The I Theory of Money*

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Abstract

A theory of money needs a proper place for financial intermediaries. Intermediaries create inside money and their ability to take risks determines the money multiplier. In downturns, intermediaries shrink their lending activity and fire-sell their assets. Moreover, they create less inside money. As the money multiplier shrinks, the value of money rises. This leads to a Fisher disinflation that hurts intermediaries and all other borrowers. The initial shock is amplified, volatility spikes up and risk premia rise. An accommodative monetary policy in downturns, focused on the assets held by constrained agents, recapitalizes intermediaries and hence mitigates these destabilizing adverse feedback effects. A monetary policy rule that accommodates negative shocks and tightens after positive shocks, provides an ex-ante insurance, mitigates financial frictions, reduces endogenous risk and risk premia but it also creates moral hazard.

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1 Introduction

A theory of money needs a proper place for financial intermediaries. Financial institutions are able to create money, for example by accepting deposits backed by loans to businesses and home buyers. The amount of money created by financial intermediaries depends crucially on the health of the banking system and the presence of profitable investment opportunities. This paper proposes a theory of money and provides a framework for analyzing the interaction between price stability and financial stability. It therefore provides a unified way of thinking about monetary and macroprudential policy.

Intermediaries serve three roles. First, intermediaries *monitor* end-borrowers. Second, they *diversify* by extending loans to and investing in many businesses projects and home buyers. Third, they are active in maturity *transformation* as they issue short-term (inside) money and invest in long-term assets. Intermediation involves taking on some risk. Hence, a negative shock to end borrowers also hits intermediary levered balance sheets. Intermediaries' individually optimal response to an adverse shock is to lend less and accept fewer deposits. As a consequence, the amount of inside money in the economy shrinks. As the total demand for money as a store of value changes little, the value of outside money increases, i.e. disinflation occurs.

The disinflationary spiral in our model can be understood through two extreme polar cases. In one polar case the the financial sector is undercapitalized and cannot perform its functions. As the intermediation sector does not create any inside money, money supply is scarce and the value of money is high. Savers hold only outside money and risky projects. Savers are not equipped with an effective monitoring technology and cannot diversify. The value of safe money is high. In the opposite polar case, intermediaries are well capitalized. Intermediaries mitigate financial frictions and channel funds from savers to productive projects. They lend and invest across in many loans and projects, exploiting diversification benefits and their superior monitoring technology. Intermediaries also create short-term (inside) money and hence the money multiplier is high. In this polar case the value of money is low as inside money supply supplements outside money.

As intermediaries are exposed to end-borrowers' risk, an adverse shock also lowers the financial sector's risk bearing capacity. It moves the economy closer to the first polar regime with high value of money. In other words, a negative productivity shock leads to deflation of Fisher (1933). Financial institutions are hit on both sides of their balance sheets. On the asset side, they are exposed to productivity shocks of end-borrowers. End-borrowers' fire sales depress the price of physical capital and liquidity spirals further erode intermediaries' net worth (as shown in Brunnermeier and Sannikov (2014)). On the liabilities side, they are hurt by the Fisher disinflation. As intermediaries cut their lending and create less inside money, the money multiplier collapses and the real value of their nominal liabilities expands. The Fisher disinflation spiral amplifies the initial shock and the asset liquidity spiral even further.

Monetary policy can work against the adverse feedback loops that precipitate crises, by affecting the prices of assets held by constrained agents and redistributing wealth. Since monetary policy softens the blow of negative shocks and helps the reallocation of capital to productive uses, this wealth redistribution is not a zero-sum game. It can actually improve welfare. It can reduce endogenous (self-generated) risk and overall risk premia.

Simple interest rate cuts in downturns improve economic outcomes only if they boost prices of assets, such as long-term government bonds, that are held by constrained sectors. Wealth redistribution towards the constrained sector leads to a rise in economic activity and an increase in the price of physical capital. As the constrained intermediary sector recovers, it creates more (inside) money and reverses the disinflationary pressure. The appreciation of long-term bonds also mitigates money demand, as long-term bonds can be used as a store of value as well. As interest rate cuts affect the equilibrium allocations, they also affect the long-term *real* interest rate as documented by Hanson and Stein (2014) and term premia and credit spread as documented by Gertler and Karadi (2014). From an ex-ante perspective long-term bonds provide intermediaries with a hedge against losses due to negative macro shocks, appropriate monetary policy *rule* can serve as an insurance mechanism against adverse events.

Like any insurance, "stealth recapitalization" of the financial system through monetary policy creates a moral hazard problem. However, moral hazard problems are less severe as the moral hazard associated with explicit bailouts of failing institutions. The reason is that monetary policy is a crude redistributive tool that helps the strong institutions more than the weak. The cautions institutions that bought long-term bonds as a hedge against downturns benefit more from interest rate cuts than the opportunistic institutions that increased leverage to take on more risk. In contrast, ex-post bailouts of the weakest institutions create strong risk-taking incentives ex-ante.

Related Literature. Our approach differs in important ways from both the prominent New Keynesian approach but also from the monetarist approach. The New Keynesian approach emphasizes the interest rate channel. It stresses role of money as unit of account and price and wage rigidities are the key frictions. Price stickiness implies that a lowering nominal interest rates also lowers the real interest rate. Households bring consumption forward and investment projects become more profitable. Within the class of New Keynesian models Christiano, Moto and Rostagno (2003) is closest to our analysis as it studies the disinflationary spiral during the Great Depression.

In contrast, our I Theory stresses the role of money as store of value and a redistributional channel of monetary policy. Financial frictions are the key friction. Prices are fully flexible. This monetary transmission channel works primarily through capital gains, as in the asset pricing channel promoted by Tobin (1969) and Brunner and Meltzer (1972). As assets are not held symmetrically in our setting, monetary policy redistributes wealth and thereby mitigate debt overhang problems. In other words, instead of emphasizing the substitution effect of interest rate changes, in the I Theory wealth/income effects of interest rate changes are the driving force.

Like in monetarism (see e.g. Friedman and Schwartz (1963)), an endogenous reduction of money multiplier (given a fixed monetary base) leads to disinflation in our setting. However, in our setting outside money is only an imperfect substitute for inside money. Intermediaries, either by channeling funds through or by underwriting and thereby enabling firms to approach capital markets directly, enable a better capital allocation and more economic growth. Hence, in our setting monetary intervention should aim to recapitalize undercapitalized borrowers rather than simply increase the money supply across the board. A key difference to our approach is that we focus more on the role of money as a store of value instead of the transaction role of money. The latter plays an important role in the "new monetarists economics" as outlined in Williamson and Wright (2011) and references therein.

Instead of the "money view" our approach is closer in spirit to the "credit view" à la Gurley and Shaw (1955), Patinkin (1965), Tobin (1969, 1970), Bernanke (1983) Bernanke and Blinder (1988) and Bernanke, Gertler and Gilchrist (1999).[1]

As in Samuelson (1958) and Bewley (1980), money is essential in our model in the sense of Hahn (1973). In Samuelson households cannot borrow from future not yet born generations. In Bewley and Scheinkman and Weiss (1986) households face explicit borrowing limits and cannot insure themselves against idiosyncratic shocks. Agent's desire to self-insure through precautionary savings creates a demand for the single asset, money. In our model households

[1]The literature on credit channels distinguishes between the bank lending channel and the balance sheet channel (financial accelerator), depending on whether banks or corporates/households are capital constrained. Strictly speaking our setting refers to the former, but we are agnostic about it and prefer the broader credit channel interpretation.

can hold money and physical capital. The return on capital is risky and its risk profile differs from the endogenous risk profile of money. Financial institutions create inside money and mitigate financial frictions. In Kiyotaki and Moore (2008) money and capital coexist. Money is desirable as it does not suffer from a resellability constraint as physical capital does. Lippi and Trachter (2012) characterize the trade-off between insurance and production incentives of liquidity provision. Levin (1991) shows that monetary policy is more effective than fiscal policy if the government does not know which agents are productive. More recently, Cordia and Woodford (2010) introduced financial frictions in the new Keynesian framework. The finance papers by Diamond and Rajan (2006) and Stein (2012) also address the role of monetary policy as a tool to achieve financial stability.

More generally, there is a large macro literature which also investigated how macro shocks that affect the balance sheets of intermediaries become amplified and affect the amount of lending and the real economy. These papers include Bernanke and Gertler (1989), Kiyotaki and Moore (1997) and Bernanke, Gertler and Gilchrist (1999), who study financial frictions using a log-linearized model near steady state. In these models shocks to intermediary net worths affect the efficiency of capital allocation and asset prices. However, log-linearized solutions preclude volatility effects and lead to stable system dynamics. Brunnermeier and Sannikov (2014) also study full equilibrium dynamics, focusing on the differences in system behavior near the steady state, and away from it. They find that the system is stable to small shocks near the steady state, but large shocks make the system unstable and generate systemic endogenous risk. Thus, system dynamics are highly nonlinear. Large shocks have much more serious effects on the real economy than small shocks. He and Krishnamurthy (2013) also study the full equilibrium dynamics and focus in particular on credit spreads. In Mendoza and Smith's (2006) international setting the initial shock is also amplified through a Fisher debt-disinflation that arises from the interaction between domestic agents and foreign traders in the equity market. In our paper debt disinflation is due to the appreciation of inside money. For a more detailed review of the literature we refer to Brunnermeier et al. (2013).

This paper is organized as follows. Section 2 sets up the model and derives first the solutions for two polar cases. Sect ion 3 presents computed examples and discusses equilibrium properties, including capital and money value dynamics, the amount of lending through intermediaries, and the money multiplier for various parameter values. Section 4 introduces long-term bonds and studies the effect of interest-rate policies as well as open-market operations. Section 5 showcases a numerical example of monetary policy. Section 6 concludes.

2 The Baseline Model Absent Policy Intervention

The economy is populated by two types of agents: households and intermediaries. Each household can use capital to produce either good a or good b, but can only manage a single project at a time. Each project carries both idiosyncratic and aggregate good-specific risk. The two goods are then combined into an aggregate good that can be consumed or invested. Intermediaries help fund households that produce good a by buying their equity. Intermediaries pool these equity stakes in order to diversify the idiosyncratic risk, and obtain funding for these holdings by accepting money deposits. Households that produce good b cannot get outside funding.

Households can split their wealth between one project of their choice and money. There is outside money - currency, whose supply is fixed in the absence of monetary policy - and inside money - currency claims issued by intermediaries to finance their investments in equity of households that use technology a. The dynamic evolution of the economy is driven by the effect of shocks on the agents' wealth distribution, as reflected through their portfolio choice. The model is solved using standard portfolio choice theory, except that asset prices - including the price of money - are endogenous.

Technologies. All physical capital K_t in the world is allocated between the two technologies. If capital share ψ_t is devoted to produce good a, then goods a and b combined make $y(\psi)K_t$ of the aggregate good. Function $y(\psi)$ is concave and has an interior maximum, an example is the standard technology with constant elasticity of substitution s,[2]

$$y(\psi) = A\left(\frac{1}{2}\psi^{\frac{s-1}{s}} + \frac{1}{2}(1-\psi)^{\frac{s-1}{s}}\right)^{\frac{s}{s-1}}.$$

In competitive markets, the aggregate good $f(\psi)$ is divided between the two inputs according to the formulas

$$y^a(\psi) = (1-\psi)y'(\psi) + y(\psi) \quad \text{and} \quad y^b(\psi) = -\psi y'(\psi) + y(\psi),$$

when the price of each good reflects its marginal contribution to the aggregate good.[3]

[2]For $s = \infty$ the outputs are perfect substitutes, for $s = 0$ there is no substitutability at all, while for $s = 1$ the substitutability corresponds to that of a Cobb-Douglas production function.

[3]If total output is $y(\psi)K$, then an ϵ amount of capital devoted to technology a would change total output by

$$y\left(\frac{\psi K + \epsilon}{K + \epsilon}\right)(K + \epsilon).$$

Physical capital k_t is subject to shocks that depend on the technology in which it is employed. If used in technology a capital follows

$$\frac{dk_t}{k_t} = (\Phi(\iota_t) - \delta)\, dt + \sigma^a\, dZ_t^a + \tilde{\sigma}^a\, d\tilde{Z}_t, \tag{2.1}$$

where dZ_t^a is the sector-wide Brownian shock and $d\tilde{Z}_t$ are independent project-specific shocks, which cancel out in the aggregate. A similar equation applies if capital is used in technology b. Sector-wide shocks dZ_t^a and dZ_t^b are independent of each other. The investment function Φ has the standard properties $\Phi' > 0$ and $\Phi'' \leq 0$, and the input for investment ι_t is the aggregate good.

Preferences. All agents have identical logarithmic preferences with a common discount rate ρ. That is, any agent maximizes the expected utility of

$$E\left[\int_0^\infty e^{-\rho t} \log c_t\, dt\right],$$

subject to individual budget constraints, where c_t is the consumption of the aggregate good at time t.

Financing Constraints. We assume that households who produce good b and intermediaries cannot issue equity, but may possibly borrow money, i.e. issue claims with return identical to the return on money. These claims, or *inside money*, are therefore as safe as currency, the *outside money*.

Households that produce good a can issue equity to intermediaries, but they must retain a fraction $\chi_t \geq \underline{\chi}$ of equity. For our results, we can consider the limit as $\underline{\chi} \to 0$, i.e. there is no constraint on equity issuance, but to see clearly how the chain of intermediation functions, it is useful to consider a small positive value of $\underline{\chi}$, as we do below.

Assets, Returns and Portfolios. Each household can manage only to a single project using technology a or b, and cannot diversify the project's idiosyncratic risk. Intermediaries can hold the equity of households with projects in technology a, and can fully diversify the

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Differentiating with respect to ϵ at $\epsilon = 0$, we obtain

$$y'(\psi)\frac{K + \epsilon - (\psi K + \epsilon)}{(K+\epsilon)^2}(K+\epsilon) + y(\psi) = y'(\psi)(1-\psi) + y(\psi).$$

Likewise, the marginal contribution of capital devoted to technology b would be $y(\psi) - \psi y'(\psi)$. The sum of the two terms is $y(\psi)$ since the production technology is homogenous of degree 1.

idiosyncratic risks of these projects. Everybody can hold money or create money by borrowing money from other agents. Physical money is called *outside* money, whereas monetary IOUs created by other agents are called *inside money.* The two types of money are equivalent in terms of the returns that they earn. In the baseline model, there is a fixed amount of fiat money in the economy that pays zero interest.

Assume that the price of capital per unit follows a Brownian process of the form

$$\frac{dq_t}{q_t} = \mu_t^q \, dt + (\sigma_t^q)^T \, dZ_t, \tag{2.2}$$

where $dZ_t = [dZ_t^a, \, dZ_t^b]^T$ is the vector of aggregate technology shocks. Then the capital gains component of the return in capital, $d(k_t q_t)/(k_t q_t)$, can be found using Ito's lemma. The dividend yield is $(y^a(\psi) - \iota)/q_t$ for technology a and $(y^b(\psi) - \iota)/q_t$ for technology b.

The total return of an individual project in technology a is

$$dr_t^a = \frac{y^a(\psi_t) - \iota_t}{q_t} \, dt + \left(\Phi(\iota_t) - \delta + \mu_t^q + (\sigma_t^q)^T \sigma^a 1^a\right) dt + (\sigma_t^q + \sigma^a 1^a)^T \, dZ_t + \tilde{\sigma}^a \, d\tilde{Z}_t,$$

where 1^a is the column coordinate vector with a single 1 in position a. This return is split between the household that manages this project and the intermediary that finances it, but the split may not be even. Since the market is segmented, inside and outside equity holders generally demand different risk premia. Denote the required return on outside equity held by intermediaries by

$$dr_t^I = dr_t^a - \lambda_t \, dt.$$

Then households who choose to retain inside equity fraction χ_t earn the return of[4]

$$dr_t^\chi = dr_t^a + \frac{1 - \chi_t}{\chi_t} \lambda_t \, dt. \tag{2.3}$$

Together we have

$$dr_t^a = \chi_t \, dr_t^\chi + (1 - \chi_t) \, dr_t^I.$$

The return on technology b that the rest of the households earn is

$$dr_t^b = \frac{y^b(\psi_t) - \iota_t}{q_t} \, dt + \left(\Phi(\iota_t) - \delta + \mu_t^q + (\sigma_t^q)^T \sigma^b 1^b\right) dt + (\sigma_t^q + \sigma^b 1^b)^T \, dZ_t + \tilde{\sigma}^b \, d\tilde{Z}_t.$$

[4] In this equation, χ_t is the household's choice, and it is optimal to issue the maximal amount of equity, i.e. set $\chi_t = \underline{\chi}$, if $\lambda_t > 0$. Otherwise, $\lambda_t = 0$ and $dr_t^\chi = dr_t^a$ for all χ_t.

The optimal investment rate ι_t, which maximizes the return of any technology, is given by the first-order condition $1/q_t = \Phi'(\iota_t)$. We denote the investment rate that satisfies this condition by $\iota(q_t)$.

Total money supply is fixed absent monetary policy. The value of all money depends on the size of the economy. Denote the value of money by $p_t K_t$, and assume that p_t follows a Brownian process of the form

$$\frac{dp_t}{p_t} = \mu_t^p \, dt + (\sigma_t^p)^T \, dZ_t. \tag{2.4}$$

The law of motion of aggregate capital is

$$\frac{dK_t}{K_t} = (\Phi(\iota_t) - \delta) \, dt + \underbrace{\psi_t \sigma^a \, dZ_t^a + (1 - \psi_t)\sigma^b \, dZ_t^b}_{(\sigma_t^K)^T \, dZ_t}, \tag{2.5}$$

and the return on money is given just by the capital gains rate

$$dr_t^M = \frac{d(p_t K_t)}{p_t K_t} = \left(\Phi(\iota) - \delta + \mu_t^p + (\sigma_t^p)^T \sigma_t^K\right) dt + \underbrace{(\sigma_t^K + \sigma_t^p)^T \, dZ_t}_{(\sigma_t^M)^T \, dZ_t}.$$

When a household chooses to produce good a, its net worth follows

$$\frac{dn_t}{n_t} = x_t^a \, dr_t^\chi + (1 - x_t^a) \, dr_t^M - \zeta_t^a \, dt, \tag{2.6}$$

where x_t^a is the portfolio weight on its inside equity, ζ_t^a is its propensity to consume (i.e. consumption per unit of net worth), and dr_t^χ is given by (2.3). The net worth of a household who produces good b follows

$$\frac{dn_t}{n_t} = x_t^b \, dr_t^b + (1 - x_t^b) \, dr_t^M - \zeta_t^b \, dt. \tag{2.7}$$

The net worth of an intermediary follows

$$\frac{dn_t}{n_t} = x_t \, d\bar{r}_t^I + (1 - x_t) \, dr_t^M - \zeta_t \, dt, \tag{2.8}$$

where $\bar{r}_t^I$ denotes the return on households' outside equity dr_t^I with idiosyncratic risk diversified away, i.e. removed. If intermediaries use leverage, i.e. issue inside money, then of course

$x_t > 1$.

Equilibrium Definition. The agents start initially with some endowments of capital and money. Over time, they trade - they choose how to allocate their wealth between the assets available to them. That is, they solve their individual optimal consumption and portfolio choice problems to maximize utility, subject to the budget constraints (2.6), (2.7) and (2.8). Individual agents take prices as given. Given prices, markets for capital, money and consumption goods have to clear.

If the net worth of intermediaries is N_t and the world wealth is $(q_t + p_t)K_t$, then the intermediaries' net worth share is denoted by

$$\eta_t = \frac{N_t}{(q_t + p_t)K_t}. \tag{2.9}$$

Denote by α_t the fraction of households who choose to produce good a.

Definition. *Given any initial allocation of capital and money among the agents, an equilibrium is a map from histories $\{Z_s,\ s \in [0,t]\}$ to prices p_t, q_t and λ_t, the households' wealth allocation α_t, inside equity share $\chi_t \geq \underline{\chi}$, portfolio weights (x_t^a, x_t^b, x) and consumption propensities $(\zeta_t^a, \zeta_t^b, \zeta_t)$, such that*

(i) all markets, for capital, equity, money and consumption goods, clear

(ii) all agents choose technologies, portfolios and consumption rates to maximize utility (households who produce good a also choose χ_t).

One important choice here is that of households: each household can run only one project either in technology a or b. They must be indifferent between the two choices. Household who choose to produce good a must also choose how much equity to issue. If outside equity earns less than the return of technology a, these household would want to issue the maximal amount of outside equity, retaining only fraction $\chi_t = \underline{\chi}$. This happens in equilibrium only if intermediaries are willing to accept this supply of equity at a return discount, so that inside equity earns a premium. This is the case only if the intermediaries are well-capitalized. Otherwise, inside and outside equity of technology a earns the same return as technology a. In this case, households are indifferent with respect to the amount of equity they issue, so the equity issuance constraint does not bind.

2.1 Equilibrium Conditions.

Logarithmic utility has two well-known tractability properties. First, an agent with logarithmic utility and discount rate ρ consumes at the rate given by ρ times net worth. Thus, $\zeta_t = \zeta_t^a = \zeta_t^b = \rho$ and the market-clearing condition for the consumption good is

$$\rho(q_t + p_t)K_t = (y(\psi_t) - \iota_t)K_t. \tag{2.10}$$

Second, the excess return of any risky asset over any other risky asset is explained by the covariance between the difference in returns and the agent's net worth.

From (2.7), the net worth of households who employ technology b is exposed to aggregate risk of

$$\sigma_t^{Nb} = x_t^b \underbrace{\left(\sigma^b 1^b + \sigma_t^q - \sigma_t^M\right)}_{\nu_t^b} + \sigma_t^M$$

and idiosyncratic risk $x_t^b \sigma^j \, dZ_t^{b,j}$. Taking the covariance with the excess risk of technology b over money, we find that the expected excess return of technology b over money is

$$\frac{E_t[dr_t^b - dr_t^M]}{dt} = (\nu_t^b)^T \sigma_t^{Nb} + x_t^b \tilde{\sigma}^2. \tag{2.11}$$

The risk on technology a is split between inside equity holders, households who face aggregate risk of

$$\sigma_t^{Na} = x_t^a \underbrace{\left(\sigma^a 1^a + \sigma_t^q - \sigma_t^M\right)}_{\nu_t} + \sigma_t^M$$

and idiosyncratic risk of $x_t^a \tilde{\sigma}^a \, d\tilde{Z}_t$, and outside equity holders, intermediaries who face the aggregate risk of

$$\sigma_t^N = x_t \nu_t + \sigma_t^M.$$

The risk is split according to shares $[\chi_t, 1-\chi_t]$. In order for technology a to earn the required rate of return, we must have

$$\frac{E_t[dr_t^a - dr_t^M]}{dt} = (1-\chi_t)(\nu_t)^T \sigma_t^N + \chi_t((\nu_t)^T \sigma_t^{Na} + x_t^a (\tilde{\sigma}^a)^2) \tag{2.12}$$

Since the required return of intermediaries is given by

$$\underbrace{\frac{E_t[dr_t^I - dr_t^M]}{dt}}_{\frac{E_t[dr_t^a - dr_t^M]}{dt} - \lambda_t} = (\nu_t)^T \sigma_t^N, \tag{2.13}$$

it follows that

$$\lambda_t = \chi_t((\nu_t)^T \sigma_t^{Na} + x_t^a (\tilde{\sigma}^a)^2 - (\nu_t)^T \sigma_t^N) = \chi_t((x_t^a - x_t)|\nu_t|^2 + x_t^a(\tilde{\sigma}^a)^2) \geq 0,$$

with equality if $\chi_t > \underline{\chi}$. In this case the required returns of households and intermediaries who hold inside and outside equity stakes of projects in technology a are the same.

Households must be indifferent between investing in technologies a and b. The following proposition summarizes the relevant condition

Proposition 1. *In equilibrium*

$$(x_t^a)^2(|\nu_t|^2 + (\tilde{\sigma}^a)^2) = (x_t^b)^2(|\nu_t^b|^2 + (\tilde{\sigma}^b)^2). \tag{2.14}$$

Proof. See Appendix. □

Portfolio weights, given the net worth share of intermediaries and households, have to be consistent with the allocation of the fraction ψ_t of capital to technology a. Denote by

$$\pi_t = \frac{p_t}{q_t + p_t}$$

the fraction of the world wealth that is in the form of money. Then

$$x_t = \frac{(1 - \chi_t)\psi_t(1 - \pi_t)}{\eta_t}.$$

Furthermore, the net worth of households who employ technologies a and b, together, must add up to $1 - \eta_t$, i.e.,

$$\frac{\psi_t \chi_t (1 - \pi_t)}{x_t^a} + \frac{(1 - \psi_t)(1 - \pi_t)}{x_t^b} = 1 - \eta_t. \tag{2.15}$$

Finally, we have to describe how the state variable η_t, which determines prices of capital and money p_t and q_t, evolves over time. The law of motion of η_t, together with the specification of prices and allocations as functions of η_t, constitute the full description of equilibrium: i.e. the map from any initial allocation and a history of shocks $\{Z_s\ s \in [0,t]\}$ into the description of the economy at time t after that history. The following proposition characterizes the equilibrium law of motion of η_t.

Proposition 2. *The equilibrium law of motion of η_t is given by*

$$\frac{d\eta_t}{\eta_t} = (1-\eta_t)\left(x_t^2|\nu_t|^2 - (x_t^b)^2(|\nu_t^b|^2 + (\tilde{\sigma}^b)^2)\right)\,dt + (x_t\nu_t + \sigma_t^\pi)((\sigma_t^\pi)^T\,dt + dZ_t). \tag{2.16}$$

The law of motion of η_t is so simple because the earnings of intermediaries and households can be expressed in terms of risks they take on and the required equilibrium risk premia. The first term on the right-hand side reflects the relative earnings of intermediaries and households determined by the risks they take on. The second term on the right-hand side of (2.16) reflects mainly the volatility of η_t, due to the imperfect risk sharing between intermediaries and households.

Proof. The law of motion of total net worth of intermediaries, given the risks that they take, must be

$$\frac{dN_t}{N_t} = dr_t^M - \rho\,dt + x_t(\nu_t)^T(\underbrace{(x_t\nu_t + \sigma_t^M)}_{\sigma_t^N}\,dt + \,dZ_t). \tag{2.17}$$

The law of motion of world wealth $(q_t+p_t)K_t$, the denominator of (2.9), can be found from the total return on world wealth, after subtracting the dividend yield of ρ (i.e., aggregate consumption). To find the returns, we take into account the risk premia that various agents have to earn. We have

$$\frac{d((q_t+p_t)K_t)}{(q_t+p_t)K_t} = dr_t^M - \rho\,dt + (1-\pi_t)\underbrace{(\sigma_t^K + \sigma_t^q - \sigma_t^M)^T}_{(\sigma_t^q - \sigma_t^p)^T}\,dZ_t +$$

$$(1-\pi_t)(\psi_t\underbrace{((1-\chi_t)(\nu_t)^T\sigma_t^N + \chi_t((\nu_t)^T\sigma_t^{Na} + x_t^a(\tilde{\sigma}^a)^2))}_{\frac{E_t[dr_t^a - dr_t^M]}{dt}} + (1-\psi_t)\underbrace{((\nu_t^b)^T\sigma_t^{Nb} + x_t^b(\tilde{\sigma}^b)^2)}_{\frac{E_t[dr_t^b - dr_t^M]}{dt}}).$$

Recall that

$$\sigma_t^N = x_t \nu_t + \sigma_t^M, \quad \sigma_t^{Na} = x_t^a \nu_t + \sigma_t^M \quad \text{and} \quad \sigma_t^{Nb} = x_t^b \nu_t^b + \sigma_t^M$$

and note that

$$\psi_t \nu_t + (1-\psi_t)\nu_t^b = \sigma_t^q - \sigma_t^p.$$

Therefore, the law of motion of aggregate wealth can be written as[5]

$$\frac{d((q_t+p_t)K_t)}{(q_t+p_t)K_t} = dr_t^M - \rho\, dt + \underbrace{(1-\pi_t)(\sigma_t^q - \sigma_t^p)^T}_{-(\sigma_t^\pi)^T} (\sigma_t^M\, dt + dZ_t) +$$

$$(1-\pi_t)\left(\psi_t \; \left((1-\chi_t)\, x_t |\nu_t|^2 + \chi_t\, x_t^a (|\nu_t|^2 + (\tilde{\sigma}^a)^2)\right) + (1-\psi_t)\, x_t^b (|\nu_t^b|^2 + (\tilde{\sigma}^b)^2)\right) \, dt =$$

$$dr_t^M - \rho\, dt - (\sigma_t^\pi)^T\, (\sigma_t^M\, dt + dZ_t) + \eta_t\, x_t^2 |\nu_t|^2\, dt + (1-\eta_t)(x_t^b)^2 (|\nu_t^b|^2 + (\tilde{\sigma}^b)^2)\, dt,$$

where we used (2.15) and the indifference condition of Proposition 1.

Thus, using Ito's lemma, we obtain[6]

$$\frac{d\eta_t}{\eta_t} = (1-\eta_t)\left(x_t^2 |\nu_t|^2 - (x_t^b)^2 (|\nu_t^b|^2 + (\tilde{\sigma}^b)^2)\right)\, dt + (x_t \nu_t + \sigma_t^\pi)^T (\sigma_t^\pi\, dt + \, dZ_t)$$

□

3 Risk and the Value of Money.

We begin by discussing the determinants of risk, prices, the value of money and the agents' welfare in this model. This model shares the general property of economies with financial frictions - as in He and Krishnamurthy (2012) and (2013) and Brunnermeier and Sannikov

[5] Ito's lemma implies that $\sigma_t^\pi = (1-\pi)(\sigma_t^p - \sigma_t^q)$ and $\mu_t^\pi = (1-\pi)(\mu_t^p - \mu_t^q) - \sigma^\pi \sigma^p + (\sigma^\pi)^2$.

[6] If processes X_t and Y_t follow

$$dX_t/X_t = \mu_t^X\, dt + \sigma_t^X\, dZ_t \quad \text{and} \quad dY_t/Y_t = \mu_t^Y\, dt + \sigma_t^Y\, dZ_t,$$

then

$$\frac{d(X_t/Y_t)}{X_t/Y_t} = (\mu_t^X - \mu_t^Y)\, dt + (\sigma_t^X - \sigma_t^Y)^T (dZ_t - \sigma_t^Y\, dt).$$

(2014a) and (2014b) - that economic sectors are exposed to aggregate risk due to their activities. Some of the risk is endogenous due to changes in the valuations of assets. Specialization in the economy leads to natural protections and terms-of-trade hedges. For example, an undercapitalized sector earns higher risk premia due to the concentration of risk within the sector. Endogenous risk exacerbates asset misallocation, but may raise the level of earnings and thus faster recovery.

In addition these properties, new phenomena appear due to the introduction of money. The value of money is determined endogenously in equilibrium and, with debts denominated in money, this becomes an additional source of endogenous risk that plays a role in the above dynamics. In this section, abstracting from the risk of η_t, we study how the value of money is determined in this economy. The key determinant of the value of money, of course, is the level of idiosyncratic risk included in the model - an element that is generally absent from other models of economies with frictions. Exposure to idiosyncratic risk creates the demand for safety - the demand for money.

To keep things simple, in the first benchmark we do not consider the intermediary sector explicitly. Intermediaries reduce the amount of idiosyncratic risk in the economy, so we can consider the effects of a healthy intermediary sector indirectly by varying parameter $\tilde{\sigma}$ in an economy populated by households only. That is, we fix $\eta = 0$ and study how the prices of capital and money p and q - these are constant - depend on model parameters.

"Money regime:" Equilibrium in the absence of intermediaries. Our first benchmark allows us to understand idiosyncratic risk and the value of money in a very simple setting, in which many quantities can be computed in closed form. Assume that $\sigma^a = \sigma^b = \sigma$, $\tilde{\sigma}^a = \tilde{\sigma}^b = \tilde{\sigma}$ and that $\max_\psi\ y(\psi) = \bar{y}$ is maximized at $\psi = 1/2$. Then half of all households produce good a, and the rest, good b. Aggregate capital in the economy follows

$$\frac{dK_t}{K_t} = (\Phi(\iota_t) - \delta)\, dt + \frac{\sigma}{2}\, dZ_t^a + \frac{\sigma}{2}\, dZ_t^b.$$

The risk of aggregate capital equals the risk of money, since p is constant - with the total volatility of $\bar{\sigma} = \sqrt{\sigma^2/2}$. Any household who invests in technology a or b faces incremental risk of $\hat{\sigma} = \sqrt{\tilde{\sigma}^2 + \sigma^2/2}$ which is orthogonal to the risk of money/aggregate capital.

Effectively, the economy is equivalent to a single-good economy with aggregate risk σ

and project-specific risk $\hat{\sigma}$. In this economy, the market-clearing condition for output (2.10)

$$\bar{y} - \iota(q) = \rho \underbrace{(p + q)}_{q/(1-\pi)}. \tag{3.1}$$

Each household puts portfolio weight $1-\pi$ on capital, so its net worth is exposed to aggregate risk σ and project-specific risk $(1 - \pi)\hat{\sigma}$. The excess return on capital over money is the dividend yield of $(\bar{y} - \iota(q))/q$, since the capital gains rates are the same. Therefore, the asset-pricing condition of capital relative to money is

$$\frac{\bar{y} - \iota(q)}{q} = (1 - \pi)\hat{\sigma}^2 \quad \Rightarrow \quad \pi = 1 - \sqrt{\rho}/\hat{\sigma}. \tag{3.2}$$

We see that money can have value in equilibrium only if $\hat{\sigma}^2 > \rho$. As $\hat{\sigma}$ increases, the value of money relative to capital rises.

For a special investment function of the form $\Phi(\iota) = \log(\kappa\iota + 1)/\kappa$, we can also get closed-form expressions for the equilibrium prices of money and capital. Then (3.1) implies that

$$q = \frac{\kappa\bar{y} + 1}{\kappa\sqrt{\rho}\hat{\sigma} + 1} \qquad \text{and} \quad p = \frac{\hat{\sigma} - \sqrt{\rho}}{\sqrt{\rho}} q. \tag{3.3}$$

When the investment adjustment cost parameter κ is close to 0, i.e. $\Phi(\iota)$ is close to 1, then the price of capital q is goes to 1 (this is Tobin's q). As κ becomes large, the price of capital depends on dividend yield $\bar{y}$ relative to the discount rate ρ and the level of idiosyncratic risk that affects the value of money.

There is always an equilibrium in which money has no value. In that equilibrium the price of capital satisfies $\bar{y} - \iota(q) = \rho q$, which implies that

$$q = \frac{\kappa\bar{y} + 1}{\kappa\rho + 1}. \tag{3.4}$$

In this economy, the dividend yield on capital is $(\bar{y} - \iota_t)/q = \rho$ and expected return on capital is $\rho + \Phi(\iota_t) - \delta$. Subtracting the idiosyncratic risk premium of $\hat{\sigma}^2$ the required return on an asset that carries the same risk as the whole economy, or K_t, is

$$\rho - \hat{\sigma}^2 + \Phi(\iota_t) - \delta.$$

If this rate is lower than the growth rate of the economy, i.e. $\Phi(\iota_t) - \delta$, then an equilibrium in which money has positive value exists. Lemma ? in the Appendix generalizes these results to the case when $\sigma^a \neq \sigma^b$ and $\tilde{\sigma}^a \neq \tilde{\sigma}^b$.

This benchmark allows us to anticipate how the value of money may fluctuate in an economy with intermediaries. When η_t becomes close to 0, households face high idiosyncratic risk in both sectors, leading to a high value of money. In contrast, when η_t is large enough, then most of idiosyncratic risk is concentrated in sector b, as households in sector a pass on the idiosyncratic risk to intermediaries. This leads to a lower value of money.

Intermediary net worth and the value of money will generally fluctuate due to aggregate shocks Z^a and Z^b. Relative to world wealth - recall that η_t measures the intermediary net worth relative to total wealth - intermediaries are long shocks Z^a and short shocks Z^b when they invest in equity of households who produce good a. A fundamental assumption of our model is that intermediaries cannot hedge this aggregate risk exposure. Due to this, they may suffer losses, and losses force them to stop investing in equity of households who use technology a. The intermediary sector may become undercapitalized.

To be able to interpret more fully the results that follow, we would like to investigate what happens in this economy if intermediaries always can function perfectly - this leads to our second benchmark in which we assume that perfect sharing of aggregate risk between intermediaries and households is possible.

Economy with Perfect Sharing of Aggregate Risk. What happens if intermediaries and households can trade contracts based on systemic risk, i.e. risk of the form

$$(\sigma^a 1^a - \sigma^b 1^b)^T \, dZ_t?$$

Then agents share aggregate risk perfectly, so that aggregate risk exposure of both households and intermediaries is proportional to σ_t^K, and η_t, p_t and q_t have no volatility. Furthermore, perfect sharing of aggregate risk implies that households who produce good a will retain the minimal allowed fraction of equity, $\underline{\chi}$.

The following proposition characterizes the function $\pi(\eta)$ through a first-order differential equation, together with ψ_t, household leverage x_t^a and x_t^b, price q_t and the dynamics of η.

Proposition 3. *The function $\pi(\eta)$ satisfies the first-order differential equation*

$$\mu_t^\pi = \frac{\pi'(\eta)}{\pi(\eta)} \eta \mu_t^\eta, \tag{3.5}$$

where

$$\mu_t^\eta = -(1-\eta)(x_t^b)^2(\tilde{\sigma}^b)^2, \quad \mu_t^\pi = \rho + \mu_t^\eta, \tag{3.6}$$

and ψ_t, x_t^a, x_t^b *and* q_t *satisfy*

$$y(\psi_t) - \iota(q_t) = \frac{\rho q_t}{1-\pi_t}, \quad \frac{\psi_t \underline{\chi}\, \tilde{\sigma}^a/\tilde{\sigma}^b + 1 - \psi_t}{x_t^b} = \frac{1-\eta_t}{1-\pi_t} \quad x_t^a \tilde{\sigma}^a = x_t^b \tilde{\sigma}^b \quad \text{and} \tag{3.7}$$

$$\frac{y^a(\psi_t) - y^b(\psi_t)}{q_t} = \psi_t(\sigma^a)^2 - (1-\psi_t)(\sigma^b)^2 + \underline{\chi}\, x_t^a(\tilde{\sigma}^a)^2 - x_t^b(\tilde{\sigma}^b)^2. \tag{3.8}$$

Proof. See Appendix. □

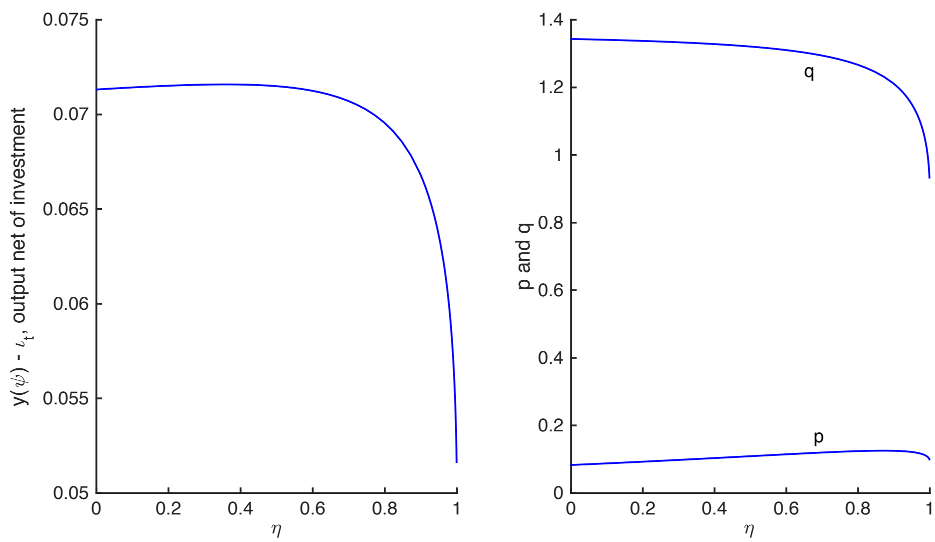

Figure 1: Equilibrium with perfect sharing of aggregate risk.

Figure 1 shows prices in the benchmark of perfect aggregate risk sharing for parameter values $\rho = 5\%$, $A = 0.5$, $\sigma^a = \sigma^b = 0.4$, $\tilde{\sigma}^a = 1.2$, $\tilde{\sigma}^b = 0.6$, $s = 0.8$, $\Phi(\iota) = \log(\kappa\iota+1)/\kappa$ with $\kappa = 2$, and $\underline{\chi} = 0.001$. The horizontal axis corresponds to the intermediary net worth share η_t. Due to perfect sharing of aggregate risk, intermediaries hold all available outside equity of households who produce good a, hedging risks perfectly, regardless of their net worth. As intermediary net worth rises, the net worth of households, and thus their capacity to absorb idiosyncratic risks, falls. Output falls as η_t rises, as seen in the left panel. The right panel shows how the prices of money and capital change with η_t. It is noteworthy that the value of money is very low relative to both the money regime and the full equilibrium that we describe in the next section. The value of money π_t relative to total wealth rises. The drift of η_t is always negative, as shown on the right panel and seen from (3.6).

In contrast, without intermediaries (3.3) implies that the prices of capital and money would be $q = 1.0655$ and $p = 3.79$ (see Lemma ? in the Appendix). The value of money is significantly higher under the benchmark without intermediaries who provide insurance against some of idiosyncratic risk of technology a. This fact creates the possibility of a significant deflationary spiral in our full model, in which the intermediaries have to absorb some of aggregate risk, and their capacity to function depends on having sufficient net worth.

4 Equilibrium in the Dynamic Model.

In this section, we explain the dynamics in the full equilibrium of the game, as well as welfare that players achieve. The computational procedure we employ, both with and without monetary policy, is described in Appendix . . . .

To facilitate comparison with the benchmarks of the previous section, we take the same parameter values that we used in Figure 1, i.e. $\rho = 0.05$, $A = 0.5$ $\sigma^a = \sigma^b = 0.4$, $\tilde{\sigma}^a = 1.2$, $\tilde{\sigma}^b = 0.6$, $s = 0.8$, $\Phi(\iota) = \log(\kappa\iota + 1)/\kappa$ with $\kappa = 2$, and $\underline{\chi} = 0.001$.

We start by looking at the allocation of capital, comparing it to the benchmark of perfect sharing of aggregate risk. The production of good a depends on intermediaries, and so it declines when intermediaries cannot fully hedge systemic risk and so must absorb some of it. The decline in the production of good a is particularly pronounced when intermediaries are undercapitalized. When η_t is very low, many households choose to produce good a without maximizing equity issuance to intermediaries, and so they inefficiently absorb idiosyncratic risk. See Figure 2.

Figure 3 shows the prices $p(\eta)$ and $q(\eta)$ of money and capital in equilibrium. At $\eta = 0$, the values of p and q converge to those under the benchmark without intermediaries, $q = 1.0532$ and $p = 3.4151$. As η rises, the price of capital rises and the price of money drops (although both fall near $\eta = 1$). Money becomes less valuable as η rises mainly because intermediaries create money. The inside money on the liabilities sides of the intermediaries' balance sheets is a perfect substitute to outside money. Even for high values of η, money is more valuable than under the benchmark of perfect sharing of aggregate risk.

Figure 4 illustrates the equilibrium dynamics through the drift and volatility of the state

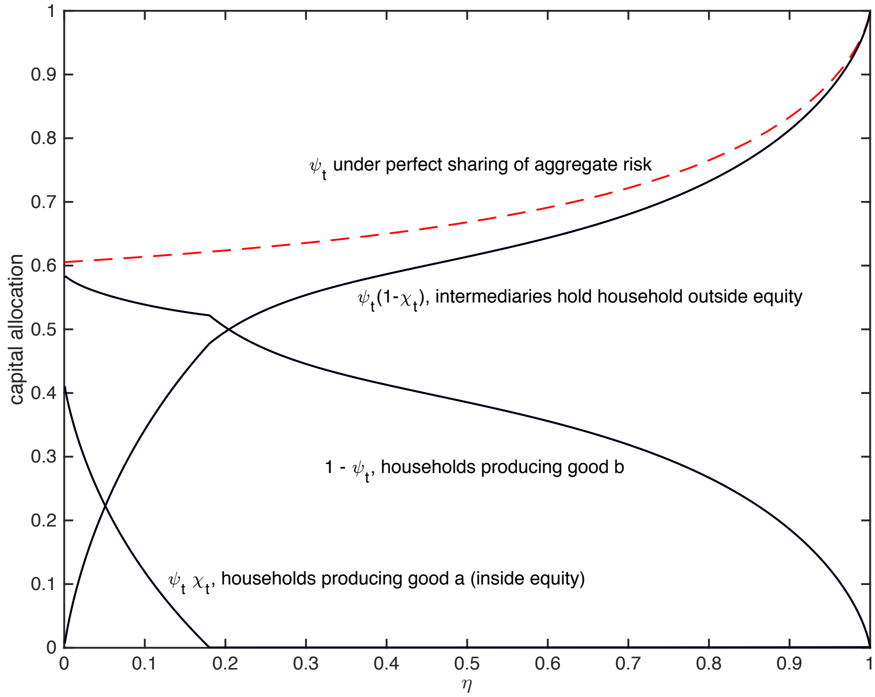

Figure 2: Equilibrium allocations.

variable η. From Proposition 2, the volatility of η_t is given by

$$\sigma_t^\eta \eta = \eta \left(x_t(\sigma^a 1^a - \sigma_t^K) + \sigma_t^\pi \left(1 - \frac{x_t}{1-\pi_t} \right) \right) dZ_t.$$

Variable η_t has volatility for two reasons: from the mismatch between the fundamental risk of assets that intermediaries hold, $\sigma^a dZ_t^a$, and overall risk in the economy and from amplification because of the endogenous fluctuations of $\pi(\eta_t)$ (the price of money relative to capital). As long as the intermediaries' portfolio share of households' equity is greater than $1-\pi_t$, the world capital share, and as long as $\pi'(\eta) < 0$, amplification exists. Figure 4 shows both the portion of volatility of η_t that arises from fundamental risk only, and total volatility that includes the effects of amplification. Amplification becomes prominent when intermediaries are undercapitalized.

The function $\pi(\eta) = p(\eta)/(q(\eta) + p(\eta))$ captures two amplification channels. First, the traditional amplification channel works on the asset sides of the intermediary balance sheets: as the price of physical capital $q(\eta)$ drops following a negative shock when η is low. In addition, shocks hurt intermediaries on the liability sides of the balance sheets through the

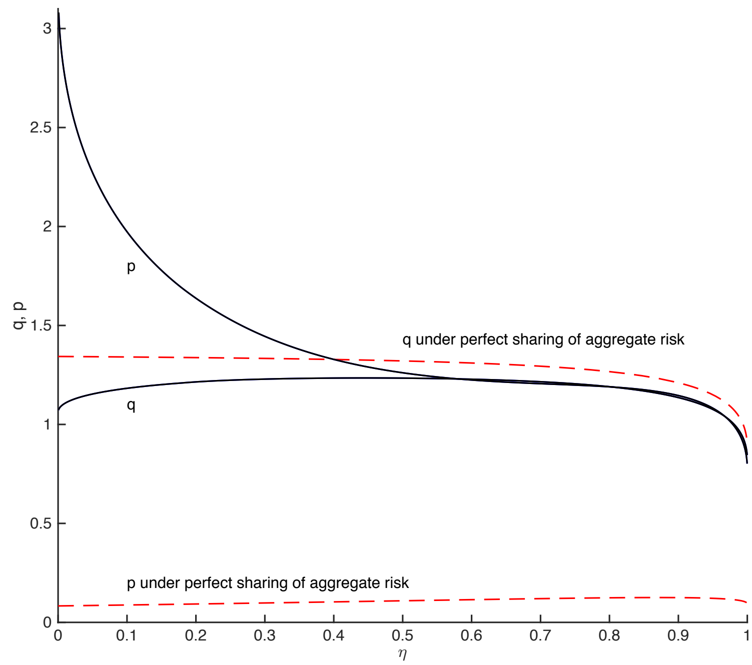

Figure 3: Equilibrium prices of capital and money.

Fisher disinflationary spiral. As we can observe from the money price curve $p(\eta)$ in Figure 3, money appreciates following a negative shock.

The drift of η_t is given by

$$\mu_t^\eta \eta = \eta(1-\eta)\left(x_t^2|\nu_t|^2 - (x_t^b)^2(|\nu_t^b|^2 + \tilde{\sigma}^2)\right) + (x_t\nu_t + \sigma_t^\pi)(\sigma_t^\pi)^T.$$

The first term captures the relative risk premia that intermediaries and households earn on their portfolios relative to money. As intermediaries become undercapitalized, the price of and return from producing good a rises, leading intermediaries to take on more risk. The opposite happens when intermediaries are overcapitalized - then the price of good b and the households' rate of earnings rises. The stochastic steady state of η_t is the point where the drift of η_t equals zero - at that point the earnings rates of intermediaries and households balance each other out. For comparison, Figure 4 also shows the drift of η_t under perfect sharing of aggregate risk - under those conditions, the earnings of intermediaries are always lower than those of households.

The dynamics in Figure 4, together with prices and allocations as functions of η in Figures 2 and 3 characterize the behavior of the economy in equilibrium. One prominent feature of

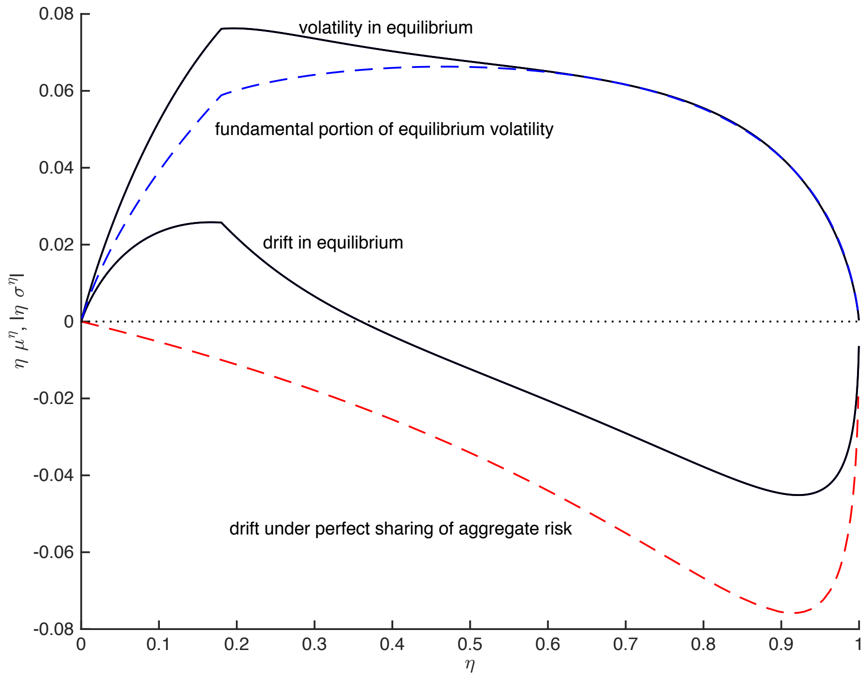

Figure 4: Equilibrium dynamics.

this behavior is the rise in the value of money when intermediaries become undercapitalized. The reason for the disinflationary pressure when intermediaries are undercapitalized is as follows. As intermediaries suffer losses, they contract their balance sheets. Thus, they take fewer deposits and create less inside money.[7] The total supply of money (inside and outside) shrinks and the money multiplier collapses, but the demand for money does not change significantly since saving households still want to allocate a portion of their savings to safe money. As a result, the value of money goes up.

4.1 Inefficiencies and Welfare.

In this section, we develop formal methodology to calculate welfare in our model. Before we formalize the computation of welfare - we propose two methods of computing it - we describe the sources of inefficiency in our model. We also emphasize relevant trade-offs with the intention of preparing ground for thinking about policy.

First, there is inefficient sharing of idiosyncratic risk. Some of it can be mitigated through

[7]In reality, rather than turning savers away, financial intermediaries might still issue demand deposits and simply park the proceeds with the central bank as excess reserves.

the use of intermediaries who can hold equity of households producing good a and diversify some of idiosyncratic risk. Consequently, cycles that can cause intermediaries to be undercapitalized can be harmful. Inefficiencies connected with idiosyncratic risks are also mitigated with the use of money - both inside and outside. Money allows households to diversify their wealth, but high value of money results in lower price of capital and potential inefficiency due to underinvestment.

Second, there is inefficient sharing of aggregate risk, which can cause whole sectors to become undercapitalized, e.g. intermediaries. If intermediaries become undercapitalized, barriers to entry into the intermediary sector help the intermediaries: the prices of goods 1 through I rise when η_t, mitigating the intermediaries risk exposures and allowing the intermediaries to recapitalize themselves. Thus, the limited competition in the intermediary activities creates a *terms-of-trade* hedge, which depends on the extent to which intermediaries cut back production in downturns, the extent to which households enter, and the substitutability s among the intermediate goods.

Finally, there is productive inefficiency: when intermediaries or households are undercapitalized, then production may be inefficiently skewed towards good a or good b. Even at the steady state production can be inefficient due to financial frictions, e.g. imperfect sharing of idiosyncratic risks.

To understand the cumulative effect of all these inefficiencies, one needs a proper welfare measure. We finish this section by proposing two methods to compute welfare in these types of models. The first method, which we call the *investment return method*, evaluates welfare by quantifying the quality of investment opportunities available to each class of agents. The second method, which we call the *economy size method*, focuses on how the growth of the whole economy and the changing wealth distribution affect welfare. We obtain two equivalent representations, with each providing a distinct set of intuitions about factors that affect welfare.

To evaluate welfare, one complicating factor is heterogeneity. We cannot focus on a representative household, since different households are exposed to different idiosyncratic risks. Some households become richer, while others become poorer. Both methods of evaluating welfare have to take this into account.

The Investment Return Method. The following proposition evaluates the welfare of any agent as a function of his/her investment opportunities.

Proposition 4. *The welfare of an agent with wealth n_t who can invest only in money takes* *the form $h^M(\eta_t) + \log(\rho n_t)/\rho$, where $h^M(\eta_t)$ satisfies*

$$h^M(\eta_t) + \frac{\log p_t}{\rho} = E\left[\int_t^\infty e^{-\rho(s-t)}\left(\log(p_s) + \frac{\Phi(\iota_s) - \delta - \rho - |\sigma_s^K|^2/2}{\rho}\right) ds\right] \tag{4.1}$$

The welfare of an intermediary with net worth n_t is $h^I(\eta_t) + \log(\rho n_t)/\rho$, where

$$h^I(\eta_t) - h^M(\eta_t) = E\left[\int_t^\infty e^{-\rho(s-t)}\frac{x_s^2|\nu_s|^2}{2\rho}\, ds\right] \tag{4.2}$$

The welfare of a household is $h^H(\eta_t) + \log(\rho n_t)/\rho$, where $h^H(\eta_t) - h^M(\eta_t)$ satisfies equation (4.2) with the term $x_s^2|\nu_s|^2$ replaced by $(x_s^b)^2(|\nu_s^b|^2 + (\tilde{\sigma}^b)^2)$.

Equation (4.2) evaluates the welfare of an individual agent by inferring the Sharpe ratio that the agent earns on risky investment from the risk that the agent chooses to take. The expectations (4.1) and (4.2) can be found numerically through an ordinary differential equation, since

$$g(\eta_t) = E\left[\int_t^\infty e^{-\rho(s-t)} y(\eta_s)\, ds\right] \quad\Rightarrow\quad \rho g(\eta) = y(\eta) + g'(\eta)\mu_t^\eta \eta + \frac{g''(\eta)|\eta\sigma_t^\eta|^2}{2}. \tag{4.3}$$

Note that, given the form of equation (4.2), it makes sense why Proposition 1 gives the right condition for the household to be indifferent between the production of goods a and b. Under condition (2.14), the household has the same welfare regardless of the technology it chooses to pursue.

Proof. With log utility, if the wealth of the agent increases by a factor of y, then his/her utility increases by $\log(y)/\rho$, since the agent increases consumption by a factor of y in perpetuity and keeps portfolio weights the same.

We can write the utility of an agent with wealth n_t who can invest only in money in the form $h^M(\eta_t) + \log(\rho)/\rho + \log(n_t)/\rho$, and if we express $n_t = p_t k_t$, then

$$\frac{dk_t}{k_t} = (\Phi(\iota_t) - \delta - \rho)\, dt + \sigma_t^K\, dZ_t,$$

since the agent consumes at rate ρ. Then the agent's utility has to satisfy the equation

$$\rho\left(h^M(\eta_t) + \frac{\log(\rho n_t)}{\rho}\right) = \log(\rho n_t) + \frac{E\left[d\left(h^M(\eta_t) + \log(k_t)/\rho + \log(p_t)/\rho\right)\right]}{dt} \quad\Rightarrow$$

$$\rho\left(h^M(\eta_t)+\frac{\log(p_t)}{\rho}\right)=\log(p_t)+\frac{\Phi(\iota_t)-\delta-\rho}{\rho}-\frac{|\sigma_t^K|^2}{2\rho}+\frac{E\left[d\left(h^M(\eta_t)+\log(p_t)/\rho\right)\right]}{dt}$$

so using Ito's lemma we can show that $h^M(\eta)+\log(p_t)/\rho$ satisfies (4.1).

The net worth of this agent follows $dn_t/n_t = dr_t^M - \rho\, dt$, and recall that the net worth of an intermediary follows (2.17), i.e.

$$\frac{dn_t^I}{n_t^I}=dr_t^M-\rho\,dt+x_t(\nu_t)^T(\underbrace{(x_t\nu_t+\sigma_t^M)}_{\sigma_t^N}\,dt+\,dZ_t).$$

If we write the utilities of these agents as $h_t^M+\log(n_t)/\rho$ and $h_t^I+\log(n_t^I)/\rho$ then we have

$$\rho\left(h_t^M+\frac{\log(\rho n_t)}{\rho}\right)=\log(\rho n_t)+\frac{1}{\rho}E\left[\frac{dn_t}{n_t}\right]/dt-\frac{|\sigma_t^M|^2}{2\rho}+E[dh_t^M]\quad\text{and}$$

$$\rho\left(h_t^I+\frac{\log(\rho n_t^I)}{\rho}\right)=\log(\rho n_t^I)+\frac{1}{\rho}E\left[\frac{dn_t^I}{n_t^I}\right]/dt-\frac{|\sigma_t^N|^2}{2\rho}+E[dh_t^I].$$

Subtracting, we find that

$$\rho(h_t^I-h_t^M)=\underbrace{\frac{2x_t(\nu_t)^T(x_t\nu_t+\sigma_t^M)}{2\rho}-\frac{|\sigma_t^N|^2-|\sigma_t^M|^2}{2\rho}}_{x_t|\nu_t|^2/(2\rho)}+E[dh_t^I-dh_t^M].$$

It follows that $h^I(\eta_t)-h^M(\eta_t)=h_t^I-h_t^M$ is represented by the stochastic expectation (4.2).

The logic for the characterization of the welfare of households is analogous. □

The Economy Size Method. The following proposition provides another way to evaluate the welfare of intermediaries and households.

Proposition 5. *The equilibrium utility of an intermediary with net worth n_t is $h^I(\eta_t)+\log(\rho n_t)/\rho$, where*

$$h^I(\eta_t)=E\left[\int_t^\infty e^{-\rho(s-t)}\left(\log(\eta_s(p_s+q_s))+\frac{\Phi(\iota_s)-\delta}{\rho}-\frac{|\sigma_s^K|^2}{2\rho}\right)\,ds\right]-\frac{\log(\eta_t(p_t+q_t))}{\rho}. \tag{4.4}$$

The equilibrium utility of a household is $h^H(\eta_t)+\log(\rho n_t)/\rho$, where

$$h^H(\eta_t)=E\left[\int_t^\infty e^{-\rho(s-t)}\left(\log((1-\eta_s)(p_s+q_s))+\frac{\Phi(\iota_s)-\delta}{\rho}-\frac{|\sigma_s^K|^2}{2\rho}\right)\,ds\right]-$$

$$\frac{\log((1-\eta_t)(p_t+q_t))}{\rho} + \frac{1}{2\rho}E\left[\int_t^\infty e^{-r(s-t)} \left(\left|\frac{\eta_s \sigma_s^\eta}{1-\eta_s}\right|^2 - |x_s^b|^2(|\nu_s^b|^2 + (\tilde{\sigma}^b)^2)\right)\, ds\right]. \quad (4.5)$$

The intuition behind equations (4.4) and (4.5) is as follows. Note that an intermediary with a unit net worth at time t will have the net worth of

$$\frac{\eta_s(p_s+q_s)K_s}{\eta_t(p_t+q_t)K_t}$$

at time $s \geq t$ and will consume ρ times net worth. The utility of consumption is

$$\log(\rho\eta_s(p_s+q_s)) - \log(\eta_t(p_t+q_t)) + \log\frac{K_s}{K_t},$$

and equation (4.4) reflects exactly that: the utility of an intermediary through the evolution of η_t and world capital.

Equation (4.5) follows the same logic, but adjusts for the risk that individual households take - including idiosyncratic risk - relative to the risk of $1-\eta_t$. Note that from (4.5), it is also clear why the condition of Proposition 1 is the right condition for households to be indifferent between producing goods a and b.

Proof. Consider an intermediary with net worth $n_t = y\eta_t(p_t+q_t)K_t$. The intermediary will consume $\rho y\eta_t K_t$, so

$$\rho\left(h^I(\eta_t) + \frac{\log(\rho\, y\, \eta_t(p_t+q_t)K_t)}{\rho}\right) =$$

$$\log(\rho\, y\, \eta_t(p_t+q_t)K_t) + \frac{E\left[d\left(h^I(\eta_t) + \log(\eta_t(p_t+q_t))/\rho + \log(K_t)/\rho\right)\right]}{dt} \quad \Rightarrow$$

$$\rho\left(h^I(\eta_t) + \frac{\log(\eta_t(p_t+q_t))}{\rho}\right) =$$

$$\log(\eta_t(p_t+q_t)) + \frac{\Phi(\iota_t)-\delta}{\rho} - \frac{|\sigma_t^K|^2}{2\rho} + \frac{E\left[d\left(h^I(\eta_t) + \log(\eta_t(p_t+q_t))/\rho\right)\right]}{dt} \quad \Rightarrow$$

$$h^I(\eta_t) + \frac{\log(\eta_t(p_t+q_t))}{\rho} = E\left[\int_t^\infty e^{-r(s-t)}\left(\log(\eta_s(p_s+q_s)) + \frac{\Phi(\iota_s)-\delta}{\rho} - \frac{|\sigma_s^K|^2}{2\rho}\right) ds\right],$$

which implies (4.4).

Now, consider a household with net worth $y(1-\eta_t)(p_t+q_t)K_t$. If the net worth of this agent had evolved like the total net worth of all households, i.e.

$$\frac{dN_t^H}{N_t^H} = dr_t^M - \rho\, dt + \frac{(1-\pi_t)\psi_t\chi_t}{1-\eta_t}((\nu_t)^T(\sigma_t^{Na}\, dt + \, dZ_t) + x_t^a(\tilde{\sigma}^a)^2\, dt)$$

$$+\frac{(1-\pi_t)(1-\psi_t)}{1-\eta_t}((\nu_t^b)^T(\sigma_t^{Nb}\, dt + \, dZ_t) + x_t^b(\tilde{\sigma}^b)^2\, dt),$$

then, likewise, the utility of this agent would be $h^{1-\eta}(\eta_t) + \log(\rho n_t)/\rho$, where

$$h^{1-\eta}(\eta_t) = E\left[\int_t^\infty e^{-r(s-t)}\left(\log((1-\eta_s)\frac{p_s}{\pi_s}) + \frac{\Phi(\iota_s)-\delta}{\rho} - \frac{|\sigma_s^K|^2}{2\rho}\right)\, ds\right] - \frac{\log((1-\eta_t)p_t/\pi_t)}{\rho}.$$

Now, of course the net worth of a household that specializes in good b follow instead

$$\frac{dn_t^b}{n_t^b} = dr_t^M - \rho\, dt + x_t^b((\nu_t^b)^T(\sigma_t^{Nb}\, dt + \, dZ_t) + x_t^b(\tilde{\sigma}^b)^2\, dt + \tilde{\sigma}_t^b\, d\tilde{Z}_t).$$

By (2.15),

$$\frac{\psi_t\chi_t(1-\pi_t)}{1-\eta_t}\frac{x_t^b}{x_t^a} = x_t^b - \frac{(1-\psi_t)(1-\pi_t)}{1-\eta_t}. \tag{4.6}$$

Thus, the difference in drifts of n_t^b and N_t^H is

$$D_t^\mu = \frac{\psi_t\chi_t(1-\pi_t)}{(1-\eta_t)x_t^a}x_t^b((\nu_t^b)^T(x_t^b\nu_t^b+\sigma_t^M)+x_t^b(\tilde{\sigma}^b)^2) - \frac{(1-\pi_t)\psi_t\chi_t}{(1-\eta_t)x_t^a}x_t^a((\nu_t)^T(x_t^a\nu_t^a+\sigma_t^M)+x_t^a(\tilde{\sigma}^a)^2)$$

$$= \frac{\psi_t\chi_t(1-\pi_t)}{(1-\eta_t)x_t^a}(x_t^b\nu_t^b - x_t^a\nu_t)^T\sigma_t^M$$

by Proposition 1.

The difference between squared volatilities of n_t^b and N_t^H is

$$D_t^\sigma = |\sigma_t^M + x_t^b\nu_t^b|^2 + |x_t^b|^2(\tilde{\sigma}^b)^2 - \left|\sigma_t^M + \frac{(1-\pi_t)\psi_t\chi_t}{1-\eta_t}\nu_t + \frac{(1-\pi_t)(1-\psi_t)}{1-\eta_t}\nu_t^b\right|^2 =$$

$$2x_t^b\nu_t^b\sigma_t^M + |x_t^b|^2|\nu_t^b|^2 + |x_t^b|^2(\tilde{\sigma}^b)^2 - 2\sigma_t^M\left(\frac{(1-\pi_t)\psi_t\chi_t}{1-\eta_t}\nu_t + \frac{(1-\pi_t)(1-\psi_t)}{1-\eta_t}\nu_t^b\right) -$$

$$\left|\frac{\eta_t\sigma_t^\eta}{1-\eta_t}\right|^2 = 2\frac{\psi_t\chi_t(1-\pi_t)}{(1-\eta_t)x_t^a}(x_t^b\nu_t^b - x_t^a\nu_t)^T\sigma_t^M + |x_t^b|^2(|\nu_t^b|^2 + (\tilde{\sigma}^b)^2) - \left|\frac{\eta_t\sigma_t^\eta}{1-\eta_t}\right|^2,$$

where we used (4.6) and the fact that

$$(1-\pi_t)(\psi_t \nu_t + (1-\psi_t)\nu_t^b) = -\sigma_t^\pi \quad \Rightarrow$$

$$\frac{(1-\pi_t)\psi_t\chi_t}{1-\eta_t}\nu_t + \frac{(1-\pi_t)(1-\psi_t)}{1-\eta_t}\nu_t^b = \frac{-\sigma_t^\pi}{1-\eta_t} - \underbrace{\frac{(1-\pi_t)\psi_t(1-\chi_t)}{1-\eta_t}\nu_t}_{\eta_t x_t \nu_t/(1-\eta_t)} = -\frac{\eta_t\sigma_t^\eta}{1-\eta_t}$$

Thus,

$$D_s^\mu - D_s^\sigma/2 = \frac{1}{2}\left(\left|\frac{\eta_t\sigma_t^\eta}{1-\eta_t}\right|^2 - |x_t^b|^2(|\nu_t^b|^2 + (\tilde{\sigma}^b)^2)\right) \quad \text{and}$$

$$h_t^H(\eta_t) - h^{1-\eta}(\eta_t) = E\left[\frac{1}{2\rho}\int_t^\infty e^{-r(s-t)}\left(\left|\frac{\eta_s\sigma_s^\eta}{1-\eta_s}\right|^2 - |x_s^b|^2(|\nu_s^b|^2 + (\tilde{\sigma}^b)^2)\right)\, ds\right].$$

This completes the proof. □

We finish this section by providing an example of how these methods of computing welfare can be applied in the "money regime" benchmark without intermediaries. We also comment informally on the goals of monetary policy, through the prism of welfare equations.

Welfare in the "money regime." Again, consider the version of our model without intermediaries, and assume that $\sigma^a = \sigma^b = \sigma$, $\tilde{\sigma}^a = \tilde{\sigma}^b = \tilde{\sigma}$, and that $\max_\psi\ y(\psi) = \bar{y}$ is attained at $\psi = 1/2$. In this case Section 3 provides closed-form expressions for the equilibrium values of money and capital. What about welfare? How does welfare in equilibrium with money compare to welfare in the money-less equilibrium? If the regulator can control the value of money by specifying a money holding requirement of the agents, will the money under optimal policy have greater value than in equilibrium, or lower value? Note that higher value of money allows agents to reduce their idiosyncratic risk exposure, but creates a distortion on the investment front, since the value of capital becomes lower.

First, we illustrate the use of the investment return method. By Proposition 4, the welfare of someone who can only invest in money is given by

$$h^M = \frac{\log\rho}{\rho} + \frac{\Phi(\iota) - \delta - \rho - \sigma^2/4}{\rho^2},$$

since $\sigma^K = \sigma(1^a + 1^b)/2$, and where $\Phi(\iota) = \log(q)/\kappa$ and q is given by (3.3). Since $x^a = x^b =$

$1-\pi$ and $\nu = -\nu^b = \sigma(1^a - 1^b)/2$, we have

$$(x_t^a)^2(|\nu_t|^2 + \tilde{\sigma}^2) = (1-\pi)^2(\sigma^2/2 + \tilde{\sigma}^2) = (1-\pi)^2\hat{\sigma}^2 = \rho.$$

It follows that household welfare is given by

$$h^H = \frac{\Phi(\iota) - \delta - \rho - \sigma^2/4}{\rho^2} + \underbrace{\frac{(x_t^a)^2(|\nu_t|^2 + \tilde{\sigma}^2)}{2\rho^2}}_{1/(2\rho)}. \tag{4.7}$$

Using the economy size method,

$$h^H = \frac{\Phi(\iota) - \delta - \sigma^2/4}{\rho^2} - \underbrace{\frac{(x_t^a)^2(|\nu_t|^2 + \tilde{\sigma}^2)}{2\rho^2}}_{1/(2\rho)}. \tag{4.8}$$

These identical closed-form expressions for welfare illustrate the equivalence of the two methods. The economy-size method directly values the harm of risk that individual agents are exposed to, relative to aggregate risk, while the investment return method values the premium that agents must earn for taking the risk, relative to the baseline of holding money. Thus, the harm from individual agent idiosyncratic risk exposure is reflected in the low economic growth, which gets reflected in the low return on money.

What if the regulator can control π by forcing the agents to hold specific amounts of money. In this case, we have to use the economy-size method which directly takes into account the cost idiosyncratic risk exposure, without assuming that agents earn the required return for the risk that they take.[8] Since the choice of π will affect asset prices, and thus wealth, we can calculate welfare per unit of capital in the economy to be

$$h^H(\pi) + \frac{\log(\rho(p+q))}{\rho} = \frac{\log(\rho(p+q))}{\rho} + \frac{\Phi(\iota(q)) - \delta - \sigma^2/4}{\rho^2} - \frac{(1-\pi)^2(\sigma^2/2 + \tilde{\sigma}^2)}{2\rho^2}, \tag{4.9}$$

where q and p are determined by the market clearing condition for consumption goods

$$\bar{y} - \iota(q) = \rho\underbrace{(p+q)}_{q/(1-\pi)} \quad \Rightarrow \quad q = \frac{\kappa\bar{y} + 1}{\kappa\rho/(1-\pi) + 1} \quad \text{if} \quad \Phi(\iota) = \frac{\log(\kappa\iota + 1)}{\kappa}.$$

[8] When the regulator controls portfolios, it is no longer true that the excess return of any risky asset over any other asset is explained by their covariance with the agent's net worth.

Proposition 6. *Welfare in equilibrium with money is always greater than that in the moneyless equilibrium, i.e. when* $\pi = 0$*. Furthermore, if* $\Phi(\iota) = \log(\kappa\iota + 1)/\kappa$*, then relative to the value of* π *in the equilibrium with money, optimal policy raises* π *if and only if* $\kappa \geq \rho$ *or*

$$\hat{\sigma} < \frac{2\sqrt{\rho}}{1 - \kappa\rho} \tag{4.10}$$

when $\kappa < \rho$*.*

Condition (4.10) reflects the trade-off between the role of money as an insurance asset, and the distortionary effect of investment of rising money value. When adjustment costs κ are large enough, these distortions are minimal. When the investment technology is flexible - κ is low - these distortions can be significant. The regulator may want to lower the value of money when idiosyncratic risk is large.

Proof. First, let us show that utility in the equilibrium with money is always greater than in that without money.

Comparing utility with $\pi = 0$ and in the equilibrium with money is like comparing

$$\frac{\Phi(\iota_0)}{\rho^2} - \frac{1}{2\rho(1-\pi)^2} \quad \text{vs} \quad \frac{\Phi(\iota_\pi)}{\rho^2} - \frac{1}{2\rho},$$

where $y - \iota(q) = \rho q$ and $y - \iota(q) = \rho q/(1-\pi)$.

Setting $\pi = 0$ in (4.9), we get the utility of

$$\frac{\Phi(\iota(q)) - \delta - \sigma^2/4}{\rho^2} - \frac{\hat{\sigma}^2}{2\rho^2} \leq$$

TO BE COMPLETED

Now, assume that $\Phi(\iota) = \log(\kappa\iota + 1)/\kappa$ and consider the optimal policy. Since $q = (\kappa\bar{y} + 1)/(\kappa\rho/(1-\pi) + 1)$ and $\Phi(\iota) = \log(q)/\kappa$, welfare (4.9) can be written as

$$\underbrace{\frac{\log(\kappa\bar{y}+1) - \log(\kappa\rho/(1-\pi)+1)}{\kappa\rho^2} - \frac{\delta + \sigma^2/4}{\rho^2} - \frac{(1-\pi)^2\hat{\sigma}^2}{2\rho^2}}_{h^H(\pi)} +$$

$$\underbrace{\frac{\log\rho + \log(\kappa\bar{y}+1) - \log(\kappa\rho/(1-\pi)+1) - \log(1-\pi)}{\rho}}_{\log(\rho(p+q))/\rho}$$

Maximizing welfare is equivalent to maximizing

$$-\left(\frac{1}{\rho}+\frac{1}{\kappa\rho^2}\right)\log(\kappa\rho+1-\pi)+\frac{1}{\kappa\rho^2}\log(1-\pi)-\frac{(1-\pi)^2\hat{\sigma}^2}{2\rho^2}. \tag{4.11}$$

Differentiating with respect to π we get

$$\left(\frac{1}{\rho}+\frac{1}{\kappa\rho^2}\right)\frac{1}{\kappa\rho+1-\pi}-\frac{1}{\kappa\rho^2(1-\pi)}+\frac{(1-\pi)\hat{\sigma}^2}{\rho^2}=$$

$$-\frac{1}{\rho}\frac{\pi}{(\kappa\rho+1-\pi)(1-\pi)}+\frac{(1-\pi)\hat{\sigma}^2}{\rho^2}=\frac{1}{\rho(1-\pi)}\left(\frac{-\pi}{\kappa\rho+1-\pi}+\frac{(\pi-1)^2\hat{\sigma}^2}{\rho}\right),$$

where the term in parentheses is increasing in π. For the equilibrium level of $\pi=1-\sqrt{\rho}/\hat{\sigma}$, this term becomes

$$\frac{\sqrt{\rho}/\hat{\sigma}-1}{\kappa\rho+\sqrt{\rho}/\hat{\sigma}}+1=\frac{2\sqrt{\rho}/\hat{\sigma}-1+\kappa\rho}{\kappa\rho+\sqrt{\rho}/\hat{\sigma}},$$

positive if and only if $2\sqrt{\rho}/\hat{\sigma}>1-\kappa\rho$. Thus, the welfare-maximizing policy raises π over the equilibrium level if and only if condition (4.10) holds. □

Welfare in equilibrium and preliminary thoughts on policy. Figure 5 shows welfare for parameter values we described at the beginning of this section. For an economy with K_t normalized to 1, Figure 5 shows the utility of a representative intermediary and a representative household (normalizing wealth dispersion among households to 0). Note that welfare of the form $h^I+\log(\rho n_t)/\rho$ depends on wealth $n_t=\eta_t(p_t+q_t)$, which in turn depends on the total price level.

The welfare of each agent type tends to increase in its wealth share, but only to a certain point. At the extreme, one class of agents becomes so severely undercapitalized that productive inefficiency makes everybody worse off - at those extremes redistribution towards the undercapitalized sector would be Pareto improving. Total welfare is maximized near the steady state of the system, but this property depends on the parameters we chose.

This is outline of the discussion that needs to be added. Effects of policy on welfare can be summarized by

- effect of policy on the value of money, and thus hedging of idiosyncratic risk and distortion
- redistribution of aggregate risk, and minimizing inefficiencies related to that such as underproduction when sector is undercapitalized

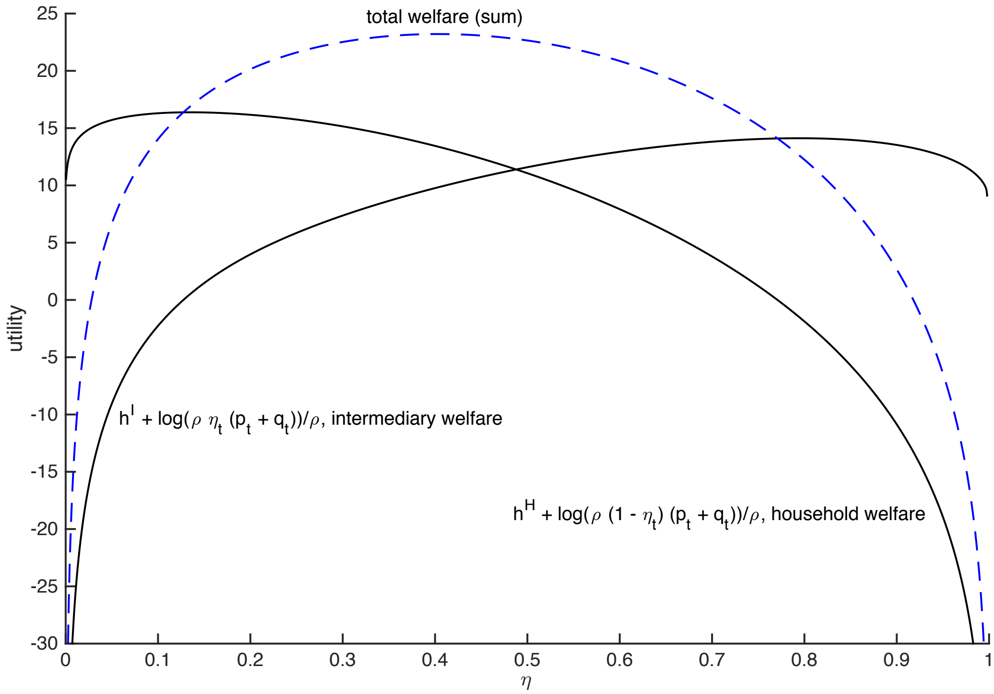

Figure 5: Equilibrium welfare.

- effects on relative competitiveness of each sector - e.g. policy could take away rents from a sector, and thus harm welfare

Conditions in the Limit as $\underline{\chi} \to 0$. Equations (2.13) and (2.12) simplify a bit if we let $\underline{\chi} \to 0$, and this version of the model also captures all the main results. Specifically, we have $\lambda_t = 0$, i.e. intermediary earn the return dr_t^a when they invest in equity of households who produce good a, but households may earn a premium in case $\chi_t = 0$. The relevant equations are

$$\frac{E_t[dr_t^a - dr_t^M]}{dt} = (\nu_t)^T \sigma_t^N \leq (\nu_t^a)^T \sigma_t^{Na} + x_t^a \tilde{\sigma}^2,$$

with strict inequality if $\chi_t = 0$, i.e. households are able to earn a premium only if there is an infinitesimal fraction of them using technology a and they sell 100% equity.

5 Monetary and Macro-prudential Policy

Policy has the potential to mitigate some of the inefficiencies that arise in equilibrium. It can undo some of the endogenous risk by redistributing wealth towards compromised sectors. It can control the creation of endogenous risk by affecting the path of deleveraging. It can also work to prevent the build-up of systemic risk in booms.

Policies affect the equilibrium in a number of ways, and can have unintended consequences. Interesting questions include: How does a policy affect equilibrium leverage? Does the policy create moral hazard? Does the policy lead to inflated asset prices in booms? What happens to endogenous risk? How does the policy affect the frequency of crises, i.e. episodes characterized by resource misallocation and loss of productivity?

We focus on several monetary policies in this section. These policies can be divided in several categories. Traditional monetary policy sets the short-term interest rate. It affects the yield curve through the expectation of future interest rates, as well as through the expected path of the economy, accounting for the supply and demand of credit. When the zero lower bound for the short-term policy rate becomes a constraint, forward guidance is an additional policy tool that is often employed in practice. The use of this tool depends on central bank's credibility, as it ties the central bank's hands in the future and leaves it less room for discretion. In this paper we assume that the central bank can perfectly commit to contingent future monetary policy and hence the interest rate policy incorporates some state-contingent forward guidance.

Several non-conventional policies have also been employed. The central bank can directly purchases assets to support prices or affect the shape of the yield curve. The central bank can lend to financial institutions, and choose acceptable collateral as well as margin requirements and interest rates. Some of these programs work by transferring tail risk to the central bank, as it suffers losses (and consequently redistributes them to other agents) in the event that the value of collateral becomes insufficient and the counterparty defaults. Other policies include direct equity infusions into troubled institutions. Monetary policy tools are closely linked to macroprudential tools, which involve capital requirements and loan-to-value ratios.

The classic "helicopter drop of money" has in reality a strong fiscal component as money is typically paid out via a tax rebate. Importantly, the helicopter drop also has redistributive effects. As the money supply expands, the nominal liability of financial intermediaries and hence the household's nominal savings are diluted. The redistributive effects are even stronger if the additional money supply is not equally distributed among the population but targeted to specific impaired (sub)sectors in the economy.

Instead of analyzing fiscal policy, we focus this paper on conventional and non-conventional monetary policy. For example, a change in the short-term policy interest rate redistributes wealth through the prices of nominal long-term assets. The redistributive effects of monetary policy depend on who holds these assets.[9] In turn, asset allocation depends on the anticipation of future policy, as well as the demand for insurance. Specifically, we introduce a perpetual long-term bond, and allow the monetary authority to both set the interest rate on short-term money, and affect the composition of outstanding government liabilities (money and long-term bonds) through open-market operations.

5.1 Extended Model with Long-term Bonds

Money and Long-Term Bonds. We extend our baseline model in two ways: we allow money to pay the floating rate interest and we introduce perpetual bonds, which pay interest at a fixed rate in money. Monetary policy sets interest $r_t \geq 0$ on money and controls the value $b_t K_t$ of all perpetual bonds outstanding. These policies are independent of fiscal policy - the monetary authority pays interest and performs open-market operations by printing money and not by using taxes.

We now denote by $p_t K_t$ the supply of all outstanding nominal assets: outside money and perpetual bonds. Also, let B_t be the endogenous equilibrium price of a single perpetual bond, which follows

$$\frac{dB_t}{B_t} = \mu_t^B \, dt + \sigma_t^B \, dZ_t. \tag{5.1}$$

Note that r_t and b_t are policy instruments, while B_t is an endogenous equilibrium process.

Returns. The expressions for the return on capital from Section 2 do not change, but money earns the return that depends on policy. If an agent holds all nominal assets in the economy - bonds and outside money - the return is

$$\left(\Phi(\iota) - \delta + \mu_t^p + \sigma_t^p (\sigma_t^K)^T\right) dt + (\sigma_t^K + \sigma_t^p) \, dZ_t.$$

This is the return on a portfolio with weights b_t/p_t on bonds and $1 - b_t/p_t$ on money. To isolate the returns on money and bonds, consider a strategy that buys bonds to earn dr_t^B by borrowing money, paying dr_t^M. We can find the payoff of this strategy by focusing on the

[9] Brunnermeier and Sannikov (2012) discuss the redistributive effects in a setting in which several sectors' balance sheets can be impaired. Forward guidance not to increase the policy interest rate in the near future has different implications than a further interest rate cut, since the former narrows the term spread while the latter widens it.

value of bonds in money. Using Ito's lemma,

$$dr_t^B - dr_t^M = \left(\frac{1}{B_t} - r_t + \mu_t^B + \sigma_t^B(\sigma_t^M)^T\right)\, dt + \sigma_t^B\, dZ_t,$$

where σ_t^M is the risk of money, which satisfies

$$\sigma_t^K + \sigma_t^p = \sigma_t^M + \frac{b_t}{p_t}\sigma_t^B \quad \Rightarrow \quad \sigma_t^M = \sigma_t^K + \sigma_t^p - \frac{b_t}{p_t}\sigma_t^B.$$

Thus, money earns the return of

$$dr_t^M = \left(\Phi(\iota) - \delta + \mu_t^p + \sigma_t^p(\sigma_t^K)^T\right) dt - \frac{b_t}{p_t}\left(\frac{1}{B_t} - r_t + \mu_t^B + \sigma_t^B(\sigma_t^M)^T\right)\, dt + \sigma_t^M\, dZ_t. \quad (5.2)$$

Equilibrium Conditions. For expositional purposes, we focus on policies that set the short-term interest rate r_t and the value of bonds b_t as functions of the state variable η_t. Then the price of bonds B_t will also be a function of η_t. Consider for concreteness policies that lead to a decreasing function $B(\eta)$, which follows from policies that cut the short-term interest rate when η_t is low, making bonds appreciate. Such a policy is designed to help intermediaries transfer some of the risk to households - by borrowing money and buying long-term bonds, intermediaries get a natural hedge that gives them insurance in the event that η_t drops and the entire intermediary sector suffer losses. The appreciation in bonds can offset partially other risks that the intermediaries face, including endogenous risks driven by amplification. In the equations below, we assume that intermediaries hold all long-term bonds as a hedge, and letter verify that this is indeed the case.

To write down the asset-pricing equations, we must first isolate the risk of buying capital (or bonds) by borrowing money. Since the return on money depends on monetary policy, we need to adjust our formulas. Thus, we have

$$\nu_t = \sigma^a 1^a - \sigma_t^K - \frac{\sigma^\pi}{1-\pi} + \frac{b_t}{p_t}\sigma^B \quad \text{and} \quad \nu_t^b = 1^b\sigma^b - \sigma_t^K - \frac{\sigma^\pi}{1-\pi} + \frac{b_t}{p_t}\sigma^B.$$

Intermediaries hold the risks of money, $\psi_t(1-\chi_t)$ of the world capital through outside equity exposure of households who use technology a, and all of the world bonds. Thus, the volatility of their net worth is

$$\sigma_t^N = \sigma_t^M + x_t\nu_t + x_t^B\sigma_t^B, \quad \text{where} \;\; x_t^B = \frac{\pi_t}{\eta_t}\frac{b_t}{p_t}$$

is the portfolio weight on bonds.

The pricing conditions for bonds is

$$\frac{1}{B_t} - r_t + \mu_t^B + (\sigma_t^B)^T \sigma_t^M = (\sigma_t^B)^T \sigma_t^N \leq (\sigma_t^B)^T (\sigma_t^M + x_t^a \nu_t),\ (\sigma_t^B)^T (\sigma_t^M + x_t^b \nu_t^b).$$

The pricing conditions for capital devoted to the production of goods a and b are

$$\frac{E_t[dr_t^a - dr_t^M]}{dt} = \underbrace{(\nu_t)^T \sigma^M + (1-\chi_t)(\nu_t)^T (x_t \nu_t + x_t^B \sigma_t^B) + \chi_t x_t^a \left(|\nu_t|^2 + \tilde{\sigma}^2\right)}_{(\nu_t)^T((1-\chi_t)\sigma_t^N + \chi_t \sigma_t^{Na}) + \chi_t x_t^a \tilde{\sigma}^2}$$

$$\text{and} \qquad \frac{E_t[dr_t^b - dr_t^M]}{dt} = (\nu_t^b)^T (\sigma_t^M + x_t^b \nu_t^b) + x_t^b \tilde{\sigma}^2.$$

The following proposition provides the law of motion of η_t with such a policy.

Proposition 7. *In equilibrium*

$$\frac{d\eta_t}{\eta_t} = (1-\eta_t)\left(\left|x_t \nu_t + x_t^B \sigma_t^B\right|^2 - (x_t^b)^2 (|\nu_t^b|^2 + \tilde{\sigma}^2)\right)\, dt +$$

$$\left(\sigma^\pi + x_t \nu_t + \frac{\pi_t - \eta_t}{\eta_t} \frac{b_t}{p_t} \sigma_t^B\right)\left(dZ_t + \left(\sigma^\pi - \frac{b_t}{p_t}\sigma^B\right)\, dt\right). \tag{5.3}$$

Proof. See Appendix. □

The effect of monetary policy on the asset-pricing conditions as well as the law of motion of η_t enters exclusively through the term $(b_t/p_t)\sigma_t^B$. If monetary policy sets the short-term interest rate r_t as well as the level of b_t as functions of η_t, then the risk of the bond price σ_t^B is collinear with $\sigma^a 1^a - \sigma_t^K$, σ_t^η and σ_t^π. Thus, monetary policy can be used to work against the endogenous risk that amplifies inefficient sharing of aggregate risk in this economy.

In fact, note that (5.3) implies that

$$\sigma_t^\eta = \frac{x_t\left(\sigma^a 1^a - \sigma_t^K\right)}{1 + (\psi_t(1-\chi_t) - \eta)\left(\frac{\pi'(\eta)}{\pi(\eta)} - (1-\pi(\eta))\frac{b_t}{p_t}\frac{B'(\eta)}{B(\eta)}\right) - \pi_t(1-\eta)\frac{b_t}{p_t}\frac{B'(\eta)}{B(\eta)}} \tag{5.4}$$

Policy gives rise to the extra term

$$\left((\psi_t(1-\chi_t) - \eta_t)(1-\pi_t) + \pi_t(1-\eta_t)\right)\frac{b_t}{p_t}\frac{B'(\eta)}{B(\eta)}$$

in this expression. This term reduces the amplification of fundamental risk, captured by the denominator of (5.4). Likewise, with policy (**??**) changes to

$$\nu_t = \left(1 - \frac{\pi'(\eta)}{\pi(\eta)(1-\pi(\eta))} + \frac{b_t}{p_t}\frac{B'(\eta)}{B(\eta)}\right)\left(\sigma^a 1^a - \sigma_t^K\right). \tag{5.5}$$

In the following section, we study the risk transfer effects of monetary policy by focusing on the term $(b_t/p_t)B'(\eta)/B(\eta)$. This one-dimensional function of η summarizes the effects of two policy tools r_t and b_t, with which any such function can be implemented in multiple ways.

5.2 Using policy to undo Endogenous Risk and Facilitate Risk Sharing.

It is illustrative to explore policies that set

$$\frac{b_t}{p_t}\sigma_t^B = \alpha_t \frac{\sigma^\pi}{1-\pi_t}, \tag{5.6}$$

thereby undoing a portion of the endogenous risk in (5.5). Given any such policy, the equilibrium can be found using the following procedure.

Computational Procedure. The function $\pi(\eta)$ can be determined by a second-order differential equation, and the following procedure provides a way to find $\pi''(\eta)$ from $(\eta, \pi(\eta), \pi'(\eta))$. First, we have to guess variables ψ and χ characterizing the allocation of capital and equity issuance, which satisfy the following conditions. Given ψ and χ, letting $w = \sigma_a 1^a - \sigma_b 1^b$, we have

$$\sigma_t^\eta \eta = \underbrace{\frac{x_t \eta(1-\psi)}{1 + (\psi(1-\chi)-\eta)(1-\alpha)\frac{\pi'(\eta)}{\pi(\eta)} - (1-\eta)\alpha\frac{\pi'(\eta)}{1-\pi(\eta)}}}_{\chi^\eta} w,$$

$$\sigma^\pi = \underbrace{\frac{\pi'(\eta)}{\pi(\eta)}\chi^\eta}_{\chi^\pi} w, \quad \sigma^\beta = \underbrace{\frac{\alpha}{1-\pi}\chi^\pi}_{\chi^\beta} w,$$

$$\nu = \underbrace{\left(1 - \psi - \frac{\chi^\pi}{1-\pi} + \chi^\beta\right)}_{\chi^\nu} w, \quad \nu^b = \underbrace{\left(-\psi - \frac{\chi^\pi}{1-\pi} + \chi^\beta\right)}_{\chi^{\nu,b}} w.$$

Define $x_t = \psi_t(1-\chi_t)(1-\pi_t)/\eta_t$, and solve for household leverage x_t^a and x_t^b from the relative pricing and indifference conditions

$$\frac{y^a(\psi_t) - y^b(\psi_t)}{q_t} - \frac{\chi^\pi}{1-\pi} w^2 = w\sigma^K - \chi^\beta w^2$$

$$+(1-\chi_t)(x_t(\chi^\nu)^2 + \frac{\pi}{\eta}\chi^\beta\chi^\nu)w^2 + \chi_t x_t^a((\chi^\nu)^2 w^2 + \tilde{\sigma}^2) - x_t^b((\chi^{\nu,b})^2 w^2 + \tilde{\sigma}^2)$$

$$\text{and} \qquad (x_t^a)^2((\chi^\nu)^2 w^2 + \tilde{\sigma}^2) = (x_t^b)^2((\chi^{\nu,b})^2 w^2 + \tilde{\sigma}^2).$$

The guesses of ψ and χ are correct if

$$\chi^\nu(x_t\chi^\nu + \frac{\pi}{\eta}\chi^\beta)w^2 \geq x_t^a((\chi^\nu)^2 w^2 + \tilde{\sigma}^2), \quad \text{with strict inequality if } \chi_t = \underline{\chi}$$

$$\text{and} \qquad \frac{\psi_t\chi_t}{x_t^a} + \frac{1-\psi_t}{x_t^b} = \frac{1-\eta_t}{1-\pi_t}.$$

Finally, find

$$\mu^\pi = (1-\pi)\left(\frac{y^b(\psi_t) - \iota_t}{q_t} + \frac{(\chi^\pi)^2 w^2}{1-\pi} - \frac{\sigma_b^2\chi^\pi}{1-\pi} + \chi^\beta(x_t\chi^\nu + \frac{\pi}{\eta}\chi^\beta)w^2 + \right.$$
$$\left.\left(\psi^b - 1 - \frac{\chi^\pi}{1-\pi}\right)(\chi^\beta w^2 - w\sigma^K) - x_t^b((\chi^{\nu,b})^2 w^2 + \tilde{\sigma}^2)\right)$$

$$\mu^\eta\eta = \eta(1-\eta)\left((x_t\chi^\nu + \frac{\pi}{\eta}\chi^\beta)^2 w^2 - (x_t^b)^2((\chi^{\nu,b})^2 w^2 + \tilde{\sigma}^2)\right) + \chi^\eta\left(\chi^\pi - \chi^\beta\right)w^2$$

$$\text{and} \quad \pi''(\eta) = \frac{2(\mu^\pi\pi(\eta) - \mu^\eta\eta\pi'(\eta))}{(\chi^\eta)^2 w^2}.$$

5.3 Welfare.

We have to adjust the equations from Section 4.1 somewhat since the baseline return on money is given by the generalized formula (5.2).

Proposition 8. *The welfare of an agent with wealth n_t who can invest only in money is* *given by* $\omega(\eta) + \log(n_t/p_t)/\rho$*, where* $\omega(\eta)$ *satisfies*

$$\rho\omega(\eta_t) = \log(\rho p_t) + \mu_t^\eta \eta\omega'(\eta) + \frac{|\sigma_t^\eta \eta|^2}{2}\omega''(\eta) +$$

$$\frac{\Phi(\iota) - \delta - \rho - (\sigma_t^K)^2 - \frac{b_t}{p_t}\sigma_t^B(x_t\nu_t + \frac{\pi_t}{\eta_t}\frac{b_t}{p_t}\sigma_t^B)^T + (\frac{b_t}{p_t}\sigma_t^B)^2/2}{\rho}. \tag{5.7}$$

The welfare of an intermediary with net worth n_t *is* $h(\eta_t) + \omega(\eta_t) + \log(n_t/p_t)/\rho$*, where*

$$\rho h(\eta) = \frac{x_t^2|\nu_t|^2}{2\rho} + h'(\eta)\mu_t^\eta\eta + \frac{h''(\eta)\eta^2\sigma_t^\eta(\sigma_t^\eta)^T}{2}. \tag{5.8}$$

The welfare of a household is $h^J(\eta_t) + \omega(\eta_t) + \log(n_t/p_t)/\rho$*, where* h^J *satisfies equation (4.2) with the term* $x_t^2|\nu_t|^2$ *replaced by* $(x_t^J)^2|\nu_t^J|^2$*.*

Proof. From the bond-pricing condition, the return on money (5.2) can be written as

$$\left(\Phi(\iota) - \delta + \mu_t^p + \sigma_t^p(\sigma_t^K)^T\right) dt - \frac{b_t}{p_t}\sigma_t^B(\sigma_t^N)\, dt + (\sigma_t^K + \sigma_t^p - \frac{b_t}{p_t}\sigma_t^B)\, dZ_t.$$

Conjecturing the form $\omega(\eta_t) + \log(n_t/p_t)/\rho$ for the value function of someone who invests only in money, the HJB equation is

$$\rho\omega(\eta_t) + \log(n_t/p_t) = \log(\rho n_t) + \mu_t^\eta\eta\omega'(\eta) + \frac{|\sigma_t^\eta\eta|^2}{2}\omega''(\eta) +$$

$$\frac{\Phi(\iota) - \delta - \rho + \mu_t^p + \sigma_t^p(\sigma_t^K)^T - \frac{b_t}{p_t}\sigma_t^B(\sigma_t^N)^T}{\rho} - \frac{(\sigma_t^K + \sigma_t^p - \frac{b_t}{p_t}\sigma_t^B)^2}{2\rho} - \frac{\mu_t^p}{\rho} + \frac{(\sigma_t^p)^2}{2\rho}.$$

Plugging in

$$\sigma_t^N = \sigma_t^M + x_t\nu_t + \frac{\pi_t}{\eta_t}\frac{b_t}{p_t}\sigma_t^B, \quad \sigma_t^M = \sigma_t^K + \sigma_t^p - \frac{b_t}{p_t}\sigma_t^B,$$

the equation for ω simplifies to (5.7). Furthermore, as in the proof of Proposition 4, the welfare of any agent with richer investment opportunities has to be adjusted by the incremental risk that the agent chooses to take. In particular, the welfare of an intermediary with net worth n_t is given by $h(\eta_t) + \omega(\eta_t) + \log(n_t/p_t)/\rho$, where $h(\eta)$ satisfies (5.8). Likewise, to get the welfare of households $h^J(\eta_t) + \omega(\eta_t) + \log(n_t/p_t)/\rho$, we have to replace the term $x_t^2|\nu_t|^2$ with $(x_t^J)^2|\nu_t^J|^2$. □

The relevant boundary conditions are the same as without policy, since the policy affects equilibrium dynamics only in the interior of the state space $[0, 1]$ and not on the boundaries.

5.4 Policies that undo Endogenous Risk.

The risk that intermediaries take on by buying more capital is captured by

$$\nu_t = \left(1 - \frac{\pi'(\eta)}{\pi(\eta)(1-\pi(\eta))} + \frac{b_t}{p_t}\frac{B'(\eta)}{B(\eta)}\right) \underbrace{\psi_t^J \left(\sigma^I \frac{1^I}{I} - \sigma^J \frac{1^J}{J}\right)}_{\text{fundamental risk}} =$$

$$(1+\alpha_t)\,\psi_t^J \left(\sigma^I \frac{1^I}{I} - \sigma^J \frac{1^J}{J}\right)$$

where the term $\frac{-\pi'(\eta)}{\pi(\eta)(1-\pi(\eta))} > 0$ amplifies the risk that intermediaries face and $\frac{b_t}{p_t}\frac{B'(\eta)}{B(\eta)}$ can mitigate it.

Likewise, the risk that households take on by buying capital is

$$\nu_t^j = (1^j\sigma^J - \sigma^K) + \left(-\frac{\pi'(\eta)}{\pi(\eta)(1-\pi(\eta))} + \frac{b_t}{p_t}\frac{B'(\eta)}{B(\eta)}\right) \underbrace{\psi_t^J \left(\sigma^I \frac{1^I}{I} - \sigma^J \frac{1^J}{J}\right)}_{\text{fundamental risk}},$$

which consists of idiosyncratic risk and average J-household risk

$$\nu_t^J = -(1-\psi^J)\left(\sigma^I \frac{1^I}{I} - \sigma^J \frac{1^J}{J}\right) + \alpha_t\psi_t^J \left(\sigma^I \frac{1^I}{I} - \sigma^J \frac{1^J}{J}\right) = (\alpha_t\psi_t^J - (1-\psi^J))\left(\sigma^I \frac{1^I}{I} - \sigma^J \frac{1^J}{J}\right)$$

It looks like $\alpha_t > 0$ in the absence of monetary policy, and monetary policy can reduce α_t. Effectively, this action shifts risk between technologies I and J - a reduction in α shifts risk in favor of I. Policy affects the world portfolio, and this (1) the total output/productive efficiency, (2) the amount of idiosyncratic risk exposure.

The agents' risk exposures are given by

$$\underbrace{\sigma_t^K + \sigma_t^q - \alpha_t\psi_t^J \left(\sigma^I \frac{1^I}{I} - \sigma^J \frac{1^J}{J}\right)}_{\sigma_t^M} + x_t^J(\alpha_t\psi_t^J - (1-\psi^J))\left(\sigma^I \frac{1^I}{I} - \sigma^J \frac{1^J}{J}\right) + x_t^J\, d\epsilon_t^j.$$

by which fundamental risk is amplified. We observe that amplification is significant, particularly below the steady state when intermediaries are undercapitalized.

In this section we consider the effect of policy that completely removes endogenous risk by setting instruments b_t and r_t in such a way that

$$\frac{\sigma^\pi}{1-\pi} = \sigma_t^p - \sigma_t^q = \frac{b_t}{p_t}\sigma_t^B. \tag{5.9}$$

This can be done in multiple ways, since we have the flexibility to choose two functions of η to match a single condition (5.9). If (5.9) holds, then

$$\nu_t = \sigma^I \frac{1^I}{I} - \sigma_t^K \quad \nu_t^i = \sigma^I 1^i - \sigma_t^K \quad \text{and} \quad \nu_t^j = \sigma^I 1^j - \sigma_t^K$$

i.e. the incremental risk that any agent faces by adding capital to his/her portfolio is only fundamental and not endogenous. In this section we show that the equilibrium dynamics that results under any such policy can be characterized in terms of a single second-order differential equation for the function $\pi(\eta)$.

First, the law of motion of η_t can be found from (5.3). Given (5.9), this reduces to

$$\frac{d\eta_t}{\eta_t} = (1-\eta_t)\left(\left|x_t\nu_t + \frac{\pi_t}{\eta_t}\frac{\sigma^\pi}{1-\pi}\right|^2 dt - |x_t^J\nu_t^J|^2\right) dt +$$
$$\left(\frac{1-\eta_t}{\eta_t}\frac{\pi_t\sigma^\pi}{1-\pi_t} + x_t\nu_t\right)\left(dZ_t - \frac{\pi\sigma^\pi}{1-\pi}\,dt\right). \tag{5.10}$$

Thus,

$$\sigma_t^\eta = x_t\left(\sigma^I\frac{1^I}{I} - \sigma_t^K\right) + \frac{1-\eta_t}{1-\pi_t}\,\pi'(\eta)\sigma^\eta \quad \Rightarrow \quad \sigma_t^\eta = \frac{x_t\left(\sigma^I\frac{1^I}{I} - \sigma_t^K\right)}{1 - \frac{1-\eta_t}{1-\pi_t}\,\pi'(\eta)},$$

where the denominator has dampening effect as long as $\pi'(\eta) < 0$, since it reduces the risk of η. Second, the function $\pi(\eta)$ it self can be found via the following procedure.

Procedure. *The function $\pi(\eta)$ that results under any policy that removes endogenous risk according to (5.9) has to satisfy the following second-order differential equation. First, given η and $(\pi(\eta), \pi'(\eta))$ the allocation of capital (ψ_t, ψ_t^J) must satisfy the conditions*

$$x_t = \frac{\psi_t(1-\pi(\eta))}{\eta_t}, \quad \sigma_t^\eta = \frac{x_t\left(\sigma^I\frac{1^I}{I} - \sigma_t^K\right)}{1 - \frac{1-\eta_t}{1-\pi_t}\,\pi'(\eta)}, \quad \sigma_t^\pi = \frac{\pi'(\eta)}{\pi(\eta)}\sigma_t^\eta\eta, \quad Y - \iota(q) = \frac{\rho q}{1-\pi(\eta)},$$

$$x_t|\nu_t|^2 + \nu_t\frac{\pi_t}{\eta_t}\frac{(\sigma^\pi)^T}{1-\pi} = x_t^I|\nu_t^I|^2, \quad \frac{(P_t^i - P_t^j)a}{q_t} = (\sigma^I 1^i - \sigma^J 1^j)(\sigma_t^K)^T + x_t^I|\nu_t^I|^2 - x_t^J|\nu_t^J|^2. \tag{5.11}$$

$$\eta + \underbrace{\frac{\psi^J(1-\pi(\eta))}{x_t^J}}_{\eta^J} + \underbrace{\frac{(1-\psi^J-\psi)(1-\pi(\eta))}{x_t^I}}_{\eta^I} = 1 \quad \textit{and} \quad (x_t^I)^2|\nu_t^I|^2 \leq (x_t^J)^2|\nu_t^J|^2,$$

with equality of $\psi + \psi^J < 1$. Then,

$$\frac{P_t^I a - \iota}{q} + \frac{\sigma^\pi}{1-\pi}\sigma_t^\eta = \nu_t \left(\sigma_t^\eta + \frac{\pi_t \sigma^\pi}{1-\pi_t} + \sigma_t^K\right)^T + \frac{\mu^\pi}{1-\pi} \tag{5.12}$$

$$\textit{and} \quad \pi''(\eta) = \frac{\mu^\pi \pi(\eta) - \pi'(\eta)\mu_t^\eta \eta}{\eta^2 \sigma_t^\eta (\sigma_t^\eta)^T/2},$$

where the volatility of η, μ_t^η, is taken from (5.10).

Proof. We only need to justify the expressions that appear in the above procedure for the first time. Note that

$$\sigma_t^N = \underbrace{\sigma_t^K + \sigma_t^p - \frac{b_t}{p_t}\sigma_t^B}_{\sigma_t^M = \sigma_t^K + \sigma_t^q} + x_t \left(\sigma^I \frac{1^I}{I} - \sigma_t^K\right) + \frac{\pi_t}{\eta_t}\frac{\sigma^\pi}{1-\pi},$$

Subtracting the capital pricing conditions for intermediaries and households who hold capital types $i = 1, \dots I$, we obtain

$$x_t|\nu_t|^2 + \nu_t \frac{\pi_t}{\eta_t}\frac{(\sigma^\pi)^T}{1-\pi} = x_t^I|\nu_t^I|^2,$$

since $\nu_t(\sigma_t^M)^T = \nu_t^i(\sigma_t^M)^T$. Furthermore, the capital pricing conditions of intermediaries and households who hold capital types $j = I+1, \dots I+J$, we find

$$\frac{(P_t^i - P_t^j)a}{q_t} + (\sigma_t^I 1^i - \sigma_t^j 1^j)(\sigma_t^q)^T = x_t^I|\nu_t^I|^2 - x_t^J|\nu_t^J|^2 + (\sigma_t^I 1^i - \sigma_t^J 1^j)(\sigma_t^K + \sigma_t^q)^T,$$

which simplifies to the second equation in (5.11).

Finally, from the capital and bond pricing conditions for intermediaries, we obtain

$$\frac{E[dr_t^I - dr_t^M]}{dt} = \frac{P_t^I a - \iota}{q} + \mu_t^q - \mu_t^p + \sigma_t^q \left(\sigma^I \frac{1^I}{I}\right)^T - \sigma_t^p(\sigma_t^K)^T + \frac{\sigma^\pi}{1-\pi}(\sigma_t^N)^T = \nu_t \sigma_t^N$$

Since

$$\sigma_t^N = \sigma_t^\eta + \frac{\pi_t \sigma^\pi}{1-\pi_t} + \sigma_t^M \quad \text{and}$$

$$\mu_t^p - \mu_t^q = \frac{\sigma^\pi}{1-\pi}\sigma^p - \frac{(\sigma^\pi)^2}{1-\pi} + \frac{\mu^\pi}{1-\pi}, \qquad \sigma_t^p - \sigma_t^q = \frac{\sigma_t^\pi}{1-\pi},$$

we have

$$\frac{P_t^I a - \iota}{q} + \frac{\sigma^\pi}{1-\pi}\sigma_t^\eta = \nu_t \left(\sigma_t^\eta + \frac{\pi_t \sigma^\pi}{1-\pi_t} + \sigma_t^K\right)^T + \frac{\mu^\pi}{1-\pi},$$

as required. □

We can verify analytically that for this policy, only intermediaries choose to hold long-term bonds. The following proposition states this fact.

Proposition 9. *Under the policy that removes endogenous risk, if $\pi'(\eta) < 0$ for all $\eta \in (0,1)$, then only intermediaries want to hold bonds at all points of the state space. That is, the required risk premium for bond holdings is lower for intermediaries than any other agents, i.e.*

$$\sigma_t^B (\sigma_t^N)^T \leq \sigma_t^B (\sigma_t^M + x_t^J \nu_t^J)^T, \ \sigma_t^B (\sigma_t^M + x_t^I \nu_t^I)^T, \tag{5.13}$$

where $\sigma_t^N = \sigma_t^M + x_t \nu_t + \frac{\pi_t}{\eta_t}\frac{\sigma^\pi}{1-\pi_t}$.

Proof. First, note that

$$\nu_t = \psi^J \left(\sigma^I \frac{1^I}{I} - \sigma_t^J \frac{1^J}{J}\right), \quad |\nu_t|^2 = \nu_t^I \nu_t = (\psi_t^J)^2 \left(\frac{(\sigma^I)^2}{I} - \frac{(\sigma^J)^2}{J}\right)$$

$$\text{and} \quad \nu_t (\nu_t^J)^T = -(1-\psi^J)\psi^J \left(\frac{(\sigma^I)^2}{I} - \frac{(\sigma^J)^2}{J}\right).$$

Furthermore,

$$\sigma_t^B = \frac{p_t}{b_t}\frac{\sigma_t^\pi}{1-\pi_t} = \underbrace{\frac{p_t}{b_t}\frac{\pi'(\eta)}{\pi_t(1-\pi_t)}\frac{\eta_t x_t}{1 - \frac{1-\eta_t}{1-\pi_t}\pi'(\eta)}}_{A<0}\nu_t.$$

Thus, (5.13) is equivalent to

$$\nu_t \left(x_t \nu_t + \frac{\pi_t}{\eta_t}\frac{\sigma^\pi}{1-\pi_t}\right)^T \geq x_t^J \nu_t (\nu_t^J)^T, \ x_t^I \nu_t (\nu_t^I)^T.$$

The first inequality holds since $x_t^J \nu_t (\nu_t^J)^T < 0$ and

$$x_t \nu_t + \frac{\pi_t}{\eta_t} \frac{\sigma^\pi}{1-\pi_t} = x_t \nu_t + \frac{\pi'(\eta)}{1-\pi_t} \underbrace{\frac{x_t}{1 - \frac{1-\eta_t}{1-\pi_t}\pi'(\eta)}}_{\sigma_t^\eta} \nu_t = x_t \underbrace{\frac{1 + \frac{\eta_t}{1-\pi_t}\pi'(\eta)}{1 - \frac{1-\eta_t}{1-\pi_t}\pi'(\eta)}}_{>0} \nu_t,$$

so $\nu_t \left(x_t \nu_t + \frac{\pi_t}{\eta_t} \frac{\sigma^\pi}{1-\pi_t} \right)^T > 0$. The second inequality holds since

$$\nu_t \left(x_t \nu_t + \frac{\pi_t}{\eta_t} \frac{\sigma^\pi}{1-\pi_t} \right) = x_t^I |\nu_t^I|^2,$$

and $|\nu_t^I|^2 > |\nu_t|^2$, specifically,

$$|\nu_t^I|^2 = |\nu_t|^2 + (\sigma^I)^2 \frac{I-1}{I}.$$

$\square$

Who holds the bonds... note also that the intermediaries' greater willingness to hold bonds depends on

$$\sigma^\pi (x\nu + \pi/\eta\sigma^\pi/(1-\pi)) < \sigma^\pi x^J \nu^J,$$

i.e. the intermediaries' required risk premium for holding bonds is lower...

Implementation. The condition (5.9) requires

$$\frac{d}{d\eta} \log \pi(\eta) = \frac{b_t(1-\pi_t)}{p_t} \frac{d}{d\eta} \log B(\eta).$$

This makes it pretty easy to solve for $\log B(\eta)$. Since B is steeper than π, the policy may need to buy access bonds when η is low (quantitative easing) and reverse the policy when η is high.

compare...

$$\frac{P_t^I a - \iota}{q_t} = \left(\nu_t - \frac{\sigma_t^\pi}{1-\pi_t} \right) (\sigma_t^\eta + \sigma_t^K)^T + \sigma_t^K (\sigma_t^\pi)^T + \left(\sigma^I \frac{1^I}{I} \right) \frac{\pi_t (\sigma^\pi)^T}{1-\pi_t} + \frac{\mu^\pi}{1-\pi}$$

$$\frac{\mu^\pi}{1-\pi} = \frac{P_t^I a - \iota}{q} - (\sigma_t^\eta + \sigma_t^K)\nu_t^T - \sigma_t^K (\sigma_t^\pi)^T - \frac{\pi \sigma_t^\pi}{1-\pi} \left(\sigma^I \frac{1^I}{I} \right)^T$$

Combining (2.13) and (**??**), we obtain $x_t|\nu_t|^2 = x_t^I|\nu_t^I|^2$ since $\nu_t(\sigma_t^K + \sigma_t^p) = \nu_t^I(\sigma_t^K + \sigma_t^p)$. Subtracting (**??**) and (**??**), we obtain

$$\frac{(P_t^I - P_t^J)a}{q} + (\sigma^I 1^i - \sigma^J 1^j)(\sigma_t^q)^T = x_t^I|\nu_t^I|^2 - x_t^J|\nu_t^J|^2 + (\sigma^I 1^i - \sigma^J 1^j)(\sigma_t^K + \sigma_t^p)^T,$$

which implies the second equation in (**??**) since $\sigma_t^p - \sigma_t^q = \sigma_t^\pi/(1-\pi)$. Finally, (2.13) is expanded to

$$\frac{P_t^I a - \iota}{q} + \mu_t^q - \mu_t^p + \sigma_t^q \left(\sigma^I \frac{1^I}{I}\right)^T - \sigma_t^p(\sigma_t^K)^T = x_t\nu_t\nu_t^T + \nu_t(\sigma_t^K + \sigma_t^p)^T \tag{5.14}$$

Since

$$\mu_t^p - \mu_t^q = \frac{\sigma^\pi}{1-\pi}\sigma^p - \frac{(\sigma^\pi)^2}{1-\pi} + \frac{\mu^\pi}{1-\pi}, \qquad \sigma_t^p - \sigma_t^q = \frac{\sigma_t^\pi}{1-\pi},$$

and $\sigma_t^\eta = x_t\nu_t + \sigma_t^\pi$, we have (5.14) can be transformed to

$$\frac{P_t^I a - \iota}{q} - \frac{\mu^\pi}{1-\pi} = (\sigma_t^\eta + \sigma_t^K)\nu_t^T + \sigma_t^\pi(\sigma_t^K)^T + \sigma^I\frac{1^I}{I}\frac{\pi\sigma_t^\pi}{1-\pi},$$

which confirms (5.12).

We have

$$\frac{(P_t^i - P_t^j)a}{q_t} + \sigma_t^q(\sigma^i 1^i - \sigma^j 1^j) = \nu_t(\sigma_t^M + x_t\nu_t + \frac{\pi_t}{\eta_t}\frac{\sigma^\pi}{1-\pi})^T - \nu_t^J(\sigma_t^M + x_t^J\nu_t^J)^T.$$

$$\frac{(P_t^i - P_t^j)a}{q_t} = (\sigma^i 1^i - \sigma^j 1^j)\sigma_t^K + x_t|\nu_t|^2 + \nu_t\frac{\pi_t}{\eta_t}\frac{(\sigma^\pi)^T}{1-\pi} - x_t^J|\nu_t^J|^2.$$

The pricing conditions for capital and money on the intermediaries balance sheets are

$$\frac{E_t[dr_t^I - dr_t^M]}{dt} = \nu_t(\sigma_t^N)^T \quad \text{and} \quad \frac{1}{B_t} - r_t + \mu_t^B + \sigma_t^B(\sigma_t^M)^T = \sigma_t^B(\sigma_t^N)^T.$$

Likewise, the households optimal allocations to capital x_t^I and x_t^J must satisfy

$$\frac{E_t[dr_t^I - dr_t^M]}{dt} = \nu_t^I(\sigma_t^M + x_t^I\nu_t^I)^T \quad \text{and} \quad \frac{E_t[dr_t^J - dr_t^M]}{dt} = \nu_t^J(\sigma_t^M + x_t^J\nu_t^J)^T.$$

We must also verify that

$$\frac{1}{B_t} - r_t + \mu_t^B + \sigma_t^B(\sigma_t^M)^T \leq \sigma_t^B(\sigma_t^M + x_t^I\nu_t^I)^T,\ \sigma_t^B(\sigma_t^M + x_t^J\nu_t^J)^T$$

$$dr_t^I = \frac{P_t^I a - \iota}{q}\, dt + \left(\Phi(\iota) - \delta + \mu_t^q + \sigma_t^q\left(\sigma^I \frac{1^I}{I}\right)^T\right) dt + \left(\sigma_t^q + \sigma^I \frac{1^I}{I}\right)\, dZ_t,$$

$$dr_t^M = \left(\Phi(\iota) - \delta + \mu_t^p + \sigma_t^p(\sigma_t^K)^T\right) dt - \frac{\sigma^\pi}{1-\pi}(\sigma_t^N)^T\, dt + \sigma_t^M\, dZ_t.$$

Ok

$$\frac{P_t^I a - \iota}{q} + \mu_t^q - \mu_t^p + \sigma_t^q\left(\sigma^I \frac{1^I}{I}\right)^T - \sigma_t^p(\sigma_t^K)^T + \frac{\sigma^\pi}{1-\pi}(\sigma_t^N)^T = \nu_t \underbrace{\left(\sigma_t^\eta + \frac{\pi_t\sigma^\pi}{1-\pi_t} + \sigma_t^M\right)}_{\sigma_t^N}$$

$$\frac{P_t^I a - \iota}{q} + \frac{\sigma^\pi}{1-\pi}\sigma_t^\eta + \frac{\sigma^\pi}{1-\pi}\frac{\pi_t\sigma^\pi}{1-\pi_t} = \nu_t\left(\sigma_t^\eta + \frac{\pi_t\sigma^\pi}{1-\pi_t} + \sigma_t^K\right)^T + \frac{\sigma^\pi}{1-\pi}\frac{\sigma^\pi}{1-\pi} - \frac{(\sigma^\pi)^2}{1-\pi} + \frac{\mu^\pi}{1-\pi}$$

$$\frac{P_t^I a - \iota}{q} + \frac{\sigma^\pi}{1-\pi}\sigma_t^\eta = \nu_t\left(\sigma_t^\eta + \frac{\pi_t\sigma^\pi}{1-\pi_t} + \sigma_t^K\right)^T + \frac{\mu^\pi}{1-\pi}$$

$$\frac{\mu^\pi}{1-\pi} = \frac{P_t^I a - \iota}{q} + \sigma_t^\eta\frac{\sigma^\pi}{1-\pi} - \left(\sigma^I \frac{1^I}{I} - \sigma_t^K\right)\left(\sigma_t^K + \sigma_t^\eta + \pi_t\frac{\sigma^\pi}{1-\pi}\right)$$

$$\mu_t^p - \mu_t^q = \frac{\sigma^\pi}{1-\pi}\sigma^p - \frac{(\sigma^\pi)^2}{1-\pi} + \frac{\mu^\pi}{1-\pi}, \qquad \sigma_t^p - \sigma_t^q = \frac{\sigma_t^\pi}{1-\pi},$$

$$\sigma_t^\eta = x_t\left(\sigma^I\frac{1^I}{I} - \sigma_t^K\right) + \underbrace{\pi_t\frac{1-\eta_t}{\eta_t}\frac{\sigma^\pi}{1-\pi}}_{\frac{\pi'(\eta)}{1-\pi}(1-\eta_t)\sigma^\eta}$$

Before we had

$$\frac{P_t^I a - P_t^j a}{q} = x_t|\nu_t|^2 + \left(\sigma^I\frac{1^I}{I} - \sigma_t^j 1^j\right)(\sigma_t^p - \sigma_t^q + \sigma_t^K)^T - x_t^J|\nu_t^j|^2$$

$$\frac{P_t^I a - \iota}{q} + \frac{(\sigma^\pi)^2}{1-\pi} - \frac{\mu^\pi}{1-\pi} + (\sigma_t^q - \sigma_t^p)\left(\sigma^I \frac{1^I}{I}\right)^T = x_t|\nu_t|^2 + \left(\sigma^I \frac{1^I}{I} + \sigma_t^q - \sigma_t^p - \sigma_t^K\right)\sigma_t^K.$$

another way to write it is

$$\frac{P_t^I a - \iota}{q} + \frac{(\sigma^\pi)^2}{1-\pi} - \frac{\mu^\pi}{1-\pi} + (\sigma_t^q - \sigma_t^p)\left(\sigma^I \frac{1^I}{I}\right)^T = \sigma_t^\eta \nu_t^T + \nu_t^T(\sigma_t^K - \sigma_t^\pi).$$

$$\frac{P_t^I a - \iota}{q} - \frac{\mu^\pi}{1-\pi} = \sigma_t^\eta \nu_t^T + \sigma_t^K \nu_t^T + \sigma_t^K \sigma_t^\pi + \frac{\pi \sigma_t^\pi}{1-\pi}\left(\sigma^I \frac{1^I}{I}\right)^T$$

$$\frac{P_t^I a - \iota}{q} - \frac{\mu^\pi}{1-\pi} = \sigma_t^\eta \nu_t^T + \left(\sigma^I \frac{1^I}{I} - \sigma_t^K\right)\sigma_t^K - \frac{\pi}{1-\pi}\sigma_t^K \sigma_t^\pi + \frac{\pi \sigma_t^\pi}{1-\pi}\left(\sigma^I \frac{1^I}{I}\right)^T$$

With policy,

$$\frac{P_t^I a - \iota}{q} - \frac{\mu^\pi}{1-\pi} = \sigma_t^\eta \left(\sigma^I \frac{1^I}{I} - \frac{\sigma^\pi}{1-\pi} - \sigma_t^K\right) + \left(\sigma^I \frac{1^I}{I} - \sigma_t^K\right)\left(\sigma_t^K + \pi_t \frac{\sigma^\pi}{1-\pi}\right)$$

or

$$\frac{P_t^I a - \iota}{q} - \frac{\mu^\pi}{1-\pi} = \sigma_t^\eta \nu_t^T + \sigma_t^K \nu_t^T + \sigma_t^K (\sigma_t^\pi)^T + \frac{\pi \sigma_t^\pi}{1-\pi}\left(\sigma^I \frac{1^I}{I}\right)^T$$

i.e. an identical formula applies in both cases.

5.5 Monetary Policy: An Example

This section provides an example of how monetary policy can affect equilibrium dynamics and welfare. We take the same parameters as in our example in Section 3. We then focus on the extent to which policy mitigates endogenous risk. Specifically, consider policies that lead to

$$\frac{b_t}{p_t}\sigma_t^B = \alpha(\eta)\frac{\sigma_t^\pi}{1-\pi},$$

so

$$\sigma_t^\eta = \frac{x_t\left(\sigma^I \frac{1^I}{I} - \sigma_t^K\right)}{1 + (\psi - \eta)(1-\alpha_t)\frac{\pi'(\eta)}{\pi(\eta)} - \alpha(\eta)\frac{1-\eta_t}{1-\pi_t}\pi'(\eta)}$$

$$\text{and} \quad \nu_t = \left(1 - (1-\alpha(\eta))\frac{\pi'(\eta)}{\pi(\eta)(1-\pi(\eta))}\right)\left(\sigma^I \frac{1^I}{I} - \sigma_t^K\right),$$

where $\alpha(\eta) \in [0,1]$. In the following example, $\alpha(\eta) = \max(0.5 - \eta, 0)$, i.e. monetary policy eliminates up to a half of endogenous risk for $\eta < 0.5$. Figure 6 compares several equilibrium quantities with and without policy, and zooms around the steady state to make the comparison clearer.

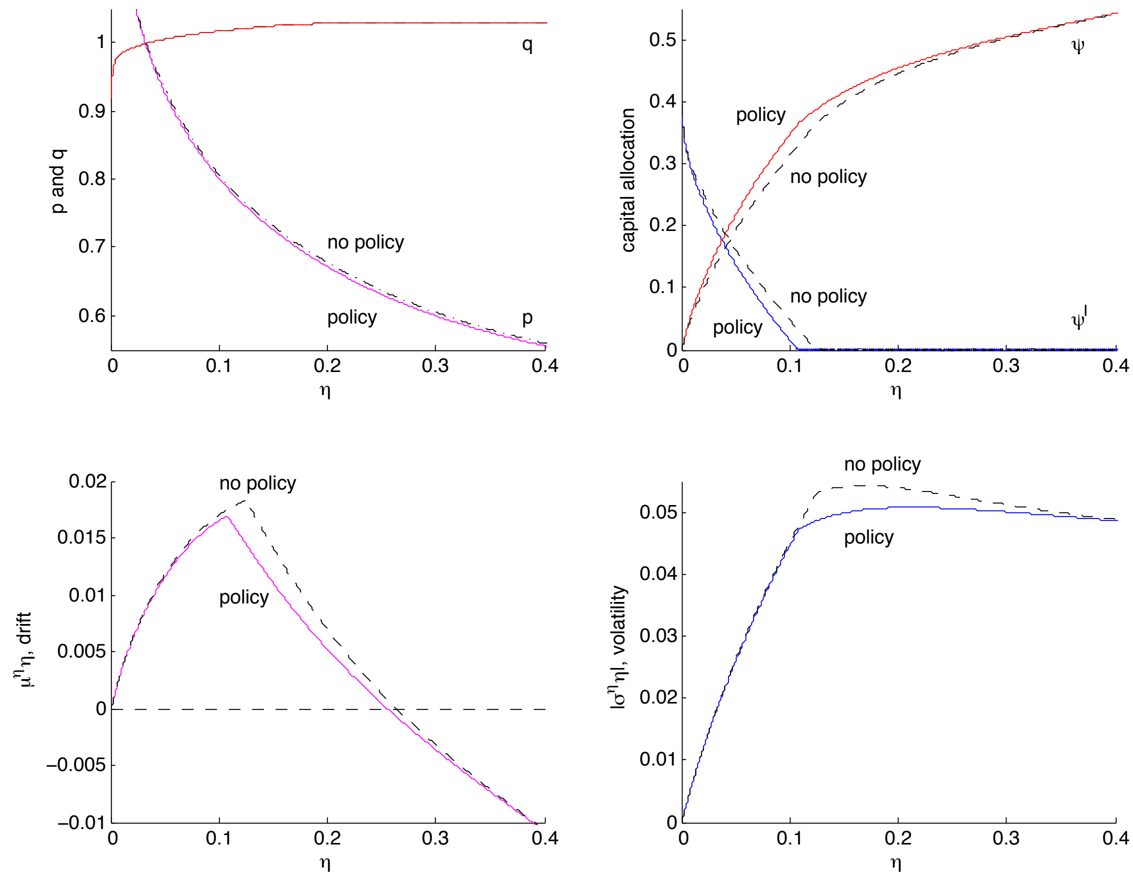

Figure 6: Equilibrium with and without policy.

The bottom right panel illustrates that policy reduces risk. As a result, intermediaries are able to employ their technologies to a greater extent even for lower levels of net worth, and households step in to inefficiently use the intermediaries' technologies at a lower level of η_t - see top right panel. At the same time, intermediaries are able to maintain higher leverage - their net worth at the steady state is reduced as shown in the bottom left panel. The top left panel shows that the value of money is somewhat lower with the policy - due to the fact that intermediaries are able to function more efficiently and create more inside money. The price of capital $q(\eta)$ does not change significantly (it becomes slightly higher with the policy).

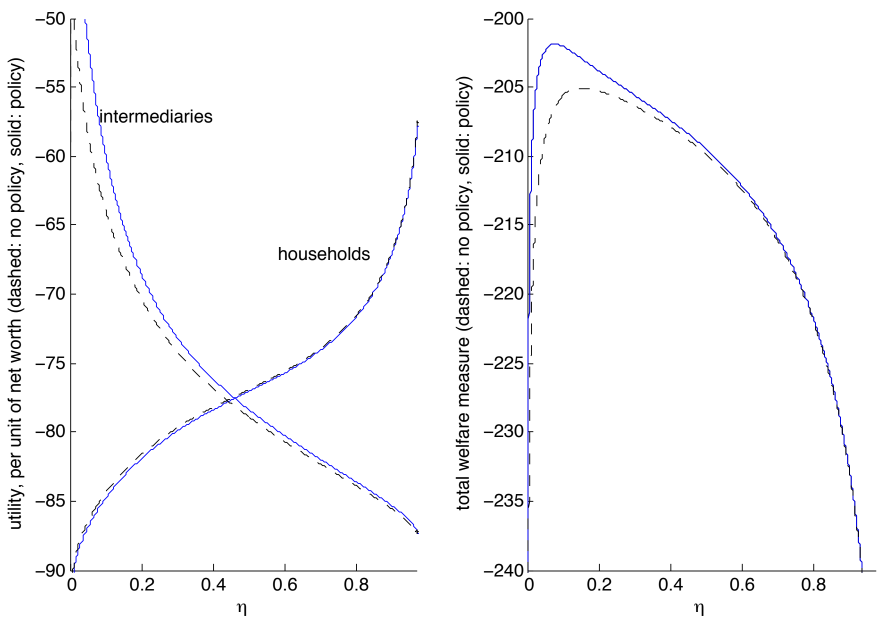

Figure 7: Welfare in Equilibrium.

Figure 7 confirms that this policy, though appropriate risk transfer, indeed improves welfare. It replicates Figure 5, and compares welfare measures with and without policy. Overall welfare clearly improves with policy, as we can see from the right panel of Figure 7. Moreover, total welfare is maximized at a lower level of η, so that the shift of the steady state to the left is not detrimental. We estimated that this improvement in welfare is equivalent to increased consumption by about 9% per year. The left panel shows the effect on individual agents. Households get slightly lower utility given any wealth level with policy, but also at the steady state they have greater wealth.

Compare with

$$\rho\omega(\eta) = \log(\rho p(\eta)) + \frac{\Phi(\iota(\eta)) - \delta - \rho - \sigma^2/2}{\rho} + \omega'(\eta)\mu_t^\eta \eta + \frac{\omega''(\eta)\eta^2\sigma_t^\eta(\sigma_t^\eta)^T}{2}.$$

Let $H_0(\eta) = \omega(\eta_t) - \log(p_t)/\rho$. Then this agent's utility can be also expressed as $H_0(\eta_t) +$

$\log(n_t)/\rho$, where H_0 satisfies the HJB equation

$$\rho H_0(\eta_t) = \log(\rho) + \frac{1}{\rho} E\left[\frac{dn_t}{n_t}\right]/dt - \frac{(\sigma_t^p + \sigma_t^K)^2}{2\rho} + H_0'(\eta)\mu_t^\eta \eta + \frac{H_0''(\eta)\eta^2 \sigma_t^\eta (\sigma_t^\eta)^T}{2}. \quad (5.15)$$

Now, consider another agent whose best investment opportunity over money has risk ν_t and who (given the returns) chooses to put portfolio weight x_t on this opportunity. Then this agent's wealth follows

$$\frac{d\tilde{n}_t}{\tilde{n}_t} = \frac{dn_t}{n_t} + x_t \nu_t (x_t \nu_t + \sigma_t^p + \sigma_t^K)^T \, dt + x_t \nu_t \, dZ_t.$$

The utility of this agent can be represented in the form $H(\eta_t) + \log(\tilde{n}_t)/\rho$, where

$$\rho H(\eta_t) = \log(\rho) + \frac{1}{\rho} E\left[\frac{dn_t}{n_t}\right]/dt + \frac{x_t \nu_t (x_t \nu_t + \sigma_t^p + \sigma_t^K)^T}{\rho}$$

$$-\frac{|\sigma_t^p + \sigma_t^K|^2}{2\rho} - \frac{x_t \nu_t (\sigma_t^p + \sigma_t^K)^T}{\rho} - \frac{x_t^2 \nu_t \nu_t^T}{2\rho} + H'(\eta)\mu_t^\eta \eta + \frac{H''(\eta)\eta^2 \sigma_t^\eta (\sigma_t^\eta)^T}{2}.$$

Subtracting (5.15), we find that the difference in the utilities of these two agents, $h(\eta_t) = H(\eta) - H_0(\eta)$, satisfies the ordinary differential equation

$$\rho h(\eta) = \frac{x_t^2 \nu_t \nu_t^T}{2\rho} + h'(\eta)\mu_t^\eta \eta + \frac{h''(\eta)\eta^2 \sigma_t^\eta (\sigma^\eta)^T}{2}. \quad (5.16)$$

6 Conclusion

We consider an economy in which household entrepreneurs and intermediaries make investment decisions. Household entrepreneurs can invest only in a single real production technology at a time, while intermediaries have the expertise to invest in a number of projects. In equilibrium intermediaries take advantage of their expertise to diversify across several investment projects. They scale up their activity by issuing demand deposits, *inside money*. Households hold this inside money in addition to *outside* money provided by the government. Intermediaries are leveraged and assume liquidity mismatch. Intermediaries' assets are long-dated and have low market liquidity - after an adverse shock the price can drop - while their debt financing is short-term. Endogenous risk emerges through amplification mechanism in form of two spirals. First, the liquidity spiral: a shock to intermediaries causes them to shrink balance sheets and "fire sale some of their assets". This depresses the price of their assets which induces further fire-sales and so on. Second, the disinflationary spiral: as intermediaries shrink their balance sheet, they also create less inside money; such a shock leads to a rising demand for outside money, i.e. disinflation. This disinflationary spiral amplifies shocks, as it hurts borrowers who owe nominal debt. It works on the liabilities side of the intermediary balance sheets, while the liquidity spiral that hurts the price of capital works on the asset side. Importantly, in this economy the money multiplier, the ratio between inside and outside money, is endogenous: it depends on the health of the intermediary sector.

Monetary policy can mitigate the adverse effects of both spirals in the presence of default-free long-term government bonds. Conventional monetary policy changes the path of interest rate earned on short-term money and consequently impacts the relative value of long-term government bond and short-term money. For example, interest rate cuts in downturns that are expected to persist for a while enable intermediaries to refinance their long-bond holding more cheaply. This recapitalizes institutions that hold these assets and and also increases the (nominal) supply of the safe asset. The resulting reduction in endogenous risk leads to welfare improvements. Of course, any policy that provides insurance against downturns could potentially create moral hazard. Indeed, intermediaries take on higher leverage, but more hazard is limited. The reason is that the "stealth recapitalization" through a persistent interest rate cut not only recapitalizes institutions with high leverage because they funded many real projects but also the ones which simply held long-term (default-free) Government bonds. The finding that moral hazard is limited might change if one were to include intermediaries with negative net worth. Including zombie banks is one fruitful direction to push this line of research further.

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A Numerical Procedure to find Equilibrium.

(to be completed)

B Proofs.

Proof of Proposition 3. Equation (3.5) follows directly from Ito's lemma. Let us justify the remaining six equations.

Relative to money, capital devoted to the production of good a earns the return of

$$dr_t^a - dr_t^M = \frac{y^a(\psi_t) - \iota_t}{q_t}\,dt + (\mu_t^q - \mu_t^p)\,dt + (\sigma^a 1^a - \sigma_t^K)^T\,dZ_t + \tilde{\sigma}^a\,d\tilde{Z}_t.$$

Likewise,

$$dr_t^b - dr_t^M = \frac{y^b(\psi_t) - \iota_t}{q_t}\,dt + (\mu_t^q - \mu_t^p)\,dt + (\sigma^b 1^b - \sigma_t^K)^T\,dZ_t + \tilde{\sigma}^b\,d\tilde{Z}_t.$$

Fraction $1 - \underline{\chi}$ of the risk of good a is borne by intermediaries, who are exposed to aggregate risk σ_t^K, and fraction $\underline{\chi}$, by households, who are exposed to aggregate risk σ_t^K and idiosyncratic risk $x_t^a\tilde{\sigma}^a$. Thus,

$$\frac{y^a(\psi_t) - \iota_t}{q_t} + \mu_t^q - \mu_t^p = (\sigma^a 1^a - \sigma^K)^T\sigma^K + \underline{\chi}\,x_t^a(\tilde{\sigma}^a)^2, \tag{B.1}$$

where $(\sigma^a 1^a - \sigma^K)^T\sigma^K$ is the risk premium for aggregate risk of this investment, and $\underline{\chi}x_t^a(\tilde{\sigma}^a)^2$ is the price of idiosyncratic risk. For good b,

$$\frac{y^b(\psi_t) - \iota_t}{q_t} + \mu_t^q - \mu_t^p = (\sigma^b 1^b - \sigma^K)^T\sigma^K + x_t^b(\tilde{\sigma}^b)^2. \tag{B.2}$$

Now, since any investment in capital will include a hedge for the aggregate risk component,

$$\nu_t = \nu_t^b = 0,$$

so the indifference condition of households (2.14) becomes, one,

$$(x_t^a)^2(\tilde{\sigma}^a)^2 = (x_t^b)^2(\tilde{\sigma}^b)^2 \quad \Leftrightarrow \quad x_t^a = x_t^b\frac{\tilde{\sigma}^b}{\tilde{\sigma}^a},$$

and the law of motion of η_t is, two,

$$\frac{d\eta_t}{\eta_t} = -(1-\eta_t)(x_t^b)^2(\tilde{\sigma}^b)^2\, dt, \tag{B.3}$$

From (2.15), we have, three,

$$\frac{\psi_t \underline{\chi}}{x_t^a} + \frac{1-\psi_t}{x_t^b} = \frac{1-\eta_t}{1-\pi_t}. \tag{B.4}$$

$$\psi_t \underline{\chi}\, \tilde{\sigma}^a/\tilde{\sigma}^b + 1 - \psi_t = x^b \frac{1-\eta_t}{1-\pi_t}.$$

Subtracting (B.2) from (B.1), we get, four,

$$\frac{y^a(\psi_t) - y^b(\psi_t)}{q_t} = (\sigma^a 1^a - \sigma^b 1^b)^T \sigma^K + \underline{\chi} x_t^a (\tilde{\sigma}^a)^2 - x_t^b (\tilde{\sigma}^b)^2. \tag{B.5}$$

The market-clearing condition for consumption goods is, five,

$$y(\psi_t) - \iota(q_t) = \frac{\rho q_t}{1-\pi_t}.$$

Finally, taking a weighted average of (B.1) and (B.2), with weights ψ and $1-\psi$, we have

$$\underbrace{\frac{y(\psi) - \iota_t}{q_t}}_{\rho/(1-\pi_t)} + \mu_t^q - \mu_t^p = (\underline{\chi}\psi\, \tilde{\sigma}^a/\tilde{\sigma}^b + 1 - \psi) x_t^b (\tilde{\sigma}^b)^2.$$

This, in combination with (B.4), and the identity $\mu_t^\pi = (1-\pi_t)(\mu_t^p - \mu_t^q) - \sigma^\pi \sigma^p + (\sigma^\pi)^2$, leads to the last equation, (3.6), six. □

(to be completed)