter and recruit new workers (on-the-job search is excluded here for simplicity). Then, firms post their wages to attract applicants. After matching, production takes place and wages are paid. Finally, each pair has an exogenous probability σ of separation. The sequence of actions starts anew in the next period. The discount factor is β for both firms and workers.

In a dynamic setting, firms' and workers' decisions are now based on gains in present values rather than one-period gains. Then, the first step is that I must specify the value function of different agents. Denote the matching rates of a type-i firm recruiting a type-k worker and a type-k worker filling a type-i vacancy as $\lambda f_k(i)$ and $\lambda w_k(i)$. The value functions from the firm's side can be defined as follows

$$V(i) = -c+\beta\left[\lambda f_H(i) J_H(i) + \lambda f_L(i) J_L(i) + (1-\lambda f_L(i) - \lambda f_H(i)) V(i)\right], \tag{3.23}$$

$$J_H(i) = p(i) A_H(i) \alpha_H(i) - w_H(i) + \beta\left[(1-\sigma) J_H(i) + \sigma V(i)\right], \tag{3.24}$$

$$J_L(i) = p(i) A_L(i) \alpha_L(i) - w_L(i) + \beta\left[(1-\sigma) J_L(i) + \sigma V(i)\right], \tag{3.25}$$

where $V(i)$ is the present value of a type-i vacancy and $J_k(i)$ is the present value of a type-i job which is filled by a type-k worker. From the discussion in the previous section, we know that $\lambda f_H(i)$ and $\lambda f_L(i)$ are

$$\lambda f_H(i) = \left(1 - e^{-x_H(i)}\right),$$

$$\lambda f_L(i) = e^{-x_H(i)}\left(1 - e^{-x_L(i)}\right).$$

The value functions from the workers' side can be defined as the following

$$U_H = \beta\left[\lambda w_H(i) W_H(i) + (1-\lambda w_H(i)) U_H\right], \tag{3.26}$$

$$U_L = \beta\left[\lambda w_L(i) W_L(i) + (1-\lambda w_L(i)) U_L\right], \tag{3.27}$$

$$W_H(i) = w_H(i) + \beta\left[(1-\sigma) W_H(i) + \sigma U_H\right], \tag{3.28}$$

$$W_L(i) = w_L(i) + \beta\left[(1-\sigma) W_L(i) + \sigma U_L\right], \tag{3.29}$$

where U_k is the value function of a type-k unemployed applicant and $W_k(i)$ is the value function of a type-k employed worker who perform a type-i task. We also know that $\lambda w_H(i)$ and $\lambda w_L(i)$ are

$$\lambda w_H(i) = \frac{1 - e^{-x_H(i)}}{x_H(i)},$$

$$\lambda w_L(i) = \frac{e^{-x_H(i)}\left(1 - e^{-x_L(i)}\right)}{x_L(i)}.$$

Throughout this paper, I will focus on the stationary equilibrium. In the following, I will define the stationary equilibrium and discuss the characterization of a stationary equilibrium in a very sketchy way (A detailed characterization of a stationary equilibrium is found in the Appendix).

Definition 3.2. *A stationary equilibrium consists of the value functions of vacant or filled type-i firms: $V(i)$ and $J_k(i)$, the value functions of employed and unemployed workers U_k and $W_k(i)$, the measure of type-i firms $M(i)$, workers' expected utilities $(U_L; U_H)$, firms' strategies $(w_k(i), \chi(i))_{k=H,L}$, workers' strategies (queue length) $x = (x_L(i); x_H(i))$, goods (tasks) price $p(i)$, task output $y(i)$ and aggregate output Y, such that*

1. *Given workers' expected utilities $(U_L; U_H)$ and other firms' strategies, each type-i firm's strategies solve the firm's problem.*

2. *Observing firms' decisions, each worker's decision obeys equations (3.26)- (3.29).*

3. *The mass of firms $M(i)$ is pinned down such that firms earn zero net profit $\pi(i) = c$.*

4. *The sum of the aggregate applicants and aggregate employment equal the endowment of the economy: $\int_0^1 \left(x_L(i) + \left(1 - e^{-x_L(i)}\right)/\sigma\right) M(i)\, di = L$ and $\int_0^1 \left(x_H(i) + \left(1 - e^{-x_H(i)}\right)/\sigma\right) M(i)\, di = H$.*

5. *The measure of different workers employed in task i is $l(i) = \left(1 - e^{-x_L(i)}\right) M(i)/\sigma$ and $h(i) = \left(1 - e^{-x_H(i)}\right) M(i)/\sigma$.*

6. *The output of task i satisfies 3.2.*

7. *Goods market clearing demand $p(i)\, y(i) = p(i')\, y(i') = PY = Y$.*

8. *Given the firms' and workers' strategies, it is straightforward to calculate the corresponding value function.*

Proposition 3.3. *The equilibrium allocation can be divided into three domains. Two threshold conditions*

$$\frac{A_H \alpha_H (I_1)}{A_L \alpha_L (I_1)} = \frac{(1 + \sigma\beta) U_H + (1 - (1 - \sigma)\beta) U_L (e^{\bar{x}_L} - 1)}{U_L [(1 - (1 - \sigma)\beta) e^{\bar{x}_L} + \beta]} \tag{3.30}$$

and

$$\frac{A_H \alpha_H (I_2)}{A_L \alpha_L (I_2)} = \frac{U_H}{U_L} \tag{3.31}$$

divide the domain $[0,1]$ into three parts: In domain $[0, I_1]$, only unskilled workers apply; in the domain(I_1, I_2), both skilled and unskilled workers apply; in the domain $[I_2, 1]$, only skilled workers apply. In the first domain, the queue length of unskilled applicants $\bar{x}_L$ is the same and given by

$$(1-\beta)\left(e^{\bar{x}_L} - \bar{x}_L - 1\right) U_L = c. \tag{3.32}$$

Similarly, in the third domain, the queue length of skilled applicants $\bar{x}_H$ is the same and given by

$$(1-\beta)\left(e^{\bar{x}_H} - \bar{x}_H - 1\right) U_H = c. \tag{3.33}$$

In the second domain, the queue lengths of different applicants are

$$(1-\beta)\left(e^{x_H(i)} - x_H(i) - 1\right) U_H + (1-\beta)\left[e^{x_H(i)}\left(e^{x_L(i)} - 1\right) - x_L(i)\right] U_L = c, \tag{3.34}$$

and

$$\frac{A_H \alpha_H (i)}{A_L \alpha_L (i)} = \frac{U_H\left[(1-(1-\sigma)\beta) e^{x_H(i)} + \beta\right] + (1-(1-\sigma)\beta) U_L e^{x_H(i)}\left(e^{x_L(i)} - 1\right)}{U_L\left[(1-(1-\sigma)\beta) e^{x_L(i)+x_H(i)} + \beta\right]}, \tag{3.35}$$

and the selection rule $\chi(i) = 1$ in this region.

One additional point should be discussed. In the dynamic setting, the type-i firms in the second domain strictly prefer skilled workers. But after they have been matched with an unskilled worker, they do not have any incentive to separate *ex post*. The reason is the following: although firms prefer skilled workers, the expected value of a job matched with an unskilled worker is positive. If they separate, the firms' expected value is zero by the free-entry condition. Therefore, the firms in the second domain do not have any incentive to separate with an unskilled worker ex post. Similarly, the unskilled workers in the second domain do not have any incentive to separate ex post. Based on this reason, I do not incorporate endogenous separation in the dynamic setting.

From proposition 3.3, we can see that the qualitative result remains unchanged, such as three domains of allocation, within and between income inequality. The main difference is quantitative. In the next subsection, I will discuss the details of calibration and quantitative results.

3.4.1 Calibration Details

In the previous subsection, I developed a dynamic directed search model and characterized the equilibrium. In this section, I calibrate the parameters of the model using aggregate data in the US labor market. Then, I will use the calibrated model to access the quantitative effect of a search friction in a Ricardian model of the labor market.

Table 3.1: Calibration

Variables	Description	Value
β	discount factor	0.996
c	vacancy cost	0.595
σ	exogenous separation rate	0.034
A_H/A_L	productivity of skilled worker	1.1
η	structure of comparative advantage	2.5
$H/(H+L)$	share of skilled workers	0.25

I choose the model period to be one month. Workers can only change employment status once a month. Here the model period is an important choice since it determines how often workers and firms will match. One period is also chosen by Menzio and Shi (2011). Given the model period to be one month, I choose the discount factor $\beta = 0.996$ to match the standard calibration of the discount factor: 0.95 one year. Shimer (2005b) measures the average separation probability from employment to unemployment in the US at $\sigma = 0.034$ per month. The vacancy cost $c = 0.595$ is chosen such that the aggregate unemployment rate is 6 percent. The next step is to calibrate the parameters which govern the production function and aggregate endowment. Since the production

function is isomorphic to the canonical model, I choose the standard calibration of the canonical model such that I can pin down the parameters governing the Ricardian model of the labor market. The most popular estimation of the coefficient of elasticity of substitution $1/(1-\rho)$ in the canonical model appears to be that of Katz and Murphy (1992), who set $1/(1-\rho)$ at 1.4.[2] From the analysis in section 3.2, we know there to be a one to one correspondence of η and ρ.
Therefore, I set the parameter η to be equal to 2.5. From Ciccone and Peri (2005), we know that in the US in 1990, the share of colleague graduates of the whole labor force is 25%; hence, I calibrate $H/(H+L)$ as 0.25 and normalize H as 1. At the same time, using the data addressed in Ciccone and Peri (2005), I set the wage ratio of the skilled and unskilled worker as 2.25. I set $A_H/A_L = 1.1$ as that implied by this income inequality and proposition 3.1.

Figures 3.5-3.8 characterize the equilibrium in the dynamic setting. The result is very similar to that in the static model. First, I will answer the question of how large is the mismatch. Here we need an index to represent the severity of mismatch. I formulate a variable to do this job. The variable is calculated as follows: given the measure of employed workers in the model with search friction, I calculate output Y_c if these workers are allocated by a competitive labor market. Since I know the output Y when there is a search friction, I let the variable $\Delta Y = \frac{Y_c - Y}{Y}$ measure the magnitude of mismatch. In the calibrated model, I obtain $\Delta Y = 0.000022$. The result means that a mismatch generates a very small output loss quantitatively. I can also proxy the magnitude of mismatch by calculating the size of the second domain. The second

[2] Caselli and Coleman (2006) estimate the aggregate elasticity of substitution between more and less educated workers using cross-country data and find a value of 1.31. Katz and Murphy (1992) estimate the aggregate elasticity of substitution between more and less educated workers using U.S. time-series data for the 1963–1987 period and find a value of 1.41. Krusell et al. (2000) also use US time-series data to estimate the short-run aggregate elasticity of substitution between more and less educated workers and find a value of 1.66. Here we choose the value 1.4 as our benchmark calibration.

domain is defined as $[I_1, I_2]$, and $I_2 - I_1 = 0.006$. Therefore, these two measurements lead to the conclusion that the magnitude of the mismatch is quantitatively small.

Table 3.2: Results of the calibrated model

Different variables	Statistics
Unemployment rate of skilled workers	5.28%
Unemployment rate of unskilled workers	6.23%
Average wage of skilled workers	0.7127
Average wage of unskilled workers	0.3051
Wage premium	2.3357

The quantitative results of the other variables are documented in the table 3.2. The different workers face different unemployment rates. Since aggregate unemployment is 6%, the unemployment rates of skilled and unskilled workers are 5.28% and 6.23%, respectively. We find that the difference in two groups of workers' employment rates is not very large. The wage premium is 2.3357 and it is slightly larger than the wage premium in the competitive market model.

3.5 Effects of Labor Market Regulations

In this section, we will analyze the effect of labor market regulations. We will focus on the effect of unemployment benefits and labor income taxation.

Let b denote the flow value of unemployment benefits paid to jobless workers, and let τ be the proportional labor income tax borne by workers. The equilibrium equations are derived in the appendix.

Proposition 3.4. *The equilibrium allocation can be divided into three domains. Two threshold conditions*

$$\frac{A_H \alpha_H (I_1)}{A_L \alpha_L (I_1)} = \frac{\beta (1-\beta) U_H + [1-(1-\sigma)\beta] \{[(1-\beta) U_H - b] + [(1-\beta) U_L - b] (e^{\bar{x}_L} - 1)\}}{\beta (1-\beta) U_L + [1-(1-\sigma)\beta] [(1-\beta) U_L - b] e^{\bar{x}_L}} \tag{3.36}$$

and

$$\frac{A_H\alpha_H(I_2)}{A_L\alpha_L(I_2)}=\frac{\beta(1-\beta)U_H+[1-(1-\sigma)\beta][(1-\beta)U_H-b]e^{\bar{x}_H}}{\beta(1-\beta)U_L+[1-(1-\sigma)\beta][(1-\beta)U_L-b]e^{\bar{x}_H}} \tag{3.37}$$

divide the domain $[0,1]$ *into three parts: In the domain* $[0,I_1]$*, only unskilled workers apply; in the domain*(I_1,I_2)*, both skilled and unskilled workers apply; in the domain* $[I_2,1]$*, only skilled workers apply. In the first domain, the queue length of unskilled applicants* $\bar{x}_L$ *is the same and given by*

$$\left(e^{\bar{x}_L}-\bar{x}_L-1\right)((1-\beta)U_L-b)=(1-\tau)c. \tag{3.38}$$

Similarly, in the third domain, the queue length of skilled applicants $\bar{x}_H$ *is the same and given by*

$$\left(e^{\bar{x}_H}-\bar{x}_H-1\right)((1-\beta)U_H-b)=(1-\tau)c. \tag{3.39}$$

In the second domain, the queue lengths of different applicants are

$$((1-\beta)U_H-b)\left(e^{x_H(i)}-x_H(i)-1\right)+((1-\beta)U_L-b)\left(e^{x_H(i)}\left(e^{x_L(i)}-1\right)-x_L(i)\right)=(1-\tau)c, \tag{3.40}$$

and

$$\frac{A_H\alpha_H(i)}{A_L\alpha_L(i)}=\frac{\beta(1-\beta)U_H+[1-(1-\sigma)\beta]e^{x_H(i)}\left\{[(1-\beta)U_H-b]+[(1-\beta)U_L-b]\left(e^{x_L(i)}-1\right)\right\}}{\beta(1-\beta)U_L+[1-(1-\sigma)\beta][(1-\beta)U_L-b]e^{x_L(i)+x_H(i)}}, \tag{3.41}$$

and the selection rule $\chi(i)=1$ *in this region.*

I put the derivation of the equilibrium and proposition 3.4 in the appendix. To get an idea of the effect of employment benefits and labor income taxation, I make a numerical experiment. I choose the unemployment benefit as 0.1 and the labor income tax rate as 20%. The equilibrium allocation is shown by figures 3.9-3.12. The increase in labor income and the unemployment benefit will increase the queue lengths, thereby reducing the matching rate of workers. This change leads to higher unemployment rates: the aggregate unemployment rate 6.6%, the unemployment rate of the skilled worker 5.47%, and the unemployment rate of the unskilled worker 7%. An increase in b and τ in this model makes workers more willing to accept an increase in the risk of unemployment in return for an increase in wages. The increasing queue length makes firms more difficult to match with a worker, thus decreasing the incentives for generating a vacancy. Figure 3.11 shows that the measures of vacancies decrease across tasks. As for the quantitative effect of mismatch, the mismatch increases as compared to the benchmark model. The range of the second domain (defined as $I_2 - I_1$) becomes 0.0077. And the output gap ΔY increases from 0.000022 to 0.000029. Although the mismatch increases a great deal expressed as a percentage, the aggregate quantitative effect of the mismatch is still very small.

Generous unemployment benefits and high taxes will make the value of unemployment artificially high. As a result, an increase in b and τ in this model makes workers more willing to accept an increase in the risk of unemployment in return for an increase in w. Therefore, we can see that firms respond by offering workers fewer jobs. This intuition can be proved from figures 3.9 and 3.11. Interestingly, the marginal effect of two labor market institutions is increasing in the magnitude of the others: the policy bundle considered has a stronger impact than the sum of the impacts of the three policies taken individually. One extreme situation is that if $b = 0$, any linear taxation will not affect the equilibrium.

3.6 Conclusions

The Ricardian model of the labor market incorporates a clear distinction between skills and tasks and allows the assignment of skills to tasks to be determined in equilibrium. But the entire analysis in the Ricardian model of the labor market relies on the assumption of a perfect labor market. In reality, the labor market is filled with different frictions, some of which are related to information and search friction. The allocation of skills to tasks is more complex in the presence of labor market imperfections.

In this paper I develop a model combining directed search and a simple Ricardian model of the labor market. Equipped with this model, I can examine the effect of search friction both qualitatively and quantitatively. Within this model, it is convenient to incorporate some realistic labor market policy variables such as unemployment benefits and labor income taxation.

The main result is that search friction leads to a mismatch: some tasks are performed by both skilled and unskilled workers. This mismatch will generate an output loss. Search friction also generates a different unemployment rate for skills and within group income inequality. Afterwards, I build a dynamic model and calibrate it and find that the effect of mismatch is small. I also introduce the unemployment benefit and labor income taxation. The results show that general unemployment benefits and high labor income taxation tend to increase the mismatch and unemployment rate.

A general finding from this paper is that even with search friction, the wage rate plays a dominant role in allocating skills to tasks. I would expect that labor market institutions such as minimum wages and labor unions which distort the wage rate will have a strong effect on allocating skills to tasks. This is worth exploring in future research and is already on my research agenda.

In this model, I only assume two types of skills. And mismatch only exists around the threshold. If I assumed a continuum of skills, would it increase the quantitative effect of mismatch? Given the technical difficulty, I have not discussed this setting in this model. But I think it is definitely an important extension worthy of further exploration.

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3.A Proofs and Numerical Algorithm

This appendix contains outlines of the proofs of the propositions and lemmas and numerical algorithm.

3.A.1 Proof of lemma 3.1 and proposition 3.2

$x_k(i)$ has two possibilities: 0 or positive. Then, there are three possibilities of the queue length of applicants. Let us analyze these possibilities one by one:

1. $x_H(i) > 0$ and $x_L(i) = 0$. The firms' profit function is

$$\pi(i) = \left(1 - e^{-x_H(i)}\right) \left[p(i) A_H \alpha_H(i) - w_H(i)\right], \tag{3.42}$$

and the constraint is

$$q_H(i) w_H(i) = U_H,$$

where

$$q_H(i) = \frac{1 - e^{-x_H(i)}}{x_H(i)}.$$

Then, the first-order condition gives

$$e^{-x_H(i)} p(i) A_H \alpha_H(i) = U_H. \tag{3.43}$$

Plugging (3.43) into (3.42), we obtain

$$\pi(i) = \left[1 - (x_H(i) + 1) e^{-x_H(i)}\right] p(i) A_H \alpha_H(i) = \left(e^{x_H(i)} - 1 - x_H(i)\right) U_H.$$

By free-entry condition $\pi(i) = \pi(i') = c$, we can easily solve $x_H(i) = \bar{x}_H$ if $x_L(i) = 0$. And $\bar{x}_H$ is given $\left(e^{\bar{x}_H} - 1 - \bar{x}_H\right) U_H = c$. At the same time, we can obtain that $\pi(i) \geq 0$, $\pi(i) = 0$ if and only if $x_H(i) = 0$.

2. $x_H(i) = 0$ and $x_L(i) > 0$. Then, the firms' profit function is

$$\pi(i) = \left(1 - e^{-x_L(i)}\right)\left[p(i) A_L \alpha_L(i) - w_L(i)\right], \tag{3.44}$$

and the constraint is

$$q_L(i) w_L(i) = U_L,$$

where

$$q_L(i) = \frac{1 - e^{-x_L(i)}}{x_L(i)}.$$

Then, the first-order condition gives

$$e^{-x_L(i)} p(i) A_L \alpha_L(i) = U_L.$$

Plugging (3.45) into (3.44), we obtain

$$\pi(i) = \left[1 - (x_L(i) + 1) e^{-x_L(i)}\right] p(i) A_L \alpha_L(i) = \left(e^{x_L(i)} - 1 - x_L(i)\right) U_L. \tag{3.45}$$

By condition $\pi(i) = \pi(i') = c$, we can easily solve $\bar{x}_L$ which is given by $(e^{\bar{x}_L} - 1 - \bar{x}_L) U_L = c$.

Now we will prove that these two possibilities will occur in equilibrium with certainty. I will prove the first case: $x_H(i) > 0$ and $x_L(i) = 0$, since the second case is analogous to this. If I pick a region close enough to task 1, i.e. $[1 - \varepsilon, 1]$, we will find that in this region, the productivity of the unskilled worker is so low (close to zero) such that given the expected utilities of unskilled workers it is not profitable to employ an unskilled worker. A similar reasoning is applied to the region $[0, \varepsilon]$. Then, I proved that in equilibrium, these two possibilities will occur with certainty. By assumption 3.2, we can see that in equilibrium, $U_H > U_L$. So we can see that $\bar{x}_L > \bar{x}_H$.

3. $x_H(i) > 0$ and $x_L(i) > 0$. Then firms' profit function is

$$\pi(i) = \left[\chi(i) + (1-\chi(i))\, e^{-x_L(i)}\right]\left[1 - e^{-x_H(i)}\right] p(i)\, A_H \alpha_H(i) - U_H x_H(i)$$
$$+ \left[1 - \chi(i) + \chi(i)\, e^{-x_H(i)}\right]\left[1 - e^{-x_L(i)}\right] p(i)\, A_L \alpha_L(i) - U_L x_L(i).$$

Rearrange the previous equation

$$\pi(i) = \chi(i)\left(1 + e^{-x_L(i)}\right)\left(1 - e^{-x_H(i)}\right)\left[p(i)\, A_H \alpha_H(i) - p(i)\, A_L \alpha_L(i)\right] - U_H x_H(i)$$
$$+ \left[1 - e^{-x_H(i)}\right] e^{-x_L(i)} p(i)\, A_H \alpha_H(i) + \left(1 + e^{-x_L(i)}\right) p(i)\, A_L \alpha_L(i) - U_L x_L(i).$$

From that expression, we can obtain FOC for $\chi(i)$, then

$$\chi(i)\begin{cases} = 1, & \text{if } A_H \alpha_H(i) > A_L \alpha_L(i) \\ = 0, & \text{if } A_H \alpha_H(i) < A_L \alpha_L(i) \\ \in [0,1], & \text{if } A_H \alpha_H(i) = A_L \alpha_L(i) \end{cases}$$

So we can divide the problem into two domains $\left[0, \bar{I}\right], \left[\bar{I}, 1\right]$ where $\bar{I}$ is defined as $A_H \alpha_H\left(\bar{I}\right) = A_L \alpha_L\left(\bar{I}\right)$.

The constraints are

$$q_L(i)\, w_L(i) = U_L,$$

$$q_H(i)\, w_H(i) = U_H,$$

where

$$q_L(i) = \frac{\left[1 - \exp(-x_L(i))\right]\left[1 - \chi(i) + \chi(i)\exp(-x_H(i))\right]}{x_L(i)},$$

$$q_H(i) = \frac{\left[1 - \exp(-x_H(i))\right]\left[\chi(i) + (1-\chi(i))\exp(-x_L(i))\right]}{x_H(i)}.$$

In the first domain, $A_H \alpha_H(i) < A_L \alpha_L(i)$, $\chi(i) = 0$. The firms' problem can be written as

$$\pi(i) = e^{-x_L(i)}\left[1 - e^{-x_H(i)}\right] p(i)\, A_H \alpha_H(i) - U_H x_H(i)$$
$$+ \left[1 - e^{-x_L(i)}\right] p(i)\, A_L \alpha_L(i) - U_L x_L(i)$$

$$\text{s.t } w_L(i) = \frac{U_L x_L(i)}{\left[1 - e^{-x_L(i)}\right]}$$

$$\text{s.t } w_H(i) = \frac{U_H x_H(i)}{\left[1 - e^{-x_H(i)}\right] e^{-x_L(i)}}.$$

Then, the first-order conditions give

$$e^{-(x_L(i)+x_H(i))} p(i) A_H \alpha_H(i) + e^{-x_L(i)} \left[p(i) A_L \alpha_L(i) - p(i) A_H \alpha_H(i)\right] = U_L$$

$$e^{-(x_L(i)+x_H(i))} p(i) A_H \alpha_H(i) = U_H.$$

From the two previous equations, we can conclude that

$$U_H < U_L.$$

In the second domain, $A_H \alpha_H(i) > A_L \alpha_L(i)$, $\chi(i) = 1$. The firms' problem can be written as

$$\begin{aligned} \pi(i) = & \left[1 - e^{-x_H(i)}\right] p(i) A_H \alpha_H(i) - U_H x_H(i) \\ & + e^{-x_H(i)} \left[1 - e^{-x_L(i)}\right] p(i) A_L \alpha_L(i) - U_L x_L(i) \end{aligned}$$

$$\text{s.t } w_L(i) = \frac{U_L x_L(i)}{e^{-x_H(i)} \left[1 - e^{-x_L(i)}\right]}$$

$$\text{s.t } w_H(i) = \frac{U_H x_H(i)}{\left[1 - e^{-x_H(i)}\right]}.$$

Then, the first-order conditions give

$$e^{-(x_L(i)+x_H(i))} p(i) A_L \alpha_L(i) + e^{-x_H(i)} \left[p(i) A_H \alpha_H(i) - p(i) A_L \alpha_L(i)\right] = U_H, \tag{3.46}$$

$$e^{-(x_L(i)+x_H(i))} p(i) A_L \alpha_L(i) = U_L. \tag{3.47}$$

From the two above equations, we can conclude that

$$U_H > U_L.$$

From the above analysis, we can see that there is no possibility that $U_H < U_L$ and $U_H > U_L$ are satisfied at the same time. Based on assumption 3.2, we know that $U_H > U_L$. Then, in the first domain $\left[0, \bar{I}\right]$, there is no probability of skilled and unskilled workers being lined up in the same firm. So, there are only unskilled applicants in the first domain. If there are skilled and unskilled applicants, we can only observe this in the second domain $\left[\bar{I}, 1\right]$.

Since we have proved that possibilities (1) and (2) will occur in equilibrium, we will prove that possibility (3) will definitely occur. If we assume that possibility (3) does not occur in equilibrium, there are only two possibilities of queue length $x_H(i) = 0$, $x_L(i) = \bar{x}_L$ or $x_L(i) = 0$, $x_H(i) = \bar{x}_H$. Then, there is a threshold $\hat{I}$ separating two domains. In the threshold, no arbitrage condition will hold: the type-$\hat{I}$ firm will be indifferent in employing skilled or unskilled workers. From the above analysis, we know that type-$\hat{I}$ firms prefer either skilled workers or unskilled workers. It is a contradiction, so there must exist a domain where two kinds of workers apply.

From the above analysis, we know that three domains lie in the interval $[0, 1]$. In the first domain $[0, I_1]$, only the unskilled worker applies, in the second domain $[I_1, I_2]$ both workers apply and in the third domain $[I_2, 1]$, only the skilled worker applies. It is important to point out that $I_1 \geq I$.

Now let us characterize three domains one by one. In the first domain $[0, I_1]$, by the free-entry condition, we obtain

$$\pi(i) = \pi(i') = c.$$

We can get $x_L(i) = x_L(i') = \bar{x}_L$, i.e $w_L(i) = w_L(i') = \bar{w}_L$. Combined with equation (3.43), we know that

$$p(i) A_L \alpha_L(i) = p(i') A_L \alpha_L(i').$$

From the goods market clearing condition $p(i) y(i) = p(i') y(i')$, we get

$$\frac{y(i)}{y(i')} = \frac{\alpha_L(i)}{\alpha_L(i')}. \tag{3.48}$$

Given the production function $y(i) = A_L \alpha_L(i) l(i)$, and $l(i) = \left(1 - e^{-x_L(i)}\right) M(i)$, the following equation is straightforward:

$$y(i) = \left(1 - e^{-x_L(i)}\right) A_L \alpha_L(i) M(i) = \left(1 - e^{-\bar{x}_L}\right) A_L \alpha_L(i) M(i). \tag{3.49}$$

Then, by equations (3.48) and (3.49)

$$M(i) = M(i') = M(0).$$

Similarly, we can characterize the equilibrium in the third domain where only skilled workers apply. By the same logic, we know that in the third domain

$$x_H(i) = x_H(i') = \bar{x}_H;$$

$$w_H(i) = w_H(i') = \bar{w}_H;$$

$$M(i) = M(i') = M(1).$$

In the second domain, both skilled and unskilled workers apply.

Plugging equations (3.46) and (3.47) into the profit function, we know that

$$\pi(i) = \left[e^{x_H(i)} - x_H(i) - 1\right] U_H + \left[e^{x_H(i)}\left(e^{x_L(i)} - 1\right) - x_L(i)\right] U_L,$$

and we can also obtain

$$\frac{A_L \alpha_L(i)}{A_H \alpha_H(i)} = \frac{e^{x_L(i)} U_L}{U_H - \left(1 - e^{x_L(i)}\right) U_L}. \tag{3.50}$$

By the free-entry condition, we have

$$\left[e^{x_H(i)} - x_H(i) - 1\right] U_H + \left[e^{x_H(i)}\left(e^{x_L(i)} - 1\right) - x_L(i)\right] U_L = c. \tag{3.51}$$

Then, we can solve $x_L(i)$ and $x_H(i)$ by equations (3.50) and (refA.10).

From equation (3.50), we get the two threshold conditions

$$\frac{A_L \alpha_L (I_1)}{A_H \alpha_H (I_1)} = \frac{e^{\bar{x}_L} U_L}{U_H - (1 - e^{\bar{x}_L}) U_L}$$

and

$$\frac{A_L \alpha_L (I_2)}{A_H \alpha_H (I_2)} = \frac{U_L}{U_H}.$$

From equation (3.50), we obtain

$$\frac{p(i)}{p(i')} = \frac{e^{x_L(i)+x_H(i)}}{e^{(x_L(i')+x_H(i'))}} \frac{\alpha_L(i')}{\alpha_L(i)}. \tag{3.52}$$

The goods market clearing demands

$$p(i)\, y(i) = p(i')\, y(i'). \tag{3.53}$$

By the production function, we get

$$y(i) = \left[\left(1 - e^{-x_H(i)}\right) A_H \alpha_H(i) + e^{-x_H(i)} \left(1 - e^{-x_L(i)}\right) A_L \alpha_L(i)\right] M(i). \tag{3.54}$$

It is straightforward to obtain the following equations from equations (3.53) and (3.54)

$$\frac{p(i)}{p(i')} = \frac{y(i')}{y(i)} = \frac{\left[\left(1 - e^{-x_H(i')}\right) A_H \alpha_H(i') + e^{-x_H(i')} \left(1 - e^{-x_L(i')}\right) A_L \alpha_L(i')\right] M(i')}{\left[\left(1 - e^{-x_H(i)}\right) A_H \alpha_H(i) + e^{-x_H(i)} \left(1 - e^{-x_L(i)}\right) A_L \alpha_L(i)\right] M(i)}. \tag{3.55}$$

From equations (3.52) and (3.55), we obtain

$$\frac{M(i)}{M(i')} = \frac{\left[e^{x_L(i')} \left(e^{x_H(i')} - 1\right) A_H \alpha_H(i') + \left(e^{x_L(i')} - 1\right) A_L \alpha_L(i')\right] \alpha_L(i)}{\left[e^{x_L(i)} \left(e^{x_H(i)} - 1\right) A_H \alpha_H(i) + \left(e^{x_L(i)} - 1\right) A_L \alpha_L(i)\right] \alpha_L(i')}. \tag{3.56}$$

In this domain, we can prove that $x_L(i)$ is decreasing by equation (3.50) and $x_H(i)$ is increasing by equation (3.51).

3.A.2 Numerical algorithm of the one shot model

Throughout the whole computation, I must do the numerical integration. I discrete the domain $[0, 1]$ and do the integration by Simpson's rule.

1. Guess $\bar{x}_L$ and $\bar{x}_H$, here $\bar{x}_H$ is smaller than $\bar{x}_L$.

2. Given $\bar{x}_L$ and $\bar{x}_H$, we can calculate U_L and U_H by conditions

$$\left(e^{\bar{x}_H} - \bar{x}_H - 1\right) U_H = c,$$

$$\left(e^{\bar{x}_L} - \bar{x}_L - 1\right) U_L = c,$$

and I_1 and I_2 by

$$\frac{A_L \alpha_L (I_1)}{A_H \alpha_H (I_1)} = \frac{e^{\bar{x}_L} U_L}{U_H - \left(1 - e^{\bar{x}_L}\right) U_L},$$

$$\frac{A_L \alpha_L (I_2)}{A_H \alpha_H (I_2)} = \frac{U_L}{U_H}.$$

We can also solve $x_L(i)$ and $x_H(i)$ in the second domain by the following two equations

$$\frac{A_L \alpha_L (i)}{A_H \alpha_H (i)} = \frac{e^{x_L(i)} U_L}{U_H - \left(1 - e^{x_L(i)}\right) U_L},$$

$$\left[e^{x_H(i)} - x_H(i) - 1\right] U_H + \left[e^{x_H(i)} \left(e^{x_L(i)} - 1\right) - x_L(i)\right] U_L = c.$$

Assume the measure of firms in the first domain to be 1. Then, we can calculate the measure of firms in task i by equation (3.56) as follows

$$\frac{M(i)}{M(I_1)} = \frac{\left[e^{x_L(I_1)} \left(e^{x_H(I_1)} - 1\right) A_H \alpha_H (I_1) + \left(e^{x_L(I_1)} - 1\right) A_L \alpha_L (I_1)\right]}{\left[e^{x_L(i)} \left(e^{x_H(i)} - 1\right) A_H \alpha_H (i) + \left(e^{x_L(i)} - 1\right) A_L \alpha_L (i)\right]} \frac{\alpha_L (i)}{\alpha_L (I_1)}$$

$$\frac{M(I_1)}{M(I_2)} = \frac{\left(e^{\bar{x}_H} - 1\right) A_H \alpha_H (I_2)}{\left(e^{\bar{x}_L} - 1\right) A_L \alpha_L (I_2)}.$$

3. By the above calculation, we can aggregate the measure of two types of workers $L = \int_0^1 x_L(i) M(i)\, di$ and $H = \int_0^1 x_H(i) M(i)\, di$. We can calculate $s_0 = \frac{H}{H+L}$. Here $M(i)$ are calculated by assuming $M(0) = 1$, but s_0 is independent of the value of $M(0)$.

4. We fix $\bar{x}_L$, and use the bisection method to update the guess of $\bar{x}_H$ until $s_0 = s$.

5. Now we have the guess of $\bar{x}_L$ and $\bar{x}_H$ which satisfy the relative endowment of skilled and unskilled workers. The next step is to solve $\bar{x}_L$. Assume that $M(0) = 1$. Then we can solve aggregate output and output in task i. We can thus solve $p(0)$ by the goods market clearing condition $p(0)\, y(0) = Y$. Then we can solve a new $\bar{x}_L$ by condition: $\left[1 - (\bar{x}_L + 1)\, e^{-\bar{x}_L}\right] p(0)\, A_L \alpha_L(0) = c$. If the new $\bar{x}_L$ is the same as the guessed $\bar{x}_L$, stop. Otherwise, update the guess of $\bar{x}_L$ and repeat the calculation from step 1 until it converges.

6. After the convergence, we need to pin down $M(0)$. We can calculate the measure of unskilled worker $\hat{L}$ if $M(0) = 1$. Obviously, $M(0) = \frac{L}{\hat{L}}$.

7. With $M(0)$, $\bar{x}_L$, and $\bar{x}_H$, we can solve all variables in the equilibrium based on the above equations.

3.A.3 Proof of proposition 3.3

Given the value function of the firm with a vacant and filled position, respectively.

$$V(i) = -c + \beta\left[\lambda f_H(i)\, J_H(i) + \lambda f_L(i)\, J_L(i) + (1 - \lambda f_L(i) - \lambda f_H(i))\, V(i)\right];$$

$$J_H(i) = p(i)\, A_H(i)\, \alpha_H(i) - w_H(i) + \beta\left[(1-\sigma)\, J_H(i) + \sigma V(i)\right];$$

$$J_L(i) = p(i)\, A_L(i)\, \alpha_L(i) - w_L(i) + \beta\left[(1-\sigma)\, J_L(i) + \sigma V(i)\right],$$

combined with the free-entry condition

$$V(i) = 0.$$

We obtain

$$J_H(i) = \frac{p(i)\, A_H(i)\, \alpha_H(i) - w_H(i)}{1 - \beta(1-\sigma)};$$

$$J_L(i) = \frac{p(i)\, A_L(i)\, \alpha_L(i) - w_L(i)}{1 - \beta(1-\sigma)}.$$

We can also define the $\lambda f_H(i)$ and $\lambda f_L(i)$ by the queue length of the applicants in task i

$$\lambda f_H(i) = 1 - e^{-x_H(i)},$$
$$\lambda f_L(i) = e^{-x_H(i)}\left(1 - e^{-x_L(i)}\right).$$

The value functions for individuals are the following

$$U_H = \beta\left[\lambda w_H(i) W_H(i) + (1 - \lambda w_H(i)) U_H\right],$$

$$U_L = \beta\left[\lambda w_L(i) W_L(i) + (1 - \lambda w_L(i)) U_L\right].$$

We can rewrite them

$$(1-\beta) U_H = \beta \frac{1 - e^{-x_H(i)}}{x_H(i)}\left[W_H(i) - U_H\right],$$

$$(1-\beta) U_L = \beta \frac{e^{-x_H(i)}\left(1 - e^{-x_L(i)}\right)}{x_L(i)}\left[W_L(i) - U_L\right],$$

and the employed workers' value function is

$$W_H(i) = w_H(i) + \beta\left[(1-\sigma) W_H(i) + \sigma U_H\right],$$

$$W_L(i) = w_L(i) + \beta\left[(1-\sigma) W_L(i) + \sigma U_L\right],$$

i.e.

$$W_H(i) = \frac{w_H(i) + \beta\sigma U_H}{1 - (1-\sigma)\beta},$$
$$W_L(i) = \frac{w_L(i) + \beta\sigma U_L}{1 - (1-\sigma)\beta}.$$

Let us characterize the stationary equilibrium step by step. First, I consider the task i where only the skilled worker applies. Consider a deviation by a firm to a wage $w_H(i)^d$. Observing the deviation, applicants modify their application strategies so that the queue length for the deviator is $x_H(i)^d$. The present value of this vacancy is $V_i\left(w_H(i)^d\right)$ and if it is filled, the present value is $W_H\left(w_H(i)^d\right)$. The best deviation that the firm can have is the solution to the following problem:

$$\max\ V_i\left(w_H(i)^d\right)$$

$$\text{s.t. } \frac{1-e^{-x_H(i)^d}}{x_H(i)^d}\left[W_H\left(w_H(i)^d\right)-U_H\right]\geq\frac{(1-\beta)U_H}{\beta}.$$

Then, in a stationary equilibrium with dynamic recruiting, wages and queue lengths are

$$w_H(i)=(1-\beta)U_H\left[1+\frac{(1-(1-\sigma)\beta)}{\beta}\frac{x_H(i)}{1-e^{-x_H(i)}}\right],$$

$$\lambda f_H(i)J_H(i)=\left(1-e^{-x_H(i)}\right)\frac{p(i)A_H\alpha_H(i)-w_H(i)}{1-\beta(1-\sigma)}.$$

By the implicit theorem $\frac{\partial V(i)}{\partial x_H(i)}=-\frac{\partial F/\partial x_H(i)}{\partial F/\partial V}$, $F(V,x)$ is defined as

$$-c-\left(1-\beta e^{x_H(i)}\right)V(i)+\beta\left[1-e^{-x_H(i)}\right]\frac{p(i)A_H\alpha_H(i)-w_H(i)}{1-\beta(1-\sigma)}.$$

Then, the first-order condition is

$$p(i)A_H\alpha_H(i)=(1-\beta)U_H\left[(1/\beta-(1-\sigma))e^{x_H(i)}+1\right]. \tag{3.57}$$

Therefore, we can get

$$\beta\lambda f_H(i)J_H(i)=\beta\left(1-e^{-x_H(i)}\right)\frac{p(i)A_H\alpha_H(i)-w_H(i)}{1-\beta(1-\sigma)}=(1-\beta)\left(e^{x_H(i)}-x_H(i)-1\right)U_H.$$

By the free-entry condition, we know that $\beta\lambda f_H(i)J_H(i)=c$ i.e.

$$(1-\beta)\left(e^{x_H(i)}-x_H(i)-1\right)U_H=c. \tag{3.58}$$

Similarly, we can solve the equilibrium in the first domain where only unskilled workers apply. The following conditions are obtained

$$p(i)A_L\alpha_L(i)=(1-\beta)U_L\left[(1/\beta-(1-\sigma))e^{x_L(i)}+1\right], \tag{3.59}$$

$$(1-\beta)\left(e^{x_L(i)}-x_L(i)-1\right)U_L=c. \tag{3.60}$$

Second, in the second domain, there are both skilled and unskilled applicants and the firms' problem can be written as

$$\max\ V_i\left(w_H(i)^d,w_L(i)^d\right)$$

$$\text{s.t. } \frac{1-e^{-x_H(i)^d}}{x_H(i)^d}\left[W_H\left(w_H(i)^d\right)-U_H\right] \geq \frac{(1-\beta)U_H}{\beta}$$

$$\text{s.t. } \frac{e^{-x_H(i)}\left(1-e^{-x_L(i)}\right)}{x_L(i)}\left[W_L\left(w_L(i)^d\right)-U_L\right] \geq \frac{(1-\beta)U_L}{\beta}.$$

Then

$$w_L(i) = (1-\beta)U_L\left[1+\frac{(1-(1-\sigma)\beta)}{\beta}\frac{x_L(i)}{e^{-x_H(i)}\left(1-e^{-x_L(i)}\right)}\right],$$

$$w_H(i) = (1-\beta)U_H\left[1+\frac{(1-(1-\sigma)\beta)}{\beta}\frac{x_H(i)}{1-e^{-x_H(i)}}\right].$$

We have first-order conditions

$$p(i)A_L\alpha_L(i) = (1-\beta)U_L\left[\frac{(1-(1-\sigma)\beta)e^{x_L(i)+x_H(i)}}{\beta}+1\right], \tag{3.61}$$

$$p(i)A_H\alpha_H(i) = (1-\beta)U_H\left[\frac{(1-(1-\sigma)\beta)}{\beta}e^{x_H(i)}+1\right]+\frac{(1-(1-\sigma)\beta)}{\beta}(1-\beta)U_Le^{x_H(i)}\left(e^{x_L(i)}\right. \tag{3.62}$$

from equations (3.61) and (3.62), we obtain

$$\frac{A_H\alpha_H(i)}{A_L\alpha_L(i)} = \frac{U_H\left[(1-(1-\sigma)\beta)e^{x_H(i)}+\beta\right]+(1-(1-\sigma)\beta)U_Le^{x_H(i)}\left(e^{x_L(i)}-1\right)}{U_L\left[(1-(1-\sigma)\beta)e^{x_L(i)+x_H(i)}+\beta\right]}. \tag{3.63}$$

From this condition, we know two thresholds I_1 and I_2 that can be pinned down by equations

$$\frac{A_H\alpha_H(I_1)}{A_L\alpha_L(I_1)} = \frac{(1+\sigma\beta)U_H+(1-(1-\sigma)\beta)U_L\left(e^{\bar{x}_L}-1\right)}{U_L\left[(1-(1-\sigma)\beta)e^{\bar{x}_L}+\beta\right]}, \tag{3.64}$$

$$\frac{A_H\alpha_H(I_2)}{A_L\alpha_L(I_2)} = \frac{U_H}{U_L}. \tag{3.65}$$

By the free-entry condition, we know that $\beta\left[\lambda f_H(i)J_H(i)+\lambda f_L(i)J_L(i)\right] = c$ i.e.

$$(1-\beta)\left(e^{x_H(i)}-x_H(i)-1\right)U_H+(1-\beta)\left[e^{x_H(i)}\left(e^{x_L(i)}-1\right)-x_L(i)\right]U_L = c. \tag{3.66}$$

Now, I will turn to solving the measure of entried firms in each domain. By equation (3.58), we can derive that $x_H(i) = x_H(i') = \bar{x}_H$ and $w_H(i) = w_H(i') = \bar{w}_H$. Combined with equation (3.57), we know that

$p(i) A_H \alpha_H(i) = p(i') A_H \alpha_H(i')$. With goods market clearing condition $p(i) y(i) = p(i') y(i')$, we know that

$$\frac{y(i)}{y(i')} = \frac{\alpha_H(i)}{\alpha_H(i')}$$

since

$$y(i) = A_H \alpha_H(i) h(i)$$

where $h(i)$ is the measure of the skilled worker. Then, by the stationary condition, we know that

$$\sigma h(i) = \left(1 - e^{-x_H(i)}\right) M(i).$$

Then, we know that $M(i) = M(i')$. Therefore, the measure of the employed worker is $h(i) = (1 - e^{-\bar{x}_H}) M(i)/\sigma$ and the measure of applicants in task i is $\bar{x}_H M(i)$ in every period.

A similar derivation can be done for the first domain. I can show that $M(i) = M(i')$ in the first domain. The measure of the employed worker is $l(i) = (1 - e^{-\bar{x}_L}) M(i)/\sigma$ and the measure of applicants in task i is $\bar{x}_L(i) M(i)$ in every period.

In the second domain, it is more complicated. By equation (3.61), we obtain

$$\frac{p(i)}{p(i')} = \frac{(1-(1-\sigma)\beta) e^{x_L(i)+x_H(i)} + \beta}{(1-(1-\sigma)\beta) e^{x_L(i')+x_H(i')} + \beta} \frac{\alpha_L(i')}{\alpha_L(i)}. \tag{3.67}$$

Goods market clearing demands $p(i) y(i) = p(i') y(i')$, and the output in task i is

$$y(i) = A_H \alpha_H(i) h(i) + A_L \alpha_L(i) l(i)$$

where

$$h(i) = \left(1 - e^{-x_H(i)}\right) M(i)/\sigma,$$
$$l(i) = e^{-x_H(i)} \left(1 - e^{-x_L(i)}\right) M(i)/\sigma.$$

We can derive that

$$\frac{p(i)}{p(i')} = \frac{y(i')}{y(i)} = \frac{\left[A_H \alpha_H(i') \left(1 - e^{-x_H(i')}\right) + A_L \alpha_L(i') e^{-x_H(i')} \left(1 - e^{-x_L(i')}\right)\right] M(i')}{\left[A_H \alpha_H(i) \left(1 - e^{-x_H(i)}\right) + A_L \alpha_L(i) e^{-x_H(i)} \left(1 - e^{-x_L(i)}\right)\right] M(i)}. \tag{3.68}$$

Then, combining equations (3.61) and (3.62), we obtain

$$\frac{M(i)}{M(i')} = \frac{\left[A_H \alpha_H(i')\left(1-e^{-x_H(i')}\right) + A_L \alpha_L(i')\, e^{-x_H(i')}\left(1-e^{-x_L(i')}\right)\right]\left[(1-(1-\sigma)\beta)\, e^{x_L(i')+x_H}\right.}{\left[A_H \alpha_H(i)\left(1-e^{-x_H(i)}\right) + A_L \alpha_L(i)\, e^{-x_H(i)}\left(1-e^{-x_L(i)}\right)\right]\left[(1-(1-\sigma)\beta)\, e^{x_L(i)+x_H(i)}\right.}$$

Then we can see that

$$\frac{M(I_1)}{M(I_2)} = \frac{M(0)}{M(1)} = \frac{A_H \alpha_H(I_2)\left(1-e^{-\bar{x}_H}\right)}{A_L \alpha_L(I_2)\left(1-e^{-\bar{x}_L}\right)} \frac{(1-(1-\sigma)\beta)\, e^{\bar{x}_H}+\beta}{(1-(1-\sigma)\beta)\, e^{\bar{x}_L}+\beta}.$$

3.A.4 Numerical algorithm of dynamic model

The numerical algorithm is very similar to that applied in the case of a one shot model. Throughout the whole computation, I have to do the numerical integration. I discrete the domain $[0,1]$ and do the integration by Simpson's rule.

1. Guess $\bar{x}_L$ and $\bar{x}_H$, here $\bar{x}_H$ is smaller than $\bar{x}_L$.

2. Given $\bar{x}_L$ and $\bar{x}_H$ we can calculate U_L and U_H by equations (3.58) and (3.60) and I_1 and I_2 by equations (3.64) and (3.65). We can also solve $x_L(i)$ and $x_H(i)$ in the second domain by equations (3.63) and (3.66). Assume the measure of firms in the first domain to be 1. Then, we can calculate the measure of firms in task i as follows

$$\frac{M(i)}{M(I_1)} = \frac{A_L \alpha_L(i)\left(1-e^{-\bar{x}_L}\right)\left[(1-(1-\sigma)\beta)\, e^{\bar{x}_L}+\beta\right]}{\left[A_H \alpha_H(i)\left(1-e^{-x_H(i)}\right) + A_L \alpha_L(i)\, e^{-x_H(i)}\left(1-e^{-x_L(i)}\right)\right]\left[(1-(1-\sigma)\beta)\, e^{x_L(i)+x}\right.}$$

$$\frac{M(I_1)}{M(I_2)} = \frac{M(0)}{M(1)} = \frac{A_H \alpha_H(I_2)\left(1-e^{-\bar{x}_H}\right)}{A_L \alpha_L(I_2)\left(1-e^{-\bar{x}_L}\right)} \frac{(1-(1-\sigma)\beta)\, e^{\bar{x}_H}+\beta}{(1-(1-\sigma)\beta)\, e^{\bar{x}_L}+\beta}.$$

3. By the above calculation, we can aggregate the measure of two types of workers $L = \int_0^1 \left[x_L(i) + \left(1-e^{-x_L(i)}\right)/\sigma\right] M(i)\, di$ and $H = \int_0^1 \left[x_H(i) + \left(1-e^{-x_H(i)}\right)/\sigma\right] M(i)\, di$. We can calculate $s_0 = \frac{H}{H+L}$.. Here $M(i)$ are calculated by assuming $M(0)=1$, but s_0 is independent of the value of $M(0)$.

4. We fix $\bar{x}_L$, and use the bisection method to update the guess of $\bar{x}_H$ until $s_0 = s$.

5. Now we have the guess of $\bar{x}_L$ and $\bar{x}_H$ which satisfy the relative endowment of skilled and unskilled workers. The next step is to solve $\bar{x}_L$. Assume that $M(0) = 1$. Then we can solve aggregate output and output in task i. We can thus solve $p(0)$ by goods market clearing condition $p(0)\,y(0) = Y$. Then we can solve a new $\bar{x}_L$ by condition: $p(0)\,A_L\alpha_L(0)\,(e^{\bar{x}_L} - \bar{x}_L - 1) - [(1/\beta - 1 + \sigma)\,e^{\bar{x}_L} + 1]\,c = 0$. If the new $\bar{x}_L$ is the same as guessed $\bar{x}_L$, stop. Otherwise, update the guess of $\bar{x}_L$ and repeat the calculation from step 1 until it converges.

6. After convergence, we need to pin down $M(0)$. We can calculate the measure of unskilled worker $\hat{L}$ if $M(0) = 1$. Obviously, $M(0) = \frac{L}{\hat{L}}$.

7. With $M(0)$, $\bar{x}_L$, and $\bar{x}_H$, we can solve all variables in the equilibrium based on the above equations.

3.A.5 Proof of proposition 3.4

When we introduce the unemployment benefit and labor income taxation, we need to revise the value function of workers, but the value function of the firms will not change.

The value function for workers is the following

$$U_H = b + \beta\left[\lambda w_H(i)\,W_H(i) + (1 - \lambda w_H(i))\,U_H\right],$$

$$U_L = b + \beta\left[\lambda w_L(i)\,W_L(i) + (1 - \lambda w_L(i))\,U_L\right],$$

and

$$W_H(i) = (1 - \tau)\,w_H(i) + \beta\left[(1 - \sigma)\,W_H(i) + \sigma U_H\right],$$

$$W_L(i) = (1 - \tau)\,w_L(i) + \beta\left[(1 - \sigma)\,W_L(i) + \sigma U_L\right].$$

Let us characterize the stationary equilibrium step by step. First I consider the task i where only the skilled worker applies. Consider a deviation by a firm to a wage $w_H(i)^d$. Observing the deviation, applicants modify their application strategies so that the queue length for the deviator is $x_H(i)^d$. The present value of this vacancy is $V_i\left(w_H(i)^d\right)$ and if it is filled, the present value is $W_H\left(w_H(i)^d\right)$. The best deviation that the firm can have is the solution to the following problem:

$$\max V_i\left(w_H(i)^d\right) \text{ s.t. } b+\frac{1-e^{-x_H(i)^d}}{x_H(i)^d}\beta\left[W_H\left(w_H(i)^d\right)-U_H\right]\geq(1-\beta)U_H.$$

Then, in a stationary equilibrium with dynamic recruiting, wages and queue lengths satisfy

$$w_H(i)=\frac{1}{1-\tau}\left\{(1-\beta)U_H\left[1+\frac{(1-(1-\sigma)\beta)}{\beta}\frac{x_H(i)}{1-e^{-x_H(i)}}\right]-\frac{(1-(1-\sigma)\beta)x_H(i)b}{\beta\left(1-e^{-x_H(i)}\right)}\right\},$$

$$\lambda f_H(i)J_H(i)=\left(1-e^{-x_H(i)}\right)\frac{p(i)A_H\alpha_H(i)-w_H(i)}{1-\beta(1-\sigma)}.$$

By the implicit theorem $\frac{\partial V(i)}{\partial x_H(i)}=-\frac{\partial F/\partial x_H(i)}{\partial F/\partial V}$ where $F(V,x)$ is defined as

$$-c-\left(1-\beta e^{x_H(i)}\right)V(i)+\beta\left[1-e^{-x_H(i)}\right]\frac{p(i)A_H\alpha_H(i)-w_H(i)}{1-\beta(1-\sigma)}.$$

Then

$$p(i)A_H\alpha_H(i)=\frac{1}{(1-\tau)}\left\{(1-\beta)U_H\left[(1/\beta-1+\sigma)e^{x_H(i)}+1\right]-(1/\beta-1+\sigma)be^{x_H(i)}\right\}, \tag{3.69}$$

$$\beta\lambda f_H(i)J_H(i)=\beta\left(1-e^{-x_H(i)}\right)\frac{p(i)A_H\alpha_H(i)-w_H(i)}{1-\beta(1-\sigma)}=\frac{(1-\beta)U_H-b}{(1-\tau)}\left(e^{x_H(i)}-x_H(i)-1\right.$$

By the free-entry condition, we know that $\beta\lambda f_H(i)J_H(i)=c$ i.e.

$$\left(e^{x_H(i)}-x_H(i)-1\right)((1-\beta)U_H-b)=(1-\tau)c. \tag{3.70}$$

Similarly, we can solve the first domain where only unskilled workers apply. The following conditions are obtained

$$p(i)A_L\alpha_L(i)=\frac{1}{(1-\tau)}\left\{(1-\beta)U_L\left[(1/\beta-1+\sigma)e^{x_L(i)}+1\right]-(1/\beta-1+\sigma)be^{x_L(i)}\right\}, \tag{3.71}$$

$$\left(e^{x_L(i)}-x_L(i)-1\right)((1-\beta)U_L-b)=(1-\tau)c. \tag{3.72}$$

Second, if there are skilled and unskilled applicants

$$\max\ V_i\left(w_H(i)^d, w_L(i)^d\right)$$

$$\text{s.t. } b + \frac{1-e^{-x_H(i)^d}}{x_H(i)^d}\beta\left[W_H\left(w_H(i)^d\right) - U_H\right] \geq (1-\beta)U_H$$

$$\text{s.t. } b + \frac{e^{-x_H(i)}\left(1-e^{-x_L(i)}\right)}{x_L(i)}\beta\left[W_L\left(w_L(i)^d\right) - \beta U_L\right] \geq (1-\beta)U_L.$$

We have two first-order conditions

$$p(i)A_L\alpha_L(i) = \frac{(1-\beta)U_L}{1-\tau} + \frac{1-(1-\sigma)\beta}{\beta(1-\tau)}\left[(1-\beta)U_L - b\right]e^{x_L(i)+x_H(i)}, \tag{3.73}$$

$$p(i)A_H\alpha_H(i) = \frac{(1-\beta)U_H}{1-\tau} + \frac{1-(1-\sigma)\beta}{\beta(1-\tau)}\left\{\left[(1-\beta)U_H - b\right]e^{x_H(i)} + \left[(1-\beta)U_L - b\right]e^{x_H(i)}\left(e^{x_L(i)}-1\right)\right. \tag{3.74}$$

Therefore,

$$\frac{A_H\alpha_H(i)}{A_L\alpha_L(i)} = \frac{\beta(1-\beta)U_H + \left[1-(1-\sigma)\beta\right]e^{x_H(i)}\left\{\left[(1-\beta)U_H - b\right] + \left[(1-\beta)U_L - b\right]\left(e^{x_L(i)}-1\right)\right\}}{\beta(1-\beta)U_L + \left[1-(1-\sigma)\beta\right]\left[(1-\beta)U_L - b\right]e^{x_L(i)+x_H(i)}}. \tag{3.75}$$

From equation (3.75), we know two thresholds I_1 and I_2 that can be pinned down by

$$\frac{A_H\alpha_H(I_1)}{A_L\alpha_L(I_1)} = \frac{\beta(1-\beta)U_H + \left[1-(1-\sigma)\beta\right]\left\{\left[(1-\beta)U_H - b\right] + \left[(1-\beta)U_L - b\right]\left(e^{\bar{x}_L}-1\right)\right\}}{\beta(1-\beta)U_L + \left[1-(1-\sigma)\beta\right]\left[(1-\beta)U_L - b\right]e^{\bar{x}_L}},$$

$$\frac{A_H\alpha_H(I_2)}{A_L\alpha_L(I_2)} = \frac{\beta(1-\beta)U_H + \left[1-(1-\sigma)\beta\right]\left[(1-\beta)U_H - b\right]e^{\bar{x}_H}}{\beta(1-\beta)U_L + \left[1-(1-\sigma)\beta\right]\left[(1-\beta)U_L - b\right]e^{\bar{x}_H}}.$$

By the free-entry condition, we know that $\beta\left[\lambda f_H(i)J_H(i) + \lambda f_L(i)J_L(i)\right] = c$ i.e.

$$\left((1-\beta)U_H - b\right)\left(e^{x_H(i)} - x_H(i) - 1\right) + \left((1-\beta)U_L - b\right)\left(e^{x_H(i)}\left(e^{x_L(i)}-1\right) - x_L(i)\right) = (1-\tau)c. \tag{3.76}$$

Now I will characterize the measure of entried firms in each domain. By equation (3.70), we get $x_H(i) = x_H(i') = \bar{x}_H$ and $w_H(i) = w_H(i')$. Combined with equation (3.69), we can see that $p(i)A_L\alpha_L(i) = p(i')A_L\alpha_L(i')$. With goods market clearing condition $p(i)y(i) = p(i')y(i')$, we know that

$$\frac{y(i)}{y(i')} = \frac{\alpha_H(i)}{\alpha_H(i')},$$

with

$$y(i) = A_H \alpha_H(i) h(i),$$

where $h(i)$ is the measure of the skilled worker. Then, by the stationary condition we know that

$$\sigma h(i) = \left(1 - e^{-x_H(i)}\right) M(i).$$

Therefore, we can prove that $M(i) = M(i')$.. Then, the measure of employed workers is $h(i) = (1 - e^{-\bar{x}_H}) M(1)/\sigma$ and the measure of applicants in task i is $\bar{x}_H M(1)$. And the measure of workers allocated to task i is $[(1 - e^{-\bar{x}_H})/\sigma + \bar{x}_H] M(1)$.

A similar derivation can be done for the first domain. The measure of the employed worker is $l(i) = (1 - e^{-\bar{x}_L}) M(0)/\sigma$ and the measure of the applicants in task i is $\bar{x}_L M(0)$. So the measure of the worker allocated to task i is $[(1 - e^{-\bar{x}_L})/\sigma + \bar{x}_L] M(0)$.

In the second domain, it is more complicated. By equation (3.73), we obtain

$$\frac{p(i)}{p(i')} = \frac{\beta(1-\beta)U_L + (1-(1-\sigma)\beta)[(1-\beta)U_L - b] e^{x_L(i)+x_H(i)}}{\beta(1-\beta)U_L + (1-(1-\sigma)\beta)[(1-\beta)U_L - b] e^{x_L(i')+x_H(i')}} \frac{\alpha_L(i')}{\alpha_L(i)}. \tag{3.77}$$

Goods market clearing condition demand

$$p(i) y(i) = p(i') y(i'),$$

and by the production function, we know that

$$y(i) = A_H \alpha_H(i) h(i) + A_L \alpha_L(i) l(i),$$

$$h(i) = \left(1 - e^{-x_H(i)}\right) M(i)/\sigma,$$

$$l(i) = e^{-x_H(i)} \left(1 - e^{-x_L(i)}\right) M(i)/\sigma.$$

So

$$\frac{p(i)}{p(i')} = \frac{y(i')}{y(i)} = \frac{\left[A_H \alpha_H(i') \left(1 - e^{-x_H(i')}\right) + A_L \alpha_L(i') e^{-x_H(i')} \left(1 - e^{-x_L(i')}\right)\right] M(i')}{\left[A_H \alpha_H(i) (1 - e^{-x_H(i)}) + A_L \alpha_L(i) e^{-x_H(i)} (1 - e^{-x_L(i)})\right] M(i)}. \tag{3.78}$$

Then, from equations (3.77) and (3.78), we obtain

$$\frac{M(i)}{M(i')} = \frac{\left[A_H \alpha_H(i')\left(1 - e^{-x_H(i')}\right) + A_L \alpha_L(i') e^{-x_H(i')}\left(1 - e^{-x_L(i')}\right)\right]\left\{\beta(1-\beta)U_L + (1-(1-\sigma)\beta)\left[(1\right.\right.}{\left[A_H \alpha_H(i)\left(1 - e^{-x_H(i)}\right) + A_L \alpha_L(i) e^{-x_H(i)}\left(1 - e^{-x_L(i)}\right)\right]\left\{\beta(1-\beta)U_L + (1-(1-\sigma)\beta)\left[(1-\right.\right.}$$

3.B Figures

Figure 3.1: Queue length of skills in each task in the one shot model

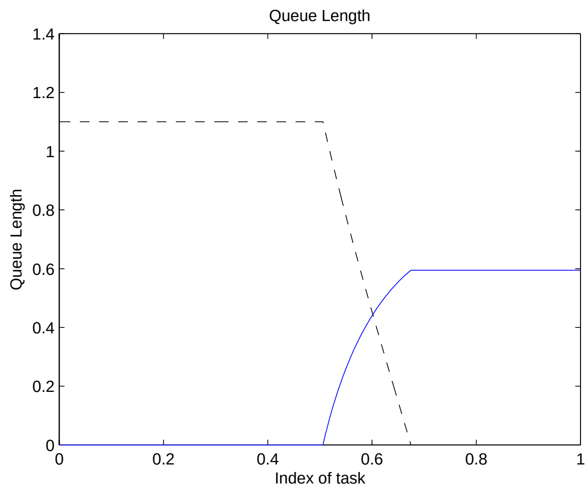


Figure 3.2: Wage rate of skills in each task in the one shot model

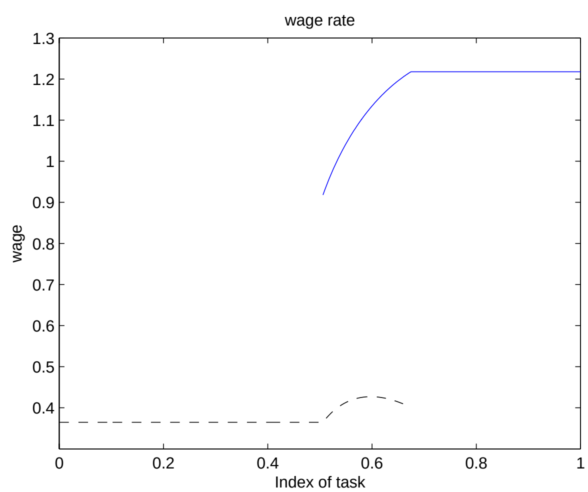


Figure 3.3: Measure of entried firms in each task in the one shot model

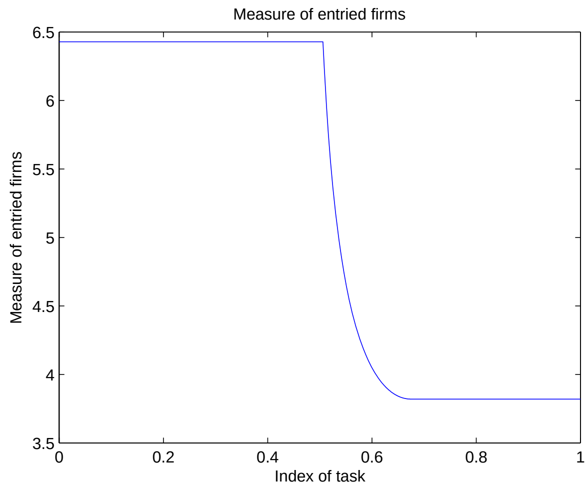


Figure 3.4: Measure of employed workers in each task in the one shot model

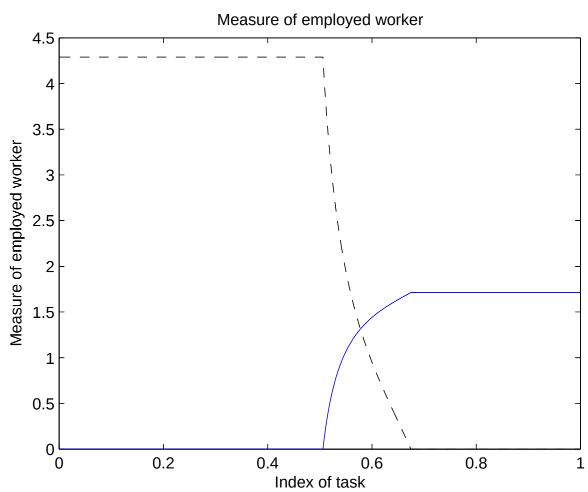


Figure 3.5: Queue length of skills in each task in the dynamic model

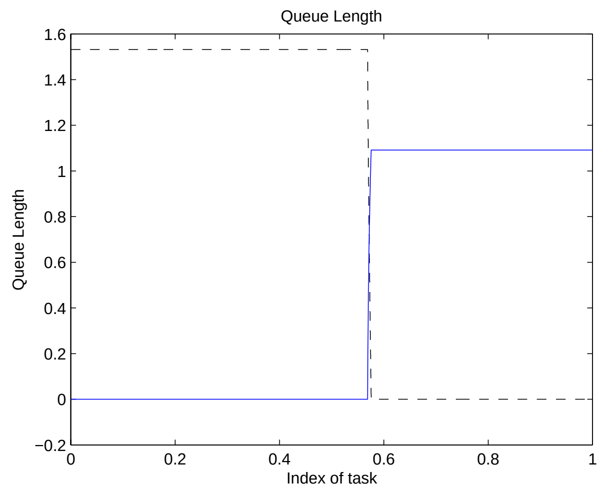


Figure 3.6: Wage rate of skills in each task in the dynamic model model

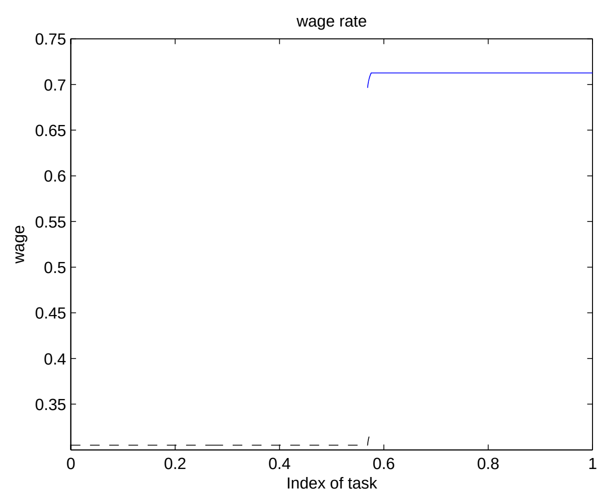


Figure 3.7: Measure of entried firms in each task in the dynamic model

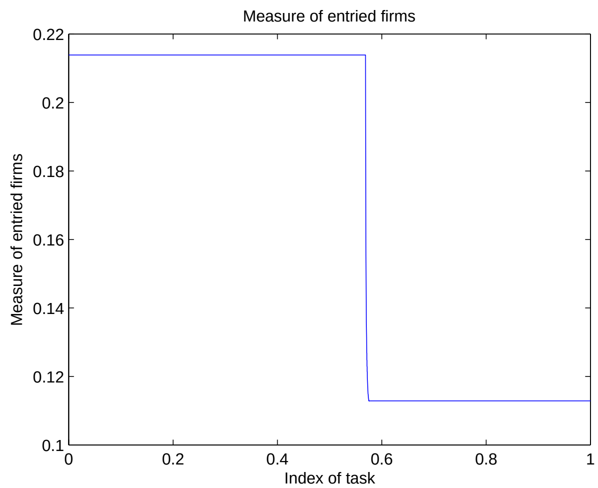


Figure 3.8: Measure of employed workers in each task in the dynamic model

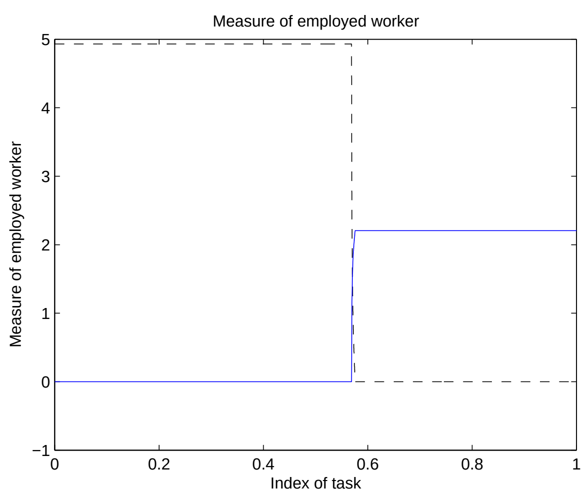


Figure 3.9: Queue length of skills in each task in the dynamic model model with unemployment benefit and income tax

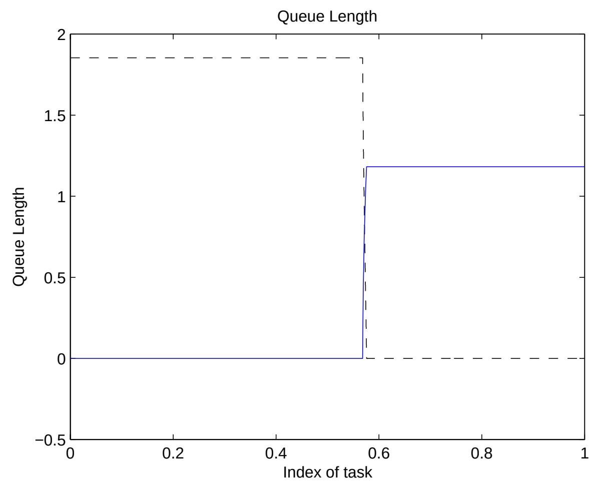


Figure 3.10: Wage rate of skills in each task in the dynamic model model with unemployment benefit and income tax

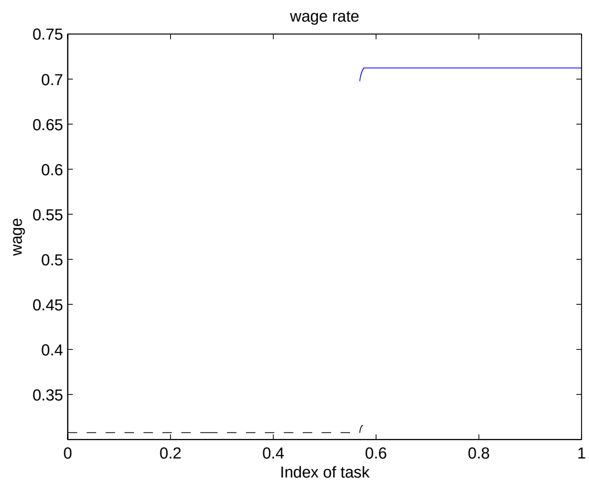


Figure 3.11: Measure of entried firms in each task in the dynamic model model with unemployment benefit and income tax

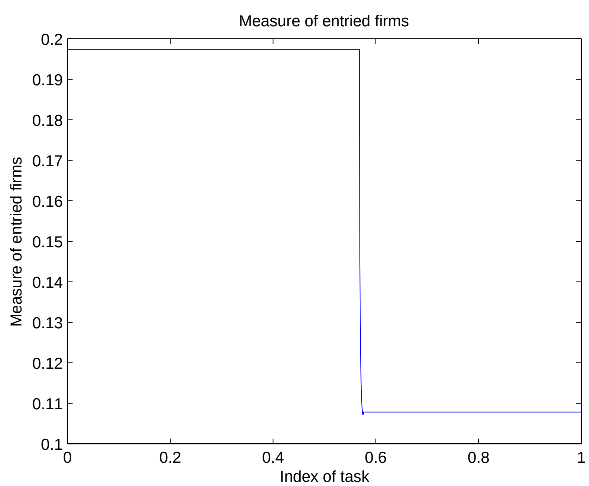
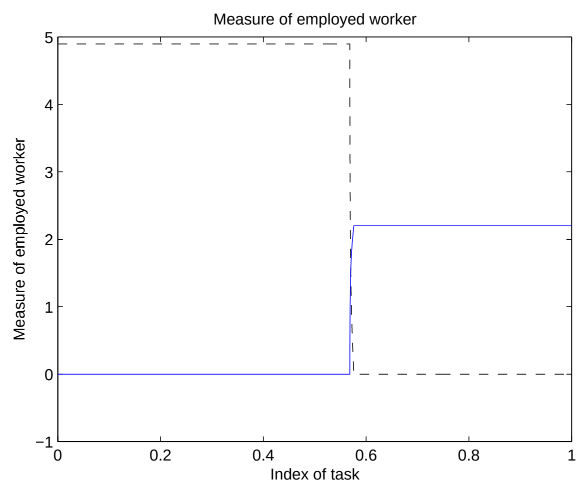


Figure 3.12: Measure of employed workers in each task in the dynamic model model with unemployment benefit and income tax



Chapter 4

A Dynamic Analysis of the Free-Rider Problem

4.1 Introduction

Lots of political, social, or economic activities are carried out by groups. However, because of the free-rider problem, individuals bear only partially the adverse consequences of reducing their efforts. Consequently, collective effort usually falls below the optimal level. The free-rider problem is pervasive and exists in a wide variety of situation in which coordination is necessary. In the political economy literature, trade unions, lobbies, and local public goods supply are standard examples.

As Olson (1965) celebrated thesis noted long ago, lobbying activities of special interests group entail a free-rider problem: all members of a group benefit, irrespective of whether they contribute to the lobbying. But in reality, we observe that some groups successfully overcome the free-rider problem, others do not. The workhorse model of lobbying which is introduced by the seminal work Grossman and Helpman (1994, 1995) still are abstracted away from modeling the organization of special interest. Therefore, the lobbying model leaves some crucial issues aside. For instance, Persson and Tabellini (2000) write:

> The major problem is that we lack a precise model of the process whereby some groups get politically organized and

> others not. This is a difficult question to which there is still no satisfactory answer. The asymmetries driving the misallocation of public goods must thus be assumed or defended on empirical grounds rather than explained. (Persson and Tabellini (2000), 175).

In this paper, I try to build a model to show that how the distorted government policy helps special interest group overcome the free rider problem to be organized. In the equilibrium, under some conditions the special interest group can sustain the equilibrium looks like the one whereby there is no free rider problem. In some sense, our analysis extends Becker's (1985) and Persson and Tabellini (2000)'s insight. Becker writes:

> A satisfactory analysis of the choice of method must consider whether the influence function itself depends on the methods used. (Becker's (1985), 338).

Persson and Tabellini (2000) write:

> An interesting conjecture is that groups are organized and solve their free-rider problem also thanks to government policy. Just as public policy confers monopoly privileges on some economic actors, it can also preserve and promote organized groups, from whom politicians then draw political or economic benefits. (Persson and Tabellini (2000), 175).

In this paper, I am going to argue that groups organize and overcome their free-rider problem thanks to distorted government policy. The government policy itself helps groups solve this free rider problem through regulation or other policy actions. As public policy confers monopoly privileges on a special interest group, it can also preserve and promote organized groups, from whom politicians then draw political or economic benefits. The key in my story is a dynamic incentive: when distorted policy generates rents for an interest group, each member of the group wish to make contributions not just to raise their rents today; they want to sustain their cooperation so that they will be able to influence policy in the future. Interest group contribute money to politician to influent policy not only considering one period rent, but also taking into account of the effect of today's contribution on the future rents. If this dynamic consideration is strong enough, it will help the special interest group organized. In some cases, the equilibrium will be identical to the one with no free rider problem; it appears that the special interest group is organized.

We present a model that builds on two important assumptions. First, the political system can not commit today to future policies, since they will be determined by whoever has political power in the future. Second, we assume that the special interest group is regulated by a distorted government policy such as entry limitation. These two assumption are proved to be crucial to the dynamic incentive to sustain corporation.

Our theory also predicts inefficient policy persistence. Different from the previous explanation, our explanation emphasize that today's inefficient policy lead the special interest group overcome the free-rider problem such that they could lobby sufficiently in the future to sustain the inefficient policy.

Our article is organized as follows. The following section discusses the related literature. Then in section 3, we outline a simple static model in which the political equilibrium with organized and unorganized special inserest group are characterized. The next section I build a dynamic model and reveal that the inefficient government policy increase the incenctive of cooperation for the group member. We then discuss the key role of government policies in organization of the special interest group. Finally, in the last section, We conclude the paper. Proofs of the main results are provided in the Appendix.

4.2 Related literature

There is a long tradition of research on static free-rider problems started by Samuelson (1954) and Olson (1965), and further development such as Palfrey and Rosenthal (1984) and Bergstrom, Blume and Varian (1985). Levhari and Mirman (1980) and Fershtman and Nitzan (1991) characterize a Markov equilibrium in a n-persom voluntary public good contribution game (public service is provide by public capital, and public capital is accumulated and a state variable). Recently, Battaglini, Nunnari and Palfrey (2012) solve the Markov equilibrium with more general utility functions and irreversible investments.

Our paper is also related to the literatures in explaining the use of relatively inefficient policies to redistribute income towards special interest groups. Tullock (1983) propose the "disguised" transfer argument. When a policy is not ostensibly aimed at redistribution, those who bear the costs may be ignorant of the redistribution and thus less likely to oppose it if the policy also has some social benefit. Coate and Morris (1995) elegantly formalize this idea and clarify when exactly it works. Another interpretation is proposed by Acemoglu and Bobinson (2001). They argue that compared to efficient methods inefficient redistribution makes it more attractive to stay in or enter a group that receives subsidies. When political institutions cannot credibly commit to future policy, and when the political influence of a group depends on its size, inefficient redistribution is a tool to sustain political power.

4.3 The Basic Static Model

In this section, I lay down a simple stylized static model in which I show the different equilibrium with and without free-rider problem. This simple model provides a start point to the dynamic model.

4.3.1 Economics Setting

We consider an economy with a special interests group with n members. And the public policy before confers monopoly privileges on this special interest group. Suppose that each firm (member) belong to the special interests group can produce output $y_i = \bar{y}$ every period with the production cost $c_i = \bar{c}$. Assume that the demand function of the good is $P = a - bQ$, and aggregate supply of the good $Q = \sum_i y_i$. we can easily derive the profit of each firm as a function of number of firms:

$$\Pi_i = Py_i - c_i = (a\bar{y} - \bar{c}) - b\bar{y}^2 n \tag{4.1}$$

From equation (4.1), we can see that each firm's profit is a decreasing function of the firms' number. At the same time, we can also conclude that there is a maximum of the firm's number. This maximum number can be derived from zero profit condition $\Pi_i\left(\bar{N}\right) = 0$, i.e. $\bar{N} = (a\bar{y} - \bar{c})/b\bar{y}^2$. To simplify the notation, we can rewrite the profit function as

$$\Pi_i = A - Bn$$

where $A = (a\bar{y} - \bar{c})$, and $B = b\bar{y}^2$. Therefore, the maximum number of firms is $\bar{N} = A/B$.

The policy make can set regulation policy to restrict the entry of new firm into the sector. Here the restrictive entry policy can be considered as license restriction or different entry barriers to the new firms. Here I assume that the regulation policy is represented by variable E which describes how many firms enter the market. Policy decisions are assumed to be centralized in the hands of a semi benevolent government, but the government can be influenced by organized interest groups. The Grossman and Helpman (1994) assume that government choosing the policy by maximizes a weighted sum of social welfare and contributions from the special interest group. In this model, the social welfare demands free entry, but the incumbent firms make contributions to limit the entry of new firm to collect monopoly rents. To focus on the key mechanism of this paper, I will model the political decision of the government as a reduced form function of sector's aggregate contribution. My key result will not be changed by abstracting from explicitly model the government policy decision making.

I assume that the regulation policy set by the politician is described as the reduced form function of size of contribution, i.e. $E = g(T)$, where T is the size of the group's contribution. Policy function $g(T)$ is a decreasing function of aggregate contribution T. It means that more contribution the politician get, more restrictive entry regulation he choose. Given the maximum number $\bar{N}$ and the number of incumbent firms n, there will be $\bar{N} - n$ new entried firms if there is no any entry barriers. When there are more potential entried firms, to prevent new entries the incumbent firms need to contribute more to induce a more restrictive entry regulation policy. In particular, I assume the policy function $g(T)$ as

$$g(C) = \left(\bar{N} - n\right) - dT.$$

The maximum contribution is $\left(\bar{N} - n\right)/d$. When incumbent firms contribute $\left(\bar{N} - n\right)/d$, there is going to be no new firm entry.

Now let us document the timing within one period. At the beginning of a period, firms' number n is predetermined, and then the group members decide the contribution and commit to provide contributions (or "bribes") contingent on future policies before the politician has made any policy choice. After receiving the contribution, the politician chooses the regulation policy as he has promised depending on the amount of contribution. After the regulation policy is decided, new firms entry. In the end of this period, production start and the special interest group's payoffs are realized. Then agent's monopoly rents can be obtained as

$$\Pi_i = A - Bn - B\left[\bar{N} - n - dT\right],$$

where the monopoly rents Π_i is an increasing function of the aggregate contribution to politician. I also assume that the agents' utility function is a linear function monetary income.

In this paper, I always assume the symmetric equilibrium, i.e. every member of the special interest group will play the same strategy.

4.3.2 The Static Problem with and without Free-rider Problem

As a benchmark with which to compare the equilibrium allocations, we first analyze equilibrium in which the special interest group is organized. The contribution policy is chosen by a "benevolent" planner who maximizes the sum of utilities of the agents. This is the welfare optimum of the special interest group. The "benevolent" planner's problem of n incumbent firms can be represented as:

$$\max_{\tau_i} - \sum_{i=1}^{n} \tau_i + n \left\{ A - Bn - B \left[\frac{A}{B} - n - d \sum_{i=1}^{n} \tau_i \right] \right\}, \tag{4.2}$$

$$s.t\ \frac{A}{B} - n - d \sum_{i=1}^{n} \tau_i \geq 0,$$

$$\tau_i \geq 0,$$

where $\sum_{i=1}^{n} \tau_i$ is the aggregate contribution and τ_i is each agent's contribution. The second term in equation (4.2) define the aggregate rents enjoyed by the group which is determined by the number of incumbent firms and new entried firms. Explicitly solving the problem gives us the Lemma 1.

Lemma 1 *If $Bdn \geq 1$, the equilibrium allocation will be that every group member contributes $(A/B - n)/nd$ every period and there is going to be no new firm entry. If $Bdn < 1$, the equilibrium allocation will be that every group member contribute 0 and $\bar{N} - n$ new firms will entry.*

The derivation of Lemma 1 is straight forward, the first order condition of τ_i is

$$-1 + Bdn. \tag{4.3}$$

If $-1+Bdn \geq 0$, each member will contribute the maximum $(A/B - n)/nd$ to the politician. If $-1+Bdn < 0$, each member will contribute the minimum, i.e. 0 to the politician.

To compare the difference of the equilibrium with and without free-rider problem, let us switch to the problem with free-rider problem. Differently form the "benevolent" planner case, in equilibrium no agent can directly choose the level of aggregate contribution T, instead an agent i chooses only his own level of contribution to the politician. Therefore, agent i's problem can be written as:

$$\max_{\tau_i} -\tau_i + \left[A - Bn - B\left[\frac{A}{B} - n - d\left(n-1\right)\tau_{-i} - d\tau_i\right]\right]$$

$$\frac{A}{B} - n - d\left(n-1\right)\tau_{-i} - d\tau_i \geq 0$$

$$0 \leq \tau_i$$

where τ_{-i} is the other agents' contribution given the symmetric equilibrium requirement. Explicitly solving the problem gives us the Lemma 2.

Lemma 2 *If $Bd \geq 1$, the equilibrium allocation will be that every group member contribute $\left(A/B - n\right)/nd$ every period and there is going to be no new firm entry. If $Bd < 1$, the equilibrium allocation will be that every group member contribute 0 and $\bar{N} - n$ new firms will entry.*

The derivation of Lemma 1 is straight forward, the first order condition of τ_i is

$$-1 + Bd. \tag{4.4}$$

If $-1+Bd \geq 0$, each member will contribute the maximum $\left(A/B - n\right)/nd$ to the politician. If $-1 + Bd < 0$, each member will contribute the minimum, i.e. 0 to the politician.

From the above analysis, we can see the clear difference between to scenario. The existence of free-rider problem decreases the incentive of contributing for every member of the group. The results in lemma 1 and 2 show that the condition for efficient lobbying for the special interesting group is quite demanding: $Bd \geq 1$, but the condition for efficient lobbying for an organized group is much easier to match: $Bdn \geq 1$.

The analysis presented in this section is quite standard. The aim of analysis is to provide a comparison to the analysis of dynamic model. With the equilibrium result in memory, it is easy to see the difference between the static equilibrium and Markov perfect equilibrium. In the next section, I will show that the dynamic model will greatly change the result.

4.4 Dynamic Analysis with Free-rider Problem

In this section, I will explore the implication of the dynamic model. I will show that the dynamics of incentive of cooperation is crucial in overcoming the free-rider problem when the political systems lack the ability to make commitments to future policy. Group member wish to take actions not just to raise their welfare today; they want to sustain their cooperation so chat they will be able to influence policy in the future in order to exploit the monopoly rents.

Before stepping into the formal analysis of dynamic model, I will lay down an important assumption.

Assumption 1 $1/\bar{N} \leq Bd < 1$.

Form the analysis of last section, we conclude that: first, the assumption make sure that with no free-rider problem, each group member will contribute enough to sustain their monopoly rents such that there will be no firm entry in every period in the static setting. Second, the assumption predicts that with free-rider problem, each group member will contribute zero such that equilibrium is characterized as firm's free entry and zero monopoly rents.

In this dynamic model, the state variable will be the number of incumbent firm in period t, i.e. n_t. The number of incumbent firms in period $t+1$ is determined by the number of incumbent firms and the regulation policy in period t. The law of motion of n_t is defined as

$$n_{t+1} = n_t + E_t, \tag{4.5}$$

where E_t is number of the entried new firms in period t.

The agent's problem has a recursive representation in which n is the state variable, and $V(n)$, the agent's value function can be represented recursively as:

$$V(n) = \max_{\tau_i} -\tau_i + \left[A - Bn - B\left[\frac{A}{B} - n - d(n-1)\tau_{-i} - d\tau_i\right]\right] + \beta V(n') \tag{4.6}$$

$$s.t\ n' = \frac{A}{B} - d(n-1)\tau_{-i} - d\tau$$

where τ_{-i} is the other member's contribution.

4.4.1 Definition of the Symmetric Markov Perfect Equilibrium

To study the properties of the dynamics of the free-rider problem, we focus on symmetric Markov Perfect equilibrium, where all group member use the same contribution strategies, and these strategies are function of payoff relevant state variable n. A strategy is agents' contribution policy $h(n)$. Given these strategies, by symmetry, the next period state variable is $n' = \frac{A}{B} - dnh(n)$.

The focus on Markov equilibrium seems particularly appropriate for this class of dynamic games. Free rider problems are often intended to represent situations in which a large number of agents autonomously and independently contribute to a public good. So it is natural to focus on an equilibrium that is anonymous and independent from the action of any single agents. The Markov perfect equilibrium satisfies this property by payoff relevant state contingent strategies.

A symmetric Markov equilibrium is therefore fully described in this scenario by two functions: individual contribution policy $h(n)$, and an associated value function $V(n)$. Hence the equation (4.6) can be rewritten as

$$V(n) = \max_{\tau_i} -\tau_i + \left[A - Bn - B\left[\frac{A}{B} - n - d(n-1)h(n) - d\tau_i\right]\right] + \beta V(n') \tag{4.7}$$

$$s.t\ n' = \frac{A}{B} - d(n-1)h(n) - d\tau$$

where agent i takes the other agents strategy $h(n)$ as given.

The equilibrium demands two conditions must be satisfied. First, the agent contribution strategy must solve equation (4.7) given $V(n)$. The second condition for an equilibrium requires the value function $V(n)$ to be consistent with the agents' strategies. Each agent receives the same benefit for the contribution to the politician. This implies:

$$V(n) = -h(n) + \left[A - Bn - B\left[\frac{A}{B} - n - ndh(n)\right]\right] + \beta V(n'(n)) \quad (4.8)$$

$$n' = \frac{A}{B} - dnh(n) \quad (4.9)$$

We can therefore define:

Definition 1 *A Symmetric Markov Perfect equilibrium consists of a value function* $V(n)$ *and the policy function* $\tau = h(n)$ *such that* $h(n)$ solves the following problem

$$V(n) = \max_{\tau_i} -\tau_i + \left[A - Bn - B\left(\frac{A}{B} - n - d\tau_i - (n-1)dh(n)\right)\right] + \beta V(n') \quad (4.10)$$

$$s.t\ n' = n + E = \frac{A}{B} + d(\tau_i + (n-1)h(n))$$

given the value function $V(n)$*; and* $V(n)$ *solves equations (4.8) and (4.9)given* $h(n)$.

In the dynamic setting, the incentive to contributing to politician is stronger. The intuition behind this claim is that the group member wishes to contribute not only to raise the monopoly rents but also to sustain their group's cohesion to sustain their monopoly. Therefore, the dynamic concern increases the incentive to cooperate. It can be shown by the first order condition. The first order condition with respect to τ_i is

$$-1 + Bd - \beta dV'(n') \quad (4.11)$$

The value function $V(n)$ is a no increasing function of n. The reason is that if n is larger the average rents will be lower and cooperation will be harder because of free-rider problem. Then the term $-\beta dV'(n')$ will be no negative. If term *(4.11)* is larger than zero, all members contribute to sustain group's monopoly privilege. Compared with the static case, successive blocking new entries demand

$$-1 + Bd \geq 0,$$

but now the intention to sustain the future monopoly power which is embedded in the term $-\beta dV'(n')$ make the cooperation easier. To solve the equilibrium explicitly and make the mechanism more transparent, in the next two I will characterize the equilibrium.

4.4.2 A Case with Analytical Solution

I will start with a special case that is easy to solve and have a analytical solution. To achieve this goal, we make one simplification.

Suppose that the policy variable can only have two values, one is $E_t = 0$, that mean no entry is allowed, another is free entry, i.e $E_t = \bar{N} - n_t$. Furthermore, I assume that when the aggregate contribution T_t is smaller than $\left(\bar{N} - n_t\right)/d$, $E_t = 0$, i.e. no entry is allowed; when the aggregate contribution T_t is no less than $d\left(\bar{N} - n_t\right)$, $E_t = \bar{N} - n_t$, i.e. free entry. Given these simple policy reaction function, we know that the equilibrium contribution only have two possible values, one is $T_t = 0$, another is $T_t = \left(\bar{N} - n_t\right)/d$. In the other words, with symmetric Markov strategy, the contribution choice of agents only have two values, one is 0, another is $\left(\bar{N} - n_t\right)/dn_t$.

This simplificationn makes us obtain the following proposition.

Proposition 1 *If the following condition is satisfied*

$$(Bd - 1) + \frac{\beta (A - Bn)}{(1 - \beta) Bn} (Bdn - 1) > 0, \tag{4.12}$$

the group will contribute the maximum amount to politician to block all the potential entried firms and sustain the same group size, i.e.

$$n' = n,$$

$$\tau = d (A/B - n)/n.$$

Proof. See the Appendix.

We can solve a threshold condition N^* which is the maximum number satisfy equation *(4.12)*. If $n \leq N^*$, the each member of special interest group will pay politician $(A/B - n)/dn$ every period, therefore, the size of the special interest group will keep constant. In the static case, because of free-rider problem the special interest group can not lobby efficiently to block new entries (we assume that $Bd - 1 < 0$). But in the dynamic model, we find that if the condition in proposition hold, the equilibrium will mimic the equilibrium without free-rider problem in the static setting. To the extreme parameter values such as $\beta \to 1$, condition *(4.12)* will boil down to $(Bdn - 1) > 0$. This condition coincides with the condition of successful lobbying with "benevolent" planner in static model. It is also easy to show that $N^* = f(\beta)$ is an increasing function, then it means that if the agents care more about future it is easier for the group to overcome free-rider problem to provide efficient lobbying.

In this simplified case, I show the primary result and the intuition behind the dynamic model. In the next subsection, I will extend the result to the generalized model in which restriction on policy variable is lifted.

4.4.3 The General Case

In the previous subsection, I use a special case with an analytical solution to show the basic result. In this section, I will entend the result to a generalized model in which the agent can choose the different contribution to determine the next period state variable. Here the next period state variables can be chosen within the whole range of the domain $\left[n, \bar{N}\right]$ instead of the two point case.

In this general case, we provide the following proposition which characterize the symmetric Markov equilibrium.

Proposition 2 *There exist a N^* such that if $n = N^*$, $n' = n$. N^* is the mamimum n which satisfies the following condition*

$$(Bd - 1) + \frac{\beta (A - Bn)}{(1 - \beta) Bn} (Bdn - 1) > 0.$$

If $n > N^*$, $n' = \bar{N}$. And when If $n < N^*$, n' will not lies in the domain $(N^*, \bar{N}]$.

Proof. See the Appendix.

Proposition 2 tell us if $n \leq N^*$, in the future the state variable will never lies in the domain $(N^*, \bar{N}]$. That means the special interest group can always sustain their monopoly rents via lobbying.

To fully charactterize the equilibrium, we need to use numerical method. The procedure to do is similar to repeat the steps in the proof of proposition 2.

4.4.4 Discussion

After the formal analysis, we think we should continue to discuss our key assumptions and their implication for our results. And the prediction and policy implication of our theory will also be addressed in this subsection.

First, our model highlight the effect of distorted government policy, here the policy is modeled as entry barriers. This helps us understand the question: why redistribution to the special interest group often takes an inefficient form. The main mechanism appears in this paper hinge on the specific form of the redistribution policy. If we assume that the redistribution policy is a lump sum income transfer, and then the dynamic concern will not show up. Therefore, in this case the cooperation of group still suffers from free-rider problem. Our view emphasizes the key role of distorted government policy on overcoming the free-rider problem for the special interest groups. Our formal analysis echoes the insight appear in Persson and Tabellini (2000).

Second, the assumption that the political system can not commit today to future policies is also crucial to our conclusion. Suppose that politician can commit to the future policy, then the lobbying problem that we address before boil down to a static problem. Then the standard free-rider argument still applies. No commitment of political system is crucial to induce incentive to contributing to politician which makes them be able to influence policy in the future.

Third, the dynamic structure of the model is also very important to reach our conclusion. Our analysis show that the dynamic modeling of the organization of special interest group provides more clues to fully understands the behavior of special interest group.

Last, but not the least. Our analysis predicts the policy persistence. There is a conventional wisdom in political economy that once an economic policy is introduced, it is likely to persist even when its original rationale is no longer valid. Our analysis provides a rationale for this phenomenon. Suppose that government start to regulate or protect a specific sector, and this policy help the special interest group overcome the free-rider problem (in the model, this correspond to the situation that $n \leq N^*$). Then in the future, this organized special interest group can lobby efficiently and sustain this distorted policy. We provide an alternative explanation to the policy persistence. The other explanation is discuss in Rodrik (1991), Brainard and Verdier (1994) and Coate and Morris (2000). They argue that if protection is granted to a declining industry in the current period, less adjustment will be undertaken, increasing the demand for protection in future periods. Our view is that our explanation is complimentary to their theory.

4.5 Conclusion

We have developed the idea that the distorted government policy help the special interest group overcome the free-rider problem. Each member of the group wishes to make contributions not only to raise their welfare today but also to sustain their monopoly power so that they are able to influence policy in the future. In order to do this, they may take current actions that would not be optimal if there were no concern for the future. Our analysis also predicts the inefficient redistribution policy to a specific special interest group tends to persist. We argue that this explanation is very important to understanding the organization of the special interest group.

In this paper, we focus on the analysis of one kind of distorted government policy, namely entry barriers. But we believe that mechanism we proposed in this paper is quite general. One important extension will be applying this approach to the trade union. Such as how labor market policy which generate insider-outsider difference (Lindbeck and Snower (1989)) help the trade union overcome the free-rider problem.

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4.A Proof of Propositions

In this appendix, I show the proofs of the propositions in this paper.

4.A.1 Proof of Proposition 1

The equilibrium can be solved by guess and verify. There are two possibilities of next period state variable, one is $n' = n$, another is $n = \bar{N}$. If $n' = n$, we can calculate the corresponding value function $V(n)$

$$\begin{aligned} V(n) &= \frac{1}{1-\beta}\left(-\tau + \left[A - Bn - B\left(\frac{A}{B} - n - d\tau n\right)\right]\right) \\ &= \frac{A - Bn}{1-\beta}\left(1 - \frac{1}{Bdn}\right), \end{aligned} \tag{4.13}$$

where $\tau = \frac{A/B - n}{dn}$ such that the law of motion $n' = n$ is satisfied. If $n' = \bar{N}$, we can calculate the corresponding value function $V(n)$

$$V(n) = 0 + \frac{1}{1-\beta} V\left(\bar{N}\right) = 0.$$

We need to solve the conditions that lead the special interest group organized. The symmetric Markov perfect equilibrium can be solved by standard guess and verify method. First, we need a guess of policy function or equivalently law of motion of state variable. Given this policy function, we can derived the corresponding value function as described before. Given the value function, we solve the optimal policy function which give us the law of motion of n. If the guess and resulted policy function are consistent, the problem is solved.

If the law of motion (policy function) is $n' = f(n) = n$, the value function $V(n)$ is given by equation ??. Given this value function, the representive member of special interest group decide the optimal level of personal contribution level. Since the periodic payoff function is a linear function of τ, the first order condition will be either positive or negative. Then the condition which make the optimal solution consistent with the guess is that first order condition is larger than zero, i.e.

$$-1 + Bd - \beta d\left(V\left(\bar{N}\right) - V(n)\right) > 0. \tag{4.14}$$

Plug in equation *(4.13)*, we obtain

$$(Bd-1)+\frac{\beta\left(A-Bn\right)}{\left(1-\beta\right)Bn}\left(Bdn-1\right)>0.$$

4.A.2 Proof of Proposition 2

The symmetric Markov perfect equilibrium can be solved as following: First, since we know that $V\left(\bar{N}\right)=0$, we could solve the equilibrium when $n=\bar{N}-1$. In the next step, we will know $V\left(\bar{N}-1\right)$ and $V\left(\bar{N}\right)$, then we can solve the equilibrium when $n=\bar{N}-1$. Repeat the same procedure, we can solve the eqilibrium for all n.

Now we can start the procedure to solve the equilibrium. Start form the state $n=\bar{N}-1$, from the proof of the proposition 1 we can conclude that

$$n'=\bar{N}, \text{ and } V\left(\bar{N}-1\right)=0,$$

if n does not satisfy condition *(4.12)*. If $n=\bar{N}-1$ satisfy condition *(4.12)*, we obtain that

$$N^{*}=\bar{N}-1,$$

$$n'=\bar{N}-1,$$

$$V\left(\bar{N}-1\right)=\frac{A-B\left(\bar{N}-1\right)}{1-\beta}\left(1-\frac{1}{Bd\left(\bar{N}-1\right)}\right).$$

If $N^{*}<\bar{N}-1$, let us move forward to the state $n=\bar{N}-2$. Since $V\left(\bar{N}-1\right)=V\left(\bar{N}\right)=0$, from individual's first order condition

$$-1+Bd-\beta dV\left(n\right),$$

we can conclude that $n'=\bar{N}$ is always preferred by individual than $n'=\bar{N}-1$. Then individual only need to make a choice between $n'=n$ or $n\prime=\bar{N}$. Therefore, the logic enbedded in proposition still applies. If $n=\bar{N}-2$ satisfies condition *(4.12)*, $n^{*}=\bar{N}-2$. If $n=\bar{N}-2$ does not satisfy condition *(4.12)*, we obtain

$$n'=\bar{N}, \text{and } V\left(\bar{N}-2\right)=0.$$

We can repeat the previous procedure until we reach the point that $n = N^*$ such that N^* is the maximum number satisfies condition *(4.12)*. From the previous analysis, we can easily conclude that

$$n' = \bar{N}, \text{and } V(n) = 0 \text{ if } n > N^*.$$

And at the state N^*, we have that

$$n' = N^*, \text{ and } V(n) = \frac{A - BN^*}{1 - \beta}\left(1 - \frac{1}{BdN^*}\right).$$

If $n < N^*$, we can prove that next period state variable can not be larger than N^*. The reason is simple, if next period state variable is N^*, the first order condition can not be satisfied.

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