### ESSAYS ON CULTURE AND TRADE

by

Ulrika Stavlöt



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## ABSTRACT

This thesis consists of three self-contained essays. The first two essays address the consumption of culture and are closely related in terms of the theoretical framework used. The third essay is a separate analysis of international trade and competition.

The studies of culture are motivated by the special treatment of culture consumption in most modern societies: there are usually large, government-provided subsidies, the aim of which is to stimulate both the production and the consumption of culture. The purpose of the present work is to explore reasons for this special treatment. Using a stylized theoretical framework, the essays contrast culture with another, generic, good or activity. Culture is thus regarded as an "experience good": previous consumption of the good enhances the current appreciation of the good. The generic good is one where experience is assumed not to be at all relevant for the appreciation of the good. For experience goods, decisions made today will influence future utility and future choices. This makes the intertemporal preferences essential. If, in particular, consumers have time-inconsistent preferences of the type that can be characterized as a present-bias—modeled with "multiple selves" using quasi-geometric discounting—as opposed to standard, time-consistent preferences, there will be a case for government subsidies. The first essay explores this possibility in detail in a framework where experience is mainly of importance in the short run. The second essay then studies cases where experience is more potent and can cause persistent diversity in culture consumption across individuals.

Culture and Control: Should There Be Large Subsidies to Culture? studies the circumstances under which public support for culture is warranted. A policy example is designed to illustrate important aspects of public support systems currently in place, and is calibrated to Swedish data. The essay concludes that, given presentbiased agents with self-control problems, public support of culture can work as a commitment device and improve long-run welfare. Furthermore, it is demonstrated that welfare-maximizing subsidies to culture can be substantial if the present-bias is profound and the taste-cultivation property of culture consumption is pronounced. Origins of the Diversity of Culture Consumption analyzes the diversity of culture consumption among individuals. If the culture good and the generic good are sufficiently close substitutes in a static sense, very large and persistent differences in the consumption of highbrow culture across consumers can be explained by differences in initial experience levels alone. Moreover, slight differences in preferences and time endowments can cause significant diversity between individuals, both in the long- and short-run levels of culture consumption. In addition, if consumers have time-inconsistent preferences, further diversity can be rationalized. If there is a present-bias, there may also be Pareto-ranked multiple equilibria with "optimism" and "pessimism": high (low) culture consumption of the current self is rationalized, based on the belief that future culture consumption will be high (low).

Has International Competition Increased? Estimates of Residual Demand Elasticities in Export Markets studies the impact of the last decades of intense economic integration on the competitive conduct of Swedish export industries. The functional relationship between the inverted residual demand elasticity and the Lerner index is used to estimate markups in eight industries. The econometric evidence suggests a deviation from competitive behavior in all industries. Moreover, the results demonstrate a trend of decreasing market power.

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Stockholm, December 2004

Ulrika Stavlöt

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# Chapter 1

# Introduction

### 1 Introduction

This thesis consists of three self-contained essays. The first two essays address the consumption of culture and are closely related in terms of the theoretical framework used. The third essay is a separate analysis of international trade and competition.

#### 1.1 Culture

The studies of culture are motivated by the special treatment of culture consumption in most modern societies: there are usually large, government-provided, subsidies, the aim of which is to stimulate both the production and the consumption of culture. These public subsidies are frequently debated and questioned, especially since redistribution often seems to favor a relatively wealthy and well-educated minority. The purpose of the present work is to explore reasons for this special treatment. Using a stylized theoretical framework, the essays contrast culture with another, generic, good or activity. Culture is thus regarded as an "experience good" in the sense that previous consumption of the good enhances the current appreciation. Put differently, a refined mind is needed to grasp culture. The generic good is one where experience is assumed not to be relevant for the appreciation of the good, i.e., the good gives instant utility, but generates no intertemporal value-added. For experience goods, decisions made today will influence future utility and future choices, which makes the intertemporal preferences essential. If, in particular, consumers have time-inconsistent preferences of the type that can be characterized as a present-bias—modelled with "multiple selves" using quasi-geometric discounting as opposed to standard, time-consistent preferences, there will be a case for government subsidies. Chapter 2 explores this possibility in detail in a framework where experience is mainly of importance in the short run. Chapter 3 then studies cases where experience is more potent and can cause persistent diversity in culture consumption across individuals.

In *Culture and Control: Should There Be Large Subsidies to Culture?*, the two goods are interpreted as highbrow and lowbrow culture. The "experience good", or taste cultivation, assumption of highbrow culture is prevalent within cultural economics and is expressed already in 1891 by Alfred Marshall as

It is therefore no exception to the Law (of diminishing marginal utility) that the more good music a man hears, the stronger is his taste for it likely to become.

As already discussed, this taste cultivation property of the utility function produces intertemporal effects, since current consumption decisions will affect past and future utility. Depending on whether the consumer is forward-looking or presentbiased, consumer behavior could be time-inconsistent. There is a recent literature, motivated by psychological and experimental evidence, dealing with quasihyperbolic discounting, which is a straightforward way of modeling time-inconsistent preferences. This form of discounting sets up a conflict between the preferences of different intertemporal selves, and introduces a need for self-control. It seems reasonable to deduce that time-inconsistency might have important implications for the consumption of experience goods such as highbrow cultural goods since, unlike standard goods, highbrow cultural goods give intertemporal utility effects.

This study outlines a theoretical framework that can nest these important qualities of culture consumption and can serve as a framework for policy analysis. It is shown that time-inconsistency reduces the steady-state level of consumption of highbrow culture and increases the consumption of lowbrow culture. The reason for this is that present-biased agents prefer the higher instant utility from consumption of lowbrow culture to long-run investments in highbrow culture, although the latter alternative would give a larger welfare over time. If the agent has a self-control problem and is incapable of internalizing these "internalities", this study shows that there is a rationale for the government to intervene. A policy example is designed to illustrate some important aspects of public support systems of highbrow culture currently in place, and is calibrated to Swedish data. In Sweden, government support to highbrow culture is rather extensive, e.g., the average box office coverage rate of the national theatres is only around 13 percent. A key finding from this exercise is that large tax-financed subsidies could indeed be substantial, if the level of present-bias is extreme and the taste-cultivation property of highbrow culture is pronounced. However, although tax-financed subsidies appear to be a potent device in restoring consumption to time-consistent levels, the welfare gains seem to be rather small in general, irrespective of calibration.

Origins of the Diversity of Culture Consumption analyzes the diversity of culture consumption among individuals. If the culture good and the generic good are sufficiently close substitutes in a static sense, very large and persistent differences in the consumption of culture across consumers can be explained by differences in initial experience levels alone. Formally, the model delivers a law of motion implying multiple steady states. Thus, the model delivers "endogenous" long-run diversity in culture consumption. This occurs if the two goods are relatively close substitutes: over time, then, consumers either move toward complete specialization in the consumption of the generic good, or toward a mix with a significant emphasis on culture. If, on the other hand, the two goods are not close substitutes, then long-run differences in culture consumption can only be explained by fundamental differences in preferences or constraints, not by initial experience: there is a unique steady state, which is reached from all initial conditions.

If the two goods are close substitutes, there might also be a unique steady state with complete specialization on one of the goods. Here, notwithstanding the level of initial experience, the long-run outcome will be the same. Long-run specialization on the culture good will occur if the constraint set—the constraint that binds the consumption of both goods—is sufficiently generous and the goods are close enough substitutes: then current culture consumption can be set quite high and therefore, induce future consumption in a manner which is beneficial even if the initial experience is very low. This can be understood from the perspective of complementarity between present and future consumption: if this complementarity is sufficiently strong, it will lead rational individuals to take advantage of it. Longrun specialization on the generic good, in contrast, results when the constraint is tight, because the complementarity is then not sufficiently powerful. Thus, there is a "scale effect" in culture consumption.

Large *short-run* differences between two individuals in their consumptions of culture can also result from small differences in their initial experience levels. This only occurs if the two goods are close enough substitutes. Formally, this is also a case of multiple steady states but, in addition, the law of motion for the evolution of experience in culture is discontinuous. In other words, there is a cutoff level of initial culture experience such that the individual is indifferent between a large and a small level of current consumption, where each of these levels then persists over time, and with slightly lower (higher) initial experience there is a strict preference for the lower (higher) culture accumulation path.

If consumers have time-inconsistent preferences, further diversity can be rationalized. If there is a present-bias, there may also be Pareto-ranked multiple equilibria with "optimism" and "pessimism": high (low) culture consumption of the current self is rationalized, based on the belief that future culture consumption will be high (low). More precisely, in the dynamic game, there are sometimes multiple equilibria—multiple decision rules each of which is a Markov-perfect equilibrium that can be ranked in terms of welfare. Thus, the model offers an explanation of differences in culture consumption that cannot be based on observables. Moreover, in this case, there could be a different role for government policy: an appropriate policy could potentially eliminate the bad equilibrium or equilibria. Second, there is another source of long- as well as short-run differences in culture consumption: whereas the model with standard time-consistent preferences generically delivers either one or three steady-state culture consumption levels and at most one discontinuity in the decision rule for culture accumulation, the model with time inconsistency can deliver an equilibrium decision rule with several jumps and more than three steady states. Third, there are parameter configurations for which pure-strategy equilibria

#### Chapter 1. Introduction

do not exist; i.e., culture consumption diversity arises from endogenous uncertainty.

#### 1.2 Trade

Increased openness is widely believed to induce competitive effects. Intense competition, inefficient firms exiting the markets, and difficulties in sustaining collusive behavior will lead to lower prices, and thus higher welfare. However, economic theory gives an ambiguous correlation between exposure to international competition and competitive conduct in a market. Whereas the theoretical predictions on import competition are negative, given imperfect competition in domestic markets, the competitive conduct of an export competing industry could be adversely affected. If the exporting industry is a price-taker on the world market, it will be constrained to competitive conduct in the same way as the import-competing industry. Conversely, if the industry can exert some market power, an enlarged world market may enhance profitability by allowing firms to spread fixed costs over a larger production volume and taking advantage of economies of scale in production. Declining marginal costs and a sustained price level, i.e., a larger markup, is an explicit confirmation of increasing market power. Moreover, if the exporting firm can discriminate between domestic and international markets, firms in the industry can maintain their price above the world market price. Mergers, acquisitions, and collusion triggered as defensive responses to internationalization can be regarded as a market failure on the international markets and will clearly counteract the competitive pressures of trade openness. Hence, the impact of globalization on competition depends on the underlying assumptions of the theoretical analysis and is ultimately an empirical question.

Has International Competition Increased? Estimates of Residual Demand Elasticities in Export Markets investigates the competitive conduct of the Swedish manufacturing industries over the period 1969-1994. The Bresnahan approach, which involves estimating the residual demand elasticities, is employed to derive the standard measure of competition, i.e., the Lerner index. After some robustness checks, the changes in competitive conduct over time are estimated.

Theoretically, the most straightforward method for measuring the Lerner index is of course to directly calculate the markup over marginal cost, i.e., accounting markups. However, since this is normally impossible due to data deficiency, estimating the right-hand side of the profit-maximization condition by the residual demand approach may be the second best alternative. The evidence from this econometric study suggests a deviation from competitive behavior in all industries. Moreover, the results demonstrate a trend of decreasing market power. It is uncertain whether the increasing competition will apply to the entire international market. One interpretation of the results might be that competition has hardened for Swedish industries, but not for the markets as such, e.g., assuming a foreign price-setter and the Swedish industry as part of a competitive fringe.

Finally, the results are compared with two conventional methods of calculating the price-cost margin; accounting markups and the Solow-Hall approach. The validity of the residual demand analysis is neither confirmed nor rejected since all three approaches give contrasting outcomes. However, the methods are not completely comparable due to different market definitions etc. in the datasets, but it may still be interesting to compare the outcomes, as long as the limitations of the analysis are kept in mind.

Overall, although drawing definite conclusions regarding the absolute markups seems hazardous, the Bresnahan approach does add evidence on the general competitive conduct.

# Chapter 2

# Culture and Control: Should There Be Large Subsidies to Culture?

\*

### 1 Introduction

In most Western countries, public support to culture is substantial, including subsidies to the performing arts, maintenance of museums and, in some countries, salaries paid directly to creative artists and tax exemptions for private donations to the higher arts. The form of cultural expression that appears to be receiving the bulk of public support is sometimes referred to as highbrow culture, including opera, drama, ballet and classical music. The public subsidies are frequently debated and questioned, especially since redistribution often favors a relatively wealthy and welleducated minority.

To some debaters, it seems self-evident that the fine arts constitute a "special case", meriting special treatment and special encouragement from the government, whereas others, among them many economists, do not see any intrinsic qualities making cultural goods more special than any other good.

Over the years, there have been some concerns about underconsumption, or even the survival, of highbrow culture. Economists such as William Baumol and Staffan

<sup>\*</sup> I thank Per Krusell, Henrik Horn, Harry Flam, Jonas Björnerstedt and participants at the IIES workshop for valuable comments and suggestions. I also thank Christina Lönnblad for excellent editorial assistance.

Burenstam-Linder have addressed these issues. They have examined potential extinction threats of highbrow culture by focusing on its production side. Various empirical studies have tried to establish trends in the performing arts markets, with mixed results. Attempts at finding a rationale for public support systems have been extensive. The justifications for support for culture have been based on various grounds, such as various inefficiencies or forms of market failure, externalities, the "market for lemons"-problems, or meritocracy; none of these arguments seem to have gained full acceptance, however, neither theoretically nor empirically. Some of the most popular arguments seem to be based on paternalism and irrationality of agents, which is certainly not in accordance with the standard analytical frameworks used in economics. Moreover, the arguments based on paternalism and irrationality are also not systematically founded either on theory or on empirical analysis.

The overall objective of this study is to put forward a theoretical framework specifying how the demand for culture (i) is qualitatively different than the demand for other goods and (ii) might be subject to a form of inefficiency not previously been proposed in this context. As part of the study, a policy analysis is performed and various policy instruments are discussed. An example is designed to illustrate some substantively important aspects of public support systems in practice. To assess the quantitative magnitudes, the model is simulated and calibrated to match Swedish data.

Cultural goods and services are often claimed to possess a number of properties distinguishing them from other goods. The role of "experience" in consuming culture is often mentioned, as are "intrinsic values" in consumption such as option and bequest values. Moreover, there are arguments for there being large economies of scale in culture production, externalities in the form of "spin-off effects", network externalities, public good characteristics, and so on. In the present study, the focus is on culture demand; I entirely abstract from the supply side. The focus chosen is that of viewing cultural goods as experience goods. The idea that culture is "hard to appreciate" without training/prior consumption is rather commonly accepted, and though there are also other goods with similar characteristics that might not be labeled culture goods, it seems useful to use the experience aspect to distinguish culture goods from other goods. In the model, there are two kinds of goods or activities: culture goods and generic goods. They can also be considered as "highbrow" culture goods and "mass culture", or "lowbrow" culture goods. Significant for the cultural good is the taste cultivation property, which means that past consumption has a positive effect on current and future consumption decisions. Put differently, a refined/experienced mind is needed to grasp culture. The generic good, in contrast, gives instant utility but generates no intertemporal value-added. The taste cultivation assumption adds an investment aspect to cultural consumption: current consumption of culture is beneficial for current utility but also gives the byproduct of enhancing the future appreciation of culture. Being forward-looking is of importance for the appreciation of this second aspect.

The idea of culture as an experience good is not new. In *Principles of Economics*, Alfred Marshall writes

It is therefore no exception to the Law (of diminishing marginal utility) that the more good music a man hears, the stronger is his taste for it likely to become.

In an article in the New York Times, Ernest van den Haag sought to distinguish lowbrow culture from highbrow culture.<sup>1</sup> He suggested that lowbrow culture provides entertainment rather than the sort of gratification provided by genuine art. Highbrow culture is "aesthetically superior" in that it "reveals and changes one's experience of reality and possibility", while popular culture provides entertainment which "diverts the tired businessman or worker from reality".<sup>2</sup> A number of empirical studies have been undertaken to confirm this property of culture.<sup>3</sup> These studies, which use a wide variety of data, in most cases compiled from audience surveys, have in general affirmed that early exposure to the arts results in later participation.

<sup>&</sup>lt;sup>1</sup> van den Haag labels them "popular culture" and "high culture".

<sup>&</sup>lt;sup>2</sup> Cited from David, Cwi, "Public Support of the Arts: Three Arguments Examined", Chapter 31, Cultural Economics II. Original reference in van den Haag, Ernest, "Should We Subsidize Popular Arts? No - An Elitist View", New York Times, February 9, 1975, Section II,1,17.

<sup>&</sup>lt;sup>3</sup> For example, Abbé-Decarroux and Grin (1992), Cameron (1999), Dobson and West (1988), Ekelund and Ritenour (1999), Gray (1998), Kurabayashi and Ito (1992), Lévy-Garbou and Montmarquette (1996), McCain (1995), Morrisson et al (1996), Prieto-Rodriquez and Fernández-Blanco (2000), Smith (1998) and West and McKee (1983).

George Stigler and Gary Becker (1977) tried to interpret and model Marshall's idea by adopting Becker's household production model to study the cultivation of taste, where the household maximizes a utility function of commodities (interpreted as the "appreciation" of a good) that is produced with market goods, time, skills, training and other human capital. The consumption of certain "addictive" goods leads to the accumulation of "consumption capital", which generates changes in taste over time by changing marginal utilities. Becker and Kevin M. Murphy (1988) develop a theory of rational addiction related to the Stigler and Becker model, according to which past consumption capital which, in this case, is an argument in the utility function, and does not have any effect via a household production function. These two studies are often cited as possible model frameworks for studies of culture.

The assumption of culture as an experience good produces intertemporal effects, since current consumption decisions will affect past and future utility. A recent literature points to certain "intrapersonal frictions" in making forward-looking decisions: consumers' attitudes toward the future are argued to be "time-inconsistent". In short, time inconsistency means that a consumer heavily discounts the future, but does not apply this heavy discounting between adjacent future dates. That is, in the eves of a consumer today, the comparison of today and tomorrow—with a strong bias for today—is very different than the comparison between tomorrow and the day after tomorrow—where tomorrow is only viewed as marginally more important. Thus, a plan made currently for accumulation decisions tomorrow—deciding on allocations between tomorrow and the times after that—would be abandoned tomorrow, when present-bias once more takes over. There is a recent literature, motivated by psychological and experimental evidence, dealing with "quasi-hyperbolic" discounting, which is a straightforward way of modeling preferences with this kind of time inconsistency. This form of discounting sets up a conflict between the preferences of different "intertemporal selves", and introduces a need for commitment. It seems reasonable to deduce that time-inconsistency might have important implications on the consumption of experience goods such as culture goods since, unlike standard goods, these goods give intertemporal utility effects. Intuitively, this phenomenon can be represented by the guilty conscience people tend to show when asked about

their recent record of attending the theatre or the opera: "Not as often as I should" seems to be the prevalent answer, at least in my own experience.

Jonathan Gruber and Botond Köszegi (2001) integrate the Becker-Murphy framework with the Laibson version of hyperbolic discounting in order to study the implications of incorporating time-inconsistent preferences into models of addiction. They find that although the forward-looking behavior generated by time-inconsistent agents is consistent with the standard version of the model with time-consistent preferences, the policy implications can radically diverge. Instead of the benchmark result that the optimal tax on cigarettes only depends on the associated externalities of this product, they find that governmental intervention should also internalize the "internalities" caused by lacking self-control.

The theoretical approach of the present model is closely related to the Gruber and Köszegi model. Although cultural consumption and negative addiction can both be studied within a Becker-Murphy framework, they are two distinct goods, with crucial differences in partial derivatives which give significant qualitative and quantitative differences in economic behavior. Here, there is no direct effect on future utility of present culture consumption: it only appears if culture is consumed in the future. Thus, the intertemporal effects of culture consumption are rather narrow and precise here, with the entire focus on changing the marginal utilities of consuming culture over time.

There could be direct effects on future utility of current culture consumption. However, interpreting direct utility effects of the capital stock is not as straightforward as in the case of a negative addiction, where it can be explained as long-run negative health effects. Here, positive utility from previous consumption of culture could, for example, be considered as accumulated human capital or some sort of "life knowledge", i.e., insights about how to live a better life. It would be interesting to formalize these effects, which in themselves form an additional hypothesis, but this is an issue better considered for a separate paper.

The present study specifies how a lower degree of forward-looking, or patience, as well as a present-bias reduce the steady-state level of consumption of culture and increase the consumption of the generic good, if the utility function is characterized by intertemporal linkages such as taste cultivation properties. Conversely, if the utility function lacks taste cultivation properties, the intertemporal aspects of preferences have no effect on consumption behavior. In the case where preferences are time-consistent, the model can be used to analyze how culture demand depends on parameters of the model. However, in that case, no government intervention is called for. In contrast, when there is a time-inconsistency, consumers can be viewed as "underconsuming" culture. In particular, from the perspective of the current self of the consumer, the future selves do not consume enough culture, since they are less forward-looking than the current self. By implication, the current self underconsumer is modelled as consisting of different selves with conflicting preferences, it is not clear how to evaluate welfare. However, several alternatives have been suggested in the literature, and they are discussed and compared in the paper.

Due to the intrapersonal friction, various forms of government interventions could help the consumer, if these interventions help induce more culture consumption. In particular, a constant subsidy to culture consumption would be beneficial; the current consumer may be induced to consume more culture than he/she wants, but the increase in future culture consumption is definitely appreciated. In other words, a constant subsidy to some extent plays the role of a commitment mechanism for setting culture consumption at a higher level in the future than what would materialize without the intervention.

Thus, the main purpose of the present study is to advance and formalize the hypothesis that subsidies to culture can be motivated using rather standard economic analysis: the argument relies on accepting the two key assumptions, namely that culture is an experience good (more than are other goods) and that individuals tend to have a present-bias. Both these assumptions are rather well-documented separately, but their interrelation has not previously been examined in the literature. However, a second and not less important purpose of the paper is to attempt a quantitative assessment of the size of the government subsidies needed to encourage culture consumption in an optimal way. This assessment, which is here performed using calibration analysis, involves several important inputs: an assessment of the degree of substitutability between the culture good and other goods, measuring the extent of forward-looking of consumers, and an assessment of how large is the present-bias. These key factors are not estimated here but the associated parameter values are borrowed from the existing literature.

The calibration exercise shows that large subsidies—in the order of those observed might be optimal, if the level of time-inconsistency is extreme and the taste-cultivation properties of the utility function are pronounced. It is, however, doubtful that the present-bias is as strong as needed to give these result; for a standard calibration of the present-bias, the optimal subsidy is significantly lower than at least in the Swedish data. Moreover, although tax-financed subsidies appear to be a potent device in restoring consumption to time-consistent levels, the actual welfare gains from not implementing the subsidies are in general small.

It might be considered that the mechanism proposed here—that culture consumption is too low because it is a good involving "capital accumulation" and preferences are present-biased—would also have some bearing on related goods with long-term effects, such as durable goods. Should such goods also be subsidized? Indeed, if undersaving is a general phenomenon due to present-biased preferences, the answer ought to be yes. However, this conclusion seems unwarranted. Markets allow most consumers to avoid this outcome, because durable goods are usually offered along with a loan. A typical example is car loans offered by car dealers, which encourage the individual to enjoy the capital in advance and repay the seller later. Here, the consumer does not need to forgo consumption early on to enjoy the capital good later. In fact, it rather seems that durable goods of this sort instead ought to be taxed, because it is likely that consumers with a present-bias are tempted to sign up for the loan in order to enjoy the good now, thus accepting to pay, and reducing other forms of consumption, only later. The fundamental difference between durable goods and experience goods, like experience, or more generally human capital, is that the latter are intangible and cannot be transferred in advance: they must be painfully accumulated.

The rest of the paper is divided into four sections. Section 2 outlines the theoretical framework of cultural consumption and investigates steady states and dynamic implications for forward-looking and present-biased agents. In section 3, various policy instruments are discussed and a numerical example calibrated to Swedish data is presented. Section 4 concludes.

### 2 A theory of cultural consumption

In this section, a basic model framework of cultural consumption will be outlined, equilibrium will be defined and the characteristics of the model examined.

#### 2.1 The basic model

#### 2.1.1 The case with time-consistent preferences

The analysis will start with a characterization of equilibrium in a standard setting, where the representative agent has standard time-consistent preferences. Time is discrete and infinite. As already discussed, the agent can consume two types of cultural goods, a highbrow good, y, or a generic good, x. The utility she receives from consuming culture depends on the previous consumption of the good, summarized by the stock of consumption capital, k. The preferences of this agent are given by  $\sum_{t=0}^{\infty} \delta^t u(x_t, y_t, k_t)$ . There is no source of uncertainty in the economy, so consumption will be a function of k. The current period utility function is assumed to have the following properties;  $u_x > u_y > 0$ ,  $u_k > 0$ ,  $u_{xx}, u_{yy}, u_{kk} \leq 0$ ,  $u_{xy}, u_{xk} \leq 0$  and  $u_{yk} > 0$ . The last assumption, i.e., setting the cross derivative of the state and control variable as positive, is the essence of the taste formation models. By letting the marginal utility of capital,  $u_k$ , be positive, this model diverges from the models of addiction.

The agent maximizes utility subject to a constraint in each period which can be regarded as a time or budget constraint. The endowment in each period is unity and prices are constant over time, so that feasible allocations are those satisfying

$$x + y = 1. \tag{2.1}$$

The stock of consumption capital is accumulating according to the investment equation

$$k' = h(y,k). \tag{2.2}$$

This expression is supposed to capture depreciation and addition to the capital stock through current consumption of y. The idea here is that k' increases in y and k such that  $h_y, h_k \in (0, 1)$ . Since the preferences are time-consistent, the individual's dynamic program is a standard recursion. The consumer's planning problem, with  $k^0$  given, is

$$V(k) = \max_{x,y} \{ u(x, y, k) + \delta V(k') \} \qquad s.t. \qquad (2.3)$$
  

$$x + y = 1$$
  

$$k' = h(y, k).$$

The optimal choice, given this recursion, is given by control variable y with the policy function y = g(k), such that

$$y = g(k) \in \arg \max \left\{ u \left( 1 - y, y, k \right) + \delta V \left( h(y, k) \right) \right\},\$$

which solves the problem for all k. Unlike the standard savings-capital model, the propensity to consume, g'(k), is not necessarily positive. For a sufficiently large capital stock, the wealth effect could be dominating, such that the agent prefers to consume the generic good x rather than maintaining the stock of consumption capital. Here, g(k) will typically be increasing, which is the case for adequate values of  $u_{yk}$ .

The current period utility function is assumed to be strictly increasing in both arguments and strictly concave and the set  $\{(k', k) : k' = h(y, k), x = 1 - y, y \in \mathbb{R}^n\}$ is convex and compact. To solve the maximization problem, substitute the constraint (2.1) and technology (2.2) in the Bellman equation (2.3) and maximize with respect to y. The first-order condition can be written as

$$-u_x (1 - y, y, k) + u_y (1 - y, y, k) - \delta h_y V'_k (h(y, k)) = 0.$$
(2.4)

 $V'_{k}(h(y,k))$  can be derived using the envelope theorem. In short, the total derivative of the value function V with respect to a parameter k is equal to the partial derivative of the objective function with respect to the parameter evaluated at the optimal policy g(k). Using this result,

$$V_{k}(k) = u_{k}(1 - y, y, k) + \delta h_{k} V_{k}'(h(y, k)).$$
(2.5)

Next, use the first-order condition (2.4) to solve for  $V'_k(h(y,k))$ 

$$V'_{k}(h(y,k)) = \frac{1}{\delta h_{y}} \left( -u_{x} \left( 1 - y, y, k \right) + u_{y} \left( 1 - y, y, k \right) \right).$$

Substitute this expression in equation (2.5), transpose one period ahead and substitute into the first-order condition. First, define

$$\Delta(k) = u_x(1 - g(k), g(k), k) - u_y(1 - g(k), g(k), k).$$

This is the difference between marginal utility of x and marginal utility of y. In a statical model, this difference is zero, while in a model with intertemporal links, it is in general different from zero. Here, the agent experiences instant utility from consuming x and instant as well as future utility from consuming y.  $\Delta(k)$  can be interpreted in terms of the future utility from accumulated capital, which follows from the consumption of y.

Using this notation, the Euler equation takes the form

$$\Delta(k) = \delta h_y(k, g(k)) \left( u_k(1 - g(k'), g(k'), k') + \Delta(k') \frac{h_k(k', g(k'))}{h_y(k', g(k'))} \right).$$

The interpretation of the Euler equation is that the cost of consuming y instead of x in time period t, in terms of lost utility, must equal the discounted direct effect of the increased capital stock on utility in time period t + 1 and the discounted change of marginal utilities that will follow from the consumption of y in the previous period. More intuitively, the agent will choose her consumption bundle so that the instant utility loss from consuming culture instead of the generic good, will equate the discounted future gains of the investment in culture today. Put differently, the Euler equation is the relation between  $\Delta$  today and  $\Delta$  tomorrow. Clearly, in this model framework, current consumption is not only history dependent but is also affected by future consumption decisions. Changes in the past affect current consumption by changing the current stock of capital, whereas changes in the future affect current consumption.

#### 2.1.2 Time-inconsistent preferences

If the agent is present-biased and places a larger value on current consumption, preferences are time-inconsistent. Time-inconsistent preferences mean that a single individual can be viewed as a collection of selves, each in a different time period and each with a different set of preferences. Here, time-inconsistency will be modeled in terms of quasi-hyperbolic, or quasi-geometric, discounting, as labeled by Laibson (1997) or Krusell and Smith (2003).<sup>4</sup> With these kinds of preferences, the current one-period discount rate is higher than the future ones. This means that the agent will, in general, deviate from the policy rules derived at any earlier time period. More specifically, self t and self t + 1 agree on the discounting between all time-periods, except for periods t+1 and t+2, and so on. Here, individuals are assumed to be aware of this feature of their own behavior and to choose economic behavior taking into account the behavior of their future selves.

I will consider intrapersonal games and use a Markov perfect equilibrium concept with  $k_t$  as the state variable, i.e., all decisions made today will depend on the initial capital stock,  $k_t$ , and no other aspects of the past.<sup>5</sup> Formally stated in a recursive setting, a consumer foreseeing that her future selves have different preferences, solves the problem

$$W(k) \equiv \max_{x,y} \left\{ u(x,y,k) + \beta \delta V(k') \right\}$$
(2.6)

subject to (2.1) and the dynamic equation (2.2). The value function V is the indirect utility of capital from the next period and onwards and must satisfy the functional equation,

$$V(k) \equiv u(1 - g(k), g(k), k) + \delta V'(h(g(k), k)).$$
(2.7)

Thus,

$$V(k_t) = u(x_t, y_t, k_t) + \delta u(x_{t+1}, y_{t+1}, k_{t+1}) + \delta^2 u(x_{t+2}, y_{t+2}, k_{t+2}) + \cdots$$

so that  $\delta V(k_{t+1})$  captures the indirect utility for self t of leaving a stock of  $k_{t+1}$  to

<sup>&</sup>lt;sup>4</sup> Laibson refers to experimental and psychological evidence as the motivation for quasihyperbolic discounting. See e.g. Laibson (1997).

<sup>&</sup>lt;sup>5</sup> Reputational equilibrium where current actions depend on additional information from the past, not captured in  $k_t$ , can also exist in this framework.

self t+1. Given V, the consumer chooses control variable y with the policy function y = g(k).

The  $\beta$  parameter represents the prospect of present-biased agents with control problems and lies between 0 and 1. With  $\beta = 1$ , the model nests the standard time-consistent case with geometric preferences and sophisticated behavior. In this case, V(k) would coincide with W(k). If  $\beta \neq 1$ , there is time-inconsistency: a consumer disagrees with his past and future selves about how to consume and save, and solving the problem is now non-trivial.

The resulting Euler equation takes the form

$$\Delta(k) = \beta \delta h_y(k, g(k)) \left( u_k(1 - g(k'), g(k'), k') + \Delta(k') \left( \left( \frac{1}{\beta} - 1 \right) g_k(k') + \frac{1}{\beta} \frac{h_k(k', g(k'))}{h_y(k', g(k'))} \right) \right),$$
(2.8)

where  $\Delta(k)$  is defined as before, see A.1 for derivations. Comparing the Euler equation in this time-inconsistent setting with the standard case above, two new effects appear. First, we have a direct effect on the shadow price of consumption capital, insofar as the discounting is stronger in the latter set-up and hence, the valuation of future effects of capital on utility is smaller. Second, there are indirect effects on the shadow price of capital. A new term appears in the Euler equation;  $\Delta(k')\left(\frac{1}{\beta}-1\right)g_k(k')$ . The term consists of three parts; the marginal utility gain of larger k, the degree of disagreement between selves and the response of the next self, i.e., her propensity to invest. Since  $\Delta(k')$  and  $\left(\frac{1}{\beta}-1\right)$  are positive and  $g_k(k')$ is assumed to be positive, the new term is positive. The intuition is as follows: the present self considers that the next self accumulates too little capital, since the next self is present-biased. Therefore, it is in the interest of the current self to leave somewhat more capital than she would otherwise prefer, to indirectly influence the actions of the next self.

#### 2.1.3 The parametric example

To more closely examine and illustrate the characteristics of the model, let the utility function take the following form

$$u\left(x,y,k\right) = \alpha_0 + \alpha_x x + \alpha_y y + \alpha_k k + \frac{\alpha_{xx}}{2} x^2 + \frac{\alpha_{yy}}{2} y^2 + \frac{\alpha_{kk}}{2} k^2 + \alpha_{xy} xy + \alpha_{xk} xk + \alpha_{yk} yk.$$

This is a quadratic utility function with parameter values set to satisfy the above assumptions:  $\alpha_x$ ,  $\alpha_y > 0$ ,  $\alpha_{xx}$ ,  $\alpha_{yy} \le 0$ ,  $\alpha_{yk} > 0$  and, for simplicity,  $\alpha_0$ ,  $\alpha_{xk}$ ,  $\alpha_{xy}$ ,  $\alpha_k$ ,  $\alpha_{kk} = 0$ . A quadratic utility function apprehends the previously discussed features in a straightforward way. Cross-effects and curvature are explicitly visible in the functional form and closed-form solutions can be derived. In addition, the linearquadratic specification has the benefit of allowing the easy inclusion of stochastic shocks to the setup, since certainty equivalence holds. I briefly discuss a stochastic environment in section 2.6.

Adding a cubic term in a standard consumption-savings model without tastecultivation properties has no major qualitative consequences. In a model with habits, however, it can have drastic consequences, such as multiple steady states and thus, heterogeneity in consumption. In addition, it can lead to discontinuous solutions for g(k) and a large number of steady states if preferences are time-inconsistent. These issues are investigated in chapter 3.

The form of the capital accumulation equation remains to be established. Letting  $h(y_t, k_t)$  be linear such that

$$k_{t+1} = h(y_t, k_t) = c_0 + c_y y_t + c_k k_t,$$

the model will be linear-quadratic.

#### 2.2 Equilibrium

The equilibrium in the intrapersonal game consists of a decision rule g(k) and value functions V(k), W(k), such that these functions solve the dynamic program for the individual:

- given g(k), V(k) satisfies equation (2.7)
- given V(k), W(k) and g(k) satisfy equation (2.3).

#### 2.3 Solution of the model

Substituting out k' and x with constraints (2.1) and (2.2), and replacing y with the guessed decision rule y = a + bk, the Euler equation can be rewritten in terms of the

state variable k, in the form  $A+Bk = 0.^6$  Since this condition must be met for all k, A and B must be zero, which produces two equations in the two unknown decision rule parameters, a and b. This system is non-linear but can easily be characterized. For the numerical examples, I solve for a and b with a typical non-linear equationsolving routine available in MATLAB. For the general linear-quadratic model, these two equations are specified in Appendix A.2.

#### 2.4 Steady states

A steady state is a stationary point,  $\{\bar{x}, \bar{y}, \bar{k}\}$ , of the policy function, such that  $x' = x \equiv \bar{x}, y' = y \equiv \bar{y}$  and  $k' = k \equiv \bar{k}$ . The steady state can thus be solved from  $\bar{k} = h\left(g\left(\bar{k}\right), \bar{k}\right)$ . In the linear-quadratic case, this becomes a simple linear equation,  $\bar{k} = c_0 + c_y \left(a + b\bar{k}\right) + c_k \bar{k}$ , i.e.,  $\bar{k} = (c_0 + c_y a) / (1 - c_y b - c_k)$ . Naturally, recall that a and b are endogenous. Steady-state consumption is easily found, since we have the policy rule and the steady-state consumption capital.

Depending on the parameter values of the utility function and the dynamic equation of consumption capital, the steady states demonstrate different properties.<sup>7</sup> This steady state, which is typically unique, can be stable or unstable, depending on the parameter values. Comparative statics confirm that the model behaves as expected when the parameter values are changed. I do not perform general comparative statics, but for the ranges of parameter values I have studied and that I deem reasonable, I find the consumption capital level of a stable steady state to increase in  $\alpha_k$ ,  $\alpha_y$ ,  $\alpha_{yk}$ ,  $c_k$ ,  $c_y$ ,  $|\alpha_{xx}|$  and  $\delta$  and decrease in  $\alpha_x$ ,  $|\alpha_{yy}|$ ,  $|\alpha_{kk}|$ ,  $|\alpha_{xy}|$  and  $|\alpha_{xk}|$ . Time-inconsistency decreases the steady-state level as long as the utility function encompasses intertemporal linkages. This means that for  $\alpha_k = \alpha_{kk} = \alpha_{yk} = \alpha_{xk} = 0$ , time-inconsistency has no effect on steady-state consumption or capital levels.

 $<sup>^{6}</sup>$  It is a well-known result that quadratic utility functions give linear decision rules, which can readily be proven by backward induction.

<sup>&</sup>lt;sup>7</sup> The procedure for controlling the stability of the steady states is described in Appendix A.3. The applied approach for confirming the concavity of the model is reported in Appendix A.4.

#### 2.5 Dynamics

The dynamics in the basic linear-quadratic model are fully described by the slope coefficient in the reduced form capital accumulation equation,  $k' = c_0 + c_y (a + bk) + c_k k$ , i.e.,  $c_y b + c_k$ , which depends non-trivially on all primitive parameters in the model. If the absolute value of the slope coefficient is less than 1, the steady states are stable, while if its absolute value is larger than 1, the steady states are unstable. Only stable dynamics is associated with optimizing behavior. In a typical case, the slope is positive, giving rise to monotonous dynamics. The flatter the slope, the more rapid is the convergence to steady state. The speed of convergence is captured in how close to zero the coefficient is. If zero, the convergence is immediate.

In one interesting case which I study later, the dynamics are given by figure 2.1.<sup>8</sup> It is apparent that in this case, consumption of culture will be homogenous since all initial levels of consumption capital give the same long-run outcome.

By changing each parameter in the model while keeping the others constant, the dynamic qualities of this model are examined. As stated in the section above, no general comparative statics are performed, but the following inference can be made for ranges of parameters that seem reasonable. Starting with the utility function, we see that the coefficients of the linear piece of the quadratic utility function,  $\alpha_x$ ,  $\alpha_y$  and  $\alpha_k$ , generate a shift in the decision rule, while the slope remains constant. This result is easily grasped since these coefficients operate as constants in optimum, which should be clear after differentiating the utility function.

The second derivatives of the utility function will have an impact on the slope of the policy function, which is likewise apparent from the form of the utility function. As in the standard consumption-smoothing model, the curvature of the utility function with regard to consumption goods x and y slow down convergence. This is captured in the model as high absolute values on  $\alpha_{xx}$  and  $\alpha_{yy}$ . Intuitively, high curvature requires consumption smoothing and hence, slows down convergence.

Since this model excludes monetary saving, the intertemporal linkages solely evolve from consumption capital. This means that parameters  $\alpha_{yk}$ ,  $\alpha_{xk}$ , and  $\alpha_{kk}$  are

<sup>&</sup>lt;sup>8</sup> The following parameter values are used:  $\alpha_x = 1$ ,  $\alpha_y = 0.04$ ,  $\alpha_k = 0$ ,  $\alpha_{xx} = -1.008$ ,  $\alpha_{yy} = -1.008$ ,  $\alpha_{kk} = 0$ ,  $\alpha_{xy} = 0$ ,  $\alpha_0 = 0$ ,  $\alpha_{xk} = 0$ ,  $\alpha_{yk} = 0.25$ ,  $c_0 = 0$ ,  $c_y = 0.15$ ,  $c_k = 0.85$ ,  $\beta = 1$  and  $\delta = 0.96$ . The values are chosen to make the utility function concave and marginal utilities positive.

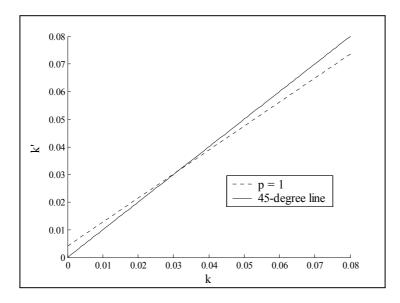


Figure 2.1: Decision Rule

determinants of the dynamics in terms of consumption. If all three parameters were zero, the convergence to steady-state consumption would be immediate, since the decision rule would be independent of consumption capital. While a high absolute value on  $\alpha_{kk}$  makes the consumption capital less important in the utility function, the other two parameters,  $\alpha_{yk}$  and  $\alpha_{xk}$ , reinforce the intertemporal linkages and more dynamics are introduced.

Parameter  $\alpha_{yx}$  affects the dynamics, depending on whether culture and the generic good are substitutes or complements, such that the stronger is the substitutability in consumption, the more dynamics, whereas the stronger is the complementarity in consumption, the less dynamics.

Next, examine the two remaining preference parameters, i.e., discount factors  $\beta$  and  $\delta$ . Low discount factors,  $\beta$  and  $\delta$ , both decrease the slope and increase the speed of convergence. The motivation behind this is that low discount rates imply that the agent puts less value on the future. The model then resembles a static model and convergence will thus be faster.

The parameters determining consumption capital accumulation,  $c_y$  and  $c_k$ , have opposite effects on dynamics. A high value of  $c_k$  has a similar effect as a high discount factor. The closer to 1, the more dynamics and the slower the speed of convergence. Conversely, when the weight on y is high, the internal dynamics of k becomes less important than the immediate effect of y on k and the speed of convergence is increasing.

It should be noted that most parameters both shift and turn the curve, which means that the time to convergence is not only dependent on the slope b, but also on the constant a and obviously, on the initial capital,  $k_0$ .

#### 2.6 Introducing shocks in the economy

Observations of cultural consumption in real life reveal significant differences in the level of consumption among individuals. There are alternative explanations to this heterogeneity of cultural consumption. One obvious, although not very sophisticated, theory is that individuals have different tastes, interpreted as dissimilar utility functions. Various long- or short-run shocks to the individual could also explain consumption patterns in culture. By solving the model with shocks, alternative explanations of heterogeneity can be compared. In terms of the theoretical framework outlined above, there are two alternative shocks to the demand side that seem reasonable. First, there could be an additive shock to technology, such that  $k' = c_0 + c_y y + c_k k + \varepsilon$ . In a way, this shock could also be regarded as a change in tastes, since a larger capital stock involves altered marginal utilities. Possible stories for explaining this kind of shock in real life could be a new opera loving acquaintance enthusiastically promoting her interest, or a governmental information campaign with the aim of fostering the cultural education of the population.

Second, there could be shocks to time or income that cannot be predicted by the consumer, such that  $x + y = 1 + \varepsilon$ . The time series properties of  $\varepsilon$  are important in discriminating between different models and identifying the taste cultivation effects.

Various shocks and heterogeneity in preferences can explain changing consumption patterns in culture and the question is how these can be separated. Without formally showing this, the following can intuitively be argued.

Heterogeneity in preferences would certainly explain low or high consumption of culture, but would generate constant consumption patterns over time, i.e., level effects.

Permanent shocks in demand would affect prices and quantities, but the change

would be constant over time, i.e., a level effect. A temporary shock would change consumption as long as the shock lasts, but it would immediately jump back to the old level when the shock is reversed, since there are no saving opportunities.

With taste cultivation properties, dynamics and inertia are introduced. Assume that the quantity demanded increased after a positive demand or supply shock. In case of a permanent shock, consumption would increase further in the long run and, in the case of a temporary shock, it would not immediately return to its previous size, since it will take some time for the consumption capital, which was build up during the temporary demand peak, to depreciate to the old level.

In the next section, the time-series properties of the basic model will be contrasted with those of a standard model, both solved with shocks.

#### 2.7 Autocorrelation

As already stated, the inclusion of past consumption in the utility function gives intertemporal effects and the model features inherent serial correlation in consumption. By eliminating the effects of second derivatives related to k, i.e., by letting  $\alpha_{yk} = \alpha_{xk} = \alpha_{kk} = 0$ , the serial correlation can be extinguished. However, correlation over time can also develop from shocks in the economic environment. Technology shocks will obviously have no effect if second derivatives are set to zero, but shocks to the constraint could generate serial correlation. Consider an additive shock to the time constraint,  $x + y = 1 + \varepsilon$ , with  $\varepsilon$  following a Markov process of the form  $\varepsilon' = \rho \varepsilon + u'$ , where u is iid normal with mean zero and variance  $\sigma^2$ . The computational details of how the model is solved with shocks are reported in A.7. The dotted line in figure (2.2) depicts the autocorrelation generated from an AR(1) shock with  $\rho = 0.5$  and  $\sigma_{\varepsilon}^2 = 1.0e - 5.9^{10}$ 

In contrast, if serial correlation is generated from taste cultivation properties instead of a Markov shock, the autocorrelation process will follow an ARMA(1,1) process. To see this property, consider a shock to the time constraint as above, but let  $\varepsilon$  be iid normal with the mean zero and the variance  $\sigma_{\varepsilon}^2$ . The decision rule can be written as

$$y_t = \lambda_0 + \lambda_1 k_t + \lambda_2 \varepsilon_t$$

and, hence, the capital accumulation equation

$$k_{t+1} = c_0 + c_y \left(\lambda_0 + \lambda_1 k_t + \lambda_2 \varepsilon_t\right) + c_k k_t$$
  
=  $c_0 + c_y \lambda_0 + \left(c_y \lambda_1 + c_k\right) k_t + c_y \lambda_2 \varepsilon_t$   
=  $A + Bk_t + C\varepsilon_t$ 

where

$$A = (c_0 + c_y \lambda_0)$$
$$B = (c_y \lambda_1 + c_k)$$
$$C = c_y \lambda_2.$$

Substitute the expression for  $k_t$  in the decision rule and iterate

 $^{10}$  For computational details, see the Appendix. The correlogram is calculated as

$$\rho = \frac{\sum_{t=1}^{T} (y_t - \overline{y})(y_{t-1} - \overline{y})}{T} \text{ where }$$

$$\overline{y} = \frac{\sum_{t=1}^{T} y_t}{T} \text{ and }$$

$$\widehat{\sigma}_y^2 = \frac{\sum (y_t - \overline{y})^2}{T - 1}.$$

<sup>&</sup>lt;sup>9</sup> The following parameter values are used:  $\alpha_x = 1$ ,  $\alpha_y = 0.03$ ,  $\alpha_k = 0$ ,  $\alpha_{xx} = -1.005$ ,  $\alpha_{yy} = -1.005$ ,  $\alpha_{kk} = 0$ ,  $\alpha_{xy} = 0$ ,  $\alpha_0 = 0$ ,  $\alpha_{xk} = 0$ ,  $\alpha_{yk} = 0$ ,  $c_0 = 0$ ,  $c_y = 0.15$ ,  $c_k = 0.85$ ,  $\beta = 1$  and  $\delta = 0.96$ .

$$y_{t} = \lambda_{0} + \lambda_{1} \left( A + Bk_{t-1} + C\varepsilon_{t-1} \right) + \lambda_{2}\varepsilon_{t}$$
  

$$= \lambda_{0} + \lambda_{1} \left( A + B \left( \left( A + Bk_{t-2} + C\varepsilon_{t-2} \right) \right) + C\varepsilon_{t-1} \right) + \lambda_{2}\varepsilon_{t}$$
  

$$= \lambda_{0} + \lambda_{1} \left( A + B \left( \left( A + B \left( \left( A + Bk_{t-3} + C\varepsilon_{t-3} \right) \right) + C\varepsilon_{t-2} \right) \right) + C\varepsilon_{t-1} \right) + \lambda_{2}\varepsilon_{t}$$
  

$$= \lambda_{0} + \lambda_{1} A \left( 1 + B + B^{2} + \cdots \right) + \lambda_{1} C \left( \varepsilon_{t-1} + B\varepsilon_{t-2} + B^{2}\varepsilon_{t-3} + \cdots \right) + \lambda_{2}\varepsilon_{t}.$$

Derive  $y_{t-1}$  in the same manner and multiply by B.

$$By_{t-1} = B\lambda_0 + B\lambda_1 A \left( 1 + B + B^2 + \cdots \right) + B\lambda_1 C \left( \varepsilon_{t-2} + B\varepsilon_{t-3} + B^2 \varepsilon_{t-4} + \cdots \right) + B\lambda_2 \varepsilon_{t-1}$$

Subtract  $By_{t-1}$  from  $y_t$  and rewrite in terms of lag operators.

$$y_t - By_{t-1} = (1 - B) \left( \lambda_0 + \frac{\lambda_1 A}{1 - B} \right) + \lambda_1 C \varepsilon_{t-1} + \lambda_2 \varepsilon_t - B \lambda_2 \varepsilon_{t-1}$$
  

$$(1 - BL) y_t = D + (\lambda_2 + (\lambda_1 C - B \lambda_2) L) \varepsilon_t$$
  

$$(1 - BL) y_t = D + \lambda_2 \left( 1 + \left( \frac{\lambda_1}{\lambda_2} C - B \right) L \right) \varepsilon_t$$
  

$$(1 - BL) \hat{y}_t = \left( 1 + \left( \frac{\lambda_1}{\lambda_2} C - B \right) L \right) \hat{\varepsilon}_t$$

which can be written as

$$(1 - aL)\,\hat{y}_t = (1 - bL)\,\hat{\varepsilon}_t$$

where

$$a = B = (c_y \lambda_1 + c_k)$$
  

$$b = -\left(\frac{\lambda_1}{\lambda_2}C - B\right) = -\left(\frac{\lambda_1}{\lambda_2}c_y \lambda_2 - (c_y \lambda_1 + c_k)\right) = c_k.$$

Thus, y follows an ARMA(1,1) process. A familiar result of the ARMA(1,1) process is that the autocorrelation function will look similar to that of an MA(1) process between time 0 and 1, whereas from time 1 and onwards, the autocorrelations will display an AR(1) behavior. If the roots of the AR and MA polynominals are close to each other, the ARMA autocorrelations are very close to those of the white

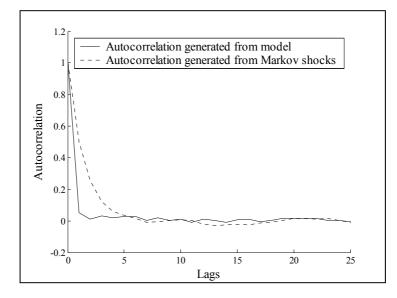


Figure 2.2: Correlogram

noise, i.e., the process is almost uncorrelated. A closer look at a and b reveals that the roots are identical except for the term  $c_y\lambda_1$ . This term is always less than one since  $c_y < 1$  and  $\lambda_1 < 1$ . However, with the parameter values used in the calibration in the later analysis, this term will be insignificant and the result will be close to white noise; see the solid line in figure 2.2.

Consequently, it is difficult to use the autocorrelation properties as a device for identifying or falsifying the model.

### 2.8 Welfare

From the preceding exercises, it seems as if the presence of time-inconsistency in a model with taste cultivation does not have any major effects on behavior.<sup>11</sup> Steady-state consumption is decreasing and the convergence to steady state is delayed. However, as will be shown, time-inconsistency will indeed have far-ranging implications on cultural consumption and public support. Whereas the previous section has primarily been concerned with characterizing behavior, the welfare implications

<sup>&</sup>lt;sup>11</sup> This property called "observational equivalence" could easily be demonstrated in a simpler model of time-inconsistency. See Krusell, Kuruşçu and Smith, (2002), for further analysis.

will be discussed below.

Since the consumer's different selves disagree, there is a conceptual problem in terms of welfare comparisons. There is not obvious according to which self's preferences the evaluation should be done. In a standard model with time-consistent preferences, policy evaluation is straightforward: the model is solved with various policy alternatives, the computed utility levels can be ordinally compared and the welfare maximizing policy identified. In this economy with time-inconsistent preferences and lack of commitment, implemented policy instruments have different effects on the set of selves. In principle, the problem is captured by the two welfare measures derived above in equations (2.6) and (2.7). Function W is the welfare of the present self at the consumption capital level k. The function V is the indirect utility function used by the past consumer to evaluate the present self's welfare at the consumption capital level k. One point worth repeating is that whereas the entire future is discounted at the rate  $\beta\delta$  from the perspective of the present self, V is computed at the discount rate  $\delta$ . Welfare analysis in a time-inconsistent context is therefore ill-defined. In the literature, there exist two different approaches for welfare evaluation in a time-inconsistent context. According to Laibson (1997), the welfare effects for the entire set of selves should be computed and considered, i.e.,  $V(k_0)$  and, in addition, the range  $W(k_0)$ ,  $V(k_1)$ ,  $W(k_1)$ , ...,  $W(k_{ss})$ . Gul and Pesendorfer (2004) show axiomatically that  $V(k_0)$  is the proper measure to use, and this is also the approach used in this paper, although  $W(k_0)$  and  $W(k_{ss})$  will, in general, also be presented.

### 3 Tax-financed subsidy

There is a broad international consensus on the necessity of public support of the arts and the bulk of public support seems to be directed towards highbrow culture. In Sweden, the total consumption of culture amounts to 53.2 billion kronor, 16.1 billion of which, i.e., approximately 1.2 percent of the government budget, originate from public consumption. The theatre and dance sector is supported with 1.9 billion kronor. Approximately 13 percent of the budgets of the national theatres consist of box office income. Other theatre groups finance approximately 30 percent of their

budget by box office income.

Apart from direct monetary support to the production side of culture in order to suppress consumer prices to tolerable levels, there are various other forms of public support. The quality of the art experience is nourished by the maintenance of artistic schools or salaries paid directly to creative artists. Children's creative minds and abilities to appreciate culture are fostered by direct campaigns towards schools, in forms of easy access for school classes to the theater, teaching aids for studying plays etc.

All these mentioned policy instruments can be easily introduced into the model framework described above and it permits us to consider fiscal policies of practical interest. For whatever reasons governments choose to engage in public funding of the fine arts, the financially affluent and numerically small audience could present political problems, as discussed in the introduction. In those countries where the fine arts mainly are mainly promoted by taxes forgone or forgiven, e.g., the United States, public support seems to be less controversial than when tax income is used to give grants to highbrow art institutions. This section will discuss an example of the latter type of policy instrument, chosen to illustrate some substantively important aspects of public support of the arts in Western Europe in general and Sweden in particular.

Consider an economy with a representative agent who lives forever and faces no uncertainty. Preferences are time-inconsistent and the period utility function is quadratic. Individuals live in an endowment economy and the endowment in every time period is given by I. The budget constraint faced by an individual is given by

$$x_t + p_t y_t = I_t - \tau_t, \tag{2.9}$$

where the price of x is normalized to 1, p is the relative price of y, including subsidies, and  $\tau$  is a lump-sum tax paid by the individual. The idea in the latter policy experiment is that the subsidy is fully financed with a balanced budget, period by period. Moreover, I will consider constant subsidy rates, i.e.,  $p_t = p \forall t$ .

Now, the changes in individual behavior due to the introduction of a constant subsidy will be studied. Following the recursive method from above, maximizing (2.6) subject to (2.2) and (2.9), the Euler equation can be derived as,

$$\Delta(k) = \beta \delta h_y(k, g(k)) \left( u_k(I - \tau (g(h(g(k), k))) - p'g(k'), g(k'), k') + \Delta(k') \left( (\frac{1}{\beta} - 1)g_k(k') + \frac{1}{\beta} \frac{h_k(k', g(k'))}{h_y(k', g(k'))} \right) \right)$$
  

$$\Delta(k) = p u_x(I - \tau (g(k)) - pg(k), g(k), k) - u_y(I - \tau (g(k)) - pg(k), g(k), k).$$
(2.10)

### 3.1 Equilibrium with a constant subsidy policy

The equilibrium in the intrapersonal game is a decision rule g(k) and value functions V(k), W(k) where g(k) satisfies Euler equation (2.10) with  $\tau = \tau(g(k))$  and  $\tau' = \tau(g(h(g(k), k)))$  and V(k) and W(k) satisfy the associated value function equations.

### 3.2 The government's problem

There is an infinitely lived government that gives subsidies (1-p) to the cultural good y, and taxes endowment in every period with  $\tau$ . There is no government debt. In every time period, the governmental budget constraint is satisfied such that

$$(1 - p_t) y_t = \tau_t. (2.11)$$

The benevolent government maximizes consumer welfare by choosing p subject to (2.10). Notice that this is a one-dimensional maximization problem which will be solved with numerical methods, see A.6 for computational details. As discussed in section 2.8, consumer welfare can be measured in different ways. In most of the analysis below, I will simply tabulate V and W for different values of p and discuss the differences in changing p on different selves.

### 3.3 Calibration

To obtain numerical solutions to the model, particular parameter values must be chosen. The model is calibrated under the assumption of the model period being one year. The discount parameter  $\delta$  is set to 0.96, which approximately corresponds to an interest rate of 4 percent, as is standard in the macroeconomic literature. Various experimental studies suggest a coefficient of time-inconsistency around 0.5 to be a reasonable base case.

The growth of consumption capital accumulation, represented by parameters  $c_y$  and  $c_k$ , is taken from Ravn, Schmitt-Grohe and Uribe (2004) and set, according to their estimations, to 0.15 and 0.85.<sup>12</sup> Although the estimated coefficients in Ravn et al correspond to consumption habits in general, it seems to be a reasonable approximation for the consumption of culture. Endowment, I, is normalized to 1.

Some theoretical restrictions on the preference coefficients follow directly from the model: To capture the essence of the model that the effect of increasing marginal utility in past consumption is positive, i.e.,  $\alpha_{yk} > 0$ , and all remaining coefficients separating this model from a standard time-separable model are set to zero such that  $\alpha_k = \alpha_{kk} = 0$ . Per definition, x and k have no reciprocal effect and thus,  $\alpha_{xk} =$ 0. The conventional coefficients of the quadratic utility function are constrained to be as unbiased as possible, in the sense of both cultural goods being treated equally. This means that the second-order effects are neutralized by letting  $\alpha_{xx} =$  $\alpha_{yy}$ . The first-order coefficient  $\alpha_x$  is normalized to 1, whereas  $\alpha_y$  is used to regulate the consumption shares. There is no inherent assumption in the model on the substitutability between x and y and thus,  $\alpha_{yx}$  is set free to vary. In addition, the parameters must be set so that the fundamental qualities of the utility function and solution are met: the marginal utilities are positive, i.e.,  $u_x, u_y, u_k > 0$ , the consumption of y is increasing in k, i.e.,  $g_k(k) = b > 0$ , the utility function is concave and the steady states are stable. The remaining preference parameters  $\alpha_{u}$ ,  $\alpha_{yy}, \alpha_{yx}$  and  $\alpha_{yk}$ , which correspond to four degrees of freedom, are set so as to match the following target statistics:

Average share of consumption of highbrow culture relative to lowbrow culture,  $\frac{y}{x}$ . The data on the time spent on entertainment<sup>13</sup> relative to the time spent on watching TV is used to calibrate this ratio to 0.05. The statistics is taken

 $<sup>^{12}</sup>$   $c_k$  corresponds to their parameter  $\rho$  and  $c_y$  corresponds to  $(1 - \rho)$ . Using U.S. quarterly data spanning the period 1967 - 2003 on consumer expenditure on durables, Ravn et al, estimate  $\rho$  to be 0.85.

<sup>&</sup>lt;sup>13</sup> Entertainment is defined as attendance at concerts, the theatre, the cinema, exhibitions, libraries, sports events etc.

from The Swedish Time Use Survey, SCB.

Price elasticity of demand,  $\varepsilon_p = \frac{\delta(y/x)}{\delta p} \frac{p}{(y/x)}$ . A number of empirical studies have estimated this variable on various cultural goods and have delivered broadly consistent elasticities. The elasticities of highbrow culture are mainly inelastic and the statistics is set to be approximately -0.5.<sup>14</sup> In the model, two types of elasticities can be derived. Long-term elasticity depicts the response of steady-state consumption at a permanent price change, whereas short-term elasticity captures the change in consumption between two periods at a one-period price change. In general, the empirical studies assess the former type of elasticity and the calibration will therefore be set to match long-term elasticity. However, both long-term and short-term elasticities will be reported in the exercises below.

Box office coverage ratio, p. The remaining parameters are chosen to match the box office coverage ratio of Swedish theatre, which is 0.13 - 0.30 percent.

The three target statistics above are not sufficient to pin down the values of the four remaining parameters. However, a sensitivity analysis will be performed to control for the effects of individual coefficients.

In all exercises below, the model is recalibrated according to the above target statistics.

### **3.4** Results

The section is introduced by the examination of some properties of an economy where the agents exhibit time-consistent preferences, i.e.,  $\beta = 1$ . This economy will serve as a point of comparison in the later analysis of public support of the arts in economies with time-inconsistent agents.

#### 3.4.1 Time-consistent preferences

The time-consistent economy is, as pointed out above, calibrated to match the empirically observed highbrow/lowbrow consumption ratio of approximately 0.05

 $<sup>^{14}</sup>$  See e.g. Abbé-Decarroux (1994), Bille Hansen (1991), Corning-Levy (2002), Frey et al (1989), Gapinsky (1984) (1986), Moore (1966), O'Hagan (1994), Schimmelpfenning (1997), Throsby (1983), Urrutiaguer (2002) and Withers (1980)

at a price subsidy of 70 percent.<sup>15</sup> The long-term and short-term elasticities in the model are inelastic and equal to -0.4902 and -0.3418.

Table (2.1) shows the steady-state properties of this economy at various government subsidies of culture. With a 30 percent subsidy, the steady-state consumption of y is 0.0490 and the y/x consumption ratio is 0.0516.<sup>16</sup> As the price is decreasing, a monotonic increase in the consumption of culture and consumption capital can be observed. Subsidizing the price of culture by 70 percent, steady-state consumption is increasing by 60 percent.

			-		
p	$\frac{y_{ss}}{x_{ss}}$	$k_{ss}$	$V\left(k_{0} ight)$	$W\left(k_{0} ight)$	$V(k_{ss}), W(k_{ss})$
1.0	0.0315	0.0306	12.4190	12.4190	12.4190
0.9	0.0332	0.0321	12.4189	12.4189	12.4190
0.8	0.0351	0.0339	12.4188	12.4187	12.4189
0.7	0.0373	0.0360	12.4184	12.4184	12.4186
0.6	0.0399	0.0384	12.4178	12.4178	12.4181
0.5	0.0430	0.0413	12.4168	12.4168	12.4173
0.4	0.0468	0.0447	12.4153	12.4152	12.4158
0.3	0.0516	0.0490	12.4127	12.4127	12.4133
0.2	0.0576	0.0545	12.4086	12.4086	12.4092
0.1	0.0656	0.0615	12.4018	12.4018	12.4021

 Table 2.1: Effects of Public Subsidies with Time-Consistent Preferences

The fourth and fifth columns of Table 2.1 examine the welfare of past and present selves, as defined in section 2.8, if the government introduces a subsidy; see A.5 for computational details. As shown above, welfare is maximized at a zero subsidy and is monotonically decreasing in price. The key to this outcome is the lack of externalities or internalities in the consumer's optimization problem, which means that the government can never improve the agent's allocation. The last column shows the welfare for past and present selves, not considering the utility loss during the transition, which is the case in the formerly reported welfare measures. Thus, the welfare measures are based on different capital stocks. The former is based on original capital stock,  $k_0$ , whereas the latter is based on the final steady-state

<sup>&</sup>lt;sup>15</sup> The parameters are set to  $\alpha_x = 1$ ,  $\alpha_y = 0.04$ ,  $\alpha_k = 0$ ,  $\alpha_{xx} = -1.008$ ,  $\alpha_{yy} = -1.008$ ,  $\alpha_{kk} = 0$ ,  $\alpha_{xy} = 0$ ,  $\alpha_0 = 0$ ,  $\alpha_{xk} = 0$ ,  $\alpha_{yk} = 0.25$ ,  $c_0 = 0$ ,  $c_y = 0.15$ ,  $c_k = 0.85$  and  $\delta = 0.96$ .

<sup>&</sup>lt;sup>16</sup> Note that for  $c_y + c_k = 1$ , it is trivial to show that y = k in steady state. Use the capital accumulation equation and solve for  $k^{ss}$ .

capital stock,  $k_{ss}$ . The utility loss induced during the transition, can be regarded as the effort cost in terms of lost consumption to build up the capital stock. In this case, welfare is maximized at a minor subsidy. However, this welfare criterion is not relevant, unless the new steady-state capital stock is costlessly transferred to the agent in the time period when the subsidy is introduced.

In the next section, the behavioral and welfare effects of public subsidies of culture when preferences are time-inconsistent are examined. Finally, some sensitivity analysis is performed.

#### 3.4.2 Time-inconsistent preferences

Consider behavior in an economy populated by agents with time-inconsistent preferences. Since there is a conflict between the preferences of different intertemporal selves, a need for self-control is introduced. A public support system could be viewed as a commitment device with the purpose of constraining the agents' future choices with time-inconsistent preferences.

The benchmark case Before reporting the results of the policy experiment, a standard is establish against which the effects of public subsidies can be compared. The time-inconsistent economy is recalibrated to match the empirically observed highbrow/lowbrow consumption ratio of approximately 0.05 at a price subsidy of 70 percent, and a time-inconsistency level corresponding to  $\beta = 0.5$ .<sup>17</sup> Table 2.2 summarizes the steady-state consumption ratio, the level of capital, the welfare of past and present selves and long and short-run elasticities at different levels of time-inconsistency for p = 1, and constant parameter values.

Table 2.2: Default Case

$\beta$	$\frac{y_{ss}}{x_{ss}}$	$k_{ss}$	$V\left(k_{0} ight)$	$W\left(k_{0} ight)$	$V\left(k_{ss}\right)$	$W\left(k_{ss}\right)$	$\varepsilon_{p,LR}^{y}$	$\varepsilon_p^y,_{SR}$	
1.0	0.0277	0.0270	12.4501	12.4501	12.4501	12.4501	-0.6496	-0.3695	
0.5	0.0249	0.0243	12.4498	6.4739	12.4498	6.4739	-0.5806	-0.3596	
0.1	0.0227	0.0222	12.4495	1.6931	12.4495	1.6931	-0.5110	-0.3506	

<sup>&</sup>lt;sup>17</sup> The following parameter values are used;  $\alpha_x = 1.$ ,  $\alpha_y = 0.03$ ,  $\alpha_k = 0$ ,  $\alpha_{xx} = -1.005$ ,  $\alpha_{yy} = -1.005$ ,  $\alpha_{kk} = 0$ ,  $\alpha_{xy} = 0$ ,  $\alpha_0 = 0$ ,  $\alpha_{xk} = 0$ ,  $\alpha_{yk} = 0.4$ ,  $c_0 = 0$ ,  $c_y = 0.15$ ,  $c_k = 0.85$  and  $\delta = 0.96$ .

The last two columns show that long-run elasticities are increasing in timeinconsistency, whereas short-run elasticities persist on the same level. Present-biased consumers tend to put less weight on the future, which generates a shift in the demand function as well as a tilt. The effect is of a similar nature as when the discount rate,  $\delta$ , is decreasing. Nesting the model to a standard model with no intertemporal linkages, confirms this claim: the elasticities are constant, irrespective of time-inconsistency.

Next, for the same parameter values as those underlining table 2.2, let us examine the effects on behavior and welfare when introducing the public support system described above, in the form of subsidies to the cultural good, y. Table 2.3 summarizes the economic effects when the subsidized price is altered.

			- 11			
p	$\frac{y_{ss}}{x_{ss}}$	$k_{ss}$	$V\left(k_{0} ight)$	$W\left(k_{0} ight)$	$V\left(k_{ss}\right)$	$W\left(k_{ss}\right)$
1.0	0.0249	0.0243	12.44983	6.473913	12.44983	6.47391
0.9	0.0265	0.0258	12.44992	6.473949	12.45001	6.47400
0.8	0.0283	0.0275	12.44993	6.473945	12.45013	6.47407
0.7	0.0305	0.0296	12.44983	6.473879	12.45015	6.47408
0.6	0.0331	0.0321	12.44953	6.473711	12.45001	6.47400
0.5	0.0364	0.0351	12.44891	6.473381	12.44958	6.47378
0.4	0.0404	0.0389	12.44777	6.472780	12.44866	6.47330
0.3	0.0457	0.0437	12.44571	6.471712	12.44684	6.47236
0.2	0.0528	0.0502	12.44197	6.469799	12.44332	6.47053
0.1	0.0628	0.0591	12.43494	6.466247	12.43636	6.46691

Table 2.3: Effects of Public Support with Time-Inconsistent Preferences

Particularly noticeable in table 2.3 is that public subsidies of culture are actually welfare increasing for an agent with time-inconsistent preferences. In this explicit calibration, maximum welfare for the present self is achieved at p = 0.9, which corresponds to a subsidy of 10 percent. The past self prefers more subsidies, in which case the welfare maximizing price is 0.8. It can be shown that the past self always prefers a lower price and thus, more subsidies than the present self. This should be expected, since time-inconsistency implicates that different selves may disagree in the ranking of policy arrangements. The future selves prefer a higher subsidy of 30 percent at the completion of the transition path, since they need not consider the accumulation of consumption capital. Although the welfare levels diverge, the welfare maximizing subsidy will always coincide. The results from this exercise demonstrate that the steady-state consumption of culture and the steady-state capital stock are monotonically decreasing in price, just as in the time-consistent case, see table 2.1. The changes in cultural consumption are fairly substantial: a 70 percent subsidy, as in the Swedish case, will increase consumption by almost 80 percent.

A further observation to be made is that although the increase in the steadystate level of consumption of y is considerable and the level is more than 60 percent higher than the time-consistent solution, public policy does not have any large effects on welfare levels. With this parameterization, the welfare gains for the past and present selves are almost negligible, when subsidies are sized to maximize welfare, and negative when subsidies correspond to the Swedish level at 70 percent of the price. In order to establish the importance of the calibration on results, I will more closely examine the set of coefficients in the next section.

#### 3.4.3 Sensitivity analysis

In this section, the specific impact of the parameters of the utility function, capital accumulation and discounting on economic behavior, public policy and welfare are examined. The first row in table 2.4 once more shows the default case with a finer grid on p to make it possible to see subtle effects from the comparative statics. Now, to test the taste cultivation properties of the utility function, nest the model to a standard time-separable model with  $\alpha_{yk} = 0$ , and, as before,  $\alpha_k = \alpha_{kk} = 0$ . The rest of the parameter values are the same as those underlining table 2.3.

The second row in the table displays a consumption ratio of 0.0177 and a consumption capital level at 0.0174. As expected, the agent chooses to consume a lower amount of y, which is in line with the steady-state analysis in section 2.4. Particularly interesting is the finding that the optimal policy is now not to subsidize prices.

Before making any more profound comments on the latter finding, continue the exercise and let  $\alpha_{xx}$  and  $\alpha_{yy}$  vary, i.e., introduce additional decreases in marginal utilities. Thus, by increasing the absolute value of the two parameters and keeping the constraint  $\alpha_{xx} = \alpha_{yy}$ , the consumption level of y is higher as compared to the default case. The welfare maximizing subsidy is now larger and the long-run price

elasticity smaller.

Next, see how the parameters of the capital accumulation equation affect the results. Let the depreciation rate of consumption capital decrease so that  $c_k$  increases from 0.85 to 0.90, followed by an increase in the "saving" rate  $c_y$  from 0.15 to 0.20. Both parameters have a similar effect on the steady-state behavior, public policy and welfare: the levels of consumption and capital, the long-run elasticities and the welfare maximizing subsidies are larger than in the benchmark case.

The next exercise is to drop the constraints on  $\alpha_k$  and  $\alpha_{kk}$  and allow these parameters to affect the outcome. To isolate the effects of each parameter, let  $\alpha_k = 0$  and start by letting  $\alpha_k = 0.01$ , i.e., let the agent experience utility from the consumption capital. Compared to the new default case in the second row in table 2.4, steady-state consumption of y and consumption capital are larger. Although  $\alpha_{yk}$  is zero, there is a case for welfare improving subsidies. Next, let  $\alpha_{kk} = -0.1$ , i.e., introduce a decreasing marginal utility of cultural capital. This makes the consumption of y smaller than the default case and the agent prefers to tax consumption of y, which depends on the negative marginal utility of k.

 $p^{\ast}$  is referring to the welfare maximizing price of the past self.

The exercise has shown that the parameters separating this model from standard models of consumption,  $\alpha_k$ ,  $\alpha_{kk}$ , and  $\alpha_{yk}$ , have important effects on the result. There is a case for welfare improving subsidies with  $\alpha_k$ ,  $\alpha_{yk} > 0$ .

Not only the taste cultivation properties of the utility function have been demonstrated to be of importance for public policy decisions. Time-inconsistency, and thus the  $\beta$  parameter, also affect optimal policy. Keep the calibration from above when  $\alpha_{yk} \neq 0$  and decrease  $\beta$  to 0.4. The results clearly demonstrate that the magnitude of time-inconsistency is highly significant in determining the size of public subsidies. The optimal price is monotonously decreasing in time-inconsistency as represented by  $\beta$ .

So far, I have not commented on variable I, which is normalized to 1 in all previous analyses. What is the effect of income on consumption and preferred public policy? Comparative statics of I shows the consumption ratio of culture to the generic good to be increasing in income. Moreover, the preferred subsidies are increasing the larger is income.

	Table 2.4:	Effects of	l laste-Culti	vation Coeffi	cients	
Exercise	$\frac{y_{ss}}{x_{ss}}\Big _{p=1}$	$k_{ss} _{p=1}$	$V\left(k_{0}\right) _{p=1}$	$W(k_0) _{p=1}$	$p^*$	t
Default	0.0249	0.0243	12.4498	6.4739	0.83	0.0046
$a_{yk} = 0$	0.0177	0.0174	12.4451	6.4715	1.00	0.0000
$\alpha_{xx} = -1.01$	0.0284	0.0276	12.3910	6.4433	0.80	0.0061
$c_{k} = 0.9$	0.0308	0.0448	12.4546	6.4764	0.82	0.0062
$c_y = 0.2$	0.0290	0.0376	12.4528	6.4755	0.81	0.0062
$\alpha_{k} = 0.01$	0.0187	0.0184	12.4474	6.4726	0.86	0.0027
$\alpha_{kk} = -0.1$	0.0174	0.0171	12.4448	6.4713	1.06	-0.0010
$\beta = 0.4$	0.0256	0.0250	12.4535	5.2803	0.7200	0.0084
I = 1.01	0.0335	0.0328	12.4603	6.4794	0.71	0.0109
cont.	$\frac{y^*}{x^*}$	$k^*$	$V^{*}\left(k_{0}\right)$	$W^{*}\left(k_{0} ight)$	$\varepsilon_{p,LR}^{y}$	$\varepsilon_p^y,_{SR}$
	0.0277	0.0270	12.4499	6.4740	-0.5806	-0.3596
	0.0177	0.0174	12.4451	6.4715	-0.3635	-0.3319
	0.0315	0.0305	12.3911	6.4434	-0.4707	-0.2932
	0.0359	0.0520	12.4549	6.4765	-0.7674	-0.3783
	0.0337	0.0435	12.4531	6.4756	-0.7123	-0.3695
	0.0198	0.0194	12.4474	6.4727	-0.3715	-0.3390
	0.0170	0.0167	12.4448	6.4713	-0.3537	-0.3292
	0.0308	0.0299	12.4539	5.2804	-0.5773	-0.4082
	0.0386	0.0375	12.4607	6.4796	-0.4078	-0.2904

 Table 2.4: Effects of Taste-Cultivation Coefficients

The above findings can be summarized into a central conclusion of this study: time-inconsistent preferences and taste cultivation properties of the utility function are necessary and sufficient conditions for welfare enhancing funded subsidies. However, quantitatively large effects are hard to come by, without drastic changes in the model or unrealistic parameter values.

#### 3.4.4 Could huge subsidies be optimal in the case of Sweden?

The results from the previous exercises suggest that the size of  $\alpha_{yk}$ ,  $\alpha_{xx}$ ,  $\alpha_{yy}$  and  $\beta$  are crucial in designing an optimal support system for culture. To examine the

sensitivity of welfare maximizing subsidies, I experiment with various parameter combinations with the purpose of matching available moments from Swedish data. As before, I will try to match the following targets; a consumption ratio of approximately 0.05 and a price elasticity of demand, approximately around -0.5. It is also necessary for the utility function to be concave, marginal utilities to be positive, the steady state stable and cultural consumption increasing in capital, i.e.,  $g_k(k) > 0$ . The first exercise is to find the set of parameters that will motivate a 70 percent subsidy on the price of the arts, under the above conditions. First, the search will be restricted to  $\beta = 0.5$ , since this size of the coefficient appears to be standard in the literature.

The simplest approach for finding a set of parameters to match the desired data is the trial-and-error approach. This method is obviously time-consuming and all possible combinations cannot be controlled for. Despite a thorough search, I have not found any parameter combination yielding the optimal price of 0.3, under the condition that marginal utilities are positive. The best hit gave a welfare-maximizing price of 0.80 for past and present selves, see the result in table  $2.5^{18}$ . Since this result far from matches Swedish data, the price elasticity is now allowed to deviate from the default value fixed at around -0.5. For twice as inelastic demand, the optimal price can be reduced to 0.65; the results are noted in the second row.<sup>19</sup>

Relaxing the restriction on  $\beta$  by assuming the agent's preferences to be heavily time-inconsistent, i.e., letting  $\beta = 0.1$ , and slightly adjusting the coefficients, a welfare maximizing price of 0.50 is obtained.<sup>20</sup>

Although the theoretical and intuitive meaning of a positive direct utility of consumption capital,  $\alpha_k > 0$ , has only been briefly discussed earlier in the paper, I will manipulate this coefficient to see whether it is possible to match an optimal subsidy of 70 percent.<sup>21</sup> Keeping the strong time-inconsistent preferences and letting

<sup>&</sup>lt;sup>18</sup> The following parameter values are used;  $\alpha_x = 1, \alpha_y = 0.025, \alpha_k = 0, \alpha_{xx} = -1.01, \alpha_{yy} = -1.01, \alpha_{kk} = 0, \alpha_{xy} = 0, \alpha_{xk} = 0, \alpha_{yk} = 0.45, c_y = 0.15, c_k = 0.85$  and  $\delta = 0.96$ .

<sup>&</sup>lt;sup>19</sup> The following parameter values are used;  $\alpha_x = 1$ ,  $\alpha_y = 0.02$ ,  $\alpha_k = 0$ ,  $\alpha_{xx} = -1.024$ ,  $\alpha_{yy} = -1.024$ ,  $\alpha_{kk} = 0$ ,  $\alpha_{xy} = 0$ ,  $\alpha_{xk} = 0$ ,  $\alpha_{yk} = 0.5$ ,  $c_y = 0.15$ ,  $c_k = 0.85$  and  $\delta = 0.96$ .

<sup>&</sup>lt;sup>20</sup> The following parameter values are used;  $\alpha_x = 1, \alpha_y = 0.025, \alpha_k = 0, \alpha_{xx} = -1.01, \alpha_{yy} = -1.01, \alpha_{kk} = 0, \alpha_{xy} = 0, \alpha_{xk} = 0, \alpha_{yk} = 0.6, c_y = 0.15, c_k = 0.85$  and  $\delta = 0.96$ 

<sup>&</sup>lt;sup>21</sup> The following parameter values are used;  $\alpha_x = 1$ ,  $\alpha_y = 0.06$ ,  $\alpha_k = 0.08$ ,  $\alpha_{xx} = -1.0$ ,  $\alpha_{yy} = -1.0$ ,  $\alpha_{kk} = -0.20$ ,  $\alpha_{xy} = 0$ ,  $\alpha_{xk} = 0$ ,  $\alpha_{yk} = 0.03$ ,  $c_y = 0.15$ ,  $c_k = 0.85$  and  $\delta = 0.96$ 

 $\alpha_k = 0.08$  and  $\alpha_{kk} = -0.20$ , the welfare maximizing price is 0.15 for the past and 0.35 for the present self.

This exercise has demonstrated the sensitivity of parameterization on outcome. However, the results appear to confirm that the level of time-inconsistency and intertemporal linkages, in the form of taste formation properties, is positively correlated to the magnitude of public subsidies, which leads me next to the main conclusion of this paper: With high levels of time-inconsistency and strong taste cultivation properties of the utility function, a public subsidy of more than 70 percent could be optimal.

Consequently, whether public subsidies are welfare enhancing for present-biased agents is ultimately a quantitative question.

	Table 2.9. Sensitivity Analysis							
β	$\frac{y_{ss}}{x_{ss}}\Big _{p=1}$	$k_{ss}\big _{p=1}$	$V\left(k_{0}\right) _{p=1}$	$W\left(k_{0}\right)\big _{p=1}$	$p^*$	t		
0.5	0.0261	0.0254	12.3882	6.44186	0.80	0.0056		
0.5	0.0342	0.0331	12.2221	6.3555	0.65	0.0130		
0.1	0.0264	0.0257	12.3907	1.6851	0.50	0.0178		
0.1	0.0346	0.0334	12.5871	1.7118	0.15	0.0498		
cont.	$\frac{y^*}{x^*}$	$k^*$	$V^{*}\left(k_{0} ight)$	$W^{*}\left(k_{0} ight)$	$\varepsilon_p^y,_{LR}$	$\varepsilon_p^y,_{SR}$		
	0.0290	0.0282	12.3883	6.44192	-0.4757	-0.2781		
	0.0385	0.0371	12.2224	6.3556	-0.2527	-0.1355		
	0.0369	0.0356	12.3918	1.6852	-0.4808	-0.2768		
	0.0622	0.0585	12.6034	1.7127	-0.5223	-0.4645		

Table 2.5: Sensitivity Analysis

### 4 Conclusions

This study has focused on public support systems of culture. The analysis may contribute to a better understanding of consumption behavior and the welfare implications of cultural consumption and has offered a plausible motivation for public subsidies. A number of key features have been explored, such as forward-looking but present-biased consumers and taste cultivation properties of preferences. A theoretical model has been outlined which depicts relevant qualities of cultural consumption and serves as a framework for policy analysis. A policy example is designed to illustrate some substantially important aspects of public support systems of culture in practice and is calibrated to Swedish data.

The central findings of this study can be summarized as follows: consumption of culture is repressed and there is a substantial welfare cost if preferences are significantly present-biased and the utility function has taste cultivation properties. These two latter properties, time-inconsistency and intertemporal linkages in the utility function, are necessary and sufficient conditions for tax-financed subsidies to be welfare improving. If the level of time-inconsistency is extreme and the taste cultivation properties of the utility function are pronounced, large subsidies could be optimal. However, although tax-financed subsidies appear to be a potent device in restoring consumption to time-consistent levels, the welfare gains seem to be rather small in general, irrespective of the calibration.

Whereas the direct impact on utility of past consumption in the form of consumption capital is a central assumption in the addiction literature, this topic has only been briefly discussed here. Attempts at incorporating this effect in the model, without formalization, have shown important quantitative effects on the design of public support systems. In fact, when adding a positive direct utility of past consumption, subsidies of 70 percent or more of the gross price could be optimal, which matches the Swedish data. Exploring this channel more deeply is an interesting task which, however, must be left for future work.

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### 5 Appendix

### 5.1 Derivation of the Euler equation in the model with timeinconsistent preferences

In order to derive the Euler equation in the model with time-inconsistent preferences, the following algorithm is used:

With a recursive formulation, the consumer's maximization problem is

$$W(k) = \max_{x,y} \left\{ \begin{array}{l} u(x,y,k) + \beta \delta V(k') \\ x+y = 1 \\ k' = h(y,k) \end{array} \right\},$$

given

$$V(k) \equiv u(1 - g(k), g(k), k) + \delta V(h(g(k), k))$$
  
=  $u_t + \delta u_{t+1} + \delta^2 u_{t+2} + \dots$ 

The agent chooses control variable y with policy function  $y^* = g(k)$ . Substitute the restrictions into the welfare function

$$\max_{y} u(1-y, y, k) + \beta \delta V(h(y, k))$$

and derive the first-order condition

$$-u_x + u_y + \beta \delta V'_k h_y = 0, \qquad (2.12)$$

where a prime denotes the function evaluated in the next period. Next, find  $V'_k$ . Start by deriving  $V_k$ .

$$V_k = (-u_x + u_y + \delta V'_k h_y)g_k + u_k + \delta V'_k h_k.$$

Since the same is true in the next period, transpose one period ahead

$$V'_{k} = (-u'_{x} + u'_{y} + \delta V''_{k} h'_{y})g'_{k} + u'_{k} + \delta V''_{k} h'_{k}.$$
(2.13)

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Transpose the first-order condition in equation (2.12) and solve for  $V_k''$ , which gives

$$V_k'' = \frac{1}{\beta \delta h_y'} (u_x' - u_y').$$
(2.14)

Use equations (2.14) and (2.13) and  $V'_k$  can be written as

$$V'_{k} = (-u'_{x} + u'_{y} + \delta \left(\frac{1}{\beta \delta h'_{y}}(u'_{x} - u'_{y})\right)h'_{y})g'_{k} + u'_{k} + \delta \left(\frac{1}{\beta \delta h'_{y}}(u'_{x} - u'_{y})\right)h'_{k}$$
  
$$= (u'_{x} - u'_{y})(\frac{1}{\beta} - 1)g'_{k} + u'_{k} + (u'_{x} - u'_{y})\frac{1}{\beta}\frac{h'_{k}}{h'_{y}}.$$
 (2.15)

Substitute this expression into the first-order condition

$$-u_x + u_y + \beta \delta \left( (u'_x - u'_y)(\frac{1}{\beta} - 1)g'_k + u'_k + (u'_x - u'_y)\frac{1}{\beta}\frac{h'_k}{h'_y} \right) h_y = 0$$

and with the complete notation, the Euler equation can be written

$$\Delta(k) = \beta \delta h_y(k, g(k)) \begin{pmatrix} u_k(1 - g(k'), g(k'), k') \\ +\Delta(k') \left( \left(\frac{1}{\beta} - 1\right)g_k(k') + \frac{1}{\beta}\frac{h_k(k', g(k'))}{h_y(k', g(k'))} \right) \end{pmatrix} (2.16)$$
  
$$\Delta(k) = u_x(1 - g(k), g(k), k) - u_y(1 - g(k), g(k), k).$$

### 5.2 Solution of the model with time-inconsistent preferences

Solve the decision rule, g(k), for a linear-quadratic case when the instant utility function is given by

$$u(x, y, k) = a_0 + a_x x + a_y y + a_k k + \frac{a_{xx}}{2} x^2 + \frac{a_{yy}}{2} y^2 + \frac{a_{kk}}{2} k^2 + a_{xy} xy + a_{xk} xk + a_{yk} yk$$

and the capital accumulation function is

$$k_{t+1} = h(y_t, k_t) = c_0 + c_y y_t + c_k k_t.$$

Since the model is linear-quadratic, it is easily shown that the policy function is linear in k, such that y = a + bk. Derive marginal utilities and first-order derivatives of the capital accumulation function, solve for x from the resource constraint (2.1),

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and substitute in the Euler equation (??). Rewrite the first-order condition as A + Bk = 0, and we will have

$$A = \alpha_x - \alpha_y - a\alpha_{yy} - \alpha_{xy} (1 - 2a) + \alpha_{xx} (1 - a)$$
  

$$-\beta \delta c_y \left( \alpha_k + \alpha_{xk} + (\alpha_{yk} - \alpha_{xk}) (a + b (c_0 + ac_y)) + (c_0 + ac_y) \alpha_{kk} + \left( b \left( \frac{1}{\beta} - 1 \right) + \frac{1}{\beta} \frac{c_k}{c_y} \right) \times \left( \alpha_x - \alpha_y + (\alpha_{xk} - \alpha_{yk}) (c_0 + ac_y) + (2\alpha_{xy} - \alpha_{yy} - \alpha_{xx}) (a + b (c_0 + ac_y)) - \alpha_{xy} + \alpha_{xx} \right) \right)$$
  

$$B = (\alpha_{xk} - \alpha_{yk} + b(2\alpha_{xy} - \alpha_{xx} - \alpha_{yy}))$$
  

$$-\beta \delta c_y \left( (c_k + bc_y) (\alpha_{kk} - b (\alpha_{xk} - \alpha_{yk})) + \alpha_{xk} - \alpha_{yk} \right) (c_k + bc_y) \right).$$

Since A + Bk = 0 must be met for all k, A and B must be zero. This produces a non-linear equation system in the two unknown decision rule parameters a and b.

### 5.3 Derivation of welfare over the transition path

To evaluate welfare by implementing a subsidy, solve

$$\max W(k) = u(x, y, k) + \delta\beta V(h(y, k)),$$

under the budget constraint and capital accumulation equation for p = 1 and p < 1. Find the initial steady-state consumption capital,  $k_0$  and the subsidy funded tax,  $\tau$ . Find decision rules

$$g_x (k, (p, t)) = a_x + b_x k$$
  

$$g_y (k, (p, t)) = a_y + b_y k$$
  

$$g_k (k, (p, t)) = a_k + b_k k$$

such that

$$a_k = c_0 + c_y a_y,$$
  

$$b_k = c_y b_y + c_k \text{ and }$$
  

$$a_x = I - t - p a_y,$$
  

$$b_x = -p b_y.$$

Evaluate

$$V(k) = u(g_x(k, (p, t)), g_y(k, (p, t)), k) + \delta V(g_k(k, (p, t))),$$

which can be rewritten as

$$V\left(k\right) = \gamma_0 + \gamma_1 k + \gamma_2 k^2,$$

and in the next period as

$$V(k') = \gamma_0 + \gamma_1 (a_k + b_k k) + \gamma_2 (a_k + b_k k)^2.$$

Evaluate the value function

$$V(k) = u(x, y, k) + \delta V(k')$$
  
=  $\alpha_0 + \alpha_x x + \alpha_y y + \alpha_k k + \frac{\alpha_{xx}}{2} x^2 + \frac{\alpha_{yy}}{2} y^2 + \frac{\alpha_{kk}}{2} k^2 + \alpha_{xy} xy + \alpha_{xk} xk + \alpha_{yk} yk$   
+ $\delta \left[ \gamma_0 + \gamma_1 k' + \gamma_2 (k')^2 \right]$ 

in the decision rules and V(k'), which gives

$$\begin{split} LHS &= \gamma_{0} + \gamma_{1}k + \gamma_{2}k^{2} \\ RHS &= \alpha_{0} + \alpha_{x}\left(a_{x} + b_{x}k\right) + \alpha_{y}\left(a_{y} + b_{y}k\right) + \alpha_{k}\left(k\right) + \frac{\alpha_{xx}}{2}\left(a_{x} + b_{x}k\right)^{2} \\ &+ \frac{\alpha_{yy}}{2}\left(a_{y} + b_{y}k\right)^{2} + \frac{\alpha_{kk}}{2}\left(k\right)^{2} + \alpha_{xy}\left(a_{x} + b_{x}k\right)\left(a_{y} + b_{y}k\right) \\ &+ \alpha_{xk}\left(a_{x} + b_{x}k\right)\left(k\right) + \alpha_{yk}\left(a_{y} + b_{y}k\right)\left(k\right) \\ &+ \delta\left[\gamma_{0} + \gamma_{1}\left(a_{k} + b_{k}k\right) + \gamma_{2}\left(a_{k} + b_{k}k\right)^{2}\right]. \end{split}$$

This can be written as

$$A + Bk + Ck^2 = 0$$

where

$$A = \gamma_0 - \alpha_0 - a_x \alpha_x - a_y \alpha_y - \frac{1}{2} a_x^2 \alpha_{xx} - \frac{1}{2} a_y^2 \alpha_{yy} - a_x a_y \alpha_{xy} - \delta \left(\gamma_0 + \gamma_1 a_k + \gamma_2 a_k^2\right)$$
  

$$B = \gamma_1 - \alpha_k - b_x \alpha_x - b_y \alpha_y - a_x b_x \alpha_{xx} - \alpha_{xy} \left(a_x b_y + a_y b_x\right) - a_y b_y \alpha_{yy} - a_x \alpha_{kx}$$
  

$$-a_y \alpha_{ky} - \delta \left(\gamma_1 b_k + 2\gamma_2 a_k b_k\right)$$
  

$$C = \gamma_2 - \frac{1}{2} b_x^2 \alpha_{xx} - b_x b_y \alpha_{xy} - \frac{1}{2} \alpha_{kk} - \frac{1}{2} b_y^2 \alpha_{yy} - b_x \alpha_{kx} - b_y \alpha_{ky} - \delta \gamma_2 b_k^2.$$

Solve for  $\gamma_0, \gamma_1$ , and  $\gamma_2$ : using the fact that  $A + Bk + Ck^2 = 0$ , we obtain A = B = C = 0, and

$$\gamma_{2} = \frac{1}{1-\delta b_{k}^{2}} \left( b_{x}\alpha_{kx} + b_{y}\alpha_{ky} + b_{x}b_{y}\alpha_{xy} + \frac{1}{2}\alpha_{kk} + \frac{1}{2}b_{x}^{2}\alpha_{xx} + \frac{1}{2}b_{y}^{2}\alpha_{yy} \right)$$

$$\gamma_{1} = \frac{1}{1-\delta b_{k}} \left( \begin{array}{c} \alpha_{k} + b_{x}\alpha_{x} + b_{y}\alpha_{y} + a_{x}\alpha_{kx} + a_{y}\alpha_{ky} + \alpha_{xy}\left(a_{x}b_{y} + a_{y}b_{x}\right) \\ + a_{x}b_{x}\alpha_{xx} + a_{y}b_{y}\alpha_{yy} + \delta 2\gamma_{2}a_{k}b_{k} \end{array} \right)$$

$$\gamma_{0} = \frac{1}{1-\delta} \left( \alpha_{0} + a_{x}\alpha_{x} + a_{y}\alpha_{y} + a_{x}a_{y}\alpha_{xy} + \frac{1}{2}a_{x}^{2}\alpha_{xx} + \frac{1}{2}a_{y}^{2}\alpha_{yy} + \delta\left(\gamma_{1}a_{k} + \gamma_{2}a_{k}^{2}\right) \right).$$

Finally, evaluate V(k) at  $k_0$ .

### 5.4 Solving the model with the governmental budget constraint holding for all t

To solve the model when the governmental budget constraint is required to hold in every time period, the following algorithm is used.

Calculate the steady state for  $p = 1 : \{\bar{k}_1, V(k)\}$ . Let p < 1 and solve for the new steady state and endogenous tax,  $\tau$ . Next, solve the transition path for  $k_0 = \bar{k}_1$ , p and  $\tau_0$  with standard dynamic programming and derive the sequences for k and y. Let T be the sequence  $\{(1-p) y_0, (1-p) y_1, \dots, \tau_0\}$ . For a transition length of 10 periods, let the 10th element be  $\tau_0$  and then the 9th  $< \tau_0$  and so forth. The 11th element is  $\tau_0$ , the 12th element is  $\tau_0$  and so on. Solve for period 10 and forward with standard dynamic programming, i.e., use the solution derived above. Now, solve for  $y_9 = \arg \max u(x, y, k) + \beta \delta V_{10}$ , where  $V_{10} = \gamma_{0,10} + \gamma_{1,10} k_{10} + \gamma_{2,10} (k_{10})^2$ is the solution derived earlier. Continue and find  $V_8 \cdots$ , and so forth. With the computed decision rules, update the sequences for y over the transition and find T. If the old and the new sequences T are the same, repeat the procedure with a new p along a grid. Otherwise, find a new sequence of  $\tau$ s and iterate. When the solution has converged, compare welfare derived at the initial capital stock for different p,  $V_{10}(\bar{k}_1)$ , and find the welfare-optimizing subsidy.

### 5.5 Solving the model with shocks

First, guess that the value function has the same functional form as the period utility function,

$$V(k) = \gamma + \gamma_k k + \gamma_{\varepsilon} \varepsilon + \frac{\gamma_{kk}}{2} k^2 + \frac{\gamma_{\varepsilon\varepsilon}}{2} \varepsilon^2 + \gamma_{k\varepsilon} k\varepsilon,$$

where the  $\gamma$  coefficients are unknown and to be determined in the maximization process.

Next, write the right-hand side of the Bellman equation as

$$\max_{y} \alpha_{x}x + \alpha_{y}y + \alpha_{k}k + \frac{\alpha_{xx}}{2}x^{2} + \frac{\alpha_{yy}}{2}y^{2} + \frac{\alpha_{kk}}{2}k^{2} + \alpha_{xy}xy + \alpha_{xk}xk + \alpha_{yk}yk + \beta\delta E \left[\gamma + \gamma_{k}k' + \gamma_{\varepsilon}\varepsilon' + \frac{\gamma_{kk}}{2}(k')^{2} + \frac{\gamma_{\varepsilon\varepsilon}}{2}(\varepsilon')^{2} + \gamma_{k\varepsilon}k'\varepsilon'|\varepsilon\right],$$

subject to  $x + y = 1 + \varepsilon$  and  $k' = c_0 + c_y y + c_k k$ . The first-order condition can be derived as

$$-\alpha_x + \alpha_y - \alpha_{xx}(1 + \varepsilon - y) + \alpha_{yy}y + \alpha_{xy}(1 + \varepsilon - 2y) - \alpha_{xk}k + \alpha_{yk}k + \beta \delta E \left[\gamma_k c_y + \gamma_{kk} c_y(c_0 + c_y y + c_k k) + \gamma_{k\varepsilon} \varepsilon' c_y |\varepsilon\right] = 0$$

Take expectations with respect to  $\varepsilon$ , which gives

$$-\alpha_x + \alpha_y - \alpha_{xx}(1 + \varepsilon - y) + \alpha_{yy}y + \alpha_{xy}(1 + \varepsilon - 2y) - \alpha_{xk}k + \alpha_{yk}k + \beta\delta \left[\gamma_k c_y + \gamma_{kk} c_y(c_0 + c_y y + c_k k) + \gamma_{k\varepsilon} c_y \rho\varepsilon\right] = 0$$

Solve for y

$$y = \begin{pmatrix} \alpha_x - \alpha_y + \alpha_{xx} - \alpha_{xy} - \beta \delta (c_y \gamma_k + c_0 c_y \gamma_{kk}) \\ -k (\alpha_{ky} - \alpha_{kx} + \beta \delta c_k c_y \gamma_{kk}) \\ -\varepsilon (\alpha_{xy} + \rho \beta \delta c_y \gamma_{k\varepsilon} - \alpha_{xx}) \end{pmatrix} \times \frac{1}{(\alpha_{xx} - 2\alpha_{xy} + \alpha_{yy} + \beta \delta c_y^2 \gamma_{kk})}$$
$$= \lambda_0 + \lambda_1 k + \lambda_2 \varepsilon$$

where

$$\lambda_{0} = \frac{\alpha_{x} - \alpha_{y} + \alpha_{xx} - \alpha_{xy} - \beta \delta (c_{y} \gamma_{k} + c_{0} c_{y} \gamma_{kk})}{(\alpha_{xx} + \alpha_{yy} - 2\alpha_{xy} + \beta \delta \gamma_{kk} c_{y}^{2})}$$

$$\lambda_{1} = \left(-\frac{(\alpha_{ky} - \alpha_{kx} + \beta \delta c_{k} c_{y} \gamma_{kk})}{(\alpha_{xx} + \alpha_{yy} - 2\alpha_{xy} + \beta \delta \gamma_{kk} c_{y}^{2})}\right)$$

$$\lambda_{2} = \left(-\frac{(\alpha_{xy} + \rho \beta \delta c_{y} \gamma_{k\varepsilon} - \alpha_{xx})}{(\alpha_{xx} + \alpha_{yy} - 2\alpha_{xy} + \beta \delta \gamma_{kk} c_{y}^{2})}\right).$$

Next, the unknown coefficients,  $\gamma$ , must be determined. For this purpose, take  $V(k, \epsilon) = u(1 + \epsilon - y, y, k) + \delta E[V(k', \epsilon'|\epsilon)]$ , and use the linear expression for y and the expression for k'. Notice that this functional equation has  $\delta$ , not  $\beta \delta$ , which is the time inconsistency.

$$\begin{split} V(k,\epsilon) &= u(1+\varepsilon-y,y,k) + \delta E \left[ V(k',\varepsilon'|\varepsilon) \right] \\ &= \alpha_x(1+\varepsilon-y) + \alpha_y y + \alpha_k k + \frac{\alpha_{xx}}{2}(1+\varepsilon-y)^2 + \frac{\alpha_{yy}}{2}y^2 \\ &+ \frac{\alpha_{kk}}{2}k^2 + \alpha_{xy}(1+\varepsilon-y)y + \alpha_{xk}(1+\varepsilon-y)k + \alpha_{yk}yk \\ &+ \left[ \begin{array}{c} \delta \left(\gamma + \gamma_k \left(c_0 + c_y y + c_k k\right)\right) + \delta \gamma_{\varepsilon} \rho \varepsilon + \delta \frac{\gamma_{kk}}{2} \left(c_0 + c_y y + c_k k\right)^2 \\ &+ \delta \frac{\gamma_{\varepsilon\varepsilon}}{2} \sigma_{\varepsilon}^2 + \delta \frac{\gamma_{\varepsilon\varepsilon}}{2} \left(\rho \varepsilon\right)^2 + \delta \gamma_{k\varepsilon} \left(c_0 + c_y y + c_k k\right) \rho \varepsilon \end{array} \right] \end{split}$$

where

$$E\left[\delta\frac{\gamma_{\varepsilon\varepsilon}}{2}(\varepsilon')^2 \mid \varepsilon\right] = \delta\frac{\gamma_{\varepsilon\varepsilon}}{2}\sigma_{\varepsilon}^2 + \delta\frac{\gamma_{\varepsilon\varepsilon}}{2}(\rho\varepsilon)^2.$$

Substitute for y and rewrite as

$$\begin{split} V(k,\epsilon) &= \alpha_x (1+\varepsilon - (\lambda_0 + \lambda_1 k + \lambda_2 \varepsilon)) + \alpha_y (\lambda_0 + \lambda_1 k + \lambda_2 \varepsilon) + \alpha_k k \\ &+ \frac{\alpha_{xx}}{2} (1+\varepsilon - (\lambda_0 + \lambda_1 k + \lambda_2 \varepsilon))^2 + \frac{\alpha_{yy}}{2} (\lambda_0 + \lambda_1 k + \lambda_2 \varepsilon)^2 \\ &+ \frac{\alpha_{kk}}{2} k^2 + \alpha_{xy} (1+\varepsilon - (\lambda_0 + \lambda_1 k + \lambda_2 \varepsilon)) (\lambda_0 + \lambda_1 k + \lambda_2 \varepsilon) \\ &+ \alpha_{xk} (1+\varepsilon - (\lambda_0 + \lambda_1 k + \lambda_2 \varepsilon)) k + \alpha_{yk} (\lambda_0 + \lambda_1 k + \lambda_2 \varepsilon) k \\ &+ \left( \begin{array}{c} \delta \left(\gamma + \gamma_k \left(c_0 + c_y \left(\lambda_0 + \lambda_1 k + \lambda_2 \varepsilon\right) + c_k k\right)\right) + \delta \gamma_\varepsilon \rho \varepsilon \\ &+ \delta \frac{\gamma_{kk}}{2} \left(c_0 + c_y \left(\lambda_0 + \lambda_1 k + \lambda_2 \varepsilon\right) + c_k k\right)^2 \\ &+ \delta \frac{\gamma_{\varepsilon\varepsilon}}{2} \sigma_{\varepsilon}^2 + \delta \frac{\gamma_{\varepsilon\varepsilon}}{2} \left(\rho \varepsilon\right)^2 + \delta \gamma_{k\varepsilon} \left(c_0 + c_y \left(\lambda_0 + \lambda_1 k + \lambda_2 \varepsilon\right) + c_k k\right) \rho \varepsilon \end{array} \right). \end{split}$$

Use  $V(k,\varepsilon) = \gamma + \gamma_k k + \gamma_{\varepsilon}\varepsilon + \frac{\gamma_{kk}}{2}k^2 + \frac{\gamma_{\varepsilon\varepsilon}}{2}\varepsilon^2 + \gamma_{k\varepsilon}k\varepsilon$ , and rewrite the equation as:

$$A + Bk + C\varepsilon + Dk^2 + E\varepsilon^2 + Fk\varepsilon = 0.$$

Since A, B, C, D, E, and F must each be zero for the equation to hold for all values of k and  $\varepsilon$ , the equation system can be used to solve for  $\gamma$ . We have

$$A = \lambda_0^2 \left( \alpha_{xy} - \frac{1}{2} \alpha_{x^2} - \frac{1}{2} \alpha_{y^2} - \frac{1}{2} \delta c_y^2 \gamma_{k^2} \right) + \lambda_0 \left( \alpha_x - \alpha_y - \alpha_{xy} - \delta c_y \gamma_k + \alpha_{x^2} - \delta c_0 c_y \gamma_{k^2} \right) + \gamma - \alpha_x - \delta \left( \gamma + c_0 \gamma_k \right) - \frac{1}{2} \alpha_{x^2} - \frac{1}{2} \delta c_0^2 \gamma_{k^2} - \frac{1}{2} \delta \sigma_{\varepsilon}^2 \gamma_{\varepsilon^2}$$

$$B = \lambda_0 \lambda_1 \left( 2\alpha_{xy} - \alpha_{x^2} - \alpha_{y^2} - \delta c_y^2 \gamma_{k^2} \right) + \lambda_1 \left( \alpha_x - \alpha_y - \alpha_{xy} - \delta c_y \gamma_k + \alpha_{x^2} - \delta c_0 c_y \gamma_{k^2} \right) + \lambda_0 \left( \alpha_{kx} - \alpha_{ky} - \delta c_k c_y \gamma_{k^2} \right) + \gamma_k - \alpha_k - \alpha_{kx} - \delta c_k \gamma_k - \delta c_0 c_k \gamma_{k^2}$$

$$C = \lambda_0 \lambda_2 \left( 2\alpha_{xy} - \alpha_{x^2} - \alpha_{y^2} - \delta c_y^2 \gamma_{k^2} \right) + \lambda_2 \left( \alpha_x - \alpha_y - \alpha_{xy} - \delta c_y \gamma_k + \alpha_{x^2} - \delta c_0 c_y \gamma_{k^2} \right) + \lambda_0 \left( \alpha_{x^2} - \rho \delta c_y \gamma_{k\varepsilon} - \alpha_{xy} \right) + \gamma_{\varepsilon} - \alpha_x - \rho \delta \gamma_{\varepsilon} - \rho \delta c_0 \gamma_{k\varepsilon} - \alpha_{x^2}$$

$$D = \lambda_1^2 \left( \alpha_{xy} - \frac{1}{2} \alpha_{x^2} - \frac{1}{2} \alpha_{y^2} - \frac{1}{2} \delta c_y^2 \gamma_{k^2} \right) \\ + \lambda_1 \left( \alpha_{kx} - \alpha_{ky} - \delta c_k c_y \gamma_{k^2} \right) \\ + \frac{1}{2} \gamma_{k^2} - \frac{1}{2} \alpha_{k^2} - \frac{1}{2} \delta c_k^2 \gamma_{k^2}$$

$$E = \lambda_2^2 \left( \alpha_{xy} - \frac{1}{2} \alpha_{x^2} - \frac{1}{2} \alpha_{y^2} - \frac{1}{2} \delta c_y^2 \gamma_{k^2} \right) \\ + \lambda_2 \left( \alpha_{x^2} - \rho \delta c_y \gamma_{k\varepsilon} - \alpha_{xy} \right) \\ + \frac{1}{2} \gamma_{\varepsilon^2} - \frac{1}{2} \alpha_{x^2} - \frac{1}{2} \rho^2 \delta \gamma_{\varepsilon^2}$$

$$F = \lambda_1 \lambda_2 \left( 2\alpha_{xy} - \alpha_{x^2} - \alpha_{y^2} - \delta c_y^2 \gamma_{k^2} \right) + \lambda_2 \left( \alpha_{kx} - \alpha_{ky} - \delta c_k c_y \gamma_{k^2} \right) + \lambda_1 \left( \alpha_{x^2} - \rho \delta c_y \gamma_{k\varepsilon} - \alpha_{xy} \right) + \gamma_{k\varepsilon} - \alpha_{kx} - \rho \delta c_k \gamma_{k\varepsilon}.$$

## Chapter 3

# Origins of the Diversity of Culture Consumption

### 1 Introduction

Culture consumption is treated differently than the consumption of other goods in most modern societies: it often receives various forms of government support. From the perspective of economics, one must ask why this is the case. One important element of this inquiry is to understand consumer preferences for culture. Here, we believe it to be important to study differences in culture consumption across individuals. In particular, what explains the diversity in the population of the consumption of culture? We argue that culture goods are not like other consumption goods and, especially, that differences in the consumption of culture may be explained by experience: in other words, the taste for culture is in important parts cultivated. In this essay, we propose a theory focusing on this possibility and examine conditions under which culture diversity can arise due to the experience factor.

Our theory contrasts culture with another, generic, good or activity, which does not require taste cultivation in order to be appreciated. We make the model stylized so that culture is the polar opposite in this regard: without any previous experience in culture consumption, current culture consumption is not appreciated at all. However, in all other respects, these goods are symmetric in utility. Thus, one can first

<sup>\*</sup> This essay is joint work with Per Krusell. We thank Christina Lönnblad for excellent editorial assistance.

imagine a static version of the theory where the differences in culture consumption between two consumers with the same preferences and the same constraints are only due to differences in their past consumption of culture: the one with higher experience in culture consumption will choose higher current consumption. Moreover, the effect of a given difference in experience depends on how close substitutes the two goods are, and if the goods are quite close substitutes, experience becomes a very important determinant of the consumption differences between individuals.

In the dynamic model considered, experience is accumulated as a standard capital good: "investment" is represented by current culture consumption, and there is also depreciation, which we assume to be geometric as in standard capital theory. Thus, forward-looking consumers take into account how current culture consumption enhances the future enjoyment of culture consumption. An increase in current culture consumption therefore leads to an induced increase in future culture consumption. How important experience is—how strong the intertemporal complementarities in culture consumption are—for explaining differences in the choices between the culture good and the generic good depends on the substitutability between the goods and also on other features of preferences and of the individual's constraints.

For each possible current value of the individual's culture experience, the model generates an endogenous choice of culture and, residually, of the generic good. An implication of this behavior is an endogenous law of motion for culture experience: a mapping from the level of experience prior to this period into the experience level at the end of this period. The shape of this law of motion is the main focus of this study. It reveals, among other things, to what extent initial experience differences between individuals, and hence culture consumption levels, can persist and possibly be amplified.

One of the main findings here is that significant *long-run* differences in culture consumption can arise between individuals, even if the only difference between these individuals is the initial experience in consuming culture. Formally, the model delivers a law of motion implying multiple steady states. Thus, the model delivers "endogenous" long-run diversity in culture consumption. This occurs if the two goods are relatively close substitutes: over time, then, consumers either move toward complete specialization in the consumption of the generic good, or toward a mix with a significant emphasis on culture. If, on the other hand, the two goods are not close substitutes, then long-run differences in culture consumption can only be explained by fundamental differences in preferences or constraints and not by initial experience: there is a unique steady state, which is reached from all initial conditions.

If the two goods are close substitutes, there might also be a unique steady state with complete specialization on one of the goods. Here, notwithstanding the level of initial experience, the long-run outcome will be the same. Long-run specialization on the culture good will occur if the constraint set—the constraint that binds the consumption of both goods—is sufficiently generous and the goods are close enough substitutes: then current culture consumption can be set quite high and therefore, induce future consumption in a manner which is beneficial even if the initial experience is very low. This can be understood from the perspective of complementarity between present and future consumption: if this complementarity is sufficiently strong, it will lead rational individuals to take advantage of it. Longrun specialization on the generic good, in contrast, results when the constraint is tight, because the complementarity is then not sufficiently powerful. Thus, there is a "scale effect" in culture consumption.

Large *short-run* differences between two individuals in their consumptions of culture can also result from small differences in their initial experience levels. This only occurs if the two goods are close enough substitutes. Formally, this is also a case of multiple steady states but, in addition, the law of motion for the evolution of experience in culture is here discontinuous. In other words, there is a cutoff level of initial culture experience such that the individual is indifferent between a large and a small level of current consumption, where each of these levels then persists over time, and with slightly lower (higher) initial experience, there is a strict preference for the lower (higher) culture accumulation path. The discontinuity appears as a result of an objective function which is not concave when viewed over sequences of culture consumption, despite being concave in consumption at any given moment in time. The source of the nonconcavity is the complementarity between culture consumption at different points in time.

The above results apply if the individual makes the time allocation decisions in a

forward-looking manner and with preferences that are "time-consistent". A requirement for time consistency in the case where discounting is stationary, i.e., where the individual discounts consumption k periods away in a way that does not depend on what the current time period is, is that discounting is geometric. The assumption of geometric discounting has been standard for a long time. However, experimental evidence has recently cast some doubt on this assumption.<sup>1</sup> In particular, it has been argued that many individuals tend to have a "present-bias", i.e., to discount nearby periods more heavily per unit of time than faraway periods. This implies time-inconsistency of preferences, and a typical description of individuals with these preferences is made in terms of multiple selves: a given individual consists of a sequence of different selves, among whom preferences are conflicting, and these selves then play a dynamic game.<sup>2</sup> In particular, the current self thinks that the future selves are not "forward-looking enough". Motivated by these findings, the present paper also examines how present-biased preferences alter the predictions discussed above. We restrict the analysis to the case with only a finite number of feasible levels for culture capital that the individual can choose; this makes for a simpler analysis than if the domain is continuous.

Time-inconsistency has several implications in the culture accumulation model that the standard model of preferences does not admit. Moreover, these implications are only present if culture is a good featuring taste cultivation. First, a role for "optimism" and "pessimism" appears. More precisely, in the dynamic game there are sometimes multiple equilibria—multiple decision rules, each of which is a Markov-perfect equilibrium—and these equilibria can be ranked in terms of welfare. Thus, the model offers an explanation of differences in culture consumption that cannot be based on observables. Moreover, in this case, there could be a different role for government policy: an appropriate policy could potentially eliminate the bad equilibrium or equilibria. Second, there is another source of long- as well as short-run differences in culture consumption: whereas the model with standard time-consistent preferences generically delivers either one or three steady-state culture consumption levels and, at most, one discontinuity in the decision rule for

<sup>&</sup>lt;sup>1</sup> See, e.g., the discussion in Laibson (1994).

 $<sup>^{2}</sup>$  This kind of formulation was first made in Strotz (1956) and Phelps and Pollak (1968).

culture accumulation, the model with time inconsistency can deliver an equilibrium decision rule with several jumps and more than three steady states. Third, there are parameter configurations for which pure-strategy equilibria do not exist; i.e., culture consumption diversity arises from endogenous uncertainty. The nature of the findings in the model with time-consistent preferences are related to findings in Krusell and Smith (2003a,b) who study time-inconsistent preferences in the context of a consumption-savings problem. These papers find multiplicity, mixed-strategy equilibria in the case with a discrete domain, and jumps in the decision rule. However, it does not deliver a large number of steady states associated with the same equilibrium decision rule. Such an outcome, on the other hand, can be found in Krusell, Martin, and Ríos-Rull (2004), but then in a context of an optimal public policy problem where the time-inconsistency arises from the expectations formation of the private sector.

The model is stylized and abstracts from other determinants of culture consumption viewed to be important, such as the culture consumer's educational level. However, general education can, in part, be viewed as a substitute for experience, so we think that important aspects of the determinants of culture consumption can be captured with our setup.

There are also connections to the literature on addiction (see, e.g., Becker and Murphy, 1988), where the byproduct from current consumption of, say, cigarettes is modeled as having a negative influence on future utility through the accumulation of a stock. There, it is remarked that multiple steady states are possible under some conditions due to similar nonconvexities arising in the present context, where the effect on future utility of current consumption is also present, but has a positive sign. The two models have intertemporal complementarity in common: current consumption of culture/cigarettes encourages future consumption of culture/cigarettes. However, the addiction model emphasizes a negative, and in any case separate, effect on utility that is not considered here. In particular, we assume that if culture consumption is zero, the stock of culture does not at all influence utility. A version of the addiction model was also studied under time-inconsistent preferences but then in a linear-quadratic case where multiplicity or nonexistence was not explored (see Gruber and Köszegi, 2001).

Finally, since we study a dynamic optimization model which can feature a nonconcave objective function, there are connections to existing literature exploring such problems, such as Boyer (1978), Orphanides and Zervos (1993), and Skiba (1978); Wirl and Feichtinger (2000,2004) moreover show that also concave problems can have similar features to those reported here.

Section 2 introduces the general setup under the assumption that preferences are time-consistent and contains the results for that case. Section 3 then studies time-inconsistent preferences. Section 4 concludes with some remarks.

### 2 Time-consistent preferences

In this section, we will analyze a benchmark model where a culture good is viewed as a good where the current enjoyment of the good is higher if this good has been consumed in the past: it is an "experience good". There is one other good assumed not to be an experience good, and the consumer's choice between these two goods at different points in time is then studied. The constraint faced by the consumer in this simple framework is not interpreted as a monetary budget but as a time constraint. Thus, we can think of the two goods as "activities" rather than as regular goods.

### 2.1 A simple model of culture habits

First, consider a simple static model where the consumer has a choice between two activities, which we can think of as two goods. The consumer has a total time endowment of I units to spend on the two activities/goods: x + y = I, where x and y are the two goods. We will think of x as an activity which does not deserve a "culture" label and y as one that does. For example, x could be the total time spent watching Robinson, whereas y is the total time spent watching Lilla Melodifestivalen.

The preference over the two goods is given by u(x, yk), where u is a standard utility function which is strictly concave in its two arguments. We will think of uas being symmetric in its two arguments, and we will later more specifically use the formulation u(x, yk) = f(x) + f(yk). Moreover, k is a "weight" on culture that, in principle, could differ across consumers and therefore explain why some consumers like higher y levels than others. The main contribution of the present analysis is to "endogenize" k by letting k capture the previous experience in consuming the y good. In other words, culture is an "experience good", and k summarizes the "total experience", or the stock of "culture capital". Thus, we will also explicitly describe how k is increasing in the previous consumption of y, and how consumers take into account how more culture consumption increases the future appreciation of culture consumption when deciding between x and y. For this purpose, a dynamic model is needed, and we now move to the description of this model.

Time is assumed to be discrete and the consumer is assumed to live for an infinite number of time periods. Suppose also, as indicated, that we have flow utility given by a function u(x, yk), and that present-value utility is given by

$$\sum_{t=0}^{\infty} \delta^t u(x_t, k_t y_t)$$

where t subscripts denote the period. As also indicated above, we assume that  $x_t + y_t = I$ , with  $x_t \ge 0$  and  $y_t \ge 0$ , for all t. Finally, we assume that culture capital accumulates according to  $k_{t+1} = h(k_t, y_t)$ , where h is increasing in its two arguments. Specifically, we assume that

$$k_{t+1} = (1 - d)k_t + by_t,$$

with  $d \in [0, 1]$  and b > 0. The formulation implies a certain complementarity between culture consumption at different points in time: because k multiplies y in utility and k depends on past ys, high values for past ys encourage a high current y, and vice versa. As we shall see in the context of a specific example, this complementarity may or may not be sufficiently strong to render the objective non-concave.

Thus, a consumer with an initial stock of culture capital equal to  $k_0$  chooses a sequence  $\{x_t, y_t, k_{t+1}\}_{t=0}^{\infty}$  satisfying the time endowment constraint and the culture accumulation equation at all times in order to maximize the present-value utility function.

Let us analyze this maximization problem using recursive methods. A variable with a prime denotes the value of this variable next period. Thus, let y(k, k')solve k' = h(k, y(k, k')) for all (k, k'): the function y(k, k') describes the amount of culture consumption now that is consistent with starting with k now and starting next period with k'. Then, the dynamic programming problem reads

$$v(k) = \max_{k' \ge 0} u(I - y(k, k'), y(k, k')k) + \delta v(k').$$

Assuming an interior solution, this leads to

$$-u_1y_2 + ku_2y_2 + \delta v'(k') = 0,$$

and, from the envelope theorem,

$$v'(k) = -u_1y_1 + u_2(y_1k + y) = (-u_1 + u_2k)y_1 + u_2y.$$

Thus, we have

$$(-u_1 + ku_2)y_2 + \delta((-u_1' + u_2'k')y_1' + u_2'y') = 0.$$

This amounts to a difference equation which is necessary and sufficient for an optimum if u(I - y(k, k'), y(k, k')k) is concave in (k, k'). Making this assumption, let us look for a steady state. This delivers

$$(-u_1 + ku_2)(y_2 + \delta y_1) + \delta u_2 y = 0.$$

In an interior steady state, provided that  $y_2 + \delta y_1 > 0$ , we observe that  $u_2k < u_1$ , i.e., that the static marginal utility of highbrow culture is lower than that of mass culture. In a static sense, this looks like irrational overconsumption of highbrow culture: the consumer would be better served by reducing the consumption of highbrow culture toward the equalization of marginal utilities. In a dynamic sense, however, and this is the relevant sense, it is precisely rational: the consumption of highbrow culture has another benefit, and one that is realized in the future: it helps build, or maintain, the stock of culture capital, thus increasing the marginal utility of such consumption at future dates. Accordingly, the marginal benefits of consumption of the two kinds of goods are indeed equalized in a steady state.

Let us now suppose that h(k, y) = (1-d)k + by, so that in steady state y = (d/b)k. In addition, we have that y(k, k') = (-(1-d)/b)k + (1/b)k' so that  $y_1 = (-(1-d)/b)k + (1/b)k'$   $d)/b) < 0, y_2 = 1/b > 0$ , and  $y_2 + \delta y_1 = (1 - \delta + \delta d)/b > 0$ . Finally, assume that  $u(x, yk) = x^{\alpha} + (yk)^{\alpha}$ , with  $\alpha \in (0, 1)$ . Then, we have

$$\left( (I - (d/b)k)^{\alpha - 1} - k((d/b)k^2)^{\alpha - 1} \right) \frac{1 - (1 - d)\delta}{b} = \frac{\delta d}{b} k((d/b)k^2)^{\alpha - 1}.$$

Simplified, this equation gives

$$\frac{Ib}{d} - k = \left(1 + \frac{d\delta}{1 - (1 - d)\delta}\right)^{\frac{1}{\alpha - 1}} k^{\frac{2\alpha - 1}{\alpha - 1}}.$$
(3.1)

The following three subsections examine the model in more detail without arriving at a complete characterization. Section 2.1.1 looks at "candidate" steady states, i.e., levels of k satisfying the first-order condition derived above. Section 2.1.2 then studies local dynamics around steady-state candidates based on first-order conditions, thus delivering further insights regarding the possible time paths for culture capital. Finally, Section 2.1.3 shows that global concavity of this problem is met when  $\alpha \leq 0.5$  but never otherwise. However, it also analyzes conditions for local sufficiency, thus providing conditions under which a candidate steady state is indeed a steady state if the domain is sufficiently restricted. These three subsections are rather detailed and can be skipped by a reader mainly wishing to see what classes of results are possible. These are described, finally, in Section 2.2, where we show examples of decision rules for culture capital accumulation for different parameter configurations, and where we discuss how these decision rules depend on the parameters.

## 2.1.1 Steady-state candidates: existence and uniqueness

Still presuming the interior condition to be sufficient, let us investigate the existence and uniqueness of a solution to the steady-state condition: this is a candidate steady state. Notice that the exponent on k on the right-hand side,

$$\frac{2\alpha - 1}{\alpha - 1}$$

is globally decreasing in  $\alpha$  and that it equals 1 for  $\alpha = 0, 0$  for  $\alpha = 0.5$ , and  $-\infty$  for  $\alpha = 1$ . Thus, under suitable assumptions on primitives, it is easily shown that

if  $\alpha \leq 0.5$ , since the right-hand side of the steady-state equation is increasing and the left-hand side is decreasing, there exists a unique steady state.

If  $\alpha > 0.5$ , then the curve defined by the right-hand side of the steady-state relation is downward-sloping and strictly convex in k. This means that if it intersects the curve given by the left-hand side, which is linear in k, there are two intersection points (unless the curves are tangent, which would only occur as a knife-edge case in terms of the parameter space). That is, whenever  $\alpha > 0.5$ , there are two positive steady states if any steady state exists.

Notice, moreover, that notwithstanding what  $\alpha$  is, positive steady-state candidates exist if I is sufficiently large.

Finally, let us consider the possibility that k = 0 would be a steady state. This would mean that a corner solution would be obtained; that is, the first-order condition would hold with inequality:

$$(-u_1 + ku_2)y_2 + \delta((-u_1' + u_2'k')y_1' + u_2'y') < 0,$$

evaluated at k = y = 0. Given the parametric functional forms assumed above, this cannot be true for  $\alpha \leq 0.5$ , because in that case, both  $ku_2$  and  $yu_2$  are plus infinity, whereas  $-u_1$  is bounded (recall that  $y_1$  and  $y_2$  are constants). However, it is precisely the case that if  $\alpha > 0.5$ , then the expression is strictly negative for all values of the primitives, since  $ku_2$  and  $yu_2$  are then both zero. Thus, a zero-capital steady state with no culture consumption satisfies the local conditions for optimality, if and only if  $\alpha > 0.5$ .

In sum, the conclusions are as follows. If  $\alpha \leq 0.5$ , there is a unique steadystate candidate with positive culture consumption. If  $\alpha > 0.5$ , then there is always a steady-state candidate with zero culture capital and zero culture consumption. Moreover, if the period budget is sufficiently large, two additional steady-state candidates also exist in the case where  $\alpha > 0.5$ , each of which has a positive stock of culture capital and positive culture consumption.

It remains to be seen whether these candidates are indeed steady states. For this purpose, global concavity of the maximization problem would be sufficient but, as we shall see, this property is hard to confirm generally. It is also important to discuss dynamics here, and they will be explored in detail in the following sections. A preliminary hypothesis that emerges so far is: (i) in the case of a unique positive steady state, there is global convergence to it; (ii) in the case of a unique steady state which is zero, there is global convergence to it; and (iii) in the case with three steady states, the middle steady state is unstable and there is convergence to the zeroculture steady state or the steady state with culture specialization, depending on the initial conditions. Thus, case (iii) would express the idea of hysteresis in culture consumption: initial conditions, and not preferences, are crucial for understanding why consumers differ in their consumption of highbrow culture. We shall see that this hypothesis about optimal culture consumption is close to, but not entirely, correct.

## 2.1.2 Local dynamics around steady states

Defining

$$F(k,k') \equiv u(I - y(k,k'), y(k,k')k),$$

the first-order condition reads

$$F_2(k,k') + \delta F_1(k',k'') = 0.$$

Local dynamics can be analyzed by linearization. Thus, at a steady state, we have

$$F_{12}\hat{k} + (F_{22} + \delta F_{11})\hat{k}' + \delta F_{12}\hat{k}'' = 0,$$

where hats denote deviations from steady state. A linear rule sets  $\hat{k}' = \lambda \hat{k}$  for some  $\lambda$ . Thus, we have

$$F_{12} + (F_{22} + \delta F_{11})\lambda + \delta F_{12}\lambda^2 = 0,$$

which can be used to solve for  $\lambda$ . In a slightly simplified form, this equation becomes

$$\frac{1}{\delta} + \frac{F_{22} + \delta F_{11}}{\delta F_{12}} \lambda + \lambda^2 = 0.$$
 (3.2)

By ocular inspection of this second-order polynomial function, it is clear that  $\lambda$  must have positive roots if  $X \equiv \frac{F_{22}+\delta F_{11}}{\delta F_{12}} < 0$ , provided that it has roots. Moreover, under this condition, a necessary and sufficient condition for local stability of a steady state (that is, for one root above 1 and one root below 1) is thus given by the condition

$$\frac{1}{\delta} + X + 1 < 0,$$

which just requires that the quadratic function be below zero at  $\lambda = 1$ . Further, local instability (both roots larger than 1) requires that the function is above zero and decreasing at  $\lambda = 1$ , i.e., that

$$\frac{1}{\delta} + X + 1 > 0,$$

and that

$$X + 2 < 0.$$

It can also be seen that, if there are roots, one of these roots must be larger than  $1/\sqrt{\delta} > 1$ . To see this, notice that the quadratic function  $1/\delta + X\lambda + \lambda^2$  has its minimum at  $X + 2\lambda = 0$ . The requirement that there is a root thus says that  $1/\delta - 2\lambda^2 + \lambda^2 < 0$ , i.e., that  $\lambda^2 > 1/\delta$ , from which the assertion follows.

Without specific restrictions on F, the stability properties cannot be determined. In our special parametric case, where  $F(k, k') = (I - y(k, k'))^{\alpha} + (y(k, k')k)^{\alpha}$ , where y(k, k') = (-(1 - d)k + k')/b, it can be shown (see the next section) that  $F_{11} < 0$ ,  $F_{12} > 0$ , and  $F_{22} < 0$ , so that indeed X < 0 as presumed above.

## 2.1.3 Concavity and local sufficiency of first-order conditions

We have focused on interior solutions in the above discussions, especially at and around steady-state points. Here, we will first develop the conditions under which there is sufficient concavity to guarantee that these assumptions are met for all points in the domain. Then, we will discuss how to check local sufficiency of first-order conditions.

**Global concavity** We will focus on the case where, as in the special parametric case, u(x, yk) = f(x) + f(yk), f is strictly concave, and where h(k, y) is linear, so that y(k, k') is linear.

With the notation of the previous subsection, then, F(k, k') = u(I - y(k, k'), y(k, k')k) =

f(I - y(k, k')) + f(y(k, k')k). We obtain

$$F_1 = (-u_1 + ku_2)y_1 + u_2y$$

and

$$F_2 = (-u_1 + ku_2)y_2.$$

Thus,

$$F_{11} = y_1[f''(x)y_1 + f'(yk) + f''(yk)k(ky_1 + y)] + y_1f'(yk) + yf''(yk)(ky_1 + y),$$
  

$$F_{12} = y_2[f''(x)y_1 + f'(yk) + f''(yk)k(ky_1 + y)],$$

which is positive in our special case since  $y_1 < 0$  and  $f'(yk) + f''(yk)yk = \alpha(1 + \alpha - 1)(yk)^{\alpha-1} > 0$ , and

$$F_{22} = y_2^2 [f''(x) + k^2 f''(yk)].$$

For somewhat shorter expressions, we can rewrite  $F_{11}$  as

$$F_{11} = f''(x)y_1^2 + f''(yk)(ky_1 + y)^2 + 2f'(yk)y_1.$$

The condition for concavity is that  $F_{11}$  and  $F_{22}$  both be negative and that  $F_{11}F_{22} - F_{12}^2 > 0$ . We see that  $F_{22}$  is always negative and that  $F_{11}$  is negative as well. After some cancellations, we have for the cross term

$$F_{11}F_{22} - F_{12}^2 = f''(x)f''(yk)y_2^2[y_1^2k^2 + (ky_1 + y)^2] + 2f'(yk)y_1y_2^2[f''(x) + f''(yk)k^2] -y_2^2(2y_1k(ky_1 + y)f''(x)f''(yk) + f'(yk)^2 + 2f'(yk)(y_1f''(x) + k(ky_1 + y)f''(yk)).$$

This expression further simplifies to

$$y_2^2[f''(x)f''(yk)y^2 - f'(yk)^2 + 2f'(yk)y_1(f''(x) + f''(yk)k^2) - 2f'(yk)(y_1f''(x) + k(ky_1 + y)f''(yk))]$$
  
=  $y_2^2[f''(x)f''(yk)y^2 - f'(yk)^2 - 2f'(yk)kyf''(yk))].$ 

We see that this expression is ambiguous in general, since the middle term is negative, whereas the other terms are positive. For our special case, let us examine this expression in more detail. It is proportional to  $f''(x)f''(yk)y^2 - f'(yk)^2 - 2f'(yk)kyf''(yk))$ , which reads

$$\alpha^{2}(1-\alpha)^{2}(I-y)^{\alpha-2}(yk)^{\alpha-2}y^{2} - \alpha^{2}(yk)^{2\alpha-2} - 2\alpha^{2}(\alpha-1)(yk)^{2\alpha-2},$$

which is itself proportional to

$$(1-\alpha)^2 (I-y)^{\alpha-2} (yk)^{\alpha-2} y^2 + (1-2\alpha)(yk)^{2\alpha-2}.$$

Here, observe that if  $\alpha \leq 0.5$ , this expression is strictly positive and concavity is therefore globally verified. Recall that this is also the case where the steady state is unique.

If  $\alpha > 0.5$ , whether the concavity condition is satisfied depends on the value of k. For any fixed positive values of k and y, it is clear that this condition must be met if  $\alpha$  is sufficiently close to 0.5. On the other hand, suppose that we fix  $\alpha$  at some value above 0.5 and, for simplicity, suppose that we consider values where k' = k, so that y is proportional to k. Then, the expression becomes

$$A(I-y)^{\alpha-2}k^{2(\alpha-1)} - Bk^{4(\alpha-1)},$$

where A and B are positive constants. This expression can be simplified to

$$k^{2(\alpha-1)} \left( A(I-y)^{\alpha-2} - Bk^{2(\alpha-1)} \right).$$

Thus, it is clear that as k becomes closer to 0 (and y thus also becomes closer to 0), this expression must turn negative, since  $\alpha < 1$ . This implies that when  $\alpha > 0.5$ , although sufficiently large values for k mean that this expression is positive, global concavity of F(k, k') over its entire domain cannot be established.

**Local sufficiency** To verify that a steady-state candidate represents consumer maximization *locally*, we need to look at whether the objective

$$F(k,k') + \delta v(k')$$

$$v_k(k) = F_1(k, g(k))$$

for all values of k, we have that the derivative of the objective with respect to k' is

$$F_2(k,k') + \delta F_1(k',g(k')).$$

Assuming g to be differentiable, and using the notation  $\lambda = g_k$  (consistently with the previous section,  $\lambda$  is the local slope of the decision rule), we obtain that the second derivative of the objective with respect to k' equals

$$F_{22} + \delta F_{11}' + \delta F_{12}' \lambda'.$$

In other words, we see that local concavity is satisfied at a steady state, if this expression is negative, i.e., if

$$\lambda < -\frac{F_{22} + \delta F_{11}}{\delta F_{12}} = -X,$$

assuming that  $F_{12} > 0$ , which it is in our special case. That is,  $\lambda$  must be lower than X, where  $\lambda$  also solves  $1/\delta + X\lambda + \lambda^2 = 0$ . As we shall now see, the existence of a positive real root ensures concavity. There is a solution to the polynomial equation if the minimizer of the polynomial,  $\lambda = -X/2$  leads to a non-negative value. If there is a solution, the smaller solution must satisfy  $\lambda < -X/2$ , which is less than -X, thus ensuring concavity. Accordingly, we know that if we find positive steady-state candidates, they will satisfy local sufficiency.

# 2.2 Full characterization of decision rules and comparative statics

In this section, we characterize decision rules for culture accumulation, g(k). Most of the characterization is based on numerical model solution, but the insights obtained parallel those in the previous sections, where candidate steady states and local stability around steady states were examined. A general feature present in the parametric case of the model considered is that g(k) is globally increasing. The economic content of this feature is that if consumer A starts with a higher culture capital than consumer B, then consumer A will always have a higher culture capital than consumer B. In Section 3, we prove this assertion for a generalized version of our model. This property will be visualized in the decision rules computed and graphed in this section.

We would also like to know whether the amount of culture consumed, i.e., (g(k) - (1-d)k)/b, is also globally increasing in k: whether consumers with a higher culture capital stock consume more culture. This feature may hold generally—it is intuitively plausible—but we have not been able to prove it. It is verified in all the examples we have computed numerically. Clearly, it also holds in the special case where there is full depreciation, i.e., when d = 0, because then the result follows from g(k) being increasing.

The analysis is divided into two broad sections: the  $\alpha \leq 0.5$  case and the  $\alpha > 0.5$  case. For each case, we explore the shapes of decision rules and how these decision rules are influenced by the parameters of the model.

## **2.2.1** Low substitutability: $\alpha \leq 0.5$

Here, many properties of the decision rule can be ascertained based on the above results. From the inspection of equation (3.1), we know that the steady state is unique, and that it satisfies the global conditions for a maximum. Moreover, we can use standard methods to show that g(k) is a continuous function.

Regarding comparative statics, an inspection of this equation reveals that the steady-state level of culture capital increases in Ib and  $\delta$ . These effects are intuitive: I allows a higher consumption of both goods, and b raises the relative appreciation of the culture good; increased patience makes the consumer place a higher weight on the dynamic benefits of consuming culture, leading to a higher long-run culture capital stock. The effects of an increase in depreciation, d, on k are ambiguous. On the one hand, higher depreciation acts as a lowering of the return to accumulating culture capital—thus working toward a lower culture capital stock. On the other hand, for a given amount of culture capital built for next period and a given starting level of culture capital, an increased depreciation raises the required culture consumption—

y = (k' - (1 - d)k)/b—and since culture consumption is complementary over time, this increase in y required by the increased need to replenish k induces an increase in k. We will look at how these two effects play out below.

Finally, an increase in  $\alpha$ , the substitutability parameter, makes the function of kon the right-hand side of equation (3.1) both shift down (the constant) and become more curved; it becomes steeper at zero and shifts down for high values of k. What is the resulting effect on the steady state? Note that if  $k = (1+r)^{-1}$ , where  $r \equiv \frac{d\delta}{1-\delta+d\delta}$ , then the right-hand side equals  $(1+r)^{-2}$  for all values of  $\alpha$ ! I.e., for a specific value of k, the right-hand side does not vary with k, which means that a change in  $\alpha$ makes the right-hand side rotate around  $((1+r)^{-1}, (1+r)^{-2})$ . More precisely, for  $k < (1+r)^{-1}$ , the function increases in  $\alpha$ , and for  $k > (1+r)^{-1}$ , the function decreases in  $\alpha$ . Thus, the steady-state effect of changing  $\alpha$  depends on whether at  $k = (1+r)^{-1}$  the left-hand side is above or below the right-hand side, i.e., whether  $(bI)/d - (1+r)^{-1}$  is above or below  $(1+r)^{-2}$ . This, among other things, is regulated by I. So if I is sufficiently high, because the left-hand side is above the right-hand side at  $k = (1+r)^{-1}$ , the steady state is above  $(1+r)^{-1}$ , and an increase in  $\alpha$  will raise the steady state. Conversely, if I is sufficiently low, the steady state is below  $(1+r)^{-1}$ , and in this case a rise in  $\alpha$  will lower the steady state.

The intuition for the comparative statics with respect to  $\alpha$  can be phrased in terms of "scale effects": an increase in the substitutability between goods will cause more specialization in one of the goods, and if the time/resource constraint is sufficiently lax, this favors the culture good, because the enjoyment of the culture good over time allows complementarity of resources over time. To see this, suppose that you always set  $y = \lambda I$ , independently of k. Then, in the long run, k will be  $b\lambda I/d$ , and the utility enjoyment from culture per period will be  $(ky)^{\alpha} = (b\lambda^2 I^2/d)^{\alpha}$ . In contrast, the enjoyment from the generic good will be  $((1-\lambda)I)^{\alpha}$ . That is, since the enjoyment of culture involves the square of I and the enjoyment of the generic good only involves I, we can call this a scale effect. So if the scale is sufficiently large, more substitutability will favor the culture good; otherwise it will favor the generic good.

Figure 3.1 below shows the decision rule for a typical parameter configuration where the scale effect is weak (I is low), and it also shows the decision rules for a

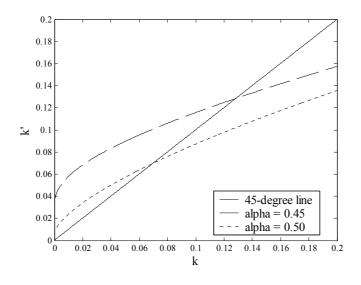


Figure 3.1:

higher and a lower value of  $\alpha$ .<sup>3</sup> As substitutability is increased, the figure reveals that the whole decision rule moves down, thereby implying a lower steady-state value for culture capital. The interpretation, as explained above, is one of scale effects in conjunction with substitutability: as there is more substitutability between the goods, the agent is more willing to forgo the culture good in order to focus on the generic good, since the scale is not sufficiently large for the intertemporal culture complementarities to pay off. Thus, culture consumption and accumulation are lower for all values of k.

In the borderline case when  $\alpha = 0.5$ , the decision rule has the same character as when  $\alpha < 0.5$ : it has g(0) > 0 and it is increasing and concave.

Figure 3.2 below shows the comparative statics in the case where the rate of depreciation is changed; an increase in d decreases cultural accumulation globally, and the steady state falls in particular.

In Figure 3.3, the discount factor is varied, and we see how an increase in the discount factor—implying more patience—globally increases culture capital accumulation and culture consumption.

<sup>3</sup> The parameter values are  $\alpha = 0.45$ , I = 1, d = 0.75, b = 0.25, and  $\delta = 0.96$ .

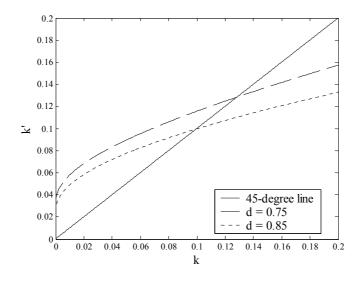


Figure 3.2:

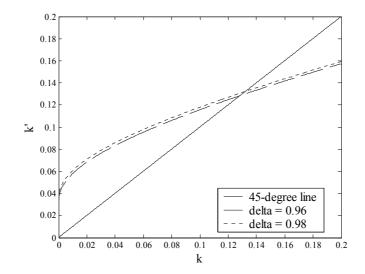


Figure 3.3:

## **2.2.2** High substitutability: $\alpha > 0.5$

When  $\alpha$  is above 0.5, so that substitutability is sufficiently strong to imply (i) that there are two positive steady-state candidates and (ii) that the objective F(k, k') is not globally concave in (k, k'), the decision rules do not only change qualitatively, but there are also different kinds of decision rules.

First, let us look at the set of candidate steady states, i.e., at equation (3.1). The right-hand side is now a convex, downward-sloping function of k. Thus, if it intersects the left-hand side, it does so twice (generically). These two steady states go further apart—the lower steady state decreases and the upper steady state increases—if Ib goes up, because that shifts the left-hand side up. If  $\delta$  goes up, the right-hand side shifts down, which has a similar effect. Thus, for the higher steady state, the comparative statics is the same as in the case where  $\alpha \leq 0.5$ , but for the lower steady state, the comparative statics reverse. Similarly, the effects of changes in d are ambiguous.

As for the comparative statics of the candidate steady states with respect to the substitutability parameter  $\alpha$ , we have the same features as before: as  $\alpha$  changes, the right-hand side of the equation rotates around a given point  $((1 + r)^{-1}, (1 + r)^{-2})$ , and the resulting changes in the steady states depend on where this point is relative to the left-hand side at  $k = (1 + r)^{-1}$ . If, say, I, is sufficiently large so that  $(bI)/d - (1 + r)^{-1} > (1 + r)^{-2}$ , then an increase in  $\alpha$  raises the lower as well as the higher steady state. If I is not sufficiently large, then an increase in  $\alpha$  raises the lower steady state and lowers the higher steady state.

Turning to the exploration of local dynamics, we verify numerically by solving equation (3.2) that the higher steady state always has real roots and is dynamically stable: the candidate decision rule slope at that steady state is always positive (and less than one). The lower steady state either yields real roots—implying a slope higher than 1—or delivers complex roots. The latter is always the case, when the lower steady state is close enough to zero for example. Thus, in the cases where there is no real root, the associated candidate steady state cannot be a steady state. Even when a candidate steady state has real roots and thus, from the analysis in Section 2.1.3, has a policy choice which is a local maximizer, it may not be a global maximizer. Our numerical computation of entire decision rules is therefore necessary for a full investigation.

We will consider three typical cases. In one case, the decision rule is continuous and intersects the 45-degree line twice for positive values, where the lower steady state is unstable and the higher steady state stable. In another case, the decision rule has a discontinuous jump at some value for k. In a third case, the decision rule is either everywhere below the 45-degree line or intersects it once. It will, however, be useful to start with a simple extreme example: that where  $\alpha = 1$ .

**Perfect substitutes** When the two goods are perfect substitutes, the period utility function is linear, given k. We will show how to solve this simple case analytically, assuming that I is sufficiently large. Guess that the value function v(k) is linear in k: v(k) = a + ck, where a and c are scalars, so long as  $k \ge k^*$ , and that  $v(k) = \bar{v}$ , where  $\bar{v}$  is a scalar, otherwise. In other words, the guess is that y = I if  $k \ge k^*$  and that x = I, otherwise. It is easily checked that  $\bar{v}$  must equal  $I/(1 - \delta)$ .

To verify that the solution is correct and show how to find a, c, and  $k^*$ , first suppose that  $k > k^*$ . We must then have that the dynamic programming equation is satisfied at the conjectured solution, i.e.,

$$a + ck = kI + \delta \{a + c((1 - d)k + bI)\}.$$

This must hold for all  $k \ge k^*$ , leading us to identify the coefficients a and c as

$$a = \frac{\delta c b I}{1 - \delta},$$

and

$$c = \frac{I}{1 - \delta(1 - d)}.$$

The cutoff value  $k^*$  is now defined as the value of k making the individual indifferent between choosing x = I, which delivers  $I/(1-\delta)$ , and y = I, which delivers  $a + ck^*$ . Substituting in the solutions for a and c, we thus have

$$\frac{I}{1-\delta} = \frac{\delta c b I}{1-\delta} + \frac{I}{1-\delta(1-d)}k^*,$$

which gives

$$k^* = (1 - \delta cb) \frac{1 - \delta(1 - d)}{1 - \delta} = 1 + \frac{\delta}{1 - \delta} (d - bI)$$

If this expression is negative, the set of k for which x = I is chosen is empty.

We now need to verify that the proposed behavior is indeed optimal. For this purpose, notice that the maximization problem reads

$$\max_{y \in [0,I]} I - y + ky + \delta \{ a + c((1-d)k + by) \},\$$

assuming that  $(1-d)k + by \ge k^*$ . In this case, it is optimal to set y = I so long as  $k - 1 + \delta cb \ge 0$ . Thus, we need to show that at  $k = k^*$ , this inequality is satisfied; if so, it is also satisfied at higher values for k. Inserting  $k^* = (1 - \delta cb) \frac{1 - \delta(1 - d)}{1 - \delta}$ , it is easily seen that  $k^* \ge 1 - \delta cb$ , with strict inequality whenever d > 0, i.e., the inequality is indeed satisfied. We also need to verify that the value of k' is indeed chosen as k exceeds  $k^*$ . This is true if  $(1 - d)k^* + bI \ge k^*$ , which (after some little algebra) delivers the restriction

$$bI \ge d. \tag{3.3}$$

Thus, the restriction is met if bI is sufficiently large or d is sufficiently small; this is natural, because it just says that replenishing k is "easy". Notice also that this restriction implies  $k^* \leq 1$ .

Finally, we need to verify that if  $k < k^*$ , it is optimal to set x = I. The choice x = I always delivers  $I/(1 - \delta)$ , notwithstanding what is k. The choice y = I gives  $kI + \delta\{a + c((1 - d)k + bI)\}$ , if k is sufficiently large so that  $k' \ge k^*$ , otherwise it gives  $kI + \delta I/(1 - \delta)$ . In the first of these cases, the objective becomes  $k(I + \delta c(1 - d)) + \delta(a + bcI) = kI/(1 - \delta(1 - d)) + \delta(a + bcI)$ , which is increasing in k and equal to  $I/(1 - \delta)$  at  $k = k^*$ ; therefore it must be lower for lower values of k. In the second case, x = I is optimal if  $k \le 1$ . But we know this to be true since  $k^* \le 1$  from above. This completes the proof.

Notice that in the case covered above, either there is specialization at all times and that the specialization is always the same: either the culture good is never consumed or the generic good is never consumed. In the case where the inequality in (3.3) is not met, the situation is different. The solution now is also to choose x = I below some cutoff value of k and y = I above that value, but in this case

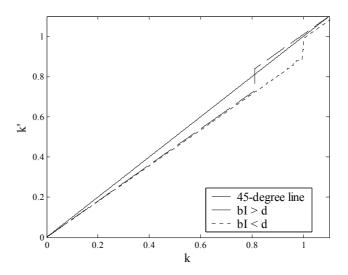


Figure 3.4:

k' < k for all k. Thus, for large values of k, k' = (1 - d)k + bI < k, and when k is sufficiently low, k' = (1 - d)k. The resulting time series means that the culture good will be consumed for a finite number of periods, after which there is a switch to the generic good, which will then be consumed forever after.<sup>4</sup> Figure 3.4 shows the decision rules for the two cases: one case where the inequality in (3.3) is met and one where it is not.<sup>5</sup>

Case 1: continuous decision rules We now look at the case of a continuous decision rule, where the point at which the right hand side of equation (3.1) pivots is above the left hand side of the equation. Figure 3.5 below displays two decision rules—for different values of  $\alpha$ —where the rule "hugs" the 45-degree line.<sup>6</sup> It makes the dynamics of culture accumulation clear: if the initial stock of culture capital is sufficiently large, culture capital will end up at a relatively high level in the long run, but if it is not sufficiently large, culture capital will converge to zero, and this

<sup>&</sup>lt;sup>4</sup> Showing this formally is somewhat more tedious; the value function is piece-wise linear with a countably infinite number of segments with increasing slopes. The slope for the smallest values of k is zero, as in the first example, then it becomes I, then  $I(1 + \delta(1 - d))$ , etc, and this sequence converges to the slope of the first example, namely  $I/(1 - \delta(1 - d))$ .

<sup>&</sup>lt;sup>5</sup> The parameter values are  $\alpha = 1$ , I = 1, and d = 0.10, b = 0.11 or d = 0.11, b = 0.10.

<sup>&</sup>lt;sup>6</sup> The parameter values are  $\alpha = 0.53$ , I = 2.3, d = 0.75, b = 0.25 and  $\delta = 0.50$ .

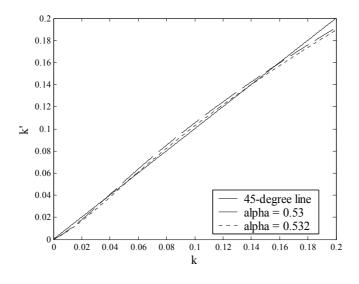


Figure 3.5:

consumer will not consume culture in the long run.

The comparative statics in the picture shows that a higher  $\alpha$  lowers the curve, implying that the higher steady state falls and the lower steady state increases, thus implying that the range of initial conditions for which culture consumption is zero in the long run increases; similarly, if culture consumption converges to a positive number, it is a smaller number for a larger value of  $\alpha$ . The explanation for this is that *I*, or *bI*, is not sufficiently large; recall the discussion of steady-state candidates at the beginning of the present section. Since scale is relevant, when *I* is not large, more substitutability tends to favor the generic good, which is the intuitive explanation for the comparative statics just noted.

If  $\alpha$  is increased enough, the decision rule would fall below the 45-degree line, thus implying that k converges to zero. For  $\alpha = 1$ , as we know, the decision rule becomes k' = (1 - d)k, as previously discussed (unless k is very large).

Figures 3.6 and 3.7 show comparative statics with respect to d and  $\delta$ . Here, as in the case of  $\alpha \leq 0.5$ , we see that higher depreciation lowers the culture accumulation rule globally, in this case implying that the range of initial conditions for which kconverges to zero increases and that the positive long-run attraction point is lower.

In contrast, and once more as in the case with lower substitutability, stronger

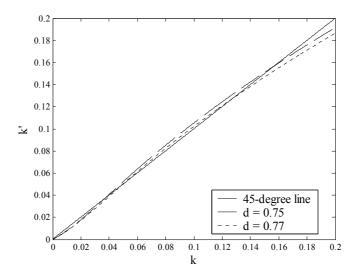


Figure 3.6:

patience raises the culture accumulation rule globally, making the range of initial conditions over which k converges to zero shrink and raising the high steady state.

Case 2: discontinuous decision rules Figures 3.8 depicts a case with discontinuous culture accumulation rules, where the point at which the right hand side of equation (3.1) pivots is above the left hand side of the equation.<sup>7</sup>

Figure 3.8 shows a case where the rule takes a jump over the 45-degree line for the middle value of  $\alpha$ ,  $\alpha = 0.55$ . Clearly, at the jump, the individual is indifferent between a high level of culture consumption and accumulation, and a much lower one. With slightly more culture capital, the individual strictly prefers the higher trajectory; with slightly less culture capital, the lower trajectory, which converges to zero, is preferred.

Interestingly, an increase in  $\alpha$  eliminates the jump. Here, with more substitutability, although a positive higher steady state exists, it does not correspond to a global maximum. Instead, it is optimal for all values of k to decrease k toward zero over time, leading to zero culture consumption in the long run. If, on the other hand,  $\alpha$  is decreased, we see that the jump disappears as well! Here, with more substi-

<sup>&</sup>lt;sup>7</sup> The parameter values are  $\alpha = 0.55$ , I = 10, d = 1, b = 0.05, and  $\delta = 0.96$ .

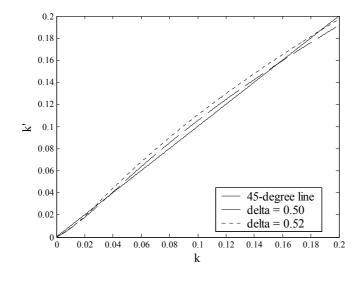


Figure 3.7:

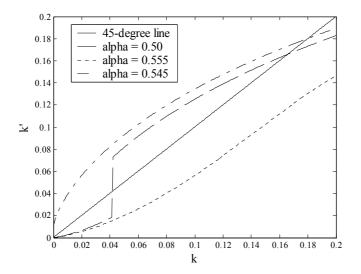


Figure 3.8:

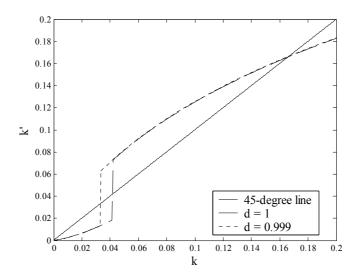


Figure 3.9:

tutability, the behavior looks like the case where  $\alpha \leq 0.5$ . The interpretation is now that the culture good is more valued, not primarily because of the complementarity in consuming it over time, but because the consumer wants a more balanced mix of the two goods.

Naturally, the changes in  $\alpha$  in Figure 3.8 that are less drastic would maintain the jump and move the point of discontinuity to the left (if  $\alpha$  decreased) or to the right (if  $\alpha$  is increased).

Figure 3.9 shows that a decrease in depreciation raises the decision rule, and moves the discontinuity to the left.

Once more, less depreciation promotes culture consumption, and the range of initial conditions for which culture consumption converges to zero shrinks; relatedly, the positive steady state is higher. The example also shows that the change in the decision rule is very slight at low and high levels of k, whereas the changes implied by the move in the point of discontinuity are drastic.

Figure 3.10, finally, illustrates a case where the decision rule falls, due to a decrease in the degree of patience. As in the previous case, the nonlinearity of the present framework leads to very small effects for small and large values of k, but drastic effects in a middle range. For example, imagine consumers to have the

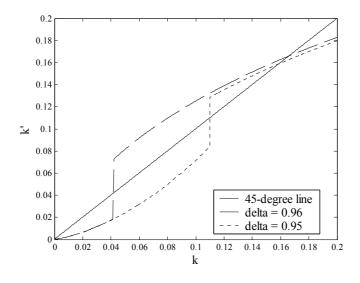


Figure 3.10:

same initial conditions for culture capital but to display differences in patience or "memory" with regard to retaining cultural experience beyond one period. Then, very large short-run and long-run differences between these consumers in culture accumulation can result, even though the differences in preferences are slight.

Case 3: unique steady states and global convergence We have seen that, for  $\alpha \in (0.5, 1)$ , it is also possible that there is a unique positive steady state. That outcome was shown in Figure 3.8. In that case, there was global convergence to a positive long-run level of culture capital. The same figure also demonstrated that, when one parameter was slightly changed, there would be global convergence to zero long-run culture consumption. The difference was in the degree of substitutability.

## 2.2.3 Summary and implications

The model has been demonstrated to possess a rich set of qualitative outcomes in terms of its law of motion for culture capital. A key parameter behind the results is  $\alpha$ : the curvature of the utility function u, which in this case regulates the substitutability of x and y, given k: the lower the  $\alpha$ , the less substitutable are x and  $y.^8$  The prediction is thus that if highbrow-culture goods and mass-culture goods are close substitutes in a static sense, there can be multiple steady states, and longrun levels of culture consumption can critically depend on the initial conditions. Moreover, short-run levels of culture can also significantly respond to the initial conditions, since the decision rules are sometimes discontinuous. Intuitively, if the goods are not close substitutes, they are both essential and low-culture "traps" are not possible: highbrow culture is simply too important to become a neglected good. If the goods are close substitutes, however, then either highbrow culture is entirely competed out (the zero-culture steady state) or the polar opposite case, namely, with specialization in highbrow culture is also possible: high consumption of these goods and low consumption of mass-culture goods.

We also saw that the other parameters of the model can be of importance. Parameter I—the time endowment, or total resources available to spend on the two goods—can be seen as a scale measure, and when it is large, more specialization on culture goods tends to occur, because culture consumption is subject to a scale effect. Higher depreciation of culture capital led, as might be expected, to lower levels of culture capital, and more patience on the part of the individual led to higher levels of culture capital. Moreover, small differences in these parameters across consumers could imply large short- and/or long-run differences in the level of culture consumption.

# **3** Time-inconsistent preferences

Suppose now that there is quasi-geometric discounting, regulated by parameter  $\beta$ , which we shall assume to be less than 1. The consumer problem becomes

$$w(k) = \max_{k' \ge 0} u(I - y(k, k'), y(k, k')k) + \beta \delta v(k'),$$

where

$$v(k) = u(I - y(k, g(k)), y(k, g(k))k) + \delta v(g(k))$$

<sup>&</sup>lt;sup>8</sup> The elasticity of substitution turns out to be  $1/(1-\alpha)$ .

and g(k) is the perceived future behavior in the infinite-horizon game: the consumer's behavior at a point in time is assumed to depend on nothing but his current culture capital stock. The behavior given by g(k) is a Markov-perfect equilibrium if g(k) solves the stated maximization problem for all k. In what follows, we will assume  $u(x, ky) = x^{\alpha} + (ky)^{\alpha}$  and that h(k, y) = (1 - d)k + by.

Assuming, as above, that u(x, yk) and y(k, k') are in the special parametric classes, we can establish an important property of an equilibrium: that the decision rule g(k) is increasing, i.e., consumers with larger initial stocks of high-brow culture capital will also have larger such stocks for the remaining time. To see this, we first establish a lemma. Once more, we will use the notation F(k, k') = u(I - y(k, k'), y(k, k')k).

**Lemma:** If  $k_2 > k_1$  and  $k'_2 > k'_1$ , then  $F(k_2, k'_2) - F(k_2, k'_1) > F(k_1, k'_2) - F(k_1, k'_1)$ .

**Proof:** Let  $y_{11} \equiv y(k_1, k'_1)$ ,  $y_{12} \equiv y(k_1, k'_2)$ ,  $y_{21} \equiv y(k_2, k'_1)$ , and  $y_{22} \equiv y(k_2, k'_2)$ , define  $F_{ij}$  in the same way, and define  $\Delta' \equiv (k'_2 - k'_1)/b$ . We know from y(k, k')being linear that  $y_{12} - y_{11} = y_{22} - y_{21} = (k'_2 - k'_1)/b = \Delta'$ . Now, we have

$$F_{22} - F_{21} = f(I - (y_{21} + \Delta')) + k_2^{\alpha} f(y_{21} + \Delta') - [f(I - y_{21}) + k_2^{\alpha} f(y_{21})]$$
$$= -[f(I - y_{21}) - f(I - (y_{21} + \Delta'))] + k_2^{\alpha} [f(y_{21} + \Delta') - f(y_{21})].$$

Similarly,

$$F_{12} - F_{11} = f(I - (y_{11} + \Delta')) + k_1^{\alpha} f(y_{11} + \Delta') - [f(I - y_{11}) + k_1^{\alpha} f(y_{11})]$$
$$= -[f(I - y_{11}) - f(I - (y_{11} + \Delta'))] + k_1^{\alpha} [f(y_{11} + \Delta') - f(y_{11})].$$

We will now compare the final expressions  $F_{22} - F_{21}$  and  $F_{12} - F_{11}$ . Note that  $y_{21} - y_{11} = -(1-d)(k_2 - k_1)/b < 0$ , which implies that  $I - y_{21} > I - y_{11}$ . Therefore, since f is strictly concave and  $\Delta' > 0$ ,  $f(I - y_{11}) - f(I - (y_{11} + \Delta'))$  is strictly greater than  $f(I - y_{21}) - f(I - (y_{21} + \Delta'))$ . It follows that  $-[f(I - y_{21}) - f(I - (y_{21} + \Delta'))] > -[f(I - y_{11}) - f(I - (y_{11} + \Delta'))]$ . That is, the first term of  $F_{22} - F_{21}$  exceeds the first term of  $F_{12} - F_{11}$ . As for the second term, once more because  $y_{21} < y_{11}$ , and due to the strict concavity of f and  $\Delta' > 0$ , we have that  $f(y_{21} + \Delta') - f(y_{21}) > f(y_{11} + \Delta') - f(y_{11})$ . Finally,  $k_2^{\alpha} > k_1^{\alpha}$  and f being strictly increasing implies that

also the second term of  $F_{22} - F_{21}$  exceeds that of  $F_{12} - F_{11}$ . QED.

Notice that this lemma would follow from F being globally concave in k and k' jointly. In our case, however, we know that it is not, unless  $\alpha < 0.5$ . Thus, the lemma is nontrivial, and relies on the functional forms adopted.

We now show that the policy function needs to be increasing, i.e., we have

**Proposition:** If  $k_2 > k_1$ , then in any Markov-perfect equilibrium, we must have that  $g(k_2) \ge g(k_1)$ .

**Proof:** Using the notation of the lemma, optimality implies

$$F(k_2, g(k_2)) + \beta \delta V(g(k_2)) \ge F(k_2, k') + \beta \delta V(k'))$$

for all feasible k'. Suppose now, by means of contradiction, that  $g(k_1) > g(k_2)$ . Presuming that choosing  $g(k_1)$  at  $k_2$  is feasible, the previous expression then yields

$$F(k_2, g(k_2)) + \beta \delta V(g(k_2)) \ge F(k_2, g(k_1)) + \beta \delta V(g(k_1))).$$

It follows that  $F(k_2, g(k_2)) - F(k_2, g(k_1)) \ge \beta \delta[V(g(k_1))) - V(g(k_2))]$ . Similarly, if choosing  $g(k_2)$  is feasible at  $k_1$ , we must have that

$$F(k_1, g(k_1)) + \beta \delta V(g(k_1)) \ge F(k_1, g(k_2)) + \beta \delta V(g(k_2))),$$

so that  $F(k_1, g(k_1)) - F(k_1, g(k_2)) \ge \beta \delta[V(g(k_2))) - V(g(k_1))]$ . This inequality implies that  $\beta \delta[V(g(k_1))) - V(g(k_2))] \ge F(k_1, g(k_2)) - F(k_1, g(k_1))$ . Now, adding the two inequalities we arrive at  $F(k_2, g(k_2)) - F(k_2, g(k_1)) > F(k_1, g(k_2)) - F(k_1, g(k_1))$ , which contradicts the lemma. Finally, we need to verify the feasibility assumed above. To see that  $g(k_1)$  is feasible from  $k_2$  and that  $g(k_2)$  is feasible from  $k_1$ , note that what is feasible from  $k_i$  is the interval  $(1 - d)k_i + [0, Ib]$ , for i = 1, 2. Thus, since  $k_2 > k_1$  and  $g(k_2) < g(k_1)$  by assumption, the required feasibility must follow. QED.

Notice that this proposition holds true independently of the value of  $\beta$ , so that it in particular applies to the standard model with time-consistent preferences. What is striking about this proposition is (i) that it holds despite F not being globally concave and (ii) whether or not preferences are time-inconsistent.

## 3.1 Continuous domain and differentiable decision rules

Assuming an interior solution, the first-order condition for this problem reads

$$(-u_1(x,ky) + ku_2(x,ky))y_2(k,k') + \beta \delta v'(k'),$$

and the envelope condition yields

$$v'(k) = (-u_1(x, ky) + ku_2(x, ky))(y_1(k, k') + y_2(k, k')g_k(k)) + u_2(x, yk)y(k, k') + \delta v'(k')g_k(k) = (-u_1(x, ky) + ku_2(x, ky))(y_1(k, k') + g_k(k)y_2(k, k')(1 - frac1\beta)) + u_2(x, yk)y(k, k').$$

Thus, the final first-order condition reads (with arguments suppressed)

$$(-u_1 + ku_2)y_2 + \beta\delta\left((-u_1' + k'u_2')(y_1' + g_k'y_2'(1 - \frac{1}{\beta})) + u_2'y'\right) = 0.$$

We see that this condition collapses to that of the time-consistent case when  $\beta = 1$ . Compared to that case, there is another benefit—a new positive term in the expression—from saving more on the margin, assuming that culture consumption is increasing in k and that there is static overconsumption of culture. This reflects an added marginal value of consuming culture as it increases future culture accumulation, which is below what the present self would choose were he able to commit.

The interior steady state cannot be found in any easy manner. If the objective function is quadratic and y(k, k') is linear, one can guess on a linear form for g(k)and verify the guess, thus in particular delivering a steady state. However, if a closed-form solution is not available, which in general it is not, the determination of a steady state is fundamentally more complex than in the case where preferences are time-consistent, because the steady state depends on the value of  $g_k$ : the long-run level of k cannot be ascertained without determining the local dynamics around this level.

In an interior steady state, we have

$$(-u_1 + ku_2)(y_2(1 + \delta g_k(\beta - 1) + \beta \delta y_1) + \beta \delta u_2 y = 0.$$

Thus, compare models which share the value of  $\beta\delta$  but where  $\beta$  and  $\delta$  differ; suppose

in one case  $\beta = 1$  and in the other  $\beta < 1$ . Then, in the latter case, an additional term appears. The additional term is  $(-u_1 + ku_2)y_2\delta g_k(\beta - 1)$ : an additional marginal return to increasing k'. It is positive, since  $-u_1 + ku_2 < 0$ ,  $g_k > 0$ , and  $\beta < 1$ . This indicates that the two models would have different steady states and that the one with a  $\beta < 1$  would have a higher steady state: there is an additional motive for accumulation of culture capital; that is, the model with time inconsistency leads to higher cultural consumption.<sup>9</sup> This is not surprising: in the two models, the short-run discount rate is the same, and the long-run discount rate is higher in the case with  $\beta < 1$ , since the  $\delta$  must be higher in that case; this is what explains the higher culture capital accumulation.

#### 3.1.1 Numerical analysis

In the case of a continuous domain and our special functional-form assumptions, we have not found a closed-form solution for any value of  $\alpha \in (0,1)$ . Therefore, in order to characterize equilibria, one would need to use numerical methods. One possible method for this is developed in Krusell, Kuruşçu, and Smith (2002). It could be used to look for a differentiable equilibrium function g satisfying the Euler equation locally. A second possibility would be to use "global methods". These rely on approximating the function q on a grid of values for k and interpolating in between grid points, either using cubic splines or some form of polynomial functions. The parameters of the cubic splines/polynomial functions would then be chosen so that the Euler equation holds on all grid points. Both these methods rely on the construction of g using the first-order condition. Thus, they do not verify sufficiency globally. This is a problem, especially in the present context of a potentially nonconcave value function, due to the complementarity of present and future culture consumption. Moreover, it is known that in itself, time-inconsistency can lead to non-concave value functions. Thus, it would require new methods to search for a differentiable Markov-perfect equilibrium, which is beyond the scope of the current analysis. However, the purpose of introducing time-inconsistency of preferences here

<sup>&</sup>lt;sup>9</sup> This conclusion will follow under concavity of F(k, k'), i.e., if  $\alpha < 0.5$ . In this case, it follows from our argument that the addition of a term on the left-hand side leads  $\delta u_2 y$  to decrease. Since the expression  $u_2 y$  is decreasing when  $\alpha < 0.5$ , this must imply that y must increase. If concavity is not met, the situation is more complicated.

is not primarily to find out how the smooth solution for g shifts in (k, k') space with parameter  $\beta$ . Rather, the purpose is to investigate whether equilibria of a different nature may exist. For this purpose, a continuous domain is not necessary.

There may also be non-differentiable equilibria in the context of the present model, when preferences are time-inconsistent. In particular, we conjecture that it is possible to construct stable steady states using step-function equilibria—equilibria where g(k) is a step function, i.e., has a sequence of flat and vertical sections following the work of Krusell and Smith (2003a). We will return to a discussion of such equilibria in the next section.

# 3.2 A discrete domain

We now look at a discrete domain, and we first focus on the very simplest case with only two possible values for culture capital. This case points to some difficulties that may arise in characterizing equilibria with time-inconsistent preferences. With a sufficiently "nice" structure, as in the case with a quadratic utility function and linear constraints, these problems can sometimes be avoided, but as we shall see in the case considered here, they do appear in other settings. After looking at the two-value setting, we consider setups with a large number of admissible values for culture capital and study equilibria using computational methods.

## 3.2.1 Two values for capital

With two values for capital, there are four possible values for the consumption of culture capital, y, which we will all consider to be feasible (i.e., the values of the parameters are such that all four values for y are between 0 and I), and hence, also four values for the other consumption good, x. Thus, we also have four values of flow utility that we denote  $F_{ij}$ , with i and j in  $\{1, 2\}$ . This case can be completely characterized: for every parameter configuration, it is possible to show what the equilibrium set will look like. However, here we will mainly show that the time inconsistency fundamentally changes the nature of outcomes, which will be illustrated with some particularly interesting cases.

Under time-consistent preferences, there is always a unique solution for the value function,  $v_1$  and  $v_2$ , from the contraction mapping theorem, and given these values,

optimal policies can be chosen without mixing.<sup>10</sup> Under time inconsistency, there is always at least one Markov-perfect equilibrium under mixed strategies; this follows from a standard fixed-point theorem.<sup>11</sup> However, (i) there may be no pure-strategy equilibrium and (ii) there may be multiple equilibria, with multiple  $(v_1, v_2)$  solving the equilibrium conditions.

We will mainly focus on pure-strategy equilibria. There are several possibilities. There may be either culture accumulation, i.e., the consumer may choose  $k' = k_2 > k_1$ , or not, i.e.,  $k' = k_1$ . This gives four possible candidates, but one case cannot be optimal, namely, choosing  $g(k_1) = k_2$  and  $g(k_2) = k_1$ , because this outcome violates the monotonicity of the decision rule: it violates the Proposition above, which was proved for a general case. Thus, there can be three kinds of equilibria: (i) choosing  $k_1$  in both states; (ii) choosing  $k_2$  in both states; and (iii) choosing to remain in whichever state is the starting point.

In an equilibrium of type (i), where  $k_1$  is always chosen,

$$F_{11} + \beta \delta v_1 \ge F_{12} + \beta \delta v_2$$

and

$$F_{21} + \beta \delta v_1 \ge F_{22} + \beta \delta v_2,$$

with  $v_1 = F_{11}/(1-\delta)$  and  $v_2 = F_{21} + \delta F_{11}/(1-\delta)$ . This implies that

$$F_{11} - F_{12} \ge \beta \delta(F_{21} - F_{11})$$

and

$$F_{21} - F_{22} \ge \beta \delta(F_{21} - F_{11}). \tag{3.4}$$

By the lemma, the former inequality is implied by (3.4). Thus, if inequality (3.4) is satisfied, an equilibrium of type (i) exists.

In an equilibrium of type (ii), where  $k_2$  is always chosen,

$$F_{22} + \beta \delta v_2 \ge F_{21} + \beta \delta v_1$$

<sup>&</sup>lt;sup>10</sup> There may be indifference between policies, but probabilities may all be set to either 0 or 1.

<sup>&</sup>lt;sup>11</sup> For example, Kakutani's fixed-point theorem can be used.

and

$$F_{12} + \beta \delta v_2 \ge F_{11} + \beta \delta v_1,$$

with  $v_2 = F_{22}/(1-\delta)$  and  $v_1 = F_{12} + \delta F_{22}/(1-\delta)$ . This implies that

$$F_{22} - F_{21} \ge \beta \delta(F_{12} - F_{22})$$

and

$$F_{12} - F_{11} \ge \beta \delta(F_{12} - F_{22}). \tag{3.5}$$

By the lemma, the former inequality is implied by (3.5). Thus, if inequality (3.5) is satisfied, an equilibrium of type (ii) exists.

Finally, in an equilibrium of type (iii), where  $g(k_1) = k_1$  and  $g(k_2) = k_2$ ,

$$F_{11} + \beta \delta v_1 \ge F_{12} + \beta \delta v_2$$

and

$$F_{22} + \beta \delta v_2 \ge F_{21} + \beta \delta v_1,$$

with  $v_1 = F_{11}/(1-\delta)$  and  $v_2 = F_{22}/(1-\delta)$ . This implies

$$F_{11} - F_{12} \ge \frac{\beta \delta}{1 - \delta} (F_{22} - F_{11}), \tag{3.6}$$

and

$$F_{22} - F_{21} \ge \frac{\beta \delta}{1 - \delta} (F_{11} - F_{22}).$$
 (3.7)

Neither of these conditions in general imply the other. Thus, for an equilibrium of type (iii) to exist, both inequality (3.6) and inequality (3.7) must be verified.

**Features of pure-strategy equilibria** Now that the conditions for each of the pure-strategy equilibria to exist have been stated, a few remarks are in order. First, inequalities (3.4) and (3.5) cannot both be met at the same time: adding the two inequalities and rearranging gives:

$$[F_{21} + F_{12} - F_{11} - F_{22}](1 - \beta \delta) \ge 0.$$

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The term in brackets, however, is strictly negative from the lemma, which thus rules out inequalities (3.4) and (3.5) from holding for the same set of parameter values. Thus, equilibria of type (i) and (ii) cannot coexist.

Second, and relatedly, both equilibria of type (i) and type (ii) have the feature that only short-run discounting is of importance, i.e., the value of  $\beta\delta$ : whether the conditions for existence of these equilibria are met only depends on the product of  $\beta$  and  $\delta$  and not on these parameters separately. Intuitively, this is because in both these types of equilibria, the next period's choice of culture capital accumulation, k'', is independent of current actions. Therefore, how two adjacent periods in the future are compared (which is determined by  $\delta$ ) relative to how the next period is compared to the present period (which is determined by  $\beta\delta$ ) is of no importance. For example, in state 2 of an equilibrium of type (i), the consumer must find it optimal not to deviate and choose little capital accumulation. Whether the deviation is optimal is only related to comparing  $F_{21} + \beta\delta F_{11}$ , which is what the equilibrium prescribes, with  $F_{22} + \beta\delta F_{21}$ , the deviation, because what the consumer then does is to choose  $k_2$ , regardless of the current actions. Equilibrium (iii) does not have this feature: whether it exists depends on  $\beta$  and  $\delta$  separately.

Third, inequality (3.4) can be rearranged as

$$F_{22} - F_{21} \le \frac{\beta \delta}{1 - \beta \delta} (F_{11} - F_{22}).$$

Similarly, inequality (3.5) can be written as

$$F_{11} - F_{12} \le \frac{\beta \delta}{1 - \beta \delta} (F_{22} - F_{11}).$$

From these expressions, we realize that in the case of time-consistent preferences, when  $\beta = 1$ , it must be that inequality (3.4) is identical to the reverse of inequality (3.7) and that inequality (3.5) is the same as the reverse of inequality (3.6). This is an illustration of the point made earlier: under time-consistent preferences, one and only one of the three kinds of equilibria must exist (unless we are in a non-generic case where the inequalities are equalities, in which case the consumer is indifferent). That is, if type (i) exists, type (ii) cannot exist and (iii) cannot exist (except in the non-generic case where (3.4) holds with equality so that (iii) gives the same values as equilibrium (i)); if (ii) exists, (i) cannot exist and (iii) cannot exist (except ...); if (iii) exists, neither can (i) or (ii) (except ...), and it is not possible that none of the equilibria exist, because if equilibrium of type (iii) does not exist, then either equilibrium of type (i) has to exist or equilibrium of type (ii) must exist.

Welfare analysis Here, compute welfare effects of some different kinds of policies that might be imagined in this economy, and compare them to the equilibria. Two different kinds of welfare measures are available: that given by the vs and that given by the ws. For simplicity, do not explicitly consider policies, but rather look at whether a given equilibrium with an associated rule g can be improved upon, either in the sense of v or in the sense of w, for one or both of the states of nature. For example, consider parameter configurations such that the equilibrium is unique and is of type (i), i.e., always consume as little culture as possible: always choose state 1. Then the question would be whether a different decision rule, e.g., choose  $k_2$  in state 2 and  $k_1$  in state 1, might lead to higher welfare even though it is not an equilibrium. If that is the case, then any (unspecified) policy inducing people to follow this alternative would be desirable. Given that  $v_1 = F_{11}/(1-\delta)$  and  $v_2 = F_{21} + \delta F_{11}/(1-\delta)$  in an equilibrium of type *i*, and the corresponding future utilities in an equilibrium of type (iii) are  $v_1 = F_{11}/(1-\delta)$  and  $v_2 = F_{22}/(1-\delta)$ , the equilibrium is worse than this specific alternative in state 2, and for the current self, if

$$F_{21} + \beta \delta \frac{F_{11}}{1 - \delta} < F_{22} + \beta \delta \frac{F_{22}}{1 - \delta}$$

and worse for the last-period self if

$$F_{21} + \delta \frac{F_{11}}{1 - \delta} < F_{22} + \delta \frac{F_{22}}{1 - \delta}.$$

Thus, one question would be whether one or both of these inequalities could be satisfied at the same time as inequality (3.4), which guarantees the existence of a type-(i) equilibrium, is satisfied. If so, it could be argued that the equilibrium is inefficient, and that the government should try to induce agents to choose state 2 instead of state 1, when they are in state 2.

Nonexistence of pure-strategy equilibria Let us now illustrate non-existence of pure-strategy equilibria under time-inconsistent preferences. First, take a case under time-inconsistent preferences where the primitives are such that an equilibrium of type (iii) exists, i.e., one with two steady states: where you start is where you end up. This implies that conditions (3.4) and (3.5), for equilibria of type (i) and (ii), respectively, are violated. Now select a new  $\delta$ , which we call  $\hat{\delta} > \delta$ , and a  $\beta < 1$ , such that  $\beta \hat{\delta} = \delta$ . Thus, conditions (3.4) and (3.5) are still violated, so that neither equilibrium of type (i) nor type (ii) can exist. Furthermore, suppose, without loss of generality, that  $F_{22} > F_{11}$ . Then it is clear that while letting  $\beta \hat{\delta} = \delta$ ,  $\hat{\delta}$  can be increased close enough to 1 (and let  $\beta$  fall) that inequality (3.6) is violated, since  $1/(1 - \hat{\delta})$  can be made to go to infinity.<sup>12</sup> Thus, with a judicious choice of discount factors, it appears that none of the equilibria exist.

Concerning the intuition for why a pure-strategy equilibrium does not exist here, note that what has happened due to the violation of condition (3.6) is that the consumer who is in state 1 now finds it worthwhile to deviate and "save more", i.e., increase culture consumption, and the reason for this is the time inconsistency produced by a very high  $\delta$ :  $\beta$  is now significantly lower than 1. Thus, this consumer disagrees with his future selves, who are not very forward-looking, and he realizes that by switching from  $k' = k_1$  to  $k' = k_2$ , he effectively ensures that  $k_2$  will be chosen forever after: one always remains in this equilibrium.

It may appear counterintuitive that the equilibrium would not simply switch to one of type (ii) now, i.e., always choose  $k_2$ . However, we know that because  $\beta\delta$  is the same as before, i.e., it is sufficiently low that it does not pay to choose  $k_2$  in state 1, since that does not give the long-run benefit that it delivers in a type-(iii) equilibrium. So in this case, instead, there is a *mixed-strategy equilibrium* where the consumer randomizes between choosing  $k_1$  and  $k_2$  in state 1. In terms of comparative statics using  $\delta$ , as it first reaches the level where condition (3.6) becomes an equality, the probability of choosing  $k_2$  in state 1 is 0, but as it increases further, this probability is increased.<sup>13</sup> As it is increased, the benefit of choosing  $k_2$ 

<sup>&</sup>lt;sup>12</sup> If  $F_{22} < F_{11}$ , use the same argument on inequality (3.7).

<sup>&</sup>lt;sup>13</sup> The probability with which the agent randomizes between  $k_1$  and  $k_2$  must be chosen precisely so that (3.6) holds with equality. Denoting the probability of choosing  $k_1$  by  $\pi$ , we have that  $v_1 = \pi (F_{11} + \delta v_1) + (1 - \pi)(F_{12} + \delta v_2)$  and that  $v_2 = F_{22}/(1 - \delta)$ . This gives that  $v_1 - v_2 =$ 

over  $k_1$  is increased because of the accentuation of time-inconsistency—a discrepancy between the  $\beta\delta$  which remains constant and the  $\delta$  which is increasing—but there is a counteracting force making the consumer remain indifferent between  $k_1$  and  $k_2$ : the long-run gain is now realized with lower and lower probability, since  $k_1$  is less and less likely to be chosen in state 1.

**Multiplicity** To illustrate the multiplicity of equilibria, let us consider a case with some analytical convenience. So suppose first that we are in a world with timeconsistent preferences satisfying the nongeneric case where (3.4) holds with equality and is identical to (3.7):  $F_{21} - F_{22} = (\beta \delta/(1-\delta))(F_{22} - F_{11})$ , with  $\beta = 1$ . Moreover, suppose that (3.6) is strictly satisfied (so that (3.5) is strictly violated). In words, this is the case where an equilibrium of type (i)—always keep culture accumulation low—"barely" exists and coexists with an equilibrium of type (iii), which also barely exists. In each case, the temptation to deviate is in state 2, where the consumer is indifferent between remaining in state 2—maintaining high culture consumption and going to state 1; the equilibrium actions are thus different in (i) and (iii) in state 2—choose  $k' = k_1$  in equilibrium (i) and choose  $k' = k_2$  in equilibrium (iii)—but because of indifference, the associated values are the same. Suppose, in addition, that  $F_{11} < F_{22}$ .<sup>14</sup>

The formal idea is now to let  $\beta$  differ from 1 while maintaining  $\beta\delta/(1-\delta)$ , so that  $F_{21} - F_{22} = (\beta\delta/(1-\delta))(F_{22} - F_{11})$  still holds; i.e., condition (3.7) still holds with equality: the type-(iii) case would thus still "barely" constitute an equilibrium. The expression  $\beta\delta/(1-\delta)$  which is to be held constant, can be written as  $[\beta\delta/(1-\beta\delta)][(1-\beta\delta)/(1-\delta)]$ . Now, consider a change such that the first of these factors,  $\beta\delta/(1-\beta\delta)$ , decreases; this decrease would make the consumer not be indifferent in state 2 of the type-(i) equilibrium, but the consumer would rather strictly prefer to remain in state 2 over going to state 1. As a result,  $(1-\beta\delta)/(1-\delta)$  would have to

 $<sup>(\</sup>pi(F_{11}-F_{12})+F_{12}-F_{22})/(1-\pi\delta)$ . This expression can be used together with equality of (3.6), which is stated as  $F_{11} + \beta\delta v_1 = F_{12} + \beta\delta v_2$ , to deliver an equation determining  $\pi$ . The solution is  $\pi = (1/\delta - \beta(F_{22}-F_{12})/(F_{11}-F_{12}))/(1-\beta)$ . Note here that this expression is still positive when  $\delta$  has reached 1. The reason is that, by assumption, we have  $F_{22} > F_{11}$  (the assumption upon which the non-existence of pure-strategy equilibria is based) and that  $F_{11} + \beta\delta F_{12} > F_{12} + \beta\delta F_{22}$  (since, by assumption, the type-(ii) equilibrium does not exist), which together imply that  $\beta(F_{22}-F_{12})/(F_{11}-F_{12}) < 1/\delta$ .

<sup>&</sup>lt;sup>14</sup> If it is not, a similar argument can be constructed.

increase, which requires  $\beta$  to be below 1 (and also makes  $\delta$  increase).

How do these equilibria differ? If the economy starts in state 1, they do not differ: they both involve choosing to remain in state 1 forever. However, the two equilibria have very different long-run characteristics if the initial state is state 2. If equilibrium (i) is played, the consumer chooses to go to state 1 and then remain there forever, and if equilibrium (ii) is played, the consumer chooses to remain in state 2 forever.

What are the associated welfare levels? From the perspective of the current self, the two equilibria are identical. To see this, note that in the type-(i) case, the consumer obtains  $w_2^i = F_{21} + (\beta \delta / (1 - \delta))F_{11}$ , whereas in the type-(iii) equilibrium, the resulting welfare is  $w_2^{iiii} = F_{22} + (\beta \delta/(1-\delta))F_{22}$ . But because of indifference in state 2 of the type-(iii) equilibrium, we see that  $w_2^i$  must equal  $w_2^{iii}$ . This feature is special and due to the way this equilibrium was constructed, there are other cases of multiplicity where the welfare of the current self differs across equilibria. Still, this case of equilibrium multiplicity is not without interest from a welfare perspective, because it could be argued that the more appropriate welfare measure is the one using  $\delta$  as the discount factor, i.e., using the perspective of yesterday's self on choices today and in the future.<sup>15</sup> Thus, we have  $v_2^i - v_2^{iii} = F_{21} - F_{22} + (\delta/(1-\delta))(F_{11} - F_{22}),$ which gives more weight to the latter term, and since  $F_{11} < F_{22}$ , this implies that equilibrium of type (iii) is better. To summarize, in this example, welfare analysis suggests that the outcome with high long-run culture consumption is to be preferred over the one with low culture consumption, even though both these outcomes are equilibria.

## 3.2.2 Summary and implications

In this section, we summarize and draw some brief conclusions.

Multiplicity: optimism and pessimism in culture consumption The multiplicity of equilibria is a quite striking feature of a decision problem. An individual, left to his or her own devices, can solve the problem in different ways, some of which are strictly better than others, yet these different ways are all rational, in the sense

 $<sup>^{15}</sup>$  See Chapter 1 for a justification and discussion.

of being equilibria to the multiple-selves game. Thus, two individuals in identical choice situations may choose to consume different amounts of culture and end up on different welfare levels. One individual chooses a great deal of culture, because the future selves of this person are expected to choose high levels of culture consumption, thereby making present culture consumption pay off, and the other chooses little culture because so will the future selves. Thus, we have yet another source of diversity in culture consumption: optimism and pessimism as multiple equilibria in the intrapersonal game.

Because of the potential for welfare-ranked equilibria, policy here seems particularly relevant to consider: governments could potentially help people not to have to fall into traps of pessimism! The idea is that a policy changes incentives and some policies may render equilibria unique and eliminate the pessimistic equilibrium. To further explore this idea, policy would need to be explicitly considered in the model; we leave this to future research.

Numerical methods for computing equilibria When preferences are not timeconsistent, Markov-perfect equilibria can be expressed as functional equations but are not contraction mappings. This means that there is no guarantee that a Markovperfect equilibrium will exist in the case of a continuous domain. For a discrete domain, these equilibria can be shown to exist if mixed strategies are allowed, but there is no guarantee that pure-strategy equilibria exist. When they do not, numerical computation is challenging; methods based on iteration on the functional equation defining equilibrium are then bound not to work. Suppose, for example, that there is an initial guess on a decision rule for the future selves, and that given this guess, we derive the future value functions, i.e., the vs. Then, the new policy rule is obtained by the maximization problem over k', given the vs. This problem will generically not deliver indifference between two alternatives. So if the equilibrium is unique and requires mixed strategies equilibrium, it will not be found with standard iteration.

## 3.2.3 Numerically computed equilibria

Here, we illustrate how more steady states—and more discontinuities in decision rules—can arise due to time inconsistency alone. This is done by numerically calculating pure-strategy equilibria, using a finite grid and forcing culture capital to lie on the grid. We will only show one example; a full characterization is beyond the scope of the present project.

Before displaying the example, we make two remarks. First, as shown in the previous section in a simple example with two grid points only, pure-strategy equilibria do not always exist. Indeed, in Krusell and Smith (2003b), a consumption-savings model with time-inconsistent preferences and a finite domain for capital is studied, and there pure-strategy equilibria did not tend to exist for fine grids. We believe that the same often holds true in this model. In particular, starting from a case with  $\beta = 1$  and a stable steady state, we found that slight decreases in  $\beta$  led to nonconvergence when using pure strategies only. For much lower values of  $\beta$ , however, such that the whole decision rule would be expected to fall below the 45-degree line, we were able to find pure-strategy equilibria, and it is one of those that we will discuss below.

Second, and relatedly, we believe that at least in the case where  $\alpha < 0.5$ , stepfunction equilibria using mixed strategies, along the lines of the findings in Krusell and Smith (2003b) can be found. Indeed, we conjecture that step-function equilibria of the sort in Krusell and Smith (2003a) also exist for a continuous domain in this case, then as pure-strategy equilibria.

**The example** We start from a case with  $\beta = 1$  and  $\alpha > 0.5$ , where the optimal law of motion for culture capital hugs the 45 degree line: there are two positive steady states. Lowering  $\beta$  to 0.83, we see a drastic change in the nature of the decision rule. Figure 3.11 shows the decision rules for these two values of  $\beta$ .<sup>16</sup>

There is a large number of jumps in the decision rule in the case of timeinconsistent preferences. Moreover, there are 7 positive steady states! Thus, if the initial level of culture capital is below the lowest positive steady state, the individual consumes less and less culture over time and converges to zero culture consumption.

<sup>&</sup>lt;sup>16</sup> The parameter values are  $\alpha = 0.53$ , I = 2.3, d = 0.75, b = 0.25, and  $\delta = 0.50$ .

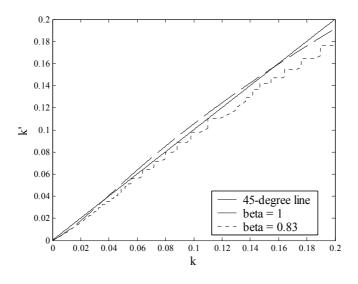


Figure 3.11:

If, however, the initial culture capital stock is higher than that, the long-run level of culture capital depends on exactly in what interval the initial level lies. In other words, we see a drastic increase in the long-run diversity of culture consumption due to time inconsistency in preferences.

To obtain intuition for the decision rule, let us focus on the typical segment between two steady states, denoted  $k_1$  and  $k_3$ , stylized in Figure 3.12 below.

To understand how this behavior can be optimal over this range, first assume that over this range, there is a perceived lower bound on k'; we will later go back and argue why there is such a lower bound. If  $k' \ge k_1$  is thus perceived as a constraint, this constraint will bind for a range of k from  $k_1$  to the value denoted by  $k_2$  in the graph. Over this range, the constraint binds strictly, and at  $k_2$ , it has ceased to bind strictly (the multiplier on the constraint at this point would be exactly zero). Thus, in the open interval  $(k_2, k_3)$ , an increase in k increases k'. In the range  $[k_1, k_2]$ , all future selves, like the current self, will choose  $k' = k_1$  and culture consumption will be  $y = dk_1/b$  forever. In the range  $(k_2, k_3)$ , all future selves will also choose  $k' = k_1$ but culture consumption will be higher in the next period than  $y = dk_1/b$  and only one period later fall to  $y = dk_1/b$  and remain there forever. When k is as high as  $k_3$ , the individual chooses to remain at that level: an upward jump.

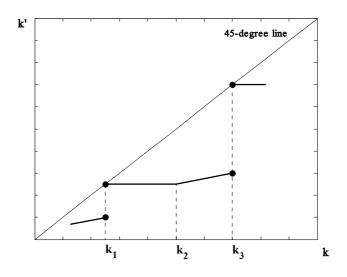


Figure 3.12:

Why is there an upward jump at  $k_3$ ? Time-inconsistency implies that the current self would like the next self to be more forward-looking, and hence choose to consume more culture goods so as to accumulate culture capital. The current self thus sees a tradeoff between the low current culture accumulation, leading to  $k'' = k_1$  (i.e., a starting level of  $k_1$  two periods from now), which is consistent with the present-bias of the current self, and an effort which goes against the present-bias but ensures that the future selves end up at a much higher level of culture consumption. At  $k_3$ , the consumer is exactly indifferent between the low and the high level of culture consumption.

To explain the lower bound assumed, i.e.,  $k' \geq k_1$ , suppose the current self to have a level of k slightly above  $k_1$ . For the same reason that the consumer is indifferent between remaining at  $k_3$  and letting it fall drastically, the consumer at  $k_1$  is indifferent between remaining at  $k_1$ , with high culture consumption, and significantly dropping culture consumption. For a consumer with a slightly higher initial value of k, thus, imagine that a choice of k' at a value slightly below  $k_1$  were considered, thus moving beyond the artificially imposed constraint. The next self would then, by construction, be close to indifferent between keeping a high and a low level of culture consumption, but would marginally choose the lower level. What is the evaluation of this possibility of the current self? The current self significantly disagrees with the next self and places a larger weight on the future than does the next self. Therefore, it would *strictly* prefer the next self not to choose the lower value. This is why a choice of k' slightly below  $k_1$  would give a jump down in utility and thus, we can locally view  $k_1$  as a corner solution. Because of the strict preference, moreover, the constraint is strictly binding, which is the reason why there must be a flat section immediately to the right of  $k_1$ . Time inconsistency is thus essential in the argument, because it embodies the disagreement leading to flat sections and jumps.

In the stylized section, notice that over the range  $[k_1, k_2)$ , current culture consumption is strictly *decreasing* in k: more experience with culture leads to lower current culture consumption.

On a general level, the nonconvexity of the maximization problem leads to multiple local peaks in the objective functions, which will lead to decision rules with jumps. Because adjacent selves then disagree, flat sections are created, and so on.

## 4 Concluding remarks

We have explored a model where culture is viewed as a good involving taste cultivation. This model rather naturally implies that significant, endogenously generated, long- and short-run diversity in culture consumption follows, as long as culture is a relatively close substitute to the alternative good or activity. High substitutability seems quite natural, particularly since some forms of culture do not seem to be consumed at all, or almost not at all, by many consumers. However, one would like to estimate this degree of substitutability using a dynamic model of culture consumption like the present one; this is an important task for the future.

Though not considered here, a case where two culture goods compete can also be considered. Consider, for example, two distinct forms of "difficult-to-appreciate" music. Here, the assumption of high substitutability is a very natural one, and it seems clear that results similar to those obtained here would arise: the initial conditions would be of great importance, and a large diversity in taste generated by the dynamics of taste cultivation could be observed. The possibility that preferences are time-inconsistent is particularly interesting when taste cultivation is important, because it suggests that consumers may benefit from government intervention. Here, it seems that government intervention may also have the effect of reducing the diversity of culture consumption, though we have only begun to analyze this issue. In particular, we did not explicitly introduce government policy here. Moreover, we did not provide a complete characterization of Markov-perfect equilibria under time-inconsistent preferences, but merely made some qualitative remarks using a simplified setup with a discretized domain for culture capital. Research that is left to future work thus includes a more complete exploration of equilibria in the present setup and similar setups as well as an explicit consideration of the role of policy.

Finally, one would also like to compare the results obtained here using the setup with multiple selves to that based on temptation and self-control developed by Gul and Pesendorfer (2001,2004). Multiplicity is less likely to result there, but interesting endogenous diversity due to the non-standard features of preferences may still be present.

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## Chapter 4

# Has International Competition Increased? Estimates of Residual Demand Elasticities in Export Markets

## 1 Introduction

An often repeated statement is that "competition has increased in international markets", a conclusion which may seem reasonable in view of the increasing internationalization in the last decades. Since the 1950's and especially since the mid-1980s, the world has experienced an immense economic integration. World trade has expanded much more than total production and there is trade among a larger number of countries. A widely believed argument is that openness will increase competition and thereby induce falling prices. However, economic theory does not give an unambiguous correlation between exposure to international competition and competitive conduct in a market. In this paper, I use two-digit industry data from the 1960's to the 1990's to study the degree of competitiveness in Swedish export markets and see whether I can find any signs of "increased competitively.

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Traditional models of trade and openness build on the concept of comparative advantage and allocative efficiency that give rise to static gains of trade. In the last two decades, an approach emphasizing the role of increasing returns to scale and imperfect competition points to alternative explanations and consequences of openness. International trade can also give dynamic welfare effects, by increasing the degree of competition. The firms are forced to adopt a competitive behavior and prices are brought down to competitive levels. However, if the domestic industries are already competitive, an increasing trade exposure will have no effect on domestic markets and will not affect the margins in factor, input and product markets. Hence, the interaction between openness and concentration in the industry is crucial for a change in competition (Levinsohn, 1993). International openness does not only affect pricing decisions in the local markets; a widely held view is that competition makes firms and industries more efficient. This could include improved use of technology and exiting from the industry of high-cost producers (Horn et al, 1993).

Theoretical analysis and empirical evidence point to different relationships between international integration and competitive behavior, depending on whether an import or an export competing industry is analyzed. The effect of import competition on the domestic firm's market power is negative, assuming imperfect competition in the domestic markets. Increases in the import share will thus lead to lower prices for various reasons, e.g., more intense competition, inefficient firms exiting, and/or, in the case of collusive behavior on the domestic market, difficulties of sustaining the collusion because of changes in the pay-off and the inability to punish the deviating part.

This contrasts with the theoretical predictions on export competition, which are ambiguous. If the exporting industry is a price-taker on the world market, it will be constrained to a competitive conduct in the same way as the import competing industry. Conversely, if the industry can exert some market power, an enlarged world market may enhance profitability by allowing firms to spread fixed costs over a larger production volume and by taking advantage of economies of scale in production. Declining marginal costs and a sustained price level, i.e., a larger markup, is an explicit confirmation of increasing market power. Moreover, if the exporting firm can discriminate between domestic and international markets, firms in the industry

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can maintain their price above the world market price (Pugel, 1980). Mergers, acquisitions and collusion triggered as defensive responses to internationalization can be regarded as a market failure on the international markets and will clearly counteract the competitive pressures of trade openness. Hence, the impact of globalization on competition depends on the underlying assumptions of the theoretical analysis and is ultimately an empirical question.

To measure the nature of competition in a market, the market power of specific firms or subgroups within the market must be evaluated. Several indicators can be used to describe the competitiveness of a market, e.g., the number of competitors, market shares, concentration, size distribution and substitutability of goods. The Lerner index, defined as the difference between price and marginal cost divided by the price, is the standard measurement of market power. The main problem with this method is the difficulty in measuring the marginal cost. Such data are rarely available and approximations using the average cost build on the restrictive assumption of constant returns to scale. A substantial amount of research has been concerned with various methods for indirectly estimating markups, which better utilize observable economic relations and economize on data. Theories on pricingto-market build on the notion that an incomplete pass-through of cost shocks to a segmented market implies market power of exporting firms. Incomplete pass-through is an indication of price-setting behavior, but an absolute level of the price-marginal cost markup cannot be derived. The Solow-Hall approach (Hall, 1988) uses the existence of business cycles to compare fluctuations in production with adjustments in inputs. The basic idea is to use instruments uncorrelated with shifts in technology when estimating the output elasticities of a reduced form production function. The residuals are interpreted as technology changes and the sum of the output elasticities equals the returns to scale. A positive correlation between the instrument variable and the Solow residual is taken as a sign of market power. Markups can be estimated with this method, but the measures may be overestimated if factor utilization is disregarded.

In this paper, I will estimate markups using the theoretical concept of residual demand, which is the relationship between price and quantity, taking into account the supply responses of all other firms in the market. The residual demand curve of a firm in a perfectly competitive industry is flat, that of a monopolist is the same as the industry demand curve, and that of a firm in a product-differentiated industry lies between these extremes. Because of the functional relationship between the price elasticity of residual demand and the Lerner index of market power, the estimation of the residual demand curve provides a direct method for calculating the degree of market power. This elasticity is identified by exogenous cost shocks that shift the supply relation of the domestic export industry, relative to foreign competitors. If preferences are identical and homothetic across countries, then differences in residual demand elasticities reflect differences in the elasticity of competitors' supply across destinations. More elastic schedules reflect the presence of many close substitutes and more intense competition.

The application of the concept of residual demand, also labeled ex ante demand or "true" demand by Helpman and Krugman, is based on an econometric technique presented by Baker and Bresnahan (1988) and extended by Goldberg and Knetter (1999) to identify residual demand in international markets. This approach does not require a specification of an oligopoly model with detailed assumptions on demand and supply functions, e.g., product differentiation, a quantity- or price setting game, or conduct parameters. Demand elasticities or cross elasticities need not be identified or estimated. Jointly with this economization of data, a great deal of information is lost. This reduced form elasticity cannot separate the structural parameters such as cost, demand or behavioral parameters. Calculations of the left-hand side of the profit-maximizing condition, i.e., price and marginal costs measured directly in accounting data, admit one Lerner index per time period. The identification of the residual demand elasticities, i.e., the right-hand side of the profit-maximizing condition, involves estimations over a time series to capture the price-quantity adjustments and the estimated markup will subsequently be an average over time.

Although international integration of product markets is high on the political agenda in Europe, few studies have empirically analyzed the degree of competition. Two factors behind the lack of attention given to this concept are, first, the notion stated above, i.e., an increasing openness is generally assumed to generate a more competitive behavior and the topic is thus not considered very interesting, and second, market power is difficult to measure. As already discussed above, the

price pressing effect on the markets will not be obvious at all and in these times of establishment and enlargement of trade agreements, merger waves etc, evaluating the consequences on competition and, in the end, welfare is of utmost importance. Since the traditional methods are not satisfactory for describing market conduct, trying out and assessing alternative frameworks and techniques is of great interest.

Other papers applying the residual demand approach to measure competition in segmented markets have used highly disaggregated data.<sup>1</sup> This study will measure the competition in the Swedish export industry on the 2-digit industry level. Estimating the markup at such a highly aggregated industry level may seem optimistic, but trying such an approach may still be worthwhile for several reasons. First, suitable micro data are only available for some specific industries; although such studies should, in principle, produce precise estimates for those industries, we obtain little information about overall behavior, which is what we are really interested in. Second, micro data are typically available for relatively short periods, while industry data are available for long periods, including several major shocks (devaluation, cost crises) which have produced large variations in the data. Third, developments in a particular, relatively concentrated industry are heavily affected by firm-specific shocks (management problems etc.). Looking at one or two industries involving a few firms is thus not likely to yield reliable results; for reliable results a lot of micro data are needed.

While it is true that micro data are preferable because the errors arising from aggregation are avoided, it cannot be inferred that an aggregate approach will not work. If the sectors are reasonably homogenous, one would expect aggregate industry data to yield reasonably good estimates of average behavior, just because there is an enourmous variation in the data. Finally, even if measurement errors etc. imply biased estimates of elasticities and hence, markups, the procedure will still be able to identify the trend in markups over time if the biases are constant.

In this paper, I will estimate the residual demand elasticity in eight Swedish manufacturing industries. If the results seem reasonable, several additional questions may be asked. First, the sectors might be compared: are markups lower in sectors

<sup>&</sup>lt;sup>1</sup> See, for exemple, Goldberg and Knetter (1999), Kamerschen and Kohler (1993), Beutel and McBride (1992), Baker and Bresnahan (1988), Haller and Cotterill (1996) and Higgins et al (1995).

with more homogenous products? Second, the sample might be split to observe possible changes over time: is there any evidence of competition having increased? Third, the markup implied by the residual demand elasticity can be compared to markups derived with other methods, e.g., by accounting markups or Hall's method. The rest of the paper is divided into four sections. Section 2 outlines the theoretical framework. Section 3 presents the econometric specification and data. Section 4 describes the empirical implementation and the results obtained from the analysis. Section 5 concludes. A description of the data sources is given in the appendix.

### 2 Model Specification

Consider a domestic firm producing a good exported to one destination country. The firm faces competition from a firm located in a third country. The goods are imperfect substitutes so that the firms perceive that their demand curves are downward-sloping. Let q, q<sup>\*</sup> be the quantity demanded in the destination market of the goods produced by the domestic and the foreign firm, p, p<sup>\*</sup> prices in terms of the importer's currency, and Y a vector of demand shifters in the destination market. Demand for the products of the two firms is assumed to have the general forms:

$$q = D\left(p, p^*, Y\right) \tag{4.1}$$

$$q^* = D^* (p^*, p, Y).$$
(4.2)

Consider first the profit-maximization problem solved by the foreign competitor:

$$\max_{p^*} \Pi^* = p^* q^* - e^* C^* (q^*, W^*)$$

$$= p^* D^* (p^*, p, Y) - e^* C^* (D^* (p^*, p, Y), W^*),$$
(4.3)

where the total costs,  $C^*$ , is a function of output and a vector of firm-specific cost shifters,  $W^*$ , expressed in the exporter's currency, and  $e^*$  is the exchange rate. Maximizing with respect to the price yields the first-order condition. Solving for the foreign price and using the foreign demand function to substitute for quantity, we can obtain the foreign firm's reaction function:

$$p^* = R^* \left( p, Y, e^* W^* \right). \tag{4.4}$$

To obtain the residual demand function, the competitor's price is replaced by the reaction function in the demand function of the domestic firm:

$$q = D(p, R^*(p, Y, e^*W^*), Y) = D^{res}(p, Y, e^*W^*).$$
(4.5)

The residual demand function depicts the demand facing the domestic firm, taking into account the competitor's supply response. From equation (4.5), we can derive the residual demand elasticity:

$$\eta^{res} = -\frac{dq\,p}{dp\,q} = -\frac{p}{q} \left[ D_p + D_{p^*} R_p^* \right]. \tag{4.6}$$

To see how residual demand is linked to the market power of the firm measured as the price-marginal cost margin, consider the profit-maximizing problem faced by the domestic firm:

$$\max_{p} \Pi = pq - e^{*}C(q, W) = pD(p, p^{*}, Y) - eC(D(p, p^{*}, Y), W).$$
(4.7)

In optimum, marginal revenue equals marginal cost:

$$q + p \left[ D_p + D_{p^*} \frac{\delta p^*}{\delta p} \right] = eMC \left[ D_p + D_{p^*} \frac{\delta p^*}{\delta p} \right].$$
(4.8)

After rearranging, we can write the first-order condition as a markup of marginal costs over price:

$$\frac{p - eMC}{p} = -\frac{q}{p} \left[ D_p + D_{p^*} \frac{\delta p^*}{\delta p} \right]^{-1}.$$
(4.9)

If we assume that the domestic firm can correctly predict the reaction of the other firm and hence is profit-maximizing according to the actual demand curve it is facing, i.e., the residual demand, the right-hand side will be identical to the inverted residual demand elasticity. Consequently, the term  $\delta p^*/\delta p$  which represents the domestic firm's belief about the effect of its own price changes on the competitors' behavior, i.e., the 'conjectural variation', must be known by the firm for the equality to hold. It is clear from equation (4.9) that the lower is the residual demand elasticity and hence, the steeper the residual demand curve, the larger is the price-marginal cost markup and vice versa.

Consider the more realistic case with more than one competitor. The reaction function of a competing firm located in the ith country is:

$$p^{i} = R^{i} \left( p, p^{j}, Y, e^{i} W^{i} \right),$$
(4.10)

where p without superscript is the price of the domestic good and  $p^j$  is a vector of prices of competing goods from countries j = 1, ..., n and  $j \neq i$ . Thus, the reaction function gives the optimal price of the ith good, given the prices of all competing goods. The n number of country-specific reaction functions and demand functions is now used to eliminate all prices except the domestic price. We will then receive n equations of the form:

$$p^{i} = \tilde{f}^{i}\left(p, Y, W^{C}\right), \qquad (4.11)$$

where the variable  $W^C$  is a vector of all cost shifters of the competing countries expressed in the importer's currency. To obtain the residual demand, the *n* equations are used to substitute for foreign prices in the demand function of the domestic firm. The residual demand function may then be written:

$$q = D\left(p, \tilde{f}^{1}\left(p, Y, W^{C}\right), ..., \tilde{f}^{n}\left(p, Y, W^{C}\right), Y\right) = D^{res}\left(p, Y, W^{C}\right), \qquad (4.12)$$

and the residual demand elasticity will be

$$\eta^{res} = -\frac{dD^{res}}{dp}\frac{p}{q} = -\frac{p}{q}\left[D_p + \sum_j D_{p^j}\tilde{f}_p^j\right].$$
(4.13)

If the domestic firm solves the profit maximization problem, the first-order condition will show the same result as in the one-competitor case. A more elastic residual demand curve implies a smaller price-cost margin and hence, will reduce the firm's ability to exercise market power. As already emphasized above, this equality rests on the assumption that the firm can perfectly predict the supply responses of the foreign firms and can also act in a corresponding way. Two oligopoly models based on these assumptions are the Stackelberg model and the dominant firm or price leadership model. The latter theory presumes one dominant price-setting firm facing constraints from fringe competitors small enough to accept the price of the dominant firm as given in determining their supply response. This model corresponds to the economy depicted earlier in this section.

In the Stackelberg model, a leading firm chooses quantity to maximize profits, given the followers' reaction functions. Given the inverted demand functions  $p = G(q, q^j, Y)$  and  $p^j = G^j(q^j, q, Y)$ , the first-order condition of the domestic firm will be

$$\frac{p - eMC}{p} = -\frac{q}{p} \left[ G_q + \sum_j G_{q^j} \frac{\delta q^j}{\delta q} \right]$$
(4.14)

and the inverse residual demand elasticity

$$\eta^{res} = -\frac{dG^{res}}{dq}\frac{q}{p} = -\frac{q}{p}\left[G_q + \sum_j G_{q^j}\tilde{g}_q^j\right],\tag{4.15}$$

where  $\tilde{g}_q^j$  is is the derivative of a function analogous to equation (4.11), but with domestic and foreign quantity as endogenous variables instead of prices. Analytically, a price-setting model will generate a more competitive equilibrium than a quantitysetting model and will thus exhibit a flatter residual demand curve. Both theories build on the concept of first mover advantage, which can rationalize the functional relationship between the domestic firm's price elasticity of residual demand and the price-marginal cost markup from the optimality condition. More generally, for firms in a consistent conjectures equilibrium (Bresnahan, 1981), there is no distinction between the residual demand curve and the demand curve considered by the firm. A direct relationship between residual demand elasticity and markup will thus hold.

Two other game-theoretic models can also be consistent with this equivalence. The residual demand elasticity depends on market demand elasticity and the pricing reactions of competing firms. If the competitors do not react to the pricing decisions of the domestic firm, for example if the products are very differentiated or there exist a large number of variations, the residual demand elasticity will be very close to the market demand elasticity of the firm and thus, the monopoly solution will be the outcome. In the case of homogenous goods and perfect competition, the firms will be price-takers and produce where price is identical to marginal cost. Consequently, residual demand will be infinitely elastic and the markup equals zero. It is true that the two latter models have the desired property of the residual demand curve being no different from the conjectured one. But this outcome is due to the fact that residual demand does not exist in the same sense in these model setups, as the conjectural variation parameter is omitted by assumption. Still, a theoretical and empirical analysis based on each of these frameworks will imply a result in line with the central proposition, i.e., the lower the residual demand elasticity facing a firm, the more market power can be exercised and hence, the less competitive is the market.

In other oligopoly models explicitly assuming a conjectural variation parameter which cannot be foreseen or controlled by the firm, the demand function met by the firm in the profit maximization problem will not correspond to the residual demand function. In a Bertrand equilibrium, the firm equates its perceived marginal revenue to marginal costs, given the correctly predicted price of the foreign substitute product. Consequently, the residual demand curve and the perceived demand curve will intersect at the equilibrium domestic price. The slope of the residual demand curve will be steeper than that of the perceived demand curve, as the products in a price-setting game are assumed to be strategic complements, i.e., a price increase in the domestic product will be followed by a price increase by the competitors. In the Bertrand game, the residual demand curve will obviously equal the demand curve perceived by the leader in a price leadership model. It seems clear that a low residual demand elasticity implies a high degree of market influence and hence, less intense competition, even though the domestic firm does not consider the residual demand curve in the profit maximizing problem. Corresponding conclusions can be drawn from a quantity-setting game, i.e., the Cournot model. The residual demand curve will intersect the perceived demand curve at the equilibrium domestic quantity, but it will have a flatter slope as the products in a quantity-setting game are assumed to be strategic substitutes. The potential residual demand in the Cournot model equals the demand curve facing a Stackelberg leader and the smaller is the residual demand elasticity, the larger is the price-marginal cost markup. For a further discussion on

why increasingly intense competition would, in general, imply a flatter residual demand function, see Lerner (1934) and Landes and Posner (1981).

## **3** Econometric specification and data

The empirical application of the above approach requires a specification of the residual demand function. What is estimated is essentially a regular demand function where competitors' prices have been replaced by variables shifting competitors' costs. Demand functions are normally estimated with quantity as the dependent variable. Goldberg and Knetter (1999) estimate the residual demand curve in inverse form, however, with price as the dependent variable. This means that the estimated markup can be read off directly as the coefficient on the quantity exported. But provided the instruments are uncorrelated with the error term, and the elasticity is a finite number, it should be of no econometrical importance which way we write the equation.<sup>2</sup>

Intuitively, it seems more natural to present a demand equation with quantity on the left-hand side, as is done here.

There is, however, one case when a demand function with quantity as the dependent variable would be misspecified, i.e., when the price elasticity is infinite, so that the firm is a price taker. In that case, the specification with price as the dependent variable would still be correct. We should be able to "detect" such cases, however, by running the "first-stage" regression; if firms are price takers, export prices will be independent of domestic costs, and largely depend on foreign costs.

The equation (4.12) is estimated in double log form so that the coefficients can be interpreted as elasticities:

$$\ln q_t = \eta \ln p_t + \beta \ln Y_t + \gamma \ln e_t W_t^C + \varepsilon_t, \qquad (4.16)$$

where  $\varepsilon$  is an iid error term, Y is the vector of demand side variables,  $W^C$  is a vector of cost shifters of the foreign competitors, p is the price set by domestic firms, q is the export volume, e is the exchange rate and the subscript t denotes the time index.

 $<sup>^{2}</sup>$  In fact, if the equation is exactly identified, we numerically get the same estimated elasticity and markup, notwithstanding if we put price or quantity on the left-hand side.

Note that the specification is expressed in Swedish currency as opposed to equation (4.12) where the residual demand function was expressed in the importer's currency. As in the theoretical section, the residual demand elasticity is denoted by  $\eta$ . Under the assumptions discussed in the previous section, the inverse elasticity will directly correspond to a measure of industry markup of price over marginal cost. Thus, if the estimate is high, the domestic industry has substantial market power and the residual demand curve will be steep. The condition for optimal output implicates the theoretical boundaries of the residual demand elasticity to range between minus one and minus infinity. The corresponding boundaries of the inverse residual demand elasticity will range between zero and minus one. Clearly, the Lerner index will never exceed one hundred per cent.

There is no clear interpretation of the other coefficients, since they may reflect both direct effects on the domestic firms' demand and indirect effects through the adjustments by foreign firms. According to economic theory, a demand function is homogenous of degree zero in prices which imply nominal neutrality, i.e., the parameters of the domestic and the foreign price should be equal. Moreover, if we assume that the cost functions of the competitors are homogenous of degree one in factor prices, the residual demand should be homogenous of degree zero in domestic prices and foreign costs. Consequently, if the cost shifters perfectly capture all costs, nominal neutrality between the foreign cost shifter and the residual demand elasticity will prevail.

The price of the domestic firms is endogenous due to the presence of simultaneity from the optimality condition, and it must therefore be instrumented for. The potential instrument must satify two conditions: first, the instrument must be uncorrelated with  $\varepsilon$  and, second, the partial correlation between p and the instrument, given the additional observed control variables, must be nonzero. The first condition states that the instrument is exogenous. The second condition is that the instrument is relevant. The exogeneity condition can not be tested, but can be identified within the model. A country-specific cost shifter would be correlated with price, but is excluded from the estimating equation. Some relevant labor cost index is a suitable and feasible instrument that should be independent of the cost shifters of the foreign countries. The relevance condition can be tested with an F-test and in the end of this section the relevance of the instruments will be discussed further.

By the same reasoning as above, foreign wages or producer price indices are natural proxies for the cost side variables, W. As a producer price index is presumably more flexible than wages and consequently, captures changes more quickly, the former variable will be used in the estimations. Demand side variables, i.e., the scale variables, used in the regression include volume indices of foreign manufacturing industries. To account for industry-specific productivity changes, a linear and a quadratic time trend are included.

Since this study focuses on the market power of the Swedish industry relative to the world, i.e., the largest trading partners within the OECD group of countries, rather than relative specific markets, a weighted and a double-weighted index must be constructed to account for the varying significance of competitors and importing countries. The demand shifters are weighted by the share of Swedish exports to the respective market. As Swedish firms simultaneously compete both with other exporters and with domestic producers in foreign markets, the cost shifters of one specific country will affect the demand of all competing countries. Therefore, to account for this interdependence, cost variables in every country must be doubleweighted, first by the import shares of apparent consumption in each market and second, by the Swedish export share to every market. The construction of the export-share weighted and the double-weighted indices and the computation of the instruments and demand- and cost shifters are reported in the data appendix.

Given the economic theory on export competition, what results might be expected from the estimations? As a small industrial highly developed economy, foreign trade is of significant importance to Sweden. The export share of production ranges between approximately 30 to 50 per cent for all industries except the less export dependent food and mineral industries. Swedish exports have traditionally been dominated by products based on domestic raw materials, i.e., the wood, paper and metal industries, but in the sample period, the relative share of these industries has decreased in favor of technology intense industries like the chemical and fabricated metal industries. The expansive growth in these latter markets would imply that export demand is less elastic than demand on a stagnating market. It can also be argued that producers of heterogeneous goods are facing a downward sloping demand curve, involving less intense competition compared to a market with a homogenous good. This would suggest that the markups in the chemical and fabricated metal industries are higher than those in the raw material based industries. Clearly, if Swedish exporters have a dominant position on the world market, this presumption may be reversed. In fact, Sweden is among the eight largest producers in the paper industry and the Swedish metal industry was price leader until the middle of the 1970's. The wood, paper and metal industries exhibit some additional features that might be significant in the analysis; business cycles are stronger than in most other industries and the production capacities are rigid, which might occasionally make exports constrained by supply.

The impact of internationalization, with expanding world demand and increasing competition from transoceanic countries on the competitive behavior in the Swedish export industry, will depend on the prior degree of competition. The concentration of Swedish industries is high relative to other western countries. Most industries are dominated by a few firms, except the chemical industry which is characterized by competition among many firms. Some industries, like the food, textile and metal industries, have received different kinds of governmental support, but mainly in the first half of the time period 1969-1994.

In the period under consideration, additional countries were joining the EU and trade barriers were removed. This might have an important impact on competition for Swedish industries since Sweden's main trading partners are in Europe. Furthermore, the end of the Tokyo trade round in 1977 entailed a substantial tariff cut on industrial products. In the sample period, the Swedish market share of world trade has decreased from 3.3 to 2.3 per cent. One reason for this is obviously the emergence of several newly industrialized countries, but it may also reflect an inflexible structure of the Swedish manufacturing industries, making it difficult to capture the increasing demand of high technology goods on the world market. It seems evident that the market power of the Swedish export sector has decreased over these 25 years and markups are thus expected to have decreased.

If, after facing increased international competition, Swedish firms focus their production to less competitive sub-markets, the estimated markups tend to underestimate the competitive impacts of internationalization. For example, the Swedish textile industry has substituted the production of competition exposed ready-made clothes with highly specialized textile products. However, the structural reform of the industries is controlled for in the construction of the export price indices. Principally, the indices are based on the same products over time and quality changes are excluded.

Prior to performing the estimations, it will be useful to look at the data themselves to reveal important features. In figures 1 to 8, the logarithmic differences between export prices and foreign prices as well as export quantity and foreign demand are plotted for each industry over time. Between 1969 to 1994, there were five periods when the competitiveness of the Swedish industry was considerably affected. i) High international inflation and economic boom characterize the first half of the 1970's. Foreign costs and inflation increased more rapidly abroad than in Sweden. ii) Swedish wage costs increased by about 40 per cent in 1975-1976, considerably more than abroad. iii) Three successive, smaller devaluations in the late 1970's were insufficient to reestablish competitiveness. iv) Two devaluations, in 1981 and 1982 of about 25 per cent made Swedish industry competitive. v) The competitive edge gradually eroded, until the currency peg was abandoned in 1992.

Within the framework of an imperfect substitutes model, we would expect that in period i), iv) and v), and to lesser extent, period iii), first, Swedish export prices would increase less than world prices and, second, that the Swedish export volume would increase considerably relative to general market conditions. Beginning with a large representative industry such as the fabricated metal industry (figure 8), we see that the development of prices and quantities confirms fairly well with what would be expected. The same seems to be true for the food (figure 1), textile (figure 2) and mineral (figure 6) industries. During the above mentioned boom periods, increases in foreign prices exceed increases in Swedish export prices and Swedish export volume growth surpasses international demand. Furthermore, the chemical industry (figure 5) matches reasonably well, although three oil price shocks, two positive ones in 1974 and 1979-1980 and a negative in 1985, also affect the price and quantity movements. The industries based on domestic raw materials, i.e., the wood (figure 3), paper (figure 4) and metal (figure 7) industries, show a different pattern. In the years 1971-1972, the world economy experienced a raw material boom. The extreme overreaction of export prices and, to some extent, quantity in the wood industry in this period may be explained by this episode. However, the last three boom periods can be identified in the figures. The same activity can be recognized in the metal industry, i.e., a raw material boom in the early 1970's and then a price and quantity development principally following the general trends. Conversely, the effects of the Swedish devaluations on the paper industry are less clear. This phenomenon can be interpreted in two ways: the industry can either be a market leader, setting prices more or less independent of exchange rates or other macroeconomic fluctuations, or international markets may be competitive, so that Swedish firms have no prospect of influencing prices. In any case, other variables seem to be more important for price and quantity developments than the general economic conditions.

The determination of export prices can be further analyzed by regressing the export price on the instruments when the other explanatory variables are controlled for, i.e., by estimating the "first-step-regression". The results are reported in Table 1. It is apparent that Swedish wage costs do explain Swedish export prices of food, minerals, metals and fabricated metals, and to some extent, textiles. They are not a suitable instrument for explaining export price movements in the wood, paper and chemical industries. Furthermore, foreign producer prices seem to be very important in all industries except the paper industry.<sup>3</sup> The last four rows in the table show the specification including an additional cost variable, oil prices. It is hardly surprising that the oil price is highly significant in the chemical industry. The variable is also significant in the wood industry, but does not appear to be vital for the remaining industries.

From the p-values it can be inferred that the Swedish wage costs in fact is relevant as instruments only in the mineral and fabricated metal industries. The weakness of the instruments should be recognized in the following estimations.

<sup>&</sup>lt;sup>3</sup> This can be interpreted as signs of pricing-to-market.

## 4 Estimation and results

Diagnostic tests performed on 2SLS estimations for every industry indicate potential problems with autocorrelation and, to some extent, heteroscedasticity in some industries. For that reason, the equations will be estimated by GMM, allowing for conditional heteroscedasticity and MA1. This estimator is robust to heteroscedasticity and moving average errors, and thus eligible and efficient for many problems, albeit an overkill in some industries.

## 4.1 Market conduct in eight Swedish manufacturing industries

Equation (4.16) was estimated for eight Swedish manufacturing industries (SNI31-SNI38) with the instruments described in the previous section. The results from the regressions are reported in Table 2. The estimated coefficients have the predicted signs and are generally significant. Although the coefficients cannot be interpreted as the correct markup per se, the magnitude of the elasticity will display the relative market power of the different industries. The markups implied by the residual demand elasticities are listed in the last row. In four industries, the residual demand elasticities are significantly different from zero, suggesting a deviation from competitive behavior. As expected, the estimations of the wood, paper and chemical industries do not perform well in terms of preciseness and explanatory degree, even though the absolute values of the price elasticities seem to be plausible. In particular, the results from the wood industry should be regarded with skepticism, considering the unlikely income and cost elasticities. Moreover, the residual demand elasticity in the metal industry exceeds the theoretical boundaries. Clearly, the time series data employed do not sufficiently explain the market performance in these last four industries. Before making any inferences of the competitive behavior in the various industries, the equations will be reestimated with some alternative cost shifters that will possibly better account for the price and quantity movements on each specific market. To save on degrees of freedom, the augmented specification will include a weighted cost-shifter

$$\ln q_t = \eta \ln p_t + \beta \ln Y_t + \gamma \left(\lambda \ln W_t^C + (1 - \lambda) \ln W_t^i\right) + \varepsilon_t$$
(4.17)

where i depicts a cost shifter specific for industry i. Finding a correct cost shifter will probably be most straightforward for the chemical industry, where the oil price has a considerable influence. One of the largest cost shares in the paper industry is the price of pulpwood. This would suggest that foreign pulpwood prices be included as an additional cost shifter and that Swedish pulpwood prices be included as an additional instrument. The wood industry consists of several sub sectors, i.e., the saw milling industry, the fibre and particleboard industry and the furniture industry. Lumber as well as pulpwood prices would thus be relevant cost shifters. Since the US constitute an important competitor to the Swedish wood industry, a relative producer price index of lumber for the US was employed in the estimation. By the same reasoning, Swedish lumber and pulpwood prices were included as instruments. However, since the cost-weighting parameter lambda exceeded its theoretical upper bound, i.e., unity, which implies that prices of foreign lumber have a negative impact on export prices, the additional foreign cost variable was dropped from the specification. Finally, there is reason to believe that the performance of the metal industry is contingent on iron ore prices. A price index of Brazilian iron ore was employed as a proxy variable to foreign industry costs. As in the wood industry, the lambda turned out to be unreasonably large and the augmented specification is therefore not considered in the rest of the analysis. In Table 3, the results from the last three estimations are presented, including the previous results from the food, textile, mineral, metal and fabricated metals industries. As can be seen, the residual demand elasticity of the chemical industry has increased but since the point estimate is imprecise, no statistical inference can be made. The point estimates in the paper industry are highly significant, although the price elasticity unfortunately exceeds the theoretical boundaries. However, the R squared has increased considerably, which is encouraging since it indicates that the latter specification better captures the performance of the paper market. The same seems to be true for the wood industry, where, in addition, the estimated price elasticity turns up as highly significant. Without making any statistically significant inference of the relative competitiveness of the various

industries, the textile, wood, mineral, fabricated metal, food and chemical industries appear to be pricing with an approximate price-cost margin of fifty per cent, the last two industries possibly moderately lower. The price elasticities of the paper and metal industries are certainly absurdly small, a frequent problem in estimations of trade elasticities, see e.g., Goldstein and Khan (1985). Without interpreting the absolute values as a de facto markup, one explanation may be that these two markets are less competitive, an assumption in line with the former dominant position of these industries on the world markets. An alternative interpretation is that the specification does quite simply not work.

The income elasticities are significant in all industries but one. The fairly large elasticities in the food, wood, paper and metal industries are in accordance with the strong cycles of these markets. The elasticities of the cost shifters are signed as predicted and are significant in all industries except in the chemical industry. A t-test cannot reject the null hypothesis that the cost elasticities are equal to the residual demand elasticities, except in the wood, paper and metal industries.

Confronted with a rather limited sample, it is convenient to take advantage of all economic theoretical restrictions applicable to save degrees of freedom and increase the preciseness of the estimations. As already pointed out above, if all relevant cost- and demand shifters are included in the specification, a somewhat bold assumption, the residual demand function will depict nominal neutrality. Estimating a specification of the form

$$\ln q_t = \eta \left( \ln p_t - \left( \lambda \ln W_t^C + (1 - \lambda) \ln W_t^i \right) \right) + \beta \ln Y_t + \varepsilon_t, \tag{4.18}$$

yields the results presented in Table 4. The results seem to be robust inasfar as the relative sizes of the price elasticities are upheld. The textile, wood, mineral and fabricated metal industries appear to be less competitive than the food and chemical industries and the two outstanding industries, paper and metal, still display low point estimates.

The static imperfect substitutes model on which the estimations are based can be viewed as inconsistent with much of modern pricing theory. Costs of changing prices and incomplete information will give rigidities in the adjustment mechanisms so that market-clearing will take place with some lags. Specifying the estimated equations with a lag structure is a simple and common approach to modeling the dynamic trade behavior. The results from lagging prices and the scale variable in specification (17) are reported in Table 5. The previous results are robust to this exercise. The residual demand elasticities are significant in all industries, except the chemical and paper industries, and the corresponding markups confirm prior inferences.

Concluding this section, we can say that the Bresnahan approach seem to work best for the industries with differentiated goods. As already discussed, the paper, wood and metal industries have stronger business cycles and higher fixed costs than other industries, and these features are neither specifically included in the model nor controlled for in the estimations.

#### 4.2 Changes in market conduct over time

To examine the change in competition over the sample period, a time dummy, D, is added to the specification. The setup is constructed to allow for a changed sensitivity of exports to both domestic prices and foreign costs. The split of the sample is selected to capture possible changes in competitiveness in the various markets as a consequence of the Tokyo round in 1977, after allowing for a five-year time lag for implementation. The specification will be

$$\ln q_t = (\eta_0 + \eta_1 D_t) \ln p_t + \beta \ln Y_t + (\gamma_0 + \gamma_1 D_t) \ln W_t^C + \alpha D_t + \varepsilon_t.$$
(4.19)

The parameter  $\eta_0$  is interpreted as the residual demand elasticity in the first part of the time period. The parameter  $\eta_1$  depicts the change in residual demand elasticity and  $\eta_0 + \eta_1$  is thus interpreted as the elasticity in the latter time period. The results from the estimations are reported in Table 6 with the corresponding markups of the former time period presented in the second last row and the markups of the latter time period presented in the last row. For all industries, except the metal and fabricated metals industries, the estimates take on a negative sign which displays evidence on increasing competition in international markets. The markups are consistent with the results from the basic specification in Table 3. In three cases, the residual demand elasticity for the total time period has a value within the markup range implied by Table 6, which would be expected with more competitive markets over time. Since the point estimates of  $\eta_1$  in all industries but two are rather imprecise, this might well also be true in the other industries. The parameters stating the interaction of the competitors' cost shifters and time are positive in seven industries and negative in the (flunking) metal industry, but they are significantly different from zero in two industries only.

It is reasonable to ask whether the results are contingent on the particular year selected to split the sample. Therefore, the breaking point for every industry was derived by estimating equation (4.19) for different years, looking at the sum of the residual sums of squares and choosing the year for which this sum reaches its minimum. The results are shown in Table 7. The breaking points are estimated to range between 1981 and 1988, i.e., in the latter half of the time period. The coefficients seem fairly stable and more precise. Now, the point estimates of the metal industry are consistent with previous results and the time change is also negatively signed.

Instead of assuming an abrupt change of the parameter, the regression model can be constructed to account for smooth changes. The time dummy in equation (4.19) is replaced by a trend variable ranging from zero to one to allow for similar parameter interpretations as in prior estimations. The results are shown in Table 8. Clearly, this specification is not as successful in explaining the market variations. The point estimates in the food, wood, paper and mineral industries have retained the expected sign and magnitude but are somewhat less determined. The parameter  $\eta_1$  in the textile industry is incorrectly signed but highly insignificant. The estimates in the chemical and metal industries are implausibly high and in some cases, incorrectly signed. Moreover, looking at the R squared in the metal industry, we see that the specification explains very little of the variation in the sample. Conversely, the fabricated metals industry now appear to have been confronted with a similar market development as the other industries, i.e., stronger international competition. Parameters  $\eta_0$  and  $\eta_1$  are both negatively signed and significant at the 10 per cent level.

To summarize the findings in this section, it can be concluded that an indication

of diminishing market power has been found in all industries.

Since the results from the empirical estimations have proved to be in accordance with the theoretical framework, I now turn to comparing this method of measuring market competitiveness with two other conventional techniques.

#### 4.3 Comparing with conventional methods

One traditional method employs accounting data to obtain a measure of priceaverage cost margins used as a proxy for the Lerner index. The methodology is straightforward but rests on rather restrictive assumptions, i.e., constant returns to scale.<sup>4</sup> Following Hakura (1998), the price-cost margins will be calculated by subtracting labor compensation from value added and dividing by the value of sales, i.e., revenues-variable costs/revenues. The accounting markups derived as an average of time series data covering 1970 – 1994 are reported in Table 9. Comparing with the markups derived in the standard specification in Table 3, it is evident that those markups throughout exceed the accounting markups. Moreover, the paper and the metal industries, which are in some sense the two outstanding industries in the residual demand analysis, do not depict any signs of stronger market power relative to the other industries. The magnitude of the markups appears to be fairly equal although the wood, paper and chemical industries eventually display somewhat higher estimates. It should be borne in mind that the assumption of constant returns to scale will give the markups a downward bias if the industry actually has increasing returns to scale. Economies of scale, or certainly also economies of scope, seem to be a reasonable assumption, at least according to the flood of motivation statements that has followed in the recent wave of merging firms.

Next, after splitting the data set in 1982, the change in accounting markups can be compared with the results presented in Table 6. Interestingly, the results emerging from this exercise are contrary to previous results. The market power is suggested to be constant or even increasing in all industries.

The second method for calculating a price-cost margin is the Solow-Hall approach. The output elasticities of a production function are estimated with the

<sup>&</sup>lt;sup>4</sup> The estimations in this paper are also based on the assumption of linear homogeneity of the cost function, but this assumption is not necessary in general in the Bresnahan approach.

help of instruments, non-correlated with technological changes. The residuals are interpreted as technology changes and the sum of the output elasticities equals the returns to scale. Since the output elasticities equal the inverse of the elasticity of cost with respect to output, the returns to scale parameter can be written

$$\gamma(q) = \frac{C(q)}{qC'(q)} = \frac{C(q)/q}{C'(q)} = \frac{AC}{MC} = \frac{p}{C'(q)} \frac{C(q)}{pg} = \mu(1 - s_{\pi}),$$

following the notation of Basu and Fernald (1997), where  $\gamma$  is the output elasticity (or the inverted cost elasticity), C(q) is a cost function,  $\mu$  is the price over the marginal cost rate and  $s_{\pi}$  is the share of pure economic profit in gross revenue. Assuming the latter parameter to be small,  $\mu$  will approximately equal  $\gamma$ . Now, it is easy to back out the Lerner index , i.e.,

$$\mu = \frac{p}{MC} \rightarrow \text{Lerner index} = \frac{p - MC}{p} = 1 - \frac{1}{\mu}.$$

In a recent study, Mikael Carlsson (2000) has estimated technology growth for Swedish manufacturing industries, using the Solow-Hall method above. Moreover, he has tried different ways of controlling for varying factor utilization, which might otherwise constitute a problem when using this method. Since we both use data on two-digit manufacturing industries in Sweden in the same time period (1967 to 1993), I have used his preferred specification to derive estimates of the Lerner index to compare the Solow-Hall approach with the Bresnahan approach. The estimates are reported in Table 9. As in the accounting markup case, the Solow-Hall markups are below the residual demand markups, although the latter display larger variability. According to these results, the food, textile, wood and fabricated metal industries exhibit some market power whereas the other markets can be described as competitive. This is in contrast with the prior result that the paper and metal industries display market power. The Solow-Hall result also differs considerably from the accounting markups, not only because the absolute level differs but, more importantly, because the evidence of lacking market power in the paper, chemical, mineral and metal industries have no correspondence in the accounting markups.

To recap the findings from these exercises, the Lerner index derived by the Bresnahan approach seems to exceed the absolute values of both alternative methods. It should be noted that the latter methods do not separate the export market from the entire industry and that hence, these markups apply to total industry conduct, which may be less profitable than the export sector alone. The potential signs of extensive market power in the paper and metal industries contrast with the results of both alternative methods, neither of which produced uniform results. As already discussed, both competitive and non-competitive outcomes are possible in theory, considering the type of industry and the existing international competitors.

However, since the three approaches are difficult to compare for various reasons, e.g., considering that different markets are studied, no further analysis will be made.

## 5 Conclusions

This paper investigates the competitive conduct of the Swedish manufacturing industries over the period 1969-1994. The Bresnahan approach, which involves estimating the residual demand elasticities, is employed to derive the standard measure of competition, i.e., the Lerner index. After some robustness checks, the changes in competitive conduct over time are estimated. Finally, the results are compared with two conventional methods of calculating the price-cost margin; accounting markups and the Solow-Hall approach.

Theoretically, the most straightforward method to measure the Lerner index is of course to directly calculate the markup over marginal cost. However, since this is normally impossible due to data deficiency, estimating the right-hand side of the profit-maximization condition by the residual demand approach may be the second best alternative. The evidence from this econometric study is in line with the empirical facts, suggesting a deviation from competitive behavior in all industries. Moreover, the results demonstrate a trend of decreasing market power. It is uncertain whether the increasing competition will apply to the entire international market. One interpretation of the results might be that competition has hardened for Swedish industries, but not for the markets as such, e.g., assuming a foreign price-setter and the Swedish industry as part of a competitive fringe.

The final part of the study, when the Bresnahan approach was compared to two alternative methods of estimating the Lerner index, neither confirmed nor rejected the validity of the residual demand analysis. All three approaches give contrasting outcomes, especially regarding the paper, chemical, mineral and metal industries after disregarding the differences in absolute values. However, as already pointed out above, the methods are not completely comparable due to different market definitions etc. It may still be interesting to compare the outcomes, as long as the limitations of the analysis are kept in mind.

Overall, although drawing definite conclusions regarding the absolute markups seems hazardous, the Bresnahan approach does add evidence on the general competitive conduct.

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## 6 Data Appendix

#### 6.1 Weight-indices

Two different kinds of weight-indices are constructed for all eight industries to account for the index properties of the price and quantity data of Swedish exports. The export-share weighted index is employed to compile demand and instrument variables into indices for the estimations. The weights reflect the country composition of Swedish exports to the 14 principal export markets: Austria, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, Netherlands, Norway, Switzerland, United Kingdom and United States, which approximately amount to 80 per cent of the total Swedish exports. The relative weights of the countries are very similar at the beginning and the end of the time period, so that fixed weights corresponding to the export shares of 1980 are used. The export and import data are obtained from the annual publication *OECD's International Trade in Commodity Statistics* after mapping the data from the product-based classification SITC to the activity-based classification SNI.

The double-weighted index weights the cost shifters corresponding to the respective importance of Swedish competitors. The weights of the competitors are derived by weighting the market share, i.e., the share of apparent consumption (production + import – export) in all countries, with the Swedish export share to the importing country in question, which can be expressed as

$$\alpha_{ji} = \sum_{i=1}^{14} m_{ijk} \omega_{jk},$$

where  $m_{ijk}$  is the market share of country *i* in country *k* in industry *j* and  $\omega_{jk}$  is the Swedish export share to country *k*. The weights of the competitors also include the consumption of domestic production, which will give the largest Swedish trading partners a significant weight in the index if the import share of apparent consumption is modest in the industry in question.

#### 6.2 Demand shifters

The volume index of foreign manufacturing industries is collected from the *OECD Main Indicators*. Due to missing data, an index of total industrial production from the OECD Main Indicators is used as a proxy for Germany for the period 1969-1991. For Denmark, an index of industrial production, collected from B.R. Mitchell's International Historical Statistics Europe, linked in 1970 to an index of the production of consumer durable goods, collected from the OECD Main Indicators, is used as a proxy for manufacturing production. The export-share weighted demand index, i.e., variable  $Y_t$  in the specification, denoted below with subscript j to show that a demand shifter for every industry is computed, is derived as

$$Y_{jt} = \sum_{k=1}^{14} \omega_{jk} y_{kt},$$

where  $y_{kt}$  is the volume index of country k at time t.

#### 6.3 Cost shifters

The producer price index of foreign manufacturing industries is collected from the OECD Main Indicators and Compensation per Employee, private sector, is collected from OECD Economic Outlook. Due to missing data, a series of Compensation of Employees from the OECD Economic Outlook is used as a proxy for Switzerland for the period 1969-1994. The variable  $W_t^C$  in the specification, denoted below with subscript j to point out that a cost shifter for every industry is computed, is derived as

$$W_{jt}^C = \sum_{k=1}^{14} \alpha_{jk} e_{kt} W_{kt},$$

where  $W_{kt}$  is the cost shifter of country k at time t and  $e_{kt}$  is the exchange rate between Sweden and country k. The exchange rates series are collected from the *OECD Main Indicators* and refer to averages of daily closing spot rates on national markets versus the USD. The time series of pulpwood prices of spruce and fir used in the augmented specification for the paper industry is collected from U.S. Timber Production, Trade, Consumption, and Price Statistics 1965-1997, table 24, FPL-GTR-116, United States Department of Agriculture. The world spot oil price series is obtained from the IMF publication International Financial Statistics.

#### 6.4 Instruments

The labor cost index for Sweden is constructed from the time series Compensation per Employee, collected from the *OECD Economic Outlook*. The additional instruments employed in the wood and the paper industries, i.e., Swedish pulpwood and sawlog prices, are collected from *Skoglig Statistikinformation*, Skogsstyrelsen in Sweden.

#### 6.5 Accounting price-cost margins

The price-cost margins are constructed by subtracting labor compensation from value added and dividing by the value of sales. The data is obtained from the *STAN industrial database* provided by the OECD. The labor compensation is current price national accounts compatible labor costs which include wages as well as the costs of supplements such as employer's compulsory pension, medical payments, etc. The value added is national accounts compatible value added in current prices and represents the contribution of each industry to national GDP. The measure for the value of sales is national accounts compatible production (gross output) in current prices.

Table 1

	Food	Textile	Wood	Paper	Chemical	Mineral	Metal	Fabr metal
$W_t^C$	0.813	0.739	1.019	0.565	0.967	0.394	1.146	0.463
rr t	(0.000)	(0.000)	(0.012)	(0.231)	(0.000)	(0.000)	(0.000)	(0.000)
$W_t^{swe}$	0.561	0.370	0.292	0.122	0.237	0.553	0.679	0.505
· · · I	(0.062)	(0.115)	(0.562)	(0.847)	(0.505)	(0.000)	(0.069)	(0.000)
$R^2$	0.99	1.00	0.98	0.97	0.99	1.00	0.99	1.00
$W_t^C$	0.966	0.919	0.647	0.452	0.626	0.389	1.074	0.508
	(0.000)	(0.000)	(0.091)	(0.388)	(0.000)	(0.000)	(0.000)	(0.000)
$W_t^{swe}$	0.646	0.485	0.038	0.033	0.107	0.550	0.610	0.550
	(0.026)	(0.017)	(0.934)	(0.961)	(0.647)	(0.000)	(0.118)	(0.000)
$W_t^{oil}$	-0.068	-0.075	0.144	0.046	0.155	0.002	0.032	-0.025
	(0.059)	(0.003)	(0.025)	(0.596)	(0.000)	(0.882)	(0.501)	(0.044)
$R^2$	1.00	1.00	0.99	0.97	1.00	1.00	0.99	1.00

Specification:  $\ln p_t = C + t + t^2 + \delta_1 \ln Y_t + \delta_2 \ln W_t^C + \delta_3 \ln W_t^{swe} + \delta_4 \ln W_t^{oil} + \varepsilon_t$ Sample period: 1969-1994. Method of estimation: OLS

P-values in brackets. Notation: C: constant, t: time trend,  $t^2$ : square of time trend,  $p_t$ : log export price index,  $Y_t$ : log-export weighted production volume index,  $W_t^C$ : log double-weighted producer price index of foreign manufacturing industries,  $W_t^{swe}$ : log labor cost index for the Swedish private sector,  $W_t^{oil}$ : log oil price.

#### Table 2

Exports of eight Swedish manufacturing industries. Specification:  $\ln q_t = \eta \ln p_t + \beta \ln Y_t + \gamma \ln W_t^C + \varepsilon_t$ Sample period: 1969-1994. Method of estimation: GMM

	Food	Textile	Wood	Paper	Chemical	Mineral	Metal	Fabr metal
С	-7.284	2.799	-52.101	4.667	2.873	3.910	-5.490	6.568
	(0.014)	(0.272)	(0.445)	(0.944)	(0.568)	(0.092)	(0.006)	(0.254)
t	-0.136	0.033	-0.136	0.197	0.024	0.103	-0.085	0.152
	(0.003)	(0.286)	(0.619)	(0.901)	(0.704)	(0.010)	(0.002)	(0.065)
t <sup>2</sup>	0.002	-0.001	-0.005	-0.005	-0.001	-0.002	0.001	-0.002
	(0.005)	(0.225)	(0.552)	(0.881)	(0.613)	(0.001)	(0.034)	(0.019)
$p_t$	-2.680	-1.964	-5.820	-3.589	-1.170	-1.834	-0.407	-1.860
	(0.041)	(0.043)	(0.378)	(0.842)	(0.335)	(0.000)	(0.226)	(0.005)
$Y_t$	3.365	1.339	14.823	1.841	0.143	1.035	2.035	0.739
ŧ	(0.002)	(0.006)	(0.414)	(0.633)	(0.856)	(0.003)	(0.000)	(0.424)
$W_t^C$	3.208	1.650	6.679	2.632	2.048	1.461	1.082	1.042
,, t	(0.006)	(0.034)	(0.313)	(0.768)	(0.083)	(0.000)	(0.004)	(0.002)
$R^2$	0.90	0.82	0.37	0.36	0.99	0.97	0.96	0.96
Markups %	37	51	17	28	85	55	246	54

P-values in brackets. Notation: C: constant, t: time trend,  $t^2$ : square of time trend,  $p_t$ : log export price index,  $q_t$ : log export volume index,  $Y_t$ : log-export weighted production volume index,  $W_t^C$ : log double-weighted producer price index of foreign manufacturing industries. Instruments: log labor cost index for the Swedish private sector, C, t,  $t^2$ ,  $Y_t$ , and  $W_t^C$ .

	Food	Textile	Wood <sup>1,2</sup>	Paper <sup>1</sup>	Chemical	Mineral	Metal	Fabr
								metal
С	-7.284	2.799	-13.188	-5.425	4.808	3.910	-5.490	6.568
	(0.014)	(0.272)	(0.081)	(0.000)	(0.721)	(0.092)	(0.006)	(0.254)
t	-0.136	0.033	-3.231	-1.164	0.407	0.103	-0.085	0.152
	(0.003)	(0.286)	(0.007)	(0.006)	(0.892)	(0.010)	(0.002)	(0.065)
$t^2$	0.002	-0.001	0.001	0.000	0.000	-0.002	0.001	-0.002
	(0.005)	(0.225)	(0.540)	(0.080)	(0.998)	(0.001)	(0.034)	(0.019)
η	-2.680	-1.964	-1.160	-0.353	-2.794	-1.834	-0.407	-1.860
	(0.041)	(0.043)	(0.024)	(0.000)	(0.560)	(0.000)	(0.226)	(0.005)
β	3.365	1.339	3.873	2.097	0.951	1.035	2.035	0.739
	(0.002)	(0.006)	(0.041)	(0.000)	(0.039)	(0.003)	(0.000)	(0.424)
γ	3.208	1.650	2.189	0.806	3.067	1.461	1.082	1.042
	(0.006)	(0.034)	(0.000)	(0.000)	(0.395)	(0.000)	(0.004)	(0.002)
λ		· /	· /	0.887	0.851		Ì,	. ,
				(0.000)	(0.000)			
$\mathbb{R}^2$	0.90	0.82	0.85	0.97	0.98	0.97	0.96	0.96
Markups%	37	51	86	283	36	55	246	54

Exports of eight Swedish manufacturing industries.

Table 3

Specification:  $\ln q_t = \eta \ln p_t + \beta \ln Y_t + \gamma (\lambda \ln W_t^C + (1 - \lambda) \ln W_t^i) + \varepsilon_t$ 

P-values in brackets. Notation: C: constant, t: time trend,  $t^2$ : square of time trend,  $p_t$ : log export price index,  $q_t$ : log export volume index,  $Y_t$ : log-export weighted production volume index,  $W_t^C$ : log doubleweighted producer price index of foreign manufacturing industries,  $W_t^{34}$ : log pulpwood prices of spruce and fir, Northern New Hampshire, USA,  $W_t^{35}$ : log oil price. Instruments: log labor cost index for the Swedish private sector, C, t,  $t^2$ ,  $Y_t$ ,  $W_t^C$ ,  $W_t^i$ , see also footnotes.

<sup>1</sup> log Swedish pulpwood prices included as an instrument. <sup>2</sup> log Swedish sawlog prices included as an instrument.

Table 4
Exports of eight Swedish manufacturing industries.
Specification: $\ln q_i = n(\ln n_i - (\lambda \ln W_i^C + (1 - \lambda) \ln W^i)) + \beta \ln \gamma$

Specification:  $\ln q_t = \eta (\ln p_t - (\lambda \ln W_t^C + (1 - \lambda) \ln W_t^I)) + \beta \ln Y_t + \varepsilon_t$ 

Sample period: 1969-1994. Method of estimation: GMM

	Food	Textile	Wood <sup>1,2</sup>	Paper <sup>1</sup>	Chemical	Mineral	Metal	Fabr
				-				metal
С	-6.156	-0.187	-2.025	-1.802	7.772	-0.068	1.388	-1.084
	(0.172)	(0.890)	(0.739)	(0.067)	(0.001)	(0.932)	(0.174)	(0.527)
t	-0.105	0.007	0.030	0.028	0.041	0.037	0.025	0.029
	(0.098)	(0.719)	(0.155)	(0.000)	(0.090)	(0.000)	(0.014)	(0.010)
t <sup>2</sup>	0.001	-0.001	-0.001	0.000	0.000	-0.001	0.000	-0.001
	(0.131)	(0.098)	(0.095)	(0.001)	(0.553)	(0.000)	(0.078)	(0.000)
$\eta$	-3.924	-1.212	-1.127	-0.385	-3.843	-1.296	-0.040	-0.968
	(0.021)	(0.029)	(0.024)	(0.000)	(0.004)	(0.000)	(0.897)	(0.001)
β	3.895	1.571	1.956	1.532	0.886	1.545	0.772	1.693
	(0.015)	(0.000)	(0.206)	(0.000)	(0.090)	(0.000)	(0.006)	(0.001)
λ	. ,	, í		0.586	0.837	· /	. ,	, ,
				(0.000)	(0.000)			
$R^2$	0.81	0.87	0.81	0.96	0.97	0.98	0.92	0.96
Markups%	25	83	89	260	26	77	2489	103

P-values in brackets. Notation: C: constant, t: time trend,  $t^2$ : square of time trend,  $D_t$ : dummy variable for 1981 and later, pi: log export price index, qi: log export volume index, Yi: log-export weighted production volume index,  $W_t^C$ : log double-weighted producer price index of foreign manufacturing industries,  $W_t^{34}$ : log pulpwood prices of spruce and fir, Northern New Hampshire, USA,  $W_t^{35}$ : log oil price. Instruments: log labor cost index for the Swedish private sector, C, t,  $t^2$ ,  $Y_t$ ,  $W_t^C$ ,  $W_t^i$ , see also footnotes.

<sup>1</sup> log Swedish pulpwood prices included as an instrument.
 <sup>2</sup> log Swedish sawlog prices included as an instrument.

	Food	Textile	Wood <sup>1,2</sup>	Paper <sup>1</sup>	Chemical	Mineral	Metal	Fabr metal
С	0.807	8.063	4.788	3.484	11.271	17.149	1.486	10.089
	(0.741)	(0.050)	(0.573)	(0.131)	(0.237)	(0.000)	(0.531)	(0.002)
t	-0.055	0.018	0.039	0.116	0.100	0.268	-0.014	0.172
	(0.157)	(0.785)	(0.611)	(0.012)	(0.199)	(0.000)	(0.725)	(0.000)
t <sup>2</sup>	0.001	0.001	-0.001	-0.002	-0.001	-0.004	0.000	-0.002
	(0.085)	(0.499)	(0.306)	(0.026)	(0.303)	(0.000)	(0.996)	(0.000)
η	-2.959	-3.949	-1.066	-0.057	-2.217	-2.174	-0.856	-2.412
	(0.056)	(0.008)	(0.010)	(0.712)	(0.568)	(0.003)	(0.016)	(0.000)
β	1.889	1.112	0.268	0.226	-0.788	-1.504	0.842	0.143
-	(0.073)	(0.286)	(0.884)	(0.531)	(0.007)	(0.000)	(0.129)	(0.815)
γ	3.003	3.200	1.163	-0.268	2.316	1.000	1.093	1.481
	(0.012)	(0.015)	(0.053)	(0.086)	(0.439)	(0.007)	(0.034)	(0.000)
λ	. ,	· /	. /	2.426	0.878		Ì,	. ,
				(0.001)	(0.000)			
$R^2$	0.82	0.56	0.83	0.93	0.99	0.93	0.96	0.96
Markups %	34	25	94	1767	45	46	117	41

Table 5 Exports of eight Swedish manufacturing industries.

P-values in brackets. Notation: C: constant, t: time trend, t<sup>2</sup>: square of time trend,  $D_t$ : dummy variable for 1981 and later,  $p_t$ : log export price index,  $q_t$ : log export volume index,  $Y_t$ : log-export weighted production volume index,  $W_t^C$ : log double-weighted producer price index of foreign manufacturing industries,  $W_t^{34}$ : log pulpwood prices of spruce and fir, Northern New Hampshire, USA,  $W_t^{35}$ : log oil price. Instruments: log labor cost index for the Swedish private sector, C, t, t<sup>2</sup>,  $Y_t$ ,  $W_t^C$ ,  $W_t^i$ , see also footnotes.

<sup>1</sup> log Swedish pulpwood prices included as an instrument.

<sup>2</sup>log Swedish sawlog prices included as an instrument.

	Food	Textile	Wood <sup>1,2</sup>	Paper <sup>1</sup>	Chemical	Mineral	Metal	Fabr metal
С	-8.660	3.563	-9.903	-3.734	2.793	-0.088	-3.998	7.473
C	(0.055)	(0.247)	(0.000)	(0.108)	(0.052)	(0.971)	(0.364)	(0.128)
t	-0.105	0.078	-0.065	-0.041	-0.013	0.059	-0.102	0.168
-	(0.050)	(0.005)	(0.002)	(0.182)	(0.156)	(0.153)	(0.282)	(0.011)
t <sup>2</sup>	0.002	-0.002	0.000	0.000	0.001	-0.001	0.000	-0.004
	(0.023)	(0.027)	(0.747)	(0.468)	(0.000)	(0.488)	(0.789)	(0.044)
$\eta_0$	-1.014	-1.125	-0.837	-0.389	-1.613	-1.123	-1.722	-1.371
.70	(0.580)	(0.156)	(0.000)	(0.002)		(0.000)	(0.643)	(0.002)
$\eta_1$	-2.792	-0.528	-1.622	-0.204	-0.844	-1.733	1.909	0.934
.71	(0.303)	(0.351)	(0.000)	(0.384)		(0.015)	(0.734)	(0.527)
β	3.279	1.016	3.355	1.944	1.074	1.739	1.982	0.245
1-	(0.000)	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)	(0.005)	(0.801)
$\gamma_0$	1.369	0.600	1.370	0.652	1.804	0.791	2.532	0.699
7.0	(0.434)	(0.209)	(0.000)	(0.000)	(0.000)	(0.000)	(0.537)	(0.034)
$\gamma_1$	1.522	0.493	1.535	0.053	0.021	1.177	-0.939	0.354
71	(0.392)	(0.415)	(0.000)	(0.520)	(0.964)	(0.000)	(0.817)	(0.483)
λ	(****)	(	(00000)	1.056	0.783	(*****)	(*****)	(0000)
				(0.000)	(0.000)			
α	9.363	0.992	3.043	1.110	5.146	5.074	-6.720	-7.030
	(0.235)	(0.758)	(0.071)	(0.398)	(0.080)	(0.104)	(0.644)	(0.307)
$R^2$	0.93	0.91	0.94	0.98	1.00	0.98	0.88	0.98
Markups %	99	89	119	257	62	89	58	73
Markups %	26	60	41	169	41	35	-532	229

Table 6Exports of eight Swedish manufacturing industries.

Specification:  $\ln q_t = (\eta_0 + \eta_1 \cdot D_t) \ln p_t + \beta \ln Y_t + (\gamma_0 + \gamma_1 \cdot D_t) (\lambda \ln W_t^C + (1 - \lambda) \ln W_t^i) + \alpha \cdot D_t + \varepsilon_t$ Sample period: 1969-1994. Method of estimation: GMM

P-values in brackets. Notation: C: constant, t: time trend,  $t^2$ : square of time trend,  $D_t$ : dummy variable for 1982 and later,  $q_t$ : log export volume index,  $p_t$ : log export price index,  $Y_t$ : log-export weighted production volume index,  $W_t^C$ : log double-weighted producer price index of foreign manufacturing industries,  $W_t^{34}$ : log pulpwood prices of spruce and fir, Northern New Hampshire, USA,  $W_t^{35}$ : log oil price. Instruments:  $W_t^{Sw}$ : log labor cost index for Swedish private sector, C, t,  $t^2$ ,  $Y_t$ ,  $W_t^C$ ,  $W_t^i$ ,  $D_t$ ,  $D_t \cdot W_t^C$ : interaction between foreign labor cost and time,  $D_t \cdot W_t^i$ : interaction between industryspecific cost shifter and time, and  $D_t \cdot W_t^{Sw}$ : interaction between Swedish labor cost and time, see also footnotes.

<sup>1</sup> log Swedish pulpwood prices included as an instrument.

<sup>2</sup> log Swedish sawlog prices included as an instrument.

	Food	Textile	Wood <sup>1,2</sup>	Paper <sup>1</sup>	Chemical	Mineral	Metal	Fabr metal
С	-6.531	3.958	-14.641	-6.440	11.678	1.025	-6.193	7.473
	(0.006)	(0.150)	(0.000)	(0.000)	(0.009)	(0.599)	(0.016)	(0.128)
t	-0.124	0.062	-0.089	-0.059	0.011	0.064	-0.094	0.168
	(0.013)	(0.014)	(0.001)	(0.006)	(0.779)	(0.050)	(0.062)	(0.011)
$t^2$	0.003	0.000	0.000	0.001	0.002	-0.001	0.001	-0.004
	(0.040)	(0.833)	(0.963)	(0.000)	(0.006)	(0.015)	(0.413)	(0.044)
$\eta_0$	-2.579	-1.312	-1.270	-0.283	-5.064	-1.265	-0.404	-1.371
	(0.191)	(0.042)	(0.000)	(0.003)	(0.001)	(0.000)	(0.448)	(0.002)
$\eta_1$	-0.443	-1.567	-1.362	-0.210	-2.026	-1.365	-0.122	0.934
• 1	(0.792)	(0.000)	(0.000)	(0.365)	(0.442)	(0.003)	(0.844)	(0.527)
β	3.363	1.086	4.500	2.364	0.971	1.453	2.087	0.245
	(0.001)	(0.000)	(0.000)	(0.000)	(0.127)	(0.000)	(0.001)	(0.801)
$\gamma_0$	2.835	0.666	1.977	0.706	4.553	1.017	1.225	0.699
	(0.093)	(0.079)	(0.000)	(0.000)	(0.001)	(0.000)	(0.114)	(0.034)
$\gamma_1$	-0.453	0.444	1.310	-0.072	-0.981	1.126	0.312	0.354
- 1	(0.789)	(0.183)	(0.000)	(0.482)	(0.634)	(0.000)	(0.662)	(0.483)
λ				0.897	0.788			
				(0.000)	(0.000)			
α	4.660	7.255	2.292	1.545	16.580	3.075	-0.803	-7.030
	(0.400)	(0.028)	(0.083)	(0.207)	(0.031)	(0.055)	(0.478)	(0.307)
$R^2$	0.92	0.96	0.92	0.99	0.95	0.98	0.97	0.98
Markups %	39	76	79	353	20	79	247	73
Markups %	33	35	38	203	14	38	190	229

Table 7 Exports of eight Swedish manufacturing industries.

Specification:  $\ln q_t = (\eta_0 + \eta_1 \cdot D_t) \ln p_t + \beta \ln Y_t + (\gamma_0 + \gamma_1 \cdot D_t) (\lambda \ln W_t^C + (1 - \lambda) \ln W_t^i) + \alpha \cdot D_t + \varepsilon_t$ 

P-values in brackets. Notation: C: constant, t: time trend,  $t^2$ : square of time trend,  $D_t$ : estimated dummy variable for the various industries, 1981 and later for the metal industry, 1982 and later for the fabricated metal industry, 1985 and later for the wood industry, 1986 and later for the food industry, 1987 and later for the textile and minerals industries, 1988 and later for the paper and chemicals indutsries, qt: log export volume index, pt: log export price index, Yt: log-export weighted production volume index,  $W_t^C$ : log double-weighted producer price index of foreign manufacturing industries,  $W_t^{34}$ : log pulpwood prices of spruce and fir, Northern New Hampshire, USA,  $W_t^{35}$ : log oil price. Instruments:  $W_t^{Sw}$ : log labor cost index for the Swedish private sector, C, t, t<sup>2</sup>, Y<sub>t</sub>,  $W_t^C$ ,  $W_t^i$ ,  $D_t$  $D_t \cdot W_t^C$ : interaction between foreign labor cost and time,  $D_t \cdot W_t^i$ : interaction between industryspecific cost shifter and time, and  $D_t \cdot W_t^{Sw}$ : interaction between Swedish labor cost and time, see also footnotes. <sup>1</sup> log Swedish pulpwood prices included as an instrument.

<sup>2</sup>log Swedish sawlog prices included as an instrument.

Sample perio							2.5 - 1	- 1
	Food	Textile	Wood <sup>1,2</sup>	Paper <sup>1</sup>	Chemical	Mineral	Metal	Fabr
								metal
С	-1.143	7.240	-19.813	-4.807	-34.286	-1.956	45.544	6.672
	(0.770)	(0.324)	(0.022)	(0.036)	(0.234)	(0.393)	(0.896)	(0.204)
t	-3.756	-2.183	3.770	-2.691	44.608	4.456	8.630	2.411
	(0.455)	(0.743)	(0.101)	(0.083)	(0.211)	(0.001)	(0.877)	(0.336)
t <sup>2</sup>	-0.006	-0.006	0.004	-0.001	0.011	0.000	-0.075	-0.005
	(0.117)	(0.232)	(0.391)	(0.555)	(0.374)	(0.791)	(0.879)	(0.081)
$\eta_0$	-1.769	-1.835	-0.969	-0.343	8.405	-0.419	19.460	-0.818
	(0.380)	(0.271)	(0.024)	(0.050)	(0.171)	(0.172)	(0.878)	(0.107)
$\eta_1$	-1.610	0.256	-2.222	-0.066	-14.555	-2.237	-25.264	-1.153
• 1	(0.520)	(0.918)	(0.000)	(0.749)	(0.171)	(0.000)	(0.870)	(0.080)
β	2.643	0.949	4.948	2.230	2.408	1.664	-4.772	0.766
	(0.000)	(0.064)	(0.000)	(0.000)	(0.121)	(0.000)	(0.919)	(0.237)
$\gamma_0$	0.902	0.576	2.549	0.457	-4.708	0.479	-33.538	-0.446
. 0	(0.644)	(0.599)	(0.090)	(0.215)	(0.110)	(0.225)	(0.880)	(0.418)
$\gamma_1$	3.665	1.408	0.732	0.694	8.529	2.055	45.774	2.329
<i>·</i> 1	(0.176)	(0.392)	(0.681)	(0.286)	(0.084)	(0.002)	(0.874)	(0.017)
				0.899	0.537			
				(0.000)	(0.000)			
R <sup>2</sup>	0.92	0.87	0.90	0.97	0.94	0.98	0.07	0.97
Markups %	57	55	103	291	-12	238	-5	122
Markups %	30	63	32	246	16	39	21	52

Table 8Exports of eight Swedish manufacturing industries.

Specification:  $\ln q_t = t + (\eta_0 + \eta_1 \cdot t) \ln p_t + \beta \ln Y_t + (\gamma_0 + \gamma_1 \cdot t)(\lambda \ln W_t^C + (1 - \lambda) \ln W_t^i) + \varepsilon_t$ Sample period: 1969-1994. Method of estimation: GMM

P-values in brackets. Notation: C: constant, t: time trend,  $t^2$ : square of time trend,  $q_i$ : log export volume index,  $p_t$ : log export price index,  $Y_t$ : log-export weighted production volume index,  $W_t^C$ : log doubleweighted producer price index of foreign manufacturing industries,  $W_t^{34}$ : log pulpwood prices of spruce and fir, Northern New Hampshire, USA,  $W_t^{35}$ : log oil price. Instruments:  $W_t^{Sw}$ : log labor cost index for the Swedish private sector, C, t,  $t^2$ ,  $Y_t$ ,  $W_t^C$ ,  $W_t^i$ ,  $t \cdot W_t^C$ : interaction between foreign labor cost and time,  $t \cdot W_t^i$ : interaction between industry-specific cost shifter and time, and  $t \cdot W_t^{Sw}$ : interaction between Swedish labor cost and time, see also footnotes.

<sup>2</sup> log Swedish sawlog prices included as an instrument.

Comparison of methods of deriving the Lerner index  $L = \frac{p - MC}{P}$ 

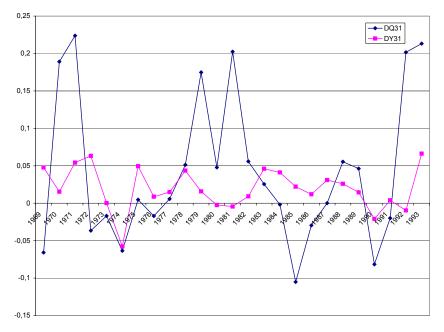
					Γ			
Method	Food	Textile	Wood	Paper	Chemical	Mineral	Metal	Fabr metal
Residual demand	0.37*	0.51*	0.86*	2.83**	0.36	0.55**	2.46	0.54**
Accounting markups	0.07	0.07	0.11	0.12	0.15	0.08	0.05	0.08
Solow-Hall	0.20*	0.29**	0.16**	0.08	-0.01	0.00	0.05**	0.16
Residual	0.99	0.89	1.19**	2.57**	0.62**	0.89**	0.58	0.73*
demand	0.26	0.60	0.41**	1.69	0.41	0.35**	-5.32	2.29
Accounting	0.04	0.05	0.11	0.12	0.13	0.04	0.03	0.06
markups	0.10	0.10	0.10	0.12	0.16	0.13	0.07	0.09

\* and \*\* indicate significance on the 5% and 1% level, respectively. H(0): Solow-Hall approach: RTS=1, Residual demand:  $\eta = 0$ .



Figure 1 Food, beverages, and tobacco, changes from previous year, log

Notation: DEP: Swedish export price, DWF: Foreign producer prices.



Notation: DQ: Swedish export volume, DY: Foreign demand shifter.

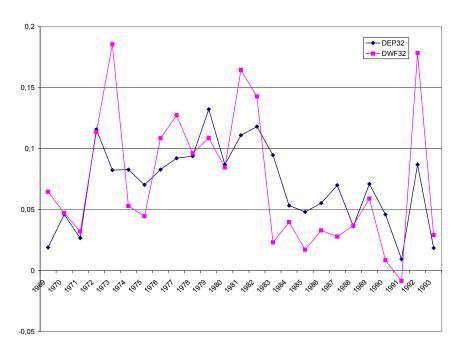
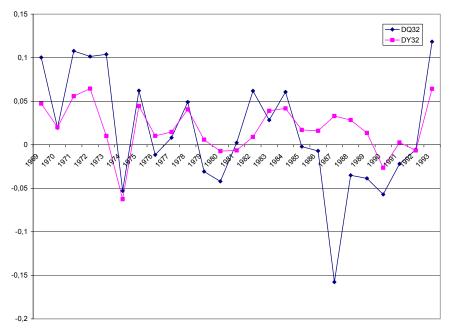


Figure 2 Textiles, changes from previous year, log

Notation: DEP: Swedish export price, DWF: Foreign producer prices.



Notation: DQ: Swedish export volume, DY: Foreign demand shifter.

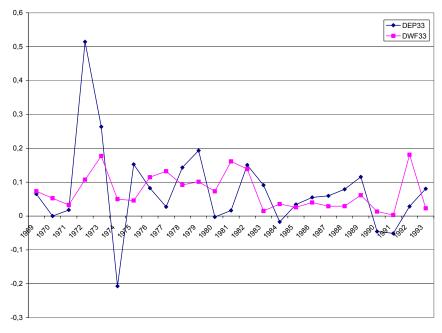
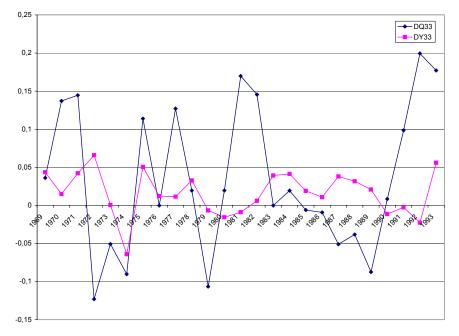


Figure 3 Wood, changes from previous year, log

Notation: DEP: Swedish export price, DWF: Foreign producer prices.



Notation: DQ: Swedish export volume, DY: Foreign demand shifter.

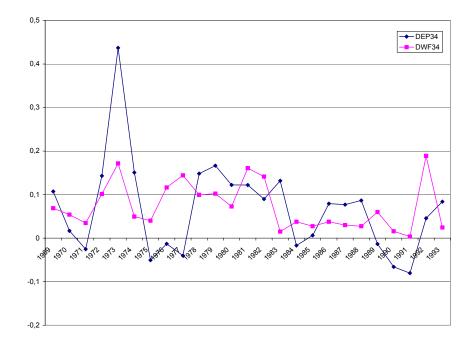
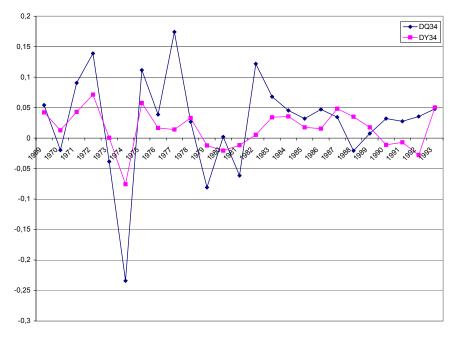
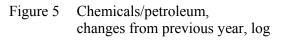


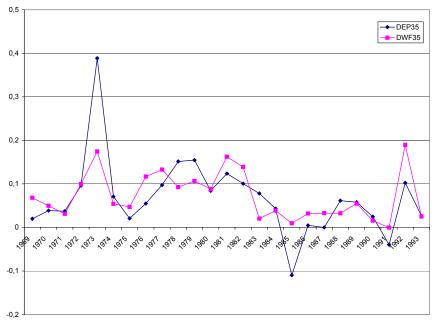
Figure 4 Paper, printing and publishing, changes from previous year, log

Notation: DEP: Swedish export price, DWF: Foreign producer prices.

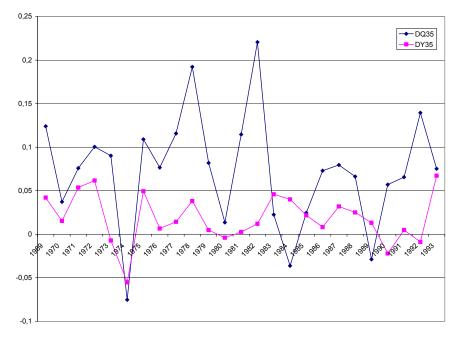


Notation: DQ: Swedish export volume, DY: Foreign demand shifter.





Notation: DEP: Swedish export price, DWF: Foreign producer prices.

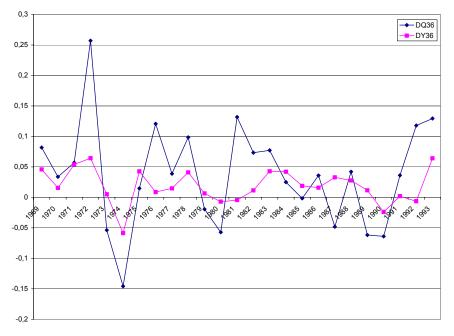


Notation: DQ: Swedish export volume, DY: Foreign demand shifter.



Figure 6 Non-metallic mineral products, changes from previous year, log

Notation: DEP: Swedish export price, DWF: Foreign producer prices.



Notation: DQ: Swedish export volume, DY: Foreign demand shifter.

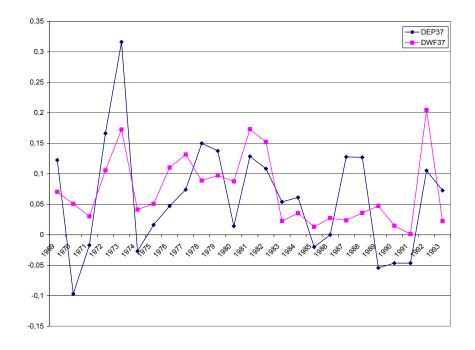
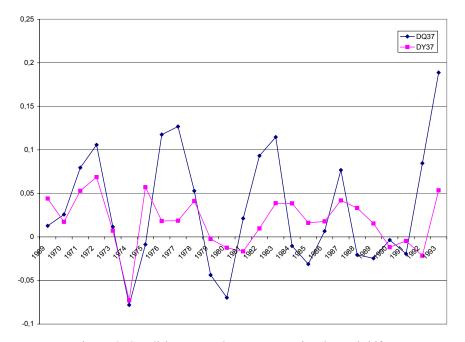


Figure 7 Primary metals, changes from previous year, log

Notation: DEP: Swedish export price, DWF: Foreign producer prices.

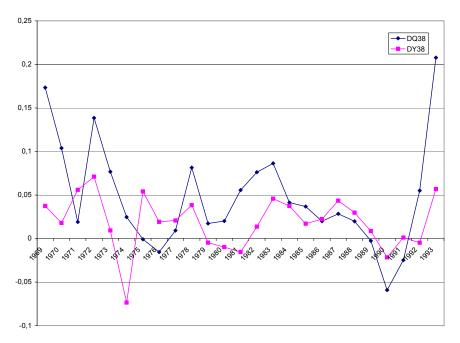


Notation: DQ: Swedish export volume, DY: Foreign demand shifter.



Figure 8 Fabricated metals, changes from previous year, log

Notation: DEP: Swedish export price, DWF: Foreign producer prices.



Notation: DQ: Swedish export volume, DY: Foreign demand shifter.

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