



Models of Electoral Competition

Three Essays in Political Economics

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Abstract

This thesis consists of three essays in political economics.

In **“The Swing Voters’ Blessing”**, I model elections with quality differences between two ideological candidates. The quality differences are only observable to a limited number of informed voters. I show that if uninformed voters follow an optimal strategy of only making their voting decisions dependent on their ideological position relative to the median voter, the candidate who is preferred by the median voter wins. Furthermore, I show that the existence of boundedly rational uninformed voters who always support the candidate whose policy offer is most attractive increases the welfare of the majority of voters. It forces candidates to announce positions closer to the median voter’s bliss point. This is "The Swing Voters’ Blessing".

“Lobbying and Elections” contributes to the literature on lobbying. This literature is large, but only a few papers allow for the interaction of post-election lobbying and the voting decision of forward-looking voters. Besley and Coate (2001) use their well-known citizen candidate framework and find that if citizen candidates with sufficiently extreme preferences are available, lobbying has no influence on equilibrium policy. I show that this result does not apply in a more realistic model with ideological parties instead of citizen candidates because the parties cannot adjust their policy positions. In a two-party system, even if forward-looking voters are aware that lobbying will take place, their choice between policies is different when lobbies do and do not exist. However, often the average and/or the median voter are better off with lobbying.

“Lexicographic Voting” reconsiders the division of the literature into models with forward-looking voters and models with backward-looking voters by developing a model that incorporates motives from both literatures. As long as there is no uncertainty about preferences and parties can commit in advance to the ideological dimension of policy, but not to a maximal level of rent extraction, voters can constrain the latter to the same extent as in a purely backward-looking model. At the same time, the policy preferred by the median voter is implemented as in a standard

forward-looking model of political competition. Voters achieve this outcome by following a simple lexicographic voting strategy. They cast their vote in favor of their favorite policy position whenever parties offer different platforms, but make their vote dependent on the incumbent parties' performance whenever they are indifferent. When uncertainty about the position of the median voter is introduced into the model, voters have to accept higher rent payments, but they still retain some control over rent extraction.

Für meine Eltern

Acknowledgments

I had never seriously thought about studying economics until I finished school and began my year of military service. Even then, I only considered economics as a minor field of study to be combined with a history or philosophy major. However, some energetic economics professor at the University of Potsdam changed my mind and I decided to become a "Diplomvolkswirt" after his talk at some information day for potential future students.¹

Nonetheless, I did not begin my studies in Potsdam but at Humboldt-Universität zu Berlin. Little did I know at that time how much difference the decision to study economics in Berlin and not in Potsdam would make. In Potsdam, I would have had a much more applied and traditional German economics education. I think the difference might very well be comparable to the difference between studying physics or engineering. It seems unlikely that I would have ended up in an American style graduate program if I had begun my studies in Potsdam.

Several years later, I became a graduate student in Stockholm. Here, I found a program with many inspiring teachers and great fellow students. I can mention only a few.

First and foremost, I would like to thank my advisor Torsten Persson for his great support and patience. He read all my research many times and made many useful proposals and suggestions. Last but not least, not only did he teach me how to do research but also how to transform it into readable papers.

I appreciate very much that he is generous with his advice but does not impose his opinion on his students and I enjoyed the freedom I had to pursue my own research. Another great advantage of having Torsten as an advisor is that he has many areas of expertise so that one can always switch to a different field of economics or from theoretical to empirical research.

I still remember when I first heard the name of my future advisor. When I told a fellow student that I was going to write a diploma thesis about "Central Banks from a Public Choice Perspective", he told me that there was a new book by Persson and Tabellini with the title "Political Economics" that I should probably have a look at. I did not since the book was not available in our university library. At that time I did not know that Torsten had begun his career as a macroeconomist and

¹After consulting the Homepage of the University of Potsdam I am almost sure that the energetic economist was Wilfried Fuhrmann.

that the book contains three chapters on monetary politics that could have been very useful for my thesis. In the end, I had the book ordered for our chair when I worked for a few months as a research assistant at the European Business School in Oestrich-Winkel and knew already that I was going to become a Ph.D student in Stockholm. Probably the book is still at the European Business School and perhaps the fact that I ordered it inspired someone who read it later to work on political economics.

Special thanks belong also to many other researchers at the IIES. David Strömberg always had time for me when I came by his office and provided many valuable insights when I presented my research in seminars. He gave me important advice on how to present my results in "The Swing Voters' Blessing".

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My beloved girlfriend Rongrong Sun read all of the papers several times and made many valuable comments and suggestions. She had always time for me when I needed her help or her advice. Last but not least, she suggested at least half of the commas you can find in the final manuscript.

Christina Lönnblad helped with editing the final version of this thesis even during her holidays and was always available when there was a question or a problem during my time at the IIES. This thesis would be much less well written without her generous help.

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I would also like to thank all the other administrators at the IIES, the department of economics and Stockholm University in general who have helped to make my time in Stockholm a more pleasant experience. I would like to thank John Hassler for being a very accessible mentor in the second year of my studies. Robert Östling and Masayuki Kudamatsu both gave valuable advice what literature could be relevant for

my research. Jens Josephson, Per Krusell, Michael Neugart and Elena Paltseva all provided important comments on "The Swing Voters' Blessing". James M. Snyder also provided comments on the paper and gave me the opportunity to present it at the MIT Political Economy Breakfast. Karl Wärneryd's course at Stockholm School of Economics inspired me to write "The Swing Voters' Blessing". Without his course I would not have been familiar with the information aggregation approach to elections. Even if the actual model was written much later, the basic idea to combine the Downsian approach to elections with the information aggregation approach was conceived already then. Jörgen Weibull several times gave me the opportunity to discuss my research with him. In addition, he was my favorite teacher (Mathematics 1) in the Stockholm graduate program.

I would like to thank Ruixue Jia for always being willing to listen to my random thoughts. Moreover, she has read most of my papers and provided great help in finishing them. Marta Lachowska was a true friend and very generous with her advice. Without her help I would still not have figured out how BibTeX can be used in combination with Scientific Workplace.² Moreover, we produced a fabulous "how to give a job market talk" video together that is available upon request.

Emilia Simeonova, Sergei Koulayev and Pedro Brinca provided great company especially in the last weeks before bringing my thesis to the printer. Heng Chen was my best friend in the first years in Stockholm and I would like to thank him for *henging out* with me before deserting to Zurich. My Italian former office mate Ettore had a great influence on me. I learned how to express myself by using my hands extensively when talking. This will come in handy when I move to Italy.

David von Below provided extremely helpful TeX advice. Without him this thesis would much less reflect my ideas about how the final product should look like. Olle Folke provided many of the templates that were needed for producing a thesis of the required format. I also want to thank all my other fellow graduate students at the IIES, the department of economics and last but not least the Stockholm School of Economics for all the help and friendship they provided.

Moreover, I would like to thank Ruchir Agarwal for making me feel at home when I spent one year of my graduate studies at the economics department of Harvard.

I would also like to mention the IIES espresso machine and thank whoever decided to buy it. As with many great things too often taken for granted, its true value is only appreciated when it does not work.

²Even with her help it took about a week and it leads to problems once you have to put your thesis document together. Think twice before you start using it.

I also intended to thank all the people who played football on Wednesdays regularly. However, I came to the conclusion that my fellow players should actually thank me for making them look good in comparison.

Last, but most importantly, I would like to thank my parents for providing me with lots of support over many years.

Stockholm, August 2010

Jan Klingelhöfer

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Chapter 1

Introduction

This thesis consists of three essays in theoretical political economics. All three essays model different aspects of elections in a democracy.

"The Swing Voters' Blessing" deals with the consequences of limited information and limited rationality of voters and come to the surprising, but positive, conclusion that such limitations might actually increase voters' welfare. "Lobbying and Elections" reconsiders the influence of interest groups on policy outcomes taking into account feedback effects on elections. Finally, "Lexicographic Voting" shows the compatibility of prospective and retrospective voting in a model with rent-seeking parties that compete on a spatial policy dimension.

With the exception of the second part of the first essay, I restrict myself to the rational choice approach of modeling elections. Voters as well as policy makers are assumed to maximize their utility and to use all information available to them in an optimal way. The first essay introduces a plausible behavioral assumption about how uninformed voters cast their votes and the results of this approach are contrasted with the rational choice outcome.

Some history of thought Rational choice is (still) part of the standard methodology in almost all of economics and also has a great influence in formal political science. It was used by all early contributors to formal models of elections. The most influential among them were the three economists Kenneth Arrow, Duncan Black and Anthony Downs.¹

Only later, mathematical models were also established in political science by

¹It should be mentioned that some first attempts to use formal methods to model group decision making go back much further in history to Condorcet (1785) who developed his famous paradox and the less well-known jury theorem. More about the jury theorem can be found Chapter 2.

William Riker. Riker called his approach "positive political theory" and was one of the first political scientists to use game theory in the modeling of politics (Riker 1962). By now, game-theoretical analysis is well established in political science and contributions to the formal modeling of elections are made by political scientists as well as economists.

A third tradition that has influenced the modeling of collective decision making is the public choice approach developed by Buchanan and Tullock (1962). Often, the term public choice is used to describe all uses of economic tools to study problems that are traditionally in the province of political science.² However, there is also a narrower definition of "public choice" which defines it as an alternative approach to standard public finance. In contrast to most of public finance, public choice rejects the assumption of a benevolent dictator as the decision maker. Instead, public choice scholars assume that politicians maximize their own utility.

Arrow's work was concerned with theoretical limitations of collective decision making summarized in his famous impossibility theorem (Arrow 1963) which can be seen as a clarification and generalizations of problems with voting procedures known ever since Condorcet (1785) pointed out his paradox of voting. Black, on the other hand, showed that with some additional assumptions on voters' preferences (single peakedness), the existence of an equilibrium in a voting game could be guaranteed. He developed the median voter theorem in the setting of committee decision making (Black 1948). Subsequently, Downs was the first researcher who applied the median voter theorem to a model of democracy. In a simple spatial model of elections with two parties solely interested in vote maximization, he used Black's median voter theorem to derive policy outcomes in a democracy directly from the preferences of the voters (Downs 1957). With this work, he laid the foundations for most rational choice models of elections.³

My essays are closely related to the Downsian approach of modeling electoral competition. They retain several of Downs' original assumptions that have become standard by now, for example the restriction to a world with only two parties.⁴

²This broad definition is more or less synonymous with my use of the term "political economics", although the latter might indicate the use of more rigorous methods than implied by the term "public choice".

³The one important exception is models where elections aggregate information rather than preferences. The interested reader can find a short introduction to this type of model in "The Swing Voters' Blessing."

⁴Naturally, there are now many models with more than two parties or candidates. Nonetheless, what could be called the core of political economics almost exclusively consists of two-party models. To confirm this, it is sufficient to have a look at the content of a standard textbook such as Persson

Part of the secret of the longevity and influence of Downs' approach is without any question that he managed to cut down his model to the bare essentials so that other researchers could build on his research by adding institutional detail to answer more specific questions.

Existing models combined in novel ways Neither do I try to provide new work-horse models that other researchers would hopefully build on, nor do I put more institutional details into existing work-horse models. What all three of my essays instead have in common is that they combine different existing and well established models in new ways.⁵ This provides a check on the consistency of the core models of electoral competition with each other. For example, standard models of rent seeking and political accountability assume voters who cast their ballots according to past performance of the incumbent, while the Downsian approach to electoral competition assumes prospective voting. Text books usually introduce both approaches without much discussion of potential conflicts. In the third essay, I show that combining the two approaches does not necessarily lead to inconsistencies. This result is not trivial because models of accountability usually assume an indifference of the voters with respect to the candidates. Introducing an additional spatial component of policy means that this indifference can no longer be taken as exogenously given.

All three models have in common a standard spatial policy dimension on which voters disagree on what policy should be implemented, just as in Downs (1957). The main difference to Downs is that politicians and parties are not assumed to be (only) office motivated, but either have their own ideology or want to acquire rents for themselves.

The Swing Voters' Blessing In the first essay, "The Swing Voters' Blessing", I add an additional valence dimension to the standard Downsian spatial policy dimension. In other words, the candidates running for election are of different quality. The idea of combining a policy dimension with candidates of different quality and ideology is not new. What is new is that I introduce imperfect information of the electorate. Not all voters can observe the quality of the candidates. Essentially, I am combining a standard spatial model of policy determination in which elections are a way of aggregating voters' preferences with the alternative approach of modeling

and Tabellini (2000). The most important exception is probably citizen candidate models that completely ignore parties (Osborne and Slivinski 1996; Besley and Coate 1997).

⁵I would like to point out that I realized this similarity in approach only after finishing the three essays.

elections as a mean of aggregating information. The standard median voter result does not apply in this combined framework because the difference in the quality of candidates gives the higher quality candidate an opportunity to deviate from the median position and to win the elections nonetheless. Surprisingly, the lack of information by some voters either has no effect on policy (as long as all voters are fully rational) or if some of the voters are boundedly rational and follow the most plausible behavioral strategy even leads to a policy closer to the standard median voter result.

Lobbying and Elections In the second essay, "Lobbying and Elections", I combine a spatial model of elections with a model of post-election lobbying. So far, post-election lobbying and elections were mostly dealt with in different models. However, Besley and Coate (2001) is an important exception. They use their well-known citizen-candidate framework and find that if citizen candidates with sufficiently extreme preferences are available, lobbying has no influence on implemented policy. I show that this result does not apply in a more realistic model with ideological parties instead of citizen candidates because the parties cannot adjust their policy positions. In a two-party system, even if forward-looking voters are aware that lobbying will take place, their choice between policies is different when lobbies do and do not exist. Nonetheless, voters are often better off with lobbying.

In addition to the main result, I provide a discussion clarifying the reasons for the differences in policy outcomes between the approaches in my paper and in Besley and Coate (2001) on the one hand, and models without elections taking place before the lobbying stage, presented for example in Grossman and Helpman (2001), on the other hand. The differences are not only due to the additional election stage, but also to the fact that my model (as well as that of Besley and Coate) is not a model of special interest, but of general interest lobbying.

Lexicographic Voting In the third essay, "Lexicographic Voting", I combine a spatial model of elections with a model of political accountability in the tradition of Barro (1973) and Ferejohn (1986). The combination of prospective and retrospective voting motives could easily be dealt with in a behavioral framework, but I show that they can also be combined within a rational choice framework. This has the advantage that the internal consistency and the discipline that the rational choice approach imposes are not lost.

I show that as long as preferences are known with certainty and parties can

commit in advance to the ideological dimension of policy, but not to a maximal level of rent extraction, voters can constrain the latter to the same extent as in a purely backward-looking model. At the same time, the policy preferred by the median voter is implemented as in a standard forward-looking model of political competition. Voters achieve this outcome by following a simple lexicographic voting strategy. They cast their vote in favor of their favorite policy position whenever parties offer different platforms, but make their vote dependent on the incumbent parties' performance whenever they are indifferent. When uncertainty about the position of the median voter is introduced into the model, voters have to accept higher rent payments, but they still retain some control over rent extraction.

A surprising pattern in the results Somewhat unexpectedly, in none of my three essays does the combination of different models lead to policies that differ more from Downs' predictions than the underlying work-horse models I use would predict. Either there is no influence on the policy dimension,⁶ or the results actually come closer to Downs' predictions for policy.⁷ On the whole, this makes for a rather optimistic assessment of the working of democracy and elections. Of course, the standard caveats apply. Even if a model comes to the conclusion that the preferred policy of the median voter will be implemented, we know that this policy is, in general, not welfare maximizing. However, as long as the distribution of voter preferences is not too asymmetric, the preferences of the median voter might provide an acceptable approximation to a welfare maximizing policy.

The purpose of this thesis is certainly not to assess the general desirability of the policy outcomes in a democratic system. Moreover, the assumptions underlying the rational choice approach might somewhat bias the results towards a positive assessment of democracy. At least this seems plausible following the critique of the rational voter assumption by Caplan (2007). However, and somewhat ironically, the one time I deviated from the rational choice framework in this thesis, I actually found that my alternative behavioral assumptions led to higher expected utility for all voters.

The existence of a stable democracy is an assumption and not an outcome in my papers. In all three essays, the rules of the democratic game are taken as given by

⁶This is the case in first part of the first essay and in the main model of the third essay.

⁷This is the case in the second part of the first essay. Moreover, it is also the case in the second essay if it is interpreted as an extension of a standard lobbying model with additional elections and not the other way around.

voters as well as politicians and they maximize their utility within this given framework. Thus, my work cannot contribute anything new to the important question where and under what circumstance democratic electoral systems can develop and be sustained. A partial exception is only chapter 3, which shows that the monetary contributions of interest groups do not necessarily disturb the policy outcome of the democratic process too much.

Some thoughts on formal modeling and rational choice The rational choice approach to politics is not without its critics (Green and Shapiro 1996; Caplan 2007). Formal models, even if they are not (entirely) based on rational choice motives, are often seen with skepticism by more empirically minded researchers, especially in political science. Rational choice models are criticized for their "unrealistic" assumptions. At this time, we also observe a trend in economics in general and political economics in particular to focus more on empirical work that is probably partly driven by this criticism. Given that there is hardly a well established theoretical alternative to rational choice, empirical research seems to be an attractive alternative for many young researchers at the beginning of their careers.

I do believe in the value of empirical research. However, I also think that theoretical research can still contribute novel and valuable insights to economics and political science. Moreover, I learned from Smith and Ricardo about the division of labor and comparative advantage and believes that my own comparative advantage is within the different fields of theoretical research.

Empirical research provides an important check on theoretical research. However, especially at times in which empirical research seems to be on a forceful rise, theoretical research also provides an indispensable check on empirical work. To give just one example from my research, consider the second essay, "Lexicographic Voting". Empirical researchers often implicitly assume without further justification that prospective and retrospective voting motives rule each other out. My essay shows this not to be the case, even within the strict limits of the rational choice framework. This is an insight that future empirical research will hopefully consider.

Of course, theoretical and empirical research can often be fruitfully combined. However, combining both in one paper restricts the research essentially to theory that leads to results that can be tested directly with existing data. Besides of such work, I see an important role left for more stylized models in the spirit of my thesis. Stylized models are not easy to test empirically. But this does not mean that they are not valuable. Arrow (1963) is a good example of problems whose existence were not

even considered before they were derived with formal methods. The models of Downs (1957) cannot capture the full richness of democratic elections and policy making, but they help to isolate some essential forces of electoral competition whose existence are hardly doubted even by the greatest sceptics of theoretical research. The results in "The Swing Voters' Blessing" may be easy to grasp intuitively once they are established. However, they are in sharp contrast to the results I expected to find before solving the model. They would be difficult to establish and to communicate without the help of formal analysis.

A further advantage of formal models is that the underlying assumptions are made explicit. This is often not the case in purely verbal argumentation and some empirical work.

Combining existing models as a robustness check on core results of political economics One possible reading of many of my results is to see them as a theoretical robustness test on some of the core models commonly used in theoretical political economics. Does combining existing models lead to contradictions or surprising new results?

In many ways, the models I combine and reexamine seem to pass the robustness test rather well. For example, "The Swing Voters' Blessing" shows that the standard information requirements of the median voter theorem are stronger than necessary to derive the standard results. This becomes clear in the extension of the main model where I show that if voters are neither informed about the quality, nor about the policy positions of the politicians the underlying logic of the model still applies and the lack of information has no consequences as long as all voters are rational. Thus, the results in the literature are more robust to lack of information than what seems to be commonly believed.

Chapter 2

The Swing Voters' Blessing*

1 Introduction

When most political economists model elections, the focus is on aggregating individual preferences. Voters disagree on questions of distribution or ideology and elections are a way of deciding which policies are implemented. This literature goes back to the seminal contributions of Black (1948) and Downs (1957).¹ Here, the problem is that voters want different things and elections decide whose preferences prevail. If candidates can commit to policy platforms, as is often assumed in the literature, elections become a way of aggregating conflicting preferences.

A different approach to modeling voting and elections goes back to the Jury Theorem by the Marquis de Condorcet (1785). The idea is to model elections as an information aggregation device.² Voters' interests and preferences are aligned and if all voters were fully informed, they would support the same proposition or candidate. Here, the problem is not that voters want different things, but that limited information creates uncertainty about the consequences of a particular election outcome. Therefore, voters who maximize their expected utility need to understand

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¹ For an overview over this literature, see Persson and Tabellini (2000).

² Condorcet himself was more interested in the verdict of a jury in a court. For an overview of the information aggregation approach, see Piketty (1999).

that their vote has an impact on the election results only if both sides obtain exactly half of the votes and their vote is pivotal for the outcome of the elections (Austen-Smith and Banks 1996; Feddersen and Pesendorfer 1996, 1997). Voters who do not consider this when making their voting decision can suffer from what Feddersen and Pesendorfer (1996) call "the swing voter's curse". Whenever such a voter actually decides an election with her vote, it is likely to turn out that her voting decision makes her worse off.

In reality, elections aggregate preferences as well as information. There is little disagreement about the fact that voters and candidates have different preferences, for example about the amount of income redistribution. However, voters also have common interests, for example in having a President who is a good crisis manager. Candidates who are aware of their superior abilities might be tempted to let the electorate "pay" by choosing relatively extreme policy positions close to their own bliss point, knowing that they will beat their opponent nonetheless. Therefore, separate modeling of preference and information aggregation might conceal important insights.

1.1 Outline of the model and the results

This paper combines the information aggregation as well as the preference aggregation aspects of elections. In contrast to a similar attempt by Feddersen and Pesendorfer (1999), I allow policy offers to be freely decided by the candidates. As in the information aggregation literature, there is a dimension on which voters agree when they are fully informed and, as in the preference aggregation literature, there is a policy dimension over which voters disagree. Specifically, I model elections with quality or valence³ differences between two ideological candidates who can commit to policies before the elections. Incomplete information plays a crucial role because the valence differences are only observed by a limited number of voters. However, given the true quality difference, voters agree on who is the candidate whom they prefer to win the election for a given policy position. Voters may prefer the candidate who is ideologically further away from their ideological bliss point if his quality

³ In the political science literature, quality differences between politicians are often referred to as valence differences. For an early use of the term "valence" in the literature, see Stokes (1963). I use both terms, valence and quality, interchangeably throughout the text.

advantage over the other candidate is sufficiently large.

I show that if uninformed voters follow a simple equilibrium strategy of basing their voting decisions on their own ideological position relative to that of the median voter, the candidate who is preferred by the informed median voter wins. Thus, the uninformed voters effectively ignore the policy platforms of the candidates.

As an example, consider the problem of an uninformed conservative voter deciding between Republican John McCain and Democrat Barack Obama in the 2008 United States presidential elections. Obama's unobserved valence advantage could be so large that the uninformed conservative voter would prefer him if she were fully informed. However, in this case, Obama would not need the conservative voter's support to win the elections because his great appeal to informed voters would ensure his victory even without her vote. The elections are only a close call if McCain has a relatively high valence as compared to Obama. Therefore, the conservative voter knows that she must prefer McCain in case her vote is pivotal.

The uncertainty among uninformed voters makes no difference for the implemented policy as compared to a situation where all voters are fully informed. The candidate with the support of the median voter wins in both cases. Therefore, the candidate with the valence advantage wins. He announces the platform that is as close as possible to his own bliss point without giving the other candidate the opportunity to win the support of the median voter. If the median voter is uninformed, it can be shown that the results for the informed median case provide a good approximation for the uninformed median case, as long as informed voters are located close to the median voter.

The proposed strategy requires the uninformed voters to have a certain amount of sophistication that not every reader might find credible. Therefore, I introduce unsophisticated swing voters into my model to check for the robustness of the results. These uninformed voters do not take into account that their vote makes a difference only if the elections are decided by just one vote. Thus, they vote for the candidate whom they prefer given the unconditional distribution of the valence difference. Because they can only observe the different policy offers by the candidates, their decision is always in favor of the candidate whose policy offer is ideologically closer to their own preferences. They are not only "swing voters" in the sense of Feddersen and Pesendorfer (1996), that is voters who make their voting decision without con-

sidering the fact that their vote only makes a difference when they are pivotal. They are also swing voters in the more common use of the term in the political science literature, that is voters who are likely to switch their support from one party or candidate to a different one. In the terminology introduced by Austen-Smith and Banks (1996), the swing voters in my model vote "sincerely", not "strategically".

It turns out that the majority of voters is better off, in expectation, if such boundedly rational uninformed voters exist. This somewhat surprising result is an application of the second-best principle that introducing an additional distortion into a model may bring the equilibrium closer to the equilibrium without distortion and increase welfare rather than reducing it further (Lipsey and Lancaster 1956). The existing valence differences between candidates "distort" political competition on the policy dimension and lead to results that are different from normal Downsian Competition. The additional distortion of boundedly rational voting brings the results closer to Downsian competition. But just as Downsian competition will not necessarily lead to welfare maximizing results, there is no guarantee that swing voters bring the outcome closer to the utilitarian optimum.

It is illuminating to consider the consequences of swing voters in the Obama versus McCain example mentioned above. An unsophisticated uninformed voter with a bliss point close to, but left of the median voter votes for the centrist McCain if Obama chooses a position far to the left. The existence of such voters forces Obama to stay closer to the median voter than he would otherwise have to in order to win the elections.

Unsophisticated voters make irrational voting decisions, but this turns out to be a blessing and not a curse. They can play a strategy that a rational voter could not commit to because it would not be time-consistent to do so and she would want to deviate after the candidates have chosen their positions. No unsophisticated swing voter has to regret her vote because the candidate with valence advantage wins nonetheless. Her vote could only make her worse off if it were not foreseen by the candidates. But because the candidates know about the existence of swing voters, they adjust their positions. The candidate with the valence advantage wins, but with a more moderate policy position than in the case of full rationality. I call this force of moderation the "swing voters' blessing".

1.2 Related approaches and literature

Feddersen and Pesendorfer (1999) combine motives from the information aggregation with the preference aggregation literature in an attempt at explaining abstentions in a setup where voters' interests are not perfectly aligned. Their main example is a plebiscite over the decision of whether to build a bridge. The main difference as compared to my setup is that the details about the building plans are exogenously given. Feddersen and Pesendorfer also mention the example of different candidates for office, but their framework is ill suited to this application since the policies proposed before elections are not exogenously given. Thus, what is missing to make the model in Feddersen and Pesendorfer an adequate framework for the analysis of elections is a stage of the game in which candidates or parties endogenously decide on policies. With exogenous policy proposals the swing voters' blessing cannot occur.

My model is similar to that in Groseclose (2001) in combining a policy dimension with a candidate quality dimension. However, I focus on uncertainty in the electorate about the quality of the politicians, while Groseclose focuses on uncertainty among candidates about the preferences of the electorate.

In a series of papers by McKelvey and Ordeshook (1985, 1986), uninformed voters use a sequence of opinion polls to infer the truth about candidate positions. However, McKelvey and Ordeshook ignore the strategic aspects of being a pivotal voter that are central to my basic model. Voters simply try to vote for their favorite candidate given their best estimate of the candidates' positions just as is done by the swing voters in the generalization of my model. If the McKelvey and Ordeshook model were formulated as a game, an uninformed voter would have to condition her estimate of the candidates' positions on herself being pivotal. Moreover, the answers to opinion poll questions may be given strategically. McKelvey's and Ordeshook's assumptions could nonetheless be a good description of how boundedly rational voters actually make their voting decision, but there is no discussion of this issue in their papers.

Another paper in the same tradition is Cukierman (1991) whose model is very similar to mine with respect to voters' preferences and information. In contrast to the papers of McKelvey and Ordeshook, voters do not only care about policy, but also about valence. Just as in my approach, some of the voters do not directly observe valence. However, as in the McKelvey and Ordeshook approach, uninformed

voters try to gauge some information from opinion polls and, once more, their voting decisions lack game-theoretic foundations.

An idea related to mine can be found in the recent paper by Bond and Eraslan (2010). These authors endogenize proposals in a Feddersen-Pesendorfer setup. However, they do not model political competition, but rather decision making within a committee as in Feddersen and Pesendorfer (1998), and there is only one offer by an agenda setter, not two offers by competing candidates. Just as in my setup, however, endogenizing positions leads to important differences in the results.

1.3 Structure of the paper

The paper proceeds as follows. In Section 2, the basic model is introduced and discussed. Section 3 allows for some generalizations. In Section 4, swing voters are introduced. The welfare implications of their existence are discussed in Section 5 and Section 6 provides an example with a continuum of voters. The paper ends with a conclusion. A technical appendix contains most of the proofs.

2 The model

Consider a polity with a one-dimensional ideological policy space on the real line $[0, 1]$, two candidates L and R and an odd number n of voters denoted by $i = 1, 2, 3, \dots, n$. Candidates have quality (also called valence) q_L and q_R , respectively, and a bliss point for implemented policy b_L and b_R , respectively.⁴ The candidates' utility is decreasing in the distance of implemented policy to their bliss point and it is given by:

$$U_J(p) = -(p - b_J)^2, \quad (1)$$

with $J = L, R$, and where p is implemented policy.

Just as the candidates, every voter i has a bliss point b_i on the policy space. By assumption, they are all distinct and no two voters have exactly the same preferences. Voters are ordered by their bliss points so that $b_1 < b_2 < b_3$ and so on.⁵ Besides

⁴ Variables with capital letter subscripts are used to denote characteristics of candidates, while variables with small letter subscripts denote characteristics of voters.

⁵ The assumption that no two voters have exactly the same bliss point is a mild one given that

the policy dimension, voters care about the quality of candidates and voter i has the utility function:

$$U_i(b_i, p, q) = -(p - b_i)^2 + q, \quad (2)$$

where $q \in \{q_L, q_R\}$ is the quality of the candidate who wins the elections.⁶ Assuming that the (dis)utility from policy does not interact with the quality of the winning candidate as is done here is the most straightforward way of modeling information and preference aggregation in one election. However, the results also hold for more general utility functions where the possible interaction of quality and distance is not ruled out. This is shown in Section 3.3 where generalizations of (1) as well as (2) are discussed.

The median bliss point of the voters is denoted by b_m with $m = \frac{n+1}{2}$. By assumption $b_L \leq b_m \leq b_R$; I thus call candidate L the left and candidate R the right candidate. The difference in quality of the two candidates is denoted by the variable $v = q_R - q_L$, which hence measures the quality advantage of the right candidate. If the left candidate has a quality advantage, v takes a negative value. The values of q_R and q_L are drawn from a continuous distribution function before the candidates announce their position. The cumulative distribution of v is given by the function $G(v)$. By assumption, the corresponding probability density function of $g(v)$ has positive support everywhere on the real line. All players, voters as well as candidates, know the basic structure of the game including the policy preferences of the parties as well as the distribution of the bliss points of the voters.⁷

The sequence of moves is the following: First, nature chooses q_R and q_L . Second, candidates announce the policy platform they propose to be implemented after observing the quality difference v . Third, elections in which every voter casts one vote are held. Some of the voters, the so-called informed voters (their number is n_I), can observe the random variable v and the policy platforms offered by the candidates before they make their voting decision. The so-called uninformed voters (their number is n_U) only observe the policy platforms before they cast their votes. Fourth, the candidate who obtains at least $\frac{n+1}{2}$ of the votes in the elections wins

the probability of two voters having exactly the same position is 0 if all of them are drawn from a continuous distribution function. It considerably simplifies the notation.

⁶ This kind of preferences can be called "one and a half dimensional" (Grosche 2007).

⁷ For the basic model it is sufficient if uninformed voters know the position of the median voter.

and his announced policy platform is implemented. Therefore, $p = p_L$ and $q = q_L$ if candidate L obtains more than half of the votes, and $p = p_R$ and $q = q_R$ if he obtains less than half of the votes.

Abstentions are not allowed. This assumption is made to simplify the notation. It is easily verified that in my model, no voter would ever want to abstain in equilibrium. By assumption, the majority of voters are informed, that is $n_I > n_U$.⁸

In the main part of the paper, the median voter is assumed to be informed. A discussion of the model with an uninformed median voter can be found in Section 3.2. There, it is also shown that the main model is a good approximation of this case for "large" electorates.

For the moment, I assume all voters to be sophisticated in the sense that they are able to understand the Bayesian Nash equilibrium of the voting game and play equilibrium strategies. In Section 4, this assumption is relaxed and boundedly rational voters are introduced into the model.

2.1 Solving the model

I begin my analysis at the last stage of the game and solve the problem of the voters after observing the platforms of the candidates. Then, I solve the problem of the candidates when announcing their policy platforms and show what is the equilibrium policy.

2.2 Informed voters

I consider equilibria where informed voters play the weakly dominating strategy of always voting for the candidate whom they favor.⁹ It is possible to determine who is the rightmost informed voter weakly in favor of the candidate with the left policy position. Specifically, the cutoff point is the bliss point b^* that makes a voter indifferent between the two candidates. This point is implicitly defined by (2). Equating the utility of voting for the left candidate and voting for the right

⁸ This assumption helps avoid implausible additional equilibria with all uninformed voters voting for one party independently of policy positions. The assumption is not necessary for the existence of the type of equilibrium analyzed below.

⁹ Without this restriction, it is possible to have equilibria where everybody votes left or everybody votes right independently of the candidates' policy positions, so that none of the voters is ever pivotal.

candidate gives:

$$\Delta U(b^*, p_L, p_R, q_L, q_R) = U(b^*, p_L, q_L) - U(b^*, p_R, q_R) = -(p_L - b^*)^2 + (p_R - b^*)^2 - v = 0. \quad (3)$$

The cutoff point b^* exists for any v as long as $p_L \neq p_R$ and it is uniquely given by:

$$b^*(p_L, p_R, v) = \frac{p_L + p_R}{2} - \frac{v}{2(p_R - p_L)} \text{ for } p_R \neq p_L. \quad (4)$$

All voters with a bliss point to the left of b^* prefer the candidate with the left position, while all voters with a bliss point to the right of b^* prefer the candidate with the right position.¹⁰ Note that the right candidate could, in principle, be located at the left position (if $p_R < p_L$), although this will never be the case in any plausible equilibrium.¹¹ The intuition for this formula is straightforward. If $v = 0$, the cutoff point is midway between the policy position of the two candidates. A positive v makes the right candidate more attractive and therefore shifts the cutoff point to the left for given policy positions as long as $p_R > p_L$. However, the marginal effect of valence differences on the position of the cutoff point is decreasing in $p_R - p_L$, i.e. the distance in policy. The further the candidates' policy positions are from each other, the more policy matters relative to valence. The reason is that the disutility of distance from the ideal policy point of a voter is quadratic while utility is linear in valence. In the case of large valence differences, it is possible that the cutoff point is to the left of the left policy position or to the right of the right policy position.

The cutoff point between preferred candidates is the same for informed and uninformed voters. The difference between the two types of voters is that uninformed voters do not know where b^* is located since they do not know the valence difference v . However, for informed voters, the voting decision only depends on b^* and therefore

¹⁰ This can be seen from the derivative of the difference in utility from the left candidate's position and the right candidate's position, with respect to a voter's bliss point:

$$d \frac{\Delta U(b, p_L, p_R)}{db} = -2(p_R - p_L) \begin{cases} < 0 \text{ if } p_R > p_L \\ > 0 \text{ if } p_R < p_L \end{cases} .$$

¹¹ In theory, a candidate who knows that he will lose against the other candidate's bliss point with any policy position could choose a position further away from his own bliss point than the other candidate's bliss point in equilibrium. Throughout the text, I will use the short expression "vote for the left (right) position" instead of the slightly more precise but cumbersome "vote for the candidate with the left (right) policy position".

$b_I^*(p_L, p_R, v) = b^*(p_L, p_R, v)$, where b_I^* denotes the cutoff point between informed voters who vote for the left position and informed voters who vote for the right position. If an informed voter is located exactly at b_I^* she is indifferent and, by assumption, votes in favor of the candidate with valence advantage.

Now, b^* can be located outside the policy space $[0, 1]$. Whenever this is the case, either all or none of the informed voters vote left or right position, respectively. If $p_L = p_R$ and $v \neq 0$, no value of b solves equation (3) because all informed voters prefer the candidate who has the valence advantage and vote for him. Therefore, $b^* = b_I^* = 0$ if $p_L = p_R$ and $v > 0$, and $b^* = b_I^* = 1$ if $p_L = p_R$ and $v < 0$. If $p_L = p_R$ and $v = 0$, equation (3) holds for arbitrary values of b since all informed voters are indifferent between the candidates independently of their respective bliss points. Without loss of generality, I make the assumption that in this case, all informed voters give their vote to the left candidate L and therefore $b_I^* = 0$.

2.3 Uninformed voters

The problem of an uninformed voter with bliss point b is that she does not know which candidate she favors, because she is not able to observe the valence difference v . Let $F_I(b)$ be the number of informed voters with a bliss point smaller than or equal to b , let $F_I^{-1}(x)$ be the bliss point of the informed voter with the x_{th} lowest bliss point b among the informed voters' bliss points, and let l_U be the number of votes for the left policy position by uninformed voters. I call p_L the left position when $p_L \leq p_R$, and call p_R the left position when $p_L > p_R$. Then I refer to:

$$b_I^d(l_U) = F_I^{-1}\left(\frac{n+1}{2} - l_U\right) \quad (5)$$

as the bliss point of the decisive informed voter given l_U .¹²

Lemma 1. *The candidate with the support of the decisive informed voter wins the elections.*

¹² I distinguish between "decisive voters" and "pivotal voters". A voter is pivotal if the winner of the elections wins with one vote and would lose if a pivotal voter changed her vote. All voters who vote for the winner in an election that is decided by one vote are therefore pivotal. If the majority is larger, there are no pivotal voters. A voter is "decisive" if the candidate whom she prefers wins the elections given the preferences of the other voters. In a standard Downsian model, the decisive voter has the median bliss point.

Proof. For a majority, $\frac{n+1}{2}$ votes are necessary and therefore at least $(\frac{n+1}{2} - l_U)$ votes by informed voters in favor of the left position for the candidate with the left position to win, and at least $(\frac{n+1}{2} - (n_U - l_U))$ votes by informed voters in favor of the right position for the candidate with the right position to win. If the decisive informed voter votes for the left position, then $b_I^d(l_U) \leq b^*(p_L, p_R, v)$, and all informed voters with a bliss point $b < b_I^d(l_U)$ vote for the left position. Thus, the left position obtains at least $F_I(b_I^d(l_U)) = \frac{n+1}{2} - l_U$ votes by informed voters. Together with the l_U votes for the left position by uninformed voters, this constitutes a majority. If the decisive informed voter votes for the right position, then $b_I^d(l_U) \geq b^*(p_L, p_R, v)$, and all informed voters with a bliss point $b > b_I^d(l_U)$ vote for the right position. Therefore, the left position obtains at most $\frac{n-1}{2}$ votes, and the right position obtains a majority. ■

The candidate preferred by the decisive informed voter wins the elections. Therefore, this voter is decisive in the same sense as the median voter in standard models with full information.

The strategies of the voters can only constitute an equilibrium if none of the voters has an incentive to deviate, given the strategies of the other players and her information. Due to the strategy of the informed voters, this implies that, in equilibrium, none of the uninformed voters would prefer to shift the position of the decisive informed voter by changing her own voting decision. A simple strategy fulfills this condition if it is followed by all uninformed voters. Let $b_U^*(p_L, p_R)$ be the cutoff point for uninformed voters: i.e. all uninformed voters with $b < b_U^*(p_L, p_R)$ support the left position and all uninformed voters with $b > b_U^*(p_L, p_R)$ support the right position. Specifically, the condition holds with $b_U^*(p_L, p_R) = b_m$ as the cutoff point. All uninformed voters with a bliss point to the left (right) of the informed median voter vote for the the candidate with the left (right) policy position.¹³ This cutoff point is independent of the policy platforms that the candidates announce.¹⁴ Moreover, with this cutoff point, the decisive informed voter is the median voter as in standard models without uninformed voters.

¹³ By assumption, there is no uninformed voter with bliss point b_m . Therefore, this cutoff point determines the voting decision of all uninformed voters.

¹⁴ However, it should be kept in mind that because the right candidate could play the left policy position, the candidate whom a voter supports in the elections is not completely independent of the policy positions.

Lemma 2. *If the cutoff point for uninformed voters is $b_U^*(p_L, p_R) = b_m$ for all combinations of p_L and p_R , the voting decision of the the informed median voter is decisive for the outcome of the elections.*

Proof.

$$b_I^d(F_U(b_m)) = b_I^d\left(\frac{n+1}{2} - F_I(b_m)\right) = F_I^{-1}(F_I(b_m)) = b_m,$$

where the first equality comes from the implicit definition of the median voter's bliss point b_m : $\frac{n+1}{2} = F_I(b_m) + F_U(b_m)$, and the second equality follows directly from the definition of b_I^d given in equation (5). The third equality follows from the fact that, by assumption, an informed voter with bliss point b_m exists. ■

Given Lemmas 1 and 2, it follows:

Lemma 3. *The cutoff point $b_U^*(p_L, p_R) = b_m$ characterizes an equilibrium strategy for uninformed voters given that informed voters play the weakly dominating strategy characterized by the cutoff point $b_I^*(p_L, p_R, v)$. As in standard models with full information, the preferences of the median voter decide the elections.*

Proof. To show the optimality of the strategy of an individual voter, it is sufficient to show that she cannot be better off by changing her strategy, given the strategies of all other voters.

Consider the case of an uninformed voter with her bliss point to the left of b_m . Since such a voter votes for the left policy position, her alternative is voting for the right position instead. From Lemma 2, we know that if she votes for the left position, the bliss point of the decisive informed voter is b_m . If she instead votes for the right position, the bliss point of the decisive informed voter changes from b_m to $b_I^d(F_U(b_m) - 1) = b_I^d\left(\frac{n+1}{2} - F_I(b_m) - 1\right) = F_I^{-1}(F_I(b_m) + 1) > b_m$. This can change the election outcome only if $b^*(p_L, p_R, v)$ takes a value such that $b_m \leq b^*(p_L, p_R, v) \leq F_I^{-1}(F_I(b_m) + 1)$. From Lemma 1, it follows that if the elections outcome changes, then it must be the case that the candidate with the right position wins instead of the candidate with the left position. Because the voter's bliss point is to the left of the median bliss point and, consequently, also to the left of the cutoff point $b^*(p_L, p_R, v)$, this would make her worse off. Therefore, an uninformed voter with bliss point $b < b_m$ is at least as well off voting for the left position as voting for the right position and thus, voting for the left position must be optimal for all possible combinations of p_L and p_R and independently of the candidates' strategies.

An analogous argument can be applied to show that a voter whose bliss point is to the right of b_m can never be better off voting for the left position, given the strategies of the other voters. In the case of $p_L = p_R$, no cutoff point for informed voters exists, but because all of them vote for the candidate with the valence advantage, this candidate wins independently of the decision of the uninformed voters. ■

To provide some intuition for the uninformed voters' strategy, it is helpful to reinterpret the voting strategy of the uninformed as a way of appointing the decisive informed voter. All uninformed voters prefer a decisive informed voter with preferences as close to their own as possible. To achieve this aim, they vote left (right) if their bliss point is to the left (right) of the bliss point of the decisive informed voter. In this way, they attempt to pull the position of the decisive informed voter closer to their own bliss point.

Another interpretation of the result is that uninformed voters ensure that they vote for their favorite (under full information) candidate whenever they are pivotal. They realize that it is not important for whom they vote, as long as their vote does not change the election outcome. If an uninformed voter follows the strategy of only making her decision dependent on her position relative to the median bliss point, she can be certain of never voting against the candidate whose election maximizes her utility when he loses by just one vote. Thus, she can avoid becoming a victim of the swing voter's curse. To see this, imagine that the elections are decided by one vote and assume (without loss of generality) that the right position wins. Because the decisive informed voter is the median voter, this implies that the median voter votes for the right position, but informed voters to the left of the median voter vote for the left position (otherwise the majority would be larger). Because the median voter prefers the right position, all uninformed voters with bliss a point to the right of the median voter who vote for the right candidate indeed have higher utility from a victory of the right candidate than they would in case the left candidate won. This interpretation provides some intuition as to why the position of the decisive informed voter does not change as compared to the standard setup. Whenever they are pivotal, uninformed voters manage to make the same voting decision as if they had full information.

2.4 The candidates

Lemma 3 shows that the candidates' problem is exactly the same as it would be in the full-information setting. The candidates need the support of the decisive informed voter to win, and the decisive informed voter turns out to be the median voter. The candidate with a valence advantage can win by offering the bliss point of the median voter as the policy proposal. However, he can do considerably better by moving as close as possible to his own bliss point without endangering his election victory.¹⁵

Proposition 1. *Together with the cutoff point $b_U^*(p_L, p_R) = b_m$ for uninformed voters and the weakly dominating strategy of informed voters the following policy platforms of the candidates constitute an equilibrium of the game:*

$$\left. \begin{array}{l} p_R^* = \min(b_R, b_m + v^{0.5}) \\ p_L^* = b_m \end{array} \right\} \text{if } v > 0, \quad (6)$$

$$\left. \begin{array}{l} p_R^* = b_m \\ p_L^* = \max(b_L, b_m - (-v)^{0.5}) \end{array} \right\} \text{if } v \leq 0,$$

and the implemented policy is:

$$p^* = \begin{cases} \min(b_R, b_m + v^{0.5}) & \text{if } v > 0 \\ \max(b_L, b_m - (-v)^{0.5}) & \text{if } v \leq 0 \end{cases}. \quad (7)$$

Proof. Proof for the case $v > 0$:

From Proposition 1, we know that the candidate with the support of the informed median voter wins the elections. The best reply of the right candidate to a policy p_L by the left candidate is therefore given by the solution to the following constrained maximization problem:

$$p_R^b(p_L, v) = \max_{[0, b_R]} p_R \text{ s.t. } -(p_R - b_m)^2 + v \geq -(p_L - b_m)^2.$$

¹⁵ An interesting aspect of the result is that usual Downsian Competition results do not hold. Due to the additional valence dimension, the winner of the elections does not implement the most preferred policy of the median voter. This is the case despite the fact that Black's theorem applies to the model and the majority's preferences are identical to those of the median voter (Groseclose 2007). The reason for the discrepancy is that (with the exception of $v = 0$) the median policy is not on offer in combination with the high quality candidate because it is not in the interest of the winning candidate to make it available. Black's theorem holds, but the Downsian version of the median voter theorem does not.

$p_R^b(p_L, v)$ gives the largest $p_R \leq b_R$ which makes the median voter at least indifferent between voting for the left candidate and voting for the right candidate and, therefore, leads to an election victory for the right candidate. A unique solution exists and is given by:

$$p_R^b(p_L, v) = \min(b_R, b_m + (v + (p_L - b_m)^2)^{0.5}).$$

$p_R^b(p_L, v) \geq p_L$ for $p_R^b(p_L, v) < b_R$. Therefore, the right candidate could not be better off with a position at which he is defeated by the left candidate and $p_R^b(p_L, v)$ is therefore a best reply to the other players' strategies. $p_R^b(b_m, v) = \min(b_R, b_m + v^{0.5})$, hence right is playing the unique best reply to the left candidate's position $p_L^* = b_m$ in equilibrium. If $-(p_R - b_m)^2 + v \geq -(p_L - b_m)^2$ holds for $p_L = b_m$, it follows that the inequality also holds for all other values of p_L because $p_L = b_m$ maximizes the right-hand side of the inequality. Therefore, the left candidate loses the elections with any reply to p_R , and $p_L = b_m$ (as well as any other p_L) is a best reply to $p_R^b(b_m, v)$. If $p_R^b(b_m, v) < b_R$, the equilibrium response of the left candidate is unique. To see this, consider any other policy platform $p_L \neq b_m$. $p_R^b(p_L, v) > p_R^b(b_m, v)$ for any $p_L \neq b_m$. But against any $p_R > p_R^b(b_m, v)$, the left candidate can win with $p_L = b_m$. Therefore, p_L cannot be a best reply to $p_R^*(p_L, v)$ for any $p_L \neq b_m$, and $p_L \neq b_m$ cannot be part of the equilibrium. If $p_R^b(b_m, v) = b_R$, then $p_R^b(p_L, v) = b_R$ for all values of p_L , and therefore, $p_R^* = b_R$ in combination with any policy platform p_L constitutes an equilibrium. All values of p_L are best replies to $p_R^* = b_R$, and $p_R^* = b_R$ is a best reply to all values of p_L . For all equilibrium combinations of p_L and p_R , the median voter either prefers the right candidate or is indifferent. Therefore, the median voter votes for the right candidate and Lemma 3 implies that the right candidate wins and policy platform p_R^* is implemented. The implemented policy $p = p_R = p_R^b(b_m)$ is therefore unique for all values of $v > 0$. For the proposition, I assume that the left candidate always chooses $p_L^* = b_m$, even when $p_R^* = b_R$ and the solution is not unique.

An analogous argument can be given for the case $v \leq 0$. ■

2.5 Interpretation as Perfect Bayesian Nash Equilibrium

So far, I have ignored the fact that voters might try to infer the quality difference of the politicians from their chosen policy position. This does not invalidate the above analysis, because the derived strategies of the uninformed voters were (weakly) optimal independently of the players' beliefs about the valence difference v . However, it is interesting that uninformed players can infer the values of v exactly as long as none of the candidates has a valence advantage that is so large that he can choose his bliss point as his policy position and win.

A system of beliefs for the uninformed voters that is consistent with the analysis so far can therefore easily be constructed with the help of the equilibrium outcomes for the candidate positions. All that the uninformed voters have to do to infer the value of v is to observe for what values of v the policy offers by the candidates constitute an equilibrium. As long as the candidates play their equilibrium strategies and neither candidate offers his bliss point, there is only one value of v that is consistent with the policy offers. If one candidate announces his bliss point as the policy platform and the other candidate announces the median bliss point, this combination of policy positions is consistent with an interval of values of v . In this case, Bayes' Theorem is applied to calculate the density of the distribution function of v :

$$\begin{aligned}
 v(p_L, p_R) &= \begin{cases} (p_R - b_m)^2 & \text{if } p_L = b_m, p_R \in [b_m, b_R] \\ -(p_L - b_m)^2 & \text{if } p_R = b_m, p_L \in (b_L, b_m] \end{cases} \\
 g(v|p_L, p_R) &= \left. \begin{cases} \frac{g(v)}{1 - G((b_m - b_R)^2)} & \text{for } v \geq (b_m - b_R)^2 \\ 0 & \text{for } v < (b_m - b_R)^2 \end{cases} \right\} \text{if } p_L = b_m, p_R = b_R \\
 & \left. \begin{cases} \frac{g(v)}{G(-(b_m - b_L)^2)} & \text{for } v \leq -(b_m - b_L)^2 \\ 0 & \text{for } v > -(b_m - b_L)^2 \end{cases} \right\} \text{if } p_R = b_m, p_L = b_L,
 \end{aligned} \tag{8}$$

where $v(p_L, p_R)$ is a value of v that leads to the combination of p_L and p_R in equilibrium. For all other (out of equilibrium) combinations of policy offers, I assume that the uninformed voters do not update their a priori beliefs of the distribution of v , so that in this case $g(v|p_L, p_R) = g(v)$.

Because the voting decision of the uninformed voters does not depend on the beliefs about v , candidates cannot manipulate the voters' beliefs to their own advantage

by choosing their policy position. Nonetheless, in equilibrium, all uninformed voters support the candidate they favor, given their beliefs about v whenever their information is sufficient to determine who this candidate is. Only if v is either so high or so low that the winning candidate can offer his bliss point and win nonetheless, an uninformed voter cannot determine the exact value of v . In this case, it can happen that an uninformed voter votes against the candidate whom she would prefer to win if she were fully informed. But since in this case the candidate wins even without her vote, this has no consequences for the outcome of the elections.

Voters do not actually need to be so sophisticated that they possess the beliefs they are required to have in the Perfect Bayesian Nash Equilibrium. As should be clear from the way in which the equilibrium was derived, their strategies are optimal, independently of the strategies of the candidates if all other voters follow the same strategy. Voters could have any conceivable beliefs about the distribution of v without their strategies ceasing to be optimal. The reason is that their vote is only decisive for certain values of v . Therefore, the belief of how likely those values of v are is of no importance for their voting decisions. The requirements for the sophistication of the uninformed voters are considerably lower than in most cases of a Perfect Bayesian Nash Equilibrium and the results are less sensitive to its strong rationality and information requirements.

2.6 Uniqueness of the equilibrium

The equilibrium derived in Sections 2.1 – 2.4 is not unique. This is not surprising considering that "implausible" equilibria with voter coordination exist already in models with fully informed voters. A trivial example is that of all voters always voting for the same candidate. Since none of the voters is pivotal, this constitutes a Bayesian Nash equilibrium if the winning candidate announces his bliss point as the policy platform (the other candidate can announce an arbitrary policy position).

A more interesting example is that all uninformed voters always vote in favor of the candidate who chooses the policy closest to the median position, while informed voters follow the strategy to vote for their preferred candidate. What uninformed voters do when both candidates choose a position with equal distance to the median voter's bliss point is of no importance, because the higher-quality candidate wins in this case with the votes of all informed voters. Candidates are forced to choose the

median bliss point as policy position, even if they have a small valence advantage. If the advantage is sufficiently large to ensure a majority with only informed votes at some more advantageous position, they choose this position. Since, in equilibrium, the higher-quality candidate chooses the median position, or wins at a position closer to his bliss point with a majority consisting only of informed voters, no uninformed voter could increase her utility by deviating. However, it remains unclear how uninformed voters should be able to coordinate on such an equilibrium and why they would want to do so. Every single uninformed voter would be better off if the candidates knew before the elections that she deviated to the strategy of making her voting decision dependent on the median position, because that would for some values of v lead to implemented policies closer to her own bliss point. Moreover, the equilibrium given in Sections 2.1 – 2.4 seems to be the relevant one, given that I am interested in solving the problem of an uninformed voter who attempts to make the correct voting decision under the condition that she is pivotal, since the other equilibria are only equilibria since no uninformed voter can ever be pivotal.

To provide some more formal justification, I focus on equilibria with a cutoff point $b_U^*(p_L, p_R)$ for uninformed voters. This means that I rule out equilibria in which an uninformed voter with bliss point b_1 votes for the right candidate while an uninformed voter with bliss point $b_2 > b_1$ votes for the left candidate. Such equilibria exist, but they require some coordination by voters that is hard to achieve in large scale elections and there is no intuition that might justify them. Moreover, I assume that the uninformed voter's beliefs are such that every value of v is considered to be possible for any combination of p_L and p_R . Let $G(v^b|p_L, p_R)$ be the cumulative distribution function of the beliefs of an uninformed voter about the value of v conditional on the policy platforms.¹⁶ I assume the corresponding density function $g(v^b|p_L, p_R)$ to be positive for every value of v . This implies that voters assign at least a small probability ε to the possibility that candidates do not play equilibrium strategies, even when they observe equilibrium positions being played. More importantly for my argument, it rules out out-of-equilibrium beliefs that assign a zero probability to certain values of v for certain combinations of candidates' policy positions without further justification. The idea is to rule out equilibria that

¹⁶ For the argument, it is not essential that all uninformed voters have the same beliefs. It is only important that they all believe all values of v to be possible.

are only justified by restrictions on voters' out-of-equilibrium beliefs. In such equilibria, voters believe that they are never pivotal, which is then, in return, justified by the equilibrium response of the candidates to the voters' strategies. Such equilibria are at odds with the basic idea of my model that voters try to make the right decision conditional on being pivotal. An example of such an equilibrium is the one mentioned above with uninformed voters voting for the candidate who offers the position closest to the median voter's bliss point.¹⁷

Lemma 4. *If voters' beliefs about the value of v are such that the probability distribution function $g(v^b|p_L, p_R) > 0$ for all combinations of v , p_L and p_R , the Bayesian Nash equilibrium given in Sections 2.1 – 2.4 is the unique equilibrium with informed voters playing their weakly dominating strategy and uninformed voters' voting determined by a cutoff point $b_U^*(p_L, p_R)$.*

Proof. See the Appendix ■

3 Generalizations

In Section 3.1, I relax the assumption that uninformed voters can observe the policy positions of the candidates. In 3.2, I show what happens if the median voter is uninformed. Finally, in Section 3.3 I show that my results are valid for more general utility functions of voters as well as candidates.

3.1 Voters who are ignorant about policy positions

So far, I have assumed that uninformed voters can observe the policy positions of the candidates. However, even if a voter does not know the policy platforms of the candidates, she can still vote for the candidate with the left (right) bliss point b_L (b_R) if her own bliss point is to the left (right) of the median bliss point. While in equilibrium all voters support the same candidate and candidates choose the same position as in Section 2, updating their beliefs about the valence difference v is

¹⁷ The standard way of restricting out-of-equilibrium beliefs in a game in extensive form is to use the Sequential Equilibrium concept by Kreps and Wilson (1982). Unfortunately, there is no straightforward extension to a setup where players have a continuum of moves to choose from as candidates have in my model.

impossible for the completely uninformed voters.¹⁸

Proposition 2. *If candidates and informed voters follow the same strategies as in Section 2.1–2.4, and completely uninformed voters vote for the left (right) candidate if they have a bliss point $b < b_m$ ($b > b_m$), these strategies constitute a Bayesian Nash equilibrium in the game with completely uninformed voters. The implemented policy is the same as in Proposition 1.*

Proof. See the Appendix ■

It is straightforward to verify that the equilibrium policy positions and the implemented policy are also robust to scenarios where some uninformed voters observe the policy positions, while some do not. Some of the voters who do not observe the policy positions could observe the valence term. If such voters ignore the information about the valence term and follow the same strategy as completely uninformed voters in Proposition 2 and all other players follow the given strategies, this once more constitutes an equilibrium.

The fact that voters may not necessarily need to know the exact announced policy positions of candidates to make an optimal voting decision was already pointed out by McKelvey and Ordeshook (1986) in a different setup. It is an important point, since it shows that the information requirements for voters are often much weaker than the standard assumption that each voter has full information.

3.2 The case of an uninformed median voter

If the median voter is not informed, but all other voters follow the strategies derived in Sections 2.2 – 2.3, she is forced to decide between a decisive informed voter with a bliss point to the left or to the right of her own bliss point. Therefore, she does not have the possibility to make a decision that is optimal for all possible beliefs about the true value of v . Nonetheless, if she follows the simple strategy to always vote in favor of the policy position closest to her bliss point, this leads to an election outcome in which she never votes against her preferred candidate. The reason is that the higher-quality candidate who wins the elections adjusts his position to win the

¹⁸ This is the case despite the fact that the equilibrium strategies of voters are formally not the same. The strategies can, by definition, not be identical because voters have different information on which to base their moves if they observe the policy positions as compared to the case when they do not.

elections, even without the support of the uninformed median voter. I assume that the uninformed median voter votes for the left policy position if both candidates have the same distance to her bliss point.

Given the vote of the uninformed median voter, the decisive informed voter has the bliss point:

$$b_I^d = \begin{cases} b_l = F_I^{-1}\left(\frac{n+1}{2} - F_U(b_m)\right) & \text{if } |p_L - b_m| \leq |p_R - b_m| \\ b_r = F_I^{-1}\left(\frac{n+1}{2} - (F_U(b_m) - 1)\right) & \text{if } |p_L - b_m| > |p_R - b_m| \end{cases} . \quad (9)$$

The informed voter with bliss point b_l is the one with the bliss point closest to the left of the median bliss point, and the informed voter with bliss point b_r is the informed voter closest to the right of the median bliss point.

The position of the decisive voter given in (9) leads to the following strategies of the candidates:

Lemma 5. *The candidates' strategies in the case of an uninformed median voter are:*

$$\left. \begin{aligned} p_L^* &= \max\left(b_m - \frac{v}{4(b_m - b_l)}, b_l\right) \\ p_R^* &= \min\left(b_R, b_l + (v + (b_l - p_L^*)^2)^{0.5}\right) \end{aligned} \right\} \text{if } v > 0 , \quad (10)$$

$$\left. \begin{aligned} p_R^* &= \min\left(b_m + \frac{-v}{4(b_r - b_m)}, b_r\right) \\ p_L^* &= \max\left(b_L, b_r - (-v + (b_r - p_R^*)^2)^{0.5}\right) \end{aligned} \right\} \text{if } v \leq 0$$

and implemented policy is:

$$p^* = \begin{cases} p_R^* & \text{if } v > 0 \\ p_L^* & \text{if } v \leq 0 \end{cases} . \quad (11)$$

Proof. See the Appendix ■

It remains to be shown that given these strategies of the candidates, the voters' decision and consistent beliefs are indeed a best reply and we have a Perfect Bayesian Nash equilibrium of the game. $V(p_L, p_R)$ denotes the set of all values of $v(p_L^*, p_R^*)$ that lead to equilibrium policy positions p_L^* and p_R^* in (10). The following is a consistent belief system for uninformed voters, given the equilibrium strategies of the candidates:

$$\begin{aligned}
& v(p_L, p_R) \text{ with probability 1} && \text{if } v(p_L, p_R) \text{ is the only element in } V(p_L, p_R), \\
& g(v|p_L, p_R) = \frac{g(v)}{\int_{v \in V(p_L, p_R)} g(v) dv} && \text{if } V(p_L, p_R) \text{ contains more than one element,} \\
& g(v|p_L, p_R) = \begin{cases} \frac{g(v)}{\int_0^\infty g(v) dv} & \text{for } v > 0 \\ 0 & \text{for } v \leq 0 \end{cases} && \text{if } V(p_L, p_R) = \emptyset \text{ and } |p_L - b_m| > |p_R - b_m|, \\
& g(v|p_L, p_R) = \begin{cases} \frac{g(v)}{\int_{-\infty}^0 g(v) dv} & \text{for } v \leq 0 \\ 0 & \text{for } v > 0 \end{cases} && \text{if } V(p_L, p_R) = \emptyset \text{ and } |p_L - b_m| \leq |p_R - b_m|,
\end{aligned} \tag{12}$$

where $g(v|p_L, p_R)$ is the probability distribution function of v conditioning on the policy positions.

The beliefs can simply be calculated from the equilibrium values for p_L^* and p_R^* given in Lemma 5. For the out of equilibrium beliefs, there are no restrictions in Perfect Bayesian Nash equilibrium and they are chosen in a way that makes them consistent with the strategies of the uninformed voters.

Proposition 3. *Together with the candidates' strategies given in Lemma 5 and the beliefs in 12, the voters' strategies constitute a Perfect Bayesian Nash equilibrium.*

Proof. See the Appendix ■

The intuition for the strategy of the median voter is simple. She punishes the candidate who deviates further from her ideal point. Since in equilibrium the candidate with the valence advantage adjusts his position in a way that ensures his victory, a situation in which the median voter regrets her vote ex post cannot occur. The candidate with valence advantage chooses his platform in a way that brings him as close as possible to his bliss point without losing the elections. The candidate with valence disadvantage chooses his platform so that he defeats the winning candidate if the latter chooses a platform even closer to his own bliss point.

It is remarkable that an uninformed median voter is actually better off as compared to the game in which she is informed. Her lack of information makes it possible for her to commit to a strategy that would otherwise not have been credible. An informed median voter cannot commit to vote against the candidate who takes a position further away from her bliss point, since that can imply that she has to vote against a candidate whom she prefers, even if that leads to the loss of this candidate.

Let $\varepsilon = \max(b_m - b_l, b_r - b_m)$ be the maximal distance of the median voter to an informed voter on either side. For $\varepsilon \rightarrow 0$, the strategies and the implemented policy given by (10) and (11) converge to the solution with an informed median given in (6) and (7). If informed voters are located "close" to the uninformed median voter, candidates' strategies in the case of the informed median provide a good approximation in the case of an uninformed median. This is likely to be the case if n is large.

3.3 Generalizing the utility functions

In the baseline model discussed so far, the utility functions of the voters as well as of the candidates are chosen to be as simple as possible. This section shows that the results are quite robust to the choice of the utility functions.

3.3.1 The utility function of the voters

First, consider the utility function of the voters given in (2). The proofs in Section 2 are based on there being a single cutoff point between informed voters who prefer the left candidate and informed voters who prefer the right candidate. As a result, all proofs hold without any major modification if there is at most one cutoff point or all informed voters prefer the same candidate. To show that a more general function leads to the same type of equilibria as in Section 2, I only need to show that the assumptions about functional form imply a unique cutoff point.

A more general utility function that depends only on distance and the quality of politicians is:

$$U_i(p, b_i) = u(d_i, q), \quad (2')$$

with $d_i = |b_i - p|$. If $u(d_i, q) = -d_i^2 + q$, (2') is identical to (2).

Sufficient restrictions on the utility function for having a unique cutoff point are that the following derivatives exist and fulfill the conditions:

- (a) $u_d(d, q) \leq 0$,
- (b) $u_{dd}(d, q) < 0$,
- (c) $u_q(d, q) \geq 0$,
- (d) $u_{qd}(d, q) \leq 0$.

Condition (a) naturally follows from saying that a point b is a bliss point. Condition (b) is somewhat stronger, but nonetheless standard. A voter suffers less from departing from her ideal point b to some alternative policy p' than from departing the same distance $|b - p'|$ away from p' to a policy p'' which has the distance $2|b - p'|$ from b . Without this restriction, for example, a high-quality Democratic candidate could be preferred by Democrats as well as very conservative voters, while moderate Republicans would prefer the low-quality Republican candidate. This would lead to at least two cutoff points. Condition (c) implies that voters never feel worse off with a higher-quality candidate *ceteris paribus*. It is necessary to ensure that, for example, very conservative voters do not prefer a low-quality Democrat to a high-quality moderate Republican. Condition (d), for example, helps to rule out cases of a very conservative voter preferring a high-quality Democrat to a moderate Republican, if the latter is preferred by moderate Republican voters.

Lemma 6. *Given the conditions on its derivatives, the generalized utility function $u(d_i, q)$ leads to at most one cutoff point in b for a given combination of q_L , q_R , p_L and p_R .*

Proof. See the Appendix ■

3.3.2 The utility function of the candidates

What about the utility function of the candidates given in (1)? Candidates are assumed to care neither about office nor about the quality of the winner. Both assumptions can be relaxed, because there is no uncertainty about the winner in the model. Concerns about victory do not enable the lower-quality candidate to win. On the other hand, even if the lower-quality candidate actually preferred to lose to let a higher quality candidate govern, he would still have an incentive to choose a position that forces the winner to make compromises with respect to his position. Therefore, the equilibria given in Section 2 do not disappear if the utility of the candidates depends on the quality of the winner of the elections or on winning the elections.

4 Swing voters

The equilibrium strategy for uninformed voters derived in Section 2.3 is relatively simple. Nevertheless, it requires a certain level of sophistication of the uninformed voters. Relaxing the sophistication requirements allows the reader, or future empirical researchers, to decide if they are indeed an attribute of a typical electorate. Moreover, modeling less sophisticated voters implies interesting effects on political competition that run counter to the common expectations regarding the effects of a less sophisticated and informed electorate. Specifically, I show that such an electorate actually leads to increased electoral control in the sense of forcing the winning candidate to a policy closer to the bliss point of the median voter. This does not only increase the welfare of the median voter, but that of a majority of all voters.

The methods I use to solve the model are similar to Section 2. First, I solve the problem of the voters and then the problem of the candidates. Finally, I show that the strategies of the sophisticated voters and the candidates together constitute a Perfect Bayesian Nash equilibrium. In Section 5, I analyze the welfare implications of having sophisticated uninformed voters instead of informed voters, as well as of having unsophisticated uninformed voters instead of sophisticated voters. For this analysis, I hold the overall distribution of voters' bliss points constant. In Section 6, I change the basic setup. Instead of a finite number of voters, I assume a continuum. This makes comparative statics possible.

4.1 The voting decision of unsophisticated voters

In this section, I introduce a third class of voters. These unsophisticated uninformed voters or swing voters vote naively without considering the fact that being the pivotal voter reveals information about the quality of politicians. Instead, they calculate their expected welfare given the policy platforms of the candidates and their a priori belief of the distribution of v . Therefore, they have the cutoff point $b_{UU}^*(p_L, p_R) = \frac{p_L + p_R}{2} - \frac{E(v)}{2(p_R - p_L)}$, which reduces to $b_{UU}^*(p_L, p_R) = \frac{p_L + p_R}{2}$ under the assumption that $E(v) = 0$ which I make from now on. All unsophisticated uninformed voters with a bliss point to the left of this cutoff point vote for the left candidate, all unsophisticated uninformed voters with a bliss point to the right of this cutoff point vote for the right candidate. To ensure the existence of an equilibrium, I assume that

a voter with bliss point $b_{UU}^*(p_L, p_R)$ votes for the left candidate if $b_{UU}^*(p_L, p_R) \leq b_m$ and for the right candidate if $b_{UU}^*(p_L, p_R) > b_m$.

4.2 The problem of the sophisticated uninformed voters

The decisive informed voter now has the bliss point:

$$b_I^d(p_L, p_R) = F_I^{-1} \left(\frac{n+1}{2} - l_{SU}(p_L, p_R) - l_{UU}(p_L, p_R) \right), \quad (13)$$

where $l_{SU}(p_L, p_R)$ is the number of sophisticated uninformed voters voting in favor of the left policy position, and $l_{UU}(p_L, p_R)$ the number of unsophisticated uninformed voters voting in favor of the left policy position. Using the same arguments that were used to derive the equilibrium in Section 2, it is possible to show that if all sophisticated uninformed voters vote for the left position if their bliss point is smaller than b_I^d , and for the right position if their bliss point is larger than b_I^d , their strategies constitute an optimal reply to the strategies of the other voters for arbitrary combinations of p_L , p_R and v . This gives the second condition:

$$l_{SU}(p_L, p_R) = F_{SU}(b_I^d(p_L, p_R)). \quad (14)$$

If conditions (13) and (14) hold, all sophisticated uninformed voters vote optimally independently of the strategies of the candidates. However, just as in the case with only sophisticated uninformed voters when the median is uninformed, it is sometimes impossible for a sophisticated uninformed voter to make her position only dependent on her own position relative to that of the decisive informed voter. The reason is that her own decision changes the decisive informed voter's identity. Consider the following cutoff point between voting left and right for sophisticated uninformed voters:

$$b_{SU}^*(p_L, p_R) = F_S^{-1} \left(\frac{n+1}{2} - l_{UU} \left(\frac{p_L + p_R}{2} \right) \right), \quad (15)$$

where $F_S(b) = F_I(b) + F_{SU}(b)$ is the cumulative distribution function of sophisticated voters (that is voters who are either informed or sophisticated uninformed), and $F_S^{-1}(x)$ gives the sophisticated voter with the x_{th} smallest bliss point b among sophisticated voters. If the sophisticated voter at $b_{SU}^*(p_L, p_R)$ is informed, she is decisive because if she votes left (right), all informed and sophisticated uninformed

voters to the left (right) of her vote left (right). Together with the unsophisticated uninformed voters who vote left (right), this constitutes a majority. Moreover, (14) holds and the voting stage of the game is in equilibrium.

If the sophisticated voter with bliss point $b_{SU}^*(p_L, p_R)$ is uninformed, she faces a situation similar to that of the uninformed median voter in Section 3.2. If she votes for the left candidate, the bliss point of the decisive informed voter is located to the left of her bliss point, and if she votes for the right candidate, the bliss point of the decisive informed voter is located to the right of her bliss point. Therefore, it is not independent of her beliefs of the value of v which decisive informed voter she prefers. I assume that she votes for the candidate whose position is closer to her own bliss point. Similarly to the case of an uninformed median voter, this turns out to be consistent with an equilibrium. The reason is once more that the candidates adjust their positions to the voters' strategies and the candidate with valence advantage wins. If both candidates have the same distance from b_{SU}^* , I assume that a sophisticated uninformed voter with this bliss point votes left if $\frac{p_L + p_R}{2} \geq b_m$ and right if $\frac{p_L + p_R}{2} < b_m$. Therefore, the decisive informed voter has the bliss point:

$$b_I^d(p_L, p_R) = \begin{cases} b_{SU}^*(p_L, p_R) & \text{if } b_{SU}^*(p_L, p_R) \in B_I \\ F_I^{-1}(F_I(b_{SU}^*(p_L, p_R))) & \text{if } b_{SU}^* \notin B_I \text{ and } \frac{p_L + p_R}{2} > b_{SU}^* \\ F_I^{-1}(F_I(b_{SU}^*(p_L, p_R))) & \text{if } b_{SU}^* \notin B_I \text{ and } \frac{p_L + p_R}{2} = b_{SU}^* \text{ and } \frac{p_L + p_R}{2} \geq b_m \\ F_I^{-1}(F_I(b_{SU}^*(p_L, p_R) + 1)) & \text{if } b_{SU}^* \notin B_I \text{ and } \frac{p_L + p_R}{2} = b_{SU}^* \text{ and } \frac{p_L + p_R}{2} < b_m \\ F_I^{-1}(F_I(b_{SU}^*(p_L, p_R) + 1)) & \text{if } b_{SU}^* \notin B_I \text{ and } \frac{p_L + p_R}{2} < b_{SU}^*, \end{cases} \quad (16)$$

where B_I is the set of bliss points of informed voters.

4.3 The problem of the candidates

The candidates face a somewhat more complicated problem than in Section 2 for there is now a trade-off in winning the support of sophisticated versus unsophisticated voters. Let $l_I(p_L, p_R, v)$ once more be the number of votes for the left position by informed voters, $l_{SU}(p_L, p_R)$ the number of votes for the left position by sophisticated uninformed voters, and $l_{UU}(p_L, p_R)$ the number of votes for the left position by unsophisticated uninformed voters. Then, there exists a best reply function for the candidate with valence advantage:

Lemma 7. *If $v > 0$, a best reply p_R^b to any p_L exists and is given by:*

$$p_R^b(p_L, v) = \begin{cases} \max_{[0, b_R]} p_R \text{ s.t. } l_I(p_L, p_R, v) + l_{SU}(p_L, p_R) + l_{UU}(p_L, p_R) < \frac{n+1}{2} & \text{if } p_L \leq b_R \\ b_R & \text{if } p_L > b_R \end{cases} \quad (17)$$

If $v \leq 0$, a best reply p_L^b to any $p_R \geq b_L$ exists and is given by:

$$p_L^b(p_R, v) = \begin{cases} \min_{[b_L, 1]} p_L \text{ s.t. } l_I(p_L, p_R, v) + l_{SU}(p_L, p_R) + l_{UU}(p_L, p_R) > \frac{n+1}{2} & \text{if } p_R \geq b_L \\ b_L & \text{if } p_R < b_L \end{cases} \quad (18)$$

Proof. See the Appendix ■

The equilibrium policy platforms of the candidates are:

Proposition 4. *Equilibrium policy platforms of the candidates are:*

$$\left. \begin{aligned} p_R^* &= \min_{p_L \in [0, 1]} p_R^b(p_L, v) \\ p_L^* &= \arg \min_{p_L \in [0, 1]} p_R^b(p_L, v) \end{aligned} \right\} \text{if } v > 0, \quad (19)$$

$$\left. \begin{aligned} p_L^* &= \max_{p_R \in [0, 1]} p_L^b(p_R, v) \\ p_R^* &= \arg \max_{p_R \in [0, 1]} p_L^b(p_R, v) \end{aligned} \right\} \text{if } v \leq 0, \quad (20)$$

and the implemented policy is:

$$p = \begin{cases} p_R^* & \text{if } v > 0 \\ p_L^* & \text{if } v \leq 0 \end{cases} \quad (21)$$

Proof. See the Appendix ■

As in Section 2, the candidate with the valence advantage wins the elections. However, without any further assumptions about the distribution of the voters, it is not possible to give a more precise solution than that in Proposition 4.

The intuition for Proposition 4 is straightforward. The candidate with the valence advantage chooses a position that is as close as possible to his own bliss point without being defeated. The lower-quality candidate chooses his position to force the winner as close to the median voter's bliss point as possible.

4.4 Voters' beliefs and equilibrium

Similarly to the case without swing voters in Section 2.5, I now formulate beliefs for the voters that are consistent with the equilibrium:

Let $V(p_L, p_R)$ once more denote the set of all values of v that would lead to equilibrium policy positions p_L^* and p_R^* in (4). The following is a consistent belief system for the uninformed voters, given the equilibrium strategies of the candidates:

$$\begin{aligned}
 &v(p_L, p_R) \text{ with probability } 1 && \text{if } v(p_L, p_R) \text{ is the only element in } V(p_L, p_R), \\
 &g(v|p_L, p_R) = \frac{g(v)}{\int_{v \in V(p_L, p_R)} g(v) dv} && \text{if } V(p_L, p_R) \text{ contains more than one element,} \\
 &g(v|p_L, p_R) = \begin{cases} \frac{g(v)}{\int_0^\infty g(v) dv} & \text{for } v > 0 \\ 0 & \text{for } v \leq 0 \end{cases} && \text{if } V(p_L, p_R) = \emptyset \text{ and} \\
 & && |p_L - b_{SU}^*(p_L, p_R)| > |p_R - b_{SU}^*(p_L, p_R)|, \\
 &g(v|p_L, p_R) = \begin{cases} \frac{g(v)}{\int_{-\infty}^0 g(v) dv} & \text{for } v \leq 0 \\ 0 & \text{for } v > 0 \end{cases} && \text{if } V(p_L, p_R) = \emptyset \text{ and} \\
 & && |p_L - b_{SU}^*(p_L, p_R)| \leq |p_R - b_{SU}^*(p_L, p_R)|.
 \end{aligned} \tag{22}$$

It remains to be shown that the voters' and the candidates' strategies, together with these beliefs, indeed constitute a Perfect Bayesian Nash Equilibrium:

Proposition 5. *Taking the nonstrategic decisions by the unsophisticated uninformed voters as given, the candidates' strategies together with the voting decisions by the sophisticated uninformed and the informed voters constitute a Bayesian Nash equilibrium. Together with the beliefs in (22), they constitute a Perfect Bayesian Nash equilibrium.*

Proof. See the Appendix ■

The interpretation of the equilibrium is similar to the equilibria in Section 2. Once more, sophisticated uninformed voters vote in a way that ensures that they vote for the candidate whom they prefer when they are pivotal. Because the candidates adjust their positions accordingly, even a sophisticated uninformed voter with bliss point b_{SU}^* never votes against her favorite candidate when she is pivotal. In fact, she is never pivotal in equilibrium and always votes against the winner.

4.5 Swing voters and equilibrium policy

The solution for the candidates' problem given in Proposition 5 is rather abstract. Nonetheless, I am able to make some welfare statements and analyze how voters'

welfare depends on the number of swing voters for a given overall distribution of voters $F(b)$. Let B be the set of all bliss points and B_I , B_{SU} and B_{UU} the sets of the bliss points of the informed, the sophisticated uninformed and the unsophisticated uninformed voters so that $B = B_I \cup B_{SU} \cup B_{UU}$.¹⁹ Then, the following results hold:

Lemma 8. *Taking the overall set of bliss points B as given, having sophisticated uninformed voters (case ') instead of informed voters (case ") at some bliss points ($B' = B''$, $B'_I \subsetneq B''_I$, $B''_{SU} \subsetneq B'_{SU}$, $B'_{UU} = B''_{UU}$) leads to equilibrium policies as close or closer to the median bliss point for all values of v ($|p^{*'}(v) - b_m| \leq |p^{*''}(v) - b_m|$).*

Proof. See the Appendix ■

Lemma 9. *Taking the overall set of bliss points B as given, having unsophisticated uninformed voters (case ') instead of sophisticated uninformed voters (case ") at some bliss points ($B' = B''$, $B'_I = B''_I$, $B'_{SU} \subsetneq B''_{SU}$, $B''_{UU} \subsetneq B'_{UU}$) leads to equilibrium policies as close or closer to the median bliss point for all values of v ($|p^{*'}(v) - b_m| \leq |p^{*''}(v) - b_m|$).*

Proof. See the Appendix ■

The lemmas show that turning an informed voter into a sophisticated uninformed voter as well as turning a sophisticated uninformed voter into an unsophisticated one, can only lead to policies closer to the median voter's bliss point for a given value of the quality difference v . So far, I have only shown that equilibrium policy can only move towards the median bliss point if it changes, not that it actually does change. It is difficult to give general rules for a change of equilibrium policy. However, examples can provide some insight into this. Therefore, I discuss two examples and then provide a further example with a continuum of voters in Section 6.

The example for the effect of having a voter switch from being informed to being sophisticated uninformed was already provided by the generalization of the basic model in Section 2 to the model with an uninformed median voter in Section 3.2. A comparison of (7) and (11) confirms the result of Lemma 8. For values of v that do not allow the high-quality candidate to achieve his bliss point in spite of an

¹⁹ Using sets is possible because of the assumption that all voters' bliss points are distinct from each other.

uninformed median voter, the implemented policy is closer to the median bliss point if the median voter is uninformed.

To provide an example of uninformed voters becoming unsophisticated informed voters, consider the case of an electorate with only unsophisticated uninformed voters and compare it with the equilibrium in Section 2. The equilibrium with only sophisticated voters is given in Proposition (1). The given equilibrium policies cannot constitute an equilibrium with unsophisticated uninformed voters if the candidate with lower valence now wins given the equilibrium positions. If $0 < v \leq (b_R - b_m)^2$, any unsophisticated uninformed voter in the interval $(b_m, b_m + v^{0.5}/2)$ forces the right candidate to choose a position closer to the median bliss point to win the elections. If he does not adjust, he loses because, in equilibrium, he wins with just one vote and the swing voters in the interval are now voting against him instead of in his favor. If $0 < (-v) \leq (b_L - b_m)^2$, any unsophisticated uninformed voter in the interval $(b_m - (-v)^{0.5}/2, b_m)$ forces the left candidate to choose a position closer to the median bliss point to win. Unsophisticated uninformed voters change the equilibrium policy for a larger interval of values of v , if they are located closer to the median bliss point.²⁰

From the last example, it should be clear that if the electorate is large and the number of swing voters is neither very small nor their distribution very different from the distribution of sophisticated voters, the equilibrium will not be the same as the equilibrium without swing voters.

5 Welfare analysis

In this section, I show the welfare impact of having swing voters in the electorate and, as a consequence, policies that are at least weakly closer to the median voter's bliss point.

Take $g(v)$ as given. Let again $p'(v)$ be an equilibrium policy and $p''(v)$ a different one resulting from the same distribution of bliss points $F(b)$, but with some informed voters instead of sophisticated uninformed voters and/or some sophisticated uninformed voters instead of unsophisticated uninformed voters. By Lemma

²⁰ The median position is not necessarily a best reply for the candidate with valence disadvantage. Therefore, the examples in the text give sufficient, but not necessary conditions for a change of equilibrium positions due to the existence of swing voters.

8 and Lemma 9, we know that the policy $p'(v)$ is at least as close to the median bliss point as the policy $p''(v)$. Thus, $(p''(v) - b_m)^2 \geq (p'(v) - b_m)^2$ for any difference in quality, v . Therefore, the median voter must be (weakly) better off with policy $p'(v)$ for every value of v . Conditioning on v , the majority of voters must be better off with $p'(v)$ instead of $p''(v)$. If $v > 0$ ($v \leq 0$), all voters to the left (right) of the median bliss point are better off with $p'(v)$ since the implemented policy is closer to their bliss point.

In an equilibrium with policy $p(v)$, the expected utility (before nature chooses the quality of candidates) of a voter with bliss point b is :

$$E[U(p, b)] = \int_{-\infty}^{\infty} -(p(v) - b)^2 g(v) + E[\max(q_L, q_R)], \quad (23)$$

where the first term is the utility from implemented policy dependent on the valence difference and the second part is the utility from the quality or valence of the winner of the elections. Since the candidate with a valence advantage always wins, the valence of the winner is the larger of the two valence factors, q_R and q_L .

The difference in ex ante expected utility from the different equilibrium policies $p''(v)$ and $p'(v)$ for a voter with bliss point b is therefore:

$$\Delta E(U, b) = E[U(p'', b)] - E[U(p', b)] = \int_{-\infty}^{\infty} (-p''(v)^2 + p'(v)^2 + 2b(p''(v) - p'(v)))g(v). \quad (24)$$

We know that the difference is weakly negative for the median voter because we know that she is better off with $p'(v)$:

$$\Delta E(U, b_m) = \int_{-\infty}^{\infty} (-p''(v)^2 + p'(v)^2 + 2b_m(p''(v) - p'(v)))g(v) \leq 0 \quad (25)$$

If $\int_{-\infty}^{\infty} (p''(v) - p'(v))g(v) > 0$, all voters with $b < b_m$ are better off with $p'(v)$ in expectation, and if $\int_{-\infty}^{\infty} (p''(v) - p'(v))g(v) < 0$, all voters with $b > b_m$ better off with $p'(v)$ in expectation. Together with the median voter, either group constitutes a majority and therefore, the majority of voters is better off with $p'(v)$.

If the expected value of $p'(v)$ is the same as the expected value of $p''(v)$, all voters are better off without exception. The intuition is simply that in this case, the volatility of policy decreases, which is good for all voters because they are risk averse, while the expected policy remains the same. If $p'(v)$ is not same as $p''(v)$,

it is possible to calculate a cutoff point between the voters who are better off and those who are worse off. This is given by:

$$\begin{aligned}\Delta E(U, b_{cut}) &= \int_{-\infty}^{\infty} (-p''(v)^2 + p'(v)^2 + 2b_{cut}(p''(v) - p'(v)))g(v) = 0 \quad (26) \\ \Rightarrow b_{cut} &= \frac{\int_{-\infty}^{\infty} (p''(v)^2 - p'(v)^2)g(v)}{2 \int_{-\infty}^{\infty} (p''(v) - p'(v))g(v)} \text{ for } \int_{-\infty}^{\infty} (p''(v) - p'(v))g(v) \neq 0.\end{aligned}$$

If $\int_{-\infty}^{\infty} (p''(v) - p'(v))g(v) > 0$, all voters with $b < b_{cut}$ ($b > b_{cut}$) are better (worse) off with $p'(v)$ in expectation and if $\int_{-\infty}^{\infty} (p''(v) - p'(v))g(v) < 0$, all voters with $b > b_{cut}$ ($b < b_{cut}$) are better (worse) off with $p'(v)$ in expectation.

A utilitarian (Benthamite) social welfare function that takes all voters equally into account is given by:

$$E\left[\sum_{i=1}^n (U(p, b_i))\right] = \sum_{i=1}^n \int_{-\infty}^{\infty} -(p(v) - b_i)^2 g(v) + E[\max(q_L, q_R)] \quad (27)$$

The difference between the aggregate welfare of policy $p''(v)$ and $p'(v)$ is given by:

$$\Delta E \sum_{i=1}^n (U, b_i) = \sum_{i=1}^n \int_{-\infty}^{\infty} (-p''(v)^2 + p'(v)^2 + 2b_i(p''(v) - p'(v)))g(v) \quad (28)$$

From this, it follows that aggregate utility is increased if the average bliss point $b_{av} = \frac{\sum_{i=1}^n b_i}{n}$ is on the side of the cutoff point where the change in policy leads to a welfare improvement for voters.²¹

6 An example with a continuum of voters

For an example with more specific assumptions about the distribution of voters, I will depart from the basic setup and assume a continuum of voters instead of a finite number. Working with a continuum of voters ensures that the strategies of the candidates become continuous in v and makes it possible to analyze the impact of a marginal change in the number of unsophisticated voters on the welfare of voters. Comparative statics analysis is possible.

²¹ This last result is due to the quadratic disutility in distance and is therefore not robust to changes in the utility function of voters.

As was just shown, the welfare analysis leads to somewhat ambiguous results. The majority of the voters is better off with swing voters, but it is possible to construct examples where the average voter is not. However, under some symmetry assumptions, I show unambiguous ex ante welfare improvements in the specific example I give. Similar welfare improvement can be expected in politics where the symmetry assumptions do not hold exactly, but give a good approximation.

The assumptions about utility functions, candidates and the sequence of moves by the players remain the same as before. Instead of a finite number, there is now a continuum of voters with mass 1. $1 - \alpha - \beta$ of these are informed, α of these are uninformed but sophisticated and β are uninformed but not sophisticated. I assume that $\alpha + \beta < \frac{1}{2}$. This implies that the winner of the elections needs the support of some informed voters and therefore, insures the existence of a decisive informed voter independently of the voting decision of the uninformed voters. Moreover, to simplify the analysis, I assume that the bliss points of all three groups of voters are uniformly distributed on the policy space $[0, 1]$. The expected value of the quality difference v is assumed to be 0 with density $g(v)$ symmetric around 0, that is $g(v) = g(-v)$ for all values of v . This implies that a certain valence advantage is as likely for the right candidate as for the left candidate. In addition, $b_R - 0.5 = 0.5 - b_L$, that is, the distance between both candidates' bliss points and the median position is the same.

Voters are assumed to vote as in Section 4. Informed voters cast their ballot for the candidate they prefer, after observing the policy positions as well as the valence factors. The unsophisticated uninformed voters vote for the candidate whose policy is closer to their bliss point. The sophisticated uninformed voters one more have to find an optimal cutoff point that determines their strategy.

To deal with the assumption of a continuum of voters, I need to make an additional tie-breaking assumption in case both candidates get exactly half of the votes. To ensure the existence of an equilibrium, I assume that in this case the candidate with the valence advantage wins. The intuition from the case with a finite number of voters still applies and therefore the continuum should be interpreted as a convenient approximation of the case of a finite number of voters. Therefore, this assumption seems innocent.

6.1 Solving the model with a continuum of voters

Given the decision of the two kinds of uninformed voters and the assumption about the distribution of voters, the bliss point of the decisive informed voter for $p_L \neq p_R$ is given by:

$$b_I^d(p_L, p_R, l_{SU}(p_L, p_R), l_{UU}(p_L, p_R)) = \frac{0.5 - l_{SU}(p_L, p_R) - \beta \frac{(p_L + p_R)}{2}}{1 - \alpha - \beta}. \quad (29)$$

Once more, the voter with bliss point b_I^d is decisive because when she votes left at least 50% of the voters vote left, and when she votes right, at least 50% of the voters vote right. Therefore, the candidate who has the support of the decisive informed voter wins the elections.

Sophisticated uninformed voters vote for the left (right) policy position if their bliss point is to the left (right) of the decisive informed voter's bliss point. They want to pull the decisive informed voter's bliss point closer to their own. From this follows the second condition:

$$F_{SU}(b_I^d(p_L, p_R, l_U)) = \alpha b_I^d(p_L, p_R, l_{SU}) = l_{SU}(p_L, p_R). \quad (30)$$

Putting (29) and (30) together, I obtain that:

$$b_I^d(p_L, p_R) = \frac{0.5 - \beta \frac{(p_L + p_R)}{2}}{1 - \beta}. \quad (31)$$

The share α of sophisticated uninformed voters drops out of the equation for the decisive informed voter because these voters manage to vote just as informed voters when they are pivotal. This implies that I can analyze the equilibrium policy with only informed voters and unsophisticated uninformed voters and know that the equilibrium policy positions must be identical for all cases with the same share β of unsophisticated uninformed voters in the electorate.

The number of votes for left in the case $\alpha = 0$ is:²²

$$\beta \frac{(p_L + p_R)}{2} + (1 - \beta) \left(\frac{p_L + p_R}{2} - \frac{v}{2(p_R - p_L)} \right). \quad (32)$$

²² To simplify the notation, I ignore the fact that the share of informed voters voting left cannot be smaller than 0 or larger than 1. This has no influence on any of the results.

What happens if $p_L = p_R$? Several assumptions seem plausible. In the case without any unsophisticated voters, the sophisticated uninformed ones had little reason to deviate from their strategy of making voting dependent only on their position relative to the median position. Here, I assume that sophisticated as well as unsophisticated uninformed voters randomize over their voting decision if $p_L = p_R$. Because there is a continuum of voters, there is nonetheless no uncertainty about the mass of votes for the left and the right candidate. I also assume that if $p_L = p_R$ and $v = 0$, all informed voters vote for the left candidate.

The equilibrium policy platforms of the candidates and the implemented policy are now:

Lemma 10.

$$\left. \begin{aligned} p_R^* &= \min((1 - \beta)^{0.5}v^{0.5} + 0.5, b_R) \\ p_L^* &= \max(p_R^* - (1 - \beta)^{0.5}v^{0.5}, 0) \\ p^* &= p_R^* \end{aligned} \right\} \text{if } v > 0, \quad (33)$$

$$\left. \begin{aligned} p_L^* &= \max(-(1 - \beta)^{0.5}(-v)^{0.5} + 0.5, b_L) \\ p_R^* &= \min(p_L^* + (1 - \beta)^{0.5}(-v)^{0.5}, 1) \\ p^* &= p_L^* \end{aligned} \right\} \text{if } v \leq 0.$$

Proof. See the Appendix ■

6.2 Electoral control and welfare analysis

The ex-ante expected utility of any voter with bliss point b given an equilibrium policy $p(v)$ is still given by (23):

$$E(U(b)) = \int_{v=-\infty}^{v=\infty} -(p(v) - b)^2 g(v) dv + E(\max(q_R, q_L)),$$

Lemma 11. *The welfare of every voter is increasing in the share of unsophisticated uninformed voters β , independently of her bliss point b . The change in expected utility of a voter (or candidate) with bliss point b due to a marginal increase in the share of boundedly rational voters is given by the formula:*

$$\frac{dE(U(b))}{d\beta} = \int_{v=-\frac{(b_L-0.5)^2}{(1-\beta)}}^{v=\frac{(b_R-0.5)^2}{(1-\beta)}} |v|g(v)dv > 0, \quad (34)$$

which is larger than 0 as long as the distribution of v is not degenerate.

Proof. See the Appendix ■

The intuition is straightforward. An increase in the number of uninformed unsophisticated voters increases electoral control and forces the politician with the valence advantage to stay closer to the median position to win the elections. This is advantageous for all voters. Even though a voter might profit from an increased likelihood of somewhat more extreme policies close to her bliss point, she always suffers more from the equally increased likelihood of extreme policies on the other side of the median position. The overall effect of a change in β is the same for all voters independently of their bliss points due to the quadratic disutility in policy.

The impact of a change in β increases in:

$$\int_{v=-\frac{(b_L-0.5)^2}{(1-\beta)}}^{v=\frac{(b_R-0.5)^2}{(1-\beta)}} |v|g(v)dv.$$

This term could be called the "adjusted" absolute deviation of the valence difference v . The reason for this "adjusted" absolute deviation being the relevant measure of dispersion is that for large absolute values of v , the winning candidate implements his bliss point. Therefore, an additional dispersion of large absolute values of v does not lead to an additional variance in implemented policy. The advantage of a larger β disappears if v is always 0, that is in a standard model of Downsian Competition without a valence component. Because every single voter is better off with an increase in β , this constitutes a Pareto improvement from an ex-ante perspective.

7 Conclusion

This paper combines elements of the two approaches in political economics that interpret elections as preference aggregation and information aggregation, respectively. I merge a Downsian model with voter disagreement on policy on a left-right scale, with a model of voter agreement over the quality of political candidates, which is not observable to all voters, however.

A lack of information on the part of some voters about the quality of politicians is shown to have no consequences at all if every uninformed voter is rational. But if there are some boundedly rational swing voters, a lack of information increases

electoral control, in the sense of pulling the implemented policy closer to the preferences of the median voter. This surprising result arises because boundedly rational voters support whoever offers them a policy closer to their own bliss point. They do not consider the fact that their vote is only of importance in a close election with both candidates obtaining exactly half of the votes. This voting strategy works as a commitment device, which forces the winning candidate to moderate his policy position. The larger is the group of unsophisticated uninformed voters, the stronger is the favorable effect.

A remarkable aspect of my findings is that always voting for the candidate of the same party is entirely rational for uninformed voters. In equilibrium, uninformed voters support a candidate whose preferences are located on the same side of the median position as their own, in spite of the fact that they do not know whether this is the candidate who they would support if they were fully informed. This forces candidates to consider the preferences of uninformed voters as much as those of informed voters and stands in stark contrast to the literature that claims that abstentions can be the result of rational choice even when voting is costless (Feddersen and Pesendorfer 1999).

I show that voting patterns that are considered to be evidence for irrational partisan behavior can be the rational response to a lack of information about the quality of candidates. Nevertheless, the belief that partisan voting can lead to less desirable policies is confirmed. This is surprising, because the belief that partisan voting leads to bad policy is usually based on the belief that it is irrational. In my model, partisan voters decrease the welfare of the majority of voters, not because they act irrationally, but because they are rational and therefore cannot commit to "punishing" the candidate who announces a policy position further away from their own preferences. The individual rationality of their decisions leads to a decrease in electoral control, policies further away from the median voter's bliss point and an expected loss in welfare for the majority of voters. Boundedly rational swing voters, on the other hand, turn out to be a blessing, not a curse.

One possibility for testing my model is offered by experiments. Similar models have already been tested experimentally (Palfrey 2009), and it might be the best way of testing if the size of the electorate is likely to have an influence on the relative number of swing voters. This is left for future research.

Appendix A

Proof Section 2

Proof Lemma 4. Suppose that there was a cutoff point $b'_U(p_L, p_R) \neq b_m$ such that $F_U(b'_U(p_L, p_R)) \neq F_U(b_m)$ for any combination of p_L and p_R .²³ Without loss of generality, I assume that an uninformed voter at this bliss point votes left. (To describe a strategy with this voter voting right given p_L and p_R , while the other voters make the same voting decision, it is always possible to choose a cutoff point slightly further to the left.)

If $F_U(b'_U(p_L, p_R)) < F_U(b_m)$, the number of votes for the left policy position by uninformed voters is smaller than in the equilibrium given in Sections 2.1 – 2.4, and at least one uninformed voter with a bliss point $b < b_m$ votes for the right policy position. The bliss point of the decisive informed voter is given by $b_I^d(l_U) = F_I^{-1}(\frac{N+1}{2} - F_U(b'_U(p_L, p_R)))$, and the bliss point of the decisive informed voter for $F_U(b'_U(p_L, p_R)) + 1$ votes for the left position by uninformed voters is given by $b_I^{d+1}(l_U) = F_I^{-1}(\frac{N+1}{2} - F_U(b'_U(p_L, p_R)) - 1)$. The assumption $F_U(b'_U(p_L, p_R)) < F_U(b_m)$ implies that $b_I^d(l_U) > b_I^{d+1}(l_U) \geq b_m$. An uninformed voter with bliss point $b < b_m$ who votes right can only be pivotal if $b_I^{d+1}(l_U) \leq b^*(p_L, p_R, v) \leq b_I^d(l_U)$. Because v^b has positive support everywhere, voters believe that the possibility of this happening is positive. An uninformed voter with bliss point $b < b_m \leq b_I^{d+1}(l_U)$ prefers the left position to win when she is pivotal. Therefore, $F_U(b'_U(p_L, p_R)) < F_U(b_m)$ cannot be a cutoff point that is consistent with a Bayesian Nash equilibrium that is consistent with $g(v^b|p_L, p_R) > 0$ for all combinations of v , p_L and p_R .

The case $F_U(b_U^*(p_L, p_R)) > F_U(b_m)$ can be ruled out by an analogous argument. Therefore, only cutoff points of informed voters with $F_U(b_U^*(p_L, p_R)) = F_U(b_m)$ can characterize an equilibrium. From Section 2.1 – 2.4, we know that in combination with the candidates' policy positions given in Proposition 1 and the weakly dominating strategies of the informed voters, the strategies characterized by this cutoff point constitute a Bayesian Nash equilibrium for all possible beliefs about $g(v^b|p_L, p_R)$. ■

²³ If $F_U(b_U^*(p_L, p_R)) = F_U(b_m)$, both cutoff points lead to the same voting decision and therefore describe the same strategy by uninformed voters. For every distribution of uninformed voters, such alternative cutoff points exist around b_m .

Appendix B

Proofs Section 3

Proof Proposition 2. Consider the case $v > 0$.

The left candidate obtains $F_U(b_m)$ votes by uninformed voters. Therefore, the left candidate wins if and only if at least $\frac{n+1}{2} - F_U(b_m) = F_I(b_m)$ informed voters vote in his favor. This will be the case if and only if the informed median voter prefers the left candidate. This implies that the candidates face the same problem as if all voters observed the policy positions that is analyzed in Sections 2.1 – 2.4. The analysis as well as the results of Proposition 1 therefore apply to the candidates' problem.

The informed voters' strategy constitutes a best reply to the other players' strategies, because they play a weakly dominating strategy. The completely uninformed voters who vote for the left candidate play a best reply to the other players' strategies since they are not pivotal. The completely uninformed voters who vote for the right candidate have a bliss point $b > b_m$. From the fact that the informed median voter votes for the right candidate in equilibrium, we know that $b^*(p_L, p_R, v) \leq b_m$. Therefore, all uninformed voters voting for the right candidate have a bliss point $b > b^*(p_L, p_R, v)$ and obtain higher utility with the right candidate. Voting right is therefore optimal for them.

The argument for the case $v \leq 0$ is analogous. ■

Proof Lemma 5. Outline of the proof: First, I show that the informed voter with bliss point b_l is decisive if the median voter votes left for the left policy position, and the informed voter with bliss point b_r is decisive if the median voter votes for the right policy position. I continue by showing that the implemented policy is as given in (11). Then, I show that the strategies are optimal for the candidates. It is straightforward to verify that for all values of v , we have $p_R^* \geq p_L^*$, so that the right candidate adopts the right policy position.

Consider the case when the median voter votes for the left policy position: If the informed voter with bliss point b_l votes for the left policy position, this implies that $b_l^* \geq b_l$. Therefore, all informed voters with a bliss point $b < b_l$ vote for the left policy position. It follows that all voters with a bliss point $b \leq b_m$ vote for the left policy position, and the candidate with the left policy position obtains at least $\frac{n+1}{2}$ votes and wins the elections. If the informed voter with bliss point b_l votes for the

right policy position, this implies that $b_I^* \leq b_l$. Therefore, all informed voters with a bliss point to the right of the median voter vote for the right policy position. This implies that all voters with a bliss point $b > b_m$ and the voter with bliss point b_l vote for the right policy position. Therefore, the candidate with the right policy position obtains at least $\frac{n+1}{2}$ votes and wins the elections.

An analogous argument shows that the informed voter with bliss point b_r is decisive if the median voter votes for the right policy position.

Implemented policy:

If $v > 0$, then $p_R^* \leq b_l + (v + (b_l - p_L^*)^2)^{0.5}$ and therefore $\Delta U(b_l, p_L^*, p_R^*) = -(b_l - p_L^*)^2 + (b_l - p_R^*)^2 - v \leq 0$. This implies that the informed voter with bliss point b_l votes for the right candidate and therefore, the right candidate obtains a majority independently of the vote of the uninformed median voter.

If $v \leq 0$, then $p_L^* \geq b_r - (-v + (b_r - p_R^*)^2)^{0.5}$ and therefore, $\Delta U(b_r, p_L^*, p_R^*) = -(b_r - p_L^*)^2 + (b_r - p_R^*)^2 - v \geq 0$. This implies that the informed voter with bliss point b_r votes for the left candidate and therefore, the left candidate obtains a majority independently of the vote of the uninformed median voter.

Candidates' strategies for the case $v > 0$:

If $p_R^*(v) = b_r$, the right candidate cannot do better with any other position. If $p_R^* < b_r$, then $\Delta U(b_l) = -(b_l - p_L^*)^2 + (b_l - p_R^*)^2 - v = 0$. Therefore, the right candidate would not have the support of the informed voter with bliss point b_l for any position $p_R > p_R^*$. If $p_L^* = b_m - \frac{v}{4(b_m - b_l)}$, then $p_R^* = b_l + (v + (b_l - b_m + \frac{v}{4(b_m - b_l)})^2)^{0.5} = b_m + \frac{v}{4(b_m - b_l)}$ and if $p_L^* = b_l$ then $p_R^* = b_l + v^{0.5} \geq 2b_m - b_l$ (where the last inequality is due to the fact that $p_L^* = b_l$ implies that $v \geq 4(b_m - b_l)^2$). In both cases $|p_R^* - b_m| \geq |p_L^* - b_m|$, and therefore for any position $p_R > p_R^*$, the median voter votes for the left candidate. Therefore, for any $p_R > p_R^*$, the decisive informed voter is the voter with bliss point b_L who votes for the left candidate, and the left candidate wins with a position $p_L^* < p_R^*$. Thus, the right candidate cannot obtain any better implemented policy than p_R^* , and his position is a best reply to the strategies of the other players.

The left candidate would only have a better reply than p_L^* to the other players' strategies if he could win with a position $p_L < p_R^*$. Consider first the case with $p_L^* = b_l$. Any position p_L such that $p_L \neq b_l$ and $p_L < p_R^*$ loses because $\Delta U(b_l, b_l, p_R^*) = (b_l - p_R^*)^2 - v \leq 0$ implies that $\Delta U(b_l, p_L, p_R^*) = -(b_l - p_L)^2 + (b_l - p_R^*)^2 - v \leq 0$ for any $p_L < p_R^*$. Second case: $p_L^* = b_m - \frac{v}{4(b_m - b_l)}$. Then $p_R^* \leq b_m + \frac{v}{4(b_m - b_l)}$ and

therefore $|p_R^* - b_m| \leq |p_L^* - b_m|$. This implies that if the left candidate chooses a position $p_L < p_L^*$, the median voter votes for the right candidate. The left candidate loses because $\Delta U(b_l, p_L^*, p_R^*) \leq 0$ implies that $\Delta U(b_r, p_L^*, p_R^*) < 0$ which implies that $\Delta U(b_r, p_L, p_R^*) < 0$ for all $p_L < p_L^*$, so that the informed voter with bliss point b_r votes for the right candidate. If the left candidate chooses a position p_L such that $p_R^* > p_L > p_L^*$, then $p_L > b_l$ and from $\Delta U(b_l, p_L^*, p_R^*) \leq 0$ it follows that $\Delta U(b_l, p_L, p_R^*) < 0$ for such a value of p_L . Therefore, with a position $p_L < p_R^*$, the left candidate can neither win the vote of the informed voter with bliss point b_l , nor the vote of the informed voter with bliss point b_r , and thus loses. For $p_R^* < b_R$, the given combination of p_R^* and p_L^* is the only one that can be part of an equilibrium. For any $p_L \neq p_L^*$, the right candidate could choose a position closer to his bliss point and win, so that p_R^* would not be a best reply to any $p_L \neq p_L^*$. However, any $p_R > p_R^*$ can be defeated by p_L^* . If $p_R^* = b_R$, any left reply is a best reply. For the Lemma, I assume that the left candidate chooses the position given in (10).

An analogous argument applies to the case $v \leq 0$. ■

Proof Proposition 3. That the strategies of the candidates are best replies is shown in Lemma 5. Informed voters choose weakly dominating strategies that are best replies to any strategy profile by other players. It remains to be shown that the strategies of the uninformed voters are best replies.

The uninformed voters with bliss points $b > b_m$ who always vote for the right policy position can make a difference by voting left only in elections where they are pivotal. In equilibrium, this only occurs if the informed voter with bliss point b_l votes for the right candidate. In this case, the uninformed voters who vote right have a bliss point $b > b_m > b_l \geq b^*$ and maximize their utility with voting for the right candidate. An analogous argument applies for uninformed voters voting left. The uninformed median voter is never pivotal in equilibrium and thus, her strategy is a best reply.

For any out of equilibrium combinations of p_L and p_R , voters believe that the candidate with a position closer to the median voter has the valence advantage. Take the case $|p_L - b_m| > |p_R - b_m|$ and $p_L < p_R$. All uninformed voters believe that $v > 0$ and that all voters with $b > b_m$ (informed as well as uninformed) vote for the right candidate and therefore constitute a majority for him. Thus, for the uninformed voters with $b > b_m$, voting right is a best reply because they believe that

$b^* < b_m$. For the uninformed voters with $b < b_m$, voting left is a best reply because they believe that the right candidate wins independently of their voting decision. A similar argument applies to the other possible cases. ■

Proof Lemma 6. Assume that $p_R \geq p_L$, as is always the case in any equilibrium (the proof is analogous for $p_R < p_L$). Then, there are two possibilities, $q_R \geq q_L$ and $q_R < q_L$. If $q_R \geq q_L$, then either every voter prefers right (and the unique cutoff point is $b^* = 0$), or there is at least one value $b \in [0, 1]$ that solves $u(|b - p_L|, q_L) = u(|b - p_R|, q_R)$. The latter follows from the mean value theorem because u is continuous in d (this is implied by the fact that u has a derivative with respect to d), and therefore also in b (because d is a continuous function of b). Let b^* denote the largest b that solves the equation. From $q_R \geq q_L$, it follows that $b^* \leq \frac{p_L + p_R}{2}$, because a higher-quality candidate is always preferred if he is located closer to a voter's bliss point. From this and the fact that b^* is the rightmost bliss point with $u(|b - p_L|, q_L) = u(|b - p_R|, q_R)$, it follows that all voters to the right of b^* prefer right. If $b^* > p_L$, all voters with a bliss point b such that $p_L \leq b < b^*$ must prefer left because their bliss point is closer to p_L and further away from p_R than for the indifferent voter with bliss point b^* . Voters with $b < \min(p_L, b^*)$ must have a preference for left because the two assumptions $u_{dd}(d, q) < 0$ and $u_{qd}(d, q) \leq 0$ ensure that $u_d(p_L, q_L) > u_d(p_R, q_R)$ everywhere to the left of p_L . Therefore, there can be only one cutoff point and all voters with a bliss point to the left of b^* prefer left.

An analogous argument can be given to show that in the case of $q_R < q_L$, the bliss point is also unique. ■

Appendix C

Proofs Section 4

The proof of Lemma 7 and Proposition 4 requires the following three lemmas:

Lemma A-1 *The candidate with valence advantage wins if he chooses a position as close or closer to the median voter's bliss point than the other candidate.*

Proof. Consider the case $v > 0$ and $p_R \geq p_L$:

If $p_R = p_L$, all informed voters vote in favor of the candidate with valence advantage. Because the informed voters are the majority of voters, the candidate with valence advantage wins the elections.

If $p_R > p_L$, it follows from the assumptions that $|p_R - b_m| \leq |p_L - b_m|$ and $p_R > p_L$ that $b_{UU}^* = \frac{p_R + p_L}{2} < b_m$. Therefore, all unsophisticated uninformed voters with a bliss point at and to the right of the median bliss point vote right. If an unsophisticated uninformed voter is located at the median bliss point, then $l_{UU}(\frac{p_L + p_R}{2}) < F_{UU}(b_m)$ and therefore from equation (15):

$$\begin{aligned} b_{SU}^*(p_L, p_R) &= F_S^{-1} \left(\frac{n+1}{2} - l_{UU} \left(\frac{p_L + p_R}{2} \right) \right) \\ &= F_S^{-1} \left(F_S(b_m) + F_{UU}(b_m) - l_{UU} \left(\frac{p_L + p_R}{2} \right) \right) \geq F_S^{-1}(F_S(b_m) + 1) > b_m. \end{aligned}$$

If a sophisticated voter has the median bliss point, then $l_{UU}(\frac{p_L + p_R}{2}) \leq F_{UU}(b_m)$ and therefore:

$$\begin{aligned} b_{SU}^*(p_L, p_R) &= F_S^{-1} \left(\frac{n+1}{2} - l_{UU} \left(\frac{p_L + p_R}{2} \right) \right) = \\ &F_S^{-1} \left(F_S(b_m) + F_{UU}(b_m) - l_{UU} \left(\frac{p_L + p_R}{2} \right) \right) \geq F_S^{-1}(F_S(b_m)) = b_m. \end{aligned}$$

In both cases, $b_{SU}^*(p_L, p_R) \geq b_m > b_{UU}^*$. Moreover, $v > 0$ together with $p_R > p_L$ implies that $b_{UU}^* > b_I^*$. Because $b_{SU}^*(p_L, p_R) > b_{UU}^*$, a sophisticated uninformed voter with bliss point b_{SU}^* votes right and thus $b_I^d \geq b_{SU}^*(p_L, p_R)$ and $b_I^d > b_{UU}^* > b_I^*$. Thus, the decisive informed voter votes for the right candidate and the right candidate wins.

An analogous argument applies to the cases $v > 0, p_R < p_L$ and $v < 0$. ■

Lemma A-2 *As long as $v > 0$ and $p_R > p_L$ the number of votes for left candidate is nondecreasing in p_R .*

Proof. The cutoff points for informed voters and unsophisticated uninformed voters b_I^* and b_{UU}^* , respectively, are increasing in p_R^* . Therefore, the number of informed and unsophisticated uninformed voters voting for the left candidate is nondecreasing in p_R . Remember that the cutoff point for uninformed voters is given by (15):

$$b_{SU}^*(p_L, p_R) = F_S^{-1} \left(\frac{n+1}{2} - l_{UU} \left(\frac{p_L + p_R}{2} \right) \right),$$

and therefore decreasing in $l_{UU}(\frac{p_L + p_R}{2})$, the number of unsophisticated uninformed voters voting left. Therefore, the number of sophisticated uninformed voters vot-

ing right is nondecreasing in p_R . However, the number of sophisticated uninformed voters with a bliss point at or to the left of the cutoff point given by $F_{SU} \left(F_S^{-1} \left(\frac{n+1}{2} - l_{UU} \left(\frac{p_L + p_R}{2} \right) \right) \right)$, and this can by the definition of $F_S^{-1}(x)$ and $F_{SU}(b)$ not decrease faster than $l_{UU} \left(\frac{p_L + p_R}{2} \right)$ increases. Moreover, a sophisticated uninformed voter at the cutoff point votes right only if $\frac{p_L + p_R}{2} < b_{SU}^*$. Therefore, the number of all uninformed voters, sophisticated and unsophisticated, voting left is nondecreasing in p_R . Given that neither the number of votes by informed voters nor the number of votes by uninformed voters can decrease, the number of votes for left candidate must be nondecreasing in p_R . ■

Lemma A-3 *The interval of values of $p_R \geq p_L$ for which $l_I(p_L, p_R, v) + l_{UU}(p_L, p_R) + l_{SU}(p_L, p_R) < \frac{n+1}{2}$ holds is closed for all values of p_L .*

Proof. Suppose the statement were to be false and that the interval of values of $p_R \geq p_L$ for which $l_I(p_L, p_R, v) + l_{UU}(p_L, p_R) + l_{SU}(p_L, p_R) < \frac{n+1}{2}$ holds is not closed for some p_L . From Lemma A-2, we know that if a value $p_R^w > p_L$ wins against p_L , any other p_R such that $p_R > p_L$ and $p_R < p_R^w$ must also win. Therefore, if the interval is not closed, there is a value \bar{p}_R with $l_I(p_L, \bar{p}_R, v) + l_{UU}(p_L, \bar{p}_R) + l_{SU}(p_L, \bar{p}_R) > \frac{n+1}{2}$, but $l_I(p_L, p_R, v) + l_{UU}(p_L, p_R) + l_{SU}(p_L, p_R) < \frac{n+1}{2}$ for all p_R such that $p_L \leq p_R < \bar{p}_R$.

Define $\bar{l}_I = l_I(p_L, \bar{p}_R, v)$. This implies that $b_I^*(\bar{p}_R, p_L, v) > F_I^{-1}(\bar{l}_I)$, where the strict inequality is due to the fact that, by assumption, indifferent informed voters vote for the candidate with valence advantage. Then, by the continuity of b_I^* in p_R (for $p_R > p_L$), there exists a $p_R^I < \bar{p}_R$ for which $b_I^*(p_R, p_L, v) > F_I^{-1}(\bar{l}_I)$, and $l_I(p_L, p_R^I, v) = \bar{l}_I$. Define $\bar{l}_{UU} = l_{UU}(p_L, \bar{p}_R, v)$. This implies that $b_{UU}^*(\bar{p}_R, p_L) > F_U^{-1}(\bar{l}_{UU})$, with $b_{UU}^* = \frac{p_L + \bar{p}_R}{2}$. The strict inequality follows from the assumption that the left candidate wins the elections. Lemma A-1 implies that in this case $|p_L - b_m| < |p_R - b_m|$ and therefore $b_{UU}^*(p_L, p_R) > b_m$. By the assumption stated on page 34, an unsophisticated uninformed voter with bliss point $b_{UU}^*(p_L, p_R)$ votes for the right candidate in this case. By the continuity of b_{UU}^* in p_R , there exists a $p_R^{\bar{l}_{UU}} < \bar{p}_R$ for which $b_{UU}^*(p_R, p_L, v) > F_U^{-1}(\bar{l}_{UU})$. For $p_R \in [p_R^{\bar{l}_{UU}}, \bar{p}_R]$, the cutoff point for sophisticated uninformed voters is $b_{SU}^*(p_L, \bar{p}_R)$.

Define $\bar{l}_{SU} = l_{SU}(p_L, \bar{p}_R, v)$. This implies $b_{SU}^*(\bar{p}_R, p_L) > F_{SU}^{-1}(\bar{l}_{SU})$. The strict inequality follows from the fact that left wins and therefore $\frac{p_L + \bar{p}_R}{2} > b_{SU}^*$. But then there must be some $p_R^{\bar{l}_{SU}}$ such that $p_R^{\bar{l}_I} < p_R^{\bar{l}_{SU}} < \bar{p}_R$ for which $\frac{p_L + p_R^{\bar{l}_{SU}}}{2} > b_{SU}^*$ and a sophisticated uninformed voter with bliss point b_{SU}^* votes right. Therefore,

$l_{SU}(p_L, p_R, v) = \bar{l}_{SU}$ for all p_R such that $p_R^{ISU} \leq p_R < \bar{p}_R$.

Putting the results together, for any $p_R \geq \max(p_R^{ISU}, p_R^I)$, $p_R < \bar{p}_R$, we have $l_I(p_L, \bar{p}_R, v) + l_{UU}(p_L, \bar{p}_R) + l_{SU}(p_L, \bar{p}_R) = \bar{l}_I + \bar{l}_{UU} + \bar{l}_{SU} > \frac{n+1}{2}$. This is a contradiction and thus, the interval is closed for all values of p_L . ■

Proof Lemma 7. If $p_L \leq b_R$, we know from Lemma A-1 that there is a $p_R \geq p_L$ for which $l_I(b_m, v) + l_{SU}(p_L, b_m) + l_{UU}(p_L, b_m) < \frac{n+1}{2}$ for all p_L and $v > 0$. From Lemma A-3, we know that the interval of values p_R for which $l_I(b_m, v) + l_{SU}(p_L, b_m) + l_{UU}(p_L, b_m) < \frac{n+1}{2}$ is closed. Moreover, it is bounded. Thus, a solution to the maximization problem (17) exists. Its solution must constitute a best reply because $p_R^b \geq p_L$ and therefore, the right candidate is weakly better off with p_R^b than with choosing a position that loses the elections and therefore leads to policy position p_L being implemented.

If $p_L > b_R$, we know from Lemma A-1 that the right candidate can win the elections with his bliss point as the policy position, so that $p_R = b_R$ is the best reply in this case.

The proof for the case $v \leq 0$ is analogous. ■

For the proof of Proposition 4 I need the following Lemma:

Lemma A-4 *The best reply functions given in Lemma 7 are continuous*

Proof. Consider the case $v > 0$:

Let $\varepsilon > 0$. Consider the two policy platforms p_L and $p'_L = p_L + \varepsilon$. First, I show that $p_R^b(p_L) + \varepsilon \geq p_R^b(p'_L)$. If $p_R^b(p_L) = b_R$ this is obvious. If $p_R^b(p_L) < b_R$, right must be losing with any $p_R > p_R^b(p_L)$ given p_L , if not $p_R^b(p_L)$ is not a best reply. Now if $p_R^b(p_L) + \varepsilon < p_R^b(p'_L)$, it follows that $b_I^*(p_L, p_R^b(p_L), v) < b_I^*(p'_L, p_R^b(p'_L), v)$ and $b_U^*(p_L, p_R^b(p_L), v) < b_U^*(p'_L, p_R^b(p'_L), v)$. But right cannot win with $p_R^b(p'_L)$ against p'_L if every $p_R > p_R^b(p_L)$ loses against p_L because there must be $p_R > p_R^b(p_L)$ that gains at least as many votes against p_L as $p_R^b(p'_L)$ against p'_L . This is a contradiction, and therefore $p_R^b(p_L) + \varepsilon \geq p_R^b(p'_L)$. Consider the reply $p_R^b(p_L) - \varepsilon$ to p'_L . Then $b_I^*(p_L, p_R^b(p_L), v) > b_I^*(p'_L, p_R^b(p'_L) - \varepsilon, v)$ and $b_{UU}^*(p_L, p_R^b(p_L), v) = b_{UU}^*(p'_L, p_R^b(p'_L) - \varepsilon, v)$ and therefore right must win. Thus $p_R^b(p'_L) \geq p_R^b(p_L) - \varepsilon$. Taking both results together, we know that if the best reply to p_L is $p_R^b(p_L)$, the best reply to $p_L + \varepsilon$ must be within distance ε of $p_R^b(p_L)$. From this, it follows that if there are two policy platforms p_L and \tilde{p}_L such that $|p_L - \tilde{p}_L| < \frac{\delta}{2}$, then $|p_R^b(p_L) - p_R^b(\tilde{p}_L)| < \delta$ and therefore $p_R^b(p_L)$ is continuous.

The proof for the case $v \leq 0$ is analogous. ■

Proof Proposition 4. Proof for the case $v > 0$:

First, I prove that p_R^* and p_L^* exist, then that they constitute best responses to each other and the voters' strategies. From Lemma A-4, we know that $p_R^b(p_L, v)$ is a continuous function. In addition, $[0, 1]$ is compact. Therefore $p_R^* = \min_{p_L \in [0, 1]} p_R^b(p_L, v)$ exists according to Weierstrass' maximum theorem. Consequently, $p_L^* = \arg \min_{p_L \in [0, 1]} p_R^b(p_L, v)$ also exists, but is not necessarily unique.

By the definition of p_R^* , there is no p_L that can obtain a majority against it given the strategies of the voters. Therefore, p_L^* must be a best reply by the left candidate. By construction, p_R^* is a best reply to p_L given the strategies of the voters.

The argument for the case $v \leq 0$ is similar. ■

Proof Proposition 5. The informed voters always vote for the candidate they prefer. That the candidates maximize their utility given the strategies of the voters was already shown in Proposition 4. The sophisticated uninformed voters with bliss points $b > b_{SU}^*(p_L, p_R)$ who vote for the right policy position can make a difference by voting left only in elections where they are pivotal. In equilibrium, this only occurs if the informed voter with bliss point $F_I^{-1}(b_{SU}^*(p_L, p_R))$ votes for the right candidate and therefore $F_I^{-1}(b_{SU}^*(p_L, p_R)) \geq b^*$. In this case, the uninformed voters who vote right have a bliss point $b > b_{SU}^*(p_L, p_R) > F_I^{-1}(b_{SU}^*(p_L, p_R)) \geq b^*$ and are better off with the right candidate for whom they vote. An analogous argument applies for uninformed voters with $b < b_{SU}^*(p_L, p_R)$ who vote for the left policy position. The uninformed voter with bliss point b_{SU} is never pivotal in equilibrium and is therefore never worse off following her strategy.

For any out of equilibrium combinations of p_L and p_R , voters believe that the candidate with a position closer to the uninformed voter with bliss point $F_I^{-1}(b_{SU}^*(p_L, p_R)) \geq b^*$ has the valence advantage. Take the case $|p_L - b_m| > |p_R - b_m|$ and $p_L < p_R$. All uninformed voters believe that $v > 0$ and that all voters with $b > F_I^{-1}(b_{SU}^*(p_L, p_R)) \geq b^*$ (informed as well as uninformed) vote for the right candidate and therefore constitute a majority for him. Thus, for the uninformed voters with $b > F_I^{-1}(b_{SU}^*(p_L, p_R))$, voting right is a best reply because they believe that $b^* < F_I^{-1}(b_{SU}^*(p_L, p_R))$. For the uninformed voters with $b < F_I^{-1}(b_{SU}^*(p_L, p_R))$, voting left is a best reply because they believe that the right candidate wins independently of their voting decision. A similar argument applies to the other possible cases. ■

Proof Lemma 8. Consider the case $v > 0$:

If $|b_m - p_R| \leq |b_m - p_L|$, right wins (by Lemma A-1) independent of the details of the distribution. Therefore, I have to check only combinations of p_L and p_R with $|b_m - p_R| > |b_m - p_L|$.

From Proposition 4, we know that the right candidate wins the elections with some position $p_R \geq b_m$. Moreover, the cutoff point for sophisticated uninformed voters $b_{SU}^*(p_L, p_R) = F_S^{-1}\left(\frac{n+1}{2} - l_{UU}\left(\frac{p_L + p_R}{2}\right)\right)$ depends only on the distribution of sophisticated and unsophisticated uninformed voters and is therefore the same independent of the exact distribution of informed and sophisticated uninformed voters among the sophisticated voters. From $p_R \geq b_m$ and $|b_m - p_R| > |b_m - p_L|$, it follows that $b_{UU}^* > b_m$. Therefore, all unsophisticated uninformed voter at and to the left of the median bliss point vote for the left candidate and if an unsophisticated uninformed voter has the median bliss point, thus $l_{UU}\left(\frac{p_L + p_R}{2}\right) \geq F_{UU}(b_m)$ and therefore using equation (15):

$$\begin{aligned} b_{SU}^*(p_L, p_R) &= F_S^{-1}\left(\frac{n+1}{2} - l_{UU}\left(\frac{p_L + p_R}{2}\right)\right) \\ &= F_S^{-1}\left(F_S(b_m) + F_{UU}(b_m) - l_{UU}\left(\frac{p_L + p_R}{2}\right)\right) \\ &\leq F_S^{-1}(F_S(b_m)) \leq b_m. \end{aligned}$$

It follows that $b_{SU}^*(p_L, p_R) < b_{UU}^*$, and therefore the voter with bliss point b_{SU}^* votes left if she is uninformed. The decisive informed voter is thus given by $b_I^{d''}(p_L, p_R) = F_I''^{-1}(F_I''(b_{SU}^*(p_L, p_R)))$ respectively $b_I^{d'}(p_L, p_R) = F_I'^{-1}(F_I'(b_{SU}^*(p_L, p_R)))$. From the fact that there are more informed voters in case (") than in case ('), it follows that $F_I'(b) \leq F_I''(b)$ which in turn implies that $F_I'^{-1}(F_I'(b)) \leq F_I''^{-1}(F_I''(b))$ for all b . Therefore, $b_I^{d'} \leq b_I^{d''}$ and every position p_R that wins given $(B_I', B_{SU}', B_{UU}', p_L)$ wins also given $(B_I'', B_{SU}'', B_{UU}'', p_L)$, but not vice versa. This implies that $|p^{*'}(v) - b_m| \leq |p^{*''}(v) - b_m|$.

The argument for the case $v \leq 0$ is analogous. ■

Proof Lemma 9. Consider the case $v > 0$:

It follows from Lemma A-1 that before and after the sophisticated uninformed voters switch to being unsophisticated uninformed, the right candidate can win with some position $p_R \geq b_m$.

If $|b_m - p_R| \leq |b_m - p_L|$, the right candidate wins (by Lemma A-1) independent

of the details of the distribution. Therefore, I focus on combinations of p_L and p_R with $|b_m - p_R| > |b_m - p_L|$.

The informed voters make the same voting decision for given p_L, p_R and v for both $(B''_I, B''_{SU}, B''_{UU})$ and (B'_I, B'_{SU}, B'_{UU}) .

From $p_R \geq b_m$ and $|b_m - p_R| > |b_m - p_L|$ follows that $b^*_{UU} = \frac{p_L + p_R}{2} > b_m$. Thus, $l_{UU}(\frac{p_L + p_R}{2}) \geq F_{UU}(b_m)$ and therefore using equation (15):

$$\begin{aligned} b^*_{SU}(p_L, p_R) &= F_S^{-1} \left(\frac{n+1}{2} - l_{UU}(\frac{p_L + p_R}{2}) \right) \\ &= F_S^{-1} \left(F_S(b_m) + F_{UU}(b_m) - l_{UU}(\frac{p_L + p_R}{2}) \right) \\ &\leq F_S^{-1}(F_S(b_m)) \leq b_m. \end{aligned}$$

From this it follows that $b^*_{SU}(p_L, p_R) \leq b_m \leq b^*_{UU}$ for (B'_I, B'_{SU}, B'_{UU}) as well as $(B''_I, B''_{SU}, B''_{UU})$.

Therefore, the voters who are sophisticated uninformed in one case and unsophisticated in the other either do not change their voting decision, or vote for the right candidate if they are sophisticated and for the left candidate when they are not sophisticated. This can be partly offset by sophisticated informed voters voting left instead of right. However, the number of votes for the right candidate by uninformed voters cannot be larger given p_L and p_R in case (') compared to case ("). This can be seen by comparing

$$b^{*''}_{SU}(p_L, p_R) = F_S^{-1''} \left(\frac{n+1}{2} - l''_{UU}(\frac{p_L + p_R}{2}) \right)$$

and

$$b^{*'}_{SU}(p_L, p_R) = F_S^{-1'} \left(\frac{n+1}{2} - l'_{UU}(\frac{p_L + p_R}{2}) \right)$$

If a sophisticated voter is turned into an unsophisticated voter, but would also vote left if he had stayed sophisticated, $F'_{SU}(b^{*'}_{SU}(p_L, p_R))$ does not increase compared to $F''_{SU}(b^{*''}_{SU}(p_L, p_R))$. If a sophisticated voter is turned into an unsophisticated voter and votes left instead of right, $F'_{SU}(b^{*'}_{SU}(p_L, p_R))$ can increase compared to $F''_{SU}(b^{*''}_{SU}(p_L, p_R))$, but not by more than $l'_{UU}(\frac{p_L + p_R}{2})$ increases compared to $l''_{UU}(\frac{p_L + p_R}{2})$, so once more the number of uninformed voters voting left cannot increase.

Therefore, the total number of votes for right cannot be larger given p_L and p_R in case (') compared to case ("). Every position p_R that wins given $(B'_I, B'_{SU}, B'_{UU}, p_L)$ wins also given $(B''_I, B''_{SU}, B''_{UU}, p_L)$, but not vice versa. This implies that $|p^{*'}(v) - b_m| \leq |p^{*''}(v) - b_m|$.

The argument for the case $v \leq 0$ is analogous. ■

Appendix D

Proofs Section 6

Proof Lemma 10.

Consider the case $v > 0$:

The left candidate can never be better off offering a policy position $p_L \geq p_R$. For $p_L < p_R$, his vote-maximizing strategy is given by:

$$p_L^{\max}(p_R) = \arg \max_{p_L \in [0, p_R]} \beta \frac{(p_L + p_R)}{2} + (1 - \beta) \left(\frac{p_L + p_R}{2} - \frac{v}{2(p_R - p_L)} \right). \quad (35)$$

The first-order condition for an interior maximum is $\frac{1}{2} - (1 - \beta) \frac{v}{2(p_R - p_L)^2} = 0$. From this, it follows that:

$$p_L^{\max}(p_R) = \max(p_R - (1 - \beta)^{0.5} v^{0.5}, 0) \quad (36)$$

With this $p_L^{\max}(p_R)$ it is possible to calculate the optimal strategy for the right candidate. He must take the position that is closest to his bliss point, subject to the constraint that he wins against $p_L^{\max}(p_R)$. Therefore:

$$p_R^*(v) = \arg \max_{p_R \in [0, b_R]} p_R \text{ s.t. } \frac{(p_L^{\max}(p_R, v) + p_R)}{2} - (1 - \beta) \frac{v}{2(p_R - p_L)} \leq 0.5. \quad (37)$$

The left-hand side of the constraint is increasing in p_R , while the right-hand side is constant. Therefore, the solution is either b_R as long as $p_L^{\max}(b_R)$ obtains less than 50% of the votes for the left candidate, or the solution to:

$$\frac{2p_R - (1 - \beta)^{0.5} v^{0.5}}{2} - (1 - \beta) \frac{v}{2((1 - \beta)^{0.5} v^{0.5})} = 0.5. \quad (38)$$

The solution to the right candidate's problem is therefore:

$$p_R^*(v) = \max((1 - \beta)^{0.5}v^{0.5} + 0.5, b_R). \quad (39)$$

Given that he cannot win the elections, the vote maximizing reply must be an optimal reply for the left candidate:

$$p_L^*(v) = \max(p_R^*(v) - (1 - \beta)^{0.5}v^{0.5}, 0). \quad (40)$$

The argument for the case $v \leq 0$ is analogous. ■

Proof Lemma 11. The expected equilibrium utility of a voter is given by equation (23). It can be rewritten to facilitate the calculation of the derivative with respect to the share of unsophisticated voters β :

$$\begin{aligned} E(U(b)) &= \int_{v=-\infty}^{v=\infty} -(p(v) - b)^2 g(v) dv + E(\max(q_R, q_L)) \quad (41) \\ &= \int_{v=-\infty}^{v=\infty} (-p(v)^2 + 2bp(v))g(v)dv - b^2 + E(\max(q_R, q_L)) \\ &= \int_{v=-\infty}^{v=\infty} -p(v)^2 g(v)dv + b - b^2 + E(\max(q_R, q_L)) \\ &= -(1 - G\left(\frac{(b_R - 0.5)^2}{(1 - \beta)}\right)) p_R^{*2} - G\left(-\frac{(b_L - 0.5)^2}{(1 - \beta)}\right) p_L^{*2} \\ &\quad - \int_{v=-\frac{(b_L - 0.5)^2}{(1 - \beta)}}^{v=\frac{(b_R - 0.5)^2}{(1 - \beta)}} p(v)^2 g(v)dv + b - b^2 + E(\max(q_R, q_L)) \\ &= -(1 - G\left(\frac{(b_R - 0.5)^2}{(1 - \beta)}\right)) p_R^{*2} - G\left(-\frac{(b_L - 0.5)^2}{(1 - \beta)}\right) p_L^{*2} \\ &\quad - \int_{v=-\frac{(b_L - 0.5)^2}{(1 - \beta)}}^{v=\frac{(b_R - 0.5)^2}{(1 - \beta)}} \left((1 - \beta)|v| + \text{sign}(v)((1 - \beta)|v|)^{0.5} + \frac{1}{4}\right)g(v)dv \\ &\quad + b - b^2 + E(\max(q_R, q_L)) \end{aligned}$$

Taking the derivative of (41) with respect to β by applying Leibniz's rule gives (34):

$$\frac{dE(U(b))}{d\beta} = \int_{v=-\frac{(b_L - 0.5)^2}{(1 - \beta)}}^{v=\frac{(b_R - 0.5)^2}{(1 - \beta)}} |v|g(v)dv > 0. \quad (42)$$

■

Chapter 3

Lobbying and Elections^{*}

1 Introduction

The influence of interest groups on decision making within a democratic society is one of the most vibrant fields in political economics. However, so far, most of the literature neglects the feedback effects of post-election lobbying on voter behavior. I develop a model of interest group influence on policy in a setup with ideological parties and voters who correctly foresee the post-election bargaining outcome.

Specifically, I consider a polity with two ideological parties which cannot commit to policy positions before elections take place and an interest group that can pay contributions to the party in office. If the party accepts the contribution, it agrees to implement a specific policy in return. In equilibrium, policy is a weighted average of the bliss points of the party in power and the interest group. A voter's utility depends on how close the implemented policy is to her bliss point. Therefore, she does not vote for the party that is ideologically closest to her, but for the party that she predicts to implement the policy closest to her bliss point. To achieve this, she must take the post-election influence of the interest group into account.

I show that in many cases, the existence of interest group influence makes the median voter better off. Even in cases where she is worse off, the negative effects on her welfare are bounded as long as the effects of lobbying are not too large. Large negative effects of lobbying can occur only when the effects of lobbying are so strong

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that they actually make one of the parties not only implement policies that are further away from its own bliss point than the bliss point of the median voter, but moreover take a position that is further away from the median bliss point than the bliss point of either party.

The median voter's welfare is only of interest as far as it provides an approximation of the average voter's welfare. This is the case if voters' bliss points are not too asymmetrically distributed around the median voter's bliss point. There is little reason to assume that lobbying only has small negative or positive effects on the welfare of the median voter, but strong negative effects on the average voter.

Since voters predict equilibrium policies, the winning party in the case of lobbying is different from the winning party without lobbying if the median voter's bliss point is closer to the implemented policy of the party whose bliss point is further away from her own. The welfare of the interest group must increase with lobbying as compared to the case without lobbying, as long as the winning party of the elections does not change. However, the effects of lobbying can easily make the position of the party closer to the interest group less attractive and lead to the victory of the other party. In this case, the interest group must be worse off if its influence is not very large.

My results are in contrast to the findings of Besley and Coate (2001), which is the most important paper in the literature that considers feedback effects of post-election lobbying on voter behavior and election outcomes. Besley and Coate's main result could be called a "lobbying irrelevance theorem". They show that as long as sufficiently extreme candidates are available, lobbying has no influence on policy at all. Therefore, it also has no influence on the welfare of voters who neither run as candidates nor contribute to lobbying efforts. The interest group is always worse off in the case of lobbying as compared to the case without lobbying if the implemented policy is the same, because it must make positive contributions to the winning candidate. The question why an interest group would ever be formed in such a setup is not asked, its existence is taken as given.¹

The reasons for the differences between my findings and those of Besley and Coate are straightforward. Their setup is very similar to mine with respect to the post-

¹ For a useful discussion of the Besley and Coate (2001) paper and its contribution to the literature, see also Dewan and Shepsle (2008).

election bargaining between interest groups and parties and with respect to rational expectations of voters. However, they use their own citizen-candidate framework introduced in Besley and Coate (1997), while I use a model with ideological parties. Political parties that seem to care at least to some degree about policies are a widely observed phenomenon, while true citizen candidates rather seem to be the exception than the rule. In a citizen-candidate framework with a continuum of candidates, the choice set of voters is a continuum of possible policies (given that a citizen candidate with the policy is willing to run) whereas in my model with political parties, the voters have to decide between two policies only. The influence of post-election lobbying by the interest group alters the implemented policies of each potential citizen candidate as well as those of both political parties. However, if the choice set only contains two policies from the beginning, lobbying changes the policy choice of voters in a significant way. With a continuum of citizen candidates, on the other hand, only relatively extreme policies become unavailable in the case of lobbying. If candidates with sufficiently extreme preferences are available, voters can completely offset the influence of the interest group and equilibrium policy does not change.

In Section 4, I allow the parties to run with candidates who differ from their own party in their preferences. Then, the "lobbying irrelevance theorem" of Besley and Coate (2001) once more becomes relevant as long as both parties have sufficiently extreme candidates available. In this case, political competition forces both parties to choose candidates who implement the median voter's bliss point after being lobbied by the interest group.

1.1 Related literature

There is a vast body of empirical as well as theoretical research on the influence of interest groups on decision making within a democratic society. A good overview of the theoretical approaches from a political economics perspective can be found in Grossman and Helpman (2001). The literature can be divided into two major strands. On the one hand, there are models where lobbies influence policy by providing information to politicians. Examples are Austen-Smith (1993), Bannedsen and Feldmann (2002) and several models discussed in Grossman and Helpman (2001). On the other hand, there are models where interest groups influence decision makers

with the help of monetary contributions. Two of the most important papers in this strand of the literature are Grossman and Helpman (1994, 1996).

In most models with monetary contributions in return for policy, elections are disregarded and only the post-election bargaining of interest groups with individual politicians (see, for example, Grossman and Helpman (1994)) or several members of a legislature (see, for example, Groseclose and Snyder (1996)) is considered. The models that incorporate interaction of lobbying and elections more often than not deal with the interaction of campaign contributions and elections (Grossman and Helpman 1996). Politicians accept contributions not as an end in themselves, as in my model, but for the financing of electoral campaigns. The feedback effects of post-election lobbying on elections outcomes have received less attention so far. This is somewhat surprising, given that they can be dealt with in a purely rational choice framework. In contrast, the campaign contribution literature needs to rely on a somewhat uneasy mix of a framework that combines standard rational choice elements with an ad hoc assumption of the existence of a group of voters that is not only uninformed about policy but, moreover, impressionable by campaign contributions as in Baron (1994) and Grossman and Helpman (1996).² Moreover, Baron (2006) provides evidence from the Center for Responsive Politics that expenditures on lobbying after elections are at least as large as spending on campaign contributions.³

The few papers which actually deal with the feedback effect on elections include the already mentioned Besley and Coate (2001) paper and two papers that build further on its citizen-candidate-cum-lobbying framework by Felli and Merlo (2006, 2007). Snyder and Ting (2008) develop a dynamic model where voters can hold parties accountable.

A possible explanation for the neglect of post-election lobbying compared to campaign contributions constitutes the focus of most of the literature on special interest politics. It is not obvious how voters should adjust their voting behavior even if they can predict the influence of post-election special interest lobbying. They can avoid voting for a farmer to reduce farm subsidies, but they may not have a

² I am not arguing against the introduction of some behavioral elements into the modelling of political economics in general, but against the ad hoc use of behavioral assumptions without further justification when they are convenient modeling devices.

³ www.opensecrets.org.

candidate available with a specific interest in low subsidies. Therefore, candidates who would completely offset the lobbying of a farming interest group are unlikely to be available. The paper by Besley and Coate, on the other hand, deals with general interest lobbying. In their case, it is the provision of a public good that benefits everybody that is influenced by interest groups. The conflict arises because citizens disagree on the exact amount of the public good that should be provided.

That their model is de facto a model of general interest lobbying rather than special interest lobbying is never stated by Besley and Coate. Nonetheless, this difference is of essential importance in explaining why they find a lobbying irrelevance result in sharp contrast with the results in other papers.

A further distinction between models of post-election lobbying and models of campaign contributions is the ability of politicians to commit to policies before elections take place. If they want to attract campaign contributions in return for their policy announcements, politicians must be able to commit to policies in advance. If, on the other hand, politicians are free to choose policies after the elections, there is no reason why an existing interest group would not want to influence them at this point rather than, or in addition to, the campaign stage of the game.

However, the different assumptions on the ability of politicians to commit to policies seem adequate once the differences between general interest and special interest lobbying are taken into account. Parties can more easily commit on special interest issues because they are unlikely to have a strong ideological bias against or in favor of them. On a general interest policy dimension, on the other hand, it seems plausible that commitment is impossible or at least more difficult because political parties are usually defined by their ideologies. It seems unlikely that, for example, a socially conservative party could make a credible commitment to implement socially progressive policies before an election takes place.

Therefore, my model does not provide an alternative theory of special interests with elections and their feedback effects taken into account. Instead, it provides a new contribution to the small literature on general interest lobbying. For real-world examples of general interest lobbying, the reader might want to consider large trade unions and large employer organizations. Such organizations often have interests on rather broad policy dimensions, in many cases in addition to special interests.

The analysis also provides a further rationale as to why general interests are not

often organized in interest groups. As discussed in Section 3.2, committing to refrain from any lobbying can actually make the potential members of an interest group better off, even if they could overcome the collective action problems described in the classic treatise of Olson (1971).

1.2 Structure of the paper

The paper proceeds as follows. In Section 2, the main model is introduced and discussed. A numerical example is given for the model and its implications. That section also discusses the welfare implication of lobbying for voters as well as the interest group. Section 3 allows for some extensions and generalizations of the model and Section 4 discusses the implications of parties running with ideological candidates. Finally, the paper ends with a concluding section.

2 The Model

There is one policy dimension and policy p is given by a point in the interval $[0, 1]$. There are two parties, L and R and one interest group. Both parties are policy motivated and have a given ideal policy $i_J \in [0, 1]$ that could, for example, reflect the average preferences of their members. By assumption, $i_L < i_R$ and therefore, L is the "left" and R the "right" party. The utility of a party $J = L, R$ is given by:

$$U_J(p, m) = -(p - i_J)^2 + m, \quad (1)$$

where $J = L, R$ and $m \geq 0$ are the monetary funds received from the interest group. The utility of the interest group is given by:

$$U_I(p, m) = -\alpha(p - i_I)^2 - m, \quad (2)$$

where $\alpha > 0$ gives the weight that the interest group attaches to policy relative to monetary contributions and i_I is its bliss point. Since the relative weight of policy relative to monetary contributions is normalized to 1 for both parties, α also measures how much lobbies care about policy relative to monetary payments relative to how much the parties care about policy relative to monetary funds. The monetary

transfers m to the party in power are costly for the interest group. Therefore, they negatively enter its utility function. The variable i_I denotes the policy bliss point of the interest group.

No commitment is possible in advance of the elections. After the elections, the winning party is not bound by any previous announcements. Let the number of voters be an odd number N . Voter n 's utility function is:

$$U_n(p) = -(p - \theta_n)^2, \quad (3)$$

where θ_n is the bliss point of voter n . I order the voters by their preferences from left to right such that θ_1 is the bliss point of the voter with the ideal point closest to 0 and θ_m , with $m = \frac{N+1}{2}$, is the bliss point of the median voter. After the elections, the interest group makes an offer to the party that won. The party accepts or rejects this offer. If it accepts the offer, it implements the agreed policy. If not, it is free to choose any policy and therefore implements its own bliss point. By assumption, the party accepts the offer if indifferent.

To summarize, the order of moves is the following: First, elections take place and the party which achieves the majority of votes wins. Second, at the lobbying stage, the interest group makes a take-it or leave-it offer to the party that has won the elections, specifying a policy p and a payment m in case this policy is accepted. Third, if the party accepts the payment, it must implement the policy proposed by the interest group. If the party does not accept the payment, it is free to choose any policy. The interest group has no possibility to commit to abstain from lobbying after the elections.

2.1 Solving the model

The interest group maximizes its utility subject to making the party indifferent between accepting the offer and implementing its favorite policy. A party J in power that does not accept monetary contributions would implement its favorite policy and achieve a utility of 0. The equilibrium policy given that party $J^* = L, R$

is in power is given by:

$$\begin{aligned} (p_J^*, m_J^*) &= \arg \max_{p, m} U_I(p, m) \text{ s.t. } U_J \geq 0 \\ \Rightarrow p_J^* &= \arg \max_p -\alpha(p - i_I)^2 - (p - i_J)^2 = \frac{\alpha i_I + i_J}{1 + \alpha}. \end{aligned} \quad (4)$$

In the equilibrium with lobbying, policy is a weighted average of the ideal point of the party in power and the interest group. The larger the relative weight of policy α in the utility function of the parties, the closer is equilibrium policy to the bliss point of the party in power. Since by assumption, $i_L < i_R$, it directly follows that $p_L^* = \frac{\alpha i_I + i_L}{1 + \alpha} < \frac{\alpha i_I + i_R}{1 + \alpha} = p_R^*$. If there is no interest group, party J maximizes its utility by implementing its bliss point i_J when in power. Therefore, if party J^* is in power, the interest group offers the payment:

$$m_J^* = (p_J^* - i_J)^2 = \left(\frac{\alpha(i_I - i_J)}{1 + \alpha} \right)^2 \quad (5)$$

for implementing policy p_J^* . Moreover, the utility of the parties and the interest group are:

$$\begin{aligned} U_{J^*} &= 0, \\ U_{-J^*} &= - \left(\frac{(i_{J^*} - i_{-J^*}) + \alpha(i_I - i_{-J^*})}{1 + \alpha} \right)^2, \\ U_I &= - \frac{\alpha}{1 + \alpha} (i_I - i_{J^*})^2, \end{aligned}$$

where $-J^*$ denotes the party out of power. Party J^* is indifferent between accepting and rejecting the offer and therefore accepts it by assumption. This is a jointly efficient outcome for the interest group and the party, as could be expected in a perfect-information set-up without frictions in the negotiations over the policy. However, the joint efficiency between the party in power and the interest group does not imply Pareto efficiency, because the utility of the voters only plays a role in so far as they are organized in the interest group or the party in power and the utility of the party out of office is disregarded completely. Voters are assumed to be able to predict the post-election outcome before they cast their ballots. What differs from most models of interest group influence on policy-making is therefore that the effects of lobbying are predicted by the voters who adjust their voting decisions accordingly.

Let

$$d(\alpha) = p_R^* - p_L^* = \frac{i_R - i_L}{1 + \alpha} \quad (6)$$

measure the distance in the policies implemented by the two parties in case they win the elections. The difference goes towards 0 when α goes to infinity, because in this case, p_J^* goes to i_I for both parties. The interest group is willing to pay any price for having its own policy bliss point implemented since the relative weight of monetary contributions as compared to policy in its utility function goes towards zero. On the other hand, when $\alpha = 0$, no lobbying takes place because the interest group attaches no weight to policy whatsoever.

I assume that all voters cast their ballots in favor of the party which they forecast to implement the policy closest to their respective bliss point. This is the only plausible strategy for a voter because it is weakly dominating. If the median voter weakly prefers a policy position, this is also preferred by either all voters with $\theta_n \leq \theta_m$ or all voters with $\theta_n \geq \theta_m$. Thus, the party which implements the policy preferred by the median voter achieves the majority of votes. The winning party in case of lobbying is thus given by:

$$J^* = \arg \min_{J \in \{L, R\}} |p_J^* - \theta_m|, \quad (7)$$

i.e., the party which implements the policy that is most attractive to the median voter. I denote the implemented policy in case lobbying is taking place by $p_I^* = p_{J^*}$. If the median voter is indifferent, she is assumed to vote for the left party L .⁴ In contrast, if there is no lobbying, a party in power implements its bliss point. Thus, the party with the bliss point closest to the median voter wins:

$$J_{-I}^* = \arg \min_{J \in \{L, R\}} |I_J - \theta_m|. \quad (8)$$

I denote the equilibrium policy without lobbying by $p_{-I}^* = i_{J_{-I}^*}$. Once more, if the median voter is indifferent, she is assumed to vote for the left party L .

In the next subsection, I provide a numerical example, while Subsections 2.3-2.5 provide some formal analysis of the welfare implications of lobbying for the median

⁴ Assuming that the median voter supports one of the parties in the case of being indifferent avoids stochastic elements in the model that would lead to some complications without giving any additional insights.

voter, the average voter and the interest group.

2.2 An example

Interestingly, the possibility of lobbying does not necessarily make the interest group better off. Consider the case of an interest group promoting the rightmost possible policy with bliss point $i_I = 1.0$ and relative weight of policy in the utility function $\alpha = 1$. Let the left party have bliss point $i_L = 0.25$ and the right party have bliss point $i_R = 0.75$:

$$\begin{aligned} U_L &= -(0.25 - p)^2 + m, \\ U_R &= -(0.75 - p)^2 + m, \\ U_I &= -(1 - p)^2 - m. \end{aligned} \tag{9}$$

It is straightforward to calculate the implemented policy conditioning on either party winning. In case of a party L victory, it is:

$$p_L^* = \arg \max_p -(0.25 - p)^2 - (1 - p)^2 = 0.625. \tag{10}$$

And in case of a party R victory, it is:

$$p_R^* = \arg \max_p -(0.75 - p)^2 - (1 - p)^2 = 0.875. \tag{11}$$

As long as the existence of the interest group does not influence the election result, the interest group is better off with respect to the policy and increases its utility by lobbying even after subtracting the cost m of lobbying. This is the case if the median voter has preferences with the bliss point either in the interval $[0, 0.5]$ or the interval $(0.75, 1]$. However, if the median voter's bliss lies in the interval $(0.5, 0.75]$, she votes for the left party rather than the right party due to the presence of the interest group and the implemented policy changes from 0.75 to 0.625. This makes the interest group worse off, even disregarding the cost of lobbying.⁵ In addition, the interest group has to pay $m_L^* = (0.625 - 0.25)^2 = 0.375^2$ to make the left

⁵ However, this is specific to the example. A lobby can be better off even if it causes its favorite party to lose the elections if its influence on the elections is sufficiently large as shown in Subsection 2.5.

party implement 0.625 instead of 0.25. If the right party wins, the payment is only $m_R^* = (0.875 - 0.75)^2 = 0.125^2$.

Whether the average and the median voter are better or worse off due to the existence of the interest group is impossible to say without further assumptions about their preferences, but both cases are plausible, especially considering the fact that even an interest group with extreme preferences can lead to a more centrist implemented policy if it changes the election outcome.

In the example, there are three different cases where the median voter is better off with than without the interest group. (1) The median voter is better off with the interest group if she supports the left party in both cases and her bliss point is closer to p_L^* than to i_L , that is, when her bliss point $\theta_m \in (0.4375, 0.5]$. (2) She is better off when she supports the right party in both cases and she is better off with p_R^* than with i_R , that is when her bliss point $\theta_m > 0.8125$. (3) Finally, she is better off if she has a bliss point $\theta_m \in (0.5, 0.6875)$. In this case, the median voter votes left instead of right if there is an interest group and she is made better off with the moderate right policy of the left party that is made available by the existence of the interest group.

If $\theta_m < 0.4375$ or $\theta_m \in (0.6875, 0.8125)$, the median voter is worse off with than without the interest group. In the first case, the reason is that a far to the left policy is no longer available, in the second case the reason is that the policy that will be implemented by the left party after being lobbied is too much to the left and the policy of the right party after being lobbied is too much to the right to be preferable to i_R . For voters in general, everything is possible because the outcome of the elections depends on the location of the median voter. Due to the fact that an interest group with a position far to the right can lead to a more leftist equilibrium policy, a voter with any bliss point θ can be made worse off or better off as long as she is not at the median position and does not determine policy.

For a possible interpretation, imagine a two-party system with an economically liberal party and a socialist party. If the socialist party leadership is known to accept monetary contributions from a business interest group for implementing more centrist policies than its leadership would otherwise prefer, this does not necessarily hurt its election prospects. On the contrary, it makes the party more attractive for centrist voters. It seems plausible that the existence of strong business interest

group organizations in the US makes the Democrats more and the Republicans less attractive for centrist voters.⁶

The mechanism at work here is somewhat related to that described by Ellman and Wantchekon (2000). In their model, it is not an interest group that influences policies, but the threat of violence. They show that in a two-party setup that is quite close to the one described in my model, the threat of violence by either party or some exogenous group can serve as a de facto commitment device to implement more centrist policies after the elections to avoid such violence. Just like in my model, it is the post-election influence on the implemented policy that stops candidates from implementing their ideal policy and might therefore help them be more attractive to centrist voters before the elections.

The interesting point is that an interest group can actually make life more difficult for the party to which it is ideologically closer, but never make it better off with respect to its electoral prospects. This is due to the forward-looking character of the model and in contrast to the effects that are commonly found in the campaign contribution literature.

In the following three subsections, some formal statements about the impact of lobbying on the welfare of voters and interest groups are derived.

2.3 The welfare of the median voter

It seems to be widely believed that lobbying is detrimental to welfare in a democracy, because voters do not get the policies they voted for. However, in my model, it can be shown that in many cases, lobbying makes the median voter better off. Moreover, the plausible upside is larger than the plausible downside. There are three cases to consider:

Case 1 (Large effects of lobbying). *Either* $i_I < \max\left(i_L - \frac{(i_R - i_L)}{\alpha}, \frac{2(\alpha+1)}{\alpha}\theta_m - \frac{(\alpha+2)}{\alpha}i_R\right)$
or $i_I > \min\left(i_R + \frac{(i_R - i_L)}{\alpha}, \frac{2(\alpha+1)}{\alpha}\theta_m - \frac{(\alpha+2)}{\alpha}i_L\right)$.

In this case, lobbying has large effects on the positions implemented by parties in office compared to the policy the same party would implement without lobbying. However, because the identity of the winning party can also change as a result of

⁶ Naturally, party positions might not be exogenous to the lobby environment of a country in the long run. This issue is beyond the scope of this paper.

the interest group influence, this does not necessarily imply large effects on policy. Consider the case with $i_I < \max\left(i_L - \frac{(i_R - i_L)}{\alpha}, \frac{2(\alpha+1)}{\alpha}\theta_m - \frac{(\alpha+2)}{\alpha}i_R\right)$. The policy that the right party implements if winning office is to the left of the median voter's bliss point and further away from it than the closer of the two parties' bliss points in the case without lobbying.

Such a large effect of lobbying seems rather implausible for most countries. On the one hand, an interest group might be expected to have rather extreme policy preferences and therefore i_I might be expected to be either very small or very large because centrist special interest groups would have more problems in solving the collective action problem. On the other hand, for small α , the values of i_I that would lead to large effects of lobbying are outside the policy space $[0, 1]$, so that even an interest group with the most extreme possible bliss point $i_I = 0$ or $i_I = 1$ would not have large effects on policy for a given party in power.

It can be shown that in the case of such large effects of lobbying, the median voter is worse off:

Proposition 1. *If lobbying has large effects, as defined in Case 1, then it decreases the utility of the median voter as compared to the case without lobbying.*

Proof. By (4) and straightforward algebra if

$i_I < \max\left(i_L - \frac{(i_R - i_L)}{\alpha}, \frac{2(\alpha+1)}{\alpha}\theta_m - \frac{(\alpha+2)}{\alpha}i_R\right)$, then either $p_R^* < i_L$ or $p_R^* < 2\theta_m - i_R$ or both. If $p_R^* = \frac{\alpha i_I + i_R}{\alpha + 1} < i_L$, then because $p_L^* < p_R^* < i_L < \theta_m$ the median voter prefers the right party and policy p_R^* is implemented. Without lobbying, $(p_{-I}^* - \theta_m)^2 \leq (i_L - \theta_m)^2 < (p_R^* - \theta_m)^2$. Thus, lobbying decreases the utility of the median voter. If $p_R^* < 2\theta_m - i_R$, then $p_L^* - \theta_m < p_R^* - \theta_m < \theta_m - i_R < 0$ and therefore $|p_L^* - \theta_m| > |p_R^* - \theta_m| > |\theta_m - i_R|$. Once more, lobbying makes the median voter worse off because i_R would be more attractive for her than either p_R^* or p_L^* .

The proof for the case $i_I > \min\left(i_R + \frac{(i_R - i_L)}{\alpha}, \frac{2(\alpha+1)}{\alpha}\theta_m - \frac{(\alpha+2)}{\alpha}i_L\right)$ is analogous.

■

Case 2 (Intermediate effects of lobbying). $i_I = \max\left(i_L - \frac{(i_R - i_L)}{\alpha}, \frac{2(\alpha+1)}{\alpha}\theta_m - \frac{(\alpha+2)}{\alpha}i_R\right)$
or $i_I = \min\left(i_R + \frac{(i_R - i_L)}{\alpha}, \frac{2(\alpha+1)}{\alpha}\theta_m - \frac{(\alpha+2)}{\alpha}i_L\right)$.

Proposition 2. *If lobbying has intermediate effects, as defined in Case 2, then it has no influence on the welfare of the median voter.*

Proof. If $i_I = \max\left(i_L - \frac{(i_R - i_L)}{\alpha}, \frac{2(\alpha+1)}{\alpha}\theta_m - \frac{(\alpha+2)}{\alpha}i_R\right)$, then either $p_R^* = i_L \geq 2\theta_m - i_R$ or $p_R^* = 2\theta_m - i_R > i_L$. If $p_R^* = i_L \geq 2\theta_m - i_R$, the left party wins without lobbying and with lobbying the right party wins with the same position, i_L , so implemented policy and thus also the utility of the median voter is the same in both cases. If $p_R^* = 2\theta_m - i_R > i_L$, the right party wins with and without lobbying and in both cases implements policies with the same distance to the bliss point of the median voter θ_m (but on opposite sides of θ_m).

The proof for the case $i_I > \min\left(i_R + \frac{(i_R - i_L)}{\alpha}, \frac{2(\alpha+1)}{\alpha}\theta_m - \frac{(\alpha+2)}{\alpha}i_L\right)$ is analogous.

■

When lobbying has intermediate effects on policy, the position that a party implements once in office changes. However, the welfare of the median voter is not influenced since either the winning party remains the same and implements a policy with the same distance to, but on the other side of the median voter's bliss point, or the winning party changes, but policy does not.

Intermediate effects of lobbying is a borderline case between large and small effects that is unlikely to have much relevance.

Case 3 (Small effects of lobbying). $\max\left(i_L - \frac{(\alpha+1)}{\alpha}(i_R - i_L), \frac{2(\alpha+1)}{\alpha}\theta_m - \frac{(\alpha+2)}{\alpha}i_R\right) < i_I < \min\left(i_R + \frac{(i_R - i_L)}{\alpha}, \frac{2(\alpha+1)}{\alpha}\theta_m - \frac{(\alpha+2)}{\alpha}i_L\right)$.

In the case of small effects of lobbying, at least one of the parties offers a position that is closer to the median voter's bliss point when it is influenced by the interest group after the elections as compared to the case where no interest group exists.

Proposition 3. *If the effect of lobbying is small, as described in Case 3, and the interest group is on the same side of the median voter as the party with the larger distance to the median (that is if either $i_I \geq \theta_m$ and $\frac{i_L + i_R}{2} \geq \theta_m$ or $i_I \leq \theta_m$ and $\frac{i_L + i_R}{2} \leq \theta_m$), the median voter is better off as compared to the case without lobbying. If the interest group is on the other side (that is if either $i_I > \theta_m > \frac{i_L + i_R}{2}$ or $i_I < \theta_m < \frac{i_L + i_R}{2}$), the median voter is better off as compared to the case without lobbying if and only if either the effect of lobbying is sufficiently large ($i_I > \frac{(1+\alpha)(2\theta_m - i^R) - i_L}{\alpha}$ if $i_I > \theta_m > \frac{i_L + i_R}{2}$ and $i_I < \frac{(1+\alpha)(2\theta_m + i^L) + i_R}{\alpha}$ if $i_I < \theta_m < \frac{i_L + i_R}{2}$) or the interest group's policy bliss point is between the two parties' bliss points ($i_L < i_I < i_R$).*

Proof. Case $i_I \geq \theta_m$ and $\frac{i_L + i_R}{2} \geq \theta_m$:

$\frac{i_L + i_R}{2} \geq \theta_m$ implies $|i_R - \theta_m| \geq |i_L - \theta_m|$. Therefore, without lobbying, the left party

wins and $p_{-I}^* = i_L$ is implemented. Because we have a small effect of lobbying, it follows from (4) that $p_L^* < \min(i_R, 2\theta_m - i_L)$. Together with $|i_R - \theta_m| \geq |i_L - \theta_m|$, this implies that $p_L^* < 2\theta_m - i_L$. It follows that $p_L^* - \theta_m < \theta_m - i_L$ and the median voter is better off with p_L^* than she would be with $p_{-I}^* = i_L$.

The proof for the case $i_I \leq \theta_m$ and $\frac{i_L+i_R}{2} \leq \theta_m$ is analogous.

Case $i_I > \theta_m > \frac{i_L+i_R}{2}$:

$i_I > \theta_m > \frac{i_L+i_R}{2}$ implies that $|i_R - \theta_m| < |i_L - \theta_m|$ and without lobbying, the right party wins and $p_{-I}^* = i_R$ is implemented. If $i_I > \frac{(1+\alpha)(2\theta_m - i^R) - i_L}{\alpha}$, then $p_L^* = \frac{\alpha i_I + i_L}{(1+\alpha)} > 2\theta_m - i_R$ and together with $p_L^* < i_R$ (what follows from the fact that the effects of lobbying are small) it follows that $|p_L^* - \theta_m| < |\theta_m - i^R|$. This implies that the median voter is better off with p_L^* than with p_{-I}^* and therefore must be better off with lobbying. If $i_I \leq \frac{(1+\alpha)(2\theta_m - i^R) - i_L}{\alpha}$, then $p_L^* \leq 2\theta_m - i_R$ and $|p_L^* - \theta_m| \geq |\theta_m - i^R|$. There are two cases to consider: If $i_L < i_I < i_R$, then $i_I < p_R^* = \frac{\alpha i_I + i_R}{(1+\alpha)} < i_R$ and the median voter is better off because small effects of lobbying imply that $p_R^* \geq 2\theta_m - i_R$.

If, on the other hand, $i_L < i_I < i_R$ is not true, then $i_I > \theta_m > \frac{i_L+i_R}{2}$ implies that $i_I \geq i_R$ and therefore $p_R^* > i_R > \theta_m$ and lobbying must make the median voter worse off because $(p_I^* - \theta_m)^2 = \min((p_R^* - \theta_m)^2, (p_L^* - \theta_m)^2) > (i_R - \theta_m)^2$.

The proof for the case $i_I < \theta_m < \frac{i_L+i_R}{2}$ is analogous. ■

Proposition 3 implies that in most cases, small effects of lobbying make the median voter better off. Moreover, it can be shown that even if the median voter is worse off her loss of utility is limited:

Lemma 1. *If the effect of lobbying is small, for given bliss points of parties i_L and i_R , the loss of utility with lobbying as compared to the case without interest group for the median voter is at most $\left(\frac{i_R - i_L}{2(1+\alpha)}\right)^2 - \min_J (i_J - \theta_m)^2$ and lobbying must have a positive effect on his welfare as long as $\alpha \geq \frac{\max|i_J - \theta_m| - \min|i_J - \theta_m|}{2 \min|i_J - \theta_m|}$.*

Proof. The utility of the median voter in the case of lobbying is:

$U_m = -\min_J \left(\frac{i_J + \alpha i_I}{1+\alpha} - \theta_m\right)^2$. For given policy positions and small effects of lobbying, the worst possible bliss point i_I^w of the interest group from the perspective of the median voter is given by:

$$i_I^w = \arg \max_{i_I} \min_J \left(\frac{i_J + \alpha i_I}{1+\alpha} - \theta_m\right)^2 \text{ s.t.}$$

1. $\max \left(i_L - \frac{(i_R - i_L)}{\alpha}, \frac{2(\alpha+1)\theta_m - (\alpha+2)i_R}{\alpha}\right) < i_I$
2. $\min \left(i_R + \frac{(i_R - i_L)}{\alpha}, \frac{2(\alpha+1)\theta_m - (\alpha+2)i_L}{\alpha}\right) > i_I$

where the constraints come from the assumption that the effects of lobbying are small. There are two possibilities. The first is that no solution exist because the constraints are binding. In this case lobbying cannot make the median voter worse off because his utility cannot be lower than in Case 2 where she is indifferent between the outcome with lobbies and the outcome without. If a solution exists, it is given by: $i_I^w = \frac{(1+\alpha)}{\alpha}\theta_m - \frac{i_L+i_R}{2\alpha}$ which leads to $p_L^*(i_I^w) = \frac{\alpha i_I^w + i_L}{1+\alpha} = \theta_m - \frac{i_R-i_L}{2(1+\alpha)}$ and $p_R^*(i_I^w) = \frac{\alpha i_I^w + i_R}{1+\alpha} = \theta_m + \frac{i_R-i_L}{2(1+\alpha)}$ and thus

$U_m = -\left(\frac{i_R-i_L}{2(1+\alpha)}\right)^2$. Given that the disutility of the median voter without lobbying is given by $\min_J (i_J - \theta_m)^2$, the maximum welfare loss due to lobbying is given by $\left(\frac{i_R-i_L}{2(1+\alpha)}\right)^2 - \min_J (i_J - \theta_m)^2$. It is positive if and only if $\left(\frac{i_R-i_L}{2(1+\alpha)}\right)^2 \geq \min_J (i_J - \theta_m)^2 \Rightarrow \max_J |i_J - \theta_m| + \min_J |i_J - \theta_m| \geq 2(1+\alpha) \min_J |i_J - \theta_m|$

$$\Leftrightarrow \alpha \leq \frac{\max_J |i_J - \theta_m| - \min_J |i_J - \theta_m|}{2 \min_J |i_J - \theta_m|}. \quad \blacksquare$$

The intuition is straightforward. If lobbying has small effects, it is impossible that a party's policy moves in the direction of the median voter, but nevertheless becomes less attractive for her because it moves too far on the other side. Because $i_L < \theta_m < i_R$ and lobbying moves implemented policy in the same direction for both parties, this implies that lobbying makes the position of at least one party more attractive for the median voter. i_I^w is the position of the interest group such that the party J_{-I}^* which would win without lobbying is just as attractive as the party that would lose. If the interest group is more central ($|i_I - \theta_m| < |i_I^w - \theta_m|$), the same party is closer to the median; if the interest group is more extreme ($|i_I - \theta_m| > |i_I^w - \theta_m|$), the other party becomes more attractive for the median voter. It should also be noted that a value of α that fulfills the condition need not exist within the range of lobbying with small effects. However, it should also be clear that the condition $\alpha \geq \frac{\max |i_J - \theta_m| - \min |i_J - \theta_m|}{2 \min |i_J - \theta_m|}$ is a sufficient but not necessary condition for lobbying to have positive effects on the welfare of the median voter.

Corollary 1. *If both parties have an equal distance to the median voter ($|i_R - \theta_m| = |i_L - \theta_m|$) and the effects of lobbying are small (Case 3), the median voter must be better off with lobbying.*

Proof. This directly follows from Lemma 1 because ($|i_R - \theta_m| = |i_L - \theta_m|$) implies that $\max_J |i_J - \theta_m| = \min_J |i_J - \theta_m|$. \blacksquare

The intuition is that lobbying moves at least one of the parties in the direction

of the median voter and if both parties' bliss points have the same distance to the median voter's bliss point, one of the parties must implement a policy closer to the median voter's bliss point if lobbying is taking place as compared to the case without lobbying, as long as the influence of the interest group is small.

2.4 The welfare of the average voter

The welfare of the median voter is interesting in its own right for the purpose of comparison with standard models of elections without lobbying. However, from a welfare economics perspective, the median voter is no more interesting than any other voter. Consider a utilitarian (Benthamite) social welfare function that gives equal weight to all voters:

$$U_B = \sum_{n=1}^N U_n(p) = \sum_{n=1}^N -(p - \theta_n)^2. \quad (12)$$

This function reaches its unique maximum with policy:

$$p_B^* = \bar{\theta} \equiv \frac{\sum_{n=1}^N \theta_n}{N}. \quad (13)$$

Thus, whenever the welfare of the voter with the average bliss point $\bar{\theta}$ is maximized, we are at the utilitarian maximum and the welfare of the average voter is also maximized.⁷

If $\bar{\theta} = \theta_m$, the results derived for the welfare of the median voter derived in Section 2.3 also apply to the average voter and overall welfare. There is no reason why $\bar{\theta} = \theta_m$ should hold exactly, but it can provide a reasonable approximation if the voters' bliss points are not too asymmetrically distributed around the median voter's bliss point.

In the literature on the determination of tax levels following the pioneering work of Meltzer and Richard (1981)⁸, it is often assumed that $\theta_m < \bar{\theta}$ and the larger θ , the lower the implemented tax level.⁹

⁷ This is a consequence of quadratic disutility in policy and is not true for more general utility functions. However, there is always a representative voter whose welfare is maximized when the welfare of the average voter is maximized.

⁸ For an overview over this literature, see Persson and Tabellini (2000).

⁹ Of course, there is no specific reason why low levels of θ should represent high levels of taxation

A modeling alternative would be to take a given distribution of voters and then make some additional assumptions about how they influence the ideologic position of the parties. In this way, the parties' policy positions could be endogenized.

2.5 The welfare of the interest group

The interest group must be better off whenever the same party wins with or without lobbying. With lobbying and party J^* winning the elections, the utility of the interest group is:

$$\begin{aligned} U_I^*(p_J^*, m_J^*) &= -\alpha(p_J^* - i_I)^2 - m_J^* & (14) \\ &= -\alpha \left(\frac{i_I - i_{J^*}}{1 + \alpha} \right)^2 - \left(\frac{\alpha(i_I - i_{J^*})}{1 + \alpha} \right)^2 \\ &= -\frac{\alpha}{1 + \alpha} (i_I - i_{J^*})^2. \end{aligned}$$

Without any lobbying and party J_{-I}^* winning the elections, the utility of the interest group is:

$$U_I(i_J, 0) = -\alpha(i_I - i_{J_{-I}^*})^2. \quad (15)$$

If the same party $J^* = J_{-I}^*$ wins with and without lobbying, the welfare effect of lobbying on the interest group is simply the difference:

$$U_I^*(p_J^*, m_J^*) - U_I(i_{J^*}, 0) = \frac{\alpha^2}{1 + \alpha} (i_I - i_{J^*})^2 > 0. \quad (16)$$

When the winner does not change as a consequence of the existence of the interest group, lobbying always makes the interest group better off. This result is not surprising given that the interest group is assumed to obtain the entire surplus from the negotiations with the party in power. If $J^* \neq J_{-I}^*$, the difference in utility of the interest group between the two cases is given by:

$$U_I(p_J^*, m_J^*) - U_I(i_{J_{-I}^*}, 0) = -\frac{\alpha}{1 + \alpha} (i_I - i_{J^*})^2 + \alpha(i_I - i_{J_{-I}^*})^2. \quad (17)$$

J^* and J_{-I}^* are different parties if $|i_{J_{-I}^*} - \theta_m| < |i_{J^*} - \theta_m|$ and $\left| \frac{i_{J_{-I}^*} + \alpha i_I}{1 + \alpha} - \theta_m \right| >$

and high levels of θ low levels of taxations and not vice versa, but given that I called party L the left party and party R the right party labeling appears consistent.

$\left| \frac{i_{J^*} + \alpha i_I}{1 + \alpha} - \theta_m \right|$.¹⁰ This implies that $i_I > i_{J^*} > \theta_m$ or $i_I < i_{J^*} < \theta_m$. Therefore, if lobbying leads to a change of winner of the elections, it must lead to the victory of the party with the bliss point further away from the interest group. Whether the lobby is nonetheless better off depends on α :

$$U_I(p_J^*, m_J^*) - U_I(i_{J^*}, 0) \stackrel{\leq}{\geq} 0 \iff \alpha \stackrel{\leq}{\geq} \left(\frac{i_I - i_{J^*}}{i_I - i_{J^*}} \right)^2 - 1. \quad (18)$$

Only when the effect of lobbying is sufficiently large because the interest group cares enough about policy as compared to monetary contributions (large α), lobbying makes the interest group better off even if it leads to the loss of the party to which it is ideologically closer.

3 Extensions of the model

To check for the robustness of the results in the main part, this section deals with several extensions of the model presented in Section 2.

3.1 Alternative surplus sharing rules

How robust are results to the sharing of the surplus between the interest group and the party in power? Due to the assumption that the interest group makes a take-it or leave-it offer to the party in power, the whole surplus is given to the interest group and the party is not better off than it would be without lobbying. An alternative assumption is that the party in power and the interest group share the surplus created by post-election bargaining and therefore:

$$m(p) = (1 - \beta)[U_I(p, 0) - U_I(i_J, 0)] - \beta[U_J(p, 0) - U_J(i_J, 0)], \quad (19)$$

¹⁰ For simplicity and without much loss of generality, I ignore the cases in which the median voter is indifferent between the parties because then she decides by assumption according to the identity of party J what complicates notation.

with $\beta \in [0, 1]$ being the interest group's share of the surplus. Then, the interest group wants to maximize its utility over p :

$$p_I^* = \arg \max_p U_I(p, m(p)) = \arg \max_p \beta[U_I(p, 0) + (U_J(p, 0)) + (1 - \beta)U_I(i_J, 0) - \beta(U_J(i_J, 0))], \quad (20)$$

while party J wants to implement:

$$p_J^* = \arg \max_p U_J(p, m(p)) = \arg \max_p (1 - \beta)[U_J(p, 0) + U_I(p, 0) - U_I(i_J, 0)] + \beta U_J(i_J, 0). \quad (21)$$

It is easily verified that the interest group as well as the party in office agree that $p_I^* = p_J^*$ should be implemented and therefore the equilibrium policy given party J in power is the same for all sharing rules. If $\beta = 1$, we have returned to the basic model in Section 2 where the interest group appropriates the entire surplus. If $\beta = 0$, we have the opposite result and the party in power gets the entire surplus from the lobbying negotiations. An alternative model with the same result would be to give the party in power the opportunity to make a take-it or leave-it offer to the interest group. As had to be expected, as long as bargaining is efficient, the sharing rule makes no difference for implemented policy. However, the welfare implications for the interest group as well as the parties are different and this would be important if there were an additional, initial stage where the interest group could commit to not getting involved in lobbying after the elections.

3.2 The case of interest groups that can commit to restrict their lobbying activity

How do the results depend on the assumptions about the ability to commit? The main reason why commitment of the interest group is not part of the main model is that it seems somewhat arbitrary to assume that an interest group can commit to abstain from interfering with policy while the politicians have no possibility to commit to a specific policy position. Parties are known to make promises, while interest groups are not known to make promises about noninterference with policies.

It is easy to show that if an interest group has the possibility of committing not to interfere with policy making, it cannot be worse off. Moreover, it must be better off in all cases that are shown in Section 2.5 to make it worse off in the case of lobbying

compared to the case without lobbying. In the latter cases, it would commit before the elections not to interfere with the policies that will be implemented once a party is in office.

3.3 Several interest groups

How robust are the results to the introduction of more interest groups? Let there be Z interest groups. Allowing only a take-it or leave-it offer by the interest groups would now severely reduce the possibilities of strategical interaction. Therefore, I now follow Grossman and Helpman (1994) who use the common agency approach of Bernheim and Whinston (1986). The ruling party is the common agent and the lobbies are the principals. I assume that interest groups offer contribution schedules that specify a weakly positive contribution for any policy p . Interest group z maximizes:

$$U_z^I(p, m) = -\alpha_z(p - i_z)^2 - m_z(p, J), \quad (22)$$

where i_z is the policy bliss point of interest group z and $m_z(p, J)$ is its monetary contribution to the ruling party given that it implements policy p and the party in power is J . Parameter α_z measures how much lobby z cares about policy relative to monetary payments. The utility functions of the parties and the voters are still given by equations (1) and (3) in Section 2 with $m = \sum_{z=1}^Z m_z$ now being the aggregate monetary payment to the party.

All truthful contribution schedules have the following form:

$$m_z(p, J) = \max(U_z^I(p) - B_z(J), 0) \text{ for some } B_z. \quad (23)$$

Let:

$$p_J^* = \frac{i_J + \sum_{z=1}^Z \alpha_z i_z}{1 + \sum_{z=1}^Z \alpha_z}, \quad (24)$$

$$p_{J,-z}^* = \arg \max_p U_J(p, 0) + \sum_{\substack{y \in Z \\ y \neq z}} (m_y(p, J)), \quad (25)$$

Following Grossman and Helpman (1994), it is possible to show that there are truth-

ful contribution schedules with:

$$\begin{aligned}
 B_z(J) &= U_z^I(p_J^*, 0) + U_J(p_J^*, 0) - U_J(p_{J,-z}^*, 0) \\
 &\quad + \sum_{\substack{y \in Z \\ y \neq z}} (m_y(p_J^*, J) - m_y(p_{J,-z}^*, J)), \tag{26}
 \end{aligned}$$

that together with policy p^* constitute an equilibrium. In this equilibrium, the monetary transfers of the lobbies are not uniquely determined because lobbies that try to pull equilibrium policies in the same direction have to solve a free rider problem. This multiplicity of equilibrium payment schedules is not unique to my paper, but seems so far to have got little or no attention in the literature.¹¹

As in the main model in Section 2, the policy is efficient in the sense that to make any of the lobbies or the ruling party better off, some of the other players would have to be made worse off. Moreover, because of the quasilinear utility functions, the efficient policy is unique.

The case $Z = 1$ essentially reduces to the 1 interest group case dealt with in Section 2 because, in equilibrium, the monetary transfer is equal to the take-it or leave-it model:

$$m_z^*(p^*, J) = U_J(i_J, 0) - U_J(p_J^*, 0).$$

The interest group does not make a take-it or leave-it offer, but the implemented policy as well as monetary transfer are exactly the same.

Moreover, a representative interest group can be used to capture all the interest group activity in the model as follows directly from the fact that $p_J^* = \frac{i_J + \sum_{z=1}^Z \alpha_z i_z}{1 + \sum_{z=1}^Z \alpha_z}$. This is exactly the same policy that would be implement if there were only one interest group with bliss point $i_I = \bar{i}_z \equiv \frac{\sum_{z=1}^Z \alpha_z i_z}{\sum_{z=1}^Z \alpha_z}$ and a weight of $\alpha = \sum_{z=1}^Z \alpha_z$ on policy relative to monetary contributions. The model with only one interest group given in Section 2 can therefore be reinterpreted as a model with a representative interest group which captures the total lobbying effort in the polity. This shows that the limitation of the basic model to one interest group only has consequences for the analysis of the welfare of the interest group and the parties, not for the more important analysis of the welfare of the voters.

¹¹ Of course, the restriction to truthful contribution schedules already restricts the number of equilibria considerably. For other forms of equilibria that lead to different policy outcomes, see Besley and Coate (2001) and especially Kirchsteiger and Prat (2001).

4 The case of parties running with candidates

An important assumption that has been made so far is that parties implement policies. An alternative and equally plausible assumption is that the candidates who run for office decide about policy and the parties only decide who is their candidate in the elections. In this section, the situation is closer to the citizen candidate approach to lobbying by Besley and Coate (2001) than to the model in Section 2. In case the candidate accepts a monetary offer from the interest group, he must share the contribution with his party according to a predetermined sharing rule.

A party's utility function is now given by:

$$U_J(p, m) = -(p - i_J)^2 + (1 - \gamma)m, \quad (1')$$

with $J = L, R$. $\gamma \in [0, 1]$ is the share of monetary transfers that goes to the candidate while $(1 - \gamma)$ is the share that goes to the party. Potential candidates have a utility function that is similar to the utility function of the parties. Just as parties do, they care about policy as well as monetary transfers. It is given by:

$$U_J^k = -(p - \lambda_J^k)^2 + \gamma m, \quad (27)$$

where $J = L, R$ denotes the party the candidate is running for, λ_J^k is the bliss point of candidate k and γ the candidate's share of the monetary contribution if he is elected.¹² The interest group and the voters are assumed to have the same utility function as in Section 2, given by (2) and (3). Moreover, the assumption that $i_L < \theta_m < i_R$ is retained.

The order of moves is now the following. First, each party decides simultaneously over a candidate who will run for the party in the elections. Then, elections take place and the candidate with the majority of votes wins. If there is an interest group, it can make a take-it or leave-it offer to the winning candidate, offering her an amount of monetary contributions for implementing a certain policy. The candidate can either accept the offer, take the payment and implement the agreed

¹² Naturally, only the candidate who is in office actually profits from the monetary contribution m . But since only the candidates in office can actually influence their own utility, I ignore this fact for notational convenience.

policy, or choose any alternative policy.

Once more, the interest group makes an offer that just leaves the winning candidate indifferent between accepting and rejecting. Therefore, the implemented policy of a candidate with bliss point λ_j^k is given by:

$$\begin{aligned} (p^*(\lambda_j^k), m^*(\lambda_j^k)) &= \arg \max_{p, m} U_I(p, m) \text{ s.t. } U_J^k \geq 0 \\ \Rightarrow p^*(\lambda_j^k) &= \arg \max_p -\alpha(p - i_I)^2 - \frac{1}{\gamma}(p - \lambda_j^k)^2 \\ &= \frac{\alpha i_I + \frac{1}{\gamma} \lambda_j^k}{\alpha + \frac{1}{\gamma}}, \end{aligned} \quad (28)$$

$$m^*(\lambda_j^k) = \frac{1}{\gamma} \left(\lambda_j^k - \frac{\alpha i_I + \frac{1}{\gamma} \lambda_j^k}{\alpha + \frac{1}{\gamma}} \right)^2 = \frac{1}{\gamma} \left(\frac{\alpha(\lambda_j^k - i_I)}{\alpha + \frac{1}{\gamma}} \right)^2 \quad (29)$$

As in the main model, implemented policy is a weighted average of the bliss points of the interest group and the policy maker. The larger the candidate's share γ of the monetary contribution, the less influence does his bliss point have on implemented policy. Define $\lambda_j^k(p)$ such that $p^*(\lambda_j^k(p)) = p$:

$$\lambda_j^k(p) = (\gamma\alpha + 1)p - \gamma\alpha i_I. \quad (30)$$

In this notation, $\lambda_j^k(p)$ gives the preferences of a candidate who would implement policy p if he is lobbied. This value is potentially not available for all p , but it is unique if it exists. Moreover, $\lambda_j^{k'}(p) = \gamma\alpha + 1 > 0$.

Let Λ_J be the set containing all available candidates' bliss points for party J and let P_J^* be the set of all post-lobbying policies available from party J , that is, $p \in P_J^*$ if and only if $\lambda_j^k(p) \in \Lambda_J$.

If there is a continuum of candidates with bliss points everywhere in the policy space $[0, 1]$ available and there is no interest group, the model is essentially identical with the classical Downsian model of two competing parties. A party can commit to any policy by just choosing a candidate who has the policy it wants to commit to as his bliss point. Consequently, both parties will choose a candidate with the same bliss point as the median voter.

That both parties have to choose such a candidate in equilibrium follows from

the same logic as the standard Downsian result.¹³ First, it is clear that both parties running with such a candidate constitutes an equilibrium, because a deviation by one of the parties would not change the policy outcome. If one party deviates, the other party wins and the bliss point of the median voter is nonetheless implemented. That there cannot be any other equilibrium follows from the fact that at least one of the candidates could always win with a position closer to her own bliss point than the winning position by deviating. That parties are ideological rather than office seeking does not change the Downsian logic that leads to full policy convergence and therefore the preferences of the median voter prevail.

If there is an interest group that tries to influence the policy after the elections, there are no fundamental differences. The only adjustment is that the parties choose the candidate who will implement the median voter's favorite policy after being lobbied instead of a candidate with the bliss point of the median voter. They choose a candidate with bliss point $\lambda_j^k(\theta_m) = (\gamma\alpha + 1)\theta_m - \gamma\alpha i_I$ who will implement the median voter's bliss point θ_m if elected. If both parties have such a candidate available, lobbying will not change the implemented policy, just as if citizen candidates were running for office as in Besley and Coate (2001). Lobbying turns out to be irrelevant for the implemented policy.

A somewhat different situation only occurs when $p^*(0) > \theta_m$ or $p^*(1) < \theta_m$. In the first case, an interest group with bliss point $i_I > \theta_m$ leads to a policy to the right of the median voter's bliss point policy even with the leftmost candidate possible. In the second case, an interest group with bliss point $i_I < \theta_m$ leads to a left of the median voter's bliss point policy even with the rightmost candidate possible. In this case, we have an equilibrium where both parties run with an extremist candidate (with bliss point 0 in the first and bliss point 1 in the second case). The interest group influences policy and the interest group irrelevance result no longer holds.

If parties are restricted in their choice of candidates, implemented policy can differ from the median voter's bliss point, even if the interest group influence on candidates is limited. This seems to be a relevant restriction, given that the members of a party usually show a certain ideological uniformity and that, for example, a far left candidate is unlikely to run for a right-of-center party.

¹³ The standard Downsian logic applies although parties in my model are not vote maximizers as in Downs (1957) but utility maximizers as in Wittman (1973).

More specifically, let $\Lambda_L = [0, \lambda_L^{\max}]$ and $\Lambda_R = [\lambda_R^{\min}, 1]$, that is, the left party L cannot run with a candidate with a bliss point to the right of λ_L^{\max} and the right party R cannot run with a candidate with a bliss point to the left of λ_R^{\min} , but there are no further restrictions on the choice of candidates. As long as $\lambda_R^{\min} \leq \theta_m \leq \lambda_L^{\max}$, the results without lobbying are not affected. However, if in addition $p^*(\lambda_R^{\min}) > \theta_m$ or $p^*(\lambda_L^{\max}) < \theta_m$, lobbying changes the equilibrium policy outcome. The reason is simply that one of the parties is no longer able to choose a candidate who is going to implement the median voter's preferred policy.

As an example, let us assume that once more there is an interest group with $i_I > i_R$ and $p^*(\lambda_R^{\min}) > \theta_m$. In this case, the left party can win with certainty by letting any candidate k_L with bliss point λ_L^k such that $|p^*(\lambda_L^k) - \theta_m| < |p^*(\lambda_R^{\min}) - \theta_m|$ run in the elections. To ensure the existence of an equilibrium, I assume that the median voter votes in favor of the candidate of party L if $\min_k |p^*(\lambda_L^k) - \theta_m| < \min_k |p^*(\lambda_R^k) - \theta_m|$ and in favor of the candidate of party R if $\min_k |p^*(\lambda_L^k) - \theta_m| > \min_k |p^*(\lambda_R^k) - \theta_m|$ whenever she is indifferent between the two candidates who are running for office. This is a purely technical assumption without economic interpretation to ensure the existence of an equilibrium. Then, in equilibrium, the left party needs to maximize its utility by solving the following problem:

$$\lambda_L^* = \arg \max_{\lambda_L^k \in \Lambda_L} -(p^*(\lambda_L^k) - i_L)^2 + (1 - \gamma)m^*(\lambda_L^k) \quad (31)$$

$$s.t. \quad |p^*(\lambda_L^k) - \theta_m| \leq |p^*(\lambda_R^{\min}) - \theta_m|, \quad (32)$$

$$\lambda_R^* = \lambda_R^{\min},$$

$$p^* = p(\lambda_L^*).$$

The left party chooses a candidate such that the implemented policy cannot be beaten by the right party. Because monetary transfers are increasing in the distance between candidate and interest group, the left party chooses not necessarily the candidate who implements the policy closest to its bliss point that can win the elections. The reason is that a candidate with preferences further left receives a larger monetary payment m^* from the interest group and therefore a tradeoff between policy and monetary contributions exists when the candidate is chosen.

In the case of candidates running for parties the interest group is never better off with lobbying. It not only has to pay monetary to just achieve the same policy that

would be implemented without lobbying, in some cases implemented policy even becomes less favorable for the interest group.

5 Conclusion

This paper argues that the interaction of post-election lobbying and elections deserves more consideration. The possibility of voters taking later attempts at lobbying into account already when they vote can at least partly offset the effects of lobbying on policy. However, in my framework, with parties instead of citizen candidates as in Besley and Coate (2001), lobbying can still influence policy. In the basic model, where parties directly decide on policy, this is the case because lobbies *de facto* change the choice set of voters. In the alternative model of Section 4, where parties only choose candidates, and candidates with different preferences implement policy, lobbying is irrelevant for policy as long as the lobbies are not too influential and parties can choose freely among candidates. If, on the other hand, there are restrictions on the candidate pool of the parties, lobbying has an influence on equilibrium policy. However, when parties cannot choose freely, lobbying influence on equilibrium policy is not in the direction the interest group would like it to be, but instead makes the interest group worse off.

Chapter 4

Lexicographic Voting^{*}

1 Introduction

Modeling elections and their impact on policy is one of the most important topics in political economics. Therefore, it is somewhat disconcerting that the existing literature is split by an unresolved fault line between theoretical models of preelection and postelection politics. In the former, candidates commit to their postelection actions before elections take place. In contrast, in postelection models, politicians are free to decide about their policies when they are in office. However, in the next elections, the voters can condition their vote on the performance of the incumbent party.¹ Models of preelection politics are especially popular for modeling spatial policy choices in the tradition of Downs (1957) where voters decide between announced policy positions, while models of postelection politics are often, but not exclusively, applied to accountability issues. Politicians are induced to put in more effort (Ferejohn 1986) or to limit rent extraction due to the possibility of losing the elections and office if they do not comply (Barro 1973).² Essentially, these accountability models apply a principal-agent framework to elections with the politicians as agents and the voters as their principals.

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¹ For an overview of both types of models, see Persson and Tabellini (2000). For an overview especially of models of accountability, see Besley (2006).

² Besides the accountability and preference aggregation function, there are at least two more functions of elections (Persson, Roland, and Tabellini 1997). In addition, elections allow citizens to select the most competent individuals for office and help aggregate information about the correct political decisions.

In this paper, I combine a simple prospective model of Downsian spatial electoral competition with policy choice and a simple retrospective model of electoral accountability with rent extraction. Specifically, parties can commit to a policy position before elections take place as in Downs (1957), but decide on the level of rent extraction once they are in office as in Barro (1973) and the simplified model of political accountability discussed in Persson and Tabellini (2000). I abstract from any details on how rents are extracted and assume that rent payments reduce a given amount of public funds which reduces every voter's utility in the same way. Voters are fully aware of how much rents are extracted.

In the basic model in Section 2, I show that having voters with divergent policy preferences does not at all restrict the possibility of holding politicians accountable, as long as there is certainty about the position of the median voter. Voters manage to hold politicians accountable as well as they would in a model with backward-looking voters but without the policy dimension. They achieve this by following a straightforward and intuitive lexicographic voting strategy. Specifically, if the parties commit to policy positions that differ in attractiveness for a voter, the voter casts her vote in favor of the party which minimizes her disutility on the policy dimension. However, when a voter is indifferent, she conditions her vote on the rent extraction of the incumbent party. She supports the incumbent party only if the rents have not exceeded a maximum acceptable level. In equilibrium, this level is positive but smaller than the maximum rent the incumbent party could take. I call this voting strategy "lexicographic" because voters cast their votes as if they had lexicographic preferences over policy and rents.

The lexicographic voting strategy forces the parties to converge on the policy dimension, but also allows for control of the incumbent's party rent extraction. Moreover, it is intuitive that a voter who is indifferent will take past actions of the parties into account, whereas it is impossible for a rational forward-looking voter to consider the past when she is not indifferent with respect to the future.

Lexicographic voting requires sophistication of the voters only with respect to the optimal determination of the acceptable level of per period rent extraction by the incumbent party. Thus, the demands with respect to the voters' sophistication are not larger than in other models of political accountability.

Generally, the equilibria in backward looking models hinge on the fact that voters

are indifferent between the incumbent party and the opposition and can therefore reward or punish past actions while playing undominated strategies. The fact that a simple strategy can solve the accountability problem in a model combining rent extraction with Downsian competition is somewhat surprising, but can be explained by the fact that competition forces both parties to choose the same platform so that voters are indeed indifferent. This is a result of the lack of uncertainty in the basic model. Section 3 of the paper shows that as soon as uncertainty about the preferences of the median voter is introduced, the accountability of politicians is reduced and the voters must accept larger rent extraction by the incumbent party. But it is still optimal for them to follow the lexicographic voting strategy. Because the incumbent party does not know the position of the median bliss point with certainty, the opposition party now has a chance of winning office by offering a different policy position than the incumbent party, even when the latter complies with voters' demands on the rent dimension. Nonetheless, the incumbent party has an incentive to accept somewhat reduced rent payments in return for being reelected whenever the voters are indifferent. The reason is that in this way, it can ensure that it will be reelected with positive probability.

In Section 4, I show that if parties are also motivated by policy and not only by rents as in the main model, the inclusion of a policy dimension into the model can even increase the accountability of politicians compared to a pure accountability model. Ideological parties give voters the additional option of threatening the incumbent party to allow the opposition party to win with policies that make the incumbent party worse off than the bliss point of the median voter. However, this requires more coordination among voters than the simple and straightforward lexicographic voting strategy given in Section 2. Therefore, the lexicographic voting strategy from the main model which continues to constitute an equilibrium in the case with ideological parties is the most plausible outcome even in the case of ideological parties. Nonetheless, the result in Section 4 shows that treating accountability and policy determination separately obscures some interesting possibilities.

A crucial assumption in the paper is that commitments to electoral platforms are credible in the policy dimension but lack credibility in the rent dimension. A first justification is that these are widely accepted standard assumptions for both types of models and that it is worth exploring if combining these leads to results that cannot

be found by looking at the models separately. Moreover, in the basic model as well as in the extension with uncertainty over the position of the median voter (Sections 2 and 3), parties have no reason to break their electoral promises with regard to policy because it does not enter their utility function. A further justification is that if parties announce policy motivated candidates who run for office, they can indeed credibly commit to policies, but not to limits of rent extraction. Osborne and Slivinski (1996) and Besley and Coate (1997) introduced citizen-candidates into the voting literature. In these models, not parties, but citizens with policy preferences run for election. Commitment to a policy position does not constitute a problem because voters vote for ideological candidates whom they know to implement their favorite policy. As long as there is a candidate with a certain ideology, voters can vote for that candidate. The principal-agent problem of the voters is solved by delegation to an agent with the right preferences. However, empirically, citizen-candidates who run independent of any parties appear to be the exception rather than the rule. The basic idea that a certain type of candidate will implement a certain kind of policy can be incorporated into models with parties if the parties have the chance of deciding before the elections who the candidate is and achieve office in case of victory and if the choice of potential candidates is sufficiently large. I do not explicitly model such a candidate choice stage, but the fact that parties usually run with candidates who have their own ideology is a good justification for the assumption that parties can commit to a policy. However, as long as there are no candidates with purely altruistic motives without interest in rent payments available, parties cannot credibly commit to refrain from rent seeking.

It is surprising that until now, there seem to have been no attempts to combine models of retrospective voting with aspects of Downsian competition. My model shows that forward-looking and backward-looking motives can be reconciled in a single model. This should be considered in future empirical research because so far, the question seems to have been if voters vote retrospectively or prospectively. If there is not necessarily a contradiction, some empirical results might have to be reevaluated.

Models of political accountability can explain the often observed incumbency advantage, as is pointed out by Austen-Smith and Banks (1989). It is hard to see how a purely policy model could account for this without assuming some asymmetries

between parties or candidates. My basic model in Section 2 leads to the implausible result that in equilibrium, the incumbent party is always reelected. In the extended model with uncertainty about the exact position of the median voter in Section 3, I find that the incumbent party always has a chance exceeding 50% of winning the elections and that its advantage depends on a measure of uncertainty about the preferences of the median voter. This result seems to be consistent with election results in many countries. Incumbent parties win more often than not, but their victory is far from certain.

The term lexicographic voting has been used before to describe similar voting strategies, for example in Dutter (1981) and Soberman and Sadoulet (2007). However, in these papers, lexicographic voting follows directly from lexicographic preferences. In my model, lexicographic voting is part of an equilibrium of the voting game although the voters' preferences are not lexicographic. My model is the first one to show that lexicographic voting can achieve a reconciliation of backward-looking and forward-looking voting.

The paper proceeds as follows. Section 2 develops the main model with certainty about the position of the median voter and discusses its equilibrium. Section 3 shows that uncertainty over the positions of the median voter leads to less electoral control. Section 4 presents an extension to policy oriented parties and strategies that are not history-independent. An Appendix contains the proofs of the results in Section 3 and the examples from Section 4.

2 The model

I consider a polity with two parties interested in winning office only for rent-seeking purposes, and an odd number N of voters $i = 1, 2, \dots, n$ interested in policy as well as rent reduction. The ideological policy space is the real line $[0, 1]$. Party $j \in \{x, y\}$ maximizes:

$$U_0^j = E_0 \sum_{t=0}^{\infty} \beta^t r_t^j, \quad (1)$$

where rents in future periods are discounted by the factor $\beta < 1$. r_t^j is the rent extracted by party j in period t . The party in government (also called the incumbent party) in period t is denoted by $I_t \in \{x, y\}$. The opposition party in period t is

denoted by $O_t \in \{x, y\}$, $O_t \neq I_t$. Parties decide how much rent $r_t \in [0, R]$ they extract in a period in which they are in office. R is the total amount of available public funds that is assumed to be constant over time and constitutes the maximum per period rent. Parties out of office cannot acquire any rents. Hence, $r_t^j = r_t$ for $j = I_t$ and $r_t^j = 0$ for $j = O_t$.

Voters $i = 1, 2, \dots, n$ maximize:

$$U_0^i = E_0 \sum_{t=0}^{\infty} \beta^t (-(p_t - b^i)^2 + (R - r_t)), \quad (2)$$

where b^i is the policy bliss point for voter i and $r_t = r_t^y + r_t^x$ the rent extraction of the incumbent party in period t . Hence, $R - r_t$ gives the amount of public funds that are used in the voters' interest. For simplicity, I assume that the utility from public good spending is uncorrelated with the ideological policy position. The variable p_t denotes the policy in period t and the vector $B = (b^1, b^2, \dots, b^N)$ the policy bliss points of the voters. $b_m = \text{median}(B)$ is the bliss point of the median voter. For the moment, it is assumed to be constant over time. In Section 3, the more general case of uncertainty about the median voter's position is discussed. Disutility in policy is quadratic in the distance to the bliss point. This standard functional-form assumption is made for convenience of notation. All the following results only depend on increasing disutility in distance of policy to a voter's bliss point. Since parties are not interested in policy in the main model, the assumption that they can commit to the policy position while they cannot commit to limit rent extraction is plausible. Parties have no incentive to break their promises on the policy dimension. Another interpretation is that parties have the possibility to let candidates with preferences different from their own (which could, for example, be the preferences of the average party member) run in the elections and in this way, they can commit to a policy. There is no reason to assume that such candidates could commit to low rent payments more credibly than a party, but they have no incentives to implement any policies that are different from their own bliss point.

2.1 The order of moves

The order of moves is the following: In any period t , the policy position p_t^I of the incumbent party $I_t \in \{x, y\}$ is implemented, then rents r_t^I and a new policy position

p_{t+1}^I are chosen by the incumbent party. An alternative policy position p_{t+1}^O is chosen by the opposition after observing the policy position of the incumbent party and the rent r_t . Then, elections take place and every voter i casts her vote $v_t^i \in \{x, y\}$. Abstentions are not possible.

Let $V_t = (v_t^1, v_t^2, \dots, v_t^N)$ be the vector containing the votes of all voters. After the elections have taken place, the new period $t + 1$ begins and the party with the majority of votes in period t becomes the incumbent party:

$$I_{t+1} = \text{mod}(V_t).$$

Period 0 is identical to all other periods, only the identity and the policy positions of the incumbent party and the opposition are exogenously given and not determined in a previous period.

The incumbent party is thus assumed to first choose its position instead of the more standard assumption that policy positions are chosen simultaneously.³ For the basic model, this is of no great importance (however, the best reply of the opposition is no longer unique), but it plays some role when I introduce uncertainty in Section 3, where it is essential for the existence of equilibria in pure strategies. The timing assumption is mostly made to keep the analysis there as simple as possible.

2.2 Strategies

To denote the entire history of a variable (or vector) z_t up to period t , I use a superscript t such that $z^t = \{z_0, z_1, z_2, \dots, z_t\}$. Let $h_t = \{p^{y,t}, p^{x,t}, I^t, V^{t-1}, r^{t-1}\}$ be the history of the game up to the beginning of period t . A strategy for a party j is the decision about a policy platform $p_{t+1}^j(h_t) \in [0, 1]$ for all possible histories with $j = I_t$ and $p_{t+1}^j(h_t, p_{t+1}^I, r_t) \in [0, 1]$ for all possible histories with $j = O_t$. In addition, the strategy contains the rent payment $r_{t,j}(h_t)$ for all possible histories with $j = I_t$. Because the opposition can observe the policy position of the incumbent party, the party that is out of office can take the policy position as well as the rent payment to the incumbent party into account when announcing its policy position, while the incumbent party cannot. A strategy for a voter i is a vote $v_t^i(h_t, p_{t+1}^y, p_{t+1}^x, r_t) \in$

³ This assumption is less common than simultaneous policy announcements, but has been made in many papers, for an early example see Wittman (1973).

$\{y, x\}$ for every period t and every possible history up to the time of her voting decision.

Definition 1. *A strategy is history-independent if all decision by a player in period t only depend on other variables that have been a) determined in the same period and b) before the decision is made.*⁴

Thus, a history independent strategy for the incumbent party implies that its platform and rent extraction do not depend on moves in past periods at all and thus, they must be constant as long as the same party j is holding office (nothing rules out a priori that the parties could play different history-independent strategies): $p_{t+1}^I = p_j^I$ and $r_t = r_j$ as long as $I = j$ for all periods t as long as j is in office. The reply of the opposition party only depends on the policy offer and rent extraction of the incumbent party and the votes only on the policy offers, the identity of the incumbent party and the rent extraction. If, in addition, both parties are assumed to play the same strategy, policy offers and rent extraction will be the same in all periods. Moreover, if the voters play pure strategies, the incumbent party is either always or never reelected.

2.3 An equilibrium with lexicographic voting

The strategies formulated in Proposition 1 below constitute an interesting equilibrium which has all the essential features of a backward-looking model in the tradition of Barro (1973) and Ferejohn (1986) as well as those of a forward-looking model in the tradition of Downs (1957). Parties converge on the ideological dimension, but voters nonetheless keep the rent payments at some level which they could achieve in a model without policy platforms. This is the result of an intuitive lexicographic voting strategy. A voter casts her ballot in favor of her preferred policy position. Only when she is indifferent in this respect does she decide according to past rent extraction by the incumbent party. It is clear that with such a strategy, she encounters no credibility or time-inconsistency problem. It also seems intuitively plausible that a voter casts her vote in this way and it is moreover consistent with the evidence that voters have prospective as well as retrospective motives.

⁴ This is often called a stationary strategy in political economics. However, it could be argued that the rent payment r_t should not play any role in a stationary strategy because it is a bygone by the time the voters cast their votes. I therefore avoid the term "stationary".

Proposition 1. *An equilibrium of the game is constituted by the following strategies:*

The parties play:

$$\begin{aligned} p_{t+1}^j &= b_m \text{ for } j = y, x \text{ in all } t, \\ r_t &= \bar{r} \text{ in all } t, \end{aligned} \quad (3)$$

where $\bar{r} = (1 - \beta)R$.

The voters play:

$$v_t^i = \begin{cases} y & \text{if } (p_{t+1}^y - b^i)^2 - (p_{t+1}^x - b^i)^2 < 0 \\ x & \text{if } (p_{t+1}^y - b^i)^2 - (p_{t+1}^x - b^i)^2 > 0 \\ I_t & \text{if } (p_{t+1}^y - b^i)^2 - (p_{t+1}^x - b^i)^2 = 0 \text{ and } r_t \leq \bar{r} \\ O_t & \text{if } (p_{t+1}^y - b^i)^2 - (p_{t+1}^x - b^i)^2 = 0 \text{ and } r_t > \bar{r} \end{cases} \text{ in all } t. \quad (4)$$

From the strategies, it follows that in equilibrium:

$$\begin{aligned} I_t &= I_0 \text{ in all } t, \\ p_t &= b_m \text{ in all } t \geq 1, \\ r_t &= \bar{r} \text{ in all } t. \end{aligned} \quad (5)$$

Proof. Given the voters' strategy, the median voter is decisive: If $v_t^m = j$, it follows that $(p_{t+1}^j - b^m)^2 - (p_{t+1}^{\tilde{j}} - b^m)^2 \leq 0$. This implies that $(p_{t+1}^j - b^i)^2 - (p_{t+1}^{\tilde{j}} - b^i)^2 \leq 0$ for all $b_i \leq b_m$ or all $b_i \geq b_m$ and therefore for a majority of voters. Thus, the majority of voters cast their vote for the same candidate as the median voter and the party with the support of the median voter wins. Given the equilibrium strategies of the parties, $(p_{t+1}^j - b^i)^2 = (p_{t+1}^{\tilde{j}} - b^i)^2$ in all periods. Because $r_t = \bar{r}$ in all periods, all voters vote for the incumbent party that remains in office and implements $p_{t+1}^I = b_m$.

Given the strategies of the parties, a voter neither influences future rents nor future policy with her vote. This is even true in the case with only one voter who is always pivotal. Therefore, a voter has no utility increasing deviation from voting for the party that offers the policy closest to her bliss point. In case a voter is indifferent with respect to policy in the next period, there is no utility increasing deviation from voting according to past performance of the incumbent because, again, it does not influence future policy or rent payments.

The fact that the opposition party cannot be better off by deviating follows from

the fact that given the position and rent extraction of the incumbent party, it either wins with certainty or has no possibility to achieve office and, moreover, it cannot influence any election results or rent payments in the future with its choice of policy position. For the incumbent party, any policy position different from $p_{t+1}^I = b_m$ leads to a loss of office (and therefore rent payments) forever because given the reply of the opposition, the latter is preferred by the median voter. The same is true for the combination of any policy position p_{t+1}^I with any rent $r_t > \bar{r}$. Therefore, reelection is only possible with $r \leq \bar{r}$. Hence, there is no possibility for the incumbent party to increase its utility by deviating with a strategy that leads to its reelection. If it accepts defeat by deviating in an arbitrary period s , the incumbent party can, at most, achieve a rent of R in the period in which it deviates and then lose office and rents forever. This gives the same utility level that the incumbent party achieves by not deviating and receiving a rent of $r_t = \bar{r} = (1 - \beta)R$ forever because the present discounted value of future rent payments in period s is the same:

$$\sum_{t=0}^{\infty} \beta^t \bar{r} = \sum_{t=0}^{s-1} \beta^t \bar{r} + \sum_{t=s}^{\infty} \beta^t \bar{r} = \sum_{t=0}^{s-1} \beta^t \bar{r} + \sum_{t=s}^{\infty} \beta^s \frac{\bar{r}}{1 - \beta} = \sum_{t=0}^{s-1} \beta^t \bar{r} + \beta^s R.$$

Therefore, no deviation from the given strategy increases the utility of the incumbent party. ■

Which party is the incumbent party in period 0 is exogenously given. This party remains in office forever, as in the standard case of backward looking models without uncertainty. However, this will no longer be the case when I introduce some uncertainty in Section 3.

Corollary 1. *There is no equilibrium with a lower present discounted value of future rent payments in any period s than R .*

Proof. Suppose that there is an equilibrium with $\sum_{t=s}^{\infty} \beta^{t+s} r_t < R$ in any period s . Then, the incumbent party in period s is better off by deviating and taking a rent of $r_s = R$. This is a contradiction. ■

Therefore, the equilibrium in Proposition 1 gives voters the maximum control over rents that can be achieved in equilibrium.⁵ It is identical to the minimum rent

⁵ There are equilibria with a lower rent payment $r_t < \bar{r}$ in period t that are sustainable because the incumbent party expects higher rent payments in the future. However, from Corollary 1, it

extraction that can be achieved in a model without a policy dimension where the only problem of the voters is to hold the parties accountable for rent extraction.

Voters play as if they were always pivotal. This seems to be a reasonable assumption for a plausible equilibrium and helps to rule out equilibria which require a great deal of coordination of voters when they cast their votes. However, Corollary 1 is valid for all possible equilibria. Therefore, restricting strategies to be history-independent does not reduce electoral control at all.

The intuition is straightforward. Nothing can stop a party in power from taking maximum rent R if this party does not expect to get at least the same present discounted value in rents in later periods. As shown in Section 4, if parties are interested in policy, there are history-dependent strategies that lead to more electoral control and lower rent payments. The reason is that ideological parties can be rewarded and punished with future policies.

As is also common in models of political accountability, the given equilibrium is not unique and other equilibria with larger rent payments exist. However, the outcome with the minimum constant rent payments is generally considered to be the most interesting outcome of a game of backward-looking voting, as it describes maximal voter control. In this sense, the equilibrium here is most in line with the literature. It shows that retrospective and prospective motives in voting are not inconsistent with each other. Voters have just one instrument, namely their single vote, but this is sufficient to control policy as well as to hold politicians accountable to a certain degree.

The following Corollary shows that convergence on the policy dimension is the rule rather than the exception, but first I derive a useful Lemma:

Definition 2. *A voter is pivotal if her vote decides about the winner of the elections because $\frac{N-1}{2}$ of the other voters vote for party x and $\frac{N-1}{2}$ vote for party y . If a voter votes as if she was pivotal she votes for a party whose victory maximizes her utility given the strategies of all players.*

Lemma 1. *If parties play symmetric history-independent strategies and voters vote as if they were pivotal even when they are not then: a) A voter votes for a party that*

follows that the present discounted value of rent extraction cannot be smaller than R . Equilibria with increasing rent payments over time seem rather implausible. The opposition party could convince the voters that it actually only demands a constant rent payment of \bar{r} once in office.

offers the bliss point minimizing her disutility from policy in the next period. b) A party's utility only depends on its being the incumbent party in the next period and the rent extraction in the current period.

Proof. History independence together with symmetry of the parties strategies imply that from period $t + 1$ onwards, policy positions and rent extraction are decided independently of past periods. The only state variable is incumbency, but voters are indifferent to which party is in office and which party offers which policy position. From this, the lemma directly follows. ■

Corollary 2. *There is no equilibrium with symmetric history-independent strategies, voters who vote as if they were pivotal, rent payments $r_t < R$ and policy $p_{t+1} \neq b^m$ in any period t .*

Proof. From Lemma 1, it follows that in any equilibrium with history-independent symmetric strategies, a party's policy position influences its utility only in so far as it determines the winner of the elections and the rent extraction. Suppose that $r_t < R$. This can only be part of an equilibrium if the incumbent party is reelected with positive probability; if not it would play $r_t = R$ because a lower rent r_t could not improve its situation once in opposition. If both parties play symmetric history-independent strategies, the incumbent party can only be reelected with positive probability if it plays $p_{t+1}^I = b_m$, because all other positions would be beaten by $p_{t+1}^O = b_m$. To see this, consider the problem of a voter who votes as if pivotal: By definition of b_m , a majority of voters must prefer b_m to any $b \neq b_m$ and in equilibrium, the opposition would have to choose a position that wins the elections to maximize its utility. Therefore, if $r_t < R$ the incumbent party offers $p_{t+1} = b_m$ and, in equilibrium, a party offering b_m wins. ■

There are equilibria with $r_t = R$ and $p_{t+1} \neq b_m$. This is due to the unusual timing assumption that the opposition party chooses its policy position after the incumbent party. There are history-independent equilibria where the incumbent party always takes R and is never reelected. In such equilibria, the incumbent party has no incentive to take the median position. However, if the incumbent party does not take the median position, the opposition party does not have to take it to win because any policy position that is different from b_m can be beaten by another policy position that is different from b_m , but slightly closer to the bliss point of the

median voter. With the standard timing assumption of simultaneous announcement of policy positions, this is not possible. However, a similar equilibrium in which policy does not converge to the median position is possible in a purely Downsian framework with the incumbent party choosing its position first and the result should therefore not be attributed to the combination of prospective and retrospective voting motives. On the contrary, only in combination with the outcome of $r_t = R$ in all periods can it be sustained in the combined model.

3 Uncertainty about the median bliss point

So far, I have assumed that the identity of the median voter is known when parties decide about their policy platforms. How robust are the results to relaxing this assumption? This section shows that voters retain some control over rent extraction in a straightforward and plausible equilibrium where voters follow the same lexicographic voting strategy as in Section 2.

The assumptions and the order of moves are the same as in Section 2. The only difference is that the favorite position of the median voter is now uncertain at the point when parties announce their policy positions. Voters keep some control over rent extraction, but the control is limited because sometimes the incumbent party loses office even when it does not deviate and therefore can demand higher rents in equilibrium.

For simplicity, I assume from now on that there is only one voter. She can be thought of as representing the decisive median voter.⁶ Her expected utility is given by:

$$U_0^m = E \sum_{t=0}^{\infty} \beta^t (-(p_t - b_t)^2 + R - r_t), \quad (2')$$

where b_t is her bliss point in period t . This bliss point is now a random variable that is only determined after the parties have announced their policy positions for period t . The value of b_t is distributed identically and independently of past bliss points. The expected utility function of the parties $j = y, x$ is identical to the expected

⁶ This avoids complications in finding the distribution of the possible median bliss points by ruling out the possibility that the identity of the median voter changes between periods.

utility function in Section 2:

$$U_0^j = E_0 \sum_{t=0}^{\infty} \beta^t r_{t,j}. \quad (1)$$

Let there be K distinct possible policy bliss points b_k of the voter, all within the policy space $[0, 1]$. They are ordered such that $b_k < b_l$ if and only if $k < l$. Let q_k be the probability that the median voter of period t has the bliss point $b_t = b_k$. By assumption, this probability is the same in every period t . Then, $F(b_k) = \sum_{l=1}^{l=k} q_l$ is the cumulative distribution function of b_k . I define:

$$b_m = \min_{k \in K} F(b_k) \text{ s.t. } F(b_k) \geq 0.5, \quad (6)$$

so that b_m is now the median of the possible bliss points of the voter.⁷ Moreover, I define for the case $K \geq 2$:

$$b^*(b_k) = \begin{cases} b_2 & \text{for } k = 1 \\ b_{K-1} & \text{for } k = K \\ b_{k-1} \text{ if } F(b_{k-1}) \geq 1 - F(b_k) \\ b_{k+1} \text{ if } F(b_{k-1}) < 1 - F(b_k) \end{cases} \text{ for } k \in \{2, 3, \dots, K-1\} \quad (7)$$

$$\pi^* = \begin{cases} F(b_m) \text{ if } b^*(b_m) > b_m \\ 1 - F(b^*(b_{m-1})) \text{ if } b^*(b_m) < b_m \end{cases} \quad (8)$$

$$r^* = \frac{((1 - 2\pi^*)\beta + 1)}{(1 - \pi^*)\beta + 1} R \quad (9)$$

If $K = 1$, then $b^* = b_m = b_1$ and $\pi^* = 1$.

Proposition 2. *An equilibrium of the game entails the following strategies:*

The parties play:

$$p_{t+1}^O = \begin{cases} p_{t+1}^I = b_m, \\ r_t = r^*, \\ b^*(p_{t+1}^I) \text{ if } r_t \leq r^* \\ p_{t+1}^O = p_{t+1}^I \text{ if } r_t > r^* \end{cases} \text{ in all } t. \quad (10)$$

⁷ Naturally, b_m was also the median of the possible median bliss points in Section 2, where the distribution of the median voter was degenerate. Therefore, there is no need to change the notation.

The voter plays:

$$v_t = \begin{cases} y & \text{if } (p_{t+1}^y - b_{t+1})^2 - (p_{t+1}^x - b_{t+1})^2 < 0 \\ x & \text{if } (p_{t+1}^y - b_{t+1})^2 - (p_{t+1}^x - b_{t+1})^2 > 0 \\ I_t & \text{if } (p_{t+1}^I - b_{t+1})^2 - (p_{t+1}^O - b_{t+1})^2 = 0 \text{ and } r_t \leq r^* \\ O_t & \text{if } (p_{t+1}^I - b_{t+1})^2 - (p_{t+1}^O - b_{t+1})^2 = 0 \text{ and } r_t > r^* \end{cases} \quad \text{in all } t. \quad (11)$$

In every period, the probability that the incumbent party wins is π^* . If the incumbent party wins, b_m is implemented, if the incumbent party loses, $b^*(b_m)$ is implemented. If $K = 1$, the expected utility of the voter is: $\frac{R-r^*}{1-\beta}$ because there is no uncertainty and her favorite policy is always implemented. In the case of $K \geq 2$, the expected utility of the voter is:

$$u_{rv} = \begin{cases} \sum_{t=0}^{\infty} (\sum_{k=1}^{m-1} q_k \beta^t (-(b_{m-1} - b_k)^2 + R - r^*) + \sum_{k=m}^K q_k \beta^t (-(b_m - b_k)^2 + R - r^*)) & \text{if } b^* = b_{m-1} \\ \sum_{t=0}^{\infty} (\sum_{k=1}^m q_k \beta^t (-(b_m - b_k)^2 + R - r^*) + \sum_{k=m+1}^K q_k \beta^t (-(b_{m+1} - b_k)^2 + R - r^*)) & \text{if } b^* = b_{m+1} \end{cases} \quad (12)$$

Proof. See the Appendix ■

The best position any incumbent party can choose is the median of the possible positions of the voter. The intuition is straightforward. The incumbent party must choose its position first. Because the incumbent party will not be reelected if the voter prefers the opponent even if it constrains itself with respect to rent extraction, the best the incumbent party can do is to choose its position so that the opposition can only achieve less than 50% of the votes. The incumbent party can achieve this by announcing the median bliss point as policy position. The opposition party will then choose a position as close to the median position as possible to ensure the victory whenever the bliss point of the median voter is on the same side of the median position. It chooses the side of the median where this probability is the largest. Therefore, the most useful measure of uncertainty about the election outcome is given by:

$$\pi^* = \min(F(b_m), 1 - F(b_{m-1})).$$

It turns out that the larger is π^* , the larger is the control of the voter over rent extraction by the parties. In the special case of no uncertainty about the bliss point of the voter, $\pi^* = 1$, an incumbent party that does not extract too high rent payments is reelected with certainty. The results of Section 2 are confirmed as a

special case of the generalized model.

Restricting strategies of parties to be history-independent and identical (that is, both parties play the same history independent strategy if their situation is identical) and letting the strategy of the voter only depend on the current policy offers and the last rent payment⁸ seems intuitively plausible as the model is completely symmetric. Under these conditions, the equilibrium stated in Proposition 2 is the one with the lowest rent payment that the voter can achieve, as is shown by the following corollary:

Corollary 3. *There is no equilibrium with a rent $r_t < r^*$ if the voter's strategy only depends on rent extraction in the last period and policy positions of the parties (that is $v_t(h_t, p_{t+1}^y, p_{t+1}^x, r_t) = v_t(r_t, I_t, p_{y,t+1}, p_{x,t+1})$), while both parties play identical history-independent strategies (that is $p_{t+1}^I(h_t) = p_I$, $r_t(h_t) = r$ and $p_{t+1}^O(h_t, r_t, p_{t+1}^I) = p_{t+1}^O(r_t, p_{t+1}^I)$).*

From the voter's perspective, it would potentially enhance expected welfare if the candidates did not choose policy positions the way they actually do. Competition drives parties "almost" to convergence, but this is not necessarily in the voter's interest from an ex ante perspective. The reason is that if she has rather extreme preferences, both parties will offer a policy position that is rather centrist and she will suffer from the lack of choice. The expected per period utility of the voter before her preferences are revealed would increase if only one party chose a centrist position and the other an extreme one.

Bernhardt, Duggan, and Squintani (2009) show that such a lack of choice in policy provided by parties uncertain about the position of the median bliss point can make voters worse off. This may not be all that surprising in the light of the literature on spatial competition (Hotelling 1929).

Equilibrium rent extraction r^* is decreasing in π^* . The intuition is straightforward: The larger is π^* , the more likely it is that the incumbent party remains in office if it does not deviate. In addition, the incumbent party is also less likely to regain office once it is lost. Therefore, the rent that has to be paid to make the incumbent party willing to forgo the maximum rent R in favor of reelection decreases.

⁸ It is important to note that if the voter also plays a stationary strategy, no control over rent extraction is possible.

The voter is essentially playing the same lexicographic strategy as in the model without uncertainty in Section 2. However, she has to accept higher rent payments because there is no longer any guarantee that the incumbent party is reelected. Moreover, an incumbent party which loses office can regain office later, which also makes losing power less costly.

3.1 Two interesting cases

There are two interesting cases with intuitive results. First, there is the case of $\pi = 1$, which can only occur if $K = 1$; otherwise there would always be at least a small probability that the incumbent party loses. In this case, we are back to the setup of Section 2 and it indeed turns out that $r^* = \frac{1-\beta}{1}R = \bar{r}$. The incumbent party once more faces the choice between either remaining in office forever or stealing R once.

The second case is $\pi = 0.5$ which happens if and only if $F(b_m) = 0.5$. Because the probability that $b_t \leq b_m$ is exactly equal to the probability that $b_t > b_m$, incumbents have no possibility of increasing their chances of reelection to more than 50% even when they accept limited rent extraction. This is also what would happen if there were a continuous function of possible positions of the median voter. In this case, $r^* = \frac{1}{0.5\beta+1}R$ or $(1+0.5\beta)r^* = R$. The reason is that when the incumbent party does extract the maximum amount of rent R , he loses $0.5\beta r^*$ in the next period, but from then onwards, it has the same chance of being the incumbent party (50%) that it would have without any deviation from its strategy.

3.2 Discussion of the timing assumption

Without the assumption of the incumbent party moving first, a lexicographic strategy by the voters can only be consistent with an equilibrium if the parties randomize over policy. The reason is that the incumbent party would always like to take the same position as the opposition and win with certainty and therefore, the opposition must randomize over its position. A somewhat similar model has been solved by Aragonés and Palfrey (2002). In their setup, voters are not indifferent because candidates differ in an exogenously given policy attribute, so that the candidate who is preferred in this dimension wins if he can take the same policy position as the

other candidate. It should therefore be possible to solve an alternative model without the timing assumption and derive similar results with respect to accountability. However, finding optimal mixed strategies is not the focus of my paper.

4 Parties with policy preferences

In this section, I go back to a world without uncertainty. The model is the same as in Section 2 with the one difference that the expected utility of the parties $j \in \{x, y\}$ is from now on:

$$U_j = E_o \sum_{t=0}^{\infty} \beta^t (r_{t,j} - (p_t - b_j)^2), \quad (1')$$

with $b_x < b_m < b_y$. In other words, The parties' utility is now influenced by the policy that is implemented and party j is better off whenever policy is close to its bliss point b_j with $j \in \{x, y\}$. It is easy to check that giving parties policy preferences does not change the fact that the strategies given in Proposition 1 continue to constitute an equilibrium because by deviating and committing to a different policy than that preferred by the median voter, a party can never win the elections.

If parties have policy preferences of their own, the question arises how a party is able to commit to a policy in advance, but not to restrictions in rent seeking.⁹ As indicated before, a plausible answer is that parties commit to certain policies by running with certain candidates who are known to have preferences for the policy. If such a party wins an election, its candidate has no incentive to deviate from his preferred policy (although the average party member might still suffer from disutility from a deviation from his or her own policy bliss point).

However, with parties with policy preferences, there are now equilibria with lower rent payments that are not possible if the principle-agent problem and the electoral competition problem are treated separately. The reason is that a party can now be punished by allowing the other party to win with a position different from the bliss point of the median voter. To demonstrate this point there are three Examples that build on each other given in the Appendix. Example 1 is a special case of lexicographic voting. It is identical to the equilibrium given in Proposition 1 in Section 2 with the one difference that the incumbent is allowed to take the maximum

⁹ The fact that partisan parties potentially have a dynamic inconsistency problem with their policy announcements was first pointed out by Alesina (1988).

amount of rents and nonetheless reelected whenever the voters are indifferent with respect to policy. Strategies are identical, just $\bar{r} = R$ instead of $\bar{r} = (1 - \beta)R$. This example constitutes an equilibrium because the voters have no reason to punish the incumbent party in spite of the fact that it extracts the maximum rent level because the opposition party does not behave better once in office.

Example 1 is not very interesting in itself, but the threat to revert to it gives parties the possibility to win with a position that is different from the bliss point of the median voter b_m as is shown in Example 2. The idea is that the median voter will accept deviations from the median bliss points if she knows that if she does not the parties will punish her with the high rent equilibrium given in Example 1.

Finally, in Example 3 it is shown that the threat with the equilibrium given in example 2 makes it possible for voters to reach an equilibrium with a per period rent that is smaller than $\bar{r} = (1 - \beta)R$. As was shown in Corollary 1, there is no such equilibrium as long as policy does not enter the parties' utility functions. The reason that this is different with ideological parties is that voters can now punish parties that do not comply with policies that they dislike. Therefore, losing office becomes more costly and lower rent payments have to be accepted. In the example, it is assumed that the parameter values are such that parties refrain from any rent seeking in equilibrium.

The examples show that by separating backward-looking and forward-looking motives, some interesting strategic possibilities for voters might be overlooked. Voters are able to decrease rent payments further from \bar{r} without accepting a more ideological policy by threatening not only to vote for the opposition party, but to do so even when it does not offer the median voter's policy bliss point. This punishment is only credible because the voters end up in an even worse situation if they do not implement it.

Example 3 demands a larger degree of coordination among voters than what seems plausible to me. Moreover, even if Example 1 constitutes an equilibrium, it is not clear why voters who are as sophisticated as in Example 3 would not manage to switch to the more attractive equilibrium given in Proposition 1 instead once they are in the "bad" equilibrium of Example 1. There is no intuition how they could coordinate and commit to punish themselves for not punishing a party that deviates from the equilibrium given in Example 3. However, the analysis of this Section

nonetheless indicates that modeling accountability issues without any consideration of policy in models with partisan parties that derive utility from implemented policy could potentially lead to wrong conclusions.

5 Conclusion

This paper combines motives from prospective and retrospective voting in a single model. As long as there is certainty about the position of the median voter, I find that on the policy dimension where commitment is possible, the usual median voter results apply, while rent extraction by politicians is limited to the same degree as in a standard model without policy dimension. Voters achieve this by following a straightforward lexicographic voting strategy. All voters cast their ballot in favor of the party that they prefer in the policy dimension. Only when voters are indifferent between the parties they use the last periods rent extraction as a tiebreaker.

If there is uncertainty about the position of the median voter, voters cannot limit rent extraction to the same degree as in the certainty case, but accountability is not completely lost either. The reason is that even when the incumbent party complies with the voters demands for limited rent extraction, it will still lose office if the opposition party commits to a policy that is more attractive for the majority of voters. Because there is uncertainty which preferences the median voter will have when the parties choose their policy positions, there is no possibility for the incumbent party to avoid losing office with certainty. The best it can do is to choose a position that maximizes the probability that the majority of voters will prefer it. To make the ruling party willing to accept a limit on rents in spite of this, the voters have to allow it to acquire more of them in equilibrium.

Finally, if parties are not only interested in rents but also in policy, voters become new possibilities because they can now punish parties for excessive rent extraction by allowing the other party to win with a position that is worse than the median position. However, such equilibria demand a lot of sophistication by the voters. Lexicographic voting continues to be an equilibrium even with ideological parties and seems a more likely outcome of the game because of its intuitive appeal. The preliminary exploration of this Section nonetheless indicates that modeling accountability issues without any consideration of policy in models with partisan parties

that derive utility from implemented policy can potentially lead to precipitant conclusions. It can, so far, not be ruled out that more convincing equilibria than given in Example 3 can be constructed that also lead to rent payments that are lower than in Proposition 1. However, a detailed exploration of this question is left for future research.

Appendix A

Proofs Section 3

Proof Proposition 2. The single deviation principle states that it is sufficient to show that no player can increase his expected utility by a single deviation to prove that the given strategies constitute a subgame perfect Nash Equilibrium. The single deviation principle applies to an infinite game when the overall payoffs are a discounted sum of the per-period payoffs that are uniformly bounded. This applies to the game in Section 3.¹⁰

First, I show that the incumbent party as well as the opposition party maximize their chances of winning the elections if they follow the given strategies. For the case of $r_t > r^*$, the opposition party wins with certainty by taking the same policy position as the incumbent party $p_{t+1}^O = p_{t+1}^I$ and, in this way, it maximizes its election prospects. In case $r_t \leq r^*$, if $p_{t+1}^O = p_{t+1}^I$ and therefore $(p_{t+1}^I - b_{t+1})^2 - (p_{t+1}^O - b_{t+1})^2 = 0$, the opposition loses with certainty. If $-(b_k - b_{t+1})^2 + (b_{k-1} - b_{t+1})^2 < 0$, then $(b_k - b_{t+1})^2 - (b_y - b_{t+1})^2 < 0$ for all $y \leq k - 1$. Therefore, if $p_{t+1}^I = b_k$ and $r_t \leq r^*$, the opposition is at least as likely to win with $p_{t+1}^O = b_{k-1}$ as with any $p_{t+1}^O < b_{k-1}$. Similarly, if $-(b_k - b_{t+1})^2 + (b_{k+1} - b_{t+1})^2 < 0$, then $(b_k - b_{t+1})^2 - (b_y - b_{t+1})^2 < 0$ for all $y \geq k + 1$ and therefore, the opposition is at least as likely to win with $p_{t+1}^O = b_{k+1}$ than with any $p_{t+1}^O > b_{k+1}$. It follows that either $p_{t+1}^O = b_{k+1}$ or $p_{t+1}^O = b_{k-1}$ maximizes the probability of the opposition winning against $p_{t+1}^I = b_k$. Therefore, by the definition of $b^*(b_k)$, a policy that maximizes the probability of the opposition party winning is given by $p_{t+1}^O = b^*(p_{t+1}^I)$. It remains to be shown that $p_{t+1}^I = b_m$ maximizes the prospects of the incumbent party given the reply $b^*(p_{t+1}^I)$. By its definition and the voter's strategy, π^* gives the probability that the incumbent party wins when $r_t \leq r^*$, $p_{t+1}^I = b_m$ and $p_{t+1}^O = b^*(p_{t+1}^I)$. By the

¹⁰ See Fudenberg and Tirole (1991) for a formal statement of the single deviation principle.

definition of b_m , $F(b_{m-1}) < 0.5$ and $1 - F(b_m) \leq 0.5$. Therefore, $\pi^* \geq 0.5$. If $p_{t+1}^I \neq b_m$, the probability of winning for the opposition by choosing b_m itself is at least 0.5 and therefore, the probability that the opposition wins with $p_{t+1}^O = b^*(p_{t+1}^I)$ for $p_{t+1}^I \neq b_m$ cannot be smaller than 0.5. Hence, $p_{t+1}^I = b_m$ maximizes the chances of the incumbent party remaining in power, given the strategies of the other players and π^* gives the probability of reelecting the incumbent party in the given equilibrium.

Given the strategies of the other players, the voter will encounter the two policy offers b_m and $b^*(b_m)$ and the rent extraction r^* in all future periods. Therefore, maximizing the current period utility as she does by voting for the party she prefers if she is not indifferent is maximizing her expected utility.

Let V denote the value of being in office and W denote the value of being out of office given the strategies. The present expected value of being out of office is determined by the value of being in office and the equilibrium probability of winning the next elections, $1 - \pi^*$:

$$W = (1 - \pi^*)\beta V + \pi\beta W \implies W = \frac{\beta(1 - \pi^*)V}{1 - \pi\beta}. \quad (13)$$

It follows that $W < V$ and being in office is better than being out of office. From this, it directly follows that deviating once from the strategy cannot make the opposition that maximizes its chances of becoming the next incumbent party better off because a single deviation cannot change the future values of being in and out of office, respectively. Therefore, maximizing its probability of achieving V instead of W in the next period is optimal. The value of being the incumbent party depends on the equilibrium rent extraction r^* and the probability of being in and out of office, respectively, in the next period:

$$\begin{aligned} V &= r^* + \beta\pi^*V + \beta(1 - \pi^*)W = r^* + \beta\pi V + \beta(1 - \pi^*)\frac{\beta(1 - \pi^*)V}{1 - \pi\beta} \quad (14) \\ &= \frac{((1 - 2\pi)\beta + 1)}{(1 - \pi^*)\beta + 1}R + \beta\pi^*V + \beta(1 - \pi^*)\frac{\beta(1 - \pi^*)V}{1 - \pi\beta} \\ \implies V &= \frac{\pi^*\beta - 1}{\pi^*\beta + \beta^2 - \pi^*\beta^2 - 1}R. \end{aligned}$$

Given that the future value of being an incumbent party and in opposition, respectively, cannot be changed by a one-time deviation, it is clear that the incumbent party should maximize the rent payment for a given probability of reelection. There-

fore, any rent payment $r_t < r^*$ cannot make the incumbent party better off, because it decreases the rent as compared to a rent of r^* without changing the reelection probability. From the fact that the incumbent party loses the elections with certainty if $r_t > r^*$ independently of its chosen policy position, the only deviation that needs to be checked is $r_t = R$ in combination with any arbitrary policy position. The reason is that if the party were to be better off with extracting any rent r such that $r^* < r < R$, it must also be better off extracting R . The expected value of deviating in this way and then being in opposition in the next period is given by the sum R and the present value in opposition in the next period:

$$\begin{aligned}
 R + \beta W &= R + \beta \frac{\beta(1 - \pi^*)V}{1 - \pi^*\beta} & (15) \\
 &= R + \beta \frac{\beta(1 - \pi^*)}{1 - \pi^*\beta} \frac{\pi^*\beta - 1}{\pi^*\beta + \beta^2 - \pi^*\beta^2 - 1} R \\
 &= \frac{\pi^*\beta - 1}{\pi^*\beta + \beta^2 - \pi^*\beta^2 - 1} R = V.
 \end{aligned}$$

This gives the party the same utility V as following the strategy given in Proposition 2. Therefore, the incumbent party has no reason to deviate. None of the players is better off with a one time deviation and therefore, the given Proposition 2 constitutes a subgame perfect Nash Equilibrium. ■

Proof Corollary 3. Because $p_{t+1}^I(h_t) = p_I$, $r_t(h_t) = r$ and $p_{t+1}^O(h_t, r_t, p_{t+1}^I) = p_{t+1}^O(r_t, p_{t+1}^I) = p_{t+1}^O(r, p_I)$ for all t , the voter's decision can neither change her future policy choice nor future rent extraction. Therefore, in equilibrium, she votes for the party that offers the policy that is closest to her bliss point. Only if both parties offer the same policy position, voting for either party is consistent with an equilibrium. This gives the opposition party the possibility of being elected with a probability of at least $1 - \pi^*$ for any rent payment r_t and the policy position of the incumbent party by offering $p_{O,t+1} = b^*(p_{I,t+1})$. The opposition party maximizes its utility by maximizing the probability of being voted into office since being in office must be better than being out of office. Only in office is any rent extraction possible and the history-independence of the strategies implies that future rents are given by some constant level r . Let r^{\min} be the smallest rent payment that is consistent with an equilibrium. The value of being in office is given by $V(r^{\min}, \pi) = \frac{(1-\pi\beta)r^{\min}}{(1-\pi\beta)^2 - \beta^2(1-\pi)^2}$, where π is the probability of reelection of the incumbent party.

V is increasing in π , and the maximum π that is consistent with equilibrium is π^* . Therefore, the maximum V that is consistent with r^{\min} and an equilibrium is given by $V(r^{\min}, \pi^*) = \frac{(1-\pi^*\beta)r^{\min}}{(1-\pi^*\beta)^2 - \beta^2(1-\pi^*)^2}$. The second condition that must hold is $R \leq r^{\min} + \beta\pi V(r^{\min}, \pi^*) + \beta(1-\pi) \frac{\beta(1-\pi)r^{\min}V(r^{\min}, \pi^*)}{1-\pi\beta}$, because otherwise the incumbent party would be better off taking R and losing office. This condition can only hold if $r^{\min} \geq r^*$, hence it follows that $r^* = r^{\min}$. ■

Appendix B

Examples Section 4

Example 1 (High rent equilibrium). *The candidates play:*

$$\begin{aligned} p_{t+1}^j &= b_m \text{ for } j \in \{x, y\} \text{ and all } t, \\ r_t &= R \text{ for all } t. \end{aligned} \quad (16)$$

The voters play:

$$v_t^i = \begin{cases} x & \text{if } (p_{t+1}^x - b^i)^2 - (p_{t+1}^y - b^i)^2 < 0 \\ y & \text{if } (p_{t+1}^x - b^i)^2 - (p_{t+1}^y - b^i)^2 > 0 \\ I_t & \text{if } (p_{t+1}^x - b^i)^2 - (p_{t+1}^y - b^i)^2 = 0 \end{cases} \quad \text{in all } t. \quad (17)$$

And therefore in equilibrium:

$$\begin{aligned} I_t &= I_0 \text{ in all } t, \\ p_t &= b_m \text{ in all } t \geq 1, \\ r_r &= R \text{ in all } t. \end{aligned} \quad (18)$$

This example constitutes an equilibrium because the voters have no reason to punish the incumbent party in spite of the fact that it extracts the maximum rent level because the opposition does not behave better when in office.

Building on the fact that there is an equilibrium with high rents, an equilibrium with a party deviating from the median position becomes possible, because voters can be "punished" with high rent payments if they do not accept the deviation:

Example 2 (Deviation from median policy equilibrium). *Assume that $(b_m - b_j)^2 < R - \bar{r}$ for $j \in \{x, y\}$. Let t_s be the period in which the incumbent party $I_{t_s} \neq I_0$ for*

the first time (if it never happens $t_s = \infty$). Then, the following strategies constitute an equilibrium:

$$\begin{aligned}
 p_{t+1, I_0} &= b_{I_0} \text{ in all } t < t_S, \\
 p_{t+1, O_0} &= b_m \text{ in all } t < t_S, \\
 r_t &= \bar{r} \text{ in all } t < t_S, \\
 p_{t+1}^j &= b_m \text{ for } j \in \{x, y\} \text{ in all } t \geq t_S, \\
 r_t &= R \text{ in all } t \geq t_S.
 \end{aligned} \tag{19}$$

The voters play:

$$\begin{aligned}
 v_t^i &= \begin{cases} I_t & \text{if } (p_{t+1}^I - b^i)^2 - (p_{t+1, O_t} - b^i)^2 \leq R - \bar{r} \\ O_t & \text{if } (p_{t+1}^I - b^i)^2 - (p_{t+1, O_t} - b^i)^2 > R - \bar{r} \end{cases} \text{ in } t < t_S \\
 v_t^i &= \begin{cases} x & \text{if } (p_{t+1}^x - b^i)^2 - (p_{t+1}^y - b^i)^2 < 0 \\ y & \text{if } (p_{t+1}^x - b^i)^2 - (p_{t+1}^y - b^i)^2 > 0 \\ I_t & \text{if } (p_{t+1}^x - b^i)^2 - (p_{t+1}^y - b^i)^2 = 0 \end{cases} \text{ in } t \geq t_S
 \end{aligned} \tag{20}$$

And therefore in equilibrium:

$$\begin{aligned}
 I_t &= I_0 \text{ in all } t, \\
 p_t &= b_{I_0} \text{ in all } t \geq 1, \\
 r_t &= \bar{r} \text{ in all } t.
 \end{aligned} \tag{21}$$

This example builds on Example 1. The high rent equilibrium in Example 1 can be used to "punish" the voters for not reelecting the incumbent party. The majority of voters are better off accepting the first incumbent party implementing its favorite policy compared to accepting a higher rent payment forever in combination with the median position as long as the condition $(b_m - b_j)^2 < R - \bar{r}$ for $j \in \{x, y\}$ holds. If the condition holds, the median voter is better off and so is also either every voter to the left or to the right of the median voter, and therefore the majority of voters.

Building on Equilibrium 2, I can now show that there is also an equilibrium without any rent payments. This is the case because if the incumbent party deviates by appropriating positive rents, he can be punished with policies that make him worse off than the median position by allowing the opposition to win with its own bliss point instead of the median position as in Example 2:

Example 3 (An equilibrium without any rents). Let t_{s_1} be the period in which the incumbent party $I_{t_s} \neq I_0$ for the first time and t_{s_2} when incumbency switches a second time. If incumbency switches at most once, $t_{s_2} = \infty$, if it never switches, $t_{s_1} = t_{s_2} = \infty$. In addition, I assume that $R < \beta^2 \frac{(b_y - b_x)^2 - (b_j - b_m)^2}{1 - \beta}$ for $j \in \{x, y\}$ and that $(b_m - b_j)^2 < R - \bar{r}$ for $j \in \{x, y\}$. Then, the following strategies constitute an equilibrium:

$$\begin{aligned}
p_{t+1, I_0} &= b_m \text{ in all } t < t_{S1}, & (22) \\
p_{t+1, O_0} &= b_m \text{ in all } t < t_{S1}, \\
r_t &= 0 \text{ in all } t < t_{S1}, \\
p_{t+1}^I &= b_{I_t} \text{ in all } t_{S1} \leq t < t_{S2}, \\
p_{t+1, O_t} &= b_m \text{ in all } t_{S1} \leq t < t_{S2}, \\
r_t &= \bar{r} \text{ in all } t_{S1} \leq t < t_{S2}, \\
p_{t+1}^j &= b_m \text{ for } j \in \{x, y\} \text{ in all } t \geq t_{S2}, \\
r_t &= R \text{ in all } t \geq t_{S2}.
\end{aligned}$$

The voters play:

$$\begin{aligned}
v_t^i &= \begin{cases} x \text{ if } (p_{t+1}^x - b^i)^2 - (p_{t+1}^y - b^i)^2 < 0 \\ y \text{ if } (p_{t+1}^x - b^i)^2 - (p_{t+1}^y - b^i)^2 > 0 \\ I_t \text{ if } (p_{t+1}^x - b^i)^2 - (p_{t+1}^y - b^i)^2 = 0 \text{ and } r_t = 0 \\ O_t \text{ if } (p_{t+1}^x - b^i)^2 - (p_{t+1}^y - b^i)^2 = 0 \text{ and } r_t > 0 \end{cases} \text{ in } t < t_{S1} \\
v_t^i &= \begin{cases} I_t \text{ if } (p_{t+1}^I - b^i)^2 - (p_{t+1, O_t} - b^i)^2 \leq R - \bar{r} \\ O_t \text{ if } (p_{t+1}^I - b^i)^2 - (p_{t+1, O_t} - b^i)^2 > R - \bar{r} \end{cases} \text{ in } t_{S1} \leq t < t_{S2} & (23) \\
v_t^i &= \begin{cases} x \text{ if } (p_{t+1}^x - b^i)^2 - (p_{t+1}^y - b^i)^2 < 0 \\ y \text{ if } (p_{t+1}^x - b^i)^2 - (p_{t+1}^y - b^i)^2 > 0 \\ I_{t-1} \text{ if } (p_{t+1}^x - b^i)^2 - (p_{t+1}^y - b^i)^2 = 0 \end{cases} \text{ in } t \geq t_{S2}
\end{aligned}$$

And therefore in equilibrium:

$$\begin{aligned}
I_t &= I_0 \text{ in all } t, & (24) \\
p_t &= b_m \text{ in all } t \geq 1, \\
r_t &= 0 \text{ in all } t.
\end{aligned}$$

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