ESSAYS ON DYNAMIC MACROECONOMICS

by

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ABSTRACT

This thesis consists of three independent papers in macroeconomics.

Why do Europeans Work so Little? concerns labor supply. Market work per person is roughly 10 percent higher in the U.S. than in Sweden. However, if we include the work carried out in home production, the total amount of work only differs by 1 percent. I set up a model with home production, and show that differences in policy - mainly taxes - can account for the discrepancy in labor supply between Sweden and the U.S. Moreover, even though the elasticity of labor supply is rather low for individual households, labor taxes are estimated to be associated with considerable output losses. I also show that policy can account for the falling trend in market work in Sweden since 1960. The largest reduction occurs from 1960 until around 1980, both in the model and the data. This is also the period when taxes were increased the most. After the early 1980s, the trends for both taxes and actual hours worked are basically flat. This is also true for hours worked in the model.

Social Security and the Equity Premium Puzzle shows that social security may be an important factor in explaining the equity premium puzzle. In the absence of shortselling constraints, the young shortsell bonds to the middle-aged and buy equity. Social security reduces the bond demand of the middle-aged, thereby restricting the possibilities of the young to finance their equity purchases. They demand less equity and the average return to equity goes up. Social security also increases the covariance between future consumption and the equity income of the young. The effect on the equity premium is substantial. In fact, a model with social security and borrowing constraints can generate a fairly realistic equity premium.

The Welfare Gains of Improving Risk Sharing in Social Security shows that improved intergenerational risk sharing in social security may imply very large welfare gains, amounting to up to 15 percent of the per-period consumption relative to the current U.S. consumption. Improved risk sharing raises welfare through a direct effect, i.e., by correcting an initially inefficient allocation of risk, and through a general equilibrium (GE) effect. The GE effect is due to the fact that the allocation of risk in the pay-as-you-go system influences the demand for capital. As a result, with an efficient risk sharing arrangement, the crowding out effect associated with an unfunded system can actually be completely eliminated. Efficient risk sharing

in social security implies highly volatile and pro-cyclical benefits, i.e., that retirees' exposure to productivity risk is increased.

ACKNOWLEDGMENTS

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Chapter 1

Introduction

As the title suggests, this thesis comprises three essays on dynamic macroeconomics. Even though the individual topics differ across these papers, they also have much in common. They all feature a strong micro foundation, a computational approach and the intention to explain some specific macroeconomic regularities. The three papers also deliver rather clear results within their respective fields by suggesting that taxes are important for explaining cross-country heterogeneity in labor supply, that social security is an important factor in explaining the equity premium puzzle and that gains from improving intergenerational risk sharing are considerable.

Chapter 2 is concerned with the question of whether policy can explain the large observed difference in labor supply across countries. I concentrate on Sweden and the U.S. and start out with the observations that market work per person is roughly 10 percent (or more than 120 hours) higher in the U.S. than in Sweden. However, when the amount of work carried out in home production is included, the total amount of work seems to differ by only 1 percent.

It is obviously a fact that households consume a large amount of services which can either be bought or produced in home production, examples of which are child care, cleaning, shopping, and preparing dinner, but also repairs of household goods and vehicles. A household's choice between buying and producing these services will generally depend on the price of market-produced services, and its productivity in market work relative to its productivity in home production. I set up a general equilibrium model capturing these features to show that policy can very well account

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for the difference in labor supply between Sweden and the U.S. The distortion comes from taxes in general, and labor taxes in particular. I also show that policy can account for the falling trend in market work in Sweden since 1960. The largest reduction occurs from 1960 until around 1980, both in the model and in the data. This is also the period when taxes were increased the most. After the early 1980s, the trends for both taxes and hours worked are basically flat. This is also true for hours worked in the model. However, the model clearly overestimates the number of hours worked between 1960 and 1975. More research is therefore needed to explain this discrepancy.

Chapter 3, analyzes the effect of social security on asset prices in general and the equity premium in particular. The potential important influence of social security is mainly due to the fact that social security is the major source of income for a majority of the aged beneficiaries in the U.S. In addition, social security is a non-traded asset containing several elements of risk, thereby affecting the financial behavior of both taxpayers and retirees.

A simple overlapping generations model is used to show that social security unambiguously increases the equity premium. The effect is substantial: the premium can be roughly 85 percent higher with than without social security. In fact, a model with social security and borrowing constraints can generate a fairly realistic equity premium.

The exact effect of social security on the equity premium depends to a large extent on whether agents are subject to shortselling constraints. In the absence of such constraints, social security influences the equity premium by changing the relative demand for equity. When agents do not receive social security, the young shortsell bonds and buy the bulk of the equity stock. Their marginal valuation of equity is therefore decisive in the pricing process, and since they value equity highly, the return is low. The middle-aged mainly hold bonds, but the difference in expected return between the two assets is small. When agents do receive social security, the need to save for retirement is reduced. In addition, since social security is a relatively safe asset, it reduces the motive to hold bonds for retirement. Consequently, middle-aged agents' demand for bonds goes down. The lower demand for bonds effectively restricts the possibilities of the young to finance their equity purchases by shortselling

bonds, therefore, they demand less equity and the price of equity goes down (and the return goes up).

It is also shown that social security increases the covariance between future consumption and equity income for the young. In fact, this is the main influence of social security on the equity premium in the presence of shortselling constraints. The higher covariance is first of all due to the arrangement with defined benefits featuring contra-cyclical taxes, implying that taxes are higher when wage income is low, and vice versa. Since equity and wage income are highly correlated, the young find equity to be a worse hedge in economies with social security, since it does not pay off when really needed. Their marginal valuation of equity will therefore be lower and the result is a lower equilibrium price and a higher average return.

Chapter 4 finally, deals with the problem of how aggregate productivity risk should be allocated between taxpayers and retirees in a pay-as-you-go (PAYGO) social security system. The question is motivated by the fact that overlapping generations (OLG) models are known to be inefficient in an *ex ante* sense, stemming from the inability of the unborn to insure themselves. In fact, with standard preferences and technology, the laissez-faire allocation of risk is inefficient by imposing too little productivity risk on retirees and too much on future generations. However, a PAYGO system has the potential of correcting these inefficiencies, due to the immanent intergenerational link found by current pension payments being immediately transferred to retirees. More precisely, productivity risk can be transferred between taxpayers and retirees through benefits being allowed to respond to macroeconomic shocks. When the economy is hit by a productivity shock, the government can keep the social security budget in balance, by adjusting benefits or contributions (or, naturally, by a combination thereof). In the former case, the exposure to productivity risk is amplified for retirees and, in the latter case, for taxpayers. Using the PAYGO system to transfer productivity risk between these two groups is thus straightforward.

The findings are that improved risk sharing in social security may imply very large welfare gains. In addition, efficient risk sharing features highly volatile and pro-cyclical benefits and tax rates. The pro-cyclical arrangement eases the risk exposure of the young, since they are subject to lower taxes in bad states and vice

versa. In this way, social security provides some insurance to the uninsured young. Highly volatile benefits are motivated by the fact that the allocation of risk in the PAYGO system has a major influence on demand for capital. More specifically, highly volatile benefits strengthen the precautionary savings motive of the middle-aged agents. As a result, they save more and hold more capital to hedge their coming volatile benefits. In fact, in the absence of shortselling constraints, the crowding out effect generally associated with an unfunded system can be completely eliminated by the use of efficient risk sharing arrangements. An intuition for the findings in this paper is that the welfare cost of exposing the old to aggregate risk is relatively small, as long as the risk is predictable and known in advance, since they have had a lifetime for setting up their hedging portfolios. In contrast, the earlier in life agents are exposed to risk, the smaller are their possibilities to hedge.

Chapter 2

Why do Europeans Work so Little?[*]

1   Introduction

International comparisons highlight quite large differences in the extent of market work across OECD countries. Fig. 2.1 below displays the average annual work hours across a subset of countries. I only find European countries at the lower and non-European countries at the higher end.[1] The difference between these ends is larger than 400 hours per year, that is, on average, people in New Zealand work 43 percent more than workers in Italy.[2]

Theoretical explanations for these differences could, for instance, be cross-country

[1] The data is for 2002 and is taken from the OECD Labor database. There are basically two reasons why I am focusing on people aged between 20-64. First of all, this is the population examined in the Swedish time use surveys. Second, the conflict of how to allocate the total amount of work time between market work and domestic work seems most relevant for people of these ages.

[2] A natural objection to the above data may be that these differences could be driven by the quite low level of employment (especially for women) in some European countries. However, the picture looks pretty much the same when we look at the average annual hours per employee. The European countries are once more found at the lower end, and the non-European countries at the higher end.
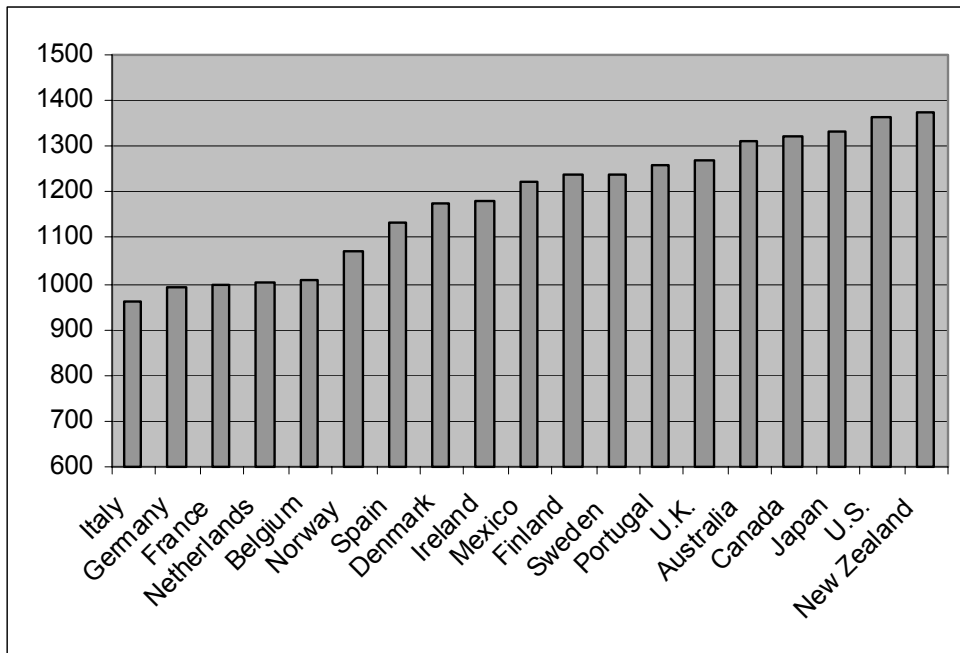
Figure 2.1: AVERAGE ANNUAL HOURS OF MARKET WORK PER PERSON (20-64)

heterogeneity in preferences, labor taxes, benefits or any other factor influencing the incentives to supply labor. Matching the data with assumptions of heterogeneity in preferences is probably quite easy. This is worth noting, but too obvious to really be of interest. More challenging are questions about actual policy such as that dealt with in this paper: Could the differences in Fig.2.1 have been generated by cross-country heterogeneity in policy? Policy will be more explicitly defined below, but taxes will be the main policy instrument of analysis and I will concentrate on Sweden and the U.S. The reason for focusing on these two countries is that, to some extent, they represent polar extremes. Among the OECD countries, the U.S. has the second highest level of market work per person, in combination with a very low tax burden.[3] Sweden, on the other hand, has the highest tax burden and a much lower level of market work.[4]

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[3] By tax burden I mean total tax revenue as a share of GDP (OECD Revenue statistics 2003). For the OECD countries, only Japan, Mexico and Korea have a lower tax burden than the U.S.

[4] Still, Sweden is at the upper end among the European countries, indicating that taxes alone

The problem in explaining differences in labor supply by cross-country hetero-geneity in labor taxes, is that the effect of labor taxation in standard models may generally be ambiguous. In particular, income and substitution effects often work in opposite directions, making the total effect sensitive to the chosen calibration. Moreover, as shown later in this paper, for a calibration of the model consistent with micro data, the net effect will generally not be sufficiently large to account for differences in labor supply of the magnitude displayed in Fig.2.1.

This paper takes into account the fact that households carry out a substantial amount of work outside their workplaces, mainly in home production. In Sweden, a working household devotes on average 44 percent of its total working time (consisting of market work and home production) to home production. The relation between the average annual hours of paid and unpaid work in Sweden and the U.S. is displayed in Fig. 2.2 below:

Market work per person is roughly 10 percent (more than 120 hours) higher in the U.S. than in Sweden, corresponding to 3 weeks of full time work per person and year, which is a substantial number. However, what is also displayed in Fig. 2.2 is that average total work only differs by 1 percent (or less than three quarters of a week of full time work).[5] The difference seems to be that Swedes allocate a larger share of their total work effort to home production.

Examples of services produced in home production are child care, cleaning, shop-ping, preparing dinner but also repairs of household goods and vehicles. In most cases, households also have the option of buying these services on the market. A

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cannot explain the differences. It has been pointed out that, besides taxes, the way the government uses its revenue may actually have a strong influence on the level of labor supply (see Lindbeck (1982) and Rogerson (2003b)). A feature of the model introduced in the next section is that labor supply is increasing in government consumption. As a result, the model will also, at least qualitatively, be consistent with Sweden's relatively high level of labor supply.

[5] The data on market work is for 2002, and is once more taken from the OECD Labor database. The data on home production in Sweden is based on the estimate that Swedish households devote, on average, 44 percent of their total working time to domestic work. This relation has actually been quite robust over time (Juster & Staford (1991) and Tid för Vardagsliv. Kvinnors och mäns tidsanvändning 1990/91 och 2000/2001 (2003)). For the U.S., the data on home production is taken from Juster & Staford (1991), wherein it is reported that Americans devote, on average, 39.4 percent of their total working time to domestic work. The Swedish data in Juster & Staford (1991) is claimed to match the U.S. classification scheme. However, the numbers must be treated with some caution. The reason is that the data is generally for men and women aged between 20-64, except for the data in Juster & Staford (1991), which is for men and women aged 25-64.
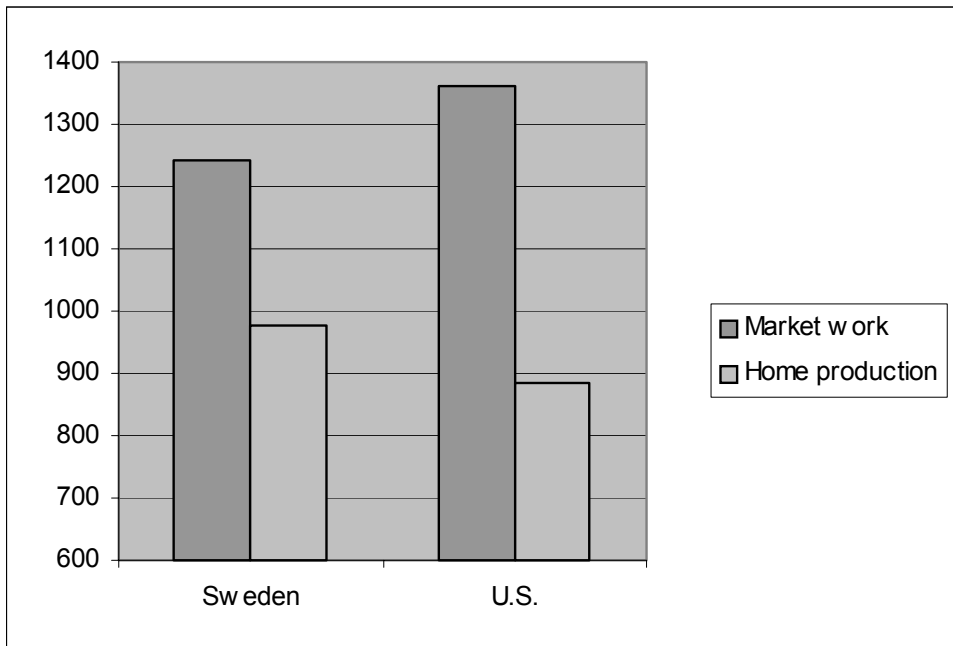
Figure 2.2: ANNUAL HOURS OF PAID AND UNPAID WORK

household's choice between buying and producing services will therefore generally depend on the price of market-produced services, and its productivity in market work relative to its productivity in home production. Taxes may distort this choice in two significant ways.[6]   First, service taxes (VAT) raise the price of market-produced services, making them a less attractive deal. Second, labor taxes distort the household's time allocation choice between working in the market and in home production. Basically, labor taxes reduce the return to market work and increase the return to home production in relative terms. As a result, households subject to labor taxes may start allocating their time to home production instead of market work, even though their productivity may be lower in the former activity than in the latter. Their allocation of total work between market work and home production will no longer be governed by the law of comparative advantage but - in the words

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[6] By now, it is quite well known that taxes may be somewhat more distortionary in the presence of home production; see, for example, Lindbeck (1981), McGrattan et al (1993), Pålsson (1997), Henrekson (1998) and Davis & Henrekson (2001).

of Davis & Henrekson (2001) - by the tax distorted law of comparative advantage.

In addition to taxes, I analyze the implications of the variance in the wage distribution for aggregate labor supply. This variance may be expected to affect the demand for market-produced services, since compressed wages may impel high-productivity households to substitute market-produced for home-produced services. As a result, market work will decrease.

Naturally, a legitimate question is whether we should actually care about cross-country differences in the hours worked? My answer is that it is important to understand whether the observed numbers could have been generated by economic incentives. GDP per capita was 31 percent higher per person aged 20-64 in the U.S. than in Sweden in the year 2002.[7] There certainly exists a need to evaluate to what extent this difference can be accounted for by the fact that Swedes spend more time in home production where, on average, they are less productive.

To test the hypothesis that differences in labor supply are driven by cross-country heterogeneity in policy, I set up a dynamic general equilibrium model consisting of many households with three arguments in their utility function: consumption goods, services and leisure. Services may be bought at the market, or produced in home production.[8] Households are assumed to be heterogenous in labor productivity and, on average, their productivity will be higher in market work than in home production. The assumption of heterogeneity allows me to analyze the effects of wage dispersion. Households in the model must thus allocate their income between consumption goods and market-produced services, and their time between market work, home production and leisure.

The model is calibrated to match a number of Swedish features, such as time spent in market work and home production. A key parameter here is the Frisch elasticity of labor supply, calibrated to be less than 0.5 in accordance with empirical estimates.[9] Another important parameter in the model, governing a household's

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[7] Data on PPP adjusted GDP from the OECD Statistical Database.

[8] This assumption separates this paper somewhat from earlier macro research on home production. See, for instance, McGrattan (1993); Greenwood et al (1994); and Klein and Jonsson (2003). An exception is Rios-Rull (1993).

[9] The Frisch is defined as the elasticity of labor supply with respect to wage, holding the marginal utility of consumption constant. The standard assumption in the literature on the effect of taxes on labor supply, is log utility in leisure. This implies that the Frisch elasticity roughly ranges

choice between market-produced and home-produced services, is the elasticity of substitution between these two objects. To estimate this elasticity, I use Swedish data on spending and estimate the fraction of total expenditures of Swedish households on market-produced services as a function of their disposable income. The elasticity is found to be 1.33, implying market-produced and home-produced services to be close to perfect complements.[10] This low elasticity supports the widespread notion among Swedes that even high-productivity households choose to engage a great deal in home production, instead of buying market-produced services. Policy parameters, such as taxes, wage dispersion and government consumption are set to their Swedish 2002 estimates. Finally, the policy parameters are changed to U.S. data to explore whether the model matches U.S. hours in market work and home production.

It is found that the model matches time spent in both market work and in home production, very well. The model predicts the GDP per capita to be 12 percent higher in the U.S. than in Sweden as a result of different policies. These results show that Swedish policy heavily influences the choice between home production and market work. The distortion comes from taxes in general, and labor taxes in particular. Even though the Frisch elasticity of labor supply is rather low in the model, labor taxes generate a substantial output loss. In fact, the model predicts that the Swedish GDP per capita would increase by almost 27 percent as a result of changing to U.S. tax rates (i.e, while holding other policy parameters constant). This is a huge number, roughly equal to 6800 dollars/person (denominated in 2001 prices).

The value added tax on services is found to be the least distortionary among the set of taxes analyzed in this paper. Reducing service taxes in Sweden from the present level of 25 to 0 percent would, according to the model, only slightly increase output (by 3 percent). This mild distortion is due to the low elasticity of substitution between market-produced and home-produced services.

A higher degree of wage dispersion is found to decrease market work, increase home production and slightly reduce output. The reason is that a higher dispersion

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between 1 and 4. This may be true for the aggregate, but at the micro level, this elasticity is estimated to be in the interval $[0, 0.5]$. See Browning et al (1999) for a survey.

[10] Which they would be if the elasticity were 1.

enhances the incentive to supply labor for high-productivity households, while it is reduced for low-productivity households. The net effect is negative, but the fall in GDP per capita stemming from the smaller work effort of low-productivity households, is almost completely offset by the increased work effort of high-productivity households.

The model is also consistent with the falling trend in hours worked since 1960. The largest reduction occurs from 1960 until around 1980, both in the model and in the data. This is also the period when taxes where increased the most. After the early 1980s, the trends for both taxes and hours worked are basically flat. This is also true for hours worked in the model. However, the model clearly overestimates the number of hours worked between 1960 and 1975.

Several recent related studies are analyzing the causes of labor supply, but they all differ from this paper in several important aspects. For instance Davis and Henrekson (2001), do not have a quanatitative approach. Neither Prescott (2003) nor Rogerson (2003a, b) consider heterogeneity in labor productivity (which may be important for the way an economy allocates time across market and nonmarket activities). Finally, Nickell (2003) concentrates on empirical findings concerning the effects of taxes and labor market institutions on the labor supply. Maybe most important, this paper is one of the first to incorporate an elasticity of labor supply consistent with micro estimates.

2 THE MODEL

2.1 CONSUMERS

Consider an economy with a continuum of infinitely lived households. The instantaneous utility for household i in period t is specified by

$$u\left(c_{it}, s_{it}, l_{it}\right) = \frac{\left(c_{it}^{\eta} s_{it}^{1-\eta}\right)^{1-\gamma}}{1-\gamma} + \varphi \frac{l_{it}^{1-\mu}}{1-\mu}, \tag{2.1}$$

where c_{it} is consumption goods, s_{it} services and l_{it} leisure.[11] Households can either buy services at the market or produce them themselves at home, i.e. in home production. For simplicity, the production function for services is assumed to be a linear function of efficiency hours $\epsilon_{hp}h_{it,hp}$ of work only.

$$f^S \left(\epsilon_{hp} h_{it,hp} \right) = \epsilon_{hp} h_{it,hp}, \tag{2.2}$$

where $h_{it,hp}$ is the time household i devotes to home production and ϵ_{hp} the productivity in that activity. I assume all households to have the same constant productivity in home production, i.e. that $\epsilon_{hp} = \bar{\epsilon}_{hp}$. Services are thus an aggregate of market-produced services $s_{it,m}$, and home-produced services $s_{it,hp}$:

$$s_{it} = \left[a \left(s_{it,m} \right)^b + \left(1 - a \right) \left(s_{it,hp} \right)^b \right]^{\frac{1}{b}}. \tag{2.3}$$

Households supply labor to the labor market, consisting of the goods market and the service sector. We may consider this economy as one where households work some hours in the goods market and some in the service sector, so that the total amount of labor supply h_{it}, equals $h_{it,g} + h_{it,s}$.

I abstract from earnings uncertainty by assuming each household to be endowed with a constant, but heterogenous level of labor productivity $\epsilon_i = e^{\xi_i}$, where ξ is a permanent component assumed to be normally distributed with the mean zero and the variance σ_ξ^2. Each household also has one unit of productive time per week, which must be divided between labor, home production and leisure

$$h_{it} + h_{it,hp} + l_{it} = 1. \tag{2.4}$$

For households to be indifferent between working in the goods sector and the service sector, the wage rate must be the same in those two sectors, i.e. $w_{t,g} = w_{t,s} = w_t$. Denoting taxes on consumption, services and labor income by θ_t^C, θ_t^S and θ_t^I, respectively, the budget constraint of household i is

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[11] As a starting point, I have assumed a Cobb-Douglas function for consumption goods and services. Naturally, this implies that they are perfect complements.

$$\left(1+\theta_t^C\right)c_{it} + \left(1+\theta_t^S\right)p_t s_{it,m} + a_{it+1} = \epsilon_i w_t h_{it}\left(1-\theta_t^I\right) + \left(1+r_t\right)a_{it} + \psi_t, \quad (2.5)$$

where a_{it+1} is the assets the household chooses to hold for the next period, p_t the market price of services, r_t the interest rate, w_t the wage rate in period t and ψ_t a transfer from the government.[12] Finally, it will be assumed that households cannot short-sell any assets[13]

$$a_{it+1} \geq 0, h_{it} \geq 0, s_{it,m} \geq 0, h_{it,hp} \geq 0. \quad (2.6)$$

Let β denote the subjective discount factor. The households' maximization problem is then to maximize their expected life-time utility,

$$U_0 = E_t \sum_{t=0}^{\infty} \beta^t u\left(c_{it}, s_{it}, l_{it}\right),$$

subject to (2.4), (2.5) and (2.6).

Finally, let $\chi(a, \xi)$ be the measure of households, and normalize the mass of households to unity.

2.2 FIRMS

The production side of the economy consists of goods and services. There is no aggregate uncertainty in the economy. Let K_t denote the period t aggregate capital stock, and $H_{t,g}$ the aggregate labor supply in efficiency units in the goods market, i.e. $H_{t,g} = \int \epsilon(\xi) h_g(a, \xi) d\chi$. The aggregate production of consumption goods is assumed to be given by

$$F\left(K_t, H_{t,g}\right) = K_t^\alpha H_{t,g}^{1-\alpha}. \quad (2.7)$$

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[12] Due to the fact that households do not face any uncertainty, the distribution of wealth will depend on the initial wealth distribution. Chatterjee (1994) shows that in this type of model, the main properties of the wealth distribution are self-perpetuating and that people do not move from one economic level to another. That is, if all households hold the same wealth today, they will also hold the same wealth tomorrow.

[13] It is not important that the shortselling restriction for asset holdings binds at zero.

Moreover, let $H_{t,s}$ be the aggregate labor supply in efficiency units in the service sector, i.e. $H_{t,s} = \int \epsilon(\xi) h_s(a,\xi) d\chi$. According to (2.2), the aggregate production of market-produced services is then given by $H_{t,s}$.

Firms are assumed to maximize their profit, while taking prices as given. Moreover, in a given period, firms must pay the period t payroll tax, θ_t^P.

2.3   THE GOVERNMENT

Let C_t denote the period t aggregate consumption of goods and $S_{t,m}$ the aggregate demand for market-produced services, i.e. $C_t = \int c(a,\xi) d\chi$, and $S_{t,m} = \int s_m(a,\xi) d\chi$. The period t income of the government is then

$$\left(\theta_t^I + \theta_t^P\right) w_t H_t + \theta_t^C C_t + \theta_t^S p_t S_{t,m}. \tag{2.8}$$

The government consumes a portion, G_t, and then makes a lump-sum transfer, Ψ_t, back to all households. For simplicity, government consumption is assumed to be useless. Moreover, the government is required to balance its budget in each period:[14]

$$\left(\theta_t^I + \theta_t^P\right) w_t H_t + \theta_t^C C_t + \theta_t^S p_t S_{t,m} - G_t = \Psi_t. \tag{2.9}$$

It has been pointed out that, besides taxes, the way the government uses its revenue may actually have a strong influence on the level of labor supply.[15] For instance, the relatively high level of labor supply in the Scandinavian countries, may to some extent be due to the fact that Scandinavian governments spend a relatively larger fraction of their money on child care (which is likely to stimulate labor supply). This mechanism will actually be present in the model, in an indirect way. Notice from (2.9) that benefits are decreasing in government consumption (i.e., holding tax revenues constant). Lower benefits will then increase the incentive to supply labor. In this way, the model will, at least qualitatively, be consistent with Sweden's relatively high level of labor supply among the European countries.

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[14] This is a natural benchmark, since the analysis will mainly concentrate on steady states.

[15] See Lindbeck (1982) and Rogerson (2003b).

3 EQUILIBRIUM

Denote the vector of taxes by $\Theta = \left[\theta^C, \theta^S, \theta^I, \theta^P\right]$. An equilibrium for this economy is then given by (i) decision rules for agents' asset holdings $a_{it+1} = \widehat{a}'(a_{i,t}, \xi_i; p, r, w, \Theta, \Psi)$, demand for market-produced services $s_{it,m} = s\left(a_{i,t}, \xi_i; p, r, w, \Theta, \Psi\right)$, hours of work in the service sector $h_{it,s} = h_s\left(a_{i,t}, \xi_i; p, r, w, \Theta, \Psi\right)$, hours of work in the goods market $h_{it,g} = h_g\left(a_{i,t}, \xi_i; p, r, w, \Theta, \Psi\right)$; (ii) aggregate values for asset holdings $A(p, r, w, \Theta, \Psi) = \int \widehat{a}'\left(a_{i,t}, \xi\right) d\chi$, the demand for market-produced services $S_m\left(p, r, w, \Theta, \Psi\right) = \int s\left(a_{i,t}, \xi\right) d\chi$, the level of efficiency units of labor supplied to the service sector $H_s\left(p, r, w, \Theta, \Psi\right) = \int \epsilon\left(\xi\right) h_s\left(a_{i,t}, \xi\right) d\chi$ and the goods sector $H_g\left(p, r, w, \Theta, \Psi\right) = \int \epsilon\left(\xi\right) h_g\left(a_{i,t}, \xi\right) d\chi$ such that the following conditions hold:

- The decision rules solve households' maximization problem

- Households are indifferent between working in the goods market and the service sector, i.e.

$$w_s = w_g = w. \tag{2.10}$$

- Factor markets clear

$$r = F_K\left(K, H_g\right) - \delta \tag{2.11}$$

$$w_{t,g} = F_{H_g}\left(K, H_g\right) \tag{2.12}$$

$$p = w, \tag{2.13}$$

where (2.13) can be obtained from the fact that profit in the service sector must be zero.

- Tax revenues equal government expenses, i.e. equation (2.9) holds

- The aggregate supply of savings is equal to firms' demand for capital,

$$A\left(p, r, w, \Theta, \Psi\right) = K\left(p, r, w, \Theta, \Psi\right). \tag{2.14}$$

- The demand for services equals the supply

$$S_m\left(p, r, w, \Theta, \Psi\right) = H_S\left(p, r, w, \Theta, \Psi\right). \tag{2.15}$$

Equation (2.13) can easily be derived from the profit maximization problem faced by firms in the service sector.

4   Computation of equilibrium

The computation of the steady state is very standard. Fix the tax rate and guess on K, H and H_s, with $H_g = H - H_s$. Compute prices, r, w, p and transfers, Ψ. Solve the households' problems. Check if aggregate savings are equal to the aggregate demand for capital, and if the implied labor supply equals the guess. Otherwise, update the guess and start again.

5   Calibration

My first goal is to calibrate the model to match a number of Swedish features. On the production side, standard values are used for the parameters, namely $\alpha = 0.3$; $\delta = 0.08$. The preference parameters are $\beta, \gamma, \mu, \eta, \varphi, a$ and b. I set $\beta = 0.96$ in order to generate an interest rate at around 4 percent. The intertemporal elasticity of substitution for consumption is set to 0.5 (which implies $\gamma = 2$), and parameter μ is set to 5, which implies that, on average, the intertemporal (Frisch) elasticity of substitution for labor will be equal to 0.48.[16] These elasticities are within the range of the existing estimates.[17]

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[16] The Frisch elasticity of labor supply is defined as the elasticity of labor supply with respect to the wage keeping the marginal utility of consumption constant. In the model, this elasticity is given by $\frac{l_{it}}{h_{it}}\frac{1}{\mu}$.

[17] See Browning et al (1999) for a survey.

Parameter η is quite tricky to pin down, because it determines the elasticity of substitution between consumption and services (both home produced and market produced), which is a non-observable number. Instead, I set out to match the fact that Swedes spend more than 3 times more of their disposable income on goods than on (market-produced) services.[18] I set $\eta = 0.5415$, which generates a ratio around 3.2.

The weight parameter on leisure, φ, is set to match the average hours of market work. For Sweden, the average hours of market work are 23.88 hours per week (see table 2.3 below) and the implied value for φ is 0.895. The productivity parameter, $\overline{\epsilon}_{hp}$, is calibrated to match the hours worked in home production. For Sweden, the relationship between market work and home production is very stable in all three studies: Swedes devote approximately 56 percent of their total working time to market work and 44 percent to home production. To match this, I calibrate the model so that "the model Swedes" devote 18.76 hours per week to home production. I set $\overline{\epsilon}_{hp} = 0.43 E\left[\epsilon_{it}\right]$.[19]

A key parameter governing a household's choice between market-produced and home-produced services is b, since the elasticity of substitution between these two goods is $\frac{1}{1-b}$. For example if $b = 1$, then $s_{it,m}$ and $s_{it,hp}$ are perfect substitutes, and if $b = 0$, then s_{it} is a Cobb-Douglas function of $s_{it,m}$ and $s_{it,hp}$, so that they are complements. In order to pin down b and a, I estimated (using a simple OLS-regression) the fraction of total expenditures households spend on market-produced services as a function of their disposable income.[20] a and b are then set to match the intercept and the slope of the estimated function.[21] The implied values are $b = 0.25$, and $a = 0.3835$.

I now turn to the policy parameters. The taxes included are set to $\theta^P = 0.3282; \theta^I = 0.3052; \theta^C = \theta^S = 0.25$. The parameter σ_ξ^2, determining the degree of wage dispersion, is calibrated with estimates from Flodén and Lindé (2001). For Sweden, they find that $\sigma_\xi^2 = 0.143$.[22] Government consumption, G, is set to 25.7

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[18] See SOU (1997), table 3.16. I have excluded house rents from private consumption.

[19] In the model, $E\left[\epsilon_{it}\right] = 1.0743$.

[20] The data, which is taken from Utgiftsbarometern 2001 (2002), table 6, is denoted for deciles. Once more, I excluded house rents.

[21] a determines the intercept and b the slope.

[22] I thus view wage dispersion as a policy instrument. Admittedly, this is a somewhat crude way

percent of GDP, which is the value reported in European Economy (2000), table 61b.

Once the model is calibrated to Sweden, I am ready to test the hypothesis that differences in labor supply are driven by cross-country heterogeneity in policy. As stated above, this is done by re-calibrating all policy parameters so that they are consistent with U.S. data. At the same time, all preference and production parameters are kept constant.

Taxes for the U.S. are set to $\theta^P = 0.138$; $\theta^I = 0.2115$; $\theta^C = 0.0825$; $\theta^S = 0.$[23] The estimate for σ_ξ^2, is once more taken from Flodén and Lindé (2001), where they find that $\sigma_\xi^2 = 0.375$. U.S. government consumption, G, is set to 14.1 percent of GDP.[24] Finally, some assumptions regarding the wealth distribution are needed. I assume the distribution of wealth to be the same in both countries, and that the fraction of wealth held by the respective quintile is $\begin{bmatrix} 0 & .01 & .08 & .19 & .72 \end{bmatrix}$.[25]

To sum up, the parameters in table 2.1 are kept constant throughout the analysis, while the policy parameters in table 2.2 are changed.

Table 2.1: Preference Parameters

β	γ	μ	φ	a	b	η	α	δ	ϵ_{hp}
0.96	2	5	0.895	0.3835	0.25	0.5415	0.3	0.08	$0.43 E\left[\epsilon_{it}\right]$

The target numbers are summarized in table 2.3 below. The data is the same as in Fig.2.2, except that the annual numbers are divided by 5200 (52 weeks times 100

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of capturing the ambition of some unions in Europe to compress wages.

[23] The employer is subject to a 7.65 percent Social Security and Health Insurance tax. In addition, the employer has to pay unemployment insurance, and workers' compensation, generating a total payroll tax of 13.8 percent. The employee pays 10 percent in federal income taxes (a household with two workers), and a 7.65 percent Social Security and Health Insurance tax. In addition, the employee pays 3.5 percent in state and local income taxes. The above tax rates are based on Schneider (2002) and Stansel (1998). The average sales tax on goods in the U.S. combining state, county and municipal levels was 8.25 percent in 1998. In most states, sales taxes on services are zero (see Isaacson & Bertoni (2000)).

Moreover, with the above tax rates, the total tax on labor in the U.S. (payroll taxes plus income taxes plus consumption taxes), is very close to the estimate reported in Nickell (2003).

[24] European Economy (2000), table 61b.

[25] The distribution is slightly adjusted, to make it consistent with the short-selling constraint. (See, for instance, Domeij and Klein (1998)). The assumption of equally distributed wealth is not at all restrictive. Using more realistic distributions does not affect the results in any significant way. I also carry out a sensitivity analysis for wealth distribution in the Appendix.

Table 2.2: Policy Parameters

	θ^P	θ^I	θ^C	θ^S	σ_ξ^2	G (SHARE OF GDP)
SWEDEN	0.3282	0.3052	0.25	0.25	0.143	0.257
U.S.	0.138	0.2115	0.0825	0	0.375	0.141

productive hours per week), and then multiplied by 100 to obtain the percentage terms. In this way, the numbers both express hours in percent and hours per week.

Table 2.3: Actual Hours Worked in Percent

	SWEDEN	U.S.
MARKET WORK	23.88	26.18
HOME PRODUCTION	18.76	17.02
GDP/CAPITA	1	1.31

6 Steady State Results

As the title says, all results in this section are steady-state analyses implying that all variables are constant over time.

6.1 A Model Without Services

A useful benchmark here is the effects of labor taxes in a model with no service sector and no home production. Standard micro theory tells us that lower income taxes will generate a substitution effect and an income effect. Since these two effects often go in opposite directions, it is generally not obvious whether a household wants to increase or decrease its labor supply in response to the new tax.

In this subsection, I analyze whether labor taxes (i.e. θ^P and θ^I) can explain the observed difference in labor supply between Sweden and the U.S. In this section, I abstract from consumption taxes (i.e., $\theta_{Sw}^C = \theta_{U.S.}^C = 0$). Government consumption, however, is changed in order not to make counterfactual assumptions about transfers.[26]

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[26] The taste parameter for leisure φ is set to 3.895 to match hours worked in Sweden.

Table 2.4: Hours Worked in the Data and the Model Without Services (in Percent)

	DATA		MODEL	
	SWEDEN	U.S.	SWEDEN	U.S.
MARKET WORK	23.88	26.18	23.88	23.18
GDP/CAPITA	1	1.31	1	.98

The result is presented in table 2.4. It shows that even though taxes and benefits are lower in the U.S. than in Sweden, labor supply is predicted to be lower in the U.S., implying that the income effect dominates the substitution effect with the chosen calibration.[27] Moreover, the effect is much too small. Even if the substitution effect had dominated, the average labor supply would still not even have come close to the observed value of 26.18, due to the low Frisch elasticity of labor supply in the model.

The result from this exercise basically confirms what was said in the introduction about standard models generally having a problem explaining differences in labor supply with cross-country heterogeneity in taxes. Let me now turn to my model with services and home production.

6.2 The Model With Services

6.2.1 Changing All Policy Parameters

In this section, the model with services and home production is used to explore whether heterogeneity in policy can explain the difference in labor supply between Sweden and the U.S. As a starting point, all policy parameters are changed simultaneously (their separate effects are analyzed in the subsequent sections). The result is presented in table 2.5.[28]

The model is clearly quite successful in replicating the data. It matches the time spent in market work and home production very well. In addition, it gets both the direction and the magnitude of the labor supply to be of the right order. Market work in the model only exceeds that in the U.S. by 3.78 percent, whereas

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[27] It is quite easy to analytically derive that this will be the case in a static environment whenever $\gamma > 1$.

[28] GDP/capita adjusted is given by GDP/capita plus the amount of home-produced services evaluated at the market price.

Table 2.5: Hours Worked in the Data and the Model With Services (in Percent)

	DATA		MODEL	
	SWEDEN	U.S.	SWEDEN	U.S.
MARKET WORK	23.88	26.18	23.88	27.17
HOME PRODUCTION	18.76	17.02	18.76	16.07
GDP/CAPITA	1	1.31	1.00	1.12
GDP/CAPITA ADJUSTED	-	-	1.19	1.30

the estimate for home production undershoots the data by 6 percent, corresponding to approximately 49 hours/year, or 8 minutes/day.

Finally, the model quantifies the output loss to be 12 percent, i.e. GDP per capita is quantified to be 12 percent higher in the U.S. than in Sweden, as a result of different policies. However, even though the model can explain a fraction of the difference in GDP per capita between Sweden and the U.S., it still leaves a large share unexplained. Let us now go deeper and disentangle which policy instrument drives these results.

6.2.2 Analyzing Policy Instruments Separately

In the above experiment, all policy parameters were changed at the same time. To analyze their individual importance, the policy instruments are treated separately in this section. The result is shown in table 2.6. The first column is once more the benchmark case Sweden. The second column shows the effect of changing government consumption only, i.e., 20.74 would be the number of hours worked in the market if Sweden would have the same level of government consumption as the U.S., while keping all the other policy parameter at their Swedish levels. Consequently, the third column shows the effect of changing wage dispersion *only* and the fourth the effect of changing taxes *only*.

Table 2.6: Analysing Policy Instruments Separately

	SWEDEN	U.S. G	U.S. PRODUCTIVITY	U.S. TAXES
MARKET WORK	23.88	20.74	22.18	31.25
HOME PRODUCTION	18.76	20.25	20.30	13.83
GDP/CAPITA	1	.88	.99	1.27
GDP/CAPITA ADJ.	1.2	1.08	1.20	1.43

Table 2.6 shows a very clear result: the dominating effect is from taxes. In this model, taxes evidently have the capacity of highly distorting the choice between home production and market work. Moreover, a considerable output loss is associated with this distortion: GDP per capita is 27 percent (26.52, to be exact) higher in the economy with U.S. than with Swedish taxes. This is a substantial number, roughly equal to 6800 dollars/person (denominated in 2001 prices).

Changing government spending decreases both market work and GDP per capita. As discussed above, it follows from the government budget constraint (2.9) that reducing government spending, while keeping taxes fixed, implies increased transfers. This, in turn, reduces the incentive to work, mainly for low-productivity households.

Changing the wage dispersion decreases market work and increases home production. However, no major output loss is associated with such a switch. The reason for this should be that a higher dispersion enhances the incentive to supply labor for high-productivity households (they are facing a higher return to labor), while it is reduced for low-productivity households (they are facing a lower return to labor). The net effect is negative, but the fall in GDP per capita stemming from the lower work effort of low-productivity households is almost completely offset by the increased work effort of high-productivity households. This result indicates that heterogeneity in productivity does not really constitute an explanation.[29]

6.2.3   Analyzing Taxes Separately

In order to evaluate the quantitative importance of each of the different taxes, their separate effects are shown in table 2.7. Once more, the second column shows the effect of changing labor taxes only and so on. Both wage dispersion and government consumption are set to Swedish data. Once more, there is a very clear result: labor taxes really distort the choice between working in the market and at home. The output loss associated with distortionary labor taxes is again substantial: GDP per capita is 17 percent higher with U.S. than with Swedish labor taxes.

The quite modest distortion associated with service taxes is somewhat surprising. Service taxes would probably be expected to have the potential to heavily distort

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[29] However, it is important to understand that the result stems from the low value of b. Moreover, b is estimated in Sweden but may, in practise, be higher in the U.S.

Table 2.7: Analyzing Taxes Separately

	SWEDEN	U.S. θ^I AND θ^P	U.S. θ^C	U.S. θ^S
MARKET WORK	23.88	28.30	25.60	25.13
HOME PRODUCTION	18.76	15.81	17.58	17.74
GDP/CAPITA	1	1.17	1.07	1.03
GDP/CAPITA ADJ.	1.2	1.36	1.24	1.21

the choice between buying and producing services. The mild distortion in the model is due to the implied elasticity of substitution between market-produced and home-produced services being rather low (slightly above 1). In reality, services differ and the elasticity of substitution will be substantially higher for some services than for others (it is infinite for perfect substitutes). A higher elasticity will obviously generate larger distortions, but this cannot be captured in the present model.

7 Transition Analysis

In this section, I analyze whether policy can account for the trend in hours worked over time. More specifically, I look at Sweden during the period 1960-2002.

The model economy is assumed to start out in a steady state, calibrated with taxes and government consumption set to their 1960 values. Taxes and government consumption are then changed according to their observed values for each year, for the period 1960-2002.[30] From 2002 and onwards, taxes and government consumption are held constant at their 2002 values. All preference and production parameters are the same as in the above analysis. The transition is announced 20 years in advance, and households have complete information on this path. The result is presented in Fig. 2.3 below.[31] The tax wedge is defined as $1 - \frac{(1-\theta^I)(1-\theta^P)}{(1+\theta^C)}$

The model is obviously consistent with the falling trend in hours worked since 1960. Both in the model and the data, the largest reduction occurs from 1960 until

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[30] Data on government consumption is taken from European Economy (2000) and SCB at www.scb.se. Data on hours worked, population and employment 20-64 from the OECD Labor Database. Income taxes are taken from Skattestatistisk årsbok 2002 (2002). And finally, the data on payroll taxes and value added taxes is taken from Skattebetalarnas Förening at www.skattebetalarnas.se.

[31] The data on average hours worked, employment and population for people between 20 and 64 was again taken from the OECD Labor Database.
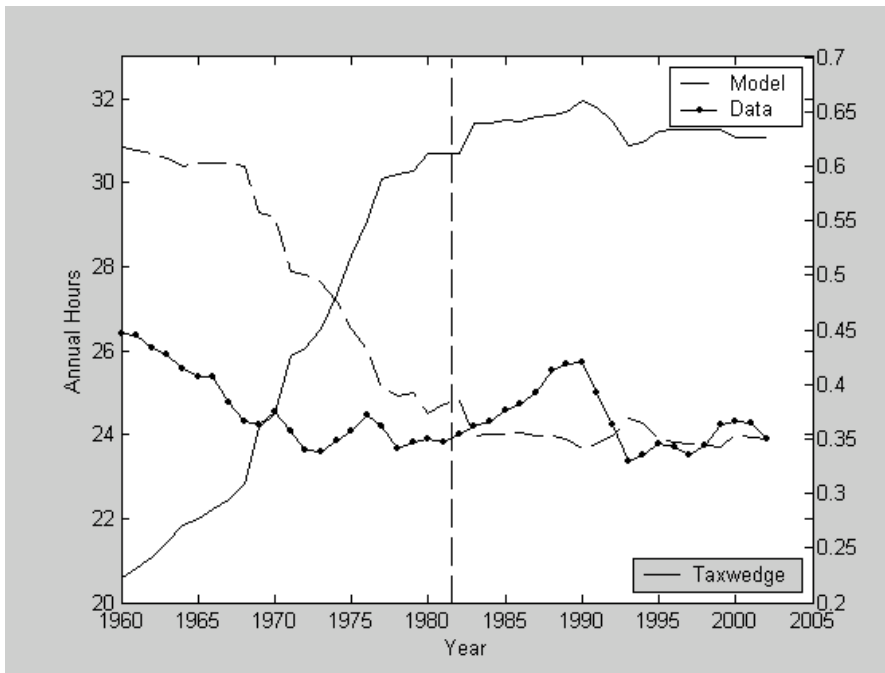
Figure 2.3: AVERAGE ANNUAL HOURS IN THE MODEL AND THE DATA.

around 1980. This strengthens the paper's hypothesis, since this is also the period when taxes increased the most. After 1981 , the trends for both taxes and actual hours worked seem to be almost flat. This is also what the model predict for hours worked.

Some statistics are shown in the correlation matrix below. As can be seen, the correlation between hours worked in the data and the tax wedge is -0.53, and the correlation between hours worked in the data and the model is 0.52.

Table 2.8: Correlation Matrix

	HOURS (MODEL)	HOURS (DATA)	TAX WEDGE
HOURS (DATA)	1	0.52	-0.995
HOURS (DATA)	0.52	1	-0.53
TAX WEDGE	-0.995	-0.53	1

Even though the model seems to be consistent with the long-run behavior, it cannot really explain the short-term movements observed in the data, which is not very

surprising, however. The model obviously abstracts from a number of potentially important features like business cycles. The discrepancy between 1985 and 1990 is, for example, most likely driven by the international boom during this period.

Finally, the model clearly overestimates the number of hours worked between 1960 and 1975. It is not really obvious how to reduce this gap. I have made some experiments with productivity growth, but that does not seem to improve the fit. However, since the low number of hours worked at the beginning of the period actually comes from the low employment/population ratio, a more promising way may be to introduce day-care in some form. I leave this for future research.

8 Conclusions

I have analyzed whether cross-country differences in labor supply could have been generated by heterogeneity in policy. The findings are that policy can account very well for the difference in average annual hours worked between Sweden and the U.S. The main distortion comes from taxes in general, and labor taxes in particular. Even though the Frisch elasticity for labor supply is low in the model, labor taxes create a substantial output loss. GDP per capita is estimated to be somewhat more than 26 percent higher with U.S. than with Swedish taxes, due to the fact that labor taxes really influence the choice between working in the market and in home production.

On the other hand, neither service taxes nor wage-dispersion seem to be associated with any significant output loss. In fact, the service tax is found to be the least distortionary tax among the set of taxes analyzed in this paper.

The model is also consistent with the falling trend in hours worked since 1960. The largest reduction occurs from 1960 until around 1980, both in the model and the data. After the early 1980s, the trends for both taxes and actual hours worked seem to be almost flat, which is as predicted by the model for hours worked. However, it clearly overestimates the number of hours worked between 1960 and 1975.

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9   Appendix

9.1   Robustness and Sensitivity analysis

In this Appendix, I evaluate how sensitive the above results are to some of the assumptions made.

9.1.1   A Higher intertemporal elasticity of substitution for consumption

To evaluate the sensitivity to this parameter, I here set $\gamma = 1$, i.e. changing the Cobb-Douglas function for consumption goods and services to a log function. To match the hours worked, the following parameters were changed to $\overline{\epsilon}_{hp} = .42 E\left[\epsilon_{it}\right]; \eta = .5430; \varphi = .0965; a_0 = .3832$

Table 2.9: Hours Worked in the Data and the Model With Services.

	DATA		MODEL	
	SWEDEN	U.S.	SWEDEN	U.S.
MARKET WORK	23.88	26.18	23.88	27.57
HOME PRODUCTION	18.76	17.02	18.76	16.35
GDP/CAPITA	1	1.31	1.00	1.17
GDP/CAPITA ADJUSTED	-	-	1.18	1.35

9.1.2 The distribution of wealth

To check how sensitive the results are to assumptions for the wealth distribution, I here make the extreme assumption that wealth is equally distributed across households (a gini coefficient of zero).

To match the hours worked, the following parameters were changed to $\bar{\epsilon}_{hp} = 0.38 E\left[\epsilon_{it}\right]; \eta = .5521; \varphi = .8450; a_0 = .41$. The result is presented in the table below.

Table 2.10: Hours Worked in the Data and the Model With Services. Wealth Equally Distributed.

	DATA		MODEL	
	SWEDEN	U.S.	SWEDEN	U.S.
MARKET WORK	23.88	26.18	23.88	26.66
HOME PRODUCTION	18.76	17.02	18.76	16.33
GDP/CAPITA	1	1.31	1.00	1.11
GDP/CAPITA ADJUSTED	-	-	1.16	1.28

I have also experimented with earnings uncertainty (which generates an endogenous wealth distribution). Preliminary results from that experiment are almost identical to the case without uncertainty.

Chapter 3

Social Security and the Equity Premium Puzzle[*]

1   Introduction

In 1985, Mehra and Prescott shed light on the question as to why the historical
equity premium is so high. They showed that the equity premium generated in
a representative consumer framework is 0.35 percent at most, in contrast to the
historical 6 percent in the U.S.[1] This paper takes a life-cycle approach and analyzes
whether the pay-as-you-go (PAYGO) social security system could be an important
factor in the explanation of the so-called equity premium puzzle. The analysis is
partly motivated by the equity premium having been substantially higher since the
introduction of the current U.S. PAYGO system in 1935. According to Mehra and
Prescott (2003), the premium in the U.S. was 3.92 percent for the period 1889-1933,
and 8.93 percent for the period 1934-2000.

The potential importance of social security is mainly due to the considerable size
of the program: in 2001, social security was the major source of income (providing at
least 50 percent of total income) for 65 percent of the aged beneficiaries, and it was
the only source of income for 20 percent of the aged population in the U.S.[2] In ad-

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    [1] Mehra and Prescott (1985).
    [2] Fast Facts and Figures about Social Security (2003).

dition, social security is a non-traded asset containing several elements of aggregate risk, thereby affecting the financial behavior of both taxpayers and retirees.

This paper sets up a simple three-period overlapping generations (OLG) model, where agents work for two periods and retire in the third, and shows that social security unambiguously increases the equity premium. Two social security arrangements with defined benefits are considered in this paper. First, the U.S. system where agents receive wage-indexed benefits based on their past income and second, the more standard arrangement in macroeconomic models with constant and completely safe benefits. The effect of social security on the equity premium is found to be substantial: up to 83 percent higher with social security than without. Since equity in the model is a claim to all risky capital in the economy, a realistic equity premium might be about 3 percent.[3] The model with wage-indexed benefits based on past income, borrowing constraints and a relative risk aversion (RRA) coefficient of 6 generates an equity premium of 2.31, which is rather close to the target.

It turns out that the exact effect of social security on the equity premium to a large extent depends on whether agents are subject to shortselling constraints. In the absence of such constraints, social security influences the equity premium by changing the relative demand for equity. When agents do not receive social security, the young shortsell bonds and buy the bulk of the equity stock.[4] Their marginal valuation of equity is therefore decisive in the pricing process, and since they value equity highly, the return to it is low. The middle-aged mainly hold bonds, but the difference in expected return between the two assets is small. When agents do receive social security, the need to save for retirement is reduced. In addition, since social security is a relatively safe asset, it reduces the motive to hold bonds for retirement. Consequently, middle-aged agents' demand for bonds goes down. The lower demand for bonds effectively restricts the possibilities of the young to finance their equity purchases by shortselling bonds. Therefore, they demand less equity and the price of equity goes down (and the return goes up).

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[3] Following Constantinides et al (2002).

[4] The large equity demand of the young is explained by the fact that they will receive some wage income in the next period, wheras the middle-aged will receive zero exogenous income. The covariance between future consumption and equity income is thus initiallty lower for the young than for the middle-aged. In the trading process, the middle-aged demand bonds and the young demand equity to reduce this discrepancy.

The main influence of social security in the presence of shortselling constraints is due to the fact that social security significantly increases the covariance between future consumption and equity income for the young.[5] At the same time, the covariance between future consumption and equity income for the middle-aged is relatively unaffected by social security. The higher covariance is first of all due to the arrangement with defined benefits featuring contra-cyclical taxes. More specifically, the government may be forced to increase taxes when wage income is low, and vice versa. Since equity and wage income are highly correlated, the young find equity to be a worse hedge in economies with social security, since it does not pay off when really needed. Their marginal valuation of equity will therefore be lower, whereas the marginal valuation of the middle-aged remains relatively unchanged. The result is a lower equilibrium price and a higher average return.

This paper adds to the literature on the importance of life-cycle effects for the equity premium. Earlier findings are that the equity premium is generally high when the equity is mainly priced by the middle-aged who face a high covariance between their future consumption and equity income. The marginal valuation of equity is low for this group, resulting in a high expected return to equity. A natural question is then why the equity would be priced by the middle-aged. Constantinides et al (2002) argue that the young would like to shortsell bonds and invest in equity, but financial frictions such as shortselling constraints may prevent them from this. Storesletten et al (2001), on the other hand, argue that the young actually choose not to hold equity, since they find it too risky. This paper offers an explanation in-between the two.[6] As in Constantinides et al (2002), the young would like to shortsell bonds and invest in equity. However, social security reduces the middle-aged agents' demand for bonds and thus, the amount of bonds the young can shortsell. As a result, the young choose to reduce their equity demand.

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[5] Recall that the Consumption Capital Asset Pricing Model (C-CAPM), predicts that the price of an asset should depend on the covariance between future consumption and the return to that asset. An asset paying off in states of high marginal utility of consumption commands a higher price than one paying off in states of low marginal utility of consumption. Moreover, marginal utility of consumption varies inversely with consumption, implying that the asset paying off in states of low levels of consumption command a higher price than the asset paying off in states of high levels of consumption.

[6] Both Constantinides et al (2002) and Storesletten et al (2001) abstract from social security.

Finally, even though the model with social security and shortselling constraints may be viewed as successful, neither shortselling constraints nor social security alone found to generate an equity premium of realistic size. This paper is also related to Abel (2003), who analyzes the price of capital in the presence of social security; and to the large literature on the effects of social security on individual and aggregate savings.[7]

2　The Economic Model

2.1　The Consumers

The basic model used is an overlapping-generations, pure exchange economy where each generation lives for three periods as young, middle-aged and old, respectively. Each generation is modeled by a representative consumer. There is one consumption good in each period, which perishes at the end of the period. Wages, consumption, dividends and coupons as well as bond and equity prices are denominated in units of the consumption good. The wage income process in the economy is stochastic and exogenous, so I am abstracting from the labor-leisure trade-off. The index $i = 0, 1$ and 2 is used to denote the young, the middle-aged and the old, respectively. A consumer born in period t receives the wage income $w_{t,0} > 0$ when young, $w_{t+1,1} > 0$ when middle-aged and a social security benefit $\widehat{\varphi}_{t+2,2} \geq 0$ when old.

There are two types of securities in the economy, bonds and equity. Both are in unit supply and infinitely lived. The bond is default-free and pays a fixed coupon $b > 0$ in every period in perpetuity. With this set-up, the bond can be considered as a proxy for long-term government debt. The *ex coupon* bond price in period t is denoted by q_t^b, and it is the price of the claim to the coupon b paid in perpetuity beginning in period $t + 1$.

The equity is a claim to a dividend stream in perpetuity and pays a net dividend d_t in period t. The *ex dividend* price of equity in period t is denoted by q_t^e, and it is

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[7] See, for instance, Feldstein (1974, 1996), who argue that social security crowds out private savings; and Hugget (1996), Hugget & Ventura (2000) and Domeij & Klein (2002) who argue that social security can explain a large degree of the heterogeneity in saving rates between households, and thereby many of the features of the wealth distribution.

the price of the claim to a perpetual dividend stream, beginning with period $t + 1$. Equity is here considered as the total sum of the claims to firms and real estate.

The consumer born in period t has zero endowment of assets. This consumer makes a portfolio decision $z_{t,0} = \left(z_{t,0}^b, z_{t,0}^e\right)$ when young; adjusts this decision to $z_{t+1,1} = \left(z_{t+1,1}^b, z_{t+1,1}^e\right)$ when middle-aged; and sells the portfolio in period $t + 2$, when old. As usual, a negative position in bonds or stocks denotes a short position in that asset.

In period t, the young consume $c_{t,0}$, the middle-aged consume $c_{t,1}$ and the old consume $c_{t,2}$. Denote the tax rate by θ_t, and the social security benefit received by the old by φ_t. The budget constraints in period t are then given by

$$c_{t,0} + z_{t,0}^b q_t^b + z_{t,0}^e q_t^e + \leq w_{t,0} \left(1 - \theta_t\right) \tag{3.1}$$

$$c_{t,1} + z_{t,1}^b q_t^b + z_{t,1}^e q_t^e \leq w_{t,1} \left(1 - \theta_t\right) + z_{t-1,0}^b \left(q_t^b + b\right) + z_{t-1,0}^e \left(q_t^e + d_t\right) \tag{3.2}$$

$$c_{t,2} \leq \widehat{\varphi}_t + z_{t-1,1}^b \left(q_t^b + b\right) + z_{t-1,1}^e \left(q_t^e + d_t\right), \tag{3.3}$$

where $(3.1), (3.2)$ and(3.3) are the budget constraints faced by the period t young, middle-aged and old, respectively, and $\widehat{\varphi}_t$ are net social security benefits.

Period utility takes the following form

$$u\left(c\right) = \frac{c^{1-\gamma} - 1}{1 - \gamma}, \tag{3.4}$$

where $\gamma > 0$ is the constant coefficient of relative risk aversion. For an agent of age $i \in \{0, 1\}$, the Bellman equation to the consumer's problem is then given by

$$V_i\left(z_{t,h}\right) = \max_{c_t, z_{t,i}^b, z_{t,i}^e} u\left(c_{t,i}\right) + \beta E_t V_{i+1}\left(z_{t+1,i+1}\right), \tag{3.5}$$

subject to the relevant budget constraints given by (3.1)-(3.3), where β is the subjective discount factor.

It is also assumed that

$$V_{t,3} \equiv 0 \ \forall t,$$

which implies that the old do not buy any assets (and that altruistic bequests are ruled out).

Aggregate income is defined as

$$y_t = w_{t,0} + w_{t,1} + b + d_t. \tag{3.6}$$

Following Constantinides et al (1998), the definition of income includes the (constant) coupon payment on government debt.[8]

2.2   The Government Sector

Three different social security arrangements are considered in this paper, beginning with the benchmark case with no social security. Naturally, the government sector is then completely abstracted from, so both taxes and benefits are set to zero. Second, I deal with the U.S. Social Security System with wage-indexed benefits based on past income and third and finally, the case with completely safe benefits. The last two cases are discussed and modeled in detail below.

2.2.1   The U.S Social Security System

The U.S. Social Security System is basically a PAYGO system, where the government collects taxes and pays out wage-indexed benefits, based on past income to the currently old. The system features defined benefits. More specifically, there are basically three important factors determining the social security benefits received by agents in the U.S.: the agent's *average income, the replacement ratio* and the evolvement of *average wages*.[9]   The first factor is important because the level of benefits when reaching the retirement age is based on lifetime earnings. For each worker, there is a calculation of the worker's Average Indexed Monthly Earnings

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[8] This is somewhat non-standard, but the assumption allows me to completely abstract from the government and thus from taxes to finance the debt in the no-social security version. In any case, the interest on government debt in the U.S. is about 3 percent of GDP and the calibration remains essentially unchanged notwithstanding if the definition of GDP includes the term b.

[9] See, for instance, the *Social Security Handbook*.

(AIME) - an average of a worker's earnings over the best 35 years of her career. One period is assumed to be 20 years in the model and each worker supplies labor for two periods, i.e. a total of 40 years. For simplicity, averages are computed over these 40 years rather than just the 35 best years. The average income history of an agent aged i is denoted by $\psi_{t,i}$. Formally, $\psi_{t,2} \equiv \frac{w_{t-2,0}+w_{t-1,1}}{2}$ is the average (pre-taxed) lifetime earnings of the period t old. The second determinant of the social security benefits is the replacement ratio, i.e. the rate at which social security replaces past earnings. This parameter will be denoted by η. Finally, the rate of return on social security is related to the aggregate wage-index, i.e., the evolvement of aggregate labor income. More specifically, an individual's earnings are indexed to the average wage level at the time of retirement.[10] The variability of this index over time influences the variability of benefits, and its comovement with other assets will be important for portfolio composition. The (pre-taxed) social security benefit of an agent retiring in period t is then

$$\varphi_t = \eta \psi_{t,2} \widehat{W}_t \tag{3.7}$$

where

$$\widehat{W}_t = \frac{W_t}{E[W]}. \tag{3.8}$$

Thus, if the wage rate in period t is above its unconditional mean, benefits are increased proportionately.

The government is required to balance its budget in each period. Budget balance in combination with defined benefits implies that the tax rate will be a stochastic variable, taking on whatever value is needed to keep the government budget in balance. In contrast to the complex means by which taxable benefits are determined in the U.S., benefits are, for simplicity, taxed at the period-t income tax rate θ_t. The government budget constraint is then

$$\theta_t (w_{t,0} + w_{t,1} + \varphi_t) \geq (1 - \theta_t) \varphi_t. \tag{3.9}$$

It is then straightforward to use (3.7), (3.9) and the fact that the government

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[10] Actually, an individual's earnings are indexed to the average wage level 2 years prior to the year of eligibility, i.e., when the agent attains the age of 62.

balances the budget to write the tax in period t as

$$\theta_t = \frac{\eta\psi_{t,2}\widehat{W_t}}{w_{t,0} + w_{t,1} + 2\eta\psi_{t,2}\widehat{W_t}}. \tag{3.10}$$

2.2.2 Safe Benefits

A natural benchmark may also be the case with completely safe benefits, more specifically, a setting where all old agents receive the same benefit and, consequently, do not face any uncertainty in their exogenous income. This is probably the most common assumption when modeling social security in macroeconomic models. Since a set-up with defined benefits is considered, taxes are stochastic, and since benefits are assumed to be constant, they cannot be taxed. Instead, retirees receive a benefit equal to the unconditional expected value of the after-taxed benefit in the model with wage-indexed benefits based on past income. The completely safe benefit is given by

$$\widehat{\varphi}_t = \varphi = E\left[\left(1 - \theta_t^{W.I}\right)\eta\psi_{t,2}\widehat{W_t}\right], \tag{3.11}$$

where $\theta_t^{W.I}$ is the period-t tax rate in the economy, with wage-indexed benefits. Finally, taxes paid by the young and the middle-aged are then given by

$$\theta_t^S = \frac{\varphi}{w_{t,0} + w_{t,1}}. \tag{3.12}$$

2.3 Shortselling Constraints

Implications for equity prices are analyzed both with and without shortselling constraints. First, Constantinides et al (2002) have argued that shortselling constraints may have important implications for asset prices, and the equity premium in a life-cycle setting. Second, borrowing constraints can be motivated on grounds of realism: it is a well known fact that human capital alone does not collateralize major loans in modern economies, for reasons of moral hazard and adverse selection. The borrowing-unconstrained economies, on the other hand, may be viewed as natural benchmark cases. In short, in borrowing-constrained economies, agent $i \in \{0, 1\}$ is subject to the following two additional constraints

$$z_{t,i}^b \geq 0 \text{ and } z_{t,i}^e \geq 0 \; \forall t. \tag{3.13}$$

3 Equilibrium

Instead of specifying joint stochastic processes for wage income and dividends $(d_t, w_{t,0}, w_{t,1})$, I specify processes for aggregate income and wage income $(y_t, w_{t,0}, w_{t,1})$. More specifically, the joint process of the detrended aggregate income and the wage income of the young and the middle-aged is modeled as a time-stationary process. Since each of these variables will only be allowed to take on a finite number of values, the triple $(y_t, w_{t,0}, w_{t,1})$ can be represented by the state variable $s_t = j$, $j \in J$. This variable is modeled as a Markov process with a non-degenerate, unique, stationary probability distribution with the transition probability matrix $\pi = (\pi_{ij})$, and is referred to as the "income state". It is then straightforward to represent a stationary equilibrium, wherein decisions made in period t and prices in period t are functions of the current income state j, and the *one period lag* of the investment decisions of the middle-aged in period-t $z_{-1} \equiv (z_{-1}^b, z_{-1}^e)$. However, in the economy with wage-indexed benefits based on past income, agents also need to keep track of the average income history of the currently old $\psi_2(j_{-1}, j_{-2})$ and middle-aged $\psi_1(j, j_{-1})$.

A stationary equilibrium of this economy is given by time-invariant decision rule for agents' asset holdings, $z_0^b(j, z_{-1}, \psi_2, \psi_1)$, $z_1^b(j, z_{-1}, \psi_2, \psi_1)$, $z_0^e(j, z_{-1}, \psi_2, \psi_1)$ and $z_1^e(j, z_{-1}, \psi_2, \psi_1)$, such that the following conditions hold:

- The decision rules solve agents' maximization problem, given by (3.5)

- $z_0^b + z_1^b = 1$ and $z_0^e + z_1^e = 1$

- The government budget constraint (3.9) is satisfied with equality

The first condition ensures that each consumer's consumption and investment policy maximizes her expected utility from the set of admissible policies (while taking the price process as given) and the second condition ensures that the bond and equity markets clear in all periods. Note that by Walras' law, condition (2) ensures that goods markets clear.

Naturally, in the presence of shortselling constraints, the following additional conditions must also have to be satisfied: $z_i^b \geq 0$ and $z_i^e \geq 0$ $\qquad i = 0, 1, 2.$

4 Numerical Computation of the Equilibrium

In the calibration, y will only be allowed to assume four different values. In addition, since it is computationally costly to keep track of income histories, w_0 and w_1 are only allowed to take on two values each. More specifically, I assume the following aggregate structure:

$$J = \begin{cases} (\overline{y} - v), (\overline{w}_i (1 - \zeta)) \\ \left(\overline{y} - \frac{v}{4}\right), (\overline{w}_i (1 - \zeta)) \\ \left(\overline{y} + \frac{v}{4}\right), (\overline{w}_i (1 + \zeta)) \\ (\overline{y} + v), (\overline{w}_i (1 + \zeta)) \end{cases} \quad i = 0, 1,$$

where $\overline{y}$ is the average aggregate income and $\overline{w}_i$, the average wage income of an agent aged i. Since wages of the young and middle-aged both can be high and low, it follows that ψ_2 is a variable with four possible values. Finally, for a given realization of $j \in J$, the variable ψ_1 can attain two different values, resulting in a total of 32 different exogenous states.[11] The stochastic process is assumed to be *i.i.d.* over time. Although it is well established that aggregate productivity shocks are highly autocorrelated at annual and quarterly frequencies, there does not seem to exist any conclusive evidence indicating such positive serial correlation at generational frequencies (i.e. 20-30 year periods).[12] As a benchmark, aggregate shocks in the model are therefore uncorrelated across time. Moreover, it is assumed that $\pi_1 = \pi_4$ and $\pi_2 = \pi_3$.

Since no closed-form solutions for portfolio policy functions and pricing functions exist, these functions are approximated by B-splines of order 4 (i.e. piecewise cubic polynomials). The system of equations is solved with a Gauss-Jacobi approach.

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[11] The numerical approximation then features more than 40000 unknowns.

[12] This assumption is also in line with several other papers analyzing OLG-models with two or three periods. See, for instance, Bohn (1999) and Smetters (2002).

5 Calibration

The preference parameters are the RRA coefficient, γ and the subjective discount factor, β. Results are presented for the values $\gamma = 4$ and 6 of the RRA coefficient. The discount factor is set to 0.44 for a period of 20 years. This corresponds to an annual subjective discount factor of 0.96, which is standard in the macro-economic literature.

In the U.S., the current payroll tax is 12.4 percent, and the replacement ratio is 44 percent.[13] The ratio of retired individuals to working people is somewhat higher in the model (0.5) than in the U.S. today (0.25), making it difficult to simultaneously match the payroll tax and replacement ratio. I set out to match the empirically observed replacement ratio of 44 percent, i.e. $\eta = 0.44$. The expected tax rate needed to finance this ratio is $E(\theta) = 0.152$.

There are 6 additional parameters to be determined: $\overline{y}, v, \overline{w}_0, \overline{w}_1, \zeta, \pi_1$.[14] These parameters are chosen to satisfy the following 6 target statistics.

(i) **The average share of income going to labor**, $E\left(\frac{w_0 + w_1}{y}\right)$. This statistic was set to be in the interval $0.66 - 0.75$, consistent with US historical experience.

(ii) **The average share of the wage income going to the young** $\frac{w_0}{w_1}$. Actual income profiles from Storesletten et al (2003) are used to calibrate this statistic to be 0.75.[15] Since $w_{t,0}$ and $w_{t,1}$ are assumed to be perfectly correlated, the ratio $\frac{w_0}{w_1}$ equals 0.75 in each period.

(iii) **The average share of income going to interest on government debt**, $\frac{b}{E(y)}$. This was set at 0.03, consistent with US historical experience.

(iv) **The coefficient of variation of the 20-year aggregate income**, $\frac{\sigma(y)}{E(y)}$. This statistic is rather problematic to calibrate since even a century-long time series only provides five non-overlapping observations, resulting in large stan-

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[13] See, for example Mchale (1999).
[14] Since $\pi_1 + \pi_2 + \pi_3 + \pi_4 = 1$, the transition matrix is uniquely defined by π_1.
[15] Profiles are estimated from PSID data.

dard errors of the point estimates. I follow Constantinides et al (2002), and calibrate this statistic to be 0.25.

(v) **The coefficient of variation of the 20-year aggregate wage** $\frac{\sigma(W)}{E(W)}$. This vital statistic represents another calibration challenge for **the reasons** mentioned above. Against this background, I once more stay close to Constantinides et al (2002), and calibrate the coefficient of variation of the 20-year aggregate income to be 0.20.

(vi) **The cross-correlation of aggregate labor income and aggregate income** $corr(y_t, w_t)$. When the production process is explicitly modelled, the standard assumption is the Cobb-Douglas production function with its constant capital and labor shares. With a single productivity shock, the implications are that labor, capital and aggregate income are all perfectly correlated. The Cobb-Douglas production function has strong empirical support in the long run. According to Baxter and Jermann (1997) and Bohn (1999), the returns to capital and labor are highly correlated in the long run, thereby supporting the Cobb-Douglas assumption with constant factor shares. With these findings in mind, I present results for $corr(y_t, w_t) = 0.90$ and 0.97.

Implied parameter values are stated in the Appendix. Since the equity in the model is the claim not just to corporate dividends but to all risky capital in the economy, a realistic equity premium is, according to Constantinides et al (1998) about 3 percent.

6 Simulation Results

All economies are simulated for 20000 periods and results from these simulations are presented in this section[16] The mean return of an asset is defined as 100 x [{mean of the 20-year holding period return}$^{1/20}$-1]. The standard deviation of the equity or bond return is defined as 100 x [std{20-year holding period return}$^{1/20}$]. The mean premium of equity return over the bond return is defined as the difference of the

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[16] The case with $corr(y_t, w_t) = 0.97$ is presented in the Appendix. All economies were simulated for 21000 periods, but the first 1000 periods were discarded.

mean return on equity and the mean return on the bond. The standard deviation of the premium of equity return over the bond return is defined as 100 x [std{20-year equity return} - std{20-year bond return}$^{1/20}$].

Table 3.1: Simulation statistics, RRA=4, corr(W,Y)=0.9

| | NO SHORTSELLING CONSTRAINTS | | |
| | NO SOC. SEC. | SOC. SEC. | |
	-	SAFE	WAGE-INDEXED
MEAN EQUITY RET.	6.59	10.73	10.08
STD OF EQUITY RET.	5.1	5.98	5.33
MEAN BOND RET	5.77	9.6	8.93
STD OF BOND RET.	4.54	5.36	4.68
MEAN PRM/BOND	0.82	1.13	1.15
STD PRM/BOND	1.55	2.00	1.97
EQUITY HELD BY THE YOUNG (%)	92	48	57
COV(c_1, EQUITY RET.)	2.25	2.30	2.36
COV(c_2, EQUITY RET.)	2.47	3.89	2.07

Table 3.2: Simulation statistics, RRA=6, corr (W,Y)=0.9

| | NO SHORTSELLING CONSTRAINTS | | |
| | NO SOC. SEC. | SOC. SEC. | |
	-	SAFE	WAGE-INDEXED
MEAN EQUITY RET.	6.33	12.66	11.51
STD OF EQUITY RET.	6.52	8.28	7.20
MEAN BOND RET	5.40	10.96	9.84
STD OF BOND RET.	6.02	7.75	6.68
MEAN PRM/BOND	0.93	1.70	1.67
STD PRM/BOND	1.34	2.00	1.93
EQUITY HELD BY THE YOUNG (%)	128	52	63
COV(c_1, EQUITY RET.)	1.99	2.12	2.17
COV(c_2, EQUITY RET.)	3.49	4.41	2.43

The first important observation across all cases reported in the tables is that social security raises the equity premium. More specifically, the equity premium is up to 83 percent higher with wage-indexed benefits than without social security when agents are not borrowing constrained, and up to 37 percent when they are. Tables 3.1 to 3.4 show that social security increases the equity premium through two main channels: a *demand* effect and a *valuation* effect.

Table 3.3: Simulation statistics, RRA=4, corr (W,Y)=0.9

	SHORTSELLING CONSTRAINTS		
	NO SOC. SEC.	SOC. SEC.	
	-	SAFE	WAGE-INDEXED
MEAN EQUITY RET.	6.54	10.76	10.04
STD OF EQUITY RET.	3.7	5.17	4.21
MEAN BOND RET	4.99	9.16	8.30
STD OF BOND RET.	3.01	4.46	3.39
MEAN PRM/BOND	1.56	1.60	1.74
STD PRM/BOND	1.57	2.03	1.99
EQUITY HELD BY THE YOUNG (%)	12	15	13
COV(c_1, EQUITY RET.)	0.41	0.97	0.72
COV(c_2, EQUITY RET.)	3.93	3.62	3.48

Table 3.4: Simulation statistics, RRA=6, corr (W,Y)=0.9

	SHORTSELLING CONSTRAINTS		
	NO SOC. SEC.	SOC. SEC.	
	-	SAFE	WAGE-INDEXED
MEAN EQUITY RET.	6.35	12.72	11.43
STD OF EQUITY RET.	4.14	6.90	5.32
MEAN BOND RET	4.62	10.50	9.12
STD OF BOND RET.	3.51	6.25	4.56
MEAN PRM/BOND	1.73	2.22	2.31
STD PRM/BOND	1.50	2.08	2.05
EQUITY HELD BY THE YOUNG (%)	11	11	9
COV(c_1, EQUITY RET.)	0.09	0.49	0.31
COV(c_2, EQUITY RET.)	4.65	4.31	4.12

The main influence of social security on the equity premium in the absence of shortselling constraints is due to the demand effect, which is seen from the fact that social security considerably influences the relative demand for equity. In the absence of social security, the young shortsell bonds and hold between 90-130 percent of the total equity stock.[17]   Their marginal valuation of equity is thus decisive in the price process, and since they value equity highly, the return to equity is low in this economy. The middle-aged mainly hold bonds, but the difference in expected

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[17] The large equity demand of the young is explained by the fact that they will receive some wage income in the next period, wheras the middle-aged will receive zero exogenous income. The covariance between future consumption and equity income is thus initiallty lower for the young than for the middle-aged. In the trading process, the middle-aged demand bonds and the young demand equity to reduce this discrepancy.

return between the two assets is small (less than 1 percent). When agents receive social security, they need not to save as much for retirement. Consequently, the middle-aged agents' bond demand is significantly reduced. The lower bond demand effectively restricts the possibilities of the young to finance their equity purchases. As a result, the young demand less equity and the price of equity goes down (and the return goes up). In fact, social security reduces their equity demand by up to as much as 60 percent. The effect on the equity premium is substantial: it is up to 83 percent higher with wage-indexed benefits, than without social security. However, wage-indexed and safe benefits generate the same equity premium in every important respect. Moreover, social security does not influence the covariances between future consumption and equity income in any significant way. Basically, the covariance increases somewhat for the young, and decreases somewhat for the middle-aged.

The main influence of social security in the presence of shortselling constraints is due to the valuation effect. This can be seen from the share of equity held by the young being close to constant, indicating that the relative demand for equity is almost unaffected by social security.[18] However, social security significantly increases the covariance between future consumption and equity income for the young - up to 6 times, as in table 3.9 in the Appendix. At the same time, the covariance between future consumption and equity income for the middle-aged is relatively unaffected by social security. This is true for both wage-indexed and safe benefits. The higher covariance is first of all due to defined benefits featuring contra-cyclical taxes. More specifically, taxpayers are (at least sometimes) forced to pay higher taxes when wage income is low and vice versa. Since equity and wage income are highly correlated, the young find equity to be a worse hedge in economies with social security because it does not pay off when really needed. Their marginal valuation of equity is therefore lower, whereas their marginal valuation of the middle-aged remains relatively unchanged. The result is a lower equilibrium price and a higher average return. Second, contra-cyclical taxes make taxpayers' consumption more volatile, which makes asset prices more volatile since the price agents are willing to pay, will vary more between states. Notice that both bond and equity returns

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[18] It is still a fact that the young find the equity very attractive and would like to invest in it. However, since their wage income is lower in the present than in the future, they choose not to buy when prevented from borrowing.

are most volatile in economies with safe benefits (where taxes are always contra-cyclical), which also increases the covariance. As a result, agents demand a higher premium for holding equities.[19]

The conclusion is that social security unambiguously increases the equity premium. None of the model economies above generate a premium equal to 3 percent. However, the economy with borrowing constraints, wage-indexed benefits based upon past income and an RRA coefficient of 6 has a premium of 2.31 percent, which is fairly close to the target of 3 percent. Moreover, the equity is never priced exclusively by the middle-aged when the model is calibrated with realistic income profiles. Instead, the price of equity is always determined in the trading process between the young and the middle-aged. Finally, social security seems to exacerbate the risk-free rate puzzle, since agents receiving social security value marginal savings less, which leads to lower asset prices and higher returns.

7   Conclusions

This paper finds that the pay-as-you-go social security system may be an important factor in the explanation of the famous equity premium puzzle. The effect of social security on the equity premium is substantial: it is up to 83 percent higher with than without social security. In addition, a model with wage-indexed benefits based on past income, borrowing constraints and an RRA coefficient of 6 can actually generate a fairly realistic equity premium.

In the absence of shortselling constraints, social security influences the equity premium by changing the relative demand for equity. When agents do not receive social security, the young shortsell bonds and buy the bulk of the equity stock. The middle-aged mainly hold bonds, but the difference in expected return between the two assets is small. When agents receive social security, the need to save for retirement, especially in bonds, is reduced. Consequently, the middle-aged agents' demand for bonds goes down. A lower bond demand implies that the amount of bonds the young can shortsell to the middle-aged is reduced. As a result, the young demand less equity and the price of equity goes down (and the return goes up).

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[19] Naturally, this effect is more pronounced when they are more risk averse.

The main influence of social security in the presence of shortselling constraints is due to the fact that social security significantly increases the covariance between future consumption and equity income for the young. At the same time, the covariance between future consumption and equity income for the middle-aged is relatively unaffected by social security. The higher covariance is first of all due to the arrangement with defined benefits featuring contra-cyclical taxes. More specifically, the government may be forced to increase taxes when wage income is low, and vice versa. Since equity and wage income are highly correlated, the young find equity to be a worse hedge in economies with social security, since it does not pay off when really needed. Their marginal valuation of equity will therefore be lower, whereas the marginal valuation of the middle-aged remains relatively unchanged. The result is a lower equilibrium price and a higher average return.

This paper adds to the literature on the importance of life-cycle effects for the equity premium. Earlier findings are that the equity premium is generally high when the equity is mainly priced by the middle-aged who face a high covariance between their future consumption and equity income. The marginal valuation of equity is low for this group, resulting in a high expected return to equity. A natural question is then why the equity would be priced by the middle-aged. Constantinides et al (2002) argue that the young would like to shortsell bonds and invest in equity, but financial frictions such as shortselling constraints may prevent them from this. Storesletten et al (2001), on the other hand, argue that the young actually choose not to hold equity, since they find it too risky. This paper offers an explanation in-between the two.[20] As in Constantinides et al (2002), the young would like to shortsell bonds and invest in equity. However, social security reduces the middle-aged agents' demand for bonds and thus, the amount of bonds the young can shortsell. As a result, the young choose to reduce their equity demand.

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8 Appendix

8.1 The Benchmark Calibration

The calibration in the benchmark case with corr $(W, Y) = 0.9$ is given by

Table 3.5: Parameter values

$\overline{y}$	v	$\overline{w}_0$	$\overline{w}_1$	ζ	π_1
10	2.8	2.89	3.86	0.2	0.187

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8.2 Tables

The results from the simulations with a higher correlation between wages and income are presented in the tables below.

Table 3.6: Simulation statistics, RRA=4, corr (W,Y)=0.97

	NO SHORTSELLING CONSTRAINTS		
	NO SOC. SEC.	SOC. SEC.	
	-	SAFE	WAGE-INDEXED
MEAN EQUITY RET.	6.71	10.81	10.12
STD OF EQUITY RET.	5.25	5.94	5.34
MEAN BOND RET	5.98	9.80	9.09
STD OF BOND RET.	4.73	5.40	4.79
MEAN PRM/BOND	0.72	1.01	1.03
STD PRM/BOND	1.32	1.73	1.69
EQUITY HELD BY THE YOUNG (%)	98	45	58
COV(c_1, EQUITY RET.)	2.23	2.17	2.28
COV(c_2, EQUITY RET.)	2.55	4.25	2.08

Table 3.7: Simulation statistics, RRA=6, corr (W,Y)=0.97

	NO SHORTSELLING CONSTRAINTS		
	NO SOC. SEC.	SOC. SEC.	
	-	SAFE	WAGE-INDEXED
MEAN EQUITY RET.	6.66	12.86	11.68
STD OF EQUITY RET.	6.52	8.33	7.25
MEAN BOND RET	5.84	11.40	10.24
STD OF BOND RET.	6.06	7.86	6.79
MEAN PRM/BOND	0.82	1.46	1.45
STD PRM/BOND	1.17	1.69	1.65
EQUITY HELD BY THE YOUNG (%)	127	51	63
COV(c_1, EQUITY RET.)	1.76	1.99	2.01
COV(c_2, EQUITY RET.)	3.87	4.80	2.56

Table 3.8: Simulation statistics, RRA=4, corr (W,Y)=0.97

	SHORTSELLING CONSTRAINTS		
	NO SOC. SEC.	SOC. SEC.	
	-	SAFE	WAGE-INDEXED
MEAN EQUITY RET.	6.64	10.89	10.17
STD OF EQUITY RET.	3.73	5.20	4.20
MEAN BOND RET	5.32	9.55	8.67
STD OF BOND RET.	3.09	4.59	3.49
MEAN PRM/BOND	1.32	1.34	1.50
STD PRM/BOND	1.32	1.74	1.72
EQUITY HELD BY THE YOUNG (%)	11	15	13
COV(c_1, EQUITY RET.)	0.48	1.12	0.83
COV(c_2, EQUITY RET.)	3.88	3.37	3.27

Table 3.9: Simulation statistics, RRA=6, corr (W,Y)=0.97

| | SHORTSELLING CONSTRAINTS | | |
| | NO SOC. SEC. | SOC. SEC. | |
	-	SAFE	WAGE-INDEXED
MEAN EQUITY RET.	6.53	13.01	11.73
STD OF EQUITY RET.	4.24	7.06	5.42
MEAN BOND RET	5.03	11.11	9.71
STD OF BOND RET.	3.63	6.48	4.77
MEAN PRM/BOND	1.5	1.90	2.02
STD PRM/BOND	1.22	1.76	1.74
EQUITY HELD BY THE YOUNG (%)	10	11	8
COV(c_1, EQUITY RET.)	0.11	0.65	0.41
COV(c_2, EQUITY RET.)	4.72	4.06	3.88

Chapter 4

The Welfare Gains of Improving Risk Sharing in Social Security[*]

1 Introduction

This paper deals with the problem of how aggregate productivity risk should be allocated between taxpayers and retirees in a pay-as-you-go (PAYGO) social security system. The question is motivated by the fact that overlapping generations (OLG) models are known to be inefficient in an *ex ante* sense, stemming from the inability of the unborn to insure themselves.[1] With standard CRRA preferences and Cobb-Douglas technology, the laissez-faire allocation of risk is inefficient by imposing too little productivity risk on retirees and too much on future generations.[2]

A PAYGO system has the potential of correcting these inefficiencies, due to the immanent intergenerational link found by current pension payments being immediately transferred to retirees. More precisely, productivity risk can be transferred between taxpayers and retirees through benefits being allowed to respond to macroeconomic shocks. When the economy is hit by a productivity shock, the government can keep the social security budget in balance, by adjusting benefits or contributions

[1] See Peled (1982) and Wright (1987).

[2] This is a very general result that will be true when the intertemporal elasticity of substitution for consumption is less than one, and/or the depreciation rate less than 100 percent. See Bohn (1998) and Bohn (2003).

(or, naturally, by a combination thereof). In the former case, the exposure to productivity risk is amplified for retirees and, in the latter case, for taxpayers. Using the PAYGO system to transfer productivity risk between these two groups is thus straightforward.[3] In the current U.S. social security system, an element of intergenerational risk sharing can be found in the mechanism of wage-indexed benefits, implying that benefits respond to younger generations' income.[4]

To analyze the importance of improved intergenerational risk sharing in social security, I set up a three-period overlapping generations model with endogenous production, aggregate uncertainty and a PAYGO system with simple (linear) schemes that can be used to allocate risk between taxpayers and retirees. Specifically, wage- and capital-indexed benefits are considered.[5] The three-period model is motivated on the grounds that three is the minimal number of periods that captures the heterogeneity of consumers across age groups that I wish to emphasize: the *uninsured* young, the *saving* middle-aged and the *dissaving* old. Moreover, the three-period model is needed to evaluate the effects of shortselling constraints.[6] The utility function developed by Epstein and Zin (1989, 1991) and Weil (1989) is used to make the model consistent with empirically revealed attitudes towards risk.

The experiment I carry out is to find the coefficients of the schemes maximizing the expected lifetime utility of a newborn agent, and compare the outcomes, both to the current U.S. and the social optimum. The analysis thus quantifies the importance of obtaining a better intergenerational risk sharing allocation, and it sheds lights on the way benefits and taxes should respond to macroeconomic shocks. I also model unexpected transitions from the current U.S. economy to the efficient equilibria, and compute the implied welfare effects.

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[3] However, the suboptimality of the risk allocation in the existing U.S. social security system has been discussed in several recent papers, including Shiller (1998), Ball and Mankiw (2001).

[4] An individual's earnings are indexed to the average wage level 2 years prior to the year of eligibility, i.e., when the agent reaches the age of 62.

[5] The production function is assumed to be of the Cobb-Douglas form, implying that the arrangement with wage-indexed benefits is identical to one where benefits are indexed to aggregate income.

[6] More specifically, the two-period model imposes implicit short-selling constraints. For instance, in that setting, the young can never take a short position in capital unless some institution (for example the government) takes a long position in capital. The case is similar for bondholdings since, in equilibrium, the young must hold the whole amount of government debt, thereby also implying a long position in bonds.

The findings are that improved risk sharing in social security may imply very large welfare gains. The welfare gain of being born into an economy with efficient wage-indexation is between 12 and almost 15 percent of per-period consumption in the current U.S., depending on whether agents are subject to shortselling constraints. In contrast, the welfare loss of being born into the inefficient economy with completely safe benefits is roughly 10 percent of the per-period consumption. Hence, welfare effects are, by all means, very large. Efficient risk sharing in social security implies highly volatile and pro-cyclical benefits and tax rates. The pro-cyclical arrangement eases the risk exposure of the young, since they are subject to lower taxes in bad states and vice versa. In this way, social security provides some insurance to the uninsured young. Highly volatile benefits are motivated by the fact that the allocation of risk in the PAYGO system has a major influence on demand for capital. More specifically, highly volatile benefits strengthen the precautionary savings motive of the middle-aged agents. As a result, they save more and hold more capital to hedge their coming volatile benefits. In fact, in the absence of shortselling constraints, the crowding out effect generally associated with an unfunded system can be completely eliminated by the use of efficient risk sharing arrangements. The welfare effects of improved risk sharing in social security can thus be decomposed into a *direct* welfare effect and a *general equilibrium* (GE) effect. The direct welfare effect raises welfare by correcting an initially inefficient allocation of risk, whereas the welfare effects stemming from the GE effect are due to the higher average level of the capital stock.[7] The welfare gains associated with the risk allocations analyzed in this paper are highly correlated with the capital stock, indicating that the major part of the welfare gain associated with improved risk sharing actually comes from the GE effect.

I also find a considerable difference between the social optimum and all other economies, including the laissez-faire economy. Despite the large welfare gains associated with improved risk sharing, no economy actually comes close to the social optimum. The capital stock in the social optimum is roughly 3.5 times the capital stock in the laissez-faire economy, indicating that the social planner builds up a very

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[7] The fact that social security may be used to change national saving and investment is also discussed in Abel (2003).

large buffer to be able to smooth the consumption of future generations.

An intuition for the findings in this paper is that the welfare cost of exposing the old to aggregate risk is relatively small, as long as the risk is predictable and known in advance. The reason is that the old have had a lifetime for setting up their hedging portfolios. In contrast, the earlier in life agents are exposed to risk, the smaller are their possibilities to hedge.

The benchmark model is calibrated to be consistent with the risk free rate, the Sharpe ratio and the capital output ratio in the U.S. However, since the coefficient of relative risk aversion (RRA) is controversial, a sensitivity experiment is also carried out to evaluate the importance of this parameter. In this exercise, CRRA utility is used and the RRA coefficient is set to 2, while the rest of the calibration is kept constant. In every important aspect, the results are very similar to those found with Epstein-Zin utility. The welfare effects are still substantial, indicating that even if agents are only moderately risk averse, they still care a great deal about the allocation of risk, mainly because of the large GE effects at stake.

The results in this paper are closely related to earlier findings on intergenerational risk sharing. First, they are in line with the findings of Storesletten et al. (1998) and Krueger and Kubler (2002), who both find the major part of the (negative) welfare effects associated with social security to come from general equilibrium effects (i.e., it crowds out capital). Second, the finding of pro-cyclical benefits and taxes resembles the findings in Smetters (2002), where it is shown that the government can use negative capital income taxes (implying pro-cyclical wage taxes) to correct the "biological trading constraint" preventing living generations from negotiating contingent contracts with the unborn. These are also in line with Bohn (1998, 2003) who find the young in OLG models to be generally to exposed to aggregate risk.

This paper abstracts from population growth, demographic uncertainty, labor supply decisions, debt policy and preferences motivating a PAYGO system. Admittedly, these are all potentially important issues. For instance, it would be more realistic to consider an endogenous labor supply, since that would give the young one more degree of freedom. However, as shown by Olovsson (2004a), the response in labor supply to a tax increase will, to a large extent, depend on whether agents also work in home production. Introducing home production significantly complicates

the analysis. Debt policy is, of course, a natural candidate for handling issues of risk sharing. The question is then how to construct the debt policy response function in an efficient way. When hit by a bad productivity shock, the government can either increase the debt and provide safe benefits to the old, or keep it constant and reduce benefits. However, due to the general similarities between unfunded social security and debt, the use of debt as an instrument to handle risk sharing can in every relevant aspect be expected to resemble one using the PAYGO system. Most important might be the fact that it is difficult to provide a rational for the PAYGO system using standard preferences. However, Gonzalez-Eiras and Niepelt (2003) show that in a model with standard preferences, a PAYGO system of realistic size may be introduced and sustained in a political equilibrium.

2   The Economic Model

2.1   The Consumers

The basic model is a three-period overlapping generations model. Each generation is modeled as a representative consumer. There is one consumption good in each period and it perishes at the end of the period. The index $i = 0, 1$ and 2 is used to denote the young, the middle-aged and the old, respectively. An agent inelastically supplies labor for two periods and retires in the third period when old. During their working years, agents are endowed with a level of productivity e_i and they receive wage W. When retired, they collect the social security benefit, $\widetilde{\varphi}_t$.

There are two types of securities in the economy, bonds and capital denoted by b and k, respectively. Bonds are assumed to be in zero net supply, while the supply of capital is endogenous. A consumer born in period t has zero endowment of assets. This consumer makes a portfolio decision $a_{t+1,0} = (b_{t+1,0}, k_{t+1,0})$ in period t, when young; adjusts this decision to $a_{t+2,1} = (b_{t+2,1}, k_{t+2,1})$ in period $t + 1$, when middle-aged; and sells the portfolio in period $t + 2$, when old. As usual, a negative position in bonds or stocks denotes a short position in that asset. The price of the bond and the gross rate of return on capital are denoted by p and R, respectively, and the tax rate is denoted by τ_t. The budget constraints in period t are then given by

$$c_{t,0} \leq (1 - \tau_t)\, e_0 W_t - k_{t+1,0} - p_t b_{t+1,0} \qquad (4.1)$$

$$c_{t,1} \leq (1 - \tau_t)\, e_1 W_t + R_t k_{t,0} + b_{t,0} - k_{t+1,1} - p_t b_{t+1,1} \qquad (4.2)$$

$$c_{t,2} \leq \widetilde{\varphi}_t + R_t k_{t,1} + b_{t,1}, \qquad (4.3)$$

where (4.1), (4.2) and (4.3) are the budget constraints faced by the period t young, middle-aged and old, respectively.

Since this paper is concerned with the welfare effects of different risk allocations, it is important to use preferences that are, at least in principle, consistent with empirically revealed attitudes towards risk. It is well known that there is no way of fitting both the level of the risk-free rate and the risk premium with standard preferences (i.e., with power utility).[8] The more flexible utility function developed by Epstein and Zin (1989,1991) and Weil (1989) will therefore be used instead. If we denote the subjective discount factor by β, the Epstein-Zin-Weil objective function can be recursively defined by

$$U_{t,i} = \frac{1}{1 - \gamma} \left[(1 - \beta)\, c_{t,i}^{\frac{1-\gamma}{\theta}} + \beta \left[(1 - \gamma)\, E_t U_{t+1,i+1} \right]^{\frac{1}{\theta}} \right]^{\theta}, \qquad (4.4)$$

where θ is defined by $\theta = (1 - \gamma)/[1 - (1/\sigma)]$, γ is the coefficient of relative risk aversion and σ is the elasticity of intertemporal substitution. Note that when $\gamma = 1/\sigma$, i.e., when $\theta = 1$, (4.4) collapses to the standard time-separable power utility function with relative risk aversion, γ.

Finally, the following assumption is made:

$$U_{t,i} \equiv 0 \quad \text{for } i \geq 3,$$

which implies that the old do not buy any assets (and that altruistic bequests are

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[8] By construction, the basic power utility model makes the elasticity of intertemporal substitution the reciprocal of the coefficient of relative risk aversion. This is restrictive, since no theoretical or empirical evidence supports such a tight link between these two concepts. The strength of the Epstein-Zin-Weil model is that it allows us to break this link.

ruled out). The maximization problem of an agent aged $i \in \{0, 1\}$ is then

$$U_{t,i} = \max_{c_{t,i}, a_{t+1,i}} \frac{1}{1-\gamma} \left[(1-\beta) c_{t,i}^{\frac{1-\gamma}{\theta}} + \beta \left[(1-\gamma) E_t U_{t+1,i+1} \right]^{\frac{1}{\theta}} \right]^{\theta}. \tag{4.5}$$

2.2 Firms

There is a representative firm, which in each period uses labor and capital to produce the consumption good, according to a constant returns to scale production function. Since firms make their decision on how much capital and labor to hire after the realization of shock Z_t, they face no uncertainty and simply maximize their current-period profits. The aggregate production function takes the form

$$Y_t = Z_t K_t^{\alpha} L^{1-\alpha}, \tag{4.6}$$

where Z_t is a stochastic productivity shock, K_t the aggregate capital stock and L the aggregate labor supply.

In order to allow the total return on capital to vary somewhat independently from the wage rate, I assume a stochastic depreciation rate. Given aggregate consumption C, and the rate of depreciation on aggregate capital δ, the law of motion for aggregate capital is given by

$$K_{t+1} = Y_t - C_t + (1 - \delta_t) K_t. \tag{4.7}$$

2.3 The Government Sector

The government administrates a PAYGO social security system, i.e. it collects taxes from workers and pays out social security benefits to the retired. In the current U.S. social security system, there are basically three important factors determining the social security benefits received by an agent: his/hers average income, the replacement ratio and the evolvement of average wages. The first is important because the level of benefits when reaching the retirement age is based on lifetime earnings. However, for computational ease, I will not base benefits on agents' income histories,

but instead on the average labor income in the economy.[9] This average is denoted by $\overline{W} = \frac{(e_0 + e_1)}{2} E[W]$, where $E[W]$ is the unconditional expected long-run wage rate in the stationary economy (and e_0 and e_1 are once more age-specific productivities).

The second determinant of the social security benefits is the replacement ratio η, i.e. the rate at which social security replaces past earnings. In a world without uncertainty, the social security benefit would thus be given by

$$\varphi = \eta \overline{W}. \tag{4.8}$$

However, benefits are also related to the aggregate wage-index, i.e., the evolvement of aggregate labor income. More specifically, an individual's earnings are indexed to the average wage level at the time of retirement.[10] The variability of the aggregate wage rate thus influences the variability of benefits. The period t benefit can then finally be specified by the following two equations

$$\widetilde{\varphi}_t = a_0 \varphi + a_w \frac{W_t}{E[W]} \varphi \ , \tag{4.9}$$

and

$$a_0 + a_w = 1. \tag{4.10}$$

The wage-indexation scheme specified by the two equations (4.9) and (4.10) constitutes an easy way of transferring aggregate (wage) risk between agents participating in the social security system.[11] Equation (4.10) implies that unconditional expected benefits always equal $\eta \overline{W}$.[12] The importance of this equation is to ensure that only risk is transferred between taxpayers and retirees. In this paper, I am concerned with the welfare gains of improving risk sharing in social security, and

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[9] This assumption can be viewed as a rough first approximation to the highly redistributive nature of the actual link between actual earnings and benefits. The assumption significantly eases the computational burden as individuals' income histories are not needed as state variables. Finally, Olovsson (2004b) shows that these histories are not really quantitatively important.

[10] Actually, an individual's earnings are indexed to the average wage level 2 years prior to the year of eligibility, i.e., when the agent reaches the age of 62.

[11] The notation in (4.9) is chosen to illustrate that the benefit φ consists of a safe part and a risky part.

[12] However, no further restrictions are placed on a_0 and a_w.

violating (4.10) would shift the focus away from the issue of risk sharing to that of the optimal size of social security. This is obviously an important issue, but beyond the scope of this paper.

When $a_w = 0$, benefits are completely safe, in the sense that all productivity risk in social security is solely borne by the tax payers. This risk can then be shifted to the retirees, and arbitrarily increased by increasing a_w. Following Bohn (1998), the current U.S. social security system is modeled by setting $a_w = 1$. Benefits thus respond to aggregate wages with a factor of 1.

It may also be interesting to consider the alternative with capital-indexed benefits. In a world where aggregate uncertainty hits the economy according to (4.6), and the returns to labor and capital are perfectly correlated, the scheme given by (4.9) would in every aspect be equivalent to one where benefits are indexed to capital income or aggregate income. In the long run, labor and capital returns are indeed highly positively correlated in the U.S. and other OECD countries.[13] However, a stochastic depreciation rate was introduced in the previous section to make it possible for the total return on capital to vary somewhat independently from the wage rate. To analyze the implications of a PAYGO scheme where benefits are instead indexed to the total return on capital, I here introduce the following alternative function

$$\widetilde{\varphi}_t = a_0 \varphi + a_k \frac{R_t K_t}{E\left[R_t K_t\right]} \varphi \ , \tag{4.11}$$

which must once more be combined with (4.10).

Finally, the government is required to balance its budget in each period. Since the government bond is in zero net supply, the government budget constraint is given by

$$\widetilde{\varphi}_t = \tau_t L W_t. \tag{4.12}$$

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[13] Baxter and Jermann (1997).

2.4   Shortselling Constraints

Shortselling constraints may be a potentially important issue, since they restrict agents to smooth consumption across states. More specifically, shortselling constraints prevent agents from borrowing against their future income, when facing a bad shock. Agents may therefore find the allocation of risk in social security more important when subject to shortselling constraints restricting them in their financial behavioral. Most likely, these constraints are most severe for the uninsured young. Both cases with and without shortselling constraints will therefore be considered. A shortselling constraint can purely be motivated on realistic grounds: it is a well known fact that human capital alone does not collateralize major loans in modern economies (for reasons of moral hazard and adverse selection).

The shortselling constraints are given by

$$b_{t+1,i} \geq 0, k_{t+1,i} \geq 0 \quad i = 0, 1, 2.$$

3   Equilibrium

3.1   The Decentralized Competitive Equilibrium

I am now searching for a stationary equilibrium where decisions made in a given period are determined by the aggregate shock $s_j = (Z_j, \delta_j)$, $s_j \in S$, the aggregate capital stock K and the current wealth of the middle-aged $a_{-1} = (b_{-1}, k_{-1})$.[14] An equilibrium can be defined to consist of market clearing prices R, W and p and a set of age-specific functions $k'_0 = k_0 (s, K, a_{-1})$; $b'_0 = b_0 (s, K, a_{-1})$ and $k'_1 = k_1 (s, K, a_{-1})$; $b'_1 = b_1 (s, K, a_{-1})$ such that

(1) *The firm's profit maximization problem is satisfied*

$$R = \alpha \frac{Y}{K} + 1 - \delta \tag{4.13}$$

$$W = (1 - \alpha) \frac{Y}{L}. \tag{4.14}$$

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[14] The young are born with zero assets.

(2) *Individual optimization problems are satisfied, i.e.* $\{U_i, c_i, b'_i, k'_i\}_{i=0}^2$ *satisfies equations (4.5)*

(3) *Markets clear and aggregate quantities result from individual decisions*

$$K' = \sum_{i=0}^{1} k_i\left(s, K, a_{-1}\right) \tag{4.15}$$

$$B' = \sum_{i=0}^{1} b_i\left(s, K, a_{-1}\right). \tag{4.16}$$

(4) *The government budget constraint (4.12) is satisfied with equality.*

(5) *The resource constraint (4.7) holds with equality.*

All the above conditions should be considered as standard.

3.2   The Command Optimum

To properly evaluate different equilibria, the social optimum needs to be solved for. This optimum gives us the best possible outcome and is therefore a natural benchmark case. The social planner's problem is to maximize the welfare function at some initial date $t = 0$:

$$W_0 = E_0\left[\sum_{t=0}^{\infty} \beta^t U_t\right],$$

subject to the resource constraint (4.7) and where the initial capital stock K_0 and the past consumption of the middle-aged and old $c_{t-1,0}$; $c_{t-1,1}$; $c_{t-2,0}$ are given. The planner is assumed to use a constant discount factor equal to the agents' subjective discount factor. Because utilities are evaluated in expectation, the allocation will be efficient *ex ante*, contingent on the initial conditions. There is some controversy in the literature about *ex ante* versus interim efficiency.[15]   Under an interim perspective, agents born in different states of nature are considered to be distinct. A Pareto improvement will then require that no birth-contingent agent in any birth state is made worse off. For the purpose of this paper, i.e. policy analysis, interim

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[15] See, for example, Wright (1987).

efficiency is rather uninteresting, since policies almost always shift resources across states of nature and are therefore not comparable by interim standards.

The planner's first-order conditions provide the necessary conditions for ex ante efficiency

$$\frac{\partial E_t U_t}{\partial c_{0,t}} = \frac{\partial E_t U_{t-1}}{\partial c_{1,t}} = \frac{\partial U_{t-2}}{\partial c_{2,t}} \tag{4.17}$$

$$\frac{\partial E_t U_t}{\partial c_{0,t}} = \beta E_t \left[\frac{\partial U_t}{\partial c_{1,t+1}} R_{t+1}^K \right]. \tag{4.18}$$

The first equation is, in the words of Bohn (1998), "the distributional optimality conditions" linking the consumption of the agents alive in period t. The second equation is "the intertemporal optimality condition" which reveals how the planner allocates resources over time. Equation (4.18) is actually identical to the individual optimality condition for savings (i.e., the Euler equation). In contrast, however, the efficiency condition (4.17) is generally not satisfied by the market allocation.[16] For time-separable utility, for example, (4.17) calls for a deterministic link between the contemporaneous consumption of all agents alive in a given period.

4   Computation of the Equilibrium

I use a spline collocation algorithm to numerically approximate the equilibrium. The strength of the three-period model is twofold: (i) it is sufficiently rich to allow agents to intergenerationally share risk and (ii) it allows me to rely on standard techniques when solving for the equilibrium. To solve their optimization problems, agents need to keep track of the aggregate capital stock, the (constant) net supply of bonds and the wealth of the middle aged (alternatively, the wealth of the old)

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[16] See Bohn (1998) for a more profound discussion.

$$\underbrace{k_{0,t-1}\left(1+r_K\right)+b_{0,t-1}}_{\text{Wealth of the mid-aged}}+\underbrace{k_{1,t-1}\left(1+r_K\right)+b_{1,t-1}}_{\text{Wealth of the old}}=K\left(1+r_K\right)+B. \qquad (4.19)$$

The approximation of the equilibrium is therefore straightforward.

A more serious challenge is the numerical computation of the social optimum. Generally, the social optimum is easier to compute than the decentralized equilibrium. With Epstein-Zin preferences, however, one also needs to keep track of consumption histories. I approximate this equilibrium by using the aggregate capital stock, the period $t-1$ consumption of the currently mid-aged and a variable summarizing the consumption history of the currently old as state variables. The result is a very large state space, consisting of three continuous endogenous state variables. In addition, the system of equations shows a very high degree of curvature (θ is equal to -51 with the chosen calibration, thereby implying that some equations are raised to the power of -52).[17]

5 Calibration

5.1 Income Profiles and Social Security

The average share of wage income going to the young $\frac{e_0}{e_1}$. Recall that the productivity of the young and the old, respectively, is given by e_0 and e_1. Labor endowments are deterministic and set so that $\frac{e_0}{e_1}=0.70$. This is basically consistent with the estimates from PSID data in Storesletten et al. (2003).

The expected replacement rate η. In the U.S., the current payroll tax is 12.4 percent, and benefits replace 43.7 percent of the average pre-taxed wage.[18] However, up to 85 percent of the received benefits may also be subject to income tax. Since I do not want to put any restrictions on the possible allocations of risk

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[17] Due to the high degree of curvature, a tensor product approach to the three-dimensional approximation is called for. The total number of unknowns then becomes: the No. of gridpoints times the No. of exogenous states times the No. of endogenous variables. Even with a simple 10x10x10 grid and 4 states, the total number of unknowns becomes 12000.

[18] See, for example, Mchale (1999).

between taxpayers and retirees, I will not assume benefits to be taxable.[19] Instead, I set $\eta = 0.4$, in the sense of benefits replacing 40 percent of the (unconditional expected) average life-time after-tax wage in the economy. This corresponds to a replacement ratio of 43.7 percent of the average pre-taxed wage, where roughly 50 percent of the received benefits are subject to income tax.

5.2   Aggregate uncertainty

Aggregate productivity is assumed to be driven by a four-state Markov process with state space $S = [s_1, s_2, s_3, s_4]$ and the transition probability matrix $\pi = (\pi_{ij})$.[20] An aggregate state is characterized by a TFP shock, and a depreciation rate $s_j = (Z_j, \delta_j)$, $s_j \in S$. The four states are in order

$$
S = \begin{cases}
(1-v), \left(\overline{\delta}+\zeta\right) \\
\left(1-\frac{v}{3.5}\right), \left(\overline{\delta}-\zeta\right) \\
\left(1+\frac{v}{3.5}\right), \left(\overline{\delta}+\frac{\zeta}{3.5}\right) \\
(1+v), \left(\overline{\delta}-\frac{\zeta}{3.5}\right)
\end{cases} ,
$$

where $\overline{\delta}$ is the average depreciation rate.[21] I set $\overline{\delta} = 0.626$, implying the average annual depreciation rate to be between 4 and 5 percent.[22] The stochastic process is assumed to be *i.i.d.* over time. Although it is well established that aggregate productivity shocks are highly autocorrelated at annual and quarterly frequencies, there does not seem to exist any conclusive evidence indicating such positive serial correlation at generational frequencies (i.e. 20-30 year periods).[23] As a benchmark, aggregate shocks are therefore assumed to be uncorrelated across time.

   With the above representation, the aggregate state s_1 is characterized by a bad TFP shock and a bad depreciation shock whereas s_4 is given by a good TFP shock,

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[19] Obviously, if benefits are taxable, both workers and retirees will be equally exposed to tax risk.

[20] This assumption implies that markets are somewhat incomplete. The setting is needed to account for a high, but not perfect, correlation between return to capital and return to labor.

[21] The four aggregate states are somewhat asymmetric since this setup makes the numerical computation of the social optimum somewhat simpler. However, the results are not sensitive to the asymmetric setup.

[22] As in Cooley and Prescott (1995).

[23] This assumption is also in line with several other papers dealing with OLG-models with two or three periods. See, for instance, Bohn (1999), Kreuger and Kubler (2002) and Smetters (2002).

and a good depreciation shock. In aggregate states s_2 and s_3, the TFP shock and the depreciation shocks move in opposite directions. It is also assumed that $\pi_1 = \pi_4$ and $\pi_2 = \pi_3$.

In order to pin down v, ζ and π_1, I set out to match the following statistics.[24]

The coefficient of variation of the 20-year aggregate income, $\frac{\sigma(y)}{E(y)}$. It is is rather problematic to calibrate this statistic, due to the fact that even a century-long time series only provides five non-overlapping observations, resulting in large standard errors of the point estimates. I follow Constantinides et al. (1998), and set the coefficient of variation of the 20-year aggregate income to 0.2.

The coefficient of variation of the 20-year aggregate capital, $\frac{\sigma(K)}{E(K)}$. Capital in this model is not just a claim to corporate dividends, but to all risky capital in the economy. According to Baxter and Jermann (1997), the return to labor is less volatile than the return to capital. In the U.S., the volatility of the return to labor is estimated to be 85 percent of the volatility of the return to capital. Lacking a better estimate, I set $\frac{\sigma^2(K)}{E(K)} = 1.25\frac{\sigma^2(Y)}{E(y)}$, which makes the model consistent with that estimate.[25]

The cross-correlation of aggregate labor income and aggregate capital income $corr\,(w_t, R_t)$. According to Baxter and Jermann (1997) and Bohn (1999), the return to capital, and the return to labor are highly correlated in the long run. With these findings in mind, I set $corr\,(w_t, R_t) = 0.9$.

5.3   Preferences

Since this paper is concerned with the welfare effects of different allocations of risk, it is important to assign values to the preference parameters that actually reflect households' attitudes to risk. The preference parameters in the model are the RRA coefficient γ, the coefficient of intertemporal substitution σ, and the subjective discount factor β. To calibrate these preference parameters, I set out to make the model consistent with the following empirical findings:

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[24] Very roughly, ν determines the variation of aggregate income, ς determines the variation of aggregate capital and π_1 determines the correlation between returns to labor and capital. The calibration procedure involved a simple grid search algorithm.

[25] Due to the Cobb-Douglas technology, aggregate income and labor income are equally volatile.

The annualized capital output ratio $\frac{K}{Y}$. This statistic is calibrated to be 3.3, which is the number reported in Cooley and Prescott (1995).

The average annual real risk free rate r_f. The risk free rate varies over time so it is not really obvious what is a reasonable level for the safe real rate. When Mehra and Prescott announced the existence of a equity premium puzzle in 1985, they found the average riskless real interest rate to be 0.8 percent for the period 1947-1976.[26] Since the mid-eighties, however, the average real risk free rate has been somewhat higher than in the period studied by Mehra and Prescott. According to Campbell (1999), the average real risk free rate was 1.955 percent for the period 1891-1995.[27]

The Sharpe ratio, $\frac{E[r_k]-r_f}{std(r_k)}$. Since in the model, capital is not just a claim to corporate dividends but to all risky capital in the economy, I do not try to match the observed equity premium, the reason, of course, being that equity returns are much more volatile than capital return. Instead, I set out to match the Sharpe ratio, which in this case is the risk premium households demand for holding risky capital, divided by the standard deviation of the return to capital. Constantinides et al. (2002) report data on 20-year holding period real returns and standard deviations for bonds and equity. The implied Sharpe ratio can then be calculated to 1.65.

I set $\gamma = 18$, $\sigma = 0.75$ and $\beta = 0.68$, which generates an annualized capital ratio of 3.3, an interest rate of 1.85 and a Sharpe ratio of 1.61 in the model without borrowing constraints and 1.67 in the model with borrowing constraints.[28] A β equal to 0.68 corresponds to an annual discount factor of 0.98. To sum up, technology and preference parameters are set to

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[26] Mehra and Prescott (1985).

[27] The annual return in the model is defined as $100\left[R_f^{1/20} - 1\right]$, since a period is assumed to be 20 years.

[28] Once more, very roughly, β determines the capital output ratio, σ determines the risk-free rate and γ determines the Sharpe ratio.

Table 4.1: P̲RODUCTION AND PREFERENCE PA̲RAMETERS

v	ζ	π_1	β	γ	σ
0.295	0.274	0.175	0.68	18	0.75

6   Results

6.1   Long-Run Equilibria

Results are presented in this section. All results presented in table 4.2 are expressed in relation to the current U.S. economy and, consequently, the capital stock and the welfare gain of being born into this economy are both normalized to 1.[29] Welfare gains are expressed from the perspective of an unborn agent. More precisely, a welfare gain of x percent of being born into a specific economy implies that the per-period consumption in the present U.S. must be increased by x percent, for the agent to be indifferent between being born into these two economies. Wage-Index* and Capital-Index* refer to economies with optimized indexation with respect to wages and capital, respectively. The laissez-faire economy is characterized by the absence of a state, taxes and thus, social security.

I summarize the results as follows.

- **General equilibrium effects are substantial**. First, the capital stock in the laissez-faire economy is roughly 64 percent higher than in the current U.S., implying the crowding out effect to be 39 percent. Interestingly, this is very close to the empirical estimate of 38 percent in Feldstein (1974).[30] Second, the allocation of risk in the PAYGO system heavily influences the demand for capital. In the absence of shortselling constraints, the capital stock is actually higher under optimized wage and capital-indexation than in the laissez-faire illustrating that the crowding out effect can be completely

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[29] All economies are simulated for 10 000 periods.

[30] More specifically, Feldstein uses U.S. data for the period 1929 through 1971 to assess how the introduction and growth of social security have affected aggregate personal savings and the national accumulation of capital. See also Feldstein (1996).

Table 4.2: CAPITAL STOCK AND WELFARE GAINS

	SHORTSELLING		NO SHORTSELLING	
	$E[K]$	WELFARE GAIN	$E[K]$	WELFARE GAIN
U.S.	1	1	1	1
SAFE BENEFITS	0.9	-10.09	0.95	-9.29
WAGE-INDEX*	1.69	14.86	1.46	12.09
CAPITAL-INDEX*	1.71	13.83	1.40	9.41
LAISSEZ-FAIRE	1.64	19.89	1.63	19.59
SOCIAL OPTIMUM	5.8	50.88	6.3	52.19

eliminated. However, welfare is still higher in the laissez-faire economy, at least partially as a result of the fact that agents in the economies with social security are forced to service the debt associated with providing an unfunded transfer to the initial generations (those who were retired when the system was introduced), whereas this is not the case for agents in the laissez-faire.[31] Finally, wage-indexation seems to be slightly preferable to capital-indexation. Recall that in an OLG settings, it is the young that need to be insured and the risk they are facing is wage-risk.

- **Welfare effects are very large**. By all standards, the welfare gains associated with the different economies are huge. Welfare gains are also obviously very highly correlated with the size of the capital stock, indicating that the major part of the welfare gain associated with improved risk sharing actually comes from the GE effect. These results are thus in line with the findings of Storesletten et. al (1998) and Krueger and Kubler (2002), who both find

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[31] The welfare gains of the laissez-faire economy should therefore be corrected for these transfer effects for it to be the relevant object of comparison (as in Storesletten et al. (1999)).

the major part of the (negative) welfare effects associated with social security to come from general equilibrium effects (i.e., it crowds out capital). Finally, welfare effects are of similar magnitude, irrespective of whether agents are subject to shortselling constraints. Thus, there seems to be no support for the view that the allocation of risk in social security is without importance, when agents can trade in several assets. On the contrary, welfare effects are actually somewhat higher in the absence of shortselling constraints.

- **Safe benefits are significantly worse than the status quo**. The welfare loss of being born into an economy with safe benefits is rather large: roughly 10 percent of per-period consumption. This welfare loss is due to the fact that the regime with safe benefits implies that the direct welfare effect and the GE welfare effects are both negative. The direct welfare effect is obviously negative, since safe benefits require contra-cyclical taxes, which inevitably increases the exposure of the young to aggregate risk.[32] The GE effect is also negative, since the capital stock is significantly lower under the regime of safe benefits.

- **A considerable difference between the social optimum and other economies, including the laissez-faire**. The capital stock in the social optimum is roughly 3.5 times the capital stock in the laissez-faire economy, indicating that the social planner builds up a very large buffer to be able to smooth the consumption of future generations. Consequently, the welfare gain of being born into the social optimum is also considerably larger than the gain of being born into the laissez-faire economy. These results illustrate the quantitative importance of *ex ante* inefficiency, and they show that agents actually find the risk of being born into the wrong state a serious issue.

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[32] As stated in the introduction, the young are already too exposed to aggregate risk in the decentralized equilibrium when the intertemporal elasticity of substitution for consumption is less than 1 (see Bohn (1998)).

6.2   The Allocation of Risk

Table 4.3 gives the optimal coefficients of the respective schemes (i.e., equations (4.9) and (4.11)). The coefficients are all positive, thereby implying that benefits,

Table 4.3: COEFFICIENTS FOR EFFICIENT RISK SHARING

	NO SHORTSELLING	SHORTSELLING
WAGE-INDEX*	$a_w = 4.39$	$a_w = 4.28$
CAPITAL-INDEX*	$a_k = 2.72$	$a_k = 3.0$

as well as taxes, should be pro-cyclical. This arrangement eases the risk exposure of the young, since they are subject to lower taxes in bad states, and vice versa. Social security thus provides some insurance to the uninsured young. As a contrast, the regime with safe benefits calls for contra-cyclical taxes which increase the risk exposure of the young, since they are forced to pay higher taxes in bad states. These results resemble the findings of Smetters (2002), who shows that the government can use negative capital income taxes (implying pro-cyclical wage taxes), to correct the "biological trading constraint" preventing living generations from negotiating contingent contracts with the unborn.[33]   These are also in line with Bohn (1998, 2003) who find the young in OLG models to be generally exposed to aggregate risk.[34]

In addition, the coefficients are all much larger than 1. Highly volatile benefits strengthen the precautionary savings motive of the middle-aged agents. As a result, they save more and hold more capital to hedge their coming volatile benefits. This is illustrated in Fig. 4.1, where capital demand is plotted as a function of a_w when agents are subject to shortselling constraints. Capital demand by the young is basically constant, whereas capital demand by the middle-aged is strictly increasing in a_w. Maybe somewhat surprising, the magnitude of the respective coefficients

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[33] A negative capital tax is valid when production takes the Cobb-Douglas form, the depreciation rate is less than 100 percent, and the intertemporal substitution elasticity is equal to one.

[34] Moreover, Bohn finds wage-indexed social security (i.e. $a_w = 1$) to be a neutral policy, implying that a_w must be larger than 1 to correct the inefficient outcome.

in table 4.3 is very much the same, irrespective of whether agents are subject to shortselling constraints.
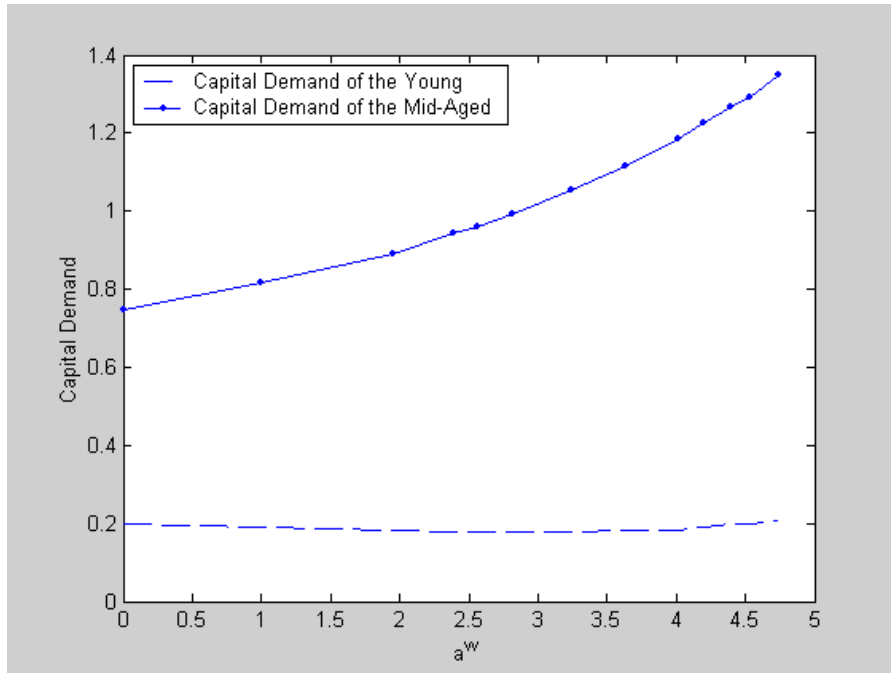

Figure 4.1: ASSET DEMAND WHEN SHORTSELLING IS NOT ALLOWED

Since the value of the RRA coefficient is controversial, I carry out a sensitivity experiment in the Appendix with respect to this parameter. Instead of Epstein-Zin preferences, power utility (CRRA) is used, i.e., the instantaneous utility for household i in period t is specified by

$$U\left(c_t\right) = \frac{c_t^{1-\gamma}}{1-\gamma}, \tag{4.20}$$

where γ is set equal to 2. The rest of the parameters are kept constant, however (only the case with short selling constraints is considered). Comparing table 4.2 and table 4.5 in the Appendix, we see that the results from this exercise with a more moderate RRA coefficient are very similar to those presented above. The actual numbers are somewhat changed but basically, all the main findings above still go through. GE effects are very large, welfare effects are substantial, safe benefits generate the worst outcome and there is still a substantial difference between the social optimum and all

other economies, including the laissez-faire. This result indicates that even if agents are only moderately risk averse, they still care a great deal about the allocation of risk, mainly because of the large GE effects at stake.

Finally, if agents are less risk averse, they respond less to changes in the allocation of risk. As a result, to generate GE effects, benefits must be made even more volatile than with Epstein-Zin utility (compare table 4.3 with table 4.6 in the Appendix).

6.3   Transitions

In this section, I carry out unexpected transitions from the current U.S. to the different economies. The results are presented in table 4.4.[35]

<div align="center">

Table 4.4: UNEXPECTED TRANSITIONS

WELFARE GAIN

	NO SHORTSELLING	SHORTSELLING
U.S.	1	1
SAFE BENEFITS	-4.29	-5.16
WAGE-INDEX*	-1.62	-0.51
CAPITAL-INDEX*	0.6	-0.13
LAISSEZ FAIR	3.17	2.14
SOCIAL OPTIMUM	8.59	9.38

</div>

Almost all welfare gains are turned into losses when the transition is considered. The main explanation is that since these transitions are unexpected, the agents alive at the introduction of the new regimes are taken from a relatively safe environment and placed in a much more risky one, without a chance of hedging themselves against

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[35] Results are averaged over 100 randomly chosen initial allocations in the benchmark economy called the U.S.

this new risk. The long-run gains found in the previous section rest upon the notion that productivity risk should be predictable and placed upon those agents who actually have the possibility of hedging this risk (i.e., the old). In the unexpected transitions, these hedging possibilities are simply not just there, obviously resulting in welfare losses.

However, since the welfare gains found in the previous section are so large, there should be some way of extracting these gains, by choosing a more sophisticated transition policy. There are several possibilities from which to choose. The most straightforward way is probably just to announce the transition several periods in advance. Another possibility could be that the government hedges the first generations in the transition to some degree, either by going into debt, or decumulating a buffer they have built up a before the transition. I leave this for future research.

7   Conclusions

The importance of improved risk sharing in social security has been analyzed. The findings are that improved risk sharing in social security may imply very large welfare gains. The welfare gain of being born into an economy with efficient wage-indexation is between 12 and almost 15 percent of per-period consumption in the current U.S., depending on whether agents are subject to shortselling constraints. In contrast, the welfare loss of being born into the inefficient economy with completely safe benefits is roughly 10 percent of the per-period consumption. Efficient risk sharing in social security implies highly volatile and pro-cyclical benefits and tax rates. The pro-cyclical arrangement eases the risk exposure of the young, since they they are subject to lower taxes in bad states and vice versa. In this way, social security provides some insurance to the uninsured young. Highly volatile benefits are motivated by the fact that the allocation of risk in the PAYGO system has a major influence on demand for capital. More specifically, highly volatile benefits strengthen the precautionary savings motive of the middle-aged agents. As a result, they save more and hold more capital to hedge their coming volatile benefits. In fact, in the absence of shortselling constraints, the crowding out effect generally associated with an unfunded system can be completely eliminated by the use of efficient risk sharing arrangements. The

welfare gains associated with the risk allocations analyzed in this paper are highly correlated with the capital stock, indicating that the major part of the welfare gain associated with improved risk sharing actually comes from the GE effect.

I also find a considerable difference between the social optimum and all other economies, including the laissez-faire economy. Despite the large welfare gains associated with improved risk sharing, no economy actually comes close to the social optimum. The capital stock in the social optimum is roughly 3.5 times the capital stock in the laissez-faire economy, indicating that the social planner builds up a very large buffer to be able to smooth the consumption of future generations.

An intuition for the findings in this paper is that the welfare cost of exposing the old to aggregate risk is relatively small, as long as the risk is predictable and known in advance. The reason is that the old have had a lifetime for setting up their hedging portfolios. In contrast, the earlier in life agents are exposed to risk, the smaller are their possibilities to hedge.

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8 Appendix

8.1 A Sensitivity Experiment

8.1.1 The Case of CRRA Utility

In this section, the RRA coefficient is set to 2, in order to evaluate the importance of this parameter. Except for this change, the rest of the calibration is kept constant. Only the case with short selling constraints is considered. The results are presented in tables 4.5 and 4.6.

Table 4.5: CAPITAL STOCK AND WELFARE GAINS, RRA=2

	NO SHORTSELLING	
	$E[K]$	WELFARE GAIN
U.S.	1	1
SAFE BENEFITS	1.02	-1.74
WAGE-INDEX*	2.00	9.20
CAPITAL-INDEX*	2.03	7.87
LAISSEZ-FAIRE	1.95	28.09
SOCIAL OPTIMUM	3.03	43.90

Table 4.6: COEFFICIENTS FOR EFFICIENT RISK SHARING, RRA=2

	NO SHORTSELLING
WAGE-INDEX*	$a_w = 6.43$
CAPITAL-INDEX*	$a_k = 4.71$

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