Political Mergers as Coalition Formation*
An Analysis of the Heisei Municipal Amalgamations

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Abstract

Local decisions regarding municipal amalgamations exhibit moral hazard if the national government provides equalization transfers. Individual preferences over municipal mergers in Japan are estimated, using moment inequalities that require neither an equilibrium selection assumption nor enumeration of all possible mergers. According to these estimates, the observed mergers saved the national government at least ¥260 billion, but another ¥130 billion could have been saved if the first-best instead of second-best transfer scheme could have been implemented.

JEL codes: C63, D71, H77

This paper examines a recent set of Japanese municipal mergers and estimates parameters to a structural model in order to assess the inefficiency of decentralized political mergers. In the Heisei Daigappei, individual Japanese municipalities could choose what merger if any they wished to participate in, given a fixed set of national government transfer policies. Due to claimed differences in efficiencies of scale, prior to the mergers smaller municipalities spent over ¥1,000,000 per capita per year providing services that larger municipalities provided for slightly over ¥100,000, with

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this difference being subsidized by the national government. These subsidies, however, lead to municipalities preferring to remain independent even when the national government would prefer that they merge. There is thus a tradeoff between correcting between-municipality inequality, and providing the right incentives for municipalities to merge. This tradeoff is particularly easy to analyze in the case of the Heisei mergers, as the national government announced its transfer policy in advance, and the nature of the policy led to mergers occurring only during the 1999-2010 period. The observed mergers can thus plausibly be treated as the outcome of a single period coalition formation game, and the transfer policy announced by the national government will be shown to have the basic characteristics of a second-best policy.

The methodological contribution of the paper consists of the use of a moment inequality framework to obtain structural parameters describing players’ preferences over coalitions when the observed coalition structure can be treated as the outcome of a Bogomolnaia and Jackson [2002] hedonic coalition formation game. Existence but not uniqueness of a stable coalition structure is guaranteed via [Ray and Vohra, 1997]. Estimation does not require any assumptions regarding the equilibrium selection rule. The moment inequality method used provides a direct link between a theoretically consistent model of jurisdiction formation and the estimating equation as actually implemented.

Nested within this coalition formation game is an Alesina and Spolaore [1997] style model of public good provision, where players’ ideal points are distributed over a geographic policy space. Surveys suggest that residents of municipalities at the geographic “edge” of a proposed merger were concerned about the distance to post-merger public facilities, while residents of more centrally located municipalities were not. The arrangement of jurisdictions is thus modelled as the result of a tradeoff between economies of scale in the provision of public goods, and heterogeneity in preferences over the physical location of those goods.

The closest related work is Gordon and Knight [2009], who examine school district mergers in Iowa. In their model, districts merge in pairs, and match quality is

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1For comparisons, ¥1=1¢ is a rough but useful approximation. During the period in which financial data is analyzed, the USD/JPY exchange rate has varied from ¥147=$1 (Aug. ’98) to ¥80=$1 (Oct. ’10). GDP per capita has remained relatively constant at ¥4,000,000.

2Although general models of coalition formation date back at least to von Neumann and Morgenstern [1944], most recent empirical work has focused on a few specific forms of coalition formation games, such as two-sided matching games [Roth, 2008]. Empirical research on more general coalition formation models has been hampered by the fact that neither existence nor uniqueness of a stable
symmetric. This results in a unique stable matching, and parameters are estimated via simulated method of moments. The approach used in this paper, on the other hand, allows for more than two partners, but does not guarantee a unique equilibrium. The two approaches are thus complementary: the model presented below is applicable to more cases, while the model used by Gordon and Knight has more desirable properties.\(^3\)

Section 1 presents a simple model of local public goods and municipal mergers. Section 2 discusses the details of local government in Japan, the recent set of municipal mergers, and how the Japanese data can be analyzed using the theoretical model. Section 3 presents a set identified estimator based on moment inequalities, and Section 4 presents and analyzes the results of this estimation.

\section{Theory}

The objective of this paper is to analyze municipal mergers in an environment where the national government chooses policies that affect which (if any) of these mergers will occur. There is an extensive literature on the provision of public goods by local governments, but in order to focus on between-municipality phenomena specifically related to mergers a very parsimonious model of within-municipality decision making will be used. The model presented below is in the style of Greenberg and Weber [1986], Demange [1994], and in particular Alesina and Spolaore [1997]: there is a tradeoff between heterogeneity in individual preferences, and efficiencies of scale in the production of public goods. The model is designed to be consistent with observed patterns of taxation and public good provision in Japan, while also guaranteeing that for any given set of municipal boundaries, within-municipality decision making is optimal from the perspective of the national government. Disagreement between the coalition structure is guaranteed, and the number of coalition structures increases exponentially with the number of players. Recently, Fox [2008] and related papers have studied coalition formation games with transferrable utility, which is particularly relevant for issues in industrial organization. This paper studies games where transfers are not possible, the case which is more relevant for issues in political economy [Acemoglu, 2003].

\(^3\)Most empirical studies of political mergers thus far focus on American school districts. Miceli [1993], the earliest example yet found, examines the trade-off that Connecticut school districts faced between efficiencies of scale and locally optimal education quality. Alesina, Baqir, and Hoxby [2004] use a much larger dataset, and examine the relationship between county-level heterogeneity and the number of school districts and other local jurisdictions. While the estimates in each of these papers imply a type of coalition formation game, they do not present an explicit coalition formation model.
national government and local governments thus arises only due to the possibility of changes in municipal boundaries.

1.1 Public Good Provision

There is a single country, populated by individuals that are distributed across a plane. The location of these individuals is fixed, and they are partitioned into municipalities. For now, suppose that this arrangement of municipalities is also fixed. Each municipality $m$ provides a public good of quality $q_m$ to its $N_m$ residents at a single location $\theta_m$ on this plane. Providing this good costs $q_m \cdot c(X_m)$, where the cost $c(X)$ of providing one quality unit of the good depends on the covariates $X$ of the municipality, such as total population. Municipality $m$ levies taxes at rate $\tau_m$ on tax base $Y_m = \sum_{i \in m} y_i$, where $i$ indexes individuals. Municipalities also receive a lump sum transfer $T(X_m)$ from the national government. Feasible ($q_m, \tau_m$) pairs are determined by the municipal budget constraint

$$q_m c(X_m) = \tau_m Y_m + T(X_m).$$

(1)

The national government obtains funds from an outside source, and spends enough of these on activities that are outside of the model that the marginal opportunity cost of providing transfers is effectively constant. Let this marginal cost of funds for the national government be $b$.

Individual utility is assumed to take the following additively separable form:

$$u_i(q_m, \tau_m, \theta_m) = \beta_0 \log((1 - \tau_m)y_i) + \beta_1 \log(q_m - \beta_3) + \beta_2 \ell(i, \theta_m) + \epsilon_m,$$

(2)

where $\beta_3$ is some minimum level of public good provision, and $\ell(i, \theta)$ is the distance between the location of individual $i$ on the plane and the location $\theta$ of the public good provided by the municipality that $i$ is a member of.

The first two terms of this utility function have Stone-Geary form, with the minimum level of the private good set to zero. As these are the only terms that contain $q_m$ or $\tau_m$, all individuals will share the same ideal point $\tau_m^*$ for taxation. To see this, note that the above equation can be rewritten to treat income as an individual fixed
effect:

\[ u_i(q_m, \tau_m, \theta_m) = \beta_0 \log(1 - \tau_m) + \beta_1 \log(q_m - \beta_3) + \beta_2 \ell(i, \theta_m) + \alpha_i + \epsilon_m, \quad (3) \]

where \( \alpha_i = \beta_0 \log(y_i) \). There will thus be unanimous support for taxing at rate

\[ \tau_m^* = 1 - \frac{\beta_0}{\beta_0 + \beta_1} \frac{Y_m + T(X_m) - \beta_3 c(X_m)}{Y_m} \quad (4) \]

and providing the public good at quality

\[ q_m^* = \frac{\beta_1}{\beta_0 + \beta_1} \frac{Y_m + T(X_m) - \beta_3 c(X_m)}{c(X_m)} + \beta_3. \quad (5) \]

On the other hand, there is no agreement among individuals regarding the location \( \theta_m \) at which the public good should be provided. The set of feasible points is a plane, and thus choosing \( \theta_m^* \) is a multidimensional policy decision, a problem which has no single accepted solution concept.

To resolve this, political decision-making at both the local and national level is assumed to take place in a probabilistic voting framework, with the standard result that the selected policy maximizes a weighted sum of individual utilities. This is discussed in more detail in Appendix A. Furthermore, at the local level, these weights are assumed to be equal for all individuals: the local politician acts as a Benthamite social planner. The national government is assumed to use equal weights for individuals within a given municipality, but might have unequal weights across municipalities. Thus, for municipality \( m \) the \( \theta_m^* \) chosen by the local government is the same as the policy the national government would want the municipality to select, but the national government might use a transfer function \( T \) that provides very large transfers to certain types of municipalities at the expense of others. An optimal transfer function will satisfy the first order condition

\[ T = \beta_3 c(X_m) - Y_m + \frac{w_m(\beta_0 + \beta_1)}{b} \quad (6) \]

for any municipality \( m \), where \( w_m \) is the total weight placed on individuals in municipality \( m \).\(^4\) In the special case where all individuals have the same \( y \), and the

\(^4\)To derive this optimization problem, plug Equations 4 and 5 into Equation 3, and drop all the terms that do not include \( T \).
government weights all individuals equally, this simplifies to

\[ T(X) = \beta_3 c(X_m) - aY_m, \quad (7) \]

where \( a = 1 - \frac{\beta_0 + \beta_1}{\gamma} \). While this transfer policy is optimal in the case where there is no potential for changing municipal boundaries, it may not be when there is such a possibility.

### 1.2 Municipal Mergers

Let \( M \) be the set of municipalities, \( S \subseteq M \) a coalition of municipalities that will merge together, and \( \mathcal{S} \) the set of all possible coalitions, including singletons.\(^5\) If merger \( S \) occurs, then the municipalities in \( S \) are permanently eliminated and a single new amalgamated municipality is created. Let \( \pi \) be a partition of \( M \) into coalitions, and \( \Pi \) the set of all possible partitions. An amalgamated municipality behaves exactly as outlined above in Section 1.1, and does not participate in any further mergers. The utility for individual \( i \) in merger \( S \) is thus as in Equation 3 above, except replacing \( m \) with \( S \):

\[
  u_i(\tau_s, q_s, \theta_S) = \beta_0 \log(1 - \tau_s) + \beta_1 \log(q_s - \beta_3) + \beta_2 \ell(i, \theta_S) + \alpha_i + \epsilon_S. \quad (8)
\]

Assume that there is perfect information with the possible exception that the national government may not be able to observe \( \epsilon \). Also assume that it is not possible for the municipalities in \( S \) to commit to a given \( \tau_s, q_s, \) or \( \theta_S \) in advance of a merger. Finally, assume that the sufficient conditions for a unique \((q^*, \tau^*, \theta^*)\) political equilibrium, described in Appendix A, hold for all potential mergers. With these assumptions, the post merger choice of \( q^*_S, \tau^*_S, \) and \( \theta^*_S \) is known in advance for every potential coalition \( S \).

Now, with some abuse of notation, let the preferences of the politician from municipality \( m \) regarding merger \( S \) be described by

\[
  u_{mS} = \beta_0 \log(1 - \tau_s^*) + \beta_1 \log(q_s^* - \beta_3) + \beta_2 \ell_m(\theta_s^*) + \alpha_m + \epsilon_S, \quad (9)
\]

\(^5\)This notation is based on that of Banerjee, Konishi, and Sönmez [2001].
where

\[ \ell_m(\theta_S^*) = \frac{1}{N_m} \sum_{i \in m} \ell(i, \theta_S^*), \]

\[ \alpha_m = \frac{1}{N_m} \sum_{i \in m} \alpha_i. \]

The first term in Equation 9 is the utility received by members of municipality \( m \) from their private consumption. Due to the log functional form, the income term itself becomes the fixed effect \( \alpha_m \), following Equation 3. The first term will thus be the same for all municipalities participating in merger \( S \), as the tax rate is by assumption the same throughout a given amalgamated municipality. Similarly, the level of public goods is also assumed to be the same within the same amalgamated municipality. Thus, the second term will also be the same for all municipalities participating in merger \( S \), as all residents are assumed to value public goods equally.\(^6\) The third term, however, takes into account distance to the location where the public service is provided, and this may differ substantially between municipalities in \( S \). For example, if residents of \( m \) would be close to the generalized median voter of \( S \), while residents of \( m' \) would be far away, then the disutility from distance will be less severe for \( m \) than for \( m' \) if merger \( S \) occurs.

Now consider the payoff for municipality \( m \) remaining a singleton. To simplify notation, the singleton merger \( \{m\} \) will be referred to simply as \( m \) when there is no possibility of confusion: \( u_{mm} \) thus represents the payoff to \( m \) of not merging with any other municipalities. The benefit to municipality \( m \) of participating in merger \( S \) can

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\(^6\)Equation 9 might appear somewhat odd at first glance: despite representing the utility of residents of municipality \( m \), the first two terms contain no \( m \) subscripts, referring instead to \( \tau_S^* \) and \( q_S^* \). This is due to the specific functional form assumption made regarding the utility function: log utility is necessary in this case because if residents with different incomes had different preferences over tax rates, then estimation becomes computationally infeasible. For example, if utility were CES, the first two terms of Equation 9 would become \( \sum_{i \in m}(\beta_0((1 - \tau_S^*)y_i)^\psi + \beta_1(q_S^* - q_i^\psi))^{1/\psi} \), and the aggregation of individuals \( i \) up to the municipal level would have to be done repeatedly each time a different \( \psi \) estimate is considered.
then be expressed as

\[ u_{mS} - u_{mm} = \beta_0 (\log(1 - \tau^*_S) - \log(1 - \tau^*_m)) \]
\[ + \beta_1 (\log(q^*_S - \beta_3) - \log(q^*_m - \beta_3)) \]
\[ + \beta_2 (\ell_m(\theta^*_S) - \ell_m(\theta^*_m)) \]
\[ + \epsilon_S - \epsilon_m. \]  

The first term is positive if the tax rate chosen in the merger is lower: \( \frac{Y_S + T(X_S) - \beta_3 c(X_S)}{Y_m + T(X_m) - \beta_3 c(X_m)} > \). The second term is positive if the quality of the public good provided is higher: \( \frac{Y_S + T(X_S) - \beta_3 c(X_S)}{c(X_S)} > \frac{Y_m + T(X_m) - \beta_3 c(X_m)}{c(X_m)} \). The third term (the difference in disutility from distance) will be zero or negative so long as distance is undesirable \((\beta_2 < 0)\), because \( \theta^*_m \) minimizes \( \ell_m \).

Municipal mergers here are being treated as a pure hedonic coalition formation game, where the payoff to each player depends only on the coalition to which it belongs, and not on what other coalitions occur. This is the game introduced by Dreze and Greenberg [1980], except without the possibility of even within-coalition transfers. Consider the case where mergers are effected via a decentralized process, where a merger requires approval from all participating municipalities, and assume that this decentralized decision-making involves each municipality making decisions based on the utility function in Equation 9. Then the inability for municipalities to negotiate transfers may prevent some coalitions from forming. To see this, rewrite Equation 9 as

\[ u_{mS} = v_{mS} + \epsilon_S \]

and consider the following example:

Example 1. Suppose that there are two municipalities with identical characteristics, the cost function \( c \) is constant returns to scale, and \( v_{mm} = v_{mS} = v \). Then if \( \epsilon_m > 0 \), \( \epsilon_m' < 0 \), \(|\epsilon_m| > |\epsilon_m'|\), the merger will not occur if transfers between municipalities are not possible.

If transfers between municipalities were possible, and \( u \) did not exhibit too much curvature in private consumption, then \( m \) would offer a transfer to \( m' \) in exchange

\footnote{As discussed below, with this sort of decentralized system, once there are multiple potential mergers involving some of the same municipalities it may no longer be obvious which mergers will occur. For now, however, consider the case where each municipality can participate in only one (non-singleton) merger.}
for \(m'\) agreeing to the merger, and the merger would occur.

If such transfers are not possible, the national government might wish to mandate mergers instead of allowing decentralized mergers. If the national government has perfect information and direct control over municipal boundaries, then in the case where \(S = \{m, m'\}\) is the only possible merger for both \(m\) and \(m'\), the national government will mandate that this merger occur when

\[
w_m (u_{mS} - u_{mm}) + w_{m'} (u_{m'S} - u_{m'm'}) > 0. \tag{12}
\]

On the other hand, if the national government does not know \(\epsilon\) then it may choose to implement decentralized mergers instead of mandating a certain pattern of municipal mergers:

**Example 2.** If, in the case described in Example 1, the national government weights all individuals equally, and does not observe \(\epsilon\), then if \(E[\epsilon_m - \epsilon_S] = E[\epsilon_{m'} - \epsilon_S] = 0\) the national government will choose to implement a decentralized merger policy.

A centralized merger policy would have expected payoff of \(v\), regardless of whether the merger is mandated or prohibited, because with the optimal transfer scheme there is no difference in total transfers. The decentralized policy, on the other hand, will result in a merger when both \(\epsilon_S - \epsilon_m > 0\) and \(\epsilon_S - \epsilon_{m'} > 0\), and no merger otherwise. This improves on either centralized policy choice, and thus the national government will choose to implement decentralized mergers even if it had the option of controlling mergers centrally.

If the government has decided to implement a decentralized merger policy, then the optimal transfer policy may not be the same as that given in the preceding section:

**Example 3.** Suppose that the situation is as described in Example 2, except now suppose that c exhibits efficiencies of scale and \(\epsilon\) is normally distributed. Then the transfer policy in Equation 6 is not optimal.

To see this, let \(u_{mS}(T)\) be the utility of municipality \(m\) in merger \(S\) when the national government implements transfer function \(T\), and let \(p_S(T)\) be the probability that merger \(S\) will occur given the transfer function \(T\). As in Equation 11, let \(v_{mS}(T)\) be the non-idiosyncratic component of \(u_{mS}(T)\). The national government’s problem
is now

\[
\max_T (1-p_T(T))(v_m(T)+E[\epsilon_m|S \not\in \pi^*, T]-2bT(X_m))+p_T(T)(v_m+\epsilon_S|S \in \pi^*, T)-bT(X_s)).
\]

(13)

where \( \pi^* \) is the partition that is actually observed, which will depend on \( \epsilon \). Now, start with the transfer policy given by Equation 6, and consider a small deviation that increases transfers to the municipalities if they merge and decreases transfers by an equivalent total amount if they do not. The cost of this deviation is that the transfer policy is no longer optimal given fixed boundaries. This cost is second-order. On the other hand, there are two first-order benefits. First, if \( c \) exhibits efficiencies of scale then the national government spends less money in expectation because \( T(X_S) < 2T(X_m) \).\(^8\) Second, in the case where one municipality prefers the merger but the other is indifferent, the indifferent municipality does not internalize the benefits of the merger to its partner, but could be encouraged to merge via a higher \( T(X_S) \). Increasing \( T(X_S) \) thus helps to resolve two externalities: specifically, local governments consider neither their partners’ payoffs nor the national budget. Thus, the transfer policy that was optimal for the national government when municipal boundaries were fixed is no longer optimal when there is the possibility of municipal mergers.

While in the preceding examples it was intuitively clear for any given \( \epsilon \) which mergers would occur if the process were decentralized, unfortunately there are other situations where the decentralized outcome is not so obvious. Consider, for example, the classic “roommates problem”:

**Example 4** (Gale and Shapley 1962). Suppose \( M = \{1, 2, 3\} \) and preferences are

\[
\begin{align*}
\{1, 2, 3\} & \prec_1 \{1\} \prec_1 \{1, 3\} \prec_1 \{1, 2\}, \\
\{1, 2, 3\} & \prec_2 \{2\} \prec_2 \{1, 2\} \prec_2 \{2, 3\}, \\
\{1, 2, 3\} & \prec_3 \{3\} \prec_3 \{2, 3\} \prec_3 \{1, 3\}.
\end{align*}
\]

(14)

Given these preferences, there is no intuitively obvious solution to this coalition formation game: Players 2 and 3 would both like to deviate from the \( \{(1, 2), \{3\}\} \)

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\(^8\)The assumption that the municipalities are identical and \( v_m = v_S \) rules out the possibility that there are diseconomies of scale, but there is a trivial case where \( c \) is constant returns to scale and distance costs are the same with the merger and without.
partition, and there are similar deviations for other partitions.

Approaches to dealing with this non-existence problem in empirical work can be divided into three general types. First, the form of preferences could be restricted so as to rule out “preference cycles”, such as the \( \{1, 2\} \prec_2 \{2, 3\} \prec_3 \{1, 3\} \prec_1 \{1, 2\} \) cycle in Example 4, thereby ensuring existence. This is the approach used by Gordon and Knight [2009], but the restrictions required on preferences are such that a geographic distance term such as \( \ell(i, \theta_m) \) in Equation 3 could no longer be included. Having preferences over mergers that differ depending on location is important to explain the Japanese merger data examined in this paper, and thus this approach of restricting preferences is not appropriate.\(^9\)

A second approach to this non-existence problem is to choose a specific non-cooperative form for the coalition formation game: for example, assume that Player \( j \) has the ability to make a proposal with probability \( p_j \), and so forth. An approach of this sort is used in Diermeier, Eraslan, and Merlo [2003]. In the case of Japanese municipal mergers, however, there is very little information on the precise structure of the game, and different assumptions would lead to different predicted solutions: in a setup similar to Example 4, the \( \{1, 2\} \) coalition might be the solution for some assumptions, and the \( \{2, 3\} \) or \( \{1, 3\} \) coalitions for others. Parameter estimates would thus likely depend on effectively arbitrary assumptions regarding the game structure.

A third approach is to relax the requirements for a partition to be considered a solution, and then deal with the resulting multiplicity problems via an appropriate set-identified estimation strategy. This is the approach used in this paper.

Ray and Vohra [1997] develop a solution concept based on only considering refinements: deviations that involve a subset of a single coalition, and thus result in moving from a coarser partition to a finer one. They then define the coarsest partitions that do not have any refinements as the solution to the coalition formation game. In the roommates example, then, the \( \{\{1, 2\}, \{3\}\} \) partition would be considered stable, as the \( \{2, 3\} \) coalition is not a subset of a coalition in the partition. There would thus be three solutions to Example 4.

The environment considered in this paper is simpler than that considered in Ray and Vohra [1997], and thus a simpler solution concept can be used:

\(^9\)A previous version of this paper estimated a model with preferences restricted so as to rule out preference cycles, and concluded that the coefficient estimates resulting from this model were not plausible.
Theorem 1. Let $\Pi^*$ be the set of partitions that do not have any deviations that are refinements or coarsenings. Then $\Pi^*$ exists, is not empty, and is unique.

Proof. See Appendix B.

Although $\Pi^*$ is unique, it may contain multiple partitions. Multiplicity of solutions is a fundamental property of “roommate-type” coalition formation games [Barberà and Gerber, 2007]. Intuitively, as Example 4 is symmetric, any plausible solution concept that gives $\{\{1,2\},3\}$ should also give the partition with $\{2,3\}$ and the partition with $\{1,3\}$ as solutions as well.

2 Japanese Context

2.1 Public Good Provision

Japan is a unitary state divided into 47 prefectures, whose boundaries have remained roughly unchanged since the 1890s. Each of these prefectures is divided into municipalities. Municipalities are responsible for providing public services in six major categories: firefighting, public works, education, welfare, industry, and administration.

Post-war Japanese national policy emphasized equalization between municipalities, and established a “national standard” reference quality for local government services. To ensure that every municipality had sufficient funds to offer services above this minimum level, the national government developed a complicated system of transfers, called the “Local Allocation Tax.”

In Japan in the period before the Heisei municipal mergers began, transfers to municipalities were determined by the formula

$$T(X_m) = \max(\tilde{c}(X_m) - .75\bar{\tau}Y_m, 0),$$

where $\tilde{c}$ is a national government estimate of the cost to a municipality with characteristics $X$ of providing public goods at the “national standard” quality, and $\bar{\tau}Y_m$ the amount of tax revenue the municipality should be able to collect if taxes are charged at the “standard” rate. The cost estimate $\tilde{c}$ is sometimes referred to as the “Standard Fiscal Need”, and is calculated by a formula equivalent to

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10The slightly-confusing name is due to the fact that it is an allocation to local governments from taxes collected by the national government.
\[
\bar{c}(X_m) = \sum_{k=1}^{24} X_{mk} \cdot \bar{c}_k (1 + \tilde{H}_k(X_m)) + \zeta_m. \tag{16}
\]

Here the public good is viewed as a sum of 24 component goods, such as firefighting, care for the elderly, resident registration, and so forth.\textsuperscript{11} Each of these component goods is associated with a quantity measure \(X_{mk}\), which for firefighting is total population, for elderly care is population over 65, for resident registration is number of families, and so forth. Each of these component goods is also associated with an estimated unit cost \(\bar{c}_k\): the estimated cost of providing firefighting for one person, elderly care for one person over 65, resident registration for one family, and so forth.

For each of these component goods \(k\), there is an “adjustment coefficient”, \(\tilde{H}_k\), created by multiplying and adding together a set of (usually) decreasing splines determined by \(X_m\).\textsuperscript{12} For example, the cost of providing the standard level of firefighting is judged to be ¥1.009 million for a city of 100,000 people, but this is increased to ¥1.029 million if the population density is 150 per km\(^2\) rather than 200. Population density also affects the estimated cost of elderly care, and resident registration, but in different ways, with the effect on firefighting costs being more severe than the effect on these other component goods. There are also some additional costs \(\zeta_m\) that are not generally subject to adjustment. These include costs related to different sorts of land (forest, farmland, etc.), or costs related to American military bases. Further details regarding Equation 16 are provided in Appendix C.

To match the Japanese municipal finance system to the theoretical model presented in Section 1, assume that each municipality must choose a single quality \(q\) at which to provide the public good: it is not possible, for example, for municipality \(m\) to choose to provide quality \(q_f^m = 5\) firefighting, but only quality \(q_r^m = 3\) resident registration. Furthermore, suppose that the cost estimate \(\bar{c}(X)\) produced by the national government has the right general form, but with possibly incorrect adjustments \(\tilde{H}\). Specifically, suppose that the true cost function for public goods is

\[
c(X_m) = \sum_{k=1}^{24} X_{mk} \cdot \bar{c}_k \cdot (1 + H_k(X_m)) + \zeta_m, \tag{17}
\]
where

$$H_k(X_m) = \beta_4 \hat{H}_k(X_m).$$

Thus, if $\beta_4 = 0$ there are in reality no efficiencies of scale, and the national government’s adjustments $\hat{H}(X_m)$ are distorting the estimated cost function $\hat{c}(X)$ away from the true (constant returns to scale) cost function $c(X)$. On the other hand, if $\beta_4 = 1$ then the national government’s estimate of the cost function precisely equal to the true cost function: $\hat{H}(X_m) = H(X_m)$ and thus $\hat{c}(X) = c(X)$.\(^{13}\)

In addition to the potential that the national government is not correctly estimating efficiencies of scale in the provision of public goods, there is also some bureaucratic imprecision regarding the $\tau Y_m$ term in Equation 15, sometimes referred to as “Standard Fiscal Revenue”. While de jure, municipalities are allowed to set their own tax rates, de facto it appears that rates less than $\tau$ may be prohibited. This issue is discussed more in Appendix C, and in the empirical analysis a model will be estimated both for the case where $\tau$ acts as a minimum tax rate, and where $\tau^*$ can be chosen freely.

The majority of local public services are provided via physical facilities, such as schools, nursing homes, libraries, and city hall itself.\(^{14}\) Surveys conducted around the time of the recent mergers reveal a widespread perception that there were unexploited efficiencies of scale in the provision of these services: the most popular response to questions regarding the potential benefits of municipal mergers was “avoid duplication of facilities / avoid useless capital expenditures” in Kyoto, “reduce expenditures by improving administrative efficiency, eliminating duplicate facilities, and reducing personnel” in Yamanashi prefecture, “reduce personnel expenses” in Akita, and “reduce personnel and other expenditures and improve efficiency” in Okinawa.\(^{15}\) It was generally understood that a municipal merger involving a smaller municipality and a larger one would result in the closure of city hall and some other facilities in the smaller municipality, and that this would result in substantial cost savings. This lead to a concern among residents of smaller municipalities that after a merger public services would only be provided at a location much further away than was the case

\(^{13}\)There are no natural units for quality, and a normalization is thus necessary. The one used here is that the $c_k$ from the national government estimates are the true unit costs.

\(^{14}\)Although many services at city hall could be accessed via mail, telephone, or the internet, it is common and in some cases required to visit in person.

\(^{15}\)Unfortunately, these surveys are difficult to analyze quantitatively, as they were only conducted in a few prefectures, and a different questionnaire was used in each prefecture.
currently. The distance term $\ell$ in Equation 8 will thus be taken to be geographic distance. This is calculated for each possible coalition $S$ using census grid square data on population. These calculations are described in more detail in Appendix C.

2.2 Municipal Mergers

Historically, although there were provisions for municipalities to merge there were limited incentive for them to do so, because if a coalition $S$ decided to form a new (amalgamated) municipality, transfers would be calculated exactly as in Equation 15:

$$T(X_S) = \max(\tilde{c}(X_S) - 0.75\tau Y_S, 0).$$

(19)

Thus savings would be passed to the national government through lower calculated values of $\tilde{H}$, with the result that historically local residents would oppose mergers more than would be warranted from a social perspective. The exact behaviour of residents was determined in part by the relationship between the true cost $c(X)$ and the national government’s estimate $\tilde{c}(X)$.

During the fiscal difficulties of the early 1990s, the Japanese national government implemented a series of reforms designed to reduce the total transfers provided to municipalities while attempting to minimize the negative effects of this decrease. First, the government substantially reduced transfers, particularly to the smallest municipalities. This was effected mainly by replacing $\tilde{H}_k$ with $\tilde{H}^{\text{new}}_k$, which was less generous towards smaller municipalities as is shown in Figure 4. This provided smaller municipalities with an incentive to merge so as to avoid having to either sharply reduce the quality of service that they provided to their residents, or increase the tax rate charged.

Second, with this new transfer function $T^{\text{new}}$ in place, the government then allowed municipalities to keep more of the savings from a merger. In particular, after 1999, the transfers for a merger $S$ would depend on how $S$ was formed. Specifically, transfers would be calculated according to the special “merger” formula

$$T^{\text{merger}}(S) = \sum_{m \in S} T^{\text{new}}_m (X_m)$$

(20)

16 In general, the division of a municipality was prohibited. In one case, such a split did occur, but both of the resulting municipalities were immediately merged with different neighbours.

17 Although this change took place around 2003, it was announced earlier.
for at least ten years starting from the date of the merger. This incentive began to be phased out in 2006, which motivated many municipalities to finalize mergers in 2005. By 2006 there were only 1,844 municipalities remaining, down from 3,255 at the start of the merger period. A small number of mergers occurred during the phase-out period, reducing the final number of municipalities to 1,750 in 2010; for the purposes of this paper, these mergers are treated as though they were finalized prior to 2006, and implementation was simply delayed for exogenous reasons.

A final incentive for mergers was the Gappei Tokureisai, special subsidized bond issues allowed for municipalities planning amalgamation. The value of these bonds is calculated based on the subsidy offered, using information from Ishihara [2000]. Municipalities are presumed to be able to save in order to equalize the quality of public services and the municipal tax rate between the decade immediately following the merger, when incentives are provided, and following decades.

Figure 6 shows the mergers that occurred in Shizuoka Prefecture. Mergers were voluntary, and needed to be approved by every participating municipality. An overview of the rules described in the “Special Municipal Merger Law” is described in Appendix A.2.

Although a large number of mergers occurred overall, very few of these mergers involved municipalities in the most metropolitan prefectures. Define a prefecture as “metropolitan” if fewer than 10% of its municipalities have a population of less than 10000, and define a prefecture as “rural” if more than 65% of its municipalities have a population of less than 10000. Table 1 shows summary statistics for municipalities in different classes of prefectures. Municipalities in metropolitan prefectures were much less likely to merge during the merger period than those in the other sorts of prefectures.

---

18 An intermediate amount between $T_{new}^{m}(X_S)$ and $T^{merger}(S)$ was offered for years 11-15 following a merger.

19 Explaining why a coalition would not form during the 1999-2005 period, but would under the progressively less-advantageous policies in place in 2006-2010 would require adding elements to the model, such as arrival of new information, that would substantially complicate the analysis. This paper treats the entire 12 years as a single period.

20 The official explanation for these bonds was to eliminate any direct financial cost of merging, such as the construction of a new city hall. The merger bonds appeared to allow significant capital expenditures beyond the actual costs of amalgamation. Relative to the incentive provided by the switch from $c_m$ to $c_{new}^{m}$ in the calculation of transfers, these bonds have a relatively small effect on incentives to merge, and thus for simplicity the the bonds are treated entirely as an additional incentive, with the direct financial costs of merging ignored.
3 Estimation

There are four parameters of interest from Equation 3: the value of private consumption ($\beta_0$), the value of public consumption ($\beta_1$), the disutility of distance ($\beta_2$), and the minimum quality for the public good ($\beta_3$). The fifth parameter of interest, $\beta_4$, which appears above in Equation 18, enters the utility function in Equation 3 through the cost function $c(X)$. These five parameters can be estimated by examining the mergers that actually occurred in Japan and comparing them to ones that could have occurred but did not, using the data on national government transfers and efficiencies of scale described in Section 2 and further explained in Appendix C.

To do this, first rewrite the utility function in Equation 11 to make explicit the fact that the values of the $\beta$ parameters affect the structural component, but do not affect the idiosyncratic component:

$$u_{ms}(\beta) = v_{ms}(\beta) + \epsilon_S.$$  

(21)

Assume that $\epsilon$ is distributed normally, with the distribution of $\epsilon_S$ identical to that of $\epsilon_{S'}$, but not necessarily independent. Furthermore, note that, as is standard in discrete choice models, multiplying $u$ by a positive constant has no effect on preferences. Thus, as a normalization, multiply such that $\epsilon \sim N(0, 1)$. Let $\beta^0$ be the true value of $\beta$. Estimation will be based on four types of moment inequalities:

1. At $\beta^0$, it should be possible to find values of the idiosyncratic shocks that rationalize the observed mergers, and are not “too extreme” relative to the $N(0, 1)$ distribution from which they are assumed to have been drawn.

2. At $\beta^0$, if the national government had not implemented the merger promotion policies, the number of mergers that would have occurred would not be “too high” relative to the number of mergers that occurred prior to 1999.

3. At $\beta^0$, the number of mergers actually observed in metropolitan prefectures is not “too low” relative to the number of mergers predicted in these prefectures by the model.

4. At $\beta^0$, the tax rates actually charged by municipalities are “similar” to those that the model predicts should be charged.
The technical definitions for terms in quotation marks will be given below. The first two types of moment inequalities should hold for metropolitan prefectures, mixed prefectures, and rural prefectures. Thus, a total of 8 (= 3 + 3 + 1 + 1) moment inequalities will be used in estimation. The remainder of this section has the following form: first, the covariance of \( \epsilon_S \) and \( \epsilon_{S'} \) is discussed, and then the details of each of the above four types of moment inequalities are presented.

### 3.1 Structure of Idiosyncratic Shocks

There are three important points regarding the set \( \mathcal{S} \) of all potential mergers. First, it is potentially very large, containing up to \( 2^M \) elements. Second, it is not always clear which coalitions should be in this set: for example, no coalitions of size greater than 15 are observed in the data, but there was also no government policy that expressly prohibited a size 50 coalition from forming. Finally, it is implausible that \( \epsilon \) is i.i.d across different coalitions: if \( S = \{m_1, m_2, \ldots, m_{14}, m_{15}\} \), and \( S' = \{m_1, m_2, \ldots, m_{14}\} \), then a reasonable econometric model should have \( \epsilon_S \) correlated with \( \epsilon_{S'} \). The following construction makes it possible to generate shocks that are \( \epsilon \sim N(0, 1) \), not independent but identically distributed, and to generate shocks for some coalitions without enumerating all potential coalitions. The basic assumption comes from the literature on ethnic fragmentation: under certain conditions, heterogenous jurisdictions produce bad results for all residents, not only those far from the median voter. While Japan is not known for extreme ethnic or linguistic heterogeneity, one could imagine even minor cultural differences playing such a role.\(^{21}\)

First, suppose that each individual resident makes an i.i.d. draw, \( \omega_i \sim N(0, 1) \), representing \( i \)'s cultural identity. For municipality \( m \) with population \( N_m \), the sample mean and sample variance of these draws will be

\[
\bar{\omega}_m = \frac{1}{N_m} \sum_{i=1}^{N_m} \omega_i \tag{22}
\]

\[
s^2_m = \frac{1}{N_m - 1} \sum_{i=1}^{N_m} (\omega_i - \bar{\omega}_m)^2, \tag{23}
\]

because there are \( N_m \) residents making i.i.d. draws. Within-municipality heterogene-

\(^{21}\)Costa’s work on civil war soldiers, etc.
ity is captured by the sample variance, so let

$$
\epsilon_m = -f(X_m) \log s_m^2,
$$

(24)

and likewise for any coalition $S$. Here $f(X) > 0$ is a function that generates weights such for any coalition $S = A \cup B$, then $f(X_S) > f(X_A)$ and $f(X_S) > f(X_B)$. That is, heterogeneity is relatively more important for larger municipalities.

Defining $\bar{\omega}_S$ and $s_S^2$ in the same way for any coalition $S$, the standard relationship for sample variances will hold:

$$
s_S^2 = \frac{\sum_{m \in S} (N_m - 1)s_m^2 + \sum_{m \in S} \sum_{m' \in S} \frac{N_m N_{m'} (\bar{\omega}_m - \bar{\omega}_{m'})^2}{N_S - 1}}{N_S - 1}
$$

(25)

Now define the vectors $\bar{\omega}_M$ and $s_M$ to be the sample means and standard deviations for all municipalities. It is possible to calculate $\epsilon_S$ for any coalition $S$ given only $\bar{\omega}_M$ and $s_M$. Let $\epsilon(\bar{\omega}_M, s_M)$ be the vector resulting from this calculation. The elements of both $\bar{\omega}_M$ and $s_M$ have known distributions, which will be helpful when computing moment inequalities.

With this construction of $\epsilon$, for any guess $\hat{\beta}$ for the parameter vector, any observed partition can be rationalized: simply choose $s_S^2$ sufficiently close to zero if $S$ is in the observed partition, and large otherwise, and then choose $\bar{\omega}_S$ and $\bar{\omega}_{S'}$ such that $(\bar{\omega}_S - \bar{\omega}_{S'})^2$ is sufficiently large to discourage any coarsenings into larger coalitions.

Finally, using the approximation

$$
\log s_S^2 \simeq \log (1 + \delta_S), \quad \delta_S \sim N(0, \frac{2}{N_S - 1})
$$

$$
\simeq \delta_S
$$

it is the case that if $f(X_S) = \sqrt{\frac{N_S - 1}{2}}$, then $\epsilon_S \sim N(0, 1)$, n.i.i.d, as desired.
3.2 Moment Inequalities Based on Observed Partition

To construct a moment inequality estimator based on rationalizing the observed coalition structure, first choose an arbitrary function \( h(\bar{\omega}, s, X) \). Then define \( h^* \) as

\[
h^*(\pi|\beta) = \min_{\bar{\omega}_M, s_M} h(\bar{\omega}_M, s_M, X) \quad \text{s.t.} \quad \pi \in \Pi^*(\epsilon(\bar{\omega}_M, s_M)|\beta)
\]

(26)

where \( \Pi^* \) is the stable set evaluated at parameters \( \beta \), and with idiosyncratic shocks \( \epsilon \) as determined by \( \bar{\omega}_M \) and \( s_M \). Thus, \( h^* \) is a lower bound for \( h \) given that partition \( \pi \) was observed, and that parameter values are \( \beta \). It is always the case that

\[
h(\bar{\omega}_M^0, s_M^0, X) \geq h^*(\pi^0|\beta^0)
\]

(27)

where \( \bar{\omega}_M^0 \) and \( s_M^0 \) are the true values that were drawn for \( \bar{\omega}_M \) and \( s_M \), respectively, and \( \pi^0 \) is the partition that resulted from those draws. This is because \( \Pi^*(\epsilon(\bar{\omega}_M^0, s_M^0)|\beta^0) \) must contain \( \pi^0 \), otherwise \( \pi^0 \) could not have been observed, and thus \( (\bar{\omega}_M^0, s_M^0) \) could be chosen in Equation 26. Consider the moment

\[
g_1(\pi, \beta|X) = Eh(\bar{\omega}_M, s_M, X) - h^*(\pi, \beta).
\]

(28)

This will be positive in expectation at the true parameter value \( \beta^0 \), because

\[
Eg_1(\pi, \beta^0|X) = Eh(\bar{\omega}_M, s_M, X) - Eh^*(\pi, \beta^0)
\]

\[
\geq 0
\]

(29)

because Inequality 27 holds at every realization of \( (\bar{\omega}_M, s_M) \) and \( \pi \), and thus also holds in expectation. A very simple example of this sort of inequality is given at the beginning of Appendix D.

3.3 Moment Inequalities Based on Counterfactual Policy

There is little debate in Japan that the large number of mergers that occurred during the 1999-2010 period were a result of policy changes made by the national government. Figure 7 shows that the merger activity is in marked contrast to the period before

\[22\text{Good choices for } h \text{ seem to be functions that will give high values when } \bar{\omega} \text{ and } s \text{ are extreme relative to the distributions from which they were assumed to have been drawn.}\]
1999: only 18 municipalities participated in mergers during the two decades preceding the implementation of merger promotion policies. The following assumption will thus be used to generate moment inequalities: in the absence of any change in national government policy, merger activity in 1999-2010 should not have been greater than merger activity in 1979-1999.

More specifically, let $F_Q(\beta)$ be the distribution of the number of municipalities that would have participated in mergers during the 1999-2010 period if the government had not implemented any new merger promotion policies. By assumption (and after making an appropriate adjustment for the fact that the 1979-1998 period is longer than the 1999-2010 period) $F_Q(\beta)$ is stochastically dominated by $F_Q^{79}(\beta)$, the distribution of the number of municipalities that participated in mergers during the 1979-1999 period. $F_Q$ is difficult to calculate directly: not only is the true equilibrium selection rule unknown but, as discussed at the beginning of Section 3.1, the precise membership of $S$ is both unknown and likely very large. Thus, instead consider only mergers of size 2.

If $S = \{m, m'\}$, and $u_{mm} > u_{mm'}, u_{m'm} > u_{m'm'}$, then any stable partition must have at least one of $m$ and $m'$ participating in a merger, because $S$ is a blocking coalition for all other partitions. Let $S_a$ be the set of size 2 mergers where the municipalities are geographically adjacent, and both municipalities prefer the merger to remaining as a singleton. This set can be used to construct an easily computable minimal number of municipalities that must be involved in mergers. Consider the following variable, which is random because the membership of $S_a$ depends on the draw of $\bar{\omega}_M$ and $s_M$:

$$Q^* = \arg\min_{Q \subseteq M} \#Q \text{ s.t. } \forall S \in S_a, S \cap Q \neq \emptyset \quad (30)$$

That is, $Q^*$ is a minimal hitting set for $S_a$: for each potential geographically contiguous size 2 merger where both participants prefer the merger relative to not merging at all, at least one of those municipalities is in $Q^*$. Let $F_Q^*(\beta)$ be the distribution of $\#Q^*$. $F_Q$ stochastically dominates $F_Q^*$, because $S_a$ is a subset of all mergers whose participants prefer the merger to remaining as a singleton, and thus any stable partition must include at least $\#Q^*$ municipalities participating in mergers regardless of equilibrium selection rule. The moment $g_2(Q, \beta|X) = Q - \mu_Q$, where $Q$ is the number of municipalities involved in mergers in the 1979-1998 period, can then be used as a
moment inequality because

\[ E_Q g_2(Q^{79}, \beta^0 | X) = \mu_Q^{79} - \mu_Q^* \]
\[ \geq \mu_Q^{79} - \mu_Q \]
\[ \geq 0 \]

due to stochastic dominance and \( Q^{79} \) having been drawn from \( F_Q^{79} \).

### 3.4 Moment Inequality Based on Metropolitan Mergers

Table 1 shows that very few mergers occurred in “metropolitan” prefectures, defined in this paper as prefectures with fewer than 10% of municipalities having a population of less than 10000. The same argument used in the previous subsection can thus be extended to mergers actually observed during the merger period: given the national government’s actually implemented policies, the number of mergers observed should not be anomalously low.

Specifically, let \( Q^{99} \) be the number of municipalities actually participating in mergers in the 1999-2010 period. Then the moment \( g_3(Q, \beta | X) = Q - \mu_Q^* \), where \( F_Q^* \) is as defined in the previous section, can be used as a moment inequality because

\[ E_Q g_3(Q^{99}, \beta^0 | X) = \mu_Q^{99} - \mu_Q^{99*} \]
\[ \geq 0 \]

due to stochastic dominance and \( Q^{99} \) having been drawn from \( F_Q \).

### 3.5 Moment Inequality Based on Tax Rates

Finally, tax rates that are actually charged are observed for all municipalities. This is particularly interesting in the merger period, where there is noticeable, although still low, dispersion in the tax rates being charged. One complication here is that \( de facto \), municipalities appear not to be able to lower their tax rate below \( \bar{r} \), although they are free to charge a higher rate. Even with this censoring, however, tax rates (after adjustment for the tax floor) should be correctly predicted by the model. Specifically,
suppose that the observed tax rates are a function of optimal tax rates plus some noise:

$$\tau^*_m = \max(\tau^*_m(\beta) + \varepsilon_m, \hat{\tau}),$$  \hspace{1cm} (33)

where \(\tau^*_m\) is taken from Equation 4. If the theoretical model is correct, then, including additional terms should not improve the fit of a Tobit regression. That is, if the restriction \(\gamma = 0\) is imposed on the model

$$\tau^*_m = \max(\tau^*_m(\beta) + \gamma X_{mk} + \varepsilon_m, \hat{\tau}),$$  \hspace{1cm} (34)

then if \(g_4(\beta, X)\) is the gradient for \(\gamma\), evaluated at \(\gamma = 0\), this can be used as a moment equality.

### 3.6 Identified Set and Confidence Sets

Constructing a 95% confidence set for \(\beta\) is simplified because assumptions regarding the distribution and correlation structure of the error terms have already been necessary in order to develop the model. The data consists of 47 prefectures, which are treated as independent coalition formation games. Prefectures are classified as “metropolitan”, “mixed”, and “rural” depending on the percentage of municipalities with a population of less than 10000. Let these sets of prefectures be \(J^{\text{metro}}, J^{\text{mixed}}, \) and \(J^{\text{rural}}\), respectively. Let \(\bar{g}^1_{\text{metro}}(\beta)\) be the sample moment of \(g_1\) with prefectures \(J^{\text{metro}}:\)

$$\bar{g}^1_{\text{metro}}(\beta) = \frac{1}{\#J^{\text{metro}}} \sum_{j \in J^{\text{metro}}} g_1(\pi^0_j, \beta)$$ \hspace{1cm} (35)

where \(\pi^0_j\) is the actually observed partition in prefecture \(j\). Construct \(\bar{g}_2\) similarly: for example, \(\bar{g}^2_{\text{mixed}}\) would be

$$\bar{g}^2_{\text{mixed}}(\beta) = \frac{1}{\#J^{\text{mixed}}} \sum_{j \in J^{\text{mixed}}} g_2(Q^{79}_j, \beta)$$ \hspace{1cm} (36)

where \(Q^{79}_j\) is the actually observed number of municipalities participating in mergers in prefecture \(j\) during the 1979-1998 period. There are thus three sample moments calculated from \(g_1\) (metro, mixed, and rural), and another three in the same way for \(g_2\). On the other hand, \(g_3\) is only calculated for “metro” (it will not bind for any value of \(\beta\) for mixed or rural, due to the large number of mergers in these types of
where \( Q_{j}^{99} \) is the actually observed number of municipalities participating in mergers in prefecture \( j \) during the 1999-2010 period. A straightforward text statistic would be

\[
T(\beta) = 2 g_{\text{metro} 1}(\beta) + 2 g_{\text{mixed} 1}(\beta) + 2 g_{\text{rural} 1}(\beta) + 2 g_{\text{metro} 2}(\beta) + 2 g_{\text{mixed} 2}(\beta) + 2 g_{\text{rural} 2}(\beta) + [g_{\text{metro} 3}(\beta)]^2 + [g_{4}(\beta)]^2
\]

(38)

where \([x]_\rho = \min(x, 0)\). As explained in Appendix C, an adjustment is made to the second and third set of moments to take into account that mergers are infrequent, and thus convergence to a normal distribution will be slow. Due to the distributional assumption already made regarding \( \phi \) and \( s \), the first six of these terms are uncorrelated. \( g_{\text{metro} 3} \), however, could well be correlated with \( g_{\text{metro} 1} \), as \( \sigma^0 \) affects both of these sample moments. The worst case scenario is that these sample moments are perfectly correlated. Similarly, \( g_{4} \) could plausibly be correlated with other moments, with the worst case scenario being perfect correlation between \( g_{4} \) and the sum of the other terms. Critical values of \( T(\beta) \), then, will be computed assuming these worst case scenarios. The identified set is

\[
\hat{\beta} = \arg\min_T T(\beta) \quad \text{(39)}
\]

and following convention the 95\% confidence set will be

\[
\{ \beta | T(\beta) < T(\hat{\beta}) + T_{0.95} \}
\]

(40)

where \( T_{0.95} \) is the 0.95 quantile of the distribution of the test statistic. In general \( T_{0.95} \) would depend on \( \beta \), but in this particular instance the distribution is the same regardless of \( \beta \).
4 Results

Results are shown in Table 2. Since mergers do not cross prefectural boundaries, each prefecture is treated as a separate coalition formation game, and asymptotics are with respect to the number of prefectures.\(^\text{23}\) Although the model is only set identified in theory, the results show that the minimizer of the test statistic is a single point. This result is standard in the literature.

The value of $\frac{\beta_1}{\beta_0 + \beta_1}$, the value placed on public goods relative to private goods, is estimated at about 0.02. If the true efficiencies of scale were those estimated by the government, then this value would indicate that the government had selected a reasonable tax rate to serve as $\bar{\tau}$; however, since government estimates of efficiencies of scale appear to be overestimates, the interpretation of this coefficient becomes more difficult. In particular, the government specified tax rate $\bar{\tau}$ is above the optimal tax rate for almost all municipalities.\(^\text{24}\) This is consistent with observed tax rates, which show that municipalities rarely select a tax rate higher than $\bar{\tau}$.

This value of $\beta_2$ roughly implies that an individual would be willing to have a municipal policy that was 1km more distant in exchange for about ¥4000 per year.\(^\text{25}\) Using this value of $\beta_2$, if the population of Japan were uniformly distributed across the country, and a social planner could set entirely new boundaries for municipalities, then the optimum size for a municipality would be

$$
\beta_0 \log\left(\frac{y_m N_m - \beta_3 c(P_m)}{y_m N_m}\right) + \beta_1 \log\left(\frac{y_m N_m - \beta_3 c(N_m)}{c(N_m)}\right) + 0.377\beta_2 \sqrt{\frac{N_m}{340}}, \quad (41)
$$

where 0.377 is a coefficient for the average distance to the centroid based on hexagonal packing, and $P_m/340$ the area in square kilometres given the population density of Japan (340 per km\(^2\)). This formula yields an optimal municipal population of about 150,000.\(^\text{26}\) This suggests roughly 800 municipalities for all of Japan, which is about half of the current number.

The estimate for $\beta_3$ indicates that the view of the government estimates of the

---

\(^{23}\)There is one exception, involving a single municipality switching prefectures. It is treated as though the municipality in question was always part of the “destination” prefecture.

\(^{24}\)One possible explanation for this distortion might be an attempt to correct a Flatters, Henderson, and Mieszkowski [1974] “fiscal externality”. This requires the possibility of migration between municipalities, which is not included in this paper.

\(^{25}\)This calculation is complicated by the fact that the municipal tax base is not directly equivalent to GDP, 0.1 percentage point decrease in taxes does not translate into 0.1% of GDP per capita.

\(^{26}\)Estimates have changed slightly since these numbers were calculated - will recalculate.
cost of providing public goods as an estimate of the cost of providing the “national minimum” appears to be correct. There is a fixed Stone-Geary style demand for 1.05 quality units of the public good, which is not statistically different from 1 (that is, the government cost estimates are the estimate for the minimum possible public expenditure), but is statistically different from zero.

On the other hand, $\beta_4$, the degree to which the central government’s estimates of efficiencies of scale in the provision of public services match the true efficiencies of scale, is estimated to be about 0.5. This means that two null hypothesis can be rejected at very high confidence levels: that there are no efficiencies of scale in the provision of public goods ($\beta_4 = 0$), and that the efficiencies of scale in the provision of public goods are equal to the initial government estimates ($\beta_4 = 1$). The hypothesis that the revised ($\tilde{H}^\text{new}$) government estimates of efficiencies of scale are equal to the true efficiencies of scale ($\beta \simeq 0.76$) can also be rejected at the 95% confidence level (further details pending).

### 4.1 Counterfactual Policies

To determine the degree to which the moral hazard problems identified in Section 1 have actual economic significance, first, it is necessary to determine what the objective the national government was attempting to optimize. Rural prefectures are heavily overrepresented in the Japanese Diet. Thus, make the following two assumptions regarding the weight $w_m$ that the government places on individuals in municipality $m$. First, assume that weights per capita are equal for all individuals in the same prefecture, and are given by the formula

$$\log w_m = \phi_0 + \log N_m + \phi_1 R_m$$

where $R_m$ is the same indicator of “ruralness” used to classify prefectures previously: the percentage of municipalities in the prefecture with population of less than 10000. $R$ (and thus per capita weights) will thus be the same for all municipalities in the same prefecture.

Next, assume that the national government’s choice of transfers to municipalities was constrained to be within a family of transfer functions like those in Equations 15 and 16. Specifically, suppose that the government is only able to choose $\psi_0$ and $\psi_1$,.
giving it two degrees of freedom:

\[ T(X_m) = \max(\bar{c}(X_m) - .75\tau Y_m, 0) \]  

\[ c(X_m) = \psi_0 \sum_{k=1}^{24} X_{mk} \cdot \bar{c}_k \cdot (1 + \psi_0 \bar{H}_k(X_m)) + \zeta_m. \]

Here Equation 16 corresponds to the case where \( \psi_0 = 1 \) and \( \psi_1 = 1 \). Now suppose that, before the merger period, the national government had chosen the optimal policy, given its weights, under the assumption that no mergers would occur. At the optimal policy, for fixed boundaries, the national government will be maximizing the objective

\[ W(T) = \sum_m w_m v_{mm}(T_m) - b \sum_m T_m \]

where, following Equation 9, \( v_{mm}(T_m) \) is the structural component of utility for municipality \( m \), where \( m \) is not participating in any merger and is receiving transfers \( T_m \). If the policy actually selected in the pre-merger period was the optimal policy for fixed boundaries, then \( \frac{\partial W}{\partial \psi_0} = 0 \) and \( \frac{\partial W}{\partial \psi_1} = 0 \) should hold at \( \psi_0 = 1 \), \( \psi_1 = 1 \), at the true parameters \( \beta^0 \). Using \( \hat{\beta} \), estimated above, \( \hat{\phi}_0 \) and \( \hat{\phi}_1 \) can then be estimated via GMM.\(^{27}\) These estimates, \( \hat{\phi}_0 = -4.7 \), \( \hat{\phi}_1 = 4.6 \), indicate a substantial bias towards more rural prefectures in the government, which is consistent with qualitative research.

Now, using these \( \hat{\phi} \) estimates, some simple counterfactual analysis can be performed. The types of counterfactual questions that can be analyzed is limited because throughout the paper no equilibrium selection assumption has been made regarding how mergers occur. In keeping with this, in this section only comparisons made will be between the no-merger case, and the mergers that actually occurred.

First, assume that during the merger period, the national government selected the optimal transfer policy, given the true equilibrium selection rule.

First, consider the computation of how much the national government benefitted from the mergers occurring, compared to the case where there were no mergers. To determine this, take the actual transfer policy used in the merger period, \( T_{\text{merger}} \), from Equation , and calculate \( V_{\text{merger}} = \sum_{S \in \pi^*} w_{S} v_{ms}(T_{\text{merger}}(S)) \). This sum \( V_{\text{merger}} \) deliberately omits the cost of public funds, \( b \), because fiscal tightening, which suggests

\(^{27}\) Estimation of \( \beta \) and \( \phi \) should be done as a single step - hopefully this will be done soon.
an increase in $b$, was a major reason the government changed the transfer scheme and encouraged municipal mergers. Now, suppose that the national government had desired to obtain this same $V^{\text{merger}}$ without allowing any municipal mergers, but instead simply by changing the transfer policy through manipulation of $\psi_0$ and $\psi_1$. At $\hat{\beta}$ and $\hat{\phi}$, the national government’s problem can be solved via constrained non-linear optimization, yielding an optimal policy of $\psi'_0 = 0.87$ and $\psi'_1 = 1.30$. That is, the national government would cut transfers more from those municipalities that have greater efficiencies of scale. Total transfers required to reach $V^{\text{merger}}$ under the old borders are ¥260 billion higher than with the new amalgamated municipalities.\footnote{Recall that ¥100 ≃ $1.}

Now, suppose that, rather than relying on decentralized mergers, the government had simply imposed the very boundaries that were actually observed post-merger. The government could then have implemented a transfer policy very different than the one it actually chose, because there are no longer any incentive problems at the municipal level regarding potential mergers. If the government desired to match the $V^{\text{merger}}$ calculated above, in the case where mergers were mandated, the optimal policy would be $\psi''_0 = 0.92$, $\psi''_1 = 1.44$. Note that both $\psi''_0 > \psi'_0$ and $\psi''_1 > \psi'_1$, because the average distance $\ell$ within a merged municipality is higher on average, and this disutility must be offset somewhere. The $\psi''_0$ transfer policy costs ¥130 billion less than the actual policy implemented. A noticeable difference is that transfers are much greater to small and thus facing high costs per capita: as mergers are mandated in this counterfactual case, there is no need to provide these smaller municipalities with an incentive to merge. Thus, if there were no issues with incentives at all, in the situation where the national government is able to change boundaries it is at least ¥390 billion cheaper to provide a total welfare of $V^{\text{merger}}$, compared to the case where mergers are not possible.

5 Conclusion

There are multiple incentive problems at the local level regarding municipal mergers. Not only do municipalities not take into account the benefits of a merger to their merger partners, but they also do not internalize the effect of a merger on the national government’s budget. To deal with these incentive problems, the national government must choose a second-best transfer policy that provides less redistribution, in order
to provide incentives for municipalities to participate in mergers.

Empirically, the effect of these incentive problems is substantial: ¥390 billion is approximately 5% of total national government transfers through the “Local Allocation Tax” transfer scheme. This calculation relies on estimates of individual preferences over political jurisdictions, obtained using revealed preference data regarding municipal merger decisions. The coefficients estimated for these preferences imply that there are substantial efficiencies of scale in the production of local public goods, although these efficiencies of scale are not as extreme as claimed by the national government. The estimates for these parameters to the individual utility function were obtained via structural estimation, using a cooperative form political coalition formation game and a moment inequalities framework. Estimation did not rely on any equilibrium selection assumptions, and did not require enumerating the entire set of all potential coalitions.
References


<table>
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<th>Units</th>
<th>Metropolitan</th>
<th>Mixed</th>
<th>Rural</th>
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<td>361.86</td>
<td>64.42</td>
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<td>(182.15)</td>
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<td></td>
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<td>(665.12)</td>
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<td></td>
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<td>(0.54)</td>
<td>(1.06)</td>
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<td>64.64</td>
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<td>2486</td>
<td>625</td>
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Table 1: Summary Statistics by Type of Prefecture
Table 2: Dependent variable is $v_{mS}$, (structural) utility to muni $m$ from merger $S$

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<tr>
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<th>tax floor</th>
<th>no tax floor</th>
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<td>CONSUMPTION ($\beta_0$)</td>
<td>200.99**</td>
<td>98.91**</td>
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<td></td>
<td>(110.0, 440.0)</td>
<td>(60.0, 350.0)</td>
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<td>GOVERNMENT ($\beta_1$)</td>
<td>4.58**</td>
<td>2.73</td>
</tr>
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<td>(2.0, 12.0)</td>
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<td>$-0.21^*$</td>
<td>$-0.25^*$</td>
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<td></td>
<td>($-0.35, -0.02$)</td>
<td>($-0.40, 0.0$)</td>
</tr>
<tr>
<td>STONE GEARY ($\beta_3$)</td>
<td>1.05**</td>
<td>1.03**</td>
</tr>
<tr>
<td></td>
<td>(0.95, 1.13)</td>
<td>(0.94, 1.25)</td>
</tr>
<tr>
<td>EFF_OF_SCALE ($\beta_4$)</td>
<td>0.49**</td>
<td>0.55**</td>
</tr>
<tr>
<td></td>
<td>(0.38, 0.61)</td>
<td>(0.29, 0.68)</td>
</tr>
</tbody>
</table>

$N$ (prefectures) 47 47

** 95% level
* 90% level

(a, b) Extreme points for this variable in (five dimensional) 95% confidence set

tax floor: municipalities cannot charge a tax rate less than $\bar{\tau}$, the national government’s reference tax rate
Figure 1: Prefectures of Japan

![Prefectures of Japan](image1)

Figure 2: Shizuoka Prefecture

![Shizuoka Prefecture](image2)
Figure 3: Personnel Costs with only Dankai Adjustment
Figure 4: Decrease in Standard Financial Need

Decrease in Standard Financial Need Population
Change in SFN (%) −40 −30 −20 −10 0

s2001
s2002
s2003
s2004
s2005
s2006
s2007
s2008

Change in SFN (%)

Population

10^3.0 10^3.5 10^4.0 10^4.5 10^5.0 10^5.5
Figure 5: Standard Fiscal Need

Figure 6: Mergers in Shizuoka Prefecture
Figure 7:
Japanese municipalities, 1970–present

Year
Number of municipalities
500
1000
1500
2000
2500
3000
1980 1990 2000 2010
A Voting Model

The variables determined by a political process at the local level, given a certain municipality $m$, are $q_m$, $\tau_m$, and $\theta_m$. The national government chooses the transfer function $T$. If there are decentralized municipal mergers, municipalities must also decide which merger to participate in. Due to the form of the utility function in Equation 3, all individuals agree on the optimal level for the public good, $q_m^*$, and the tax rate, $\tau_m^*$. For the other political choices, however, different individuals will have different ideal points, and each of these may be multidimensional: the location $\theta_m$ of the public good is geographic (latitude and longitude), the transfer function is chosen from a space of functions which later on will be assumed to be two dimensional, and a municipality could easily have many potential merger partners. A very simple model of the political process at both the local and national level gives the result that the policy selected is a weighted sum of individual utilities. To obtain this result, make the following assumptions:

1. There are two identical office-motivated candidates that run on policy platforms that they can commit to.

2. Voting in elections is determined via a probabilistic voting model where vote probabilities are linear in utility difference between the two candidates.

3. There is a continuum of voters.

4. $\forall \theta, \theta' \forall \gamma \in (0, 1)$, the set of voters $i$ such that $u_i(\gamma \theta + (1 - \gamma)\theta') > \gamma u_i(\theta) + (1 - \gamma) u_i(\theta')$ has positive measure.

With these assumptions, the unique political equilibrium is for both candidates to propose $\theta_m^*$ to maximize the sum of individual utilities: this is Theorem 4 in Banks and Duggan [2005]. An additional requirement of Theorem 4 of Banks and Duggan [2005] is that for each voter, utility is (weakly) concave with respect to the policy choice. The utility function given in Equation 3 is concave in $\theta$, although it is not strictly concave.

29 At the national level, suppose there is inequality in voting weights between different voters. Then, for the national government, $T$ will be selected to maximize a weighted sum of individual utilities.
### A.1 Japanese Context (preliminary)

In reality, municipal politics in Japan involves both a mayor and a municipal council, and thus there is in reality more than the one decision maker supposed in the theorem; however, with the exception of about a dozen “designated municipalities” the council is elected on an entirely at-large basis, without any wards or other subdivisions. The mayor has veto power, which can be overruled by a 2/3rds vote of the municipal council. Given the lack of wards in the municipal council, it is not entirely clear how or why policies proposed by council might diverge from policies proposed by the mayor, although obviously examples of this sort of conflict can be found in municipal records. Because this paper’s focus is inter- rather than intra- municipal decision-making, the following assumption will be used: mayors will veto anything other than the policy proposed in their campaign, and less than 2/3rds of council will be opposed to this (socially optimal) policy.\(^{30}\) Thus, \(\theta^*_m\) will be set to the location of the generalized median voter.

National level politics are even more complex, and thus diverge even more from this simple model. The candidates in this case would be political parties, which commit to party platforms. Here, issues with single-member constituencies and multiple houses in the legislature, with different malapportionment per house, are abstracted away from. Election of representatives is abstracted away from: individuals in areas that are overrepresented are simply assumed to be able to cast more votes, and are thus weighted more heavily.

Assumptions 3 and 4 are not quite satisfied in the data actually used: there are a large but finite number of voters, and there are a few cases (generally in the smallest municipalities), where there are locations \(\theta\) and \(\theta'\) such that all voters are indifferent between randomization between the policies, versus a convex combination. The argument regarding Assumption 3 is simply that thousands of voters is “close enough” to a continuum. Regarding Assumption 4, violations of this assumption still result in candidates proposing policies that maximize social welfare, only these policies are no longer necessarily unique. For example, with the utility function

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30Prior to the merger period, mayors were responsible for delivering hundreds of “agency delegated functions” from higher levels of government, making them bureaucrats as well as politicians, and making it possible (at least in theory) for central ministries to fire a mayor for not performing a delegated function according to specifications. “Agency delegated functions” were abolished during the merger period, and municipal policies are thus modeled as being determined by local residents through a political process.
in Equation 3 consider a municipality with exactly half of its population at one point, and exactly half at another point. Then any \( \theta^* \) between these two points is welfare maximizing, and there is not a unique political equilibrium. Empirically, actual population distributions are never this evenly balanced, and a unique \( \theta^* \) can always be computed. Given that distance enters the utility function in Equation 3 linearly, these \( \theta^* \) are points that minimize the sum of distances, points sometimes referred to as “generalized medians”.

A.2 Municipal Merger Law (preliminary)

The general rules for municipal mergers were the following:

0. Mayors of municipalities can create “voluntary merger committees” and “study committees” to gather information, but there are no regulations regarding these committees, and they are not necessary in order to proceed with a merger.

1. A petition for a specified merger from 2% of eligible voters (or the municipal council) in any single municipality forces an official response from all the municipalities included in the proposed merger, based on a debate in their municipal councils. Unanimous “yes” responses result in the creation of an “official merger committee”. There is no requirement regarding previous voluntary committees or study committees.

2. If a municipal council rejects the proposed merger committee, a petition from 1/6th of eligible voters in the relevant municipality forces a referendum on the creation of the merger committee. A majority vote in the referendum overrides the council’s rejection.

3. The merger committee produces reports on the financial situation of the municipalities and proposes some characteristics of the merger (eg. the name of the merged municipality). A majority vote in each municipal council is required to finalize the merger.

The existence of an official referendum process during the planning stage but not at the final approval stage suggests that the best strategy for politicians opposed to a merger might have been to remain silent during the initial stage, but then prevent the final resolution from passing in council. Behaviour such as this did in fact occur

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31 This discussion ignores many details, such as the distinction between hennyuu municipalities, where bylaws are inherited from one of the merger participants (normally the largest city), and shinsetsu mergers, where bylaws and regulations are developed from scratch.
in a small number of municipalities, but does not appear to have been particularly common or successful. First, the process of creating the merger committee generally attracted a considerable amount of attention, particularly in smaller municipalities. In cases where there was controversy, referendum turnout rates could exceed 90%. It was thus difficult for politicians facing a potentially controversial merger to prevent a referendum regarding the creation of the merger committee, and conditional on that referendum passing it was difficult to then vote against the final proposed merger. Furthermore, in cases where politicians did vote against mergers that appeared to have popular support, a hitherto seldom used recall process was employed to remove them from office via a majority vote in a recall referendum. Whereas there was only one recall referendum during the 1990s, there were at least 41 during the merger period.

A formal interpretation of these rules is somewhat difficult; however, a common element in all mergers is that they were approved by all municipalities in question, either via local referendum, or in the municipal council. As council resolutions were subject to veto by the mayor, this paper will assume that the binding constraint on the behaviour of a municipality is the ability of its residents to recall the mayor. Suppose that there is perfect information regarding what mergers are feasible (i.e. will be approved by all other participants). The mayor proposes a merger for the municipality to participate in, or proposes remaining independent. A single challenger then appears, and similarly proposes a policy. If the policies proposed are the same, the incumbent mayor remains in office; if the challenger’s policy is preferred, then the mayor is replaced. Once again referring to Appendix A, given the assumptions there the resulting policy will be socially optimal from the perspective of municipal residents.

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32 In about a third of cases, referenda were held. Most of these were nominally consultative, but there is only one instance in which a municipal council voted opposite to a referendum result. This case was complicated due to multiple referenda with conflicting results as well as a number of other procedural irregularities, and finally resulted in a recall of the mayor and a request to the prefectural governor to reverse the merger. The request for reversal was denied.

33 The assumption here that mayors do not have a large effect on mergers might still seem suspicious. Kawaura [2010] investigates the effect of a mayors length of tenure on merger configurations, and finds effects that are small and not statistically significant at the 95% level. While there is certainly anecdotal evidence that certain mayors may have obstructed certain mergers, there is no immediately obvious relationship in the aggregate data. The private incentive for municipal politicians to maintain the independence of their municipality in order to preserve their own employment is not as strong as might be anticipated. This is due to central government policies: for example, the length of service required to receive a pension were reduced for politicians in a municipality.
A potential objection here is that the costs of organizing a recall election could be large, and thus the mayor’s incumbency advantage significant enough to allow merger proposals far from the optimal to be enacted. There are two responses to this objection: first, the merger period was sufficiently long that at least one regularly scheduled election occurred during the merger period, and during this election the merger issue was particularly salient; second, the cost of organizing a recall does not appear to be as large as might be supposed. Specifically, in 4 of the 41 recalls, a majority voted against the recall in the referendum, and in another 6 of the recalls, the mayor was re-elected in the special election following the recall process (usually after resigning voluntarily to avoid the recall referendum). Thus, a full quarter of the organized recall referenda did not succeed in removing the mayor. If the costs of organizing a recall referendum were very high, one would expect that they would be organized only when the mayor would not have majority support in the recall referendum or the subsequent election. Thus, this paper will use the assumption that the municipal merger selected by each municipality was socially optimal for that municipality, given the other alternative mergers that were feasible.

B Stability Concept

Suppose that player \( m \in M \) has preferences \( \succeq_m \) defined over the set \( \{ S \subset M | m \in S \} \), with \( \prec_m \) indicating a strict preference. Extend these preferences to partitions in the following way: if \( \pi(m) \) is the coalition that municipality \( m \) belongs to in partition \( \pi \), then \( \pi \succeq_m \pi' \) if \( \pi(m) \succeq_m \pi'(m) \). Let \( \pi \prec_S \pi' \) for some coalition \( S \) if \( \forall m \in S, \pi \prec_m \pi' \).

The solution set is defined using the von Neumann and Morgenstern [1944] “stable set”. Although the VNM stable set was originally defined in terms of imputations rather than coalition structures, this paper follows Lars [2007] in defining the stable set over coalition structures. Specifically, the von Neumann-Morgenstern solution requires that (i) no coalition structure in the stable set be dominated by another coalition structure in the set, and that (ii) any coalition structure outside of the set is dominated by a coalition structure belonging to the set.

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34 A recall referendum required a petition by between 1/6th and 1/3rd of residents.

35 A formal definition of “feasible” will be given in Section 1.
Definition 1 (Lars 2007). Let $<$ be a dominance operator, and $\Pi^{VNM} \subseteq \Pi$. Then $\Pi^{VNM}$ is called a stable set for $(\Pi, <)$ if the following two properties hold:

1. $\forall \pi, \pi' \in \Pi^{VNM}, \pi \not< \pi'$. (Internal stability)
2. $\forall \pi \not\in \Pi^{VNM}, \exists \pi' \in \Pi^{VNM}$ where $\pi < \pi'$. (External stability)

Ray and Vohra [1997] only allow deviating coalitions to force refinements of a partition, and Diamantoudi and Xue [2007] show that this creates a stable set. The hedonic game considered in this paper is simpler than the “equilibrium coalition structures” that Ray and Vohra examine, and thus in this paper both refinements and coarsenings will be allowed. Otherwise, the theory follows that presented in Ray and Vohra. Let $\pi \nearrow S \pi'$ and $\pi \searrow S \pi'$ mean that $\pi \prec_S \pi'$, $S \in \pi'$, where $\pi'$ is a coarsening and a refinement of $\pi$, respectively. Using the terminology of Ray and Vohra, $\pi$ is blocked by $\pi'$ if either there is a set of coalitions in $\pi$ that are unanimously in favour of merging to create $\pi'$, or there is a subset of “perpetrators” in $\pi$ that are unanimously in favour of deviating from their current coalition. In the former case, $\pi'$ is the coarsening that results from the merger, while in the latter it is a refinement that includes a coalition for these perpetrators and some arrangement of the “residual” left behind when the perpetrators deviated, such that the configuration of perpetrators and residual is stable. More formally, where $\rightarrow$ should be read as “blocked by”:

Definition 2. $\pi \rightarrow \pi'$ if $\exists S$ such that either $\pi \nearrow S \pi'$ or $\pi \searrow S \pi'$, where

1. $\pi \nearrow S \pi'$ if $\pi' \setminus \pi = S$ such that $\pi \prec_S \pi'$, and
   a) $S = \bigcup Q$ for some $Q \subset \pi$, and
   b) $\exists S' \subset S$ such that $\pi' \setminus \downarrow_S S'$.
2. $\pi \searrow S \pi'$ if $\exists S \in \pi'$ such that $\pi \prec_S \pi'$, and
   a) $\pi \setminus \pi' = S'$ with $S' = \bigcup Q'$ for some $Q' \subset \pi'$, and
   b) $\exists Q$ such that $Q' \rightarrow Q$.

An alternative approach would be to allow only single player deviations, as in Greenberg [1979]. Ray and Vohra [1997] is used instead because anecdotal evidence suggests that multi-player deviations involving a refinement or a coarsening were more common than single player deviations not to a refinement or a coarsening during the coalition formation process.
The recursion is well defined since $Q'$ is a proper subset of $\pi'$. Now let $\rightarrow$ be the transitive closure of $\rightarrow$. Assume that $\Pi \neq \emptyset$.

**Proposition 1.** $\Pi^* = \{\pi | \exists \pi' \text{ such that } \pi \rightarrow \pi'\}$ is a stable set with respect to $(\Pi, \rightarrow)$.

**Proof.** By construction, $\Pi^*$ is internally stable. Now take some $\pi \notin \Pi^*$. Then $\exists \{\pi_1, \ldots, \pi_n\} \subset \Pi$ such that $\pi \rightarrow \pi_1 \rightarrow \cdots \rightarrow \pi_n$ and either $\pi_n \in \Pi^*$ or there is a cycle with $\pi_n = \pi_l$ for some $l < n$. If there is such a cycle, then it must contain both mergers and dissolutions. However, such a cycle cannot exist because $\nearrow$ is defined such that there are no refinements. $\square$

The proof of Theorem 1 in the main text is then very straightforward:

- **(existence).** Immediate by the above definition of $\Pi^*$. $\square$
- **(non-emptiness).** If $\Pi \setminus \Pi^* = \emptyset$ then $\Pi^*$ is not empty because $\Pi$ is assumed not to be empty. If $\Pi \setminus \Pi^* \neq \emptyset$ then $\Pi^*$ is not empty because external stability was shown in the proof of Proposition 1. $\square$
- **(uniqueness).** Suppose that $\Pi^{**}$ is also a stable set with respect to $(\Pi, \rightarrow)$. Consider the bipartite directed graph defined by $\rightarrow$ with $\Pi^{**} \setminus \Pi^*$ and $\Pi^* \setminus \Pi^{**}$ as the two sets of nodes. Every node must have in-degree of at least one, but there can be no cycles. The only such graph is empty, and thus $\Pi^{**} = \Pi^*$. $\square$

It can also be shown that $\Pi^*$ contains a Pareto optimal partition:

- **(PO element).** Let $\Pi^{PO} \subset \Pi$ be the set of Pareto optimal partitions, and $\leadsto$ the Pareto dominance operator. Suppose that $\Pi^{PO} \cap \Pi^* = \emptyset$ and consider the directed graph defined by $\rightarrow$ with $\Pi^{**} \setminus \Pi^*$ and $\Pi^* \setminus \Pi^{**}$ as two sets of nodes. A cycle must exist, because $\forall \pi \in \Pi^{PO}, \exists \pi' \in \Pi^*$ such that $\pi \leadsto \pi'$, but at the same time $\forall \pi \in \Pi^*, \exists \pi' \in \Pi^{PO}$ such that $\pi \leadsto \pi'$. Choose the starting point in this cycle such that $\pi_0 \leadsto \pi_1 \leadsto \cdots \leadsto \pi_n = \pi_0$. Let $S_1^+$ be the set of agents that strictly prefer $\pi_1$ to $\pi_0$. It cannot be that $\pi_1 \not\nearrow \pi_2$ because this is also a Pareto improvement. Thus $\pi_1 \not\in S_2^+$, and $S_2^+ = (S_1^+ \setminus R) \cup P$ where $R$ is some subset of the residual, and $P \neq \emptyset$ is some

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37That is, $\pi \rightarrow \pi'$ if either $\pi \rightarrow \pi'$ or $\exists \{\pi_1, \ldots, \pi_n\}$ where $\pi \rightarrow \pi_1 \rightarrow \cdots \rightarrow \pi_n \rightarrow \pi'$. To see why the transitive closure is used here, consider the case where $\pi_1 \not\in S_2^+ \nearrow \pi_3$. $\pi_1$ and $\pi_2$ should not be in the stable set, while $\pi_3$ should, but $\{\pi_3\}$ is not a VNM stable set with respect to $\rightarrow$ because $\pi_1 \not\rightarrow \pi_3$.

38To see this, attempt to iteratively construct a non-empty graph that has the desired form.
subset of the perpetrators, and \((R \cup P) \subset S'\). Since \(S_n^+ = \emptyset\), at some point the agents in \(S_2^+\) must be made worse off. This can only happen via refinements, and only if there is a residual smaller than \(S_2^+\). The latter, though, implies that either some subset of \(S_2^+\) cannot be made worse off, or that \(S_3^+\) will contain some new element. Thus, \(S^+\) can never be empty. Thus a cycle cannot exist, and there is some Pareto optimal element in \(\Pi^*\).

All partitions in \(\Pi^*\), including those that are not Pareto optimal, are treated equally, since imposing additional restrictions at this stage would mean that the solution set would no longer be the outcome of the cooperative game coalition formation process described above.\(^{39}\)

### C Data and Institutional Details (very preliminary)

Population data comes from the 1995 Japanese national census, which provides data at the kilometer grid square level. Information on municipal boundaries is taken from shape files produced by ESRI Japan, also for 1995. By combining these two data sources, the location of individuals in municipalities can be known to the kilometer grid square level.

To calculate distances, first, the population of grid squares that are on a boundary between two municipalities is divided between the municipalities in proportion to the area of each grid square in each municipality. Then, for any \(\theta_m\), the distance \(\ell(i, \theta_m)\) is calculated as the great-circle distance from the physical longitude-latitude location of individual \(i\) to \(\theta_m\). For computational simplicity, all individuals within a given census grid square are assumed to live at the centre of the square. The distances in question are small relative to the curvature of the earth, so this is effectively a straight-line distance calculation.

The location \(\theta^*_m\) chosen by a municipality will minimize the sum of these individual distances, due to the assumption that the local political process is as described in Appendix A. These \(\theta^*_m\) are calculated via standard optimization techniques. Although, as discussed in Appendix A, there are cases where \(\theta^*_m\) might not be unique,

\(^{39}\)There may be some “solutions” that seem particularly unattractive: \(\{\pi \in \Pi^* | \exists \pi' \in \Pi^*, \pi \not\sim \pi'\}\). While the theory above could likely be rewritten to shrink the stable set, eliminating these elements, it would be computationally infeasible to use any of these new restrictions in the empirical section, as they would require enumerating the entire stable set.
a unique value is in fact obtained for all municipalities. For each coalition \( S \), the optimal location \( \theta^*_S \) is calculated via exactly the same process. The value of \( \ell_m(\theta^*_S) \) in Equation 9 can then be calculated by averaging over distances \( \ell(i, \theta^*_S) \) for all individuals in \( m \). This process is computationally intensive, but \( \ell_m(\theta^*_S) \) depends neither on \( \epsilon \) nor on \( \beta \), and thus the calculation of these distances only needs to be performed once.

Data for municipal characteristics \( X_m \) comes from the Statistical Information Institute, which aggregates a variety of government sources. Where 1995 data was not available, the year closest to 1995 was used. Municipal financial information was obtained from the Ministry of Internal Affairs and Communications.

The unit costs \( c_k \) and adjustment coefficients \( \tilde{H}_k \) were more challenging to obtain, both due to the complexity of the formulae and the fact that some of the data used in the calculations is not publicly available.

Discussions with Ministry officials confirmed that formulae for \( \tilde{H}_k \) are determined by the expert opinion of Ministry officers, and are not created directly via a regression of municipal characteristics on previous municipal spending, nor by applying a specific set of a priori assumptions regarding returns to scale.\(^{40}\) First, \( c_k \) is set by considering the cost of providing component good \( k \) for a reference municipality: a city with population of 100,000, surface area of 160km\(^2\), and other standard characteristics. The number and type of local bureaucrats necessary to provide the service is then estimated, along with the cost of equipment and materials, plus any transfers to the relevant target population (e.g. child benefit payments). The number and type of bureaucrats that smaller and larger municipalities would require to provide the same level of service is then estimated.\(^{41}\) National Personnel Authority salary scales are then used to convert employee numbers to a total wage bill, which is added to an adjusted estimate for equipment and materials. By definition there are no economies of scale with respect to transfers to individuals, since the same level of service would imply the same level of transfers in the cases where there are transfer payments. The 2009 version of the exposition of these formulae (the Chihō Kōfuzei Seido Kaisetsu),

\(^{40}\)According to MIC officials, however, each year estimates are modified based on formal and informal feedback from municipalities and prefectures, observed spending patterns, and in-house research.

\(^{41}\)The sizes at which these estimates are performed varies slightly from year to year and from service to service, but in recent years estimates have generally been produced for populations of 4,000, 8,000, 12,000, 20,000, and 30,000 for municipalities below the reference size, and at 250,000, 400,000, 1,000,000, and 2,000,000 for municipalities above the reference size.
consists of 600 pages of Japanese legal text, 460 pages of formulae, and 240 pages of reference values.

The official government formula for the calculation of $\tilde{c}(X_m)$ is

$$\tilde{c}(X_m) = \sum_{k=1}^{24} X_{mk} \cdot \tilde{c}_k (1 + \tilde{H}_k(X_m)).$$  \hfill (46)

However, one pattern frequently observed is that $\tilde{H}_k$ takes the form

$$\tilde{H}_k(X_m) = \prod_{j \in J_1} \tilde{H}_j^1(X_{mj}) + \frac{1}{X_{mk}\tilde{c}_k} \sum_{j \in J_2} \tilde{H}_j^2(X_{mj}).$$  \hfill (47)

The total number of available “adjustment coefficients” available in $J_1 \cup J_2$ is 15, but all 15 are never used for the same component good $k$. One interesting feature here is that the “adjustments” based on characteristics in $J_2$ do not actually depend on the unit cost that they are supposedly adjusting, due to the division by $X_{mk}\tilde{c}_k$. Thus, de facto, the method for calculating $\tilde{c}(X_m)$ is

$$\tilde{c}(X_m) = \sum_{k=1}^{24} X_{mk} \cdot \tilde{c}_k (1 + \tilde{H}_k(X_m)) + \zeta_m,$$  \hfill (48)

where

$$\tilde{H}_k(X_m) = \prod_{j \in J_1} \tilde{H}_j^1(X_m),$$  \hfill (49)

which is Equation 16. Of the adjustment coefficients in $J_1$, by far the most important is the dankai (literally “step” or “grade”) adjustment, which is based on the scale of the service provision. The dankai adjustment is generally based on the total number of residents, but in some cases the relevant subgroup may be considered instead: the adjustment for services to the elderly is based on the number of residents over 65, the adjustment for agricultural services is based on the number of farmers, and so forth. This adjustment is substantial, with the per capita cost of providing services usually estimated to be 2 to 3 times higher for a municipality of 4000 people than one of 100000. Dankai adjustments for some important services are shown in Figure 3.

Ministry calculations of $\tilde{c}_k$ and $\tilde{H}_k$ are subject to two types of outside interference. First, the amount of transfers allocated needs to somehow match the budget agreed upon with the Finance Ministry. This is accomplished by modifying capital spend-
ing estimates, with the result that official municipal capital spending “needs” vary radically from year to year; estimates of the non-capital spending required to provide municipal services, on the other hand, change very little. This sort of variation is captured in the model presented in Section 1 through a change in $b$, the cost of public funds. A second sort of interference comes from politicians, as well as line ministries such as the Construction Ministry, and involves pressure to promote spending on local projects. Over time, this has resulted in the addition of numerous “project” adjustment coefficients, each providing a special incentive for a variety of public works project. DeWit [2002] describes the history of this interference, which makes it clear that government estimates of capital spending requirements are not closely related to actual costs. This conclusion is supported by actual capital spending patterns, which are not at all close to government estimates. This sort of variation is captured in the model presented in Section 1 through a $\beta_4$ that is less than 1, indicating that the government is exaggerating expenses. The idea that a local government might be forced by the national government to spend money on public services that it does not want is captured through a tax floor at $\tau$, one of the specifications estimated in Section 3.

The determination of $\mathcal{S}$, the set of potential alternative mergers, is problematic, but so long as mergers that could never have occurred are not accidentally included, estimates should not be biased. There are a number of large mergers observed, with the largest involving fifteen municipalities. Almost all observed mergers are geographically contiguous. However, even after restricting $\mathcal{S}$ to contiguous coalitions of size fifteen or less, there are still over $10^{16}$ possibilities. Most of these coalitions, however, look very different than the actually observed coalitions. In particular, they tend to be a thin line of municipalities, stretching almost all the way across a pre-

\[42\] Occasionally modifications are also made by adding additional expense categories. These are distinguishable from the usual expense categories by their placement at the end the list of expenses, their short lifespan, and their non-specific names. The usual expense categories have remained effectively unchanged since at least 1968.

\[43\] More specifically, there are thirteen observed mergers that are not geographically contiguous, usually because one of the participants dropped out late in the merger process. Islands with only a single municipality on them are treated as being connected to the closest municipality on the “mainland” (i.e. Hokkaidō, Honshū, Shikoku, or Kyūshū) if it is within 50km. There are, however, two cases in which municipalities on an island merged with municipalities on the mainland other than the closest one. There are also six cases where municipalities on two separate islands merged together. Thus, about 3.5% of mergers (21/588) are not contiguous. No additional mergers violating contiguity are generated as comparison coalitions, although the mergers did occur are retained in the observed partitions, and may also appear in alternative partitions.
fecture. On average, individuals in these coalitions would have very high distance $\ell$, and the coalitions are thus not likely to form.

The tax base $Y_m$ is determined from “Standard Fiscal Revenue” figures produced by the Ministry. While the model in Section 1 has each municipality choosing a single tax rate $\tau_m$, actual municipal tax revenues come from several taxes, with “fixed asset” taxes (land, housing, and some business assets) and personal income tax being the two most important types. For each type of tax, the Ministry sets a “reference” tax rate, and then calculates the total revenue each municipality would receive if it charged these reference rates on its tax base. That is, if $\bar{\tau}^k$ is the reference rate for tax type $k$, and $Y_m^k$ the tax base for this tax for municipality $m$, the Ministry “Standard Fiscal Revenue” estimate for municipality $m$ is $\sum_k \bar{\tau}^k Y_m^k$. To convert this to the single tax base that is assumed in this paper, suppose that the single tax base is income, and set $\bar{\tau} = .12$, which is total municipal Standard Fiscal Revenue as a fraction of total income. Then calculate $Y_m$ for each municipality so as to satisfy

$$\bar{\tau} Y_m = \sum_k \bar{\tau}^k Y_m^k.$$  (50)

That is, $Y_m$ is calculated so that $\bar{\tau} Y_m$ is exactly equal to the Standard Fiscal Revenue for that municipality, as reported by the Ministry. The tax rate actually observed in the municipality, $\tau_m^*$, is defined as taxes as a fraction of $Y_m$. In general, this collapsing of multiple tax bases to a single tax base would be extremely problematic, but in the Japanese case, although municipalities are de jure allowed to choose a tax rate different from the standard rate, the amount of actual variation is very low. For example, in the extreme case of Yuubari City, effectively bankrupt with a debt of over ¥3 million per capita, the income tax rate was raised from 6.0% to 6.5%, but almost all other municipalities charge the standard 6.0%. The standard fixed asset rate of 1.4% is levied by about nine out of ten municipalities, with the remaining tenth mostly charging 1.5% or 1.6%. Thus, the observed tax data that the model is attempting to explain involves all municipalities effectively charging identical tax rates, equal to the Ministry’s reference rate $\bar{\tau}$. In particular, there are no cases where

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44 A fixed asset rate of more than 1.7% requires Ministry approval, but few municipalities are at this cap. While there are Ministry caps on taxes, these are rarely binding. The sole exception is for taxes on corporations, where a sizeable number of municipalities do charge at the upper bound. These corporate taxes are a small percentage of total taxes, and thus this issues with this upper bound are not considered in this paper.
a municipality chose to charge a very high rate on a particular tax base for which the reference rate was much lower, a situation which could lead to high and nonsensical calculated values for $\tau^m$.

... Another feature of the system of public goods provision in Japan is that most public goods that generate substantial externalities appear to be provided by higher levels of government, rather than by municipalities. For example, waterways and major roads are the responsibility of prefectures. Obviously, there will still be some externalities from locally provided services: visitors driving to a home in the municipality will be driving on local roads, and the person who invited them may not internalize the benefit of higher quality local roads to their visitors. Similarly, it might be possible for residents of another municipality to enter a library in order to sit and read books, although this depends on how access is controlled.\textsuperscript{45} Continuing in this vein, it would be theoretically possible, although illegal, to dump garbage at a collection point in another municipality. Schooling is an interesting case, because the elements of the elementary and middle schools that are under municipal control (physical plant, food service, school buses/boats, etc.) are precisely those that seem less likely to generate large externalities. Teachers are hired by the prefecture, and the curriculum determined by the national Ministry of Education. While it may not be plausible to claim that there are absolutely no externalities, it seems unlikely that externalities played a major role in municipal decision-making regarding potential mergers and they are thus not included in the model.\textsuperscript{46}

... A final restriction imposed by the model described in this section is that individuals do not move or otherwise change their ideal point and population is assumed to be constant. In reality, residence choice is endogenous to government characteristics, as discussed in the literature established by Tiebout [1956] and others. A more complicated model could be incorporated in order to reflect this endogeneity, but is not for three reasons. First, the majority of Japanese households own their own home:

\textsuperscript{45}One example of library access control is at the central Tsurumai library in Nagoya. A seat chit must specifically be requested, with high school students (and textbooks) banned from the above ground air-conditioned floors.

\textsuperscript{46}The earlier Showa municipal mergers in the 1960s were determined more centrally using prefectural committees. As those committees should have internalized externalities across municipal boundaries, an ongoing project comparing the Heisei and Showa mergers may be able to offer quantitative evidence of the importance of externalities in the determination of municipal boundaries.
if all households have preferences similar those given in Equation 3, then (at least in some simple models) homeowners would vote over mergers so as to maximize land value and then choose a new location based on post-merger land prices and other municipal characteristics. The case where all households own their own home thus has similar implications but a much more complicated setup compared to the case where there is no mobility. Second, observed mobility is lower in Japan than in most other developed countries and a large portion of the inter-municipality moves reported in the census appear to be temporary. Endogenous relocation is thus less of a concern than in other countries. Third, there is no evidence of tax competition. The majority of municipalities charge a standard tax rate and even though municipalities are allowed to set a different rate (within a band), few choose to exercise this option. This is consistent with the model presented above, and combined with the national government transfer scheme results in the endogeneity problem being less severe than it would be in other contexts.  

D Computational Details and Examples

D.1 Example with two municipalities per prefecture

Let $J$ be a set of prefectures each containing only two municipalities, $A_j$ and $B_j$, with a potential merger $S_j = \{A_j, B_j\}$. For simplicity, let there be only a single idiosyncratic shock $\epsilon_j$ in each prefecture:

$$u_{A_j A_j} = u_{B_j B_j} = \beta$$  \hspace{1cm} (51)

$$u_{A_j S_j} = u_{B_j S_j} = 2\beta + \epsilon_j$$  \hspace{1cm} (52)

From an implementation perspective, endogeneity would result in future population and other characteristics of a municipal merger depending on what other mergers occurred in the surrounding area. This would change the nature of the coalition formation game from a characteristic function game to a partition function game, which is substantially more computationally intensive to estimate, and likely infeasible without further theoretical or technological developments. Population dynamics are thus ignored, with the estimates that follow focusing on a single period game with player characteristics determined based on current government data sources. Exogenously changing population is also a concern, but similar results were obtained when 2005 census data is used instead of the 1995 data currently used. Predicted future municipal population could be created based on census data, but the similarity of results obtained with data from different census years suggests that this exercise may not produce particularly interesting results.
and make the distributional assumption $\epsilon_j \sim N(0, 1)$. Here, setting the variance to one normalizes the scale for $\beta$. Let $\pi^0$ be the observed partition: the only options for prefecture $j$ are the singletons $\{A_j\}$ and $\{B_j\}$, or the merger $\{A_j, B_j\}$. Define the following stability restriction:

$$R(\epsilon, \pi|\beta) : \forall j \in J, \quad \epsilon_j \leq -\beta \text{ if } \{A_j\}, \{B_j\} \in \pi$$

$$\epsilon_j \geq -\beta \text{ if } \{A_j, B_j\} \in \pi$$

Now choose $h(\epsilon) = \sum_{j \in J} \epsilon_j^2$. Thus, $E(h) = J$. Then define

$$\epsilon^*(\beta) = \arg\min_{\epsilon} h(\epsilon) \quad \text{s.t. } R(\epsilon, \pi^0|\beta).$$

That is, $\epsilon^*(\beta)$ is the vector of idiosyncratic shocks that generate the least extreme value from $h$ while still rationalizing $\pi^0$. Let that value of $h$ be

$$h^*(\beta) = \min_{\epsilon} h(\epsilon) \quad \text{s.t. } R(\epsilon, \pi^0|\beta).$$

Let $\epsilon^0$ be the actual epsilons that were drawn and resulted in $\pi^0$ being observed. Let $\beta^0$ be the true value of $\beta$. Then

$$h^*(\beta^0) \leq h(\epsilon^0)$$

because $\epsilon^0$ being drawn resulted in partition $\pi^0$ occurring, and thus $R(\epsilon^0, \pi^0|\beta^0)$ must be satisfied. If the inequality is always satisfied, it is satisfied in expectation:

$$E(h(\epsilon)) - E(h^*(\beta^0)) \geq 0,$$

where $E(h^*(\beta^0))$ indicates the expected value of $h^*$ for a partition generated from a random draw of $\epsilon$. In this particular example, for any draw of $\epsilon$ there will only be one stable partition, but neither this uniqueness nor any particular assumptions regarding an equilibrium selection rule is required for the above inequalities to hold.

The expected fraction of prefectures with a merger is $\Phi(\beta^0)$. The expected value of $h^*(\beta)$ is

$$\Phi(\beta^0) \min(0, \beta)^2 + (1 - \Phi(\beta^0)) \max(0, \beta)^2$$

Using only the above moment inequality for identification, we will have the iden-
tified set

\[ \{ \beta \mid 1 - \Phi(\beta^0) \min(0, \beta)^2 + (1 - \Phi(\beta^0)) \max(0, \beta)^2 \geq 0 \} \]  

which corresponds to the interval \([\frac{-1}{\sqrt{\Phi(\beta^0)}}, \frac{1}{\sqrt{1 - \Phi(\beta^0)}}]\). This interval contains zero, which is a general property of this type of moment: the “entirely idiosyncratic” null hypothesis will never be rejected. In order to reject \(\hat{\beta} = 0\), some additional moments of some other type must be used. In the estimator in the main paper, these correspond to the moments comparing the expected number of mergers if the government had not changed any transfer policies to the actual number of mergers observed during the period in which the old transfer policies were in effect. Considering the specification used in the main analysis, at \(\hat{\beta} = 0\), there would have been a large number of “random” mergers, and thus this null hypothesis is easy to reject using these other moments.\(^{48}\)

\[ h_\omega(\bar{\omega}, X) \geq 0 \text{ and } h_s(s, X) \geq 0 \text{ that} \]

\(^{48}\)The above estimator may appear to be somewhat similar to other estimation approaches, such as maximizing the probability that the observed partition is stable. Estimators in this latter set, however, are not in general consistent, and will thus not necessarily be inside the identified set based on the moment inequality used above. To take an extreme example, suppose that there are \(K\) municipalities in each prefecture, and that the only mergers that are possible are \(m_1; m_2; m_3; \ldots; m_K\). Preferences are determined by

\[ u_{mk} = k - K - 1 \]  

\[ u_{mk,S} = \frac{\#S}{K} + \sqrt{K} \epsilon_j, \quad S \neq \{m_k\} \]  

\[ u_{mk,m_k} = k - \frac{K - 1}{2} \]  

where \(\#S\) is the number of municipalities in \(S\), and \(j\) indexes prefectures. Thus there is again only one idiosyncratic shock per prefecture, and only one stable partition: if \(\epsilon = 0\), for example, municipalities up to \(K/2\) will merge. The probability of any given merger being stable is thus small if \(K\) is large. Now consider the more general model

\[ u_{mk,S} = (1 - \beta)(\frac{\#S}{K} + K^{1/3} \epsilon_j) + \beta(k - \frac{K + 10}{2} + \epsilon_j^2), \quad S \neq \{m_k\} \]  

\[ u_{mk,m_k} = k - \frac{K - 1}{2} \]

When \(\beta = 1\), then, most of the time no mergers are stable, but when there is a very extreme \(\epsilon\) then all mergers are stable (indifferent municipalities prevent deviations in the definition of stability used in this paper). Now suppose that the true value of \(\beta\) is \(\beta^0 = 0\), and consider a pseudo-likelihood estimator that maximizes the probability that the observed partition is stable. Then for sufficiently large \(K\), \(\beta^{\text{PML}} = 1\), because all mergers are stable when any merger is stable, unlike the situation at \(\beta^0\). The identified set using the technique outlined above, however, would exclude \(\hat{\beta} = 1\), as this would involve \(\epsilon^*\) that are very extreme, and the moment inequality would be violated. Thus, the (inconsistent) maximizer of the pseudo-likelihood function based only on stability is not always contained in the identified set.
Example 5. Let $A$ and $B$ be disjoint coalitions, and consider $S = A \cup B$. Then

$$\bar{\omega}_S = \frac{N_A \bar{\omega}_A + N_B \bar{\omega}_B}{N_A + N_B}$$

$$s_S^2 = \frac{(N_A - 1)s_A^2 + (N_B - 1)s_B^2 + N_A N_B (\bar{\omega}_A - \bar{\omega}_B)^2}{N_A + N_B - 1}.$$ 

(D.2 Computational Details)

$h^*$ is difficult to compute, so instead use $h^{**}$, which is easier to calculate.