YOLO: Can Subjective Life-Expectancies Explain Household Investment Puzzles?

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Abstract

This paper introduces the emergent concept of *subjective* life-expectancies into the discussion of life-cycle consumption, savings, and portfolio choice. We present new evidence that individuals overestimate their mortality at short horizons and rate of survival at long horizons. For example, a 28 year old male with a 99.4% chance of surviving beyond 5 years believes he will do so with 92.8% probability. A 68 year old with a 71.4% probability of living to 78, believes he has a 82.4% chance of living that long. These findings provide the basis for a unified, empirically grounded explanation for seemingly disconnected puzzles. Relative to a benchmark life-cycle model, these expectation errors generate over-consumption and under-saving when young, and under-consumption during retirement. In addition, for reasonable levels of risk-tolerance, the required excess rate of return on equity is not too high once subjective beliefs are accounted for.

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1 Introduction

Economists and policy-makers are puzzled by the consumption and retirement savings behavior of individuals over the life-cycle. Most people save too little relative to what traditional models predict they should accrue towards retirement.[1] Upon retiring, individuals save more than is necessary given their assets and consumption.[2] These findings appear to be contradictory. While researchers have offered many appealing explanations for one regularity or the other, it is not clear that any mechanism is well-suited to jointly describe savings and consumption behavior at opposite ends of the life-cycle.[3]

A further challenge to the life-cycle model comes from efforts to reproduce the behavior of financial decision-makers, as well as observed characteristics of asset prices. Famously, while estimates of the historical return on equity have ranged between 3 - 7 percent, this premium is far too high to replicate observed portfolio choice and asset allocation decisions.given elicited levels of risk-aversion (Mehra and Prescott (1985)). Accordingly, a promising direction for the broader portfolio choice literature is the incorporation of probability weights that are too favorable toward low-likelihood events (Carlson and Lazrak (2014)).

We bring together the literature on all three of these seemingly disconnected regularities, by adopting the concept of *subjective* life-expectancies – an individual's beliefs about their likelihood of survival to and beyond a given age, henceforth SLE. SLE is a natural vantage point for studying these questions, as our subjective beliefs should bear directly on savings and consumption decisions. In fact, from a practical standpoint, conditional life-expectancies or SLEs are frequently advertised in the sale of life-cycle financial instruments, such as annuities and life-insurance.[4] At the same time, household surveys used by

[1]For example, see Laibson (1997).

[2]For example, see Nardi et al. (2010).

[3]One notable difference is the theoretical work of Groneck et al. (2013) which combines ambiguity in beliefs with a learning model.

[4]http://www.forbes.com/sites/ashleaebeling/2012/08/10/americans-clueless-about-life-expectancy-bungling-retirement-planning/

economists often publish average expected lifespan, a figure for which the principle advantage is ease of comprehension.

The first part of the paper, motivated by these considerations, presents new survey evidence on subjective life-expectancies.[5] The survey uses recent advances in belief-elicitation technologies to build a comprehensive picture of survival beliefs over the entire life-cycle. This introduces a critical new finding to the economics literature: the SLE distribution is heavy-tailed in comparison to actuarial tables provided by the Social Security Administration (SSA).[6] That is, young individuals believe their chances of survival beyond the next few years are lower than that predicted by actual data, while respondents around the age of retirement believe there is a reasonable chance of surviving to extreme old age. The expectation errors are quantitatively large, often as much as a 10 percentage points over or under-estimation. For example, a 28 year old makes between a 5 and 10 percentage point one-year ahead forecasting error while a 68 year old makes roughly a 12 percentage point ten-year ahead forecasting error. This is consistent with early findings in Hamermesh (1985), but provides evidence for the complete age range and uses a more representative sample.

We find direct evidence that these errors in subjective survival beliefs matter for key aspects of financial decision-making. Increased (decreased) expectation errors are associated with an increase (decrease) in the perceived likelihood of using one's savings more quickly, even after accounting for the respondent's age. Similarly, large subjective survival belief errors are associated with a greater propensity to not save or even an over-reliance on credit cards on a month-to-month basis. These same expectation errors are associated with a decreased tendency to describe oneself as having experience investing.

In fact, since these correlations suggest that subjective survival beliefs play an important role in determining attitudes towards savings, such evidence justifies embedding these

[5]Manski (2004) argues for the importance of subjective expectations of significant life events, elicited from survey responses.

[6]Jarnebrant and Myrseth (2013) finds evidence of a similar distribution.

subjective expectations into a canonical dynamic life-cycle model and studying the implications. We find that relative to a calibration using actual life-expectancies from the SSA, the life-cycle model with SLEs predict consumption and savings behavior that accords to the three empirical regularities we outline. On average, consumption pre-retirement increases and savings fall by 10 percent relative to the benchmark. The beliefs of some consumers – even those with high-incomes – are strong enough to elicit hand-to-mouth consumption as in Campbell and Mankiw (1989). Expectation errors persist into retirement, but flip directionally. Individuals believe they have a high likelihood of surviving many additional years. Owing to this belief, individuals consume 12 percent less during their retirement. This mechanism is independent from any income differences that may exist when entering retirement age.

In addition, our model allows individuals to allocate their savings to shares of a risky or risk-less asset. Risk-tolerance plays an essential role in asset allocation, but the expected time-horizon for which to consume out of savings is equally important. SLEs matter because they influence these expectations. We find that – in comparison to a benchmark parametrization – the return on the risky-asset has to increase by a multiple of 1.5 to achieve equivalent levels of wealth-accumulation. That is, in order to be just as well-off as an individual who plans according to the SSA actuarial tables, and who has access to a risky-asset with 4 percent annual premium, an individual with SLEs demands a return premium of 6 percent. This result implies that incorporating subjective survival beliefs brings us closer to achieving a reasonable relation between elicited levels of risk-tolerance and a high historical return on equity. As a consequence, our findings build a bridge between two literatures without requiring any modifications to the basic framework.

This paper also segues with research on the consequences of probability weighting. It is well-documented that individuals place too much weight on low-likelihood events, a finding applied to seminal theories such as Cumulative Prospect Theory (Tversky and Kahneman

(1992)). Probability weighting has been shown to have important consequences for asset prices (Barberis and Huang (2008)) and portfolio choice (Polkovnichenko (2005)). Our contribution is to demonstrate a relevant application of probability-weighting, mortality risk, an avenue for which low-probability events may have an immediate impact on decision-making.

Our paper also complements a literature such as Gan et al. (2005) or Wu et al. (2014). These papers also interpret evidence of subjective life expectancies through the lense of life-cycle models. However, their evidence is for respondents older than 50 or even the oldest old. The calibrations therefore do not fully account for the younger years of life, which are crucial for asset accumulation.

Finally, we contribute to an extensive literature that models savings, consumption, and portfolio choice over the life-cycle. The shortcomings of the standard life-cycle model (in its most basic form) are well-known, which has motivated many modifications to generate more favorable comparisons with the data.[7] In contrast to much research, which tends to use actuarial tables for calibration, our paper shows that a run-of-the-mill life-cycle model better aligns with empirical findings simply by collecting and incorporating comprehensive data on survival beliefs over the life-cycle.

This paper is organized as follows. Section 2 provides survey evidence on subjective life-expectancies and outlines their important role in determining savings decisions. Section 3 describes our life-cycle model. Section 4 presents the solution to the model and describes its findings on consumption, savings, and investment over the life-cycle. Concluding thoughts are provided in Section 5.

[7]Other explanations include pre-cautionary savings (Lusardi (1998)) and self-control problems (O'Donoghue and Rabin (1999)).

2 Survey Evidence on Subjective Life Expectancies

We are not the first to ask respondents about their subjective life expectancies. This is an important feature of canonical household datasets, namely the Survey of Consumer Finances (SCF) and the Health and Retirement Survey (HRS).

Our work differs from these surveys in several key dimensions. While the SCF only asks for the age at which respondents expect to die, guided by theory, our survey asks respondents for their survival rates on a year-by-year basis. The HRS also asks for conditional survival rates, but our survey does so for a comprehensive set of ages. Both these modifications allow us to model consumption decisions more accurately based on their subjective expectations.

2.1 Survey Description

We used an online survey to measure mortality beliefs. The survey was programmed into the Qualtrics Research Suite. We contracted Qualtrics Panels to provide us with a panel of 200 respondents, screened according to national residency and age. All respondents were required to be US residents, and they were to be evenly distributed across the following five age categories: 28, 38, 48, 58, and 68. We further requested an even gender distribution sample-wide.

The survey asked respondents to indicate their survival likelihood, either for one-, two-, five-, or ten-year horizons from the day of taking the survey. The questions were formulated so as to be as clear as possible to respondents, and they were framed in terms of survival, rather than death, as the latter tends to engender stronger beliefs in mortality (Payne et al. (2013)). Respondents were also asked to indicate their expected longevity, in a manner not unlike that of the SCF. Following questions about expected longevity and survival likelihood, alike, respondents were asked to indicate their degree of confidence in their answers.

The belief elicitation was followed by questions – in the order given – that probed respondents' thought processes with respect to financial preferences (SCF), financial literacy (Lusardi and Mitchell (2011)), and numeracy (Cokely et al. (2012)). The survey also collects demographics, which are described in Table 1. Respondents are roughly evenly distributed between 28, 38, 48, 58, and 68 year olds. About half are female and half are married. Higher levels of household income are underrepresented, and so are the fraction of respondents without a high school education. Further details about the survey are given in an online Appendix.

2.2 Survey Results

There are stark differences between actuarial tables and subjective beliefs about survival. Our most straightforward results are presents in Figure 1, which plots average subjective survival beliefs by current age and survival horizon. The average respondent is 5 to 10 percentage points more pessimistic about her survival than that predicted by actuarial tables. For instance, a 28 year old with a 99.9 percent chance of survival to 29, believes he will do so with only 94.6 percent probability. These effects are persistent as the respondent ages. The effect reverses at older ages: the average 68 year old believes his likelihood of surviving to 78 is 85.8 percent, but the data predicts it is 75.3 percent.

These findings are robust to a number of relevant data trimmings. At younger ages, males and females have similar SLEs (Figure 2). Only at age 68 do men and women have noticeably different beliefs. 68 year old men and women both underestimate their short-run survival odds, whereas women have a greater overestimation of their 5 and 10 year survival probability than men (around 5 percentage points more).

Errors in subjective survival beliefs are unlikely to be caused by poor numerical literacy. Figure 3 presents SLEs partitioned by the respondent's ability to answer basic questions related to probabilities.[8] Respondents still underestimate their survival rates at short horizons. However, for some ages (28, 38, and 48), the expectation error is smaller, roughly cut in half. Even numerically literate respondents overestimate their long-run survival probability when they are 68 years old.

The most notable source of heterogeneity comes when we ask whether respondents feel confident in their beliefs about their subjective probability of survival. Figure 4 sorts subjects as being above or below the median level of confidence in their responses. Those above the median tend to have accurate responses of their short-run survival rates. On the other hand, confident respondents are the source of overestimation in long-run survival rates.

Remarkably, respondents are unaffected by the administration of an information primer prior to taking the survey. The primer specifically provides mortality statistics and so it would be reasonable for respondents to base their beliefs on the provided data. According to Figure 5, they do not. Similarly, the survey asks some respondents to give their 1, 2, 5, and 10 year survival horizons, consecutively and on the same page. Doing so may help respondents calibrate their answers to the ranking of different survival horizons. Figure 6 provides evidence that this treatment does not have a consistent impact on SLEs.

2.3 Heterogeneity of Subjective Survival Beliefs

To gauge the extent of these expectations errors, we calculate the following statistic:

$$exp.error_i = abs\left(E_{it}\left[surv(t+l)\right] - Pr_{it}\left[surv\left(t+l\right)\right]\right)$$

[8]Figure 3 calls a respondent numerically literate if they can correctly answer the following question: "Imagine that we roll a fair, six-sided die 1,000 times. Out of 1,000 rolls, how many times do you think the die would come up even (2, 4, or 6)?" However, the effects are similar when other related questions on numerical literacy are used to partition the data.

where t is the respondent i's age and $l = \{1, 2, 5, 10\}$. The expectations operator $E[\cdot]$ indicates i's subjective beliefs about her survival to at least $t + l$, while $Pr[\cdot]$ are the survival rates indicated by the SSA actuarial tables.

The survival belief errors are large, as evidenced by Figure 7, which presents a series of histograms for $exp.error_i$. The histograms are separated by the different survival horizons asked of respondents: one, two, five, and ten years. When the survival horizon is one (two) years the median error is 2 (3) years and the 25th percentile is 9 (14). As we increase the survival horizon to five (ten) years, the expectation error becomes more uniformly distributed. The median error is 6 (9) years, while the 25th percentile is 25 (19) years.

2.4 Does Subjective Life-Expectancy Matter for Savings Decisions?

Before studying the aggregate implications of SLEs, we ask whether survival beliefs matter for the decision-making process. This section presents preliminary, but direct evidence that subjective life-expectancies are closely linked to personal feelings about savings, investing, planning, and risk-tolerance.

To test how subjective life-expectancies relate to savings decisions, we employ the following multinomial logistic model:[9]

$$f(k, i) = \beta_{0k} + \beta_{1k} exp.error_{1i} + \beta_k X_i \tag{1}$$

[9]A multinomial logit model is an appropriate specification to estimate the effect of errors in subjective beliefs on various aspects of financial decision-making. The dependent variables we are interested in have more than two categorical response options. The multinomial logit models impose the constraint that the alternative responses are mutually exclusive and exhaustive. The multinomial logit flexibly allows the model to have different slope coefficients and intercepts within each outcome category, a feature that is useful for presenting our findings graphically. We find similar results using ordered logit models (when appropriate).

where $f(k,i)$ is a prediction of the probability that observation i has outcome k. The coefficient β_{1k} captures the effect of $exp.error$ on the likelihood of choosing k.[10] The matrix X_i includes i's age and gender, a categorical adjustment for the survival horizon, as well as an indicator if i was asked to consecutively provide her subjective beliefs about the four different survival horizons. The matrix also include an indicator if i correctly answers our numeracy test, which implies the results account for any differences in numerical ability. In all instances, we cluster standard errors to allow for correlated residuals across the four different survival horizons.

Figure 8 presents the estimated effect of survival belief expectation errors on savings plans. Although $expect.error_i$ has a lower bound of zero, a useful way to gauge the magnitude of its impact is to note that a 15 percentage point increase is roughly equal to a one standard deviation increase. The estimates of Equation 1 suggest that increasing the expectation error from zero to 20 increases probability of using savings "any time now" from around 14 to 20 percent. The slope is positive, but less steep when respondents plan to use their savings in the medium term. On the other hand, an equivalent increase in expectation error reduces from 45 to 30 percent the probability of using savings at least ten years from now.

The second piece of evidence that SLEs are linked to personal decisions comes from estimates of how much respondents save per month. Respondents that claim to spend all of their income or even rely on credit cards to spend more than they earn, are more likely to do so when their subjective beliefs are misaligned with the actuarial data. For instance, increasing the expectation error from zero to 20 percentage points is associated with a 4 percentage point increased likelihood of spending all income monthly (from 23 to 27 percent). An inverse relation of similar magnitude is found between increased $exp.error$ and the decreased probability of saving at least 25 percent of i's monthly income. Figure 9 summarizes these results.

[10] We Winsorize exp.error at the 5 percent level in the upper bound.

Survival expectation errors are also associated with investing experience or acumen, as well as risk-tolerance using key survey questions borrowed from the SCF. Figure 10 presents evidence of the former. Again using a zero to 20 percentage point increase in subjective survival expectation error as a metric, the likelihood of describing oneself as very (somewhat) inexperienced increases by 5 (4) percentage points, from 25 (24) percent likelihood to 30 (28). On the other hand, the likelihood the respondent is a somewhat (very) experienced investor falls from 38 (6) to 30 (4) percent.

These same expectation errors exhibit a U-shaped relation to surveyed risk-tolerance. According to Figure 11 and using the same magnitude change in $exp.error$, the probability of not taking any financial risks increases by about 10 percentage points from 15 to 25 percent. The likelihood also increases (from 5 to 10 percent) when the respondent takes substantial financial risks. On the contrary, the propensity to take average or above average financial risk falls when subjective survival rates deviate from the data. These results bears resemblance to two notable findings in the personal investing literature: there is under-participation in the stock market (Mankiw and Zeldes (1991)), but upon entry, individual decision-making contributes to excess idiosyncratic portfolio volatility (Ahmed et al. (2013)).

Taken together, our findings on the relation between survival beliefs and intended savings behavior can be interpreted as evidence that SLEs have a direct effect on financial decision-making.

3 Modeling the Life Cycle and Subjective Beliefs

This section presents our modeling framework, a canonical dynamic life-cycle model. We highlight in this framework how SLEs impact the maximization problem as they enter the effective discount rate: while the first component of the effective discount rate is given by the constant rate of time preference, the second component is given by subjective transition probabilities. This connects our model directly to our empirical strategy where me measure these transition probabilities. We calibrate the model using standard parameters. The next section shows the dramatic effect that calibration to actual subjective transition probabilities has for consumption, portfolio choices and asset pricing puzzles in a canonical life-cycle model.

3.1 Model Setup

Our framework exactly follows the model of Love (2013). As such, agents live in discrete time t, where $t = 0, 1, 2, 3, ...T_{retire}, ..., T$ and T_{retire} denotes the date of retirement. Agents choose to maximize their expected discounted stream of utility by choosing current period consumption, as well as what share of income to allocate to either a risky or risk-free asset. Recursively, their problem can be written as follows:

$$V_t^*(X_t, P_t) = \underset{C_t, \phi_t}{max}\{u(C_t) + \beta s_t \mathbf{E_t}[V_{t+1}^*(X_{t+1}, P_{t+1})] + \beta(1 - s_t)\mathbf{E_t}[B_{t+1}(R_{t+1}(X_t - C_{t)})]\}$$

where C_t denotes consumption in period t, ϕ_t the share of wealth allocated to the risky asset, P_t permanent income, X_t cash on hand, $B_t(X_t)$ a bequest motive, β the rate of time preference and s_t the subjective probability that individuals attach to transitioning to the next period conditional on having reached the current period. Cash on hand evolves as follows:

$$X_t = R_t(X_{t-1} - C_{t-1}) + Y_t$$

where the gross rate of return on the portfolio is the weighted return of the risky and the risk free asset, that is, $R_t = \phi_t R_t^r + (1 - \phi_t)R^f$.

The process for income Y_t is given by permanent income P_{t-1}, an adjustment for the age-earnings profile G_t, a shock N_t following a log-normal distribution and a transitory shock Θ_t. This specification is due to Carroll (2011). Thus, we have that:

$$Y_t = P_{t-1} G_t N_t \Theta_t$$

and permanent income transitions according to:

$$P_t = P_{t-1} G_t N_t$$

We parameterize the utility function by choosing $u(C) = C^{1-\rho}/(1-\rho)$, and the bequest function by choosing $B(X) = b(X/b)^{(1-\rho)}/(1-\rho)$. The curvature of the utility function is prescribed by ρ, with this parameter often called the coefficient of relative risk aversion. We solve the problem numerically using the method of endogenous grid-points as described in Carroll (2011) and Love (2013).[11]

We emphasize that subjective beliefs play an important role in this framework. Individual agents make their consumption and portfolio choices today using their subjective beliefs about their transitions to the next period. These beliefs multiplicatively enter the maximization problem through the effective period discount rate, that is, through βs_t. Thus, while the first component of the effective discount rate is given by β, the constant rate of time preference, the second component is exactly given by subjective transition probabilities.

All that we are doing in this paper is to highlight that optimal decision rules are determined by subjective transition probabilities. In particular, we solve for these decision rules without having to assume any kinds of bounded rationality, such as hyperbolic discounting implemented by Laibson (1997). We thus extend the pioneering work by Hamermesh (1985), by showing in a standard lifecycle model how important these subjective beliefs – carefully mapped out in the data – can be for perfectly rational, optimizing agents, and how they can potentially address several empirical challenges to the model.

[11] We are extremely grateful to David Love for sharing his set of codes with us.

3.2 Calibration

We calibrate our model to two specifications: one for comparison, one to show the effect of subjective beliefs, which can be further differentiated to yield additional comparisons. Our calibration for comparison purposes implements a canonical life-cycle model. This calibration is consistent with Love (2013) and we follow his parameter choices exactly except that we do not allow for an explicit bequest motive. Our subjective belief specification is identical to the comparison specification with one important difference: we now use subjective survival probabilities from our survey instead of statisticial mortality data.

This difference in transition probabilities reflects the most important difference to the standard calibrations in the life-cycle literature because subjective beliefs affect the effective discount factor. Our main subjective belief calibration uses the subjective transition probabilities in Figure 1: on average, individuals who are between 28 and 38 years old believe that they are approximately 50 times less likely to live to the next period than statistical mortality data indicate. The comparison uses data from the 2007 Social Security Administration Period Life Tables. We set β, the rate of time preference, which is the other component of the effective discount factor, to be 0.98 in both specifications.

The other key parameter choices reflect typical values chosen in the life-cycle literature. While we refer the reader to Tables 1 and 2 in Love (2013) for details, we highlight our choices of several key parameter values, especially those relevant for the asset pricing aspect of our analysis. We thus set the risk-free rate to 2 percent, the excess return to 4 percent and the standard deviation of the risky asset to 18 percent. The excess return is somewhat lower than found by Campbell and Viceira (2002), which is needed so that households do not end up at a corner solution of investing 100 percent into the risky asset. This is helped by a high risk aversion parameter of 5. Following Love (2013) and Carroll and Samwick (1997), we use 1970-2007 PSID data to calibrate the income process. While there are potentially many interesting aspects to the life cycle coming from family composition and marital status, we calibrate the income process for college graduates only who live in a married household but without additional dependents. We allow for a non-zero correlation between permanent income and excess returns during the working life, but set the correlation to zero during retirement.

4 Results

This section shows the dramatic impact that introducing subjective beliefs into a run-of-the-mill lifecycle model can have. First, we find that young individuals overconsume and undersave relative to what is optimal in a benchmark based on statistical transition probabilities. Second, we find that retirees consume less than would be optimal. This holds true even when we abstract from wealth differences due to undersaving when young. Third, we find that subjective transition probabilities bring us closer to observed levels of the return on equity over the risk-free rate given reasonable degrees of risk aversion – while we continue to match undersaving of the young.

First, we find that the introduction of subjective beliefs into a canonical life-cycle model has qualitatively and quantitatively large effects on consumption, savings and asset allocations of the young. Since this period is crucial for asset accumulation for retirement, we view the large effects during this period as our main result. What do we find exactly? During much of their working life, people substantially overconsume, relative to what our benchmark specification based on statistical probabilities implies. Between ages 28 and 58, people on average consume 25 percent more based on their subjective beliefs than what they should based on statistical transition probabilities. This is equivalent to $5000 per year.

The intuition for this result lies of course in the lower than statistical, subjective transition probabilities: due to these beliefs, people discount the future more and prefer present-term consumption. This is what the change of the effective discount factor implies. Unlike other work in the life-cycle literature such as Laibson (1997), we do not require any behavioral assumptions to obtain our results. We show the result for consumption in the top left panel of Figure 12.

At the same time, a result of such large over-consumption is that people have less income to save and accumulate fewer assets when they are young. Their savings rate and total accumulated assets are below those implied by our canonical comparison model between ages 28 and 58. Then, as people approach retirement, we find that their consumption stays essentially flat. In particular, this means that their savings go up starting at age 58 since the income profile we calibrate to implies rising incomes until retirement. In fact, savings leading up to retirement are slightly higher than in the canonical model.

Second, an important finding is what happens during retirement: Strikingly, throughout retirement, people save more compared to the canonical model while they consume relatively less. Because they want to smooth consumption but cannot relie on a large asset base, this is an intuitive result. Interestingly, people have an approximately constant savings profile during retirement. They save on average \$17500 per year. These results can be seen from the top right panel and the bottom left panel of Figure 12.

To what extent are the heavy right tails in subjective beliefs at old age responsible for higher savings during retirement? We investigate this by comparing only the choices of retirees, either under subjective or statistical transition probabilities, while giving individuals exactly the same amount of assets at age 65. That is, we simulate a truncated version of the life-cycle model. We find that subjective transition probabilities still generate a pattern of oversaving and underconsumption during retirement, relative to a benchmark of statistical transition probabilities (Figure 13). This finding shows that not only a lower asset base at the start of retirement but again, subjective beliefs are crucial determinants of savings and consumption decisions. Our results agree with the findings of old age and subjective life expectancies in Wu et al. (2014).

4.1 Robustness

Our results are robust to a number of alternative parametrization, as well as alternative sets of subjective transition probabilities. Figure 14 presents evidence of the latter. The most notable differences are for subjects that have confidence in their answers to the subjective life-expectancy questions. For these individuals, the life-cycle model predicts overconsumption when young by an even greater amount. This leads to increased under-accumulation of wealth.

The results are insensitive to alternative discount factors (Figure 15). When β is increased, overconsumption of the young increases and wealth accumulation falls, implying an interaction between the discount factor and SLEs. When the coefficient of relative risk-aversion is changed from our preferred parametrization ($\rho = 5$), the model produces additional overconsumption for both increases and decreases in ρ (Figure 16). These differences are owing to changes in how individuals allocate their savings between the risky and risk-less asset. Lastly, we find little change in our results when we incorporate a bequest motive into the model (Figure 17).

4.2 Subjective Beliefs and Asset Returns

Finally, our results relate to the asset pricing and finance literature, specifically the well-studied relation between the risk aversion and the difference between the returns on a risky and risk-less asset (Mehra and Prescott (1985)). As Siegel and Thaler (1997) summarize, an additively separable utility function with constant relative risk-aversion can be reduced to a single linear equation. The equation sets the equity premium (the return on the risky asset minus the risk-free rate) equal to the covariance between consumption and asset prices scaled by a single parameter, the coefficient of relative risk-aversion, ρ. Given what the data says about returns and consumption over time, the equation implies a value of $\rho = 30$. However, $\rho = 30$ also suggests that one would concede 49 percent of their wealth just to avoid a coin-flip gamble that would either double or half their existing wealth. Hence, the relationship is difficult to reconcile with observed risk-tolerance and is therefore deemed a puzzle.

An appealing way to understand this relationship is to argue not that the return on equity is too high, but that historical bond returns are too low (Weil (1989)). This argument highlights the rate of time-preference, and draws into question whether observed asset (bond) returns are a reasonable reflection of the model-implied risk-free rate. Our subjective life-expectancies contribute to this line of inquiry, because they interact multiplicatively with the discount factor. Even in our more conservative estimates of SLEs, we show that with a transition probability of around 0.95, the weight put on the discounted stream of future consumption falls relative to today's consumption. This implies that the latent risk-free rate is actually much higher than is the case when only β is used to discount. This upward adjustment to the risk-free rate lessens the differential between it and historical equity returns, which would be more consistent with lower levels of ρ. Another way to view this result is to note that there is room for an even greater divide between equity returns and $1/\beta$ once an adjustment has been made for subjective survival beliefs.

Our interpretation, that the computation of the risk-less rate does not fully account for subjective mortality risk, is reinforced by the findings in Figure 19. The figure plots average wealth accumulation over the life-cycle for different levels of risk-tolerance, the equity premium, and two different survival functions – the actuarial data (SSA) and the survey-elicited subjective belief distribution. We highlight the time-series of wealth accumulation calibrated to the actuarial survival data and with an equity premium of 4 (with the risk-free rate set equal to $1/\beta = 1.02$, the return on equity equals 6). It is plausible to consider this time-series to be the welfare-maximizing set of outcomes over the life-cycle for a perfectly calibrated individual given these parameter values. In other words, it serves as our benchmark.

In comparison, we estimate the wealth-accumulation time-series using SLEs and vary the level of equity premium. When $\rho = 2$, the model solution using SLEs requires an equity premium close to 8 to come close to achieving benchmark levels of wealth. When $\rho = 4$, the required equity premium falls to around 6. For greater levels of ρ, specifically 10 and 20 in Figure 19, the consumer is invariant to the different levels of equity premium, and is seemingly equally well-off under the subjective and actuarial survival functions.

These results provide a useful counter-factual exercise, because they show that for reasonable levels of risk-tolerance, individuals need to be compensated with higher levels of equity returns to make up for the shortcomings of subjective belief mis-calibration. Intuitively, because they change the discount factor, subjective transition probabilities lead an implicitly higher required rate of return. Therefore, the rate of return on equity is not too high once subjective beliefs are accounted for.

5 Conclusion

This paper presents new findings on subjective survival beliefs over the life-cycle. Young people tend to overestimate their mortality risk while older individuals believe they will survive longer than actuarial data expect. We embed these survival beliefs into a canonical dynamic life-cycle model with portfolio choice. Without any modifications to the standard model and using the literature's preferred parametrization, the application of elicited survival beliefs better align the model with notable features of data aggregates. Namely, our estimates produce under-saving while young and under-consumption while old. They also allow the model's solution to feature accurate levels of elicited risk-tolerance and the historical equity premium. In sum, simply incorporating more accurate data on subjective beliefs, substantially improves the performance of the standard framework in a number of seemingly unrelated yet critical dimensions.

As a final consideration, we speculate on the origins of subjective beliefs that are poorly aligned with actuarial life-expectancies. The survey asks how respondents develop expectations of their own mortality. When developing their own survival beliefs, the average respondent places about half as much weight on unlikely occurrences such as natural disasters and animal attacks as they do on the natural course of aging and medical conditions (Table 2). If that is indeed so, it is important that we fully consider how beliefs diverge from true probability distributions, because as this paper shows, there are important consequences.

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Table 1: **Survey Respondent Characteristics**

Description: This table presents data on the characteristics of participants in the survey administered by Qualtrics.

	pct.		pct.		pct.
age		*civil status*		*gross hh income*	
28	17.4	single	24.6	less than $10k	3.7
38	19.3	partner (not co-habitating)	0.8	$10k - $20k	8.4
48	21.6	partner (co-habiting)	9.5	$20k - $35k	17.4
58	23.0	married	52.9	$35k - $50k	18.8
68	18.8	divorced	8.7	$50k - $100k	38.8
		widowed	3.4	$100k - $200k	11.5
gender				greater than $200k	1.4
female	49.3	*education*			
male	50.7	primary school	3.1		
		high school	23.4		
have children		college, no degree	30.8		
no	34.7	bachelor's degree	31.9		
yes	65.3	master's degree	8.8		
		doctorate	2.0		

$N = 357$

Table 2: **What Risk Factors Matter for Subjective Beliefs?**

Description: This table presents data on the factors respondents use to judge their own survival probabilities. They are asked to rate each factor on a scale of 0 to 100 (the weights do not have to sum to one).

"When you assessed your survival likelihood, to what extent did you place weight on the following risk factors?" (scale of 0 to 100)

variable	mean	median	std dev
The natural course of life and aging ("normal risk")	74.5	80	23.5
Medical conditions (e.g., cancer and heart disease)	69.4	78	26.4
Dietary habits (e.g. unhealthy foods)	62.3	69	28.2
Traffic accidents (e.g., car crash)	45.3	50	29.8
Physical violence (e.g., murder)	35.3	20	32.3
Natural disasters (e.g., earth quakes)	34.7	23	31.5
Animal attacks (e.g., shark attacks)	25.6	9	31.3
Risky lifestyle (e.g., base jumping)	28.3	10	33.3
"Freak events" (e.g., choking on your food)	32.7	23	30.8
			$N = 357$

Figure 1: **Subjective Life-expectancy vs. Actuarial Data**

Description: The figure presents conditional survival rates from actuarial tables (Social Security Administration). These rates are compared against a respondent's subjective beliefs about his or her chances of survival to at least the stated age.

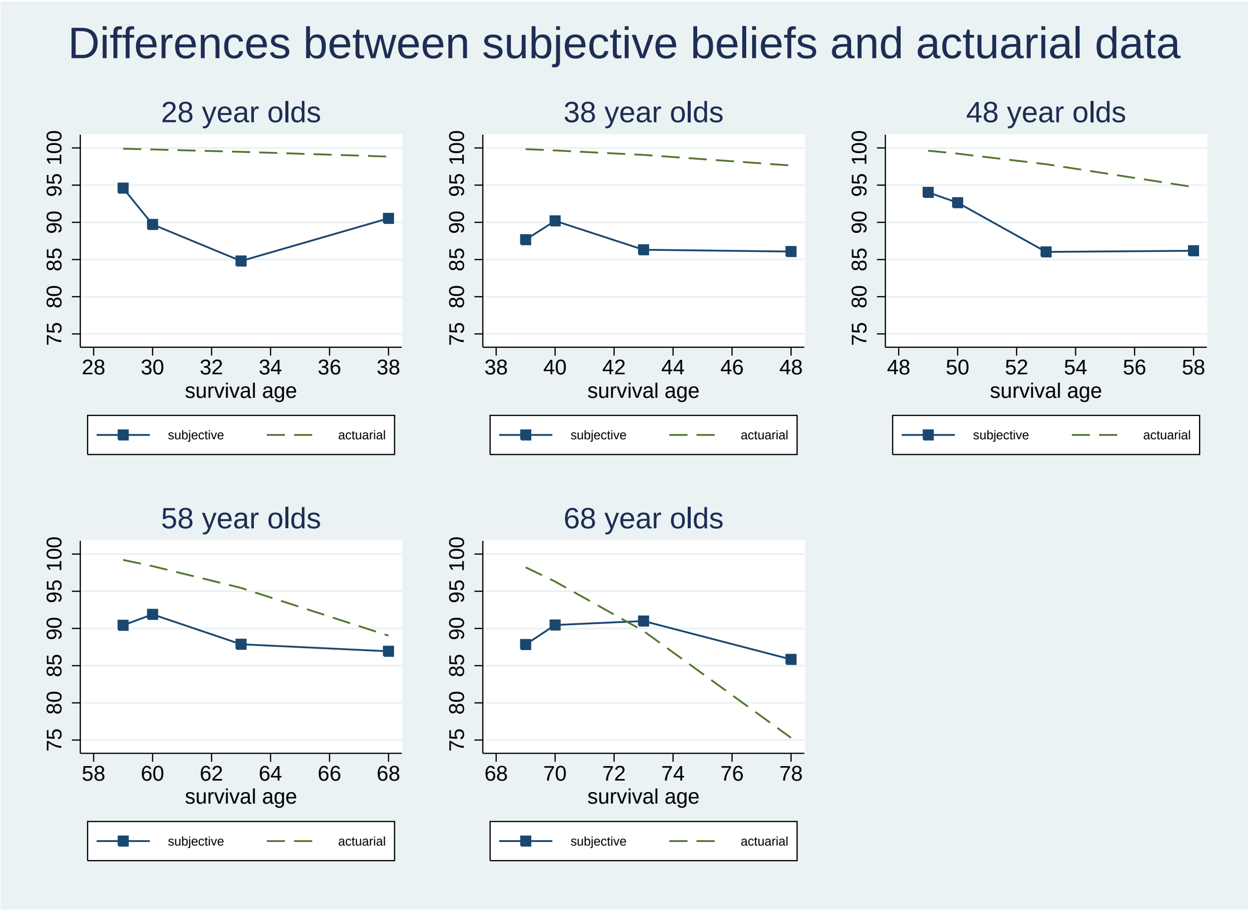

Figure 2: **Gender Differences in Subjective Life-expectancy**
Description: The description is the same as Figure 1, except the data is divided by gender.

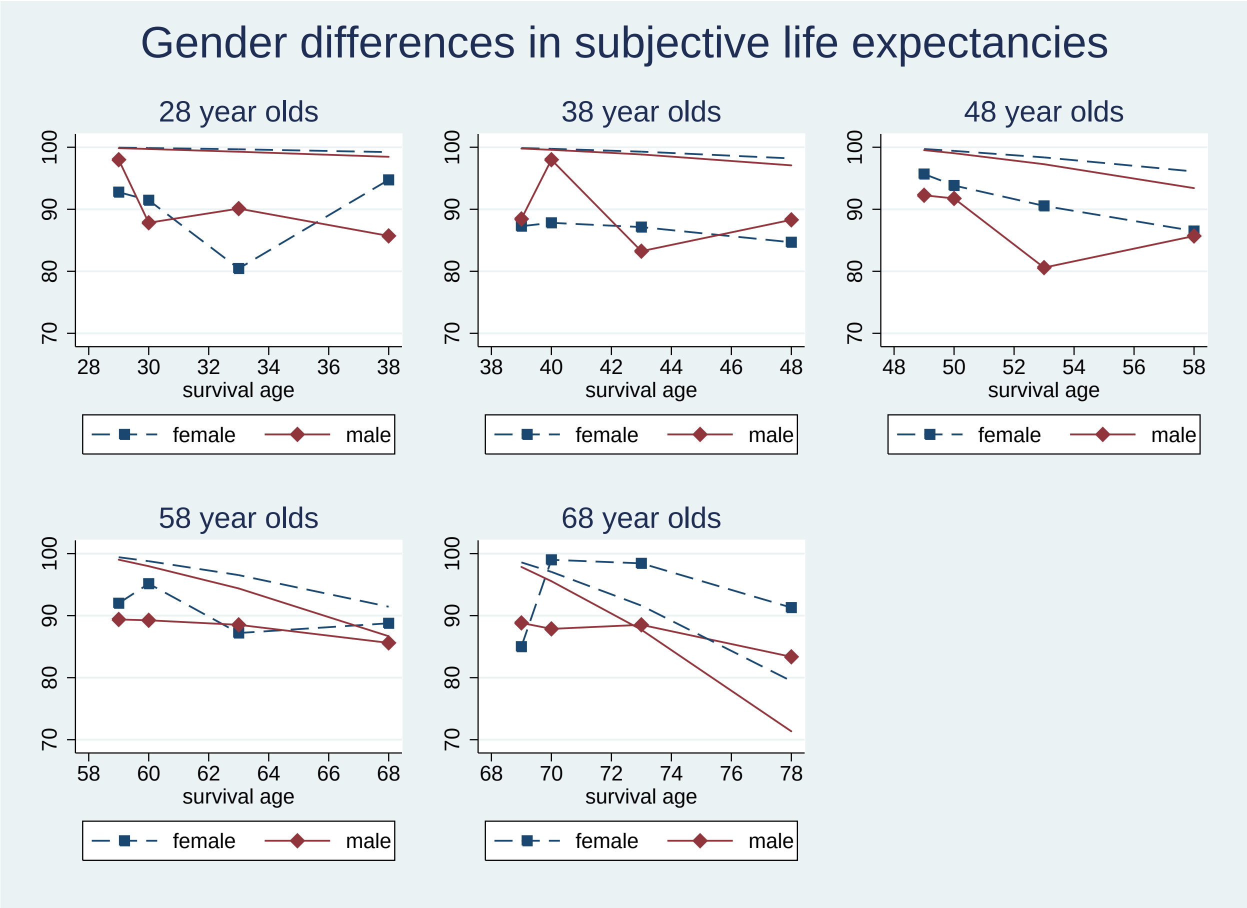

Figure 3: **SLEs when Respondents are Numerically Literate**

Description: The description is the same as Figure 1, except the data is divided by whether the respondent correctly answers questions related to their numerical ability.

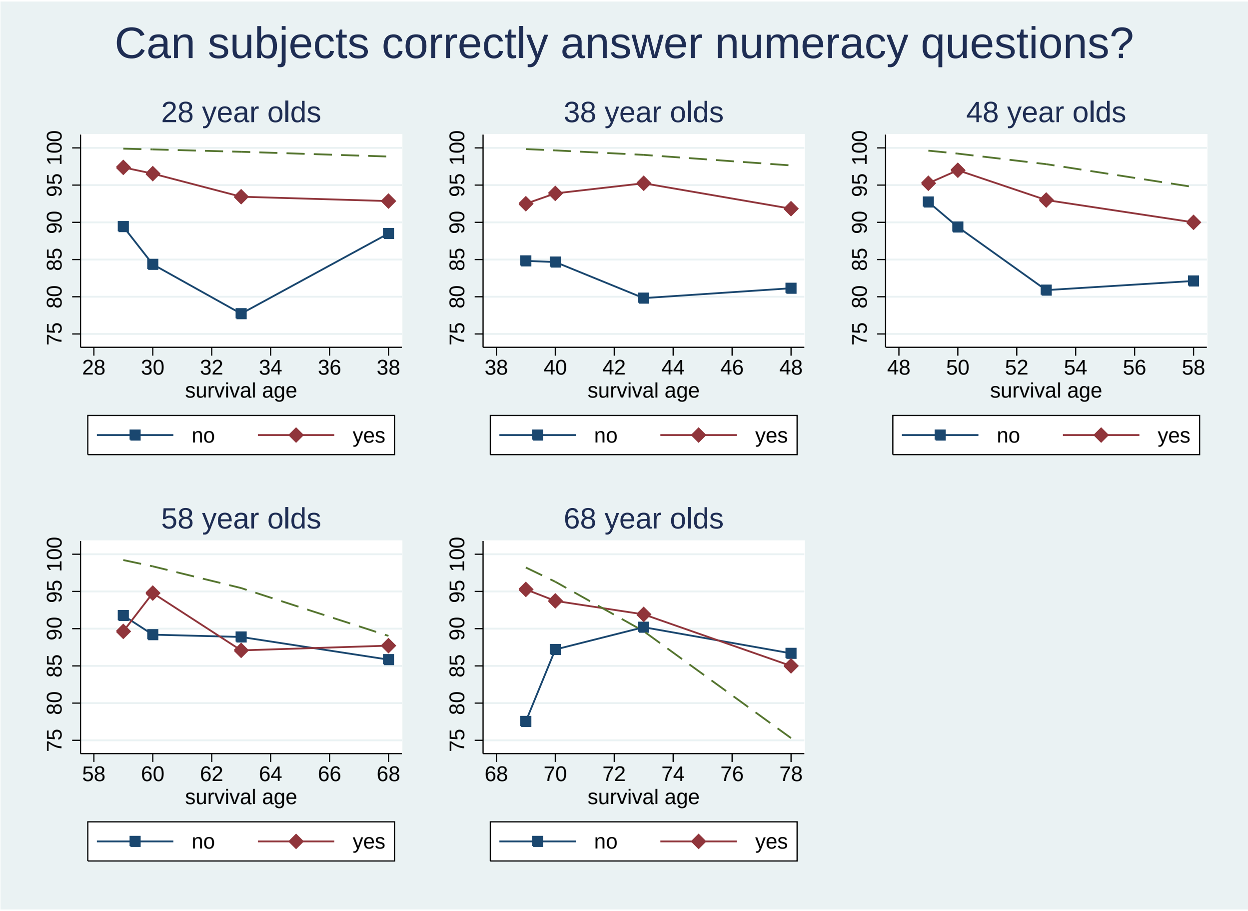

Figure 4: **SLEs when Respondents are Confident in Their Answers**
Description: The description is the same as Figure 1, except the data is divided by being above or below the median level of confidence in one's beliefs about their survival.

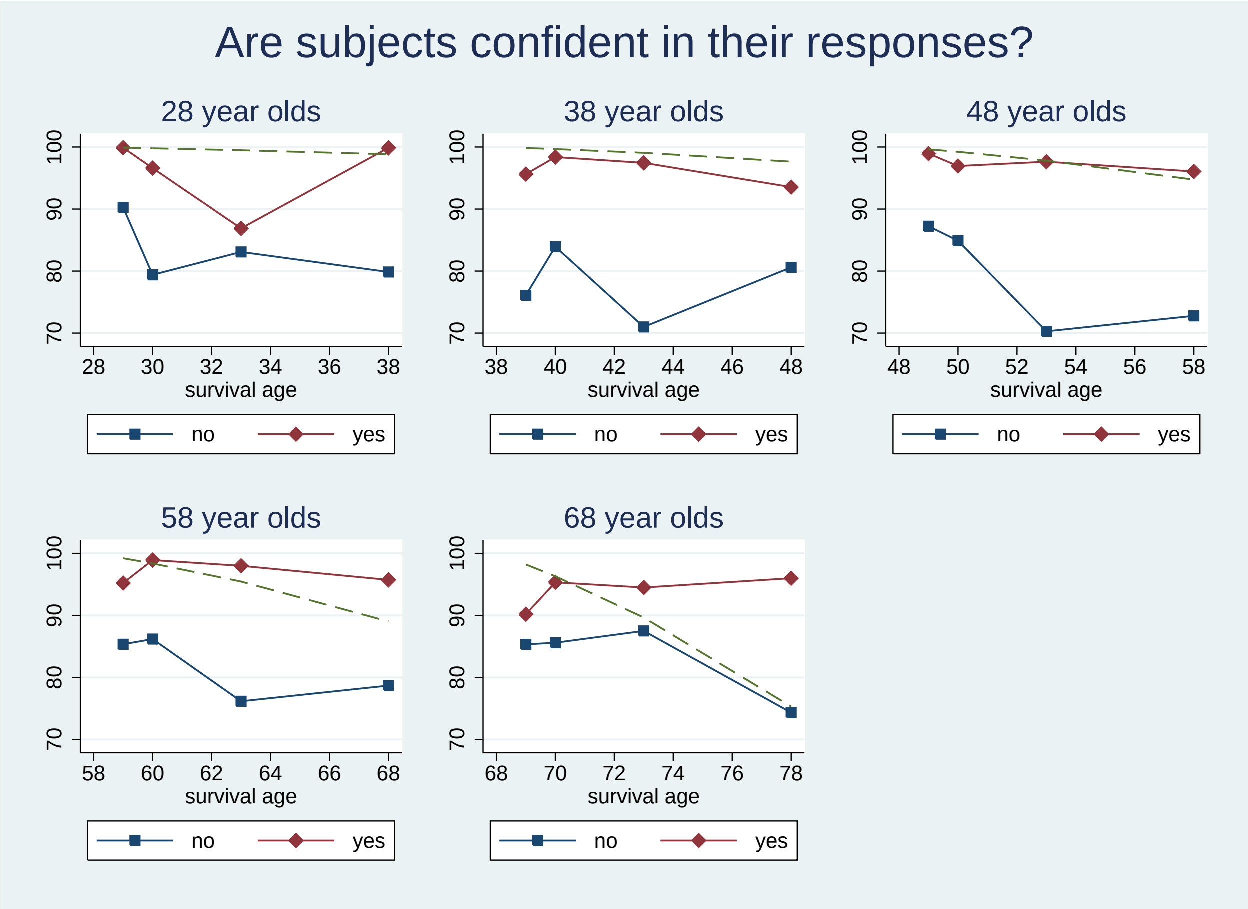

Figure 5: **SLEs when Respondents are Given an Information Treatment**
Description: The description is the same as Figure 1, except the data is divided by whether or not respondents were given an information primer that gives detail about the population survival statistics.

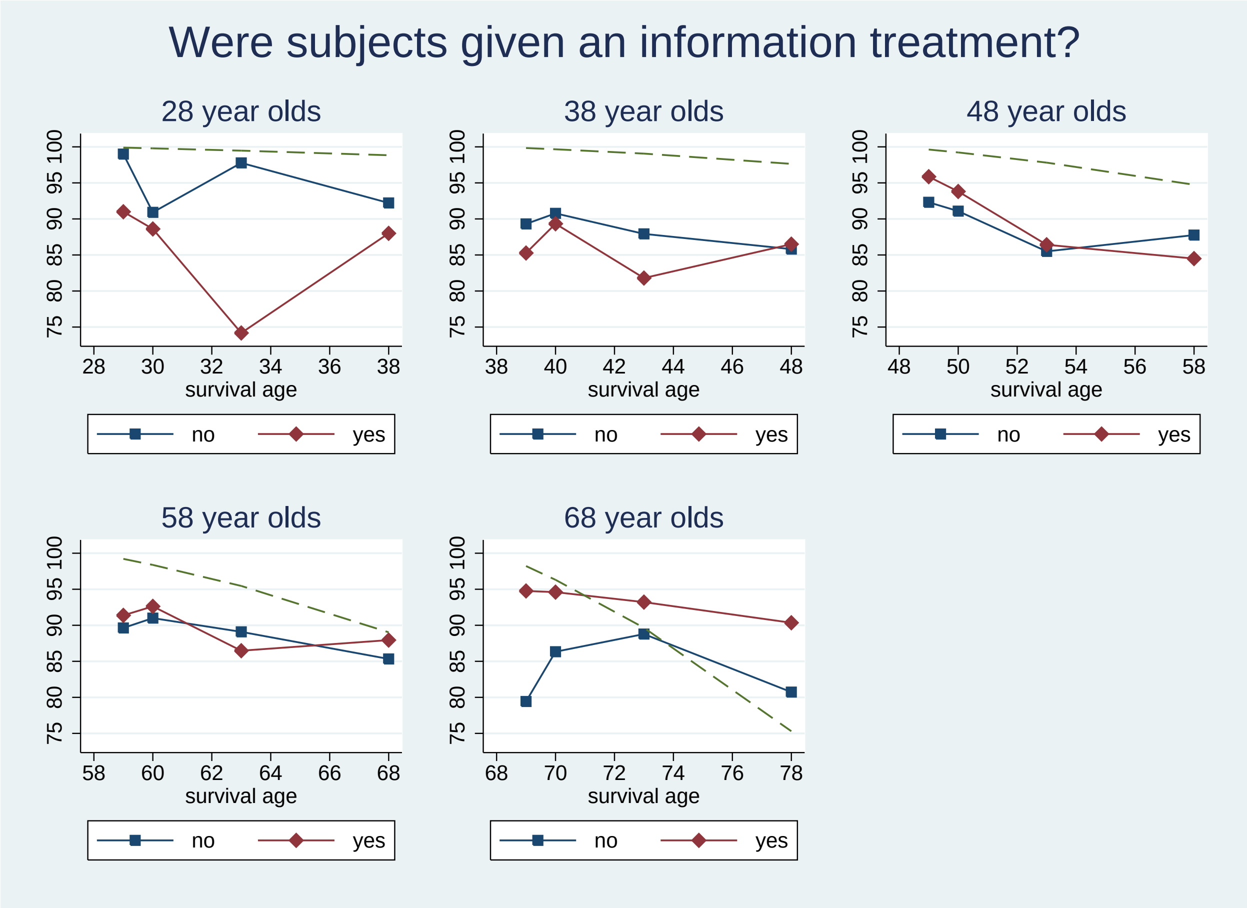

Figure 6: **SLEs when Respondents are Asked Consecutive Survival Horizons**
Description: The description is the same as Figure 1, except the data is divided by whether or not respondents are asked their beliefs over the full set of horizons (1, 2, 5, and 10 years) consecutively and on the same page. All other respondents were only asked to provide their beliefs about one of the four different survival horizons.

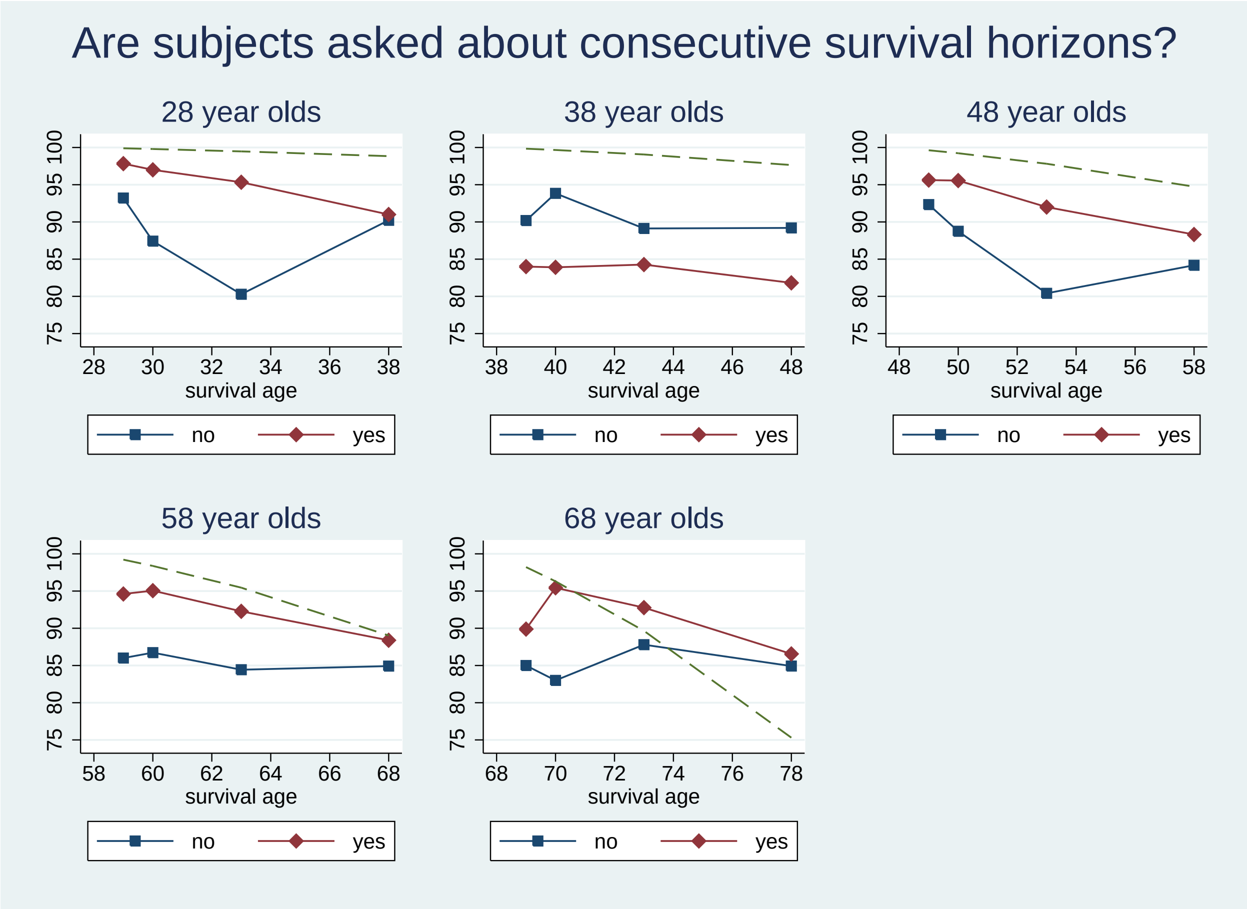

Figure 7: **Expectation Errors in Survival Beliefs**

Description: The figure presents the distribution of the following statistic:

$$exp.error_i = abs\left(E_{it}\left[surv(t+l)\right] - Pr_{it}\left[surv\left(t+l\right)\right]\right)$$

where t is the respondent i's age and $l = \{1, 2, 5, 10\}$. The expectations operator $E[\cdot]$ indicates i's subjective beliefs about her survival to at least $t + l$, while $Pr[\cdot]$ are the survival rates indicated by the SSA actuarial tables.

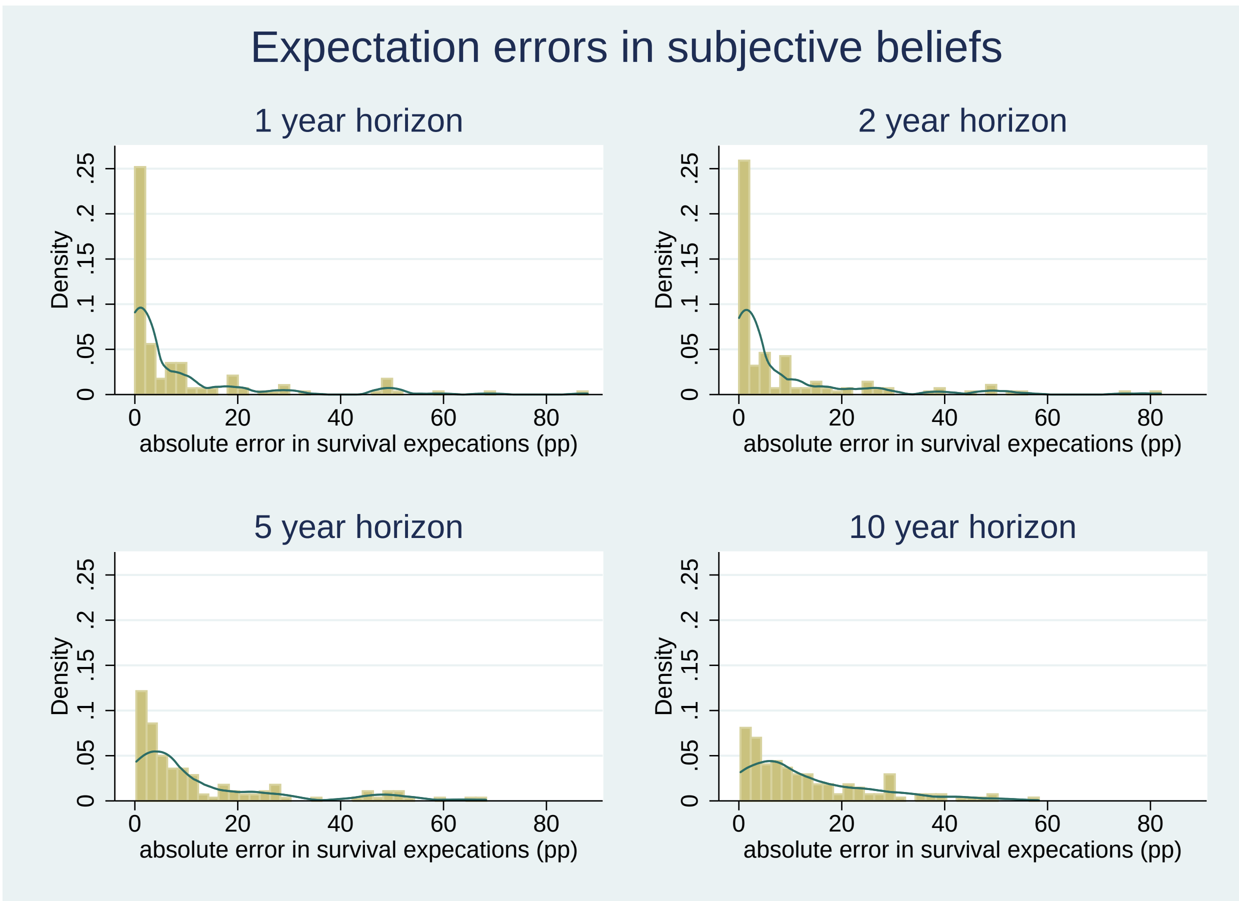

Figure 8: **Survival Beliefs and Savings Plans**

Description: The figure presents the predictions from the following multinomial logistic model:

$$f(k,i) = \beta_{0k} + \beta_{1k} exp.error_{1i} + \beta_k X_i$$

where $f(k,i)$ is a prediction of the probability that observation i has outcome k. The dependent variable is the survey question: "When do you expect to use most of the money you are now accumulating in your investments or savings?" The set of possible answers are: 1) "At any time now...so a high level of liquidity is important", 2) "Probably in the future...2-5 years from now", 3) "In 6-10 years", and 4) "Probably at least 10 years from now." The coefficient β_{1k} captures the effect of $exp.error$ on the the likelihood of choosing k. The matrix X_i includes i's age and gender, a categorical adjustment for the survival horizon, as well as an indicator if i was asked to consecutively provide her subjective beliefs about the four different survival horizons. The matrix also include an indicator if i correctly answers our numeracy test, which implies the results account for any differences in numerical ability. In all instances, we cluster standard errors to allow for correlated residuals across the four different survival horizons.

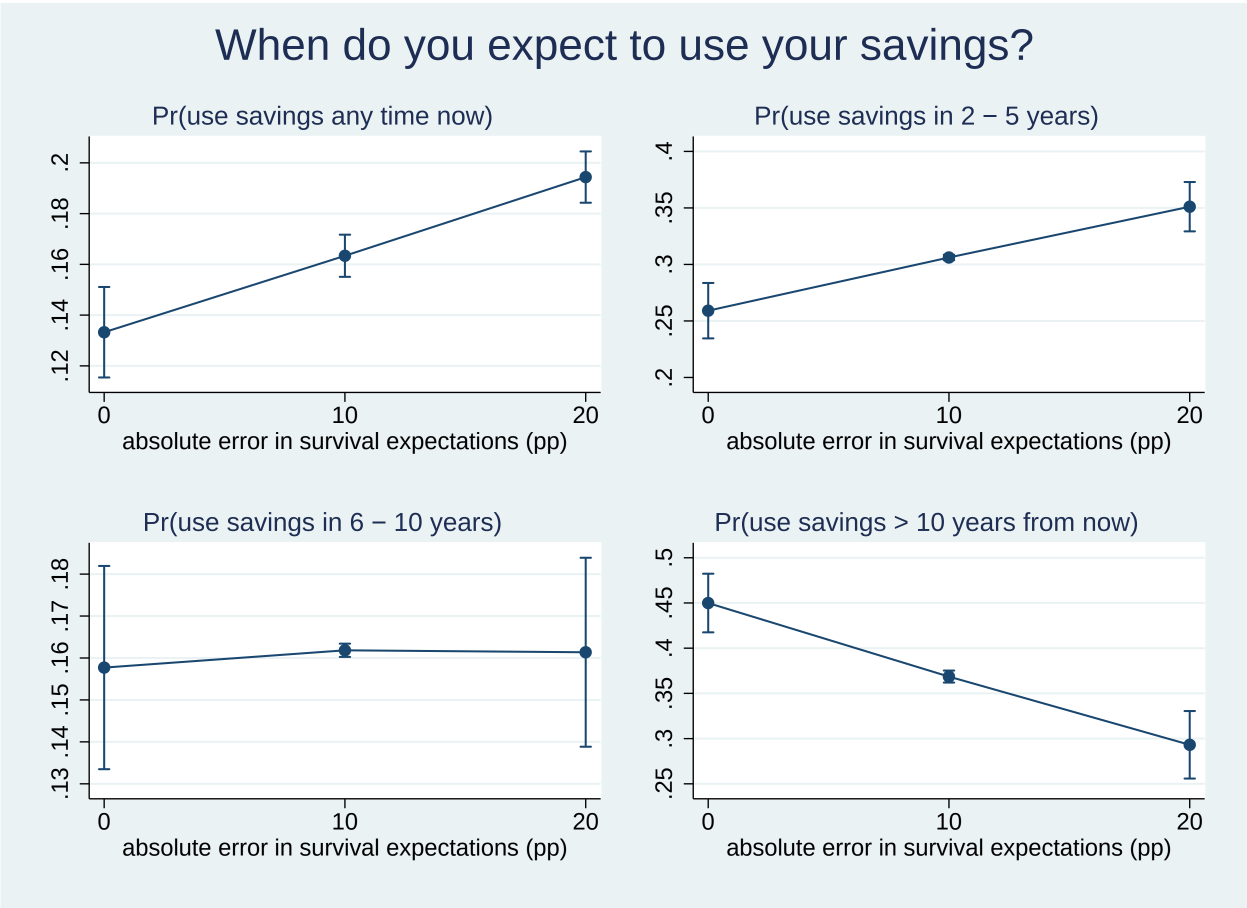

Figure 9: **Survival Beliefs and Rule-of-thumb Savings**

Description: The description of this figure is the same as Figure 8. Except, the dependent variable is the survey question: "How much of your monthly income do you save? (choose the closest answer from the following)", with the following set of possible responses: 1) "I spend more money than I earn. I often use credit cards or other loans to supplement my monthly income", 2) "I save around 25% of my monthly income", 3) "I save around 10% of my monthly income", 4) "I spend all of my income each month", or 5) "I save at least 50% of my monthly income."

Figure 10: **Survival Beliefs and Experience Investing**

Description: The description of this figure is the same as Figure 8. Except, the dependent variable is the survey question: "When it comes to investing in stocks, bonds, mutual funds, or real estate, I would describe myself as", with the following set of possible responses: 1) "Very inexperienced", 2) "Somewhat inexperienced", 3) "Experienced", 4) "Somewhat experienced", or 5) "Very experienced."

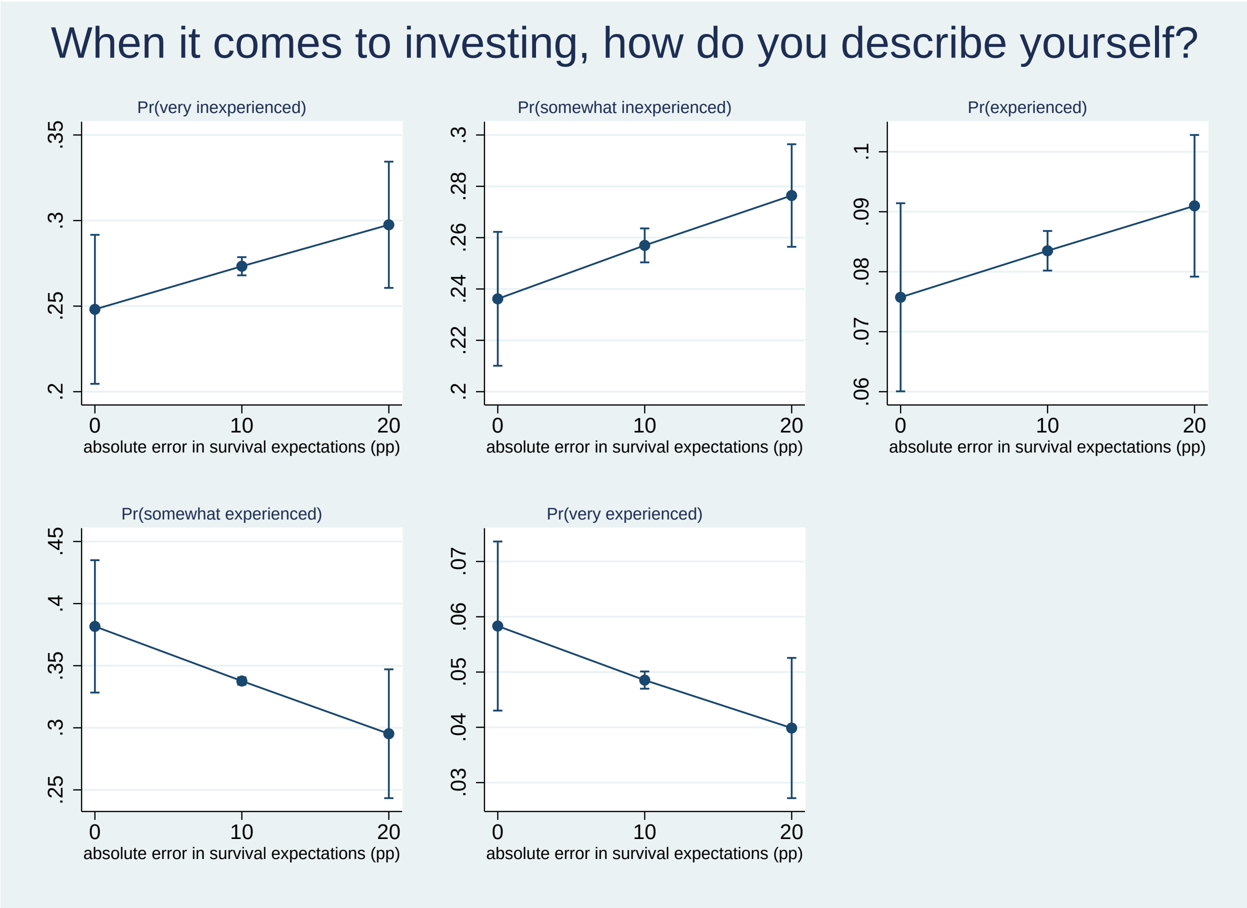

Figure 11: **Survival Beliefs and Risk-tolerance**

Description: The description of this figure is the same as Figure 8. Except, the dependent variable is the survey question: "Which of the following statements on this page comes closest to the amount of financial risk that you are willing to take when you save to make investments?", with the following set of possible responses: 1) "Take substantial financial risk expecting to earn substantial returns", 2) "Take above average financial risks expecting to earn above average returns", 3) "Take average financial risks expecting to earn average returns", or 4) "Not willing to take any financial risks."

Figure 12: **Life-Cycle Results: Subjective vs. Actual Life-Expectancies**
Description: This figure presents the solution to the life-cycle outlined in Section 3. The line "SSA" takes the survival rates from the Social Security Administration. The dotted line, "Pr(Surival)", are model estimates using survey elicited subjective survival beliefs.

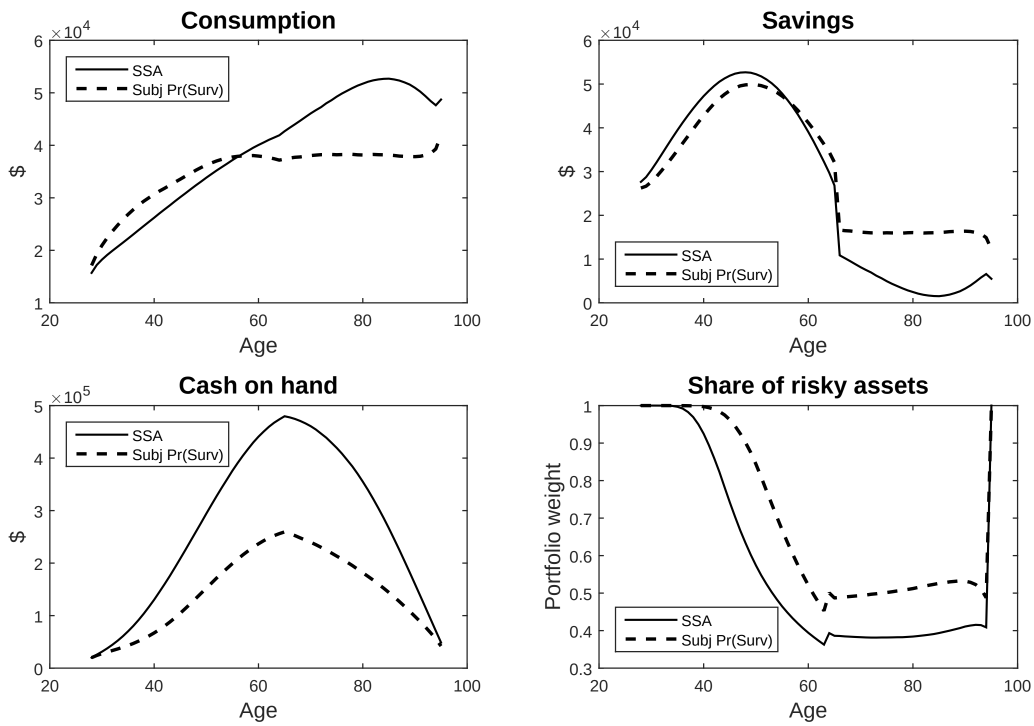

Figure 13: **Savings and Consumption into Retirement**

Description: This figure presents the solution to the life-cycle outlined in Section 3. The model is solved using survey-elicited subjective survival beliefs. We allow the representative consumer to arrive at retirement (age 65) with $525,000 accumulate wealth, which is roughly the median household in 2008, excluding housing equity and including Social Security income. We also allow consumers to plan as though they will live until 120.

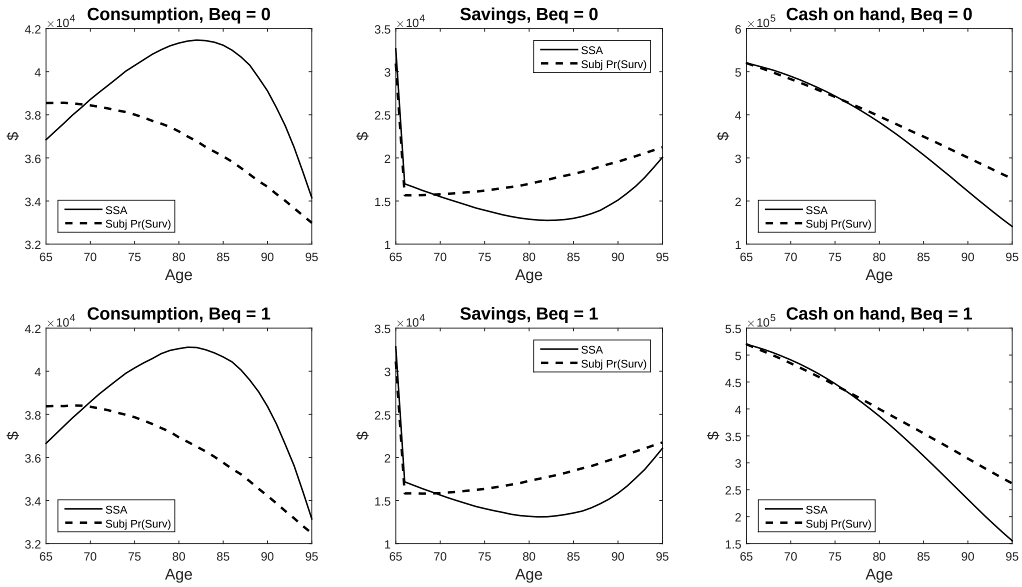

Figure 14: **Robustness to Different Subjective Beliefs**

Description: This figure presents the solution to the life-cycle outlined in Section 3. The model is solved using different sets of survey-elicited subjective survival beliefs, as described in Section 14.

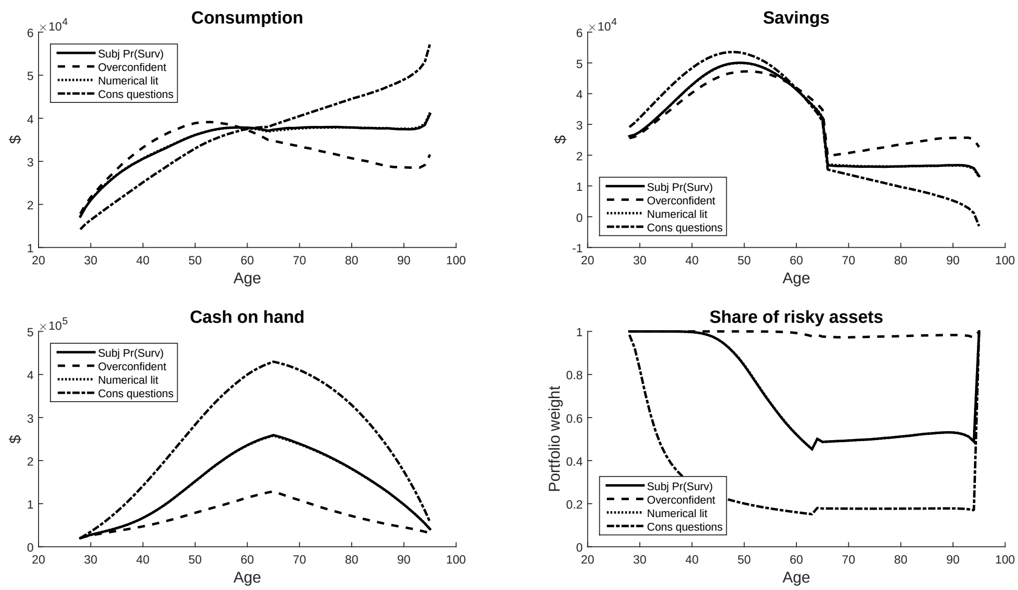

Figure 15: **Subjective Survival Beliefs and Different Discount Factors**
Description: This figure presents the solution to the life-cycle outlined in Section 3. The model is solved using survey-elicited subjective survival beliefs.

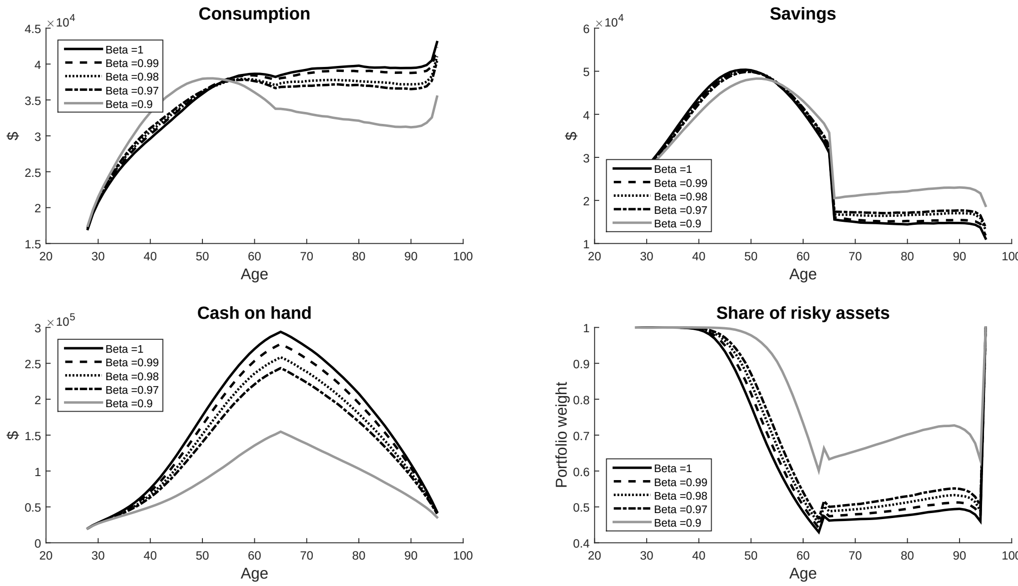

Figure 16: **Subjective Survival Beliefs and Different Levels of Risk-tolerance**
Description: This figure presents the solution to the life-cycle outlined in Section 3. The model is solved using survey-elicited subjective survival beliefs.

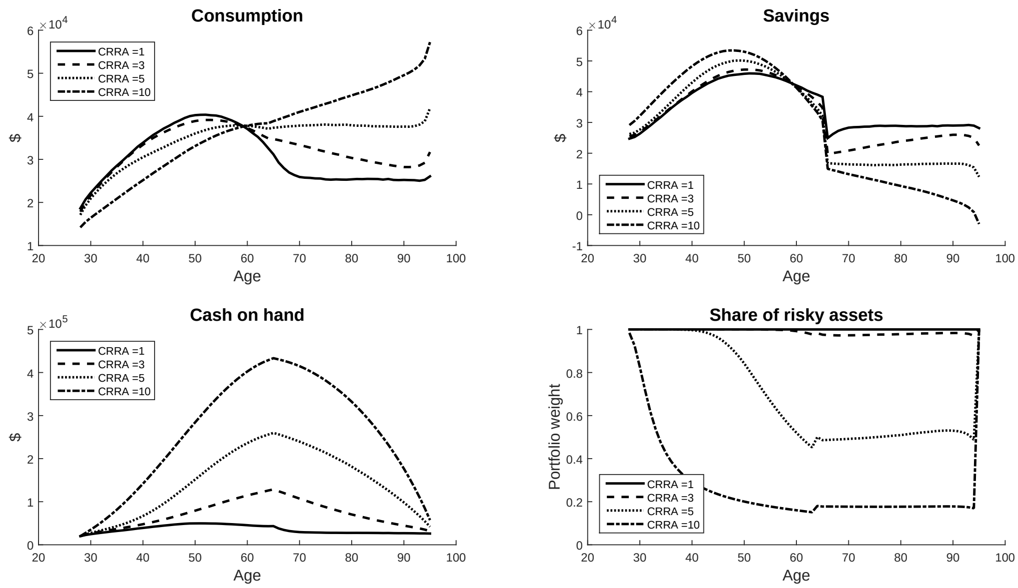

Figure 17: **Subjective Survival Beliefs and Bequests**

Description: This figure presents the solution to the life-cycle outlined in Section 3. The model is solved using survey-elicited subjective survival beliefs. The model includes a bequest motive when Bequest = 1.

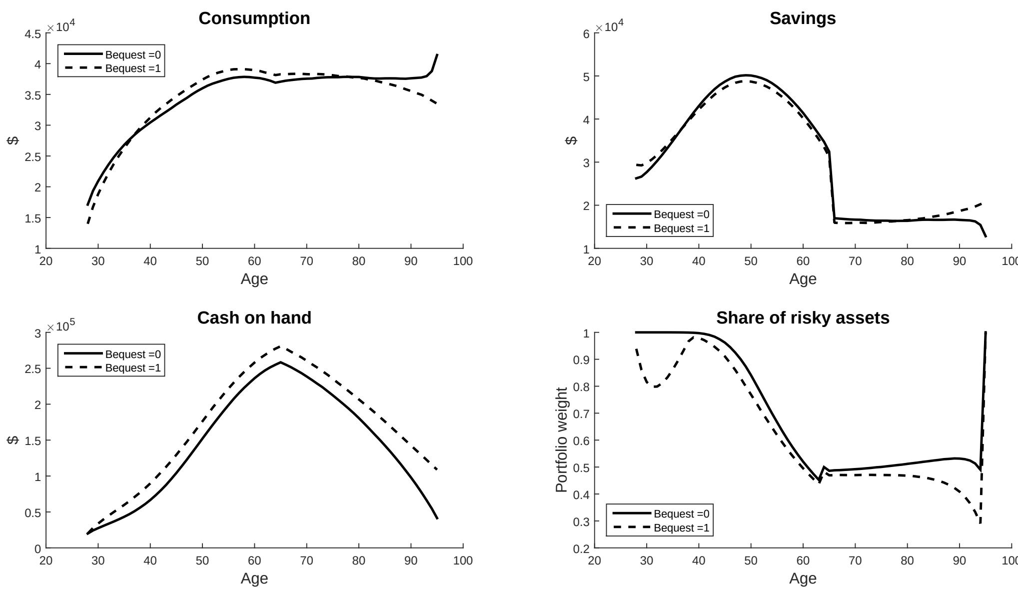

Figure 18: **Subjective Survival Beliefs and Different Levels of Education**
Description: This figure presents the solution to the life-cycle outlined in Section 3. The model is solved using survey-elicited subjective survival beliefs. Education 1, 2, and 3 is no degree, high-school, and college graduate, respectively.

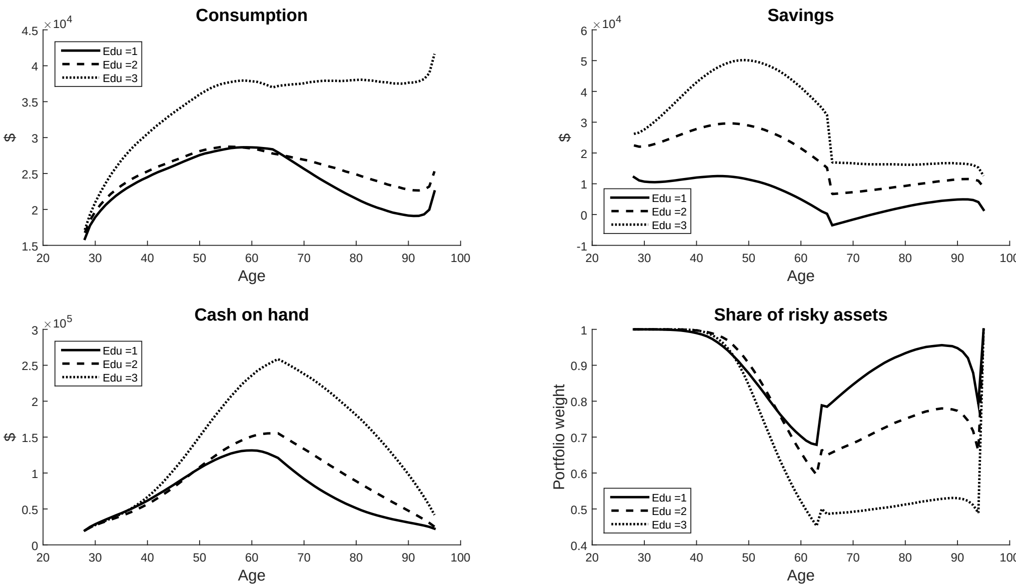

Figure 19: **Wealth Accumulation with Subjective Survival Beliefs and Different Levels of Equity Premium**

Description: This figure presents the solution to the life-cycle model outlined in Section 3. The model is solved using actuarial probabilities from the Social Security Administration (SSA). All other lines come from model solutions using the subjective survival beliefs in the Qualtrics survey, while employing different levels of the equity premium.