Essays on Public Macroeconomic Policy

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Abstract

This thesis consists of three essays on public policy in the macroeconomy.

Government Policy in the Formal and Informal Sectors quantitatively investigates the interaction between the firms' choice to operate in the formal or the informal sector and government policy on taxation and enforcement. Informality is here defined as unregistered firms in legal activities. Quantitative theory is developed, in general equilibrium, using the main determinants of informality: taxes, enforcement, and regulation. These features are incorporated in a model of heterogeneous firms, where firms differ in their productivities. A static version of Ghironi and Melitz's (2005) industry model is used to show that firms with lower productivity endogenously choose to operate in the informal sector. I use cross-country data on taxes, measures of informality, and measures of regulation (entry and compliance costs, red tape, etc) to back out how high the enforcement levels must be country by country to make the theory match the data. The model quantitatively accounts for the keys aspects in the data and allows me to back out country-specific enforcement levels. Some policy reforms on taxation and enforcement are analyzed. The result is that the welfare gains can be fairly large. I compute the shadow value of decreasing regulation and perform some counterfactual experiments. I find that welfare gains from reducing regulation are almost twice those computed for the policy reform. Finally, distortions associated with informality account for a factor of 1.5 of the output per capita difference between the richest and the poorest countries.

Determinants of Capital Intensive and R&D Intensive Foreign Direct Investment studies the determinants of capital intensity and technology content of foreign direct investment, an important economic driving force for developing countries. For this purpose, we use sectoral industry data on U.S. foreign investment abroad, and data on host countries' institutional characteristics, like investment climate, protection of property rights, labor standards and constitutional arrangements. Our regressions show that better protection of property rights has a significant positive effect on R&D but not on capital intensive capital flows. There is evidence that an increase in workers' bargaining power results in a reduction of capital and technologically intensive foreign investment. And although the evidence with respect to constitutional arrangements is not very strong, presidential regimes appear to be less able

than parliamentary ones to deliver policies attracting R&D intensive capital flows. This is consistent with recent research on the effects of constitutional arrangements on economic growth.

Ambiguity Aversion, the Equity Premium, and the Welfare Costs of Business Cycles examines the potential importance of consumer ambiguity aversion for asset prices and how consumption fluctuations influence consumer welfare. First, considering a simple Mehra-Prescott-style endowment economy with a representative agent facing consumption fluctuations calibrated to match U.S. data, we study to what extent ambiguity aversion can deliver asset prices that are consistent with data: a high return on equity and a low return on riskfree bonds. For some configurations of preference parameters—a discount factor, a degree of relative risk aversion, and a measure of ambiguity aversion—we find that it can. Then, we use these parameter configurations to investigate how much consumers would be willing to pay to reduce endowment fluctuations to zero, thus delivering a Lucas-style welfare cost of fluctuations. These costs turn out to be very large: consumers are willing to pay over 10% of consumption in permanent terms.

Aos meus pais

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Stockholm, April 2007

José Maurício Prado, Jr.

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Chapter 1

Introduction

This thesis consists of three self-contained essays on public policy in the macroeconomy. The first two papers address important questions related to development policy, specifically, about the phenomenon of informality and the attraction of foreign direct investments (FDI). The last paper is about the welfare costs of business cycles.

In recent years, informality has increased not only in developing countries, but also in Europe and the US, according to estimates by Schneider (2006). Moreover, informal production is a major component of economic activities in developing economies and therefore a subject of great importance in the public policy debates in these countries. The first essay in this thesis contributes to those debates.

As one of the ways of promoting long-term growth, governments, particularly in developing countries, try to attract foreign direct investments. The second essay studies the host countries' characteristics for attracting those highly technologically and capital intensive investments.

A third policy question analyzed in this thesis is what can be done to eliminate undesirable fluctuations in economic activity. In order to be able to better evaluate different government policies aimed at smoothing business cycles, we ask what are the welfare costs of economic fluctuations in an economy populated by agents with ambiguity aversion, a type of non-standard preference.

The main theme of this thesis is to quantitatively assess some aspects of public macroeconomic policy. Below, I proceed to summarize each of the individual essays.

Chapter 2 (Government Policy in the Formal and Informal Sectors) quantitatively investigates the interaction between the firms' choice to operate in the formal or the informal sector and the government policy on taxation and enforcement. Informality, in this essay, is defined as unregistered firms in legal activities.

Quantitative theory is developed, in general equilibrium, using the main determinants of informality: taxes, enforcement, and regulation. These features are incorporated in a model of heterogeneous firms, where firms differ in their productivities. A static version of Ghironi and Melitz's (2005) industry model is used to show that firms with lower productivity endogenously choose to operate in the informal sector.

I use cross-country data on taxes, measures of informality, and measures of regulation (entry and compliance costs, red tape, etc) to back out how high the enforcement levels must be country by country to make the theory match the data. The model quantitatively accounts for the key aspects in the data and allows me to back out country-specific enforcement levels.

Some policy reforms on taxation and enforcement are analyzed. The result is that the welfare gains can be fairly large. I compute the shadow value of decreasing regulation and perform some counterfactual experiments. Thus, I find that the welfare gains from reducing regulation are almost twice those computed for the policy reform. Finally, distortions associated with informality account for a factor of 1.5 of the output per capita difference between the richest and the poorest countries.

Chapter 3 (Determinants of Capital Intensive and R&D Intensive Foreign Direct Investment) studies the determinants of capital intensity and technology content of foreign direct investment. For this purpose, we use sectoral industry data on U.S. foreign investment abroad and data on host countries' institutional characteristics, like investment climate, protection of property rights, labor standards and constitutional arrangements.

Capital flows have increased spectacularly in the last two decades. In particular, foreign direct investment (FDI) has been growing three times as fast as total investment between 1980 and 2000. Over this period, there has also been a change in the nature of FDI flowing to developing countries. Previously, foreign investment was concentrated to the extraction of natural resources for shipment abroad. Nowadays, as developing countries become wealthier, investment diversifies into production of consumer goods for their local markets. The increasing size and variety of these flows has made both economists and policy makers interested in understanding their determinants and effects. Research, on the one hand, tries to understand how FDI affects productivity and growth, or income inequality and the environment. On the other hand, many studies try to pinpoint the host and source country and industry characteristics behind FDI flows.

A question of interest among developing countries is what policies are better at atracting much needed capital and new technologies. A number of studies has found that institutional quality to be a positive determinant of FDI (and thus, in particular, corruption has a negative effect), higher taxation reduces capital flows, and more protection of intellectual property rights attracts high-tech investment. The data shows mixed results on other dimensions of policy. For example, Rodrik (1996) found that countries with higher labor standards attract more FDI, an effect that seems to disappear when controlling for political risk (see Cho (2003)). And measures of labor costs and workers bargaing power are found to have a negative effect on FDI (Smarzynska and Spatareanu (2004) and Cooke (1997)).

Our regressions show that better protection of property rights has a significant positive effect on R&D, but not on capital intensive capital flows. There is evidence that an increase in workers' bargaining power results in a reduction of capital and technologically intensive foreign investment. And although the evidence with respect to constitutional arrangements is not very strong, presidential regimes appear to be less able than parliamentary ones to deliver policies attracting R&D intensive capital flows. This is consistent with recent research on the effects of constitutional arrangements on economic growth.

Chapter 4 (Ambiguity Aversion, the Equity Premium, and the Welfare Costs of Business Cycles) examines the potential importance of consumers' ambiguity aversion for how consumption fluctuations influence consumer welfare. Ambiguity aversion, which is a way of formalizing preferences that are consistent with the Ellsberg paradox, captures a form of violation of Savage's axioms of subjective probability. Instead, consumers behave as if a range of probability distributions is possible and as if they are averse toward the "unknown". With the typical parameterized representation of ambiguity aversion, consumers have minmax preferences, thus maximizing utility based on the worst possible belief. Thus, in an economy with a small amount of randomness, there are first-order effects on utility if there is ambiguity about this randomness. Thus, ambiguity aversion is in contrast to the standard model, where risk aversion leads to second-order effects on utility.

We first consider a simple Mehra-Prescott-style endowment economy with a representative agent facing consumption fluctuations calibrated to match U.S. data. Thus, we study to what extent ambiguity aversion can deliver asset prices that are consistent with data: a high return on equity and a low return on riskfree bonds. For some configurations of preference parameters—a discount factor, a degree of relative risk aversion, and a measure of ambiguity aversion—we find that it can. Then, we then use these parameter configurations to investigate how much consumers would be willing to pay to reduce endowment fluctuations to zero, thus delivering a Lucas-style welfare cost of fluctuations. These costs turn out to be very large: consumers are willing to pay over 10% of consumption in permanent terms.

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Chapter 2

Government Policy in the Formal and Informal Sectors*

1 Introduction

The aim of this essay is to quantitatively investigate the interaction between firms' choice to operate in the informal sector and government policy on taxation and enforcement, given a country's institutional characteristics and regulation. I follow Schneider and Enste (2000) in defining informality as "unreported income from the production of legal goods and services, either from monetary or barter transactions, hence all economic activities that would generally be taxable were they reported to the tax authorities". Emphasis here should be given to the fact I am only considering legal activities, even though the non-compliance with taxes and regulations or the lack of proper registration (when mandatory) would typify them as illegal. The size of the informal sector[1] measures the value of the production under informality. In recent years, informality has not only increased in developing countries, but also in Europe and the US, according to estimates by Schneider (2006). Moreover, informal

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[1] Throughout this essay, I interchangeably use the terms: "informal economy", "shadow economy", "underground economy", "grey economy", including its variants with "sector", instead of "economy", as referring to the same concept.

production is a major component of economic activities in developing economies and therefore a subject of great importance in the public policy debates in these countries. Thus, this essay also contributes to those debates.

The consequences of informality include, but are not limited to, two main problems concerning the government and the firms. The first is a fiscal one. Assuming the existence of public expenditures to be financed from tax collection, a smaller tax base implies a higher tax burden on formal firms. The second consequence implies that firms in the informal sector have no or less access to the courts of law. Moreover, they may be infringing regulatory, labor-market and product-market obligations. I take the view that most of these obligations or regulations are socially inefficient.[2] Therefore, I consider low regulation as an indicator of a country's institutional quality. Making a parallel to what Djankov et al (2002) name the "tollbooth" view of the public choice theory of regulation, countries with better institutional quality are those where bureaucrats are less able to extract rents or bribes through inefficient regulation.[3]

Traditionally, taxation has been blamed for the size of the informal sector. However, it cannot explain the full extent of the phenomenon of informality. An explanation should also rely on the monitoring or enforcement against firms in the informal sector, and on regulation or institutional quality.[4] Hernando de Soto's *The Other Path* (1989) is very vocal about this new strand of literature. Following de Soto's work, many papers have attempted to qualitatively explain those mechanisms involved in the determination of informal economies. However, few have quantified the effects.[5] In this essay, I develop quantitative theory using those main determinants of informality. In doing so, I am also able to analyze general equilibrium effects. My specific interest is in analyzing the elasticity of informality with respect to enforcement, taxation and regulation. I also perform some policy reforms, under

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[2] Farrell (2004) gives a more detailed description of these regulations. Among them, there are some that are socially-efficient. Those socially-efficient regulations can be considered as the benefits of formality in an economy.

[3] The government in my model can be interpreted à la Banerjee (1997), where there is a conflict of interest between the government and bureaucrats. The government maximizes household's utility at the same time as bureaucrats want to use red tape (or bad regulation).

[4] Friedman, Johnson, Kaufmann, and Zoido-Lobaton (2000) compare different views and dismiss the taxation view.

[5] Antunes and Cavalcanti (2006) and Fortin, Marceau and Savard (1997) are among those few. However, they do not focus on the government policies studied in the present work.

a public finance perspective, exploring these elasticities.

Before presenting the model, I briefly review the literature on informality. As mentioned before, the informal economy is the subject of a vast literature. A thorough review of this literature can be found in Schneider and Enste (2000).[6] Rausch (1991), followed by Fortin, Marceau and Savard (1997), Amaral and Quintin (2006), Antunes and Cavalcanti (2006), Paula and Scheinkman (2006) and many others, analyze informal economies using the "span of control" model of Lucas (1978). In these models, agents are heterogeneous in their managerial abilities.[7] In an alternative approach, I model firms with different productivities. Since my focus is not on occupational choice, a model with firms seems more appropriate. Fortin, Marceau and Savard (1997) and Sarte (2000) model firms closely to the model in the current work. However, the first paper considers a homogeneous good (while I have differentiated ones), while Sarte (2000) considers both informal and formal firms, equally dividing the production in a specific industry. In my model, a firm with productivity z produces a corresponding differentiated variety z and all firms with the same productivity level are in the same sector (formal or informal).

Rausch (1991) was probably the first to formally model the informal sector. However, he resorts to a minimum wage policy for large firms in order to create the informal sector. In Fortin, Marceau and Savard (1997), there is also a minimum wage. My model creates informality without resorting to minimum wage and still smaller firms endogenously choose to become informal. Azuma and Grossman (2003) provide a theoretical model of the informal sector where informality exists because firms' productive endowments are not perfectly observable. Then, the government cannot optimally extract resources from firms.

The model presented here does not focus on tax evasion per se but, naturally, when a firm is in the informal sector, it is evading taxes. A huge literature has dealt with tax evasion. Allingham and Sandmo (1972) is the paper which first modeled tax evasion. Andreoni, Erard and Feldstein (1998) and Niepelt (2005) are recent contributions in the area.

I consider an economy which consists of two sectors: a formal and an informal one. The sectors are structured in monopolistic competition à la Dixit-Stiglitz,

[6] An even more recent survey of the literature can be found in Antunes and Cavalcanti (2006).

[7] In the case of Fortin, Marceau and Savard (1997), the agents are, in fact, firms with different managerial abilities.

with heterogeneous firms which draw a productivity level from some given probability distribution. There are no firms with different productivities producing the same variety or different goods being produced by firms with equal productivity. The model of monopolistic competition implies that the representative household consumes all varieties. My modeling strategy closely follows the static version of the industry model of Ghironi and Melitz (2005) and Melitz (2003), both based on Hopenhayn (1992).

There exists a fixed regulation cost κ in the formal sector.[8] Further, firms in the formal sector also pay a proportional tax on production at a constant rate τ.

Another choice for the firm is to operate in the informal sector. In this case, there is no fixed cost. However, there is an enforcement cost proportional to output. This cost is the result of the probability of being caught in informality and the corresponding fine (or punishment). Fortin, Marceau and Savard's (1997) interpretation of this kind of cost is that firms engage in some costly activity to avoid being caught and pay the penalty. It is assumed that firms are better off paying the cost than risking being caught. I model this enforcement mechanism as a constant rate e on the total production of informal firms. A third interpretation is that the enforcement technology of the government destroys a fraction e of the output of informal firms. Table 2.1 summarizes the costs faced by firms in each sector.

Table 2.1: Taxes and costs associated with economic activities

	Formal Sector	Informal Sector
Regulation/Compliance cost	κ	0
Tax rate	τ	0
Enforcement rate	0	e

The government relies on taxation on formal businesses and the net revenue from enforcement. It spends its revenue on exogenous government expenditures and on the costs of enforcing informal firms. The formal sector contributes to revenue, but generates a waste in the economy, due to regulation. Thus, regulation creates a distortion in the formal sector. Since government expenditures are given, a smaller formal sector would increase the tax burden on formal firms. At the same time, enforcement reduces informality, but is costly, thereby creating another distortion

[8] We may interpret κ as a cost of complying with the formal sector, e.g. set-up costs, registration costs **and** resources spent on paper work.

in the economy. The government task is to balance these distortions on the two sectors and raise enough revenue to finance its expenditures.

I use cross-country data on taxes, measures of informality, and measures of regulation (entry and compliance costs, red tape, etc) to back out how high the enforcement levels must be country by country to make the theory match the data. The main output of this quantitative exercise consists of three things: first, the measures of enforcement can be compared with (indirect) measures of enforcement differences across countries, as a sort of "test" of the model. Second, I can ask a set of quantitative public-finance questions, for example concerning policy reforms on taxation and enforcement rates and the shadow dead-weight-loss of regulation costs. Third, I can use the model to account for how much informality reduces output per capita across countries.

The model quantitatively accounts for the degree of informality and other key aspects, such as size of government and regulation costs. The computed enforcement positively correlates with measures of tax compliance. Moreover, enforcement is positively correlated with regulation and government expenditures and, as expected, it is negatively correlated with the size of the informal sector. There is some scope for policy reforms (using e and τ as instruments). In general, most countries would do better to decrease informality, although some would benefit from increasing it. In both cases, the welfare gains can be fairly large. Countries benefiting the most are those with lower regulation costs. This suggests that reducing regulation costs is a more effective policy for increasing private consumption and reducing informality. In particular, since regulation is a distortion in the formal sector, it should be zero. However, the model here takes regulation as given and its determination are outside the scope of this essay. Nonetheless, the model allows us to measure what countries would gain from decreasing regulation (κ). This is done by computing the shadow value of decreasing regulation. Thus, we do not know how much it would cost to allow this decrease, but the model allows us to compute the benefits. Finally, I perform some counterfactual experiments by reducing the regulation costs. As a by-product of the model, I can account for how much the distortions associated with informality reduce output per capita across countries. I found that these distortions account for a factor of 1.5 of the output per capita difference between the richest and the poorest countries.

The paper is organized as follows. The next section presents a discussion about the relationship between regulation cost and informality. Section 3 presents the model, the definition and the characterization of equilibrium as well as some comparative statics. The following section brings the baseline calibration and the quantitative assessment of the model. Section 5 considers some policy reforms. First, I analyze the reallocation of taxes and enforcement and second, the shadow value of regulation and a counterfactual experiment are analyzed. In section 6, the model accounts for income differences across countries. Some concluding remarks are presented in section 7.

2 Regulation cost and informality

In this section, I focus on the relationship between regulation cost and informality. The first objective is to gather data. Djankov et al (2002) present new data on the regulation of entry for 85 countries. They calculate the official costs and the time legally required to begin operating a firm in these countries. I refer to them for detailed explanations of the procedures. They report both the monetary cost for fees and the time spent. The figure is measured as fraction of each country's per capita GDP. It seems that the data on per capita GDP from the World Bank's (2006) World Development Indicator dataset has suffered some revisions after it was first released. Some of the changes in the per capita GDP data were substantial: some countries had two-digit percentage point changes from the previous figures.

Since I have an interest in using the best data available to perform the quantitative assessment, I decided to recompute the total cost of regulation (fees + time) using updated World Bank data on the countries' per capita GDP in 1999 in current US$. The new total costs and per capita GDP in 1999 figures are shown in table 2.2. Data on the size of the informal economy as a percentage of formal GDP in 1999/2000, estimated by Schneider (2006), is also included in the table. I refer to his paper for a detailed explanation of how the size of the informal economy is estimated. In short, the informality is computed by indirect measures, like money or electricity demand and latent estimation methods using the DYMIMIC (dynamic multiple-indicators multiple-causes) model.

As can be noted from table 2.2, there is a large variation in the three variables across the selected countries. Another point worth mentioning about these figures is that the size of the informal sector is non-trivial even in developed economies, ranging between 8.6% and 25% of formal GDP. The correlation between per capita GDP and the size of the informal sector is -0.67. Loayza (1996) reports a similar correlation in his estimation of informality among Latin American countries. As a matter of fact, it is possible to group the countries in the table into categories relative to their level of informality, so as to observe similarities in the level of development of countries in each category. The first category would include "low informality" countries, with an informal sector of up to 15%. Examples of such countries are Switzerland, the U.S. and Japan. A second category would be "medium informality" countries with informal sectors of between 15% and 30%. Countries in this category include, for example, Italy, Spain, and Sweden. The next category would group countries with high informality (between 30% and 50%). These countries consist of most of Latin America and some African and Asian countries. Finally, the last category would be formed by those countries with very high informality, where the informal sector is larger than the formal sector. These patological cases include very poor economies in Africa and Asia.

Another factor, not shown in the table, but reported by Schneider (2006), is the growth of the informal sector, occurring both in developing and developed economies. A further relevant point is the correlation between the size of the informal sector (as a percentage of the formal sector) and the regulation costs. Figure 2.1 can better illustrate this point. The result is that higher regulation costs are associated with larger sizes of the informal sector. The OLS regression coefficient of this relationship is 10.13 and it is significant at the 1% level.[9]

3 The model

3.1 Basics

Firms There is a continuum of firms of measure 1. Each firm produces a differentiated good indexed by $z \in \Omega$. Firms are heterogeneous as they produce with different technologies, z, given by a distribution probability $F(z)$ with support $[z_{\min}, \infty)$ and

[9] The constant coefficient is 25.49 and is also significant at the 1% level.

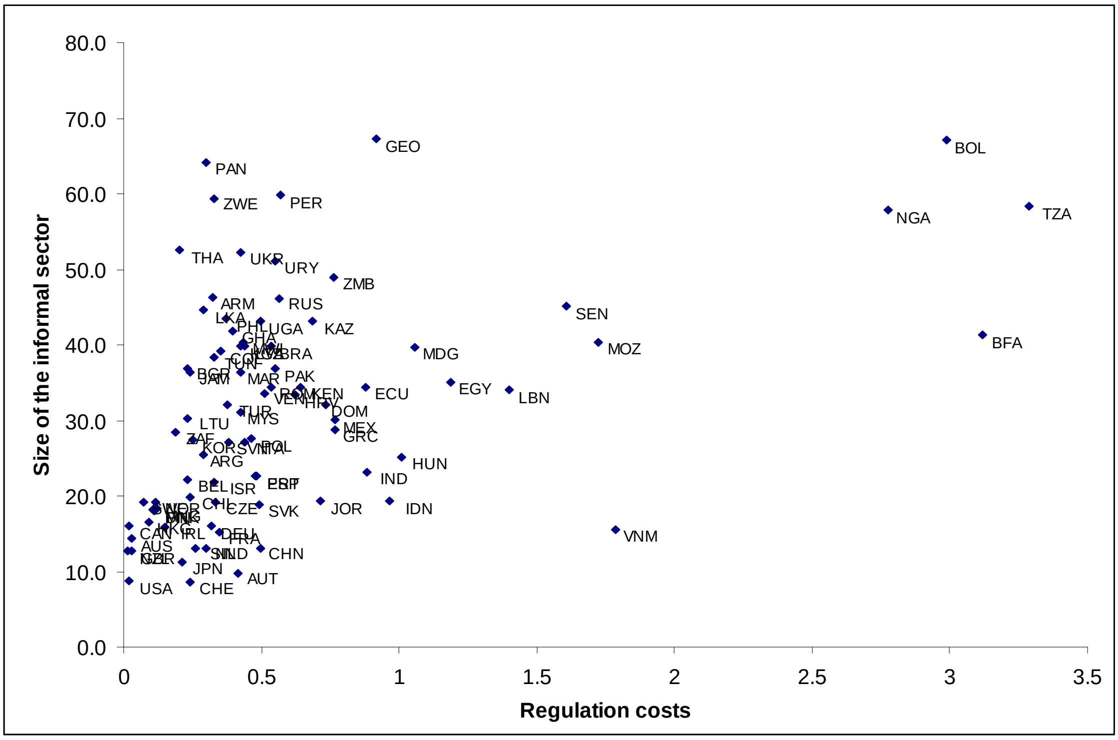

Figure 2.1: Regulation costs and the size of the informal sector

$z_{\min} > 0$. A firm with productivity z produces λz units of output per unit of labor, where λ is just a parameter (λ can be interpreted as aggregate labor productivity).[10] Productivity differences across firms then translate into differences in the unit cost of production ($w/\lambda z$). The production function can be written as

$$y(z) = \lambda z l(z), \tag{2.1}$$

where $l(z)$ is the labor employed by the firm with productivity z.

Firms can choose to operate in the formal or the informal sector. Producing in the formal sector requires the payment of a (fixed) regulation cost κ (measured in terms of labor) and the payment of a proportional tax rate τ on the firm's total output $y(z)$. Firms in the informal sector pay a proportional enforcement tax e on

[10] To clarify, z indexes both the firm's variety and its productivity. Therefore, a firm with productivity z produces a corresponding variety z.

their output. The profit maximization problem of a firm with productivity z is

$$\max_{p(z)} \Pi(z) \equiv (1 - \tau \mathcal{I}_F - e\mathcal{I}_I)p(z)y(z) - wl(z) - \mathcal{I}_F w\kappa, \tag{2.2}$$

where $\mathcal{I}_J$ is an indicator function that takes a value equal to 1 if the firm is operating in sector $J = F, I$ (formal or informal, respectively).

Representative Household The economy is populated by a unit mass of atomistic households. The representative household owns all firms and supplies L units of labor inelastically in each period at real wage w. She maximizes the utility from the composite household's consumption (C) and the level of publicly provided goods (G):

$$U \equiv u(C, G), \tag{2.3}$$

where $C \equiv \left(\int_{z\in\Omega} c(z)^{(\theta-1)/\theta} dz\right)^{\theta/(\theta-1)}$, $\theta > 1$ is the elasticity of substitution across goods and $c(z)$ is the household's consumption of good z. G takes the same aggregator form as C. Then, $G \equiv \left(\int_{z\in\Omega} g(z)^{(\theta-1)/\theta} dz\right)^{\theta/(\theta-1)}$. The utility function u is increasing in both arguments. The budget constraint of the representative household is:

$$C \leq wL + \Pi_F + \Pi_I. \tag{2.4}$$

The household earns labor income wL plus the profits in the formal (Π_F) and informal sectors (Π_I). She spends her total income buying the composite consumption C.

Government The government collects taxes and enforcement penalties. Enforcement generates a revenue $E \equiv eY_I$; however, there is a cost $\Psi(E)$ (with $\Psi'(E) > 0$) to exert this enforcement. The government spends its net revenue on the purchase of the publicly provided good G. The government budget constraint is:

$$G + \Psi(E) \leq \tau Y_F + eY_I, \tag{2.5}$$

where Y_J is total output in sector J.

Resource Constraint Define Y as total output. Then, we can write the resource constraint of this economy as:

$$Y = Y_F + Y_I = C + G + \Psi(E). \tag{2.6}$$

3.2 Prices and profits

Individual good demand Then, the individual demand for good z is $y(z)$, such that

$$y(z) = Y\left[p_J(z)\right]^{-\theta}, \tag{2.7}$$

where $p_J(z)$ is the price charged by a firm with productivity z in sector J.[11]

Prices All firms face a residual demand curve with constant elasticity θ in the output market, and they set flexible prices that reflect the same proportional markup $\theta/(\theta-1)$ over the marginal costs given by

$$p_J(z) = \frac{\theta}{(\theta-1)}\frac{1}{(1-\tau\mathcal{I}_F - e\mathcal{I}_I)}\frac{w}{\lambda z}. \tag{2.8}$$

The above price is derived from the firms' profit maximization problem (2.2) subject to individual demand (2.7). The derivation is in the appendix.

Given the price function (2.8), we can write a relation between the price in both sectors:

$$p_F(z) = \frac{(1-e)}{(1-\tau)}p_I(z). \tag{2.9}$$

Prices in the formal sector are proportionally higher to those in the informal sector if enforcement is lower or taxes are higher.

Profits Now that we have derived the equilibrium price, we can express the profit of a firm with productivity z as:

$$\Pi_J(z) = \frac{(1-\tau\mathcal{I}_F - e\mathcal{I}_I)}{\theta}\left[p_J(z)\right]^{1-\theta}Y - \mathcal{I}_F w\kappa. \tag{2.10}$$

This allows us to study how profits change with productivity

$$\frac{\partial \Pi_J}{\partial z} = (1-\tau\mathcal{I}_F - e\mathcal{I}_I)\frac{\theta-1}{\theta}\left[p_J(z)\right]^{1-\theta}Yz^{-1} > 0. \tag{2.11}$$

Since θ must be greater than 1 and so far as $z \geq 0$, which I assume, profits are monotonically increasing in productivity, as should be expected.

Now let us check the second derivative:

$$\frac{\partial^2 \Pi_J}{\partial z^2} = (1-\tau\mathcal{I}_F - e\mathcal{I}_I)\frac{(\theta-2)(\theta-1)}{\theta}\left[p_J(z)\right]^{1-\theta}Yz^{-2} \gtreqless 0 \text{ if } \theta \gtreqless 2. \tag{2.12}$$

[11] See **the** appendix for the derivation of individual demand.

The profit function can be concave or convex in z, depending on the level of θ, i.e. the elasticity of substitution across goods. When goods are highly complementary ($1 < \theta < 2$), the function is concave with respect to z, whereas the profit function is convex when goods are more substitutable ($\theta > 2$).

3.3 Definition of equilibrium

Now that the model has been described, I proceed to define and verify the existence of the equilibrium for *exogenous* policy. Before, let me state some assumptions. If $\kappa = 0$, the problem is trivial. There is a bang-bang solution, where all firms choose the formal (informal) sector if and only if $e > (<)\ \tau$. This can be seen more clearly by checking the profit expression in (2.2). To make the problem more interesting, I assume that $\kappa > 0$:

Assumption 1 *The regulation cost is positive,* $\kappa > 0$.

The next proposition describes the conditions for equilibria in the model when policy (e, τ) is exogenous.

Proposition 1 *Given Assumption 1, for* $e \leq \tau$*, all firms operate in the informal sector. For* $e > \tau$ *and a sufficiently small* $z_{\min} \geq 0$*, there exists a unique threshold value* $z^* \in [z_{\min}, \infty)$ *such that* $\Pi_F(z^*) = \Pi_I(z^*)$*, firms with* $z < z^*$ *operate in the informal sector, and firms with* $z \geq z^*$ *operate in the formal sector.*

Proof. The first result of the proposition is quite trivial. If $e \leq \tau$ and $\kappa > 0$, the profit function for the informal sector is always above that for the formal sector. Intuitively, if operating in the formal sector becomes too costly (a higher proportional and fixed cost), then no firm is willing to be formal. To prove the second part of the proposition, for now assume $z_{\min} = 0$. Then, we know that $\Pi_F(0) = -w\kappa < 0$ (by Assumption 1) and $\Pi_I(0) = 0$. Thus, $\Pi_F(0) < \Pi_I(0)$. To prove the existence of a single crossing, I need to show that the slope of the profit function in the formal sector is higher than the slope of the function in the informal sector. The slopes are given by the derivative $\dfrac{\partial \Pi_J}{\partial z}$. We need to show that

$\frac{\partial \Pi_F}{\partial z} > \frac{\partial \Pi_I}{\partial z}$:

$$\begin{aligned}
(1-\tau)\frac{\theta-1}{\theta}\left[p_F(z)\right]^{1-\theta} Y z^{-1} &> (1-e)\frac{\theta-1}{\theta}\left[p_I(z)\right]^{1-\theta} Y z^{-1} \\
(1-\tau)\left[p_F(z)\right]^{1-\theta} &> (1-e)\left[p_I(z)\right]^{1-\theta} \\
\frac{(1-\tau)}{(1-e)}\left[\frac{p_I(z)}{p_F(z)}\right]^{\theta-1} &> 1 \\
\left[\frac{(1-\tau)}{(1-e)}\right]^{\theta} &> 1 \qquad (2.13)
\end{aligned}$$

The last inequality is true *iff* $e > \tau$, which we assume. Naturally, what is left is to guarantee that $z^* \geq z_{\min}$. We assume $z_{\min}$ to be sufficiently small, so that the unique threshold always exists. In case $z_{\min}$ is not sufficiently small, then $\Pi_F(z_{\min}) \geq \Pi_I(z_{\min})$ and all firms operate in the formal sector. In that case, $z^* = z_{\min}$ and the equilibrium is still unique. ■

The reason why we need the assumption that $e > \tau$ in the second part of the proposition is quite straightforward. If the opposite occurs, the first part of the proposition shows that no formal sector exists. The individual firm faces a decision to operate in the informal sector, paying an enforcement rate e, or to operate in the formal sector, where not only the tax rate is higher, but there also exists a positive fixed cost on top. Clearly, it is not worth being formal.

The following plot illustrates the single crossing property described in Proposition 1.

Now, the definition of the equilibrium follows:

Definition 1 *An equilibrium with exogenous policy is a set of allocations of the good* $\{y(z)\}$ *and a productivity threshold* $z^\star$*, such that: (a) given exogenous government policy* (τ, e) *and wages* w*, firms maximize profit; (b) given prices* $(w, p(z))$ *and exogenous government policy* (τ, e, G)*, the representative household maximizes composite consumption* C*; (c) the budget constraint of the government holds with equality; (d) markets (for both labor and goods) clear; and, finally, (e) firms with productivity* $z < z^*$ *operate in the informal sector and firms with* $z \geq z^*$ *operate in the formal sector.*

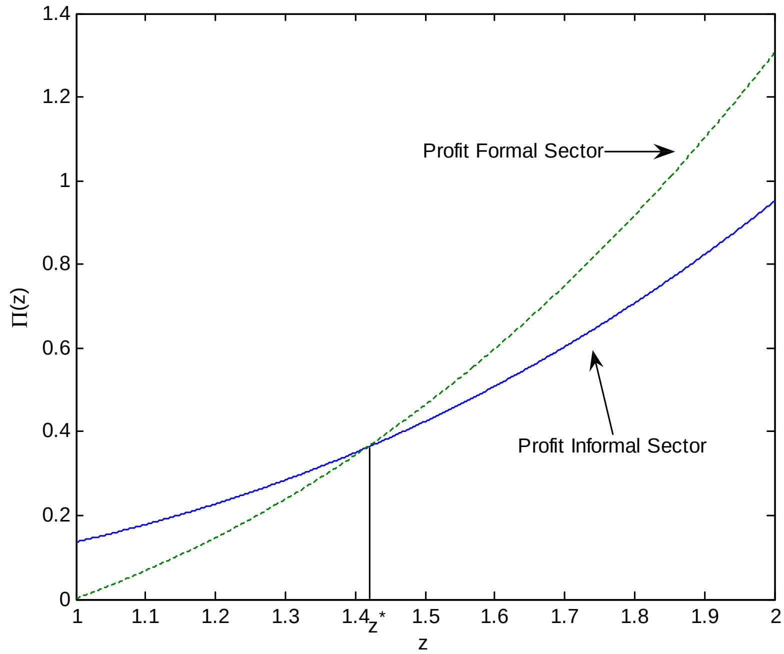

Figure 2.2: Single crossing property of the profit curves

3.4 Parametrization of the productivity distribution

I parametrize the distribution of productivities following Ghironi and Melitz (2005). They assume the distribution to be Pareto with lower bound $z_{\min}$ and shape parameter $k > \theta - 1$. Parameter k indexes the dispersion of productivity. The standard deviation of log productivity is equal to $1/k$. And the condition that $k > \theta - 1$ ensures that the variance in firm size is finite. The distribution of productivity, which is Pareto, also induces the distribution of size of firms to be Pareto. Ghironi and Melitz (2005) claim that this distribution fits firm-level data for the U.S. quite well. The cumulative distribution function is $F(z) = 1 - (z_{\min}/z)^k$ and the probability distribution function is given by

$$f(z) = kz_{\min}^k z^{-k-1}. \tag{2.14}$$

Considering the threshold equilibrium described in Proposition 1, we can compute the share of firms in the formal sector using the CDF:

$$1 - F(z^*) = (z_{\min}/z^*)^k, \tag{2.15}$$

and since there is a measure one of firms, the number of firms in the formal sector, N_F, equals $(z_{\min}/z^*)^k$.

3.5 Determination of equilibrium

This section shows the analytical solution of the equilibrium considering the parametrization of the productivity distribution given in the previous subsection. It is enough to solve for only three endogenous variables to determine the equilibrium, namely, the threshold of productivity z^*, the wage w, and total output Y. For this purpose, we need three equilibrium conditions.

The first equilibrium condition is the cutoff condition $\Pi_F(z^*) - \Pi_I(z^*) = 0$, where the two profit functions cross. Using the profit expression (2.10), the condition becomes

$$\frac{(1-\tau)}{\theta}\left[p_F(z^*)\right]^{1-\theta} Y - w\kappa = \frac{(1-e)}{\theta}\left[p_I(z^*)\right]^{1-\theta} Y. \tag{2.16}$$

Substituting for the price equation (2.9) and after having done some algebra[12] , we get the following expression:

$$\Phi_1(z^*; \tau, e, \kappa, \theta, \lambda) \equiv \left[(1-\tau)^\theta - (1-e)^\theta\right]\left(\frac{\theta}{\theta-1}\right)^{1-\theta}\frac{(\lambda z^*)^{\theta-1}}{\theta\kappa} = \frac{w^\theta}{Y}. \tag{2.17}$$

We can express the left-hand side as a function Φ_1 of the threshold z^*. The right-hand side is a simple function of the other two endogenous variables: w and Y. Remember that I consider τ and e to be exogenous policy variables. Moreover, so far, the equilibrium condition refers to the optimal choices of firms, which take these policies as given.

Another equilibrium condition to consider is the labor-market clearing, which is given by

$$\int_{z_{\min}}^{\infty} \frac{y(z)}{\lambda z} dF(z) + \kappa(1 - F(z^*)) = L. \tag{2.18}$$

This condition can also be rewritten, in a similar fashion to (2.17), as follows:

[12] The complete derivation is in the appendix.

$$\Phi_2(z^*;\tau,e,\kappa,\theta,\lambda) \equiv \frac{kz_{\min}^k \lambda^{\theta-1}}{L-\kappa(z_{\min}/z^*)^k}\left(\frac{\theta}{(\theta-1)}\right)^{-\theta} \times$$
$$\times\left((1-e)^\theta \int_{z_{\min}}^{z^*} z^{\theta-k-2}dz + (1-\tau)^\theta \int_{z^*}^{\infty} z^{\theta-k-2}dz\right) = \frac{w^\theta}{Y}, \tag{2.19}$$

where $\int_{z_{\min}}^{z^*} z^{\theta-k-2}dz = \frac{z^{*\theta-k-1} - z_{\min}^{\theta-k-1}}{\theta-1-k}$ and $\int_{z^*}^{\infty} z^{\theta-k-2}dz = \frac{-z^{*\theta-k-1}}{\theta-1-k}$. The left-hand side is expressed as a function Φ_2 of the threshold z^* and other exogenous variables. And the right-hand side is expressed as a function of w and Y.

Now, notice that the two equations (2.17) and (2.19) have the same right-hand side. Then, equating them, we get the equilibrium threshold z^* as a function of exogenous variables only:

$$\frac{kz_{\min}^k}{(L-\kappa(z_{\min}/z^*)^k)}\left((1-e)^\theta \int_{z_{\min}}^{z^*} z^{\theta-k-2}dz + (1-\tau)^\theta \int_{z^*}^{\infty} z^{\theta-k-2}dz\right) =$$
$$\left[(1-\tau)^\theta - (1-e)^\theta\right]\frac{z^{*\theta-1}}{(\theta-1)\,\kappa}. \tag{2.20}$$

Finally, we need a third equilibrium condition which is given by the goods' market clearing. The aggregate of all individual outputs equals total output in the economy:

$$Y = \left(\int_{z_{\min}}^{\infty} y(z)^{(\theta-1)/\theta} dF(z)\right)^{\theta/(\theta-1)}. \tag{2.21}$$

This condition yields the following expression of wage as a function of the threshold z^*:

$$w^{\theta-1} = kz_{\min}^k\left(\frac{(\theta-1)\,\lambda}{\theta}\right)^{\theta-1}\left\{(1-e)^{\theta-1}\frac{z^{*\theta-k-1} - z_{\min}^{\theta-k-1}}{\theta-1-k} - (1-\tau)^{\theta-1}\frac{z^{*\theta-k-1}}{\theta-1-k}\right\} \tag{2.22}$$

Given z^* (by equation 2.20), we can compute w using the above expression. And given z^* and w, we can compute Y, using either equations (2.17) or (2.19).

3.6 Comparative Statics

The equilibrium conditions allow us to do some comparative statics with respect to the fundamentals of the model. I summarize the results in the following subsubsections.

3.6.1 Tax rate τ

An increase in the tax rate makes it more costly to operate in the formal sector. At the margin, firms find it profitable to switch to the informal sector, which leads to an increase in z^*. The increase in taxes has two effects in the same direction, thereby reducing wages. The first effect is the direct effect of taxes, thereby reducing the demand for labor across sectors; the second effect is the movement of workers from formal firms to informal ones. Since informal firms have lower productivity, the marginal productivity of labor is reduced, as is the wage. For total output, the increase in taxes has three effects: (1) the direct effect of the higher tax rate, increasing Y thanks to less resources being wasted on the regulation cost κ; (2) the increase in z^*, reducing output; (3) the decrease in wages, further reducing output. The net effect on total output is therefore ambiguous. In most of the cases I studied, the first effect is larger than the sum of the last two; thus a higher Y as the tax rate rises.

3.6.2 Enforcement rate e

Compared to the tax increase, raising the enforcement rate generates an opposite effect. A higher e makes it more costly to operate in the informal sector, which makes firms on the margin switch to the formal sector, thereby decreasing z^*. Once more, there are two effects on wages. While the first effect, which reduces demand for labor, remains, the second effect is inverted, moving workers from informal to formal firms. The latter effect increases wages, since the marginal productivity of labor is higher (formal firms have higher productivity). This second effect is high for countries with *low* regulation costs[13] . If the first effect is higher, $w(e)$ is decreasing everywhere. If the first effect is higher for low levels of enforcement and lower after

[13] In the model, κ works as a softener of the effects on productivity and it directly affects the formal firms, which are the more productive ones. Analytically, $w_{e\kappa} < 0$.

some threshold $\overline{e}$, then wages become U-shaped. As before, there are three separate effects on total output. Analyzing the separate effects on output as e increases: (1) the direct effect reduces Y, because of the distortionary effect of κ; (2) a decrease in z^* increases output; and (3) there is an ambiguous effect on wages. If wages are decreased, output drops. Instead, if wages increase, output also rises. In most of the cases studied, the net effect on output is negative.

3.6.3 Regulation cost κ

An increase in regulation works in the same line as an increase in taxes. The threshold z^* increases and wages go down. Once more, the effect on output is ambiguous.

3.6.4 Elasticity of substitution across goods θ

The increase in θ can be translated as an increase in competition, since the elasticity of substitution determines the firms' markup over costs. Since there is a fixed cost in the formal sector, formal firms on the margin between being formal or informal are hurt proportionally more than the informal firms on the same margin. Then, the marginal formal firms switch to the informal sector, thus increasing z^*. The increase in θ also means that the demand for goods becomes more elastic and there is a strong increase in demand for goods with lower prices (i.e., for goods with higher productivity). This shifts labor to high productivity firms, which explains why there is an increase in wages. Finally, the increase in wages raises total output.

3.6.5 Labor supply L

An increase in L makes all firms hire more, but more jobs are proportionally created in the formal (high productivity) sector, thereby increasing wages. More workers imply more production. And, in fact, total output increases linearly with L. Informal firms on the margin switch to the formal sector, thereby reducing z^*.

3.6.6 Total factor productivity λ

In this model, parameter λ, which represents total factor productivity in the economy, only works as a scale parameter. The production function is $y(z) = \lambda z l(z)$ and total output can be written as:

$$Y = \lambda \left(\int z l(z)^{(\theta-1)/\theta} dF(z) \right)^{\theta/(\theta-1)}.$$

Then, it increases total production and since it augments labor productivity, there is an increase in wages. Since it is just a scale parameter, no effect on the threshold z^* is observed.

3.6.7 Lower bound for productivity $z_{\min}$

An increase in the minimum productivity level shifts the distribution of firm productivity to the right. Clearly, marginal productivity of labor is higher and wages increase. The effect on output is also positive. As $z_{\min}$ increases, the threshold z^* also increases. However, the size of the informal sector as a percentage of formal output remains constant.

3.6.8 Shape parameter k

Parameter k indexes the dispersion of productivity draws: dispersion decreases as k increases, and the firm productivity levels are increasingly concentrated toward their lower bound, $z_{\min}$. By definition, an increase in k decreases the marginal productivity of labor and wages go down. Since firms are more concentrated towards $z_{\min}$, the threshold z^* is reduced. The wage reduction implies that total output is also lower.

4 Quantitative assessment

So far, we have studied the mechanisms qualitatively involved in the model. In this section, I calibrate the model to 29 countries and make some quantitative experiments. The countries chosen are the OECD countries plus Brazil. The reason for using OECD countries is that the data on total government revenue is more uniform and available and the firms' characteristics are more similar when I calibrate for the distribution of productivities. Nonetheless, the cross-section of countries is quite diverse, including both developed and emerging economies.

4.1 Solving the model

The model is solved as follows. This is the implementation of the equilibrium described in subsection 3.5.

1. Given $(z_{\min}, \theta, k, L, \tau, e, \kappa)$, z^* is computed. The TFP parameter λ does not affect z^*.

2. Then, wage w and total output Y are calculated using equations (2.22) and either (2.17) or (2.19). Here, λ is just a level parameter and does not affect the results.

3. The size of the informal sector (INF) is the ratio of informal sector output Y_I and total output in the formal sector Y_F.

$$INF = \frac{Y_I}{Y_F} = \left(\frac{1-e}{1-\tau}\right)^{\theta-1} \frac{z_{\min}^{\theta-k-1} - z^{*\theta-k-1}}{z^{*\theta-k-1}} \tag{2.23}$$

   where

$$Y_F = Y * kz_{\min}^k \left[\frac{\theta}{(\theta-1)} \frac{w}{(1-\tau)\lambda}\right]^{1-\theta} \frac{z^{*\theta-k-1}}{k+1-\theta} \tag{2.24}$$

   and

$$Y_I = Y * kz_{\min}^k \left[\frac{\theta}{(\theta-1)} \frac{w}{(1-e)\lambda}\right]^{1-\theta} \frac{z^{*\theta-k-1} - z_{\min}^{\theta-k-1}}{\theta-1-k}. \tag{2.25}$$

4. Per capita GDP is formal sector output Y_F divided by L.

5. The amount of labor employed in each sector is computed, respecting that the labor market clearing condition $L_F + L_I + \kappa\left(z_{\min}/z^*\right)^k \equiv L$, where $\left(z_{\min}/z^*\right)^k$ is the proportion of formal firms in the economy.

6. Then, I calculate government expenditures as a percentage of formal GDP $(g \equiv G/Y_F)$.

4.2 Choosing the parameters

The parameters that need to be calibrated are: (1) productivity distribution parameters: $z_{\min}, k$; (2) elasticity of substitution across goods, θ; (3) regulation cost, κ; and (4) labor supply, L. The model also has two policy variables: τ and e, the tax and enforcement rates, respectively.

Table 2.3: General baseline calibration

Parameter	Economic interpretation	Value
$z_{\min}$	lowest productivity value	1
k	parameter productivity distribution	3.4
θ	elasticity of substitution across goods	3.8
L	labor supply	1
λ	total-factor productivity	1

The model period is the average life time of firms. Since this is a static model, it makes sense to consider a large time frame (about ten years). Following what Ghironi and Melitz (2005) did, I use the value of θ from Bernard et al (2003). They set $\theta = 3.8$, which is calibrated to fit U.S. plant data. They report that the standard deviation of log U.S. plant sales is 1.67. This standard deviation in the model is equal to $1/(k - \theta + 1)$. The choice of θ implies that $k = 3.4$ (which satisfies the requirement that $k > \theta - 1$). Across all computations, I normalize the size of the work force L to 1 and the lowest value of productivity $z_{\min}$ is also set to 1. Moreover, the scale parameter λ (the "TFP") is set to 1 on the baseline calibration[14] . The cost of enforcement for the government is set equal to the revenue from enforcement, $\Psi(E) = E$, so that the government only benefits from taxation on formal firms.

I match government expenditures, regulation cost and the size of the informal sector by choosing τ, e, κ. The data on government expenditures for OECD countries is the total government revenue from OECD (2003). The data on Brazil's total government revenue comes from Central Bank of Brazil. The data on the size of the informal sector is from Schneider (2006) and the data on regulation cost is from Djankov et al (2002) and my own calculations (described in Section 2). However, the calculated regulation cost is not exactly κ. It is the monetary cost (of fees and time) as a percentage of formal per capita GDP. Then, the relation between the model parameter κ and the reported regulation cost is:

$$\kappa = \frac{Y_F}{L} * \frac{\text{regulation cost}}{w}. \tag{2.26}$$

[14] In Section 6, it is calibrated to different values for each country when analyzing output per capita differences among countries.

Therefore, the model is solved with a system of six non-linear equations to determine six variables: $z^*, Y, w, \tau, e, \kappa$. The six equations are: 2.17 (or 2.19), 2.20, 2.23, 2.25, 2.26, $g = G/Y_F$.

4.3 Baseline results

Table 2.4 brings the results for the baseline calibration. The data used in the calibration is found in the first three columns: government expenditures as a percentage of formal GDP, the size of the informal economy as a percentage of formal GDP and the regulation cost. The next three columns give the results for tax and the fixed cost on formal businesses and the enforcement rate on informal business for the 29 countries in the sample. The table is sorted by the enforcement rate. Countries with lower enforcement rates are at the top of the table.

This baseline computation provides two results. The first is that I can match key facts of the data for each country. The second result is the enforcement rates that I backed out. This measure of enforcement can be compared with other (indirect) measures of enforcement differences across countries, as a sort of "test" of the model. The idea here is to check that the figures I obtain are really measuring tax enforcement. In lieu of better data, I constructed two measures of tax compliance using data on staffing of government audit offices in OECD countries (OECD, 2004). One measure is the ratio of total audit staff and total population. The second is the same ratio, but just considering the labor force in the denominator. Enforcement is strongly positively correlated with these two measures. The correlations are 0.58 and 0.59, respectively[15] . Figure 2.3 illustrates the relation between enforcement and total audit personnel per population (in million).

The computed enforcement allows us to better understand its relationship with the other variables in the model, namely, regulation, the tax rate and the size of the informal sector. Take the case of Switzerland and the United States with similar levels of informality. The U.S. have the smallest enforcement rate in the sample while Switzerland has a large one. The difference between these two countries is that the U.S. have much smaller regulation costs than Switzerland. The same can be said of Austria as compared to the United States. Austria has the largest enforcement rate

[15] Both correlation coefficients are significant at 1%.

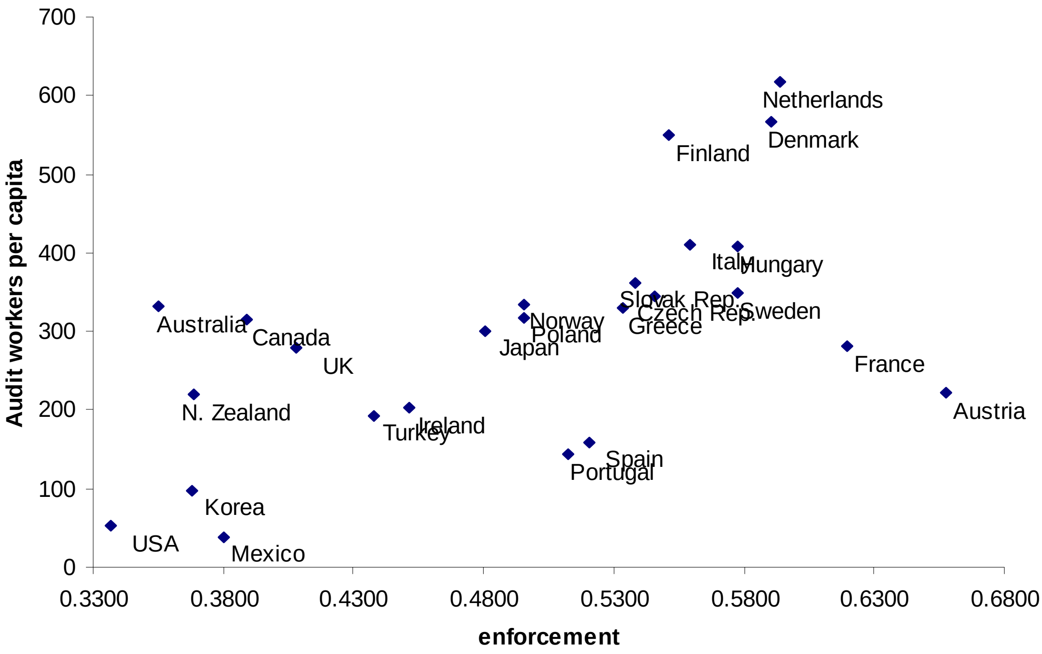

Figure 2.3: Enforcement and total audit workers per capita

in the sample and a small level of informality, like the U.S. and Switzerland. But since Austria has such high regulation costs (in the order of 20 times more), it needs to enforce the informal sector to a considerably larger extent. Otherwise, it does not create enough incentives to make firms switch to the formal sector. At the other side of the spectrum, consider countries with a large informal sector, like Brazil, Mexico and Turkey. These countries have low enforcement, as would be expected. Another point about why these countries can exert low enforcement is that taxation is not so high, especially in the case of Mexico. Greece has the same level of informality and the same amount of regulation cost as Mexico. However, these countries present very different enforcement levels. This is driven by the fact that Greece has more than twice the level of government expenditures than Mexico.

Next, table 2.5 presents some OLS estimations using the computed enforcement as a dependent variable. In column (1), we see that there is a positive correlation between the regulation fixed cost (κ) and enforcement. However, the effect is weak since other factors that are also relevant for endogenously determining the enforcement rate are omitted. The next columns present the effect of regulation on enforcement controlling for these other factors, namely the size of government

Table 2.5: Relation of enforcement and regulation, informality, and government size

	(1)	(2)	(3)	(4)
Regulation	0.097**	0.101***	0.148***	0.135***
	(0.038)	(0.022)	(0.041)	(0.022)
Size of government		0.008***		0.008***
		(0.001)		(0.001)
Informality			-0.005**	-0.004***
			(0.002)	(0.001)
constant	0.446***	0.145***	0.521***	0.215***
	(0.025)	(0.042)	(0.039)	(0.044)
Adj. R-squared	0.17	0.73	0.29	0.79

*Notes: Standard errors in parenthesis: *significant at 10%; **significant at 5%; ***significant at 1%*

(G/Y_F) and the size of the informality (Y_I/Y_F). In columns (2) to (4), the correlation of regulation and enforcement becomes very significant. Moreover, less enforcement is linked to more informality, as would be expected. Moreover, a larger size of government is associated with more enforcement. The results remain the same, even when replacing κ by the regulation measure used as the input in the computations or when using the tax rate τ instead of the size of government.

5 Policy reforms

5.1 Reallocating taxes and enforcement

After considering the baseline case, a natural question is whether the government is choosing tax and enforcement in the best possible way. I take the level of government expenditures and the regulation cost, from the previous section, as given and maximize household utility choosing the tax and enforcement rates. The problem of the government is:

$$\max_{\tau,e} \; u(C, \overline{G}), \tag{2.27}$$

where $\overline{G}$ is the level of public good given by the baseline calibration.

Since $u_C > 0$, the choice here is basically the pair (τ^*, e^*) which delivers the largest possible C. The following table gives the results for the resulting policy reform. The first four columns bring the τ, e, and the size of the informal sector, which were computed in the baseline calibration. The next two columns present the new policy in terms of taxes and enforcement. The following column presents the percentage gain of consumption with the policy reform, while the last column brings the resulting size of informality after the policy.

Comparing the enforcement to the baseline, 13 countries out of 29 increased the enforcement rate. Nine of these are the countries with the lowest regulation costs in the sample. The average gain in consumption is 1.2%. Canada and Austria had the largest increases in C. Canada benefits from a large decrease in taxes whereas Austria benefits from a decrease in enforcement, generating an increase in the informal sector. In the Canadian case, the country had low informality and low regulation. Then, a small increase in enforcement is enough to reduce further informality and allow for tax cuts in the formal sector. In the case of Austria, it had the largest enforcement rate and quite low informality. The optimal policy was to increase informality and avoid wasting resources with regulation. The economy with regulation made it possible to substantially boost private consumption, keeping the government revenue constant by increasing taxes in the formal sector by 1.7 percentage points.

The countries with lowest regulation are also those that manage to reduce their informal sectors substantially more. In particular, the U.S., New Zealand, and Canada managed to completely eliminate the informal sector with the policy reform. What happens here is that the elasticity of informality with respect to tax and enforcement is pretty large and larger for countries with lower regulation. For the other countries, most of them end up with sizes of the informal sector between 20% and 33%, except Mexico which gets an increase to 61%. The suggested reform for Mexico is to decrease the tax base in the formal sector and increase taxes, remembering that Mexico had very low taxes to start with. This allows Mexico to raise the same government revenue. Why is this policy which hikes informality up desirable? The reason is that enforcement is reduced in the informal sector, which is now 60% of the formal economy. This reduction in enforcement increases profits in the informal sector, which directly benefits the representative household's consumption, thereby reducing the distortionary effect of enforcement in the informal sector. At the same time, the distortions associated with regulation in the formal sector are also reduced. It is important to stress that regulation costs are a waste in the economy and, in the case of Mexico, they are quite high.

Concerning the choice of enforcement levels, one could ask what makes the U.S. want such a low e and Denmark, for example, a much higher one? The answer about the difference between U.S. and Denmark lies in the size of government. Remember that e must be larger than τ, otherwise no formal sector exists and the supply of public good is zero. Then, Denmark starts with a "lower bound" for enforcement that is much higher than that of the U.S. Due to the high Danish level of government expenditures as compared to the U.S., Denmark needs a tax rate almost twice the American one. Why is this preferred? Denmark manages to keep a not so large informal sector (18% in the baseline calibration; and 12% after the policy reform), having a large government and a five times larger regulation. The U.S. starts off in much better conditions: 40% smaller government size and very low regulation.

The facts that the suggested reforms in many countries are close to their previous policies and most of the countries with high regulation still keep significant levels of informality suggest that the regulation costs play an important role in determining the level of the informal sector. If these countries reduce the regulation costs, this would allow them to substantially reduce the informal sector. This is exactly what is observed for the countries with lowest regulation in the sample.

5.2 Reforming regulation

5.2.1 Shadow value of regulation

As seen in the previous section, regulation plays a significant role in determining the size of the informal sector. But it is not the scope for government policy in this model. Nonetheless, the model can be useful in measuring the benefits of reducing regulation. Then, the natural question to ask is what is the shadow value of regulation costs (κ) in the equilibrium I have computed. The next table presents the percentage consumption increase for three different changes in regulation: (1) a reduction of 1% in κ; (2) a reduction of 10% in κ; and (3) a reduction of κ by 0.01.

It is clear that reducing regulation makes informality smaller. It becomes "cheaper" to operate in the formal sector. However, production in the formal sector is also taxed. Then, the results for the first two columns (the percentage decreases in κ) show that countries with higher taxation benefit more from the reduction in regulation. Take the case of Mexico, with low taxation, which is the country that benefits the least from the policy. Meanwhile, Sweden and Denmark are the countries which benefit the most. Not by coincidence, they have the largest size of government. The third column shows results for a decrease of 0.01 in the regulation cost. This time, the countries which benefit more were exactly the countries with lower regulation. This is not surprising, since those are the countries with the largest percentage decrease in regulation.

5.2.2 A counterfactual experiment

The purpose of this counterfactual experiment is to explore the shadow value of regulation. What reduction in the regulation cost κ is needed to achieve the level of informality of Switzerland, 8.6%?[16] The following table presents the answer to this question.

Once more, the countries benefitting the most are those with **a** large regulation. The average unweighted increase in consumption is 2.1%. It is important to mention that this experiment is done keeping the level of government expenditures of the baseline calibration and holding taxes and enforcement constant. Comparing these gains to those obtained reforming taxes and enforcement gives another indication that the reduction in regulation can be a more effective policy in both increasing welfare and reducing the size of the informal sector.

6 Accounting for income differences

One interesting question that can be answered with the baseline model is to what extent the distortions associated with informality can account for the income differences among the richest and the poorest countries. Since λ, total factor productivity works as a level parameter in the model, I set it equal to 1 for all countries in the baseline calibration so that TFP differences do not influence the results. Therefore, the level of formal output computed for the baseline calibration, Y_F, only captures the effects due to regulation, enforcement, and taxation associated with the size of the informal economy in each country. When comparing Y_F to measures of actual

[16] Switzerland was chosen because **it has the informal sector with the lowest size** in the sample of 29 countries.

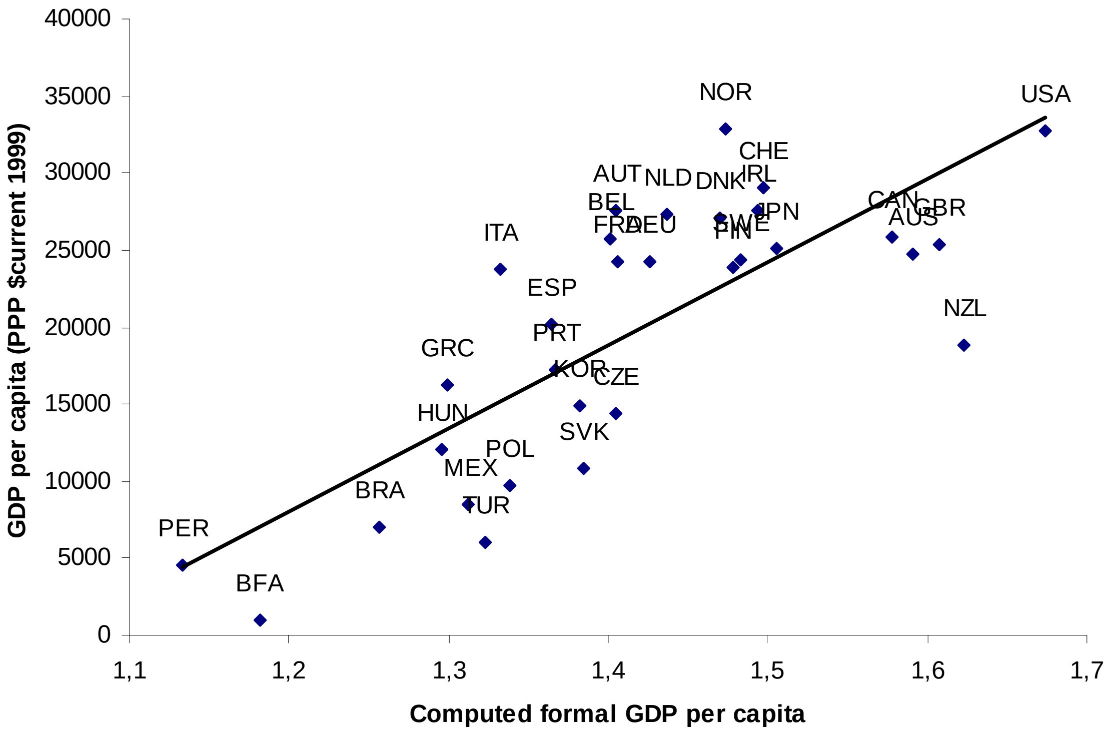

Figure 2.4: GDP per capita: data vs. model

per capita GDP, the correlation is very strong, as can be observed in the following plot. This is an indication that the distortions associated with informality play a role in accounting for income differences across countries.

Let the total-factor productivity parameter λ be calibrated so that per capita GDP in the model, Y_F, equals the value in the data. The first result is that the calibrated λ is also strongly correlated with measures of TFP in the data. This is very reassuring. The correlation between λ and a measure of TFP computed by Hall and Jones (1999) is 0.7 and significant at the 1% level.

Table 2.9 brings figures for: (1) actual GDP per capita (PPP, measured in current US$) in 1999; (2) the same figure relative to the U.S level; (3) the computed output per capita (Y_F) with $\lambda = 1$; (4) the ratio between Y_F and Y_F^{USA}, the U.S. formal output per capita; (5) the TFP λ; and (6) the Hall and Jones's (1999) TFP A relative to the U.S. Not surprisingly, the U.S. formal GDP is much higher than that of Burkina Faso. Hall and Jones (1999) report that the richest countries in the world have an output per worker that is roughly 35 times that of the poorest countries. And this is about the difference between the GDP per capita of the U.S. and Burkina Faso in the data shown below. Following Hall and Jones (1999), we can break down the differences in actual output into differences associated with savings, human capital, and total factor productivity. The last item is a remainder and can be considered to capture differences in "social infrastructure", a term used by Hall and Jones (1999). They argue that savings rates account for a factor of 1.5 and human capital accounts for a factor of 3. We observe that Y_F^{USA} is higher by a factor of 1.5 in our model. It is not huge, but it is nontrivial. Building on Hall and Jones (1999), I conclude that regulation, enforcement, and taxation of formal activities leading to a large informal sector account for roughly a factor of 1.5 of the output differences. TFP differences account for the remaining factor of 5, so that $1.5 \times 3 \times 1.5 \times 5 \approx 35$.

7 Concluding Remarks

I construct a simple general-equilibrium micro-founded model to quantitatively account for the degree of informality across countries. In the model, firms choose to which sector to belong based on proportional taxation in the formal sector, "regulation" of formal firms (fixed, red-tape cost κ), and enforcement of/punishment against informality. Sufficiently large firms find formality to be beneficial. Using the model, I back out [see before, I do not understand the use of "back out" here] what enforcement level is needed, country by country, to match the data for 29 countries. The model quantitatively accounts for the degree of informality and other key aspects, such as size of government, regulation costs, and income differences. The computed enforcement is positively correlated with indirect measures of tax compliance. Moreover, enforcement is positively correlated with regulation and government expenditures and, as expected, negatively correlated with the size of the informal sector. I find that there is some scope for policy reform (using e and τ as instruments). In general, most countries would do better to decrease informality, although some would benefit from increasing it. In both cases, the welfare gains can be fairly large. The countries benefiting the most are those with lower regulation costs.

The previous result suggests that regulation plays a significant role in the equilibrium determination and its reduction can potentially be a more effective policy for increasing private consumption and reducing informality. In particular, since regulation is a distortion in the formal sector, it should be zero. Then, I look at what countries would gain from decreasing regulation (κ), in a hypothetical exercise. I do not have a model for determining the regulation cost, but I can compute the shadow value of decreasing regulation. Thus, we do not know how much it would cost to allow this decrease, but my model allows us to compute the benefits. The result is that benefits are very large, almost twice the welfare gains of reforming taxes and enforcement. Finally, I perform some counterfactual experiments by reducing the regulation cost. I conclude that a policy reducing this waste factor in the economy has a positive impact on the supply of both private and publicly provided goods, effectively reducing the informal sector.

A by-product of the model is that I can account for how the distortions associated with informality reduce output per capita across countries. The output per capita and total-factor productivity delivered by the model are highly correlated with its counterpart in the data. I find that the aforementioned distortions account for a factor of 1.5 of the output per capita difference between the richest and the poorest countries.

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Appendix

A1 Basics of the model

Individual good demand Define the aggregate output Y is the numeraire in the economy. Then, we can maximize it subject to the constraint that the sum of the value of the required varieties should equal the total value of production.

$$\max_{y(z)} Y \equiv \max_{y(z)} \left(\int_{z\in\Omega} y(z)^{(\theta-1)/\theta} dF(z) \right)^{\theta/(\theta-1)}, \tag{2.28}$$

subject to:

$$\int_{z\in\Omega} p(z)y(z)dF(z) = Y \tag{2.29}$$

Maximizing the above problem yields the following first-order condition:

$$y(z)^{-1/\theta} \left(\int_{z\in\Omega} y(z)^{(\theta-1)/\theta} dF(z) \right)^{1/(\theta-1)} = \mu p(z),$$

where μ is the multiplier in the constraint. We can now multiply $y(z)$ on both sides of the above expression:

$$\begin{aligned} y(z)^{(\theta-1)/\theta} \left(\int_{z\in\Omega} y(z)^{(\theta-1)/\theta} dF(z) \right)^{1/(\theta-1)} &= \mu p(z)y(z) \quad (2.30) \\ \int_{z\in\Omega} y(z)^{(\theta-1)/\theta} dF(z) \left(\int_{z\in\Omega} y(z)^{(\theta-1)/\theta} dF(z) \right)^{1/(\theta-1)} &= \int_{z\in\Omega} \mu p(z)y(z)dF(z) \\ Y &= \mu Y \\ \mu &= 1. \quad (2.31) \end{aligned}$$

Now we can substitute the multiplier in the FOC:

$$\begin{aligned} y(z)^{-1/\theta} \left(\int_{z\in\Omega} y(z)^{(\theta-1)/\theta} dF(z) \right)^{1/(\theta-1)} &= p(z) \\ y(z)^{-1/\theta} Y^{1/\theta} &= p(z) \\ y(z)^{-1/\theta} &= Y^{-1/\theta} p(z) \\ y(z) &= Y p(z)^{-\theta}. \quad (2.32) \end{aligned}$$

The rest of the algebra follows directly and equation (A.6) gives the individual demand for good z.

Firms' profit maximization problem The price charged by a firm with productivity z in the sector $J = F, I$ is derived below from the firms' profit maximization problem (2.2), subject to individual demand (2.7).

$$\max_{p_J(z)} \Pi(z) \equiv (1 - \tau\mathcal{I}_F - e\mathcal{I}_I)p_J(z)y(z) - wl(z) - \mathcal{I}_F w\kappa,$$

subject to

$$y(z) = Y p_J(z)^{-\theta}.$$

We can start by replacing $l(z)$ by the production function (2.1). Then, we can replace $y(z)$ by individual demand into the objective function:

$$\Pi(z) = \max_{p_J(z)}(1 - \tau\mathcal{I}_F - e\mathcal{I}_I)p_J(z)Y p_J(z)^{-\theta} - w\frac{Y p_J(z)^{-\theta}}{\lambda z} - \mathcal{I}_F w\kappa.$$

Now, we can take a first-order condition with respect to $p_J(z)$:

$$\begin{aligned}
(1-\theta)(1 - \tau\mathcal{I}_F - e\mathcal{I}_I)Y p_J(z)^{-\theta} + \theta\frac{wY}{\lambda z}p_J(z)^{-\theta-1} &= 0 \\
(1-\theta)(1 - \tau\mathcal{I}_F - e\mathcal{I}_I)p_J(z) + \theta\frac{w}{\lambda z} &= 0 \\
(\theta-1)(1 - \tau\mathcal{I}_F - e\mathcal{I}_I)p_J(z) &= \theta\frac{w}{\lambda z}.
\end{aligned}$$

And the price of good z is:

$$p_J(z) = \frac{\theta}{(\theta-1)}\frac{1}{(1 - \tau\mathcal{I}_F - e\mathcal{I}_I)}\frac{w}{\lambda z}. \tag{2.33}$$

A2 Equilibrium conditions

Profit crossing condition We start with condition (2.16):

$$\frac{(1-\tau)}{\theta}\left[p_F(z^*)\right]^{1-\theta} Y - w\kappa = \frac{(1-e)}{\theta}\left[p_I(z^*)\right]^{1-\theta} Y.$$

Using the expression for the price condition (2.9), we get:

$$\begin{aligned}
\frac{(1-\tau)}{\theta}\left[\frac{(1-e)}{(1-\tau)}p_I(z^*)\right]^{1-\theta} Y - w\kappa &= \frac{(1-e)}{\theta}\left[p_I(z^*)\right]^{1-\theta} Y \\
\frac{(1-\tau)^{\theta}}{\theta}(1-e)^{1-\theta}\left[p_I(z^*)\right]^{1-\theta} Y - w\kappa &= \frac{(1-e)}{\theta}\left[p_I(z^*)\right]^{1-\theta} Y
\end{aligned}$$

$$\left\{(1-\tau)^{\theta}(1-e)^{-\theta}-1\right\}\frac{(1-e)\left[p_I(z^*)\right]^{1-\theta}}{\theta}=\frac{w\kappa}{Y}.$$

Now, we can use replace the price function by (2.8):

$$\begin{aligned}
\left[\left(\frac{1-\tau}{1-e}\right)^{\theta}-1\right]\frac{(1-e)\left[\frac{\theta}{(\theta-1)}\frac{1}{(1-e)}\frac{w}{\lambda z^*}\right]^{1-\theta}}{\theta} &= \frac{w\kappa}{Y}\\
\left[\left(\frac{1-\tau}{1-e}\right)^{\theta}-1\right](1-e)\left[\frac{\theta}{(\theta-1)}\frac{1}{(1-e)\lambda}\right]^{1-\theta}\frac{z^{*\theta-1}w^{1-\theta}}{\theta\kappa} &= \frac{w}{Y}
\end{aligned}$$

$$\left[\left(\frac{1-\tau}{1-e}\right)^{\theta}-1\right]\left(\frac{\theta}{\theta-1}\right)^{1-\theta}\frac{(1-e)^{\theta}\left(\lambda z^*\right)^{\theta-1}}{\theta\kappa}=\frac{w^{\theta}}{Y} \qquad (2.34)$$

Labor-market clearing condition We start with the condition (2.18) and apply

the parametrization in subsection 3.4:

$$\begin{aligned}
L_F+L_I+\kappa N^F &\equiv L\\
\int_{z_{\min}}^{\infty}\frac{y(z)}{\lambda z}dF(z)+\kappa(1-F(z^*)) &= L\\
\int_{z_{\min}}^{\infty}\frac{y(z)}{\lambda z}kz_{\min}^{k}z^{-k-1}dz+\kappa\left(z_{\min}/z^*\right)^{k} &= L
\end{aligned}$$

We can replace $y(z)$ by (2.7):

$$\begin{aligned}
\int_{z_{\min}}^{\infty}\frac{Yp_J(z)^{-\theta}}{\lambda}kz_{\min}^{k}z^{-k-2}dz+\kappa\left(z_{\min}/z^*\right)^{k} &= L\\
\int_{z_{\min}}^{z^*}p_I(z)^{-\theta}z^{-k-2}dz+\int_{z^*}^{\infty}p_F(z)^{-\theta}z^{-k-2}dz &= \frac{L-\kappa\left(z_{\min}/z^*\right)^{k}}{Yk\lambda^{-1}z_{\min}^{k}}
\end{aligned}$$

We can now use the the price functions (2.8) to get:

$$\int_{z_{\min}}^{z^*}\left[\frac{\theta}{(\theta-1)}\frac{1}{(1-e)}\frac{w}{\lambda z}\right]^{-\theta}z^{-k-2}dz+\int_{z^*}^{\infty}\left[\frac{\theta}{(\theta-1)}\frac{1}{(1-\tau)}\frac{w}{\lambda z}\right]^{-\theta}z^{-k-2}dz=$$

$$=\frac{L-\kappa\left(z_{\min}/z^*\right)^{k}}{Yk\lambda^{-1}z_{\min}^{k}}$$

$$(1-e)^{\theta}\int_{z_{\min}}^{z^*}z^{\theta-k-2}dz+(1-\tau)^{\theta}\int_{z^*}^{\infty}z^{\theta-k-2}dz=$$

$$= \left[\frac{\theta}{(\theta-1)}w\right]^{\theta} \frac{L-\kappa\left(z_{\min}/z^{*}\right)^{k}}{Yk\lambda^{\theta-1}z_{\min}^{k}}$$

We can rearrange the terms to get the desired final expression:

$$\left[(1-e)^{\theta}\int_{z_{\min}}^{z^{*}} z^{\theta-k-2}dz + (1-\tau)^{\theta}\int_{z^{*}}^{\infty} z^{\theta-k-2}dz\right]\times$$

$$\times\left[\frac{\theta}{(\theta-1)}\right]^{-\theta} \frac{kz_{\min}^{k}\lambda^{\theta-1}}{L-\kappa\left(z_{\min}/z^{*}\right)^{k}} = \frac{w^{\theta}}{Y} \tag{2.35}$$

Equilibrium condition for the threshold z^{*} We can then equate equations (2.34) and (2.35), cancel out some terms and and get:

$$\left[(1-e)^{\theta}\int_{z_{\min}}^{z^{*}} z^{\theta-k-2}dz + (1-\tau)^{\theta}\int_{z^{*}}^{\infty} z^{\theta-k-2}dz\right]\left[\frac{\theta}{(\theta-1)}\right]^{-\theta}\times$$

$$\times\frac{kz_{\min}^{k}\lambda^{\theta-1}}{L-\kappa\left(z_{\min}/z^{*}\right)^{k}} = \left[\left(\frac{1-\tau}{1-e}\right)^{\theta}-1\right]\left(\frac{\theta}{\theta-1}\right)^{1-\theta}\frac{(1-e)^{\theta}\left(\lambda z^{*}\right)^{\theta-1}}{\theta\kappa}$$

$$\left[(1-e)^{\theta}\int_{z_{\min}}^{z^{*}} z^{\theta-k-2}dz + (1-\tau)^{\theta}\int_{z^{*}}^{\infty} z^{\theta-k-2}dz\right]\frac{kz_{\min}^{k}}{L-\kappa\left(z_{\min}/z^{*}\right)^{k}} =$$

$$= \left[(1-\tau)^{\theta}-(1-e)^{\theta}\right]\frac{(1-e)^{\theta}z^{*\theta-1}}{(\theta-1)\kappa} \tag{2.36}$$

Goods' market condition

$$\begin{aligned} Y &= \left(\int_{z_{\min}}^{\infty} y(z)^{(\theta-1)/\theta}dF(z)\right)^{\theta/(\theta-1)} \\ Y &= \left(\int_{z_{\min}}^{z^{*}} y(z)^{(\theta-1)/\theta}dF(z) + \int_{z^{*}}^{\infty} y(z)^{(\theta-1)/\theta}dF(z)\right)^{\theta/(\theta-1)} \end{aligned}$$

We can replace $y(z)$ by (2.7):

$$\begin{aligned} Y &= \left(\int_{z_{\min}}^{z^{*}} \left[Yp_{I}(z)^{-\theta}\right]^{(\theta-1)/\theta}dF(z) + \int_{z^{*}}^{\infty} \left[Yp_{F}(z)^{-\theta}\right]^{(\theta-1)/\theta}dF(z)\right)^{\theta/(\theta-1)} \\ Y &= \left(Y^{(\theta-1)/\theta}\int_{z_{\min}}^{z^{*}} p_{I}(z)^{1-\theta}dF(z) + Y^{(\theta-1)/\theta}\int_{z^{*}}^{\infty} p_{F}(z)^{1-\theta}dF(z)\right)^{\theta/(\theta-1)} \\ 1 &= \left(\int_{z_{\min}}^{z^{*}} p_{I}(z)^{1-\theta}dF(z) + \int_{z^{*}}^{\infty} p_{F}(z)^{1-\theta}dF(z)\right)^{\theta/(\theta-1)} \end{aligned}$$

We use the price expressions (2.8) to substitute for $p_J(z)$:

$$\left[\frac{\theta}{(\theta-1)}\frac{w}{\lambda}\right]^{\theta-1} = (1-e)^{\theta-1}\int_{z_{\min}}^{z^*} z^{\theta-1}dF(z) + (1-\tau)^{\theta-1}\int_{z^*}^{\infty} z^{\theta-1}dF(z)$$

Rearranging some terms and applying the parametrization in subsection 3.4 yields the desired expression:

$$w^{\theta-1} = kz_{\min}^{k}\left(\frac{(\theta-1)\lambda}{\theta}\right)^{\theta-1}\left\{(1-e)^{\theta-1}\frac{z^{*\theta-k-1}-z_{\min}^{\theta-k-1}}{\theta-1-k} - (1-\tau)^{\theta-1}\frac{z^{*\theta-k-1}}{\theta-1-k}\right\}$$

A3 Tables

Table 2.2: Regulation of entry, size of the informal sector, and per capita GDP in selected economies

Country name	Regulation of Entry (time + cost)	Size of the Informal Sector (as % of formal GDP)	Per capita GDP (current US$)
Argentina	0.2917	25.4	7767
Armenia	0.3243	46.3	595
Australia	0.0292	14.3	21253
Austria	0.4140	9.8	26632
Belgium	0.2316	22.2	24555
Bolivia	2.9903	67.1	1017
Brazil	0.5362	39.8	3132
Bulgaria	0.2341	36.9	1577
Burkina Faso	3.1165	41.4	256
Canada	0.0211	16.0	21352
Chile	0.2413	19.8	4795
China	0.4959	13.1	864
Colombia	0.3518	39.1	2084
Croatia	0.6234	33.4	4375
Czech Republic	0.3324	19.1	5743
Denmark	0.1104	18.0	32548
Dominican Republic	0.7345	32.1	2134
Ecuador	0.8807	34.4	1375
Egypt, Arab Rep.	1.1881	35.1	1374
Finland	0.1071	18.1	24748
France	0.3472	15.2	24834
Georgia	0.9154	67.3	586
Germany	0.3203	16.0	26114
Ghana	0.3938	41.9	397
Greece	0.7692	28.7	11032
Hong Kong, China	0.0917	16.6	24716
Hungary	1.0068	25.1	4693
India	0.8824	23.1	452
Indonesia	0.9656	19.4	688
Ireland	0.1515	15.9	25332
Israel	0.3270	21.9	16988
Italy	0.4407	27.1	20478
Jamaica	0.2400	36.4	3041
Japan	0.2104	11.2	35160
Jordan	0.7164	19.4	1749
Kazakhstan	0.6847	43.2	1130

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Country name	Regulation of Entry (time + cost)	Size of the Informal Sector (as % of formal GDP)	Per capita GDP (current US$)
Kenya	0.6408	34.3	430
Korea, Rep.	0.2526	27.5	9554
Kyrgyz Republic	0.4238	39.8	257
Latvia	0.4381	39.9	3021
Lebanon	1.3987	34.1	5057
Lithuania	0.2306	30.3	3070
Madagascar	1.0589	39.6	236
Malawi	0.4342	40.3	158
Malaysia	0.4235	31.1	3520
Mali		42.3	227
Mexico	0.7682	30.1	4982
Mongolia	0.1184	18.4	381
Morocco	0.4272	36.4	1281
Mozambique	1.7236	40.3	227
Netherlands	0.3016	13.1	25216
New Zealand	0.0169	12.8	14982
Nigeria	2.7752	57.9	303
Norway	0.1158	19.1	35448
Pakistan	0.5517	36.8	467
Panama	0.2983	64.1	3959
Peru	0.5680	59.9	2011
Philippines	0.3728	43.4	1025
Poland	0.4641	27.6	4344
Portugal	0.4768	22.7	11313
Romania	0.5348	34.4	1585
Russian Federation	0.5635	46.1	1339
Senegal	1.6100	45.1	471
Singapore	0.2593	13.1	20592
Slovak Republic	0.4940	18.9	3778
Slovenia	0.3804	27.1	10811
South Africa	0.1900	28.4	3103
Spain	0.4846	22.7	15469
Sri Lanka	0.2887	44.6	822
Sweden	0.0746	19.2	28374
Switzerland	0.2422	8.6	37097
Taiwan,China		25.4	
Tanzania	3.2862	58.3	254
Thailand	0.2023	52.6	2010
Tunisia	0.3284	38.4	2200
Turkey	0.3781	32.1	2773
Uganda	0.4980	43.1	255
Ukraine	0.4231	52.2	636
United Kingdom	0.0290	12.7	24879

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Country name	Regulation of Entry (time + cost)	Size of the Informal Sector (as % of formal GDP)	Per capita GDP (current US$)
United States	0.0205	8.7	33028
Uruguay	0.5491	51.1	6389
Venezuela, RB	0.5108	33.6	4105
Vietnam	1.7856	15.6	370
Zambia	0.7643	48.9	299
Zimbabwe	0.3282	59.4	478

Source: *Own computations using Djankov et al's (2002) and World Bank's (2006) data. The size of the informal sector data is from Schneider (2006)*

Table 2.4: Baseline computation

	Data			Model		
Countryname	G/Y_F	Informal/Y_F	Regulation cost	τ	κ	e
United States	28.9	8.7	0.0205	0.289	0.0355	0.3369
Australia	30.8	14.3	0.0292	0.308	0.0495	0.3553
Korea	23.6	27.5	0.2526	0.236	0.3509	0.3677
New Zealand	33.9	12.8	0.0169	0.339	0.0305	0.3688
Mexico	17.3	30.1	0.7682	0.173	0.9748	0.3803
Canada	35.9	16.0	0.0211	0.359	0.0382	0.3891
United Kingdom	36.1	12.7	0.0290	0.361	0.0538	0.4081
Brazil	31.7	39.8	0.5362	0.317	0.7681	0.4344
Turkey	31.3	32.1	0.3781	0.313	0.5648	0.4376
Ireland	31.0	15.9	0.1515	0.310	0.2497	0.4513
Japan	26.4	11.2	0.2104	0.264	0.3322	0.4803
Norway	40.4	19.1	0.1158	0.404	0.2161	0.4953
Poland	35.0	27.6	0.4641	0.350	0.7503	0.4958
Portugal	34.0	22.7	0.4768	0.340	0.7806	0.5122
Spain	35.0	22.7	0.4846	0.350	0.8046	0.5206
Greece	37.0	28.7	0.7692	0.370	1.2702	0.5335
Slovak Rep	34.4	18.9	0.4940	0.344	0.8285	0.5384
Switzerland	29.8	8.6	0.2422	0.298	0.4011	0.5442
Czech Rep	38.9	19.1	0.3324	0.389	0.5974	0.5460
Finland	47.0	18.1	0.1071	0.470	0.2250	0.5508
Germany	37.7	16.0	0.3203	0.377	0.5738	0.5532
Italy	43.3	27.1	0.4407	0.433	0.8122	0.5595
Belgium	45.3	22.2	0.2316	0.453	0.4562	0.5612
Hungary	39.1	25.1	1.0068	0.391	1.7445	0.5773
Sweden	52.3	19.2	0.0746	0.523	0.1733	0.5773
Denmark	51.5	18.0	0.1104	0.515	0.2522	0.5907
Netherlands	41.2	13.1	0.3016	0.412	0.5764	0.5940
France	45.7	15.2	0.3472	0.457	0.7059	0.6197
Austria	44.0	9.8	0.4140	0.440	0.8253	0.6578

Source: *Data: Djankov et al (2002), OECD(2003), Schneider (2006); Model: own computations*

Table 2.6: Policy reforms on taxes and enforcement

	Benchmark			Optimal policy			
Countryname	τ	e	Y_I/Y_F	τ^*	e^*	$\Delta C(\%)$	Y_I/Y_F
United States	0.289	0.3369	8.7	0.2699	0.3552	1.69	0.0
Australia	0.308	0.3553	14.3	0.2784	0.3924	2.23	0.5
Korea	0.236	0.3677	27.5	0.2408	0.3591	0.06	30.9
New Zealand	0.339	0.3688	12.8	0.3055	0.3710	3.90	0.0
Mexico	0.173	0.3803	30.1	0.2041	0.3103	2.00	61.4
Canada	0.359	0.3891	16.0	0.3169	0.3975	4.77	0.0
United Kingdom	0.361	0.4081	12.7	0.3310	0.4388	2.54	0.4
Brazil	0.317	0.4344	39.8	0.3068	0.4445	0.14	33.9
Turkey	0.313	0.4376	32.1	0.3086	0.4432	0.04	29.5
Ireland	0.310	0.4513	15.9	0.3145	0.4408	0.07	18.5
Japan	0.264	0.4803	11.2	0.2828	0.4094	1.52	24.8
Norway	0.404	0.4953	19.1	0.3915	0.5112	0.46	13.4
Poland	0.350	0.4958	27.6	0.3521	0.4929	0.01	28.8
Portugal	0.340	0.5122	22.7	0.3509	0.4923	0.31	29.3
Spain	0.350	0.5206	22.7	0.3603	0.5022	0.28	28.9
Greece	0.370	0.5335	28.7	0.3764	0.5247	0.09	32.4
Slovak Rep	0.344	0.5384	18.9	0.3603	0.5025	0.85	29.2
Switzerland	0.298	0.5442	8.6	0.3187	0.4504	2.65	24.1
Czech Rep	0.389	0.5460	19.1	0.3965	0.5322	0.17	23.3
Finland	0.470	0.5508	18.1	0.4556	0.5653	0.74	12.1
Germany	0.377	0.5532	16.0	0.3892	0.5249	0.61	23.3
Italy	0.433	0.5595	27.1	0.4279	0.5652	0.06	24.5
Belgium	0.453	0.5612	22.2	0.4454	0.5700	0.18	18.6
Hungary	0.391	0.5773	25.1	0.4043	0.5564	2.66	33.2
Sweden	0.523	0.5773	19.2	0.4948	0.5911	0.48	9.0
Denmark	0.515	0.5907	18.0	0.5007	0.6028	0.84	12.2
Netherlands	0.412	0.5940	13.1	0.4256	0.5574	1.10	21.5
France	0.457	0.6197	15.2	0.4678	0.5963	0.70	21.6
Austria	0.440	0.6578	9.8	0.4571	0.5903	3.54	23.3

Table 2.7: Consumption % increase for different changes in κ

Countryname	-1%	-10%	-0.01
United States	0.05	0.50	1.42
New Zealand	0.04	0.45	1.48
Canada	0.06	0.58	1.52
United Kingdom	0.08	0.84	1.57
Australia	0.07	0.72	1.45
Sweden	0.14	1.43	0.81
Finland	0.12	1.25	0.54
Denmark	0.14	1.45	0.56
Norway	0.10	1.03	0.46
Ireland	0.08	0.81	0.31
Japan	0.07	0.73	0.21
Belgium	0.11	1.20	0.25
Switzerland	0.08	0.82	0.19
Korea	0.06	0.62	0.17
Netherlands	0.10	1.13	0.18
Germany	0.09	1.01	0.16
Czech Rep	0.10	1.05	0.17
France	0.13	1.28	0.17
Turkey	0.04	0.79	0.12
Austria	0.12	1.23	0.14
Italy	0.11	1.13	0.14
Brazil	0.07	0.81	0.11
Poland	0.08	0.89	0.11
Portugal	0.09	0.90	0.11
Spain	0.09	0.93	0.11
Slovak Rep	0.07	0.92	0.11
Greece	0.09	0.96	0.07
Mexico	0.04	0.52	0.04
Hungary	0.12	1.07	0.06

Table 2.8: Reduction in κ to achieve Swiss informality

Countryname	κ'	$\Delta\kappa$	$\Delta C(\%)$
Switzerland	0.4011	0.0	0.0
United States	0.0353	-0.6	0.0
New Zealand	0.0248	-18.7	0.1
United Kingdom	0.0432	-19.7	0.2
Canada	0.0267	-30.1	0.2
Australia	0.0367	-25.9	0.3
Sweden	0.0926	-46.6	0.7
Austria	0.6813	-17.4	0.8
Japan	0.2477	-25.4	1.0
Denmark	0.1330	-47.3	1.1
Finland	0.1188	-47.2	1.1
Norway	0.1071	-50.4	1.2
Ireland	0.1387	-44.5	1.4
Netherlands	0.3410	-40.8	1.8
Belgium	0.1638	-64.1	2.2
France	0.3404	-51.8	2.3
Germany	0.2663	-53.6	2.4
Korea	0.0986	-71.9	2.5
Czech Rep	0.2254	-62.3	2.7
Turkey	0.1169	-79.3	3.2
Italy	0.1899	-76.6	3.4
Slovak Rep	0.2800	-66.2	3.5
Brazil	0.1096	-85.7	3.6
Poland	0.1688	-77.5	3.6
Portugal	0.2159	-72.3	3.7
Spain	0.2212	-72.5	3.7
Greece	0.2231	-82.4	4.8
Mexico	0.1626	-83.3	5.0
Hungary	0.3216	-81.6	5.4

Table 2.9: Income and TFP across countries

Country	GDP per capita[a]	GDPpc/GDPpc^{USA}	Y_F	Y_F/Y_F^{USA}	λ	TFP[b]
United States	32732	1.00	1.674	1.00	19556	1.000
New Zealand	18843	0.58	1.623	0.97	11608	0.631
Canada	25811	0.79	1.577	0.94	16364	1.034
United Kingdom	25399	0.78	1.608	0.96	15796	1.011
Australia	24699	0.75	1.591	0.95	15520	0.856
Sweden	24377	0.74	1.483	0.89	16438	0.897
Finland	23900	0.73	1.478	0.88	16170	0.728
Denmark	27120	0.83	1.469	0.88	18457	0.705
Norway	32854	1.00	1.474	0.88	22289	0.699
Ireland	27556	0.84	1.494	0.89	18448	0.709
Japan	25105	0.77	1.506	0.90	16673	0.658
Belgium	25743	0.79	1.401	0.84	18374	0.978
Switzerland	28991	0.89	1.498	0.89	19359	0.883
Korea. Rep.	14849	0.45	1.382	0.83	10745	0.580
Netherlands	27332	0.84	1.437	0.86	19021	0.946
Germany	24231	0.74	1.427	0.85	16985	0.912
Czech Republic	14442	0.44	1.405	0.84	10277	0.241
France	24241	0.74	1.405	0.84	17248	1.126
Turkey	6018	0.18	1.323	0.79	4547	0.503
Austria	27534	0.84	1.405	0.84	19594	0.979
Italy	23721	0.72	1.333	0.80	17797	1.207
Brazil	6985	0.21	1.256	0.75	5561	0.758
Poland	9726	0.30	1.338	0.80	7267	0.235
Portugal	17221	0.53	1.367	0.82	12595	0.755
Spain	20187	0.62	1.365	0.82	14791	1.107
Slovak Republic	10800	0.33	1.384	0.83	7801	0.241
Greece	16269	0.50	1.299	0.78	12524	0.674
Mexico	8433	0.26	1.313	0.78	6424	0.926
Hungary	12017	0.37	1.296	0.77	9276	0.293
Peru	4561	0.14	1.133	0.68	4025	0.409
Burkina Faso	990	0.03	1.182	0.71	837	0.101

Source: [a]*World Bank (2006)*, [b]*Hall and Jones (1999)*

Chapter 3

Determinants of Capital and R&D Intensive Foreign Direct Investment*

1 Introduction

There has been a spectacular increase in capital flows in the last two decades. In particular, foreign direct investment (FDI) has been growing three times as fast as total investment between 1980 and 2000. Over this period, there has been also a change in the nature of FDI flowing to developing countries. Previously, foreign investment was concentrated to the extraction of natural resources for shipment abroad. Nowadays, as developing countries become wealthier, investment diversifies into production of consumer goods for their local markets. The increasing size and variety of these flows has made both economists and policy makers interested in understanding their determinants and effects. Research, on the one hand, tries to understand how FDI affects productivity and growth, or income inequality and the environment. On the other hand, many studies try to pinpoint the host and source country and industry characteristics behind FDI flows. A question of interest among developing countries is what policies are better at attracting much needed capital

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and new technologies. A number of studies have found that institutional quality is a positive determinant of FDI (and thus, in particular, corruption is a negative determinant), higher taxation reduces capital flows, and more protection of intellectual property rights attracts high-tech investment. The data shows mixed results on other dimensions of policy. For example, Rodrik (1996) found that countries with higher labor standards attract more FDI, an effect that seems to disappear when controlling for political risk (see Cho (2003)). And measures of labor costs and workers' bargaining power are found to have a negative effect on FDI (Smarzynska and Spatareanu (2005) and Cooke (1997)).

In this essay, we analyze the determinants of FDI by looking at the determinants of FDI composition. This is done by studying the interaction between some industry characteristics and host country characteristics. We use capital intensity and R&D expenditures for industry characteristics[1] and measures on protection of property rights, labor standards and constitutional arrangements for host country characteristics. Our regressions show that a better protection of property rights attracts high-tech investment; a result which is not surprising, given the correlation that exists between the overall protection of property rights and the degree of protection of intellectual property rights. But FDI flowing to countries with a low protection of property rights is not biased to less capital intensive sectors. We also find that countries which give workers more bargaining power attract less capital intensive and high-tech investment. Finally, we find that a country's constitutional arrangement has an effect on FDI flows. We look at whether presidential regimes and majoritarian electoral systems, as opposed to parliamentary and proportional, respectively, have a differential effect on FDI. We find evidence of there being a negative effect of presidential regimes on R&D intensive FDI. This finding is consistent with recent results of Persson (2005) which show that these political institutions have an effect on growth rates.

For our empirical analysis, we use data on US investment abroad provided by the Bureau of Economic Analysis. They provide yearly FDI data between 1999 and 2003 for 14 industry categories. Both manufacturing and services data is reported, and capital and R&D intensity are calculated from this same data source. For host

[1] Helpman et al (2004) use capital intensity and R&D intensity as proxies for unobserved industry characteristics.

country institutions, we use an average of data for the nineties, since some measures do not have data available for more recent years. Ideally we would like to perform a panel regression. But, due to the lack of data and time variation, we instead do a cross-section analysis. As a first approximation, we look at the interactions of industry characteristics with the institutional variables that constitute the focus of our study. Given that other country characteristics might have differential effects on FDI composition, we then introduce interaction terms of industry characteristics with known determinants of FDI flows.[2]

This paper is organized as follows. In section 2, we summarize the state of the current literature on determinants of FDI, with a particular emphasis on the institutional characteristics that are subject to study in this essay. In section 3, we develop the hypotheses we want to test. Section 4 presents the econometric specification and describes the data used. Section 5 presents the results and in section 6, we conclude and describe prospective further research.

2 Related literature

Researchers have had an interest in understanding FDI from two different perspectives. Trade economists are interested in FDI as a substitute for trade exports. A firm has two ways of servicing a foreign market. It can either export final goods produced at home, or it can directly set up multiple production plants in those markets. The importance of this decision can be grasped by noting that the largest 500 multinationals control approximately 50% of world trade (Rugman 1988). There are many reasons why a firm might choose the second alternative over the first. The size of a host country market, its expected growth, input costs and natural resources, as well as its policy environment, are of importance for this decision. There is also a trade-off between proximity to costumers and the advantages of scale economies from concentrated production. Riker and Brainard (1997) use US firm level data to test this last hypothesis. They find evidence that tariffs and trade costs have a negative effect on the share of exports over total sales (exports plus affiliate sales), while plant economies of scale have a positive effect on the export share. More re-

[2] We control for population size, GDP per capita, trade openness, human capital, and the size of the government as a proxy for tax rates.

cently, Helpman et al (2004) introduce intraindustry heterogeneity into a standard proximity-concentration model. To control for omitted industry characteristics, they include measures of capital intensity and R&D intensity. They find that capital intensity has a significant negative effect on the ratio of exports to FDI sales, while there is no significant effect of R&D intensity.

Development economists are interested in the effects of FDI on host countries' productivity and growth performance, and its environmental and social implications. Aitken and Harrison (1999) studied if FDI flows had an effect on Venezuelan firms and found a small effect. Haskel et al (2002) studied the effect of FDI on a sample of UK manufacturing firms and found evidence of positive FDI spillovers, although the size of these effects was not very large. Given that even if there are few spillovers, FDI still brings new technologies and management skills to the host country, governments all over the world compete for investment from multinational corporations. In order to attract these capital flows, it is important to understand the factors influencing FDI decisions as well as the determinants of the composition of such flows.

Smith (2001) studied how foreign patent rights affected US exports, affiliate sales and licenses. She found that strong patent rights increase the flow of knowledge to affiliates, as the risk of imitation is reduced. Smarzynska Javorcik (2004) finds similar results using firm data for Eastern Europe and former Soviet Union countries. It is also found that weak protection deters FDI in technology-intensive sectors, and biases investment on projects focusing on distribution rather than local production. A number of papers have shown that host countries' institutional quality in general is a significant determinant of FDI flows.[3] Alfaro et al (2003) find evidence that institutional quality is the most important predictor of capital flows for the period 1971-1998. As measures of institutional quality, they use government stability, internal conflict, corruption, observance of the law, repudiation of contracts, and expropriation risk. Of these measures, the one that received most attention in the literature is corruption. Wei (2000) finds that corruption has a large negative effect on FDI using data on flows between 12 source countries and 45 host countries. The effect found is the economic equivalent of an increase of up to 50 percentage points in the tax rate. Finally, using the same firm level data of Smarzynska Javorcik

[3] In fact, there is a strong correlation between these measures of institutional quality and the measures of protection of patent rights used in the above mentioned papers.

(2004), Smarzynska and Wei (2000) find that corruption does not only discourage inward FDI, but also shifts the ownership structure towards joint ventures. They conclude that this is evidence of the value of a local partner in minimizing the costs of bureaucratic procedures. They find no effect of corruption on R&D intensive FDI, but technologically more advanced firms retain ownership in more corrupt countries.[4]

Another series of papers has studied the impact of labor market regulations and labor standards on FDI. Cooke (1997) found that US FDI was negatively affected by the presence of high levels of union penetration, centralized collective bargaining structures, and stiff restrictions on layoffs. Conversely, Rodrik (1996) found that countries with higher labor standards (as measured by the total number of International Labor Organization conventions ratified by the country) attract more FDI. Recently, Cho (2003) showed that replicating Rodrik's regression with political stability as an added regressor eliminated the significance of labor standards on FDI flows. In her regressions, it is a higher level of political risk that discourages FDI flows. Smarzynska Javorcik and Spatareanu (2005) use firm level data for 25 European countries and find that greater flexibility in the host country's labor market (measured by flexibility of dismissals, length of notice period, and required severance payments) is associated with larger FDI flows. FDI in service sectors appears to be more affected than investment in manufactures, something they attribute to services being more labor intensive than manufactures.

There is another literature that studies the effects of constitutions on economic policymaking. Persson and Tabellini (2003 and 2004) have found systematic and quantitatively large effects of both electoral rules and forms of governments on fiscal policy and corruption. They find that the size of the government, as a percentage of GDP, is 5 percentage points lower in countries with presidential regimes and majoritarian electoral systems. There is also an effect of these constitutional variables on the composition of expenditure, with welfare spending being 2 percentage points lower in countries with presidential regimes and majoritarian electoral systems. Although this research started with the aim of empirically validating theoret-

[4] See also Henisz (2000), who examines the effect of corruption on FDI, market entry, and ownership mode for US based multinational firms, finding at most a positive effect of corruption on FDI flows. Hines (1995) also failed in finding a negative correlation between aggregate FDI inflows and corruption levels in host countries.

ical models of how the rules of policymaking affected actual policy[5] , it is spreading in new directions. Persson (2005) combines these insights with research on long-run economic development that shows certain structural policies to be essential for economic performance. He shows constitutional arrangements to have an effect on some structural policies (protection of property rights, and trade openness) that promote long-run economic growth. In particular, he finds that parliamentary democracies with proportional representation produce the most growth promoting policies.

3 Hypotheses to be tested

We are primarily interested in the determinants of the composition of FDI flows. Therefore, we need to differentiate these flows according to some dimensions that might be of interest both to the economic researcher and the policymaker. We will concentrate on two characteristics of flows that seem to be particularly relevant; capital intensity and R&D intensity. Several studies use one or both of these variables[6] , thus giving us confidence in the academic front. And FDI is seen as globalization at its best for developing countries, not only providing capital but a potent bundle of capital, managerial and technological knowledge. Thus, policymakers in developing countries would agree with us on the importance of understanding what policies attract more R&D and capital intensive FDI.

As we just saw in the previous section, better institutions in general attract more aggregate flows, and better protection of intellectual property rights in particular biases these flows towards more technology-intensive sectors. It seems natural to ask whether other dimensions of a host country's institutional strength also have a differential effect on the composition of FDI flows. Given that corruption has received substantial attention in previous works, we would like to see if countries with less corruption indeed receive more R&D intensive investments than more corrupt countries. Another measure of institutional quality that we study is expropriation risk. We would expect that the higher is this risk, the less capital and R&D intensive will foreign investments be. Observance of the law, repudiation of contracts, and

[5] See, for example, Persson and Tabellini (1999) and Lizzeri and Persico (2001).

[6] As reported above, Helpman et al (2004) use both capital and R&D intensity as proxies for industry unobservables. Smarzynska Javorcik (2004) and Smarzynska and Wei (2000) try to distinguish FDI flows according to their technological intensity.

the quality of the bureaucracy are also expected to have a similar effect on the composition of FDI flows.

Why would the composition of FDI flows be affected by labor institutions? We have seen that the literature has found a number of effects of labor market characteristics on the size of aggregate flows. It does not surprise us to see that countries with less flexible labor markets receive less investments. It has long been known that one of the driving forces behind the decision to move production abroad is to reduce input costs. If regulations make hiring labor more expensive, investment will in general be lower. Instead of looking at measures of labor market flexibility (severance payments or flexibility of dismissals), we study the effect of an increase in the power of labor negotiation on the composition of flows. As workers' bargaining power increases, the higher are their wages, especially in capital and R&D intensive industries, where there are more economic rents to bargain for. Thus, we expect to see less capital and R&D intensive foreign investment in countries with higher union penetration. We also check whether collective bargaining has an effect on the composition of FDI flows. We expect to see two opposite forces at work. On the one hand, centralized bargaining results in more union power and thus, should affect FDI composition in a similar way as union penetration. On the other hand, decentralized bargaining means that labor contracts within an industry more closely follow firms' productivity levels, thus potentially deterring capital and R&D intensive investment. Anticipating our results, we find that the former effect dominates, but the impact on FDI composition is weaker than that found for unionization.

We are finally interested in studying whether host countries' political arrangements have an effect on the level and composition of FDI. Although it seems unrealistic to think that a country would reform its constitution to change its form of government just to attract more FDI, we expect this research to be useful in two respects. First, by contributing to further understanding why some countries are better at attracting foreign investment than others. If constitutional features are part of the reason why a country fails to deliver policies that create the investment friendly environment desired by multinational corporations, there is no point in pushing the country for structural reforms. At the same time, as research finds more evidence on the social and economic costs of some forms of government, there will be a stronger case in favor of constitutional reform. Following recent work by

Persson (2005), we expect to see higher FDI flows in parliamentary democracies with proportional representation and a bias in these flows towards more capital and R&D intensive sectors.

4 Econometric specification and data

4.1 Econometric specification

Given that we want to estimate the effect of institutional variables on the composition of FDI, we should ideally use panel data with variation in source and host countries, and with data for a long period and a large number of industries. This would provide some time variation in the institutional variables of interest while, at the same time, making it possible to use country fixed effects to control for other country unobservables. Moreover, if there is time variation in the industry characteristics, such as R&D intensity, we could also control for other industry unobservables by using industry fixed effects. Having several source countries would also allow us to test whether it is host country institutions per se that are of importance, or both source and host country institutions (conveniently compared) that affect bilateral flows. The data to which we have access limits our ability to perform this analysis. We have institutional data up to the end of the nineties, and FDI outflows from a single source country, the U.S., into 56 host countries from 1999 to 2003.

Thus, we restrict ourselves to performing a cross-section study trying to get the most out of our data. For that reason, we exploit the variation in industry characteristics to see the differential effects of institutional variables on sectoral FDI, while at the same time controlling for country characteristics. Thus, to give an example, we do not directly estimate the effect of corruption on FDI, but whether more corrupt countries attract more or less capital intensive FDI.

To perform these regressions, we should take into account the existence of many zero, and even negative, values for some sector-country pairs, meaning FDI inflows. Moreover, when seeing a negative value for FDI, we are not certain of whether that value reflects the desired actual level of negative investment, or just the observed level of disinvestment given the constraints in reducing exposure in a given host country. Therefore, we treat negative values as zeroes as well and thus use a Tobit specification.[7] The regression to estimate is

$$ln(FDI_{ic}) = X_i\beta + I_cX_i\delta + \mu_c + \epsilon \tag{3.1}$$

where FDI_{ic} is investment in sector i in country c, I_c is a vector of institutional variables in country c, X_i is a vector of industry i characteristics, and δ is the regression coefficient we want to estimate: the interaction between institutions and industry characteristics on FDI flows. Finally, the μ_c are country fixed effects and the error term ϵ is assumed to be *i.i.d.* normally distributed with mean zero and variance σ^2. In this specification, there will be positive foreign investment when $X_i\beta + I_cX_i\delta + \mu_c + \epsilon > 0$, and when $X_i\beta + I_cX_i\delta + \mu_c + \epsilon \leq 0$ the realized level will be zero (and the desired level might be negative, as seen in the data).

The use of country fixed effects allows us to correctly estimate this differential effect under the hypothesis that the institutional variable of interest in the regression, corruption for example, is the only country characteristic with a differential effect on FDI composition. Given that this is a strong assumption, we perform another set of regressions. In these, we introduce interaction terms between country characteristics that have been found to affect FDI, or that we expect could possibly affect the composition of FDI, and industry characteristics. The variables we use are population, as market size is a significant determinant of capital flows, GDP per capita, as a proxy of labor costs, trade openness (measured as exports plus imports over GDP), which gives a measure of the ability to integrate production chains in a given country, government expenditure (as a fraction of GDP), to proxy for tax rates, and human capital. We denote the vector of these variables by W_c. The following is the equation we estimate

$$ln(FDI_{ic}) = X_i\beta + I_cX_i\delta + W_cX_i\gamma + \mu_c + \epsilon. \tag{3.2}$$

Finally, as a robustness check, we drop the country fixed effects and instead use the above mentioned country variables, and their interaction with industry characteristics, along with other regressors[8] . The estimated equation is

[7] Given that we use logarithm of FDI as our independent variable, we replace zeroes and negative values by small positive numbers, such that the log gives a large negative number, and we truncate the distribution just below the lowest positive observation. Performing small changes in this threshold has no significant effect on the regressions.

[8] We use continental dummies to proxy for geographical location variables that might affect FDI

$$ln(FDI_{ic}) = X_i\beta + I_cX_i\delta + W_cX_i\gamma + W_c + \epsilon. \tag{3.3}$$

As another check, we also did a regression with the same regressors as the above, replacing industry characteristics by industry fixed effects. The results are very similar in significance and size and thus, we do not report them.

4.2 Data description

The data used in this study mainly comes from three sources. The data to compute our dependent variable, the U.S. direct investment abroad (*USFDI*), comes from the Bureau of Economic Analysis (BEA) of the U.S. Department of Commerce. We use Total Capital Flows, detailed by industry and by country. The variable is measured in millions of dollars and the data available is for 56 countries plus some regional aggregates. We average the annual Total Capital Flows across years for the period 1999-2003 for each country and each industry category.

In addition, we computed two variables for each industry category: capital intensity, the ratio between capital and labor expenditures (*CAPINT*); and the ratio between R&D and capital expenditures (*RDCAP*). A list of categories and their respective characteristics is included on Table 3.1.[9]

The data on labor market indicators comes from a cross-country database described in Rama and Artecona (2002). This dataset includes 121 countries. Figures are reported for five-year period averages, from 1945-49 to 1995-1999. Our five variables of interest are classified into two broad categories: (1) trade unions and collective bargaining, and (2) labor standards. In the first category, we use the following variables[10] : total trade union membership, in percentage of the total labor force (*TUMMBR*) and workers covered by collective bargaining agreements, in percentage of total salaried or dependent workers (*TUCVGE*). In the labor standard category, we use: cumulative number of ILO (International Labor Organization) conventions ratified by the country (*ILOCNV*); ratification of the ILO convention on the right of workers and employers to establish associations or organizations of

flows, the fraction of host countries' natives that speak English, and whether the legal system is similar to the US one, as transaction costs might be reduced when speaking the same language or sharing the same legal system.

[9] All tables are in the appendix.

[10] We refer to Rama and Artecona (2002) for a detailed explanation of the variables.

their own, without government interference, and to affiliate with similar associations at the international level (*ORGNZE*); and ratification of ILO convention on the right to bargain collectively (*BRGAIN*). To build our cross-section dataset, we took averages for the last two periods: 1990-1994 and 1995-1999 for the 56 countries for which we have data on US direct investment.

The third source of data is an extended version of the cross-section described in the book by Persson and Tabellini (2003). Their data set is used to study the relation between constitutional rules and policy outcomes across democracies. Therefore, it has variables describing economic performance (e.g. GDP per capita, human capital), economic policy (openness, government consumption, protection of property rights), forms of democracy and political institutions (dummy variables for democracy, majoritarian democracy, presidential democracy), protection of property rights, and other country characteristics (continental location, colonial origin, legal origin). We extended their dataset to also include non-democracies. Variables are collected for as many countries as possible on an annual basis. A detailed description of the variables follows:

Protection of Property Rights. The primary source for the next five variables is Knack and Keefer (1995).

CORRUPTION – Variable "Corruption in Government" from the International Country Risk Guide. Lower scores indicate "high government officials are likely to demand special payments" and that "illegal payments are generally expected throughout lower levels of government" in the form of "bribes connected with import and export licenses, exchange controls, tax assessment, police protection, or loans." The variable runs from 0 to 10.

RULE OF LAW (named "Law and Order Tradition" in ICRG) – This variable "reflects the degree to which the citizens of a country are willing to accept the established institutions to make and implement laws and adjudicate disputes." Higher scores indicate: "sound political institutions, a strong court system, and provisions for an orderly succession of power." Lower scores indicate: "a tradition of depending on physical force or illegal means for settling claims." Upon changes in government new leaders "may be less likely to accept the obligations of the previous regime." The variable runs from 0 to 10.

REPUDIATION (Risk of Repudiation of Contracts by Government) – "This indicator addresses the possibility that foreign businesses, contractors, and consultants face the risk of a modification in a contract taking the form of a repudiation, postponement, or scaling down" due to "an income drop, budget cutbacks, indigenization pressure, a change in government, or a change in government economic and social priorities." Lower scores signify "a greater likelihood that a country will modify or repudiate a contract with a foreign business." The variable runs from 0 to 10.

EXPROPRIATION (Risk of Expropriation of Private Investment) – This variables evaluates the risk of "outright confiscation and forced nationalization" of property. Lower ratings "are given to countries where expropriation of private foreign investment is a likely event." The variable runs from 0 to 10.

GADP – index of government's anti-diversion policies. It is an equal-weighted average of these five categories: i) law and order, ii) bureaucratic quality, iii) corruption, iv) risk of expropriation and v) government repudiation of contracts (each of these items has higher values for governments with more effective policies towards supporting production) and ranges from zero to one.

Economic Performance.

GDPPC – Real GDP per capita in 2000 U.S. dollars (Constant price: Chain series). Primary source: Penn World Table 6.1

POPULATION – Source: Penn World Table 6.1, in thousands.

TRADE – sum of exports and imports of goods and services measured as a share of GDP. Source: The World Bank's World Development Indicators 2002.

CG – central government expenditures as a percentage of GDP, constructed using the item Government Finance – Expenditures in the IFS, divided by GDP at current prices and multiplied by 100. Source: IMF/IFS

HUMANCAPITAL – Follows Hall and Jones (1999) with data from Barro and Lee (2000).

Constitutional Variables.

MAJ – dummy variable for electoral systems. Equals 1 if the entire lower house is elected under plurality rule, 0 otherwise. Only legislative elections (lower house) are considered. Source: see Persson and Tabellini (2003)

PRES – dummy variable for forms of government, equal to 1 in presidential regimes, 0 otherwise. Only regimes where the confidence of the assembly is not necessary for the executive (even if an elected president is not the chief executive, or if there is no elected president) are included among presidential regimes. Most semi-presidential and premier-presidential systems are classified as parliamentary. Source: see Persson and Tabellini (2003).

Other Country Characteristics.

LAAM – regional dummy variable, equal to 1 if a country is in Latin America, Central America or the Caribbeans, 0 otherwise.

OECD – dummy variable, equal to 1 for all countries that were members of OECD before 1993, 0 otherwise, except for Turkey coded as 0, even though it was a member of OECD before the 1990s.

AFRICA – regional dummy variable, equal to 1 if a country is in Africa, 0 otherwise.

ASIAE – regional dummy variable, equal to 1 if a country is in East Asia, 0 otherwise.

ENGFRAC – the fraction of the population speaking English as a native language. Source: Hall and Jones (1999).

LEGOR_ UK – dummy variables for the origin of the legal system, classifying a country's legal system into Anglo-Saxon Common Law. Source: La Porta et al. (1998).

Table 3.2 brings the summary statistics for the main variables used in the regressions:

5 Empirical results

5.1 Protection of Property Rights

The empirical analysis finds substantial evidence of differential effects of the degree of protection of property rights on FDI composition. In Table 3.3, we report the results of regressions with country fixed effects and only interactions between industry characteristics, capital and R&D intensity, with measures of institutional quality. We find strong negative effects of a deterioration of the protection of property rights on R&D intensive investment, but a positive effect on capital intensive investment. As said previously, these regressions provide accurate results under the strong assumption that there are no other country characteristic with a differential effect on FDI composition. We lift this assumption and find (see Table 3.4) that all interaction terms between capital intensity and measures of protection of property rights become insignificant. R&D intensive investment is still negatively affected by a lower protection of property rights. For example, an increase of one standard deviation in *CORRUPTION* reduces R&D intensive FDI (one standard deviation above its mean) by 54.3%[11] . For the variable *GADP*, an average of all measures of protection of property rights, a deterioration of one standard deviation reduces R&D intensive investment by 47.9%.

This is an extremely important result. Not only does corruption, and other measures of a country's protection of property rights, deter aggregate FDI flows, but there is a significant reduction in the technological content of incoming flows. These results are in contrast to those of Smarzynska and Wei (2000), who find no significant interaction between corruption and technological sophistication (measured both at the firm and the industry level). While theirs is a model of the decision to invest or not, using micro data, ours is a macro result: we see how capital flows from the U.S. to a series of countries are affected by the degree of protection of property rights in these countries. Both sets of results should then be seen as addressing different questions and thus, complementing each other. Finally, Table 3.5 shows that the results remain similar in size and significance after dropping the country fixed effects and controlling for country characteristics. In the table, we only report the interaction coefficients of interest.

5.2 Labor standards

Given that our previous analysis showed that some country characteristics might have a differential effect on FDI composition, we directly report the results of the country fixed effects regression that includes these interaction terms. The results are

[11] The coefficient is positive because *CORRUPTION* is measured in such a way that higher values imply lower levels of corruption. The same holds for all other measures of protection of property rights.

reported in Table 3.6. There is a strong negative effect of unionization on both capital intensity and R&D intensity of flows. Both interaction terms are negative when we measure unionization by the dummy *ORGNZE*, and by membership *TUMMBR*. Ratification of ILO convention 87 on the right to organize reduces capital intensive FDI by 53.4% and R&D intensive FDI by 60.8%. An increase of a standard deviation in total trade union membership decreases capital intensive foreign investment by 34.6%. The effects of collective bargaining on FDI composition are less robust. Ratification of ILO convention 98 on the right to bargain collectively only has a significant effect on R&D intensive foreign investment. This is reduced by 48.3% when a country has ratified this convention, i.e. when *BRGAIN = 1*. But there is no effect on capital intensity FDI, and no effect of the coverage of collective bargaining agreements (*TUCVGE*) on either measure of FDI composition. Given that we expected to see two opposite forces at work, one increasing capital and R&D intensive FDI, and the other decreasing them, it is no surprise that the estimates are mostly insignificant. Finally, we follow Rodrik (1996) and check whether the total number of ILO conventions ratified by a country has a differential effect on FDI composition. We find negative results, thus we conclude that labor standards do not have a cumulative effect on the capital and R&D intensity of foreign investment, but what is of importance is the type of conventions that are ratified.

These results extend the findings of Smarzynska Javorcik and Spatareanu (2005). They show that labor market flexibility increases aggregate FDI flows. While they also report that FDI in services is more affected than in manufactures, their interpretation of this being due to services being more labor intensive might be wrong. In fact, the converse is true in our sample. As can be calculated from table 3.1, capital intensity in services (0.33) is higher than in manufactures (0.25). Our results should be interpreted as indicating not that higher labor costs deter labor intensive investment, but that a higher bargaining power for labor deters capital and R&D intensive investment. By allowing labor to better appropriate part of the economic rents of a project, higher bargaining power deters the most productive investments. And these are the more capital and R&D intensive ones.

5.3 Constitutional arrangements

The last series of regressions we perform relate to two constitutional features of host countries: whether the form of government is presidential or parliamentary, and whether the electoral system is proportional or majoritarian. In the regressions reported in table 3.8, we can see no significant effect of the electoral system on either capital or R&D intensive foreign investment. For presidential regimes, there is a significant negative effect in the technological content of capital flows. Countries with a presidential regime receive almost 50% less R&D intensive research than parliamentary countries.

Although the evidence is not very strong, this result supports Persson's (2004) findings that parliamentary and proportional democracies are better at promoting structural policies that lead to sustained long-run economic growth. In developing countries, FDI might be the most important way of incorporating new technologies, and thus increase their growth perspective. Thus, our result indicates that one of the forms in which presidential regimes reduce growth is by being unable to deliver policies attracting technologically intensive capital flows.

6 Conclusions

In the last twenty years, there has been an increase in the flows of FDI into developing countries. As these countries become wealthier, these flows have diversified away from the extraction of natural resources and into the production of consumer products for their local markets. Thus, there is reverse causality in that FDI goes to richer countries and, at the same time, provides these countries with the capital and technology that allow them to become richer. In this paper, we have focused on these second channels, and tried to throw some light on how host country institutions affected the capital and R&D content of capital inflows.

We saw that there are important effects of the protection of property rights on the technological content of foreign investment. Better protection of property rights results in FDI being more concentrated in technologically intensive sectors. This result strengthens the case of having an investment friendly environment, by showing that otherwise not only aggregate capital flows will be reduced, but there will be a deterioration in the technological content of incoming flows. We also saw that there is evidence that giving more power to workers results in a decrease in capital and R&D intensive foreign investment. We do not advice developing countries to reduce workers' rights, but make the point that a strengthening of workers' power should be done hand in hand with other measures compensating the negative effects on the technological content of foreign investment.

Finally, we found partial evidence of presidential regimes failing, as compared with parliamentary ones, in delivering policies attracting technologically intensive FDI. The reason for this might be that the confidence requirement inherent in parliamentary arrangements helps producing a more stable and broad legislation, for example, better protection of property rights.

We intend to explore the link between protection of property rights, and the size and composition of FDI in more detail. We are constructing a larger data set with more time, country, and industry variation to see the two channels more clearly: from protection of property rights to FDI and from constitutional arrangements to protection of property rights.

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Appendix

A1 Tables

Table 3.1: Capital Intensity and ratio of R&D and capital expenditures by industry category

Category	*CAPINT*	*RDCAP*
Mining	1.329	0.022
Utilities	1.083	0.003
Manufacturing: Food	0.236	0.151
Manufacturing: Chemicals	0.273	1.257
Manufacturing: Primary and fabricated metals	0.208	0.190
Manufacturing: Machinery	0.276	0.984
Manufacturing: Computer and electronic products	0.299	1.603
Manufacturing: Electrical eq. appliances and components	0.221	0.758
Manufacturing: Transportation equipment	0.290	0.638
Wholesale trade	0.358	0.259
Information	0.684	0.094
Financial (except depositary institutions and insurance)	0.165	0.018
Professional, scientifical, and technical services	0.146	0.691
Other industries	0.259	0.025

Source: BEA (2005)

Table 3.2: Summary Statistics

Variable	Mean	Std. Dev.	Min	Max
USFDI	157.18	727.4	-800	13619.2
CORRUPTION	6.93	2.09	3.33	10
EXPROPRIATION	9.17	0.91	6.15	10
REPUDIATION	8.57	1.21	5.59	10
RULE OF LAW	7.71	2.04	2.46	10
GADP	0.75	0.18	0.41	1
ORGNZE	0.75	0.42	0	1
BRGAIN	0.77	0.41	0	1
TUMMBR	26.87	20.9	0	85.3
TUCVGE	56.62	28.6	3.7	95
ILOCNV	54.29	28.9	4	124.4
MAJ	0.29	0.44	0	1
PRES	0.38	0.49	0	1
POPULATION	81897	211437	262.9	1189411
GDPPC	12772.6	7731.3	982.9	32785.9
HUMANCAPITAL	2.46	0.46	1.67	3.25
TRADE	73.54	54.75	17.57	355.1
CG	13.89	6.83	5.52	29.5
OECD	0.46	0.48	0	1
LAAM	0.23	0.42	0	1
ASIAE	0.14	0.35	0	1
AFRICA	0.05	0.23	0	1
ENGFRAC	0.11	0.28	0	1
LEGOR_ UK	0.29	0.45	0	1

In all the regressions, *LOGFDI* is the dependent variable and standard errors are in parenthesis: *significant at 10%; ** significant at 5%; *** significant at 1%

Table 3.3: Protection of Property Rights

	(1)	(2)	(3)	(4)	(5)
capint*corruption	-0.82**				
	(0.33)				
rdcap*corruption	0.63***				
	(0.23)				
capint*expropriation		-1.68**			
		(0.75)			
rdcap*expropriation		1.35**			
		(0.55)			
capint*repudiation			-1.14**		
			(0.57)		
rdcap*repudiation			1.20***		
			(0.40)		
capint*ruleoflaw				-0.82**	
				(0.33)	
rdcap*ruleoflaw				0.64***	
				(0.23)	
capint*GADP					-11.64***
					(4.34)
rdcap*GADP					9.82***
					(3.02)
capint	3.77	13.60**	7.99	4.47*	7.36**
	(2.34)	(6.91)	(4.91)	(2.61)	(3.47)
rdcap	-4.66***	-12.68**	-10.65***	-5.22***	-8.12***
	(1.66)	(5.11)	(3.46)	(1.88)	(2.48)
constant	-4.22***	-4.40**	2.22	-0.70	2.22
	(2.01)	(2.15)	(1.86)	(1.93)	(1.89)
N. obs.	655	655	655	655	655
Censored obs.	230	230	230	230	230
Pseudo. R2	0.06	0.06	0.06	0.06	0.06

Other controls always included: country dummies

Table 3.4: Protection of property rights and Economic Performance

	(1)	(2)	(3)	(4)	(5)
capint*corruption	-0.11				
	(0.36)				
rdcap*corruption	0.75***				
	(0.24)				
capint*expropriation		-0.60			
		(1.00)			
rdcap*expropriation		1.33*			
		(0.70)			
capint*repudiation			0.94		
			(0.66)		
rdcap*repudiation			1.26***		
			(0.44)		
capint*ruleoflaw				-0.03	
				(0.35)	
rdcap*ruleoflaw				0.44*	
				(0.24)	
capint*GADP					-1.21
					(5.80)
rdcap*GADP					13.94***
					(3.98)
capint	15.56	15.00	23.57**	16.67	15.63
	(11.10)	(10.67)	(10.98)	(11.46)	(12.48)
rdcap	-9.30	-15.02**	-9.86	-11.25	-1.33
	(7.77)	(7.55)	(7.79)	(7.99)	(8.60)
constant	1.61	4.71***	5.10***	1.90	4.92***
	(1.16)	(1.49)	(1.49)	(1.19)	(1.48)
N. Obs.	611	611	611	611	611
Censored Obs.	226	226	226	226	226
Pseudo R2	0.10	0.10	0.10	0.10	0.10

Other controls always included: country dummies, capint*log(population), rdcap*log(population), capint*log(gdppc), rdcap*log(gdppc), capint*trade, rdcap*trade, capint*cg, rdcap*cg, capint*humancapital, rdcap*humancapital

Table 3.5: Protection of Property Rights, no country dummies

	(1)	(2)	(3)	(4)	(5)
capint*corruption	-0.11				
	(0.42)				
rdcap*corruption	0.77***				
	(0.26)				
corruption	-0.11				
	(0.28)				
capint*expropriation		-0.85			
		(1.11)			
rdcap*expropriation		1.37*			
		(0.78)			
expropriation		-0.48			
		(0.79)			
capint*repudiation			1.05		
			(0.78)		
rdcap*repudiation			1.48***		
			(0.54)		
repudiation			-1.23**		
			(0.55)		
capint*ruleoflaw				-0.04	
				(0.38)	
rdcap*ruleoflaw				0.37	
				(0.26)	
ruleoflaw				-0.20	
				(0.27)	
capint*GADP					-2.07
					(6.85)
rdcap*GDP					15.07***
					(4.54)
GADP					-6.08
					(4.83)
capint	13.03	12.04	23.66*	14.33	11.98
	(12.63)	(11.77)	(12.43)	(12.74)	(14.62)
rdcap	-11.80	-16.65**	-9.45	-14.86*	-1.82
	(8.56)	(8.36)	(8.88)	(8.84)	(9.73)
constant	-31.42***	-30.04***	-38.40***	-31.42***	-36.63***
	(8.68)	(8.44)	(8.97)	(8.86)	(9.87)
N. obs.	584	584	584	584	584
Censored obs.	210	210	210	210	210
Pseudo. R2	0.07	0.06	0.06	0.06	0.07

Other controls always included: log(population), log(gdppc), humancapital, trade, cg, oecd, laam, asiae, africa, engfrac, legor_uk, capint*log(population), rdcap*log(population), capint*log(gdppc), rdcap*log(gdppc), capint*trade, rdcap*trade, capint*cg, rdcap*cg, capint*humancapital, rdcap*humancapital.

Table 3.6: Labor Market Indicators

	(1)	(2)	(3)	(4)	(5)
capint*orgnze	-2.12*				
	(1.21)				
rdcap*orgnze	-1.80**				
	(0.81)				
capint*brgain		-0.69			
		(1.03)			
rdcap*brgain		-1.28*			
		(0.67)			
capint*tummbr			-0.06**		
			(0.03)		
rdcap*tummbr			0.01		
			(0.01)		
capint*tucvge				-0.01	
				(0.02)	
rdcap*tucvge				0.00	
				(0.01)	
capint*ilocnv					-0.02
					(0.02)
rdcap*ilocnv					-0.01
					(0.01)
capint	23.43**	18.87*	13.82	19.86	16.97*
	(10.89)	(10.49)	(10.42)	(15.29)	(10.21)
rdcap	-12.85*	-14.43*	-16.72**	-12.03	-17.42**
	(7.60)	(7.50)	(7.33)	(10.59)	(7.32)
constant	4.56***	4.74***	-1.04	3.11**	4.69***
	(1.49)	(1.49)	(1.32)	(1.42)	(1.49)
N. obs.	611	611	611	400	611
Censored obs.	226	226	226	142	226
Pseudo. R2	0.10	0.10	0.10	0.10	0.10

Other controls always included: country dummies, capint*log(population), rdcap*log(population), capint*log(gdppc), rdcap*log(gdppc), capint*trade, rdcap*trade, capint*cg, rdcap*cg, capint*humancapital, rdcap*humancapital.

Table 3.7: Labor Market Indicators, no country dummies

	(1)	(2)	(3)	(4)	(5)
capint*orgnze	-1.99				
	(1.34)				
rdcap*orgnze	-1.72*				
	(0.89)				
orgnze	2.66***				
	(0.97)				
capint*brgain		-0.62			
		(1.14)			
rdcap*brgain		-1.21*			
		(0.73)			
brgain		-0.10			
		(0.79)			
capint*tummbr			-0.07**		
			(0.03)		
rdcap*tummbr			0.02		
			(0.02)		
tummbr			0.01		
			(0.02)		
capint*tucvge				-0.00	
				(0.03)	
rdcap*tucvge				0.00	
				(0.02)	
tucvge				-0.01	
				(0.02)	
capint*ilocnv					-0.02
					(0.02)
rdcap*ilocnv					-0.01
					(0.01)
ilocnv					0.02
					(0.01)
capint	21.35*	16.52	12.62	16.98	14.90
	(11.96)	(11.50)	(11.26)	(16.19)	(11.15)
rdcap	-14.79*	-17.20**	-19.98**	-12.73	-20.42**
	(8.37)	(8.22)	(8.01)	(11.22)	(8.05)
constant	-37.40***	-29.51***	-28.31***	-27.84**	-28.68***
	(8.64)	(8.30)	(8.09)	(11.29)	(8.12)
N. obs.	584	584	584	387	584
Censored obs.	210	210	210	136	210
Pseudo. R2	0.07	0.06	0.06	0.07	0.06

Other controls always included: log(population), log(gdppc), humancapital, trade, cg, oecd, laam, asiae, africa, engfrac, legor_uk, capint*log(population), rdcap*log(population), capint*log(gdppc), rdcap*log(gdppc), capint*trade, rdcap*trade, capint*cg, rdcap*cg, capint*humancapital, rdcap*humancapital.

Table 3.8: Constitutional Arrangements

	(1)	(2)	(3)	(4)
capint*maj	1.37		1.74*	
	(0.96)		(0.91)	
rdcap*maj	0.34		0.27	
	(0.59)		(0.65)	
maj			-0.47	
			(0.55)	
capint*pres		0.68		0.98
		(1.03)		(1.13)
rdcap*pres		-1.33**		-1.25*
		(0.68)		(0.75)
pres				1.13
				(0.92)
capint	7.85	11.16	2.16	5.96
	(11.03)	(11.12)	(12.07)	(12.46)
rdcap	-14.99**	-9.21	-17.17**	-11.31
	(7.00)	(7.61)	(7.67)	(8.46)
constant	4.56***	1.50	-24.70***	-28.52***
	(1.48)	(1.15)	(8.22)	(8.59)
country dummies	Yes	Yes	No	No
other characteristics	No	No	Yes	Yes
N. obs.	608	608	581	581
Censored obs.	225	225	209	209
Pseudo. R2	0.10	0.10	0.07	0.07

Other characteristics: log(population), log(gdppc), humancapital, trade, cg, oecd, laam, asiae, africa, engfrac, legor_uk

Other controls always included: capint*log(population), rdcap*log(population), capint*log(gdppc), rdcap*log(gdppc), capint*trade, rdcap*trade, capint*cg, rdcap*cg, capint*humancapital, rdcap*humancapital

Chapter 4

Ambiguity Aversion, the Equity Premium, and the Welfare Costs of Business Cycles*

1 Introduction

We examine the potential importance of consumers' ambiguity aversion in the context of macroeconomic fluctuations: we ask how consumers price risky fluctuations and how the fluctuations influence consumer welfare. Ambiguity aversion, which is a way of formalizing preferences that are consistent with the Ellsberg paradox, captures a form of violation of Savage's axioms of subjective probability. Instead, consumers behave as if a range of probability distributions are possible and as if they are averse toward the "unknown". With the typical parameterized representation of ambiguity aversion, consumers have minmax preferences, thus maximizing utility based on the worst possible belief within some given set of feasible beliefs. Thus, in an economy with a small amount of randomness, there are first-order effects on utility if there is ambiguity about this randomness. Thus, ambiguity aversion is in contrast to the standard model, where risk aversion leads to second-order effects on utility.

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The first step in our work is to look at asset pricing in a simple Mehra-Prescott-style endowment economy. Here, we demonstrate how larger equity premia can be obtained by assuming ambiguity aversion, along with low riskfree rates. The key parameter in the model is the amount of ambiguity aversion, but it interacts nonlinearly with other parameters, such as the coefficient of relative risk aversion. There is no direct evidence of which we are aware that allows us to calibrate the ambiguity parameter, but we show a range of calibrations that roughly match the average returns on risky and riskless assets.[1]

The second step of the work is to ask how consumers assess the fluctuations from a welfare point of view. Thus, we redo the Lucas (2003)-style calculation whereby it is asked by how much the representative-consumer utility would rise (expressed as a permanent increase in consumption) if all fluctuations around the trend were eliminated. The answer, in the economy with ambiguity, naturally depends on the amount of ambiguity: since ambiguity is a form of "worry" about random fluctuations, the elimination of the randomness would eliminate the worry, and consumers would be better off as a result. Here, we use asset prices as a way of calibrating the ambiguity parameter. That is, we use the first step in our work as a calibration, and then do the Lucas (2003) calculation based on it. We find the welfare costs to be of the order of magnitude of 15% of consumption. This is a huge number (Lucas found about a tenth on 1%), and it is accounted for by allowing larger risk aversion and introducing ambiguity aversion.

In assessing how ambiguity might be important in the economy, it seems relevant to consider whether there is heterogeneity in the extent to which different consumers are ambiguity-averse. The third part of our paper examines how heterogeneity in ambiguity aversion influences wealth distribution, and thus indirectly asset pricing, since consumers' influence on prices operates through (is "weighted by") their wealth holdings. We consider a simple case and assume that half of the agents display a given amount of ambiguity aversion while the rest (the "standard agents") do not. We specialize to a logarithmic period utility function and *iid* and symmetric shocks. For this particular case, we are able to show that the standard agents will increasingly dominate in the pricing of the assets over time. Furthermore, with this heterogeneity, the most ambiguity-averse agents become (almost) non-

[1] For a survey on the equity premium puzzle, see Kocherlakota (1996).

participants in the stock market over time; thus, we obtain endogenous limited participation. In conclusion, although ambiguity aversion shows great potential in providing new asset-pricing implications and in allowing us to think of a reason why the elimination of aggregate fluctuations might be quite costly, heterogeneity in the degree of ambiguity aversion will tend to limit these implications and mainly have effects on wealth distribution and the differences in portfolios across consumers.

2 The economy

This is an infinite-horizon exchange economy. Production is exogenous: the economy has a tree that pays dividends every period. The dividend grows at a random rate, which has a two-state support given by (λ_1, λ_2) and follows a first-order Markov process. The transition probabilities are given by $\phi_{ss'}$ – the probability of going to state s' if today's state is s, with $s, s' = 1, 2$.

When the consumer is ambiguous about these probabilities, he perceives them to be

$$\Phi(v) = \begin{pmatrix} \phi_{11} - v_1 & \phi_{12} + v_1 \\ \phi_{21} - v_2 & \phi_{22} + v_2 \end{pmatrix}, \tag{4.1}$$

where $v_s \in [-a, a]$ $(s = 1, 2)$ with restrictions on a such that all probabilities are in $[0, 1]$. Parameter a measures the amount of ambiguity in the economy.

Preferences are given by the maxmin formulation

$$V_t(s^t) = u\left[c(s^t)\right] + \beta \min_{\pi \in \Pi_{s^t}} E_\pi V_{t+1}(s^{t+1}), \tag{4.2}$$

where c is consumption, $u(c)$ is the period utility function, and Π_{s^t} is a set of transition probability laws given the history s^t today.

Aversion to ambiguity is captured by the "minimization" part in the utility formulation above: the consumer behaves with pessimism, i.e., he assumes the worst possible probability distribution. For an axiomatic foundation for this preference formulation, see Gilboa and Schmeidler (1989) for the static setting and Epstein and Wang (1994) and Epstein and Schneider (2003) for a multiperiod setting.

In section 3 we describe the model with a representative agent and in section 4 we look at the welfare costs of consumption variability. Finally, in section 5 we consider a model with both ambiguity-averse agents and "standard" agents who do not view the economy as ambiguous.

3 Representative-agent asset pricing

In this section and for simplicity, we first consider an ambiguity-averse representative agent with a logarithmic period utility function and discount factor β. In addition, we first assume that shocks are *iid* and symmetric, i.e., $\phi_{ss'} = 0.5$. After that, we consider a CRRA period utility function and assume serially correlated shocks. Then, we calibrate the economy and report the model's performance.

There is an equity share that is competitively traded and a riskless bond that is in zero net supply. We denote the consumer's bond and equity holdings b and e, respectively.

The representative agent holds the tree and thus, his consumption in every period is the dividend of the tree. A log-period utility function together with the assumption of *iid* shocks imply that p, the price of the tree, will be linear in d, the dividend, and independent of the state: $p(d) = \hat{p}d$.

The ambiguity-averse consumer puts a higher weight on the bad outcome than what is warranted by the objective probability; that is, he becomes pessimistic because he is worried about that outcome and does not know its probability.

We assume that $\lambda_1 > \lambda_2$ so that the bad outcome is state 2 – the outcome where the dividend is low. The objective probability of this state is 0.5, but he chooses the belief in the bad state. His belief is $\phi(v) = 0.5 - v$ and he chooses v from the set $v \in [-a, a]$. The higher is a, the more ambiguity there is in the economy.

The problem of the representative agent with wealth today given by w is

$$V(w) = \max_{e} \log\left[w - p(d)e\right] + \beta \min_{v\in[-a,a]} (\phi - v)V(w_1') + (1 - \phi + v)V(w_2')$$

subject to

$$w_1' = \left[\lambda_1 d + p(d\lambda_1)\right] e,$$

and

$$w_2' = \left[\lambda_2 d + p(d\lambda_2)\right] e.$$

Here, for ease of notation, we have excluded the bond (since bond holdings must be zero in equilibrium). Moreover, the budget constraint: $c + p(d)e + q(d)b = w$

where $w = [d + p(d)]\, e_{-1} + b_{-1}$ (e_{-1} and b_{-1} are equity and bond holdings chosen in the previous period) has been substituted away. The Euler equation for equity is

$$p(d)u'(d) =$$

$$\beta \left\{(\phi - a)[\lambda_1 d + p(\lambda_1 d)]u'(\lambda_1 d) + (1 - \phi + a)[\lambda_2 d + p(\lambda_2 d)]u'(\lambda_2 d)\right\}. \qquad (4.3)$$

Clearly, p is linear in d (a constant times d), whenever $u'(c) = c^{-\sigma}$ (here, $\sigma = 1$). Since the period utility is logarithmic, the price of equity does not depend on beliefs because the payoff and the inverse of marginal utility (u') are proportional to λd so that the payoff times marginal utility is the same in both states. Thus, $p(d) = \frac{\beta}{1-\beta} d$ solves the Euler equation above: the price of equity is independent of ϕ and a.

Trivially here, since $e = 1$ in equilibrium, $w_1' = \frac{\lambda_1 d}{1-\beta}$, $w_2' = \frac{\lambda_2 d}{1-\beta}$, then $V(w_1') > V(w_2')$, so the solution for v is a corner, i.e., $v = a$. In section 5, we show that v can be an interior solution when the economy is populated by both ambiguity-averse and standard consumers.

The Euler equation for bonds similarly gives

$$q(d)u'(d) = \beta \left\{(\phi - a)u'(\lambda_1 d) + (1 - \phi + a)u'(\lambda_2 d)\right\}. \qquad (4.4)$$

We see that q depends on beliefs:

$$q = \beta \left[\left(\frac{\phi}{\lambda_1} + \frac{1-\phi}{\lambda_2}\right) + a\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right)\right]. \qquad (4.5)$$

The higher is a – the more ambiguity aversion there is in the economy – the higher is the belief that the bad state will happen, and the higher is the present value of one unit tomorrow, since the probability weight placed on the state with a high marginal utility is higher.

The expected return on equity, ER_e, is given by

$$ER_e = \frac{\phi\lambda_1 + (1-\phi)\lambda_2}{\beta}, \qquad (4.6)$$

and it is independent of the belief. The return on bonds, R_b, decreases when ambiguity aversion increases, because $R_b = \frac{1}{q}$.

The equity premium in this economy is

$$ER_e - R_b = \frac{\phi\lambda_1 + (1-\phi)\lambda_2}{\beta} - \frac{\lambda_1\lambda_2}{\beta\left[(1-\phi)\lambda_1 + \phi\lambda_2 + a(\lambda_1 - \lambda_2)\right]}. \tag{4.7}$$

If we make $\phi = 0.5$, then the equity premium in this economy is

$$ER_e - R_b = \frac{\lambda_1 + \lambda_2}{2\beta} - \frac{\lambda_1\lambda_2}{\beta\left[0.5(\lambda_1 + \lambda_2) + a(\lambda_1 - \lambda_2)\right]}. \tag{4.8}$$

When ambiguity is most extreme, i.e., when $a = 0.5$, the equity premium becomes

$$\frac{\lambda_1 - \lambda_2}{2\beta}.$$

Using $\lambda_1 = 1.02$, $\lambda_2 = 1.01$, and $\beta = 0.98$, the equity premium is 0.5%, which is 200 times larger than the equity premium for the same parameter values when $a = 0$ – the standard model. Although this is an example, and not a calibration, it illustrates that the effect of ambiguity on asset prices/returns can be substantial.

If the period utility is $u(d) = \frac{d^{1-\alpha}}{1-\alpha}$, the price of equity depends on beliefs. In fact:

$$\hat{p} = \frac{\beta\left[(\phi - a)\lambda_1^{1-\alpha} + (1-\phi+a)\lambda_2^{1-\alpha}\right]}{1 - \beta\left[(\phi - a)\lambda_1^{1-\alpha} + (1-\phi+a)\lambda_2^{1-\alpha}\right]}. \tag{4.9}$$

3.1 Serial correlation

We now assume that the period utility is $u(c) = \frac{c^{1-\alpha}}{1-\alpha}$ and the shocks are serially correlated.

The problem of the representative agent with wealth today given by w and today's shock s is

$$V_s(w) = \max_e u\left[w - p_s(d)e\right] + \beta \min_{v_s \in [-a,a]} (\phi_{s1} - v_s)V_1(w_1') + (\phi_{s2} + v_s)V_2(w_2')$$

subject to

$$w_1' = \left[\lambda_1 d + p_1(d\lambda_1)\right] e,$$

$$w_2' = \left[\lambda_2 d + p_2(d\lambda_2)\right] e.$$

The Euler equation for equity is

$$p_s(d)u'(d) =$$

$$\beta\left\{(\phi_{s1}-v_s)[\lambda_1 d+p_1(\lambda_1 d)]u'(\lambda_1 d)+(\phi_{s2}+v_s)[\lambda_2 d+p_2(\lambda_2 d)]u'(\lambda_2 d)\right\} \tag{4.10}$$

The price of equity is still linear in d, and is now given by

$$p_s(d)=k_s d \tag{4.11}$$

where

$$k_s=\beta\left[(\phi_{s1}-v_s)\lambda_1^{1-\alpha}(1+k_1)+(\phi_{s2}+v_s)\lambda_2^{1-\alpha}(1+k_2)\right], \tag{4.12}$$

for $s=1,2$.

Explicitly solving for k_1 and k_2, we obtain:

$$k_1=\frac{\beta(\phi_{11}-a)\lambda_1^{1-\alpha}\left[1-\beta(\phi_{22}+a)\lambda_2^{1-\alpha}\right]+\beta(\phi_{12}+a)\lambda_2^{1-\alpha}+\beta^2(\phi_{12}+a)(\phi_{21}-a)(\lambda_1\lambda_2)^{1-\alpha}}{\left[1-\beta(\phi_{22}+a)\lambda_2^{1-\alpha}\right]\left[1-\beta(\phi_{11}-a)\lambda_1^{1-\alpha}\right]-\beta^2(\phi_{12}+a)(\phi_{21}-a)(\lambda_1\lambda_2)^{1-\alpha}}$$

and

$$k_2=\frac{\beta(\phi_{22}+a)\lambda_2^{1-\alpha}\left[1-\beta(\phi_{11}-a)\lambda_1^{1-\alpha}\right]+\beta(\phi_{21}-a)\lambda_1^{1-\alpha}+\beta^2(\phi_{21}-a)(\phi_{12}+a)(\lambda_1\lambda_2)^{1-\alpha}}{\left[1-\beta(\phi_{22}+a)\lambda_2^{1-\alpha}\right]\left[1-\beta(\phi_{11}-a)\lambda_1^{1-\alpha}\right]-\beta^2(\phi_{12}+a)(\phi_{21}-a)(\lambda_1\lambda_2)^{1-\alpha}}$$

Thus, wealth in the next period is:

$$w_1'=\lambda_1 d(1+k_1), \tag{4.13}$$

and

$$w_2'=\lambda_2 d(1+k_2). \tag{4.14}$$

The price of the bond is given by

$$q_s(d)=\beta\left[\phi_{s1}\frac{1}{\lambda_1^\alpha}+\phi_{s2}\frac{1}{\lambda_2^\alpha}+a\left(\frac{1}{\lambda_2^\alpha}-\frac{1}{\lambda_1^\alpha}\right)\right] \tag{4.15}$$

for $s=1,2$.

The conditional expected return on equity is

$$ER_s^e=\frac{\phi_{s1}\left[\lambda_1 d+p_1(\lambda_1 d)\right]+\phi_{s2}\left[\lambda_2 d+p_2(\lambda_2 d)\right]}{p_s(d)}-1 \tag{4.16}$$

for $s=1,2$, and the unconditional expected return on equity ER^e, is

$$\pi ER_1^e+(1-\pi)ER_2^e-1$$

where the invariant probability π solves

$$\pi = \phi_{11}\pi + \phi_{21}(1-\pi). \tag{4.17}$$

Therefore,

$$ER^e = \pi\frac{\phi_{11}\left[\lambda_1 d + p_1(\lambda_1 d)\right] + \phi_{12}\left[\lambda_2 d + p_2(\lambda_2 d)\right]}{p_1(d)} +$$

$$+(1-\pi)\frac{\phi_{21}\left[\lambda_1 d + p_1(\lambda_1 d)\right] + \phi_{22}\left[\lambda_2 d + p_2(\lambda_2 d)\right]}{p_2(d)} - 1 \tag{4.18}$$

$$ER^e = \pi\frac{\phi_{11}\lambda_1(1+k_1) + \phi_{12}\lambda_2(1+k_2)}{k_1} + (1-\pi)\frac{\phi_{21}\lambda_1(1+k_1) + \phi_{22}\lambda_2(1+k_2)}{k_2} - 1 \tag{4.19}$$

The expected return on the bond, R^b, is given by

$$\pi\frac{1}{q_1} + (1-\pi)\frac{1}{q_2} - 1 =$$

$$\frac{1}{\beta}\left[\frac{\pi}{\phi_{11}\frac{1}{\lambda_1^\alpha} + \phi_{12}\frac{1}{\lambda_2^\alpha} + a\left(\frac{1}{\lambda_2^\alpha} - \frac{1}{\lambda_1^\alpha}\right)} + \frac{(1-\pi)}{\phi_{21}\frac{1}{\lambda_1^\alpha} + \phi_{22}\frac{1}{\lambda_2^\alpha} + a\left(\frac{1}{\lambda_2^\alpha} - \frac{1}{\lambda_1^\alpha}\right)}\right] - 1. \tag{4.20}$$

Finally, the equity premium is given by

$$ER^e - R^b.$$

3.2 Calibration and evaluation of asset prices

As in Mehra and Prescott (1985), we now select the parameters of the model so that the average growth rate of per capita consumption, the standard deviation of the growth rate of per capita consumption and the first-order serial correlation of this growth rate, all with respect to the model's stationary distribution, match the sample values for the U.S. economy between 1889-1978.

The values of the parameters are $\phi = 0.43$ (where $\phi_{11} = \phi_{22} = \phi$ and $\phi_{12} = \phi_{21} = (1-\phi)$), $\lambda_1 = 1.054$, and $\lambda_2 = 0.982$.

Figure 4.1 shows the return on the risk-free bond, the expected return on equity and the equity premium for $\beta = 0.95$, $a = 0.2$, and for a range of α between 0 and 10.

The equity premium is higher as α increases. Note, for example, that for $\alpha =$

8, the risk-free return is 4.72%, the expected return on equity is 8.77%, and the resulting equity premium is 4.05%.

Figure 4.2 shows the return on the risk-free asset, the expected return on equity, and the equity premium for $\beta = 0.95$, $\alpha = 2$, and the ambiguity parameter a in a range between 0 and 0.43.

The equity premium increases with the amount of ambiguity in the economy. For example, for $a = 0.3$, the return on the bond is 4.27%, the expected return on equity is 6.98%, and the resulting equity premium is 2.71%. As a comparison, the largest equity premium that Mehra and Prescott (1985) were able to obtain was 0.35%.

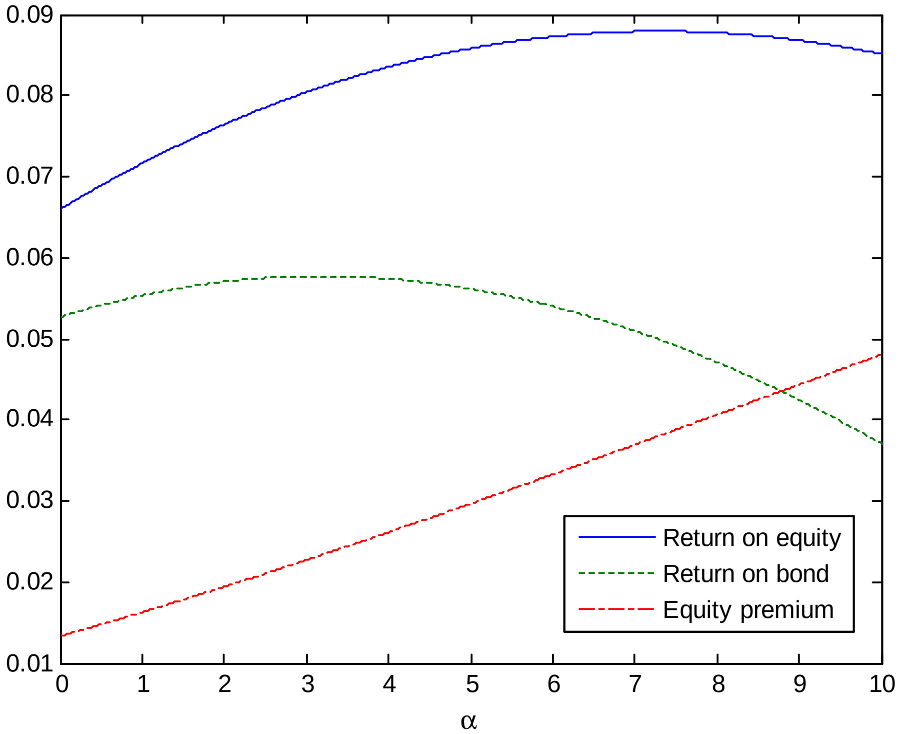

Figure 4.1: Return on equity, risk-free return, and the equity premium as a function of the risk aversion parameter (α)

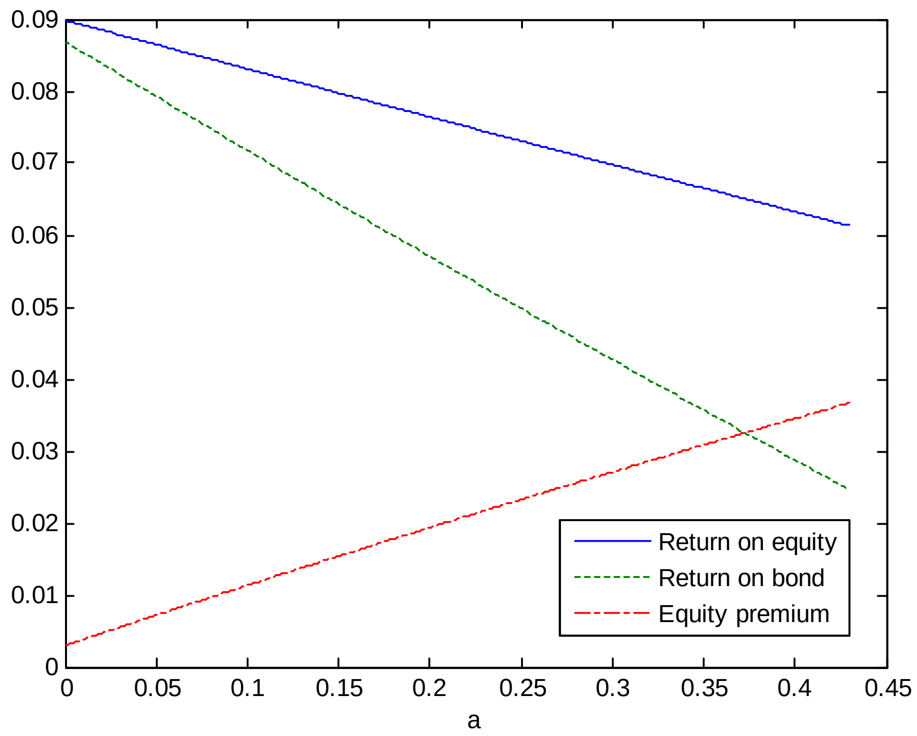

Figure 4.2: Return on equity, risk-free return, and the equity premium as a function of the ambiguity aversion parameter (a)

4 Potential benefits of eliminating consumption fluctuations

We first calculate the costs of consumption fluctuations when shocks are *iid*. The present discounted utility when the dividend today is d is given recursively by

$$V(d) = \frac{d^{1-\alpha}}{1-\alpha} + \beta \left\{ \min_{v \in [-a,a]} [\phi V(\lambda_1 d) + (1-\phi) V(\lambda_2 d)] \right\}. \tag{4.21}$$

The solution for $V(d)$ is

$$V(d) = A d^{1-\alpha}, \tag{4.22}$$

where

$$A = \frac{1}{(1-\alpha)\left\{1 - \beta\left[(\phi - a)\lambda_1^{1-\alpha} + (1-\phi+a)\lambda_2^{1-\alpha}\right]\right\}}. \tag{4.23}$$

Moreover, $v = a$ since $V(d)$ is increasing in d.

Eliminating consumption fluctuations will deliver the present value of total utility corresponding to consuming the expected value of the dividend every period. This utility is given by:

$$\sum_{t=0}^{\infty} \beta^t \frac{\left\{d\left[\phi\lambda_1 + (1-\phi)\lambda_2\right]^t\right\}^{1-\alpha}}{1-\alpha} = \frac{d^{1-\alpha}}{(1-\alpha)\left\{1-\beta\left[\phi\lambda_1+(1-\phi)\lambda_2\right]^{1-\alpha}\right\}}. \quad (4.24)$$

The costs of consumption variability are given by γ where γ solves:

$$\frac{(1-\gamma)^{1-\alpha}}{1-\beta\left[\phi\lambda_1+(1-\phi)\lambda_2\right]^{1-\alpha}} = \frac{1}{1-\beta\left[(\phi-a)\lambda_1^{1-\alpha}+(1-\phi+a)\lambda_2^{1-\alpha}\right]}. \quad (4.25)$$

Calculating the utility of the deterministic growth path is more evolving when the shocks are serially correlated. To this end, we will now introduce some notation.

Let the transition probabilities be given by

$$\Phi \equiv \begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix}, \quad (4.26)$$

let

$$\Lambda \equiv \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}, \quad (4.27)$$

and let

$$\lambda \equiv \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix}. \quad (4.28)$$

Consider the expression

$$\lambda_t^e \equiv (\Phi\Lambda)^{t-1}\,\Phi\lambda. \quad (4.29)$$

The first row of this expression, λ_t^e, is the *expected growth rate between now and t periods from now if the state now is state 1*; and the second row, λ_t^e, is the *expected growth rate between now and t periods from now if the state now is state 2*. Denote these $\lambda_t^e|_1$ and $\lambda_t^e|_2$, respectively; that is,

$$\lambda_t^e \equiv \begin{pmatrix} \lambda_t^e|_1 \\ \lambda_t^e|_2 \end{pmatrix}. \quad (4.30)$$

The utility of the deterministic growth path, where growth is equal to the expected value beginning in state s, is

$$u(d) + \beta u(d\lambda_1^e|_s) + \beta^2 u(d\lambda_2^e|_s) + \beta^3 u(d\lambda_3^e|_s) + \ldots,$$

which when we have CRRA utility equals

$$\frac{d^{1-\alpha}}{1-\alpha}\left[1 + \beta\left(\lambda_1^e|_s\right)^{1-\alpha} + \beta^2\left(\lambda_2^e|_s\right)^{1-\alpha} + \beta^3\left(\lambda_3^e|_s\right)^{1-\alpha} + \ldots\right]. \tag{4.31}$$

The present value of total utility when the dividend is d and the shock is s, is given by

$$V_s(d) = A_s d^{1-\alpha} \tag{4.32}$$

for $s = 1, 2$, and where

$$A_1 = \frac{1 + \beta\lambda_2^{1-\alpha}(\phi_{12} - \phi_{22})}{(1-\alpha)\left\{\left[1 - \beta(\phi_{22} + a)\lambda_2^{1-\alpha}\right]\left[1 - \beta(\phi_{11} - a)\lambda_1^{1-\alpha}\right] - \beta^2(\phi_{21} - a)(\phi_{12} + a)(\lambda_1\lambda_2)^{1-\alpha}\right\}},$$

and

$$A_2 = \frac{1 + \beta\lambda_1^{1-\alpha}(\phi_{21} - \phi_{11})}{(1-\alpha)\left\{\left[1 - \beta(\phi_{22} + a)\lambda_2^{1-\alpha}\right]\left[1 - \beta(\phi_{11} - a)\lambda_1^{1-\alpha}\right] - \beta^2(\phi_{21} - a)(\phi_{12} + a)(\lambda_1\lambda_2)^{1-\alpha}\right\}}.$$

Thus, the welfare cost starting from state 1 is given by the γ_1 solving

$$A_1 = \frac{(1-\gamma_1)^{1-\alpha}}{1-\alpha}\left[1 + \beta\left(\lambda_1^e|_1\right)^{1-\alpha} + \beta^2\left(\lambda_2^e|_1\right)^{1-\alpha} + \beta^3\left(\lambda_3^e|_1\right)^{1-\alpha} + \ldots\right]. \tag{4.33}$$

Similarly, the welfare cost starting from state 2 is given by the γ_2 solving

$$A_2 = \frac{(1-\gamma_2)^{1-\alpha}}{1-\alpha}\left[1 + \beta\left(\lambda_1^e|_2\right)^{1-\alpha} + \beta^2\left(\lambda_2^e|_2\right)^{1-\alpha} + \beta^3\left(\lambda_3^e|_2\right)^{1-\alpha} + \ldots\right]. \tag{4.34}$$

Figure 4.3 plots the costs of business cycles for $\beta = 0.9$, and $\alpha = 2$ as a function of a; i.e., it shows a "comparative-statics" exercise with respect to the ambiguity parameter only.

Clearly, more ambiguity aversion increases the costs of business cycles. By eliminating fluctuations (if that is possible), the government would eliminate the first-order negative effect on utility that consumers experience from random consumption.

We continue with comparative statics with respect to various parameters and

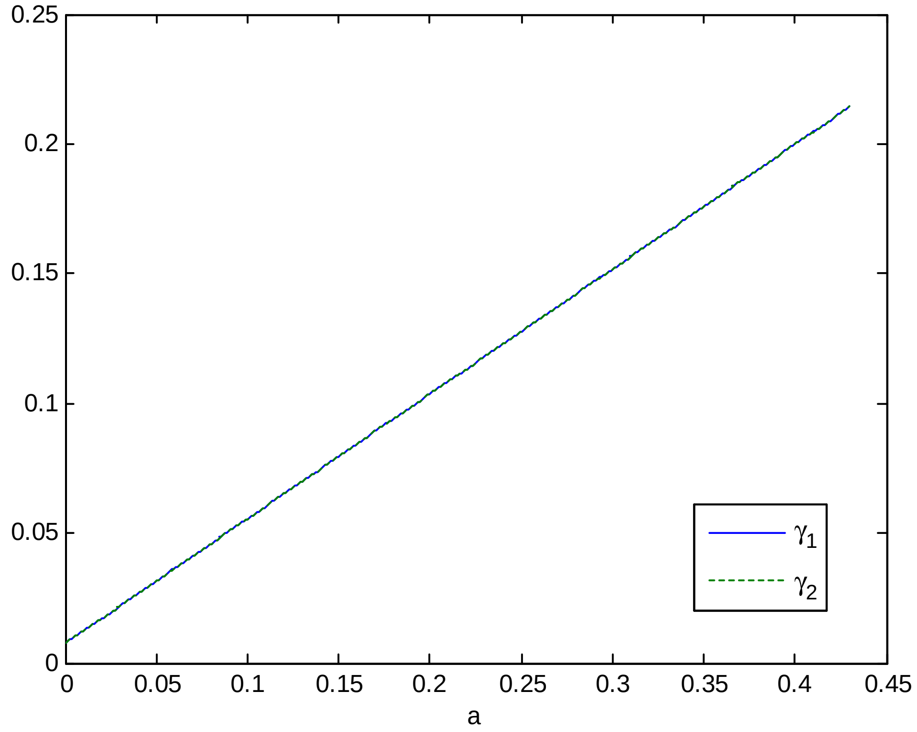

Figure 4.3: Costs of business cycles as a function of the ambiguity-aversion parameter (a)

then finally describe the welfare costs when the parameters are selected to match the asset prices.

Figure 4.4 shows the costs of business cycles for $\beta = 0.7$, and $a = 0.1$ as a function of α. Consumption fluctuations hurt more the more risk averse is a consumer. However, this result is not true for very high values of β or very high values of a.

Finally, figure 4.5 plots the costs of business cycles for $\alpha = 2$ and $a = 0.2$ as a function of β.

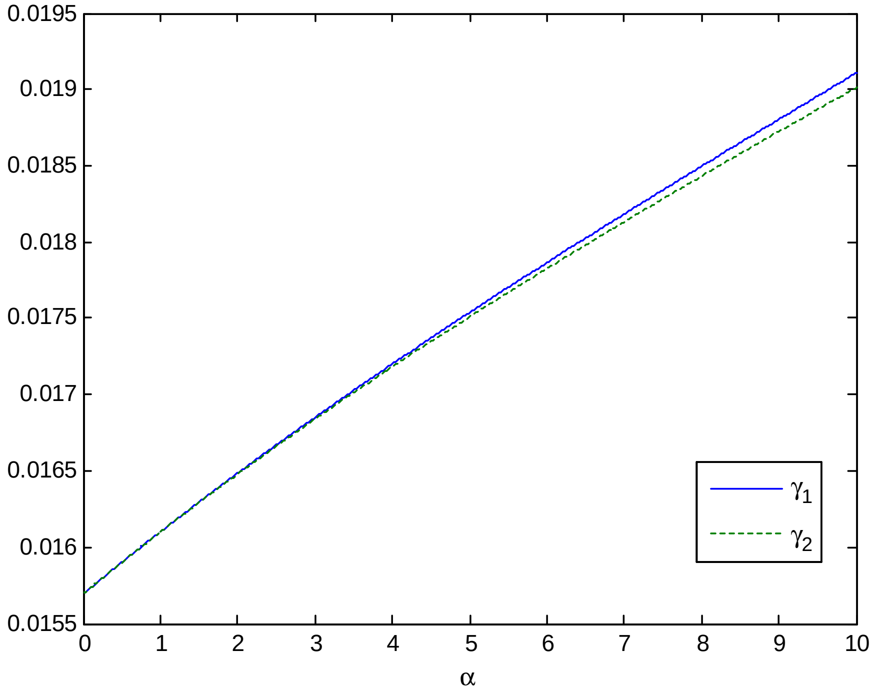

Figure 4.4: Cost of business cycles as a function of the risk-aversion parameter (α)

We now look at the costs of fluctuations when the asset prices match the data. As was discussed briefly above, this can be accomplished in different ways, and each of these is associated with a different cost. Table 4.1 illustrates that the welfare costs—or, rather, the *potential* welfare costs—of cycles are huge. They do not differ markedly across the different parameter configurations.

Table 4.1: Costs of business cycles for selected parameters and $a > 0$

β	α	a	$ER^e - R^b$	γ_1	γ_2
0.95	13.74	0.2040	6.18%	12.48%	12.46%
0.94	13.36	0.2223	6.18%	12.88%	12.86%
0.93	12.95	0.2420	6.18%	13.32%	13.30%
0.92	12.46	0.2642	6.18%	13.86%	13.85%
0.91	11.98	0.2879	6.18%	14.43%	14.41%
0.90	11.37	0.3160	6.18%	15.19%	15.17%
0.89	10.70	0.3480	6.18%	16.06%	16.05%
0.88	9.83	0.3890	6.18%	17.33%	17.32%
0.873	8.94	0.4300	6.18%	18.76%	18.75%

Finally, for comparison, we show the associated costs for $a = 0$. These are also high compared to Lucas' (2003) numbers, since α is high, but of an order of magnitude lower than above.

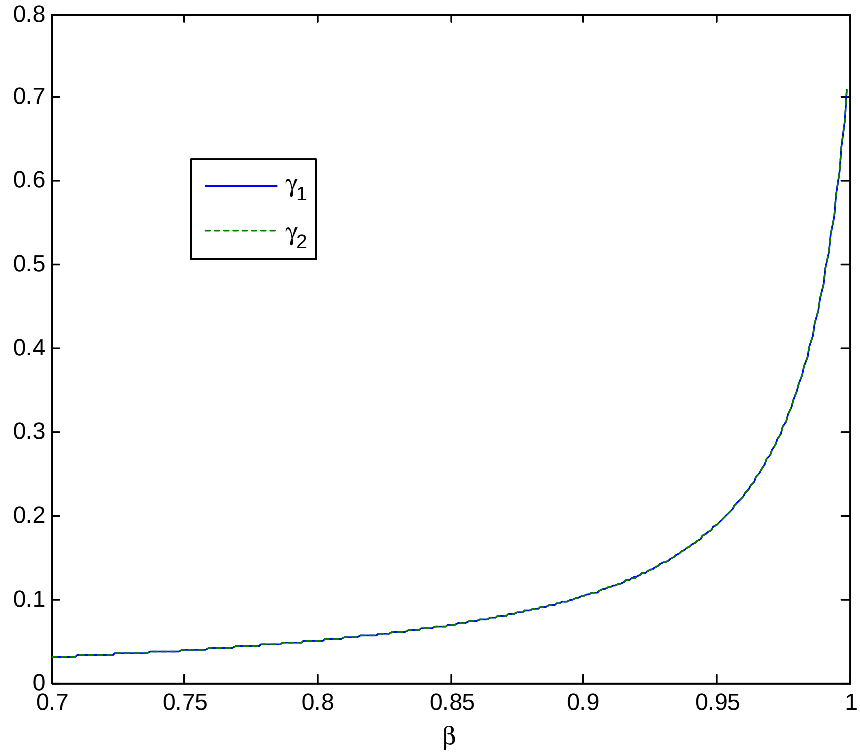

Figure 4.5: Cost of business cycles as a function of the subjective discount factor (β)

Table 4.2: Costs of business cycles for selected parameters and $a = 0$

β	α	a	R^b	$ER^e - R^b$	γ_1	γ_2
0.95	13.74	0	19.49%	4.40%	2.55%	2.53%
0.94	13.36	0	20.67%	4.25%	2.41%	2.39%
0.93	12.95	0	21.85%	4.08%	2.28%	2.26%
0.92	12.46	0	23.01%	3.88%	2.15%	2.13%
0.91	11.98	0	24.16%	3.68%	2.02%	2.01%
0.90	11.37	0	25.25%	3.43%	1.89%	1.88%
0.89	10.70	0	26.27%	3.15%	1.76%	1.75%
0.88	9.83	0	27.12%	2.80%	1.62%	1.61%
0.873	8.94	0	27.42%	2.45%	1.50%	1.49%

5 Heterogeneity in ambiguity aversion

We now consider two types of agents whose ambiguity aversions differ. We look at a general planning problem first, and then focus on the case with *iid* shocks. Later, we look at the case of serial correlation in more detail.

5.1 The planner's problem

The state vector is (d, θ, s): today's dividend, the weight the planner puts on consumer 1, and today's shock. The planner solves the problem

$$V(d, \theta, s) = \max_{c_1, c_2, z_1(s'), z_2(s')} \theta \log c_1 + (1-\theta) \log c_2 +$$

$$+\beta(\theta \min_{v^1 \in [-a^1, a^1]} \sum_{s'=1}^{2} \phi_{ss'}(v^1) z_1(s') + (1-\theta) \min_{v^2 \in [-a^2, a^2]} \sum_{s'=1}^{2} \phi_{ss'}(v^2) z_2(s')$$

subject to

$$\min_{\theta'(s')} V\left[d\lambda_{s'}, \theta'_{s'}, s'\right] - \left\{\theta'_{s'} z_1(s') + \left[1 - \theta'_{s'}\right] z_2(s')\right\} \geq 0, \tag{4.35}$$

and

$$c_1 + c_2 = d, \tag{4.36}$$

where c_i is agent i's consumption, $i = 1, 2$, and z_i is next period's present-value utility for agent i. The first constraint (4.35) makes the problem recursive and the second constraint (4.36) is the resource constraint. This formulation which is based on Lucas and Stokey (1984) is also used in Alonso (2007).

Taking FOCs with respect to the consumption of agents 1 and 2, we have

$$c_1 = \theta d, \tag{4.37}$$

and

$$c_2 = (1-\theta) d, \tag{4.38}$$

with respect to $z_1(1)$ and $z_2(1)$, we obtain

$$\frac{\theta'_1}{1-\theta'_1} = \frac{\theta(\phi_{s1} - v^1)}{(1-\theta)(\phi_{s1} - v^2)}, \tag{4.39}$$

and similarly with respect to $z_1(2)$ and $z_2(2)$ we have

$$\frac{\theta'_2}{1-\theta'_2} = \frac{\theta(\phi_{s2} + v^1)}{(1-\theta)(\phi_{s2} + v^2)}. \tag{4.40}$$

After some algebra, we can rewrite the planner's problem as

$$V(d,\theta,s) = \max_{c_1,c_2,\theta'_{s'}} \theta \log c_1 + (1-\theta)\log c_2 +$$

$$+\beta \min_{v^1,v^2} \left\{ \sum_{s'=1}^{2} \phi_{ss'} \left[\theta v^1 + (1-\theta)v^2\right] V(d\lambda_{s'}, \theta'_{s'}, s') \right\}$$

subject to

$$\theta'_{s'} = \theta \frac{\phi_{ss'}(v^1)}{\phi_{ss'}\left[\theta v^1 + (1-\theta)v^2\right]}, \tag{4.41}$$

and

$$c_1 + c_2 = d. \tag{4.42}$$

Note that $\phi_{ss'}\left[\theta v^1 + (1-\theta)v^2\right] = \phi_{ss'} - \theta v^1 - (1-\theta)v^2$ if $s' = 1$ and $\phi_{ss'} + \theta v^1 + (1-\theta)v^2$ if $s' = 2$.

5.1.1 A special case: no serial correlation and $v^2 = 0$

In the simpler case where shocks are *iid* and symmetric and consumer 2 is not ambiguity-averse ($a^2 = 0$), the planner's problem becomes:

$$V(d,\theta) = \max_{c_1,c_2,\theta'_{s'}} \theta \log c_1 + (1-\theta)\log c_2 +$$

$$+\beta \min_{v\in[-a,a]} \left\{ \sum_{s'=1}^{2} \phi_{s'}(\theta v) V(d\lambda_{s'}, \theta'_{s'}) \right\}$$

subject to

$$\theta'_{s'} = \theta \frac{\phi_{s'}(v)}{\phi_{s'}(\theta v)}, \tag{4.43}$$

and

$$c_1 + c_2 = d. \tag{4.44}$$

Using the FOCs for consumption, we obtain $c_1 = \theta d$ and $c_2 = (1-\theta)d$, so we get

$$V(d,\theta) = \log d + \log \theta^{\theta}(1-\theta)^{1-\theta} +$$

$$+\beta \min_{v} \left[(\phi - \theta v)V(d\lambda_1, \theta'_1) + (1 - \phi + \theta v)V(d\lambda_2, \theta'_2)\right]$$

with

$$\theta_1' = \theta \frac{\phi - v}{\phi - \theta v}, \tag{4.45}$$

and

$$\theta_2' = \theta \frac{1 - \phi + v}{1 - \phi + \theta v}. \tag{4.46}$$

Here, we conjecture that $V(d, \theta)$ takes the form $A \log d + W(\theta)$. This guess delivers

$$A \log d + W(\theta) = \log d + \log \theta^{\theta}(1-\theta)^{1-\theta} +$$

$$+\beta \min_{v} \left\{ (\phi - \theta v) \left[A \log(d\lambda_1) + W(\theta_1') \right] + (1 - \phi + \theta v) \left[A \log(d\lambda_2) + W(\theta_2') \right] \right\}.$$

Inspecting this functional equation, it can be seen that $A = \frac{1}{1-\beta}$ works and we can express $W(\theta)$ as

$$W(\theta) = \log \theta^{\theta}(1-\theta)^{1-\theta} +$$

$$+\beta \min_{v \in [-a,a]} \left\{ (\phi - \theta v) \left[\frac{\log \lambda_1}{1-\beta} + W\left(\theta \frac{\phi - v}{\phi - \theta v}\right) \right] + (1 - \phi + \theta v) \left[\frac{\log \lambda_2}{1-\beta} + W\left(\theta \frac{1 - \phi + v}{1 - \phi + \theta v}\right) \right] \right\}.$$

This is a one-dimensional dynamic programming problem delivering optimal v as a function of θ and hence, a law of motion for θ. The variable θ also corresponds to the fraction of the total wealth—the current dividend plus the value of the tree—owned by agent 1 in a complete-markets equilibrium. The following figures for $W(\theta)$ and $v(\theta)$ below assume the same values for the parameters as specified at the end of section 3.

Figure 4.6, for $W(\theta)$, reveals a shape similar to $\log \theta^{\theta}(1-\theta)^{1-\theta}$, which is the (constant) flow utility of a planner in a two-type economy where no consumer has ambiguity aversion.

Figure 4.7, for the optimal choice of v, shows that v is close to zero and interior at first (for small θ's), and then it increases monotonically in θ and reaches the upper bound a for a value of θ a little above 0.9. We will interpret these findings in more detail in the following sections.

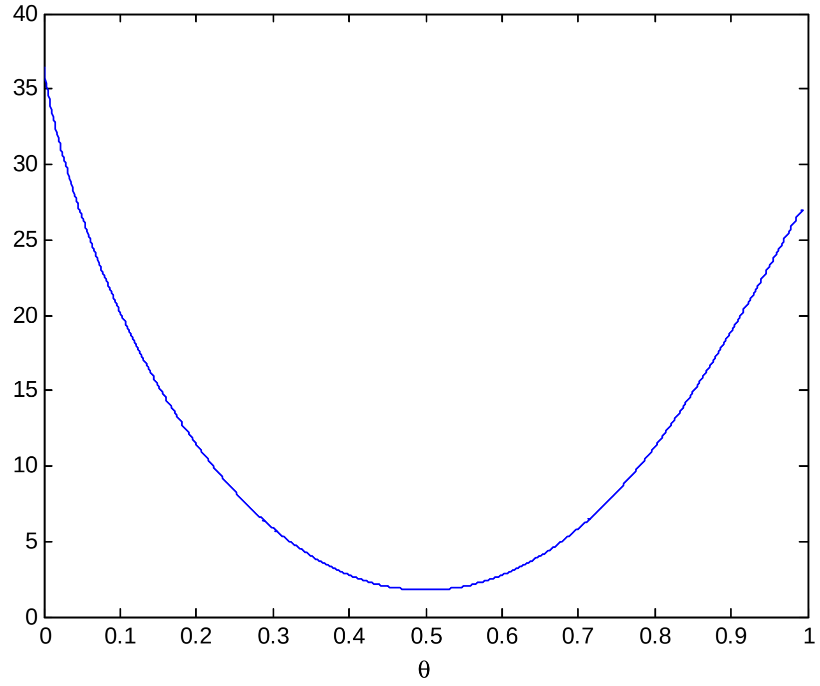

Figure 4.6: Value function $W(\theta)$

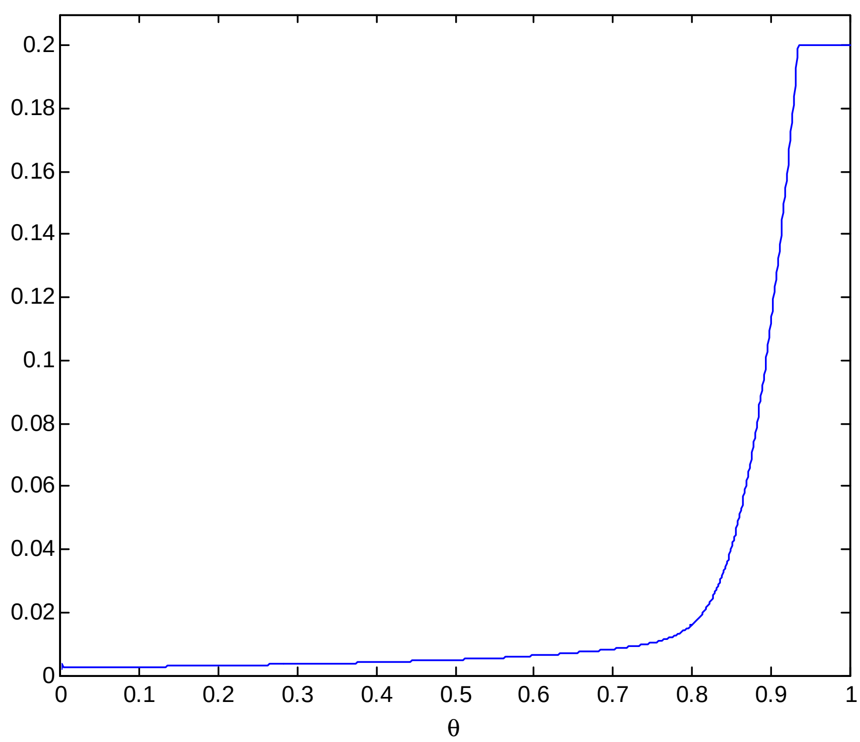

Figure 4.7: Policy function $v(\theta)$

5.2 The special (*iid*) case: the decentralized economy

Markets are complete and consumers trade in equity shares of the tree and in a riskless bond. The consumer's problem is given recursively by

$$V(d,w,\theta) = \max_{c,b,e} \left\{ \log c + \beta \min_{v} \sum_{s'=1}^{2} \phi_{s'}(v) V(\lambda_{s'} d, w'_{s'}, \theta'_{s'}) \right\},$$

subject to the budget constraint

$$c + p(d,\theta)e + q(d,\theta)b = w, \tag{4.47}$$

$$w'_{s'} = b + e\left[\lambda_{s'} d + p(\lambda_{s'} d, \theta'_{s'})\right], \tag{4.48}$$

and the law of motion for $\theta'_{s'}$ given by

$$\theta'_{s'} = g_{s'}(d,\theta), \tag{4.49}$$

where (d,w,θ) is the state vector. As before, w is the consumer's wealth today, p is the price of equity, e is the fraction of the equity share held by the consumer, q is the price today of a bond that pays one unit of the consumption good next period, and b is the holdings of the bond. (The argument d is included for g only for completeness; it will not be there under the log assumption.)

The consumers' decision rules for all (d,w,θ) are

$$c_i(d,w,\theta) \tag{4.50}$$

$$b_i(d,w,\theta) \tag{4.51}$$

$$e_i(d,w,\theta) \tag{4.52}$$

for $i \in \{1,2\}$.

Total wealth in the economy when the state variable is (d,θ) is $d + p(d,\theta)$. Thus, market clearing requires, for all values of the arguments,

$$c_1(d, \theta\left[d + p(d,\theta)\right], \theta) + c_2(d, (1-\theta)\left[d + p(d,\theta)\right], \theta) = d, \tag{4.53}$$

$$b_1(d, \theta\left[d + p(d,\theta)\right], \theta) + b_2(d, (1-\theta)\left[d + p(d,\theta)\right], \theta) = 0, \tag{4.54}$$

and

$$e_1(d, \theta\left[d + p(d,\theta)\right], \theta) + e_2(d, (1-\theta)\left[d + p(d,\theta)\right], \theta) = 1. \tag{4.55}$$

The relative wealth dynamics, finally, is given by

$$g_{s'}(d,\theta) = \frac{w'_{1s'}(d,\theta)}{w'_{1s'}(d,\theta) + w'_{2s'}(d,\theta)}, \tag{4.56}$$

where

$$w'_{1s'}(d,\theta) \equiv b_1(d, \theta\left[d + p(d,\theta)\right], \theta) + e_1(d, \theta\left[d + p(d,\theta)\right], \theta)(d\lambda_{s'} + p\left[d\lambda_{s'}, g_{s'}(d,\theta)\right]),$$

and

$$w'_{2s'}(d,\theta) \equiv b_2(d, (1-\theta)\left[d + p(d,\theta)\right], \theta) + e_2(d, (1-\theta)\left[d + p(d,\theta)\right], \theta)(d\lambda_{s'} + p\left[d\lambda_{s'}, g_{s'}(d,\theta)\right]).$$

Now we will show how to find prices and portfolio allocations in this economy. We use the planning problem and we identify the θ in that problem with the corresponding variable here: the planning weight on agent 1 equals the relative fraction of total wealth held in equilibrium by this agent.

The price of bonds, $q(d,\theta)$, then becomes

$$q(d,\theta) = \beta\left[\frac{\phi}{\lambda_1} + \frac{1-\phi}{\lambda_2} + \theta v\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right)\right] \equiv \hat{q}(\theta). \tag{4.57}$$

As shown in section 3, the price of the bond is increasing in a. In addition here, the price of bonds is increasing in θ. We show below that the ambiguity-averse agents demand the bond. The bond is more valuable when marginal utility of consumption is high (which occurs in the bad state). As θ increases, there is a higher demand for the bond, so its price goes up.

And the price of equity, $p(d,\theta)$, is given by

$$p(d,\theta) = \beta\left\{\frac{(\phi - \theta v)\left[\lambda_1 d + p(\lambda_1 d, \theta'_1)\right]}{\lambda_1} + \frac{(1-\phi+\theta v)\left[\lambda_2 d + p(\lambda_2 d, \theta'_2)\right]}{\lambda_2}\right\}, \tag{4.58}$$

where we recall that

$$\theta'_1 = \frac{\theta(\phi - v)}{\phi - \theta v} \leq \theta, \tag{4.59}$$

and

$$\theta_2' = \frac{\theta(1-\phi+v)}{1-\phi+\theta v} \geq \theta, \tag{4.60}$$

from the planning problem. (The inequalities above follow since $v \geq 0$.)

The latter laws of motion reveal that the ambiguity-averse agent gains in relative wealth when the state is bad and loses when it is good: his probability "beliefs" are tilted toward the bad state.

We see that $p(d, \theta) = d\hat{p}(\theta)$ solves this equation, delivering

$$\hat{p}(\theta) = \beta \left\{ (\phi - \theta v) \left[1 + \hat{p}(\theta_1')\right] + (1 - \phi + \theta v) \left[1 + \hat{p}(\theta_2')\right] \right\}. \tag{4.61}$$

This is a functional equation: it holds for all θ (recall that v may also depend on θ). The solution to this functional equation is

$$\hat{p}(\theta) = \frac{\beta}{1-\beta}, \tag{4.62}$$

and

$$p(d, \theta) = d\frac{\beta}{1-\beta}. \tag{4.63}$$

So the price of equity does not depend on θ.

The equilibrium holdings of equity of consumer 1, which can be obtained by using the expression for future wealth, $w_{1s'}' = b_1 + e_1(\lambda_{s'}d + p_{s'}')$, together with the equilibrium condition that $w_{1s'}' = \theta_{s'}'(d\lambda_{s'} + p_{s'}')$, are given by

$$e_1(d, \theta) = \frac{\theta_1'\lambda_1 - \theta_2'\lambda_2}{\lambda_1 - \lambda_2} \equiv \hat{e}_1(\theta). \tag{4.64}$$

Thus, the equity holdings of agent 1 are independent of the level of d. We see that if $v = 0$, in which case $\theta_1' = \theta_2' = \theta$, then $\hat{e}_1(\theta) = \theta$: the consumer's share of the tree equals his initial share of total wealth.

On the other hand, when $v > 0$ (recall that wlog we use $\lambda_1 > \lambda_2$), we know that $\theta_1' < \theta < \theta_2'$, which makes the holdings of equity lower as compared to the case when $v = 0$. That is, the ambiguity-averse agent will have a smaller share of equity holdings than his overall wealth would otherwise prescribe: this is a *portfolio composition* effect. How much his portfolio composition will be changed must be numerically examined.

We can also examine the portfolio effect by looking at the amount of bonds purchased by agent 1. Her equilibrium holdings of bonds are obtained as

$$b_1(d,\theta) = \frac{d\lambda_1}{1-\beta}\left[\theta_1' - \hat{e}_1(\theta)\right]. \tag{4.65}$$

It is interesting to note here that bond holdings are proportional to d. Naturally, they are zero in the special case $\upsilon = 0$, when $e_1 = \theta$ and $\theta' = \theta$. Moreover,

$$\theta_1' - \hat{e}_1(\theta) = \theta_1' - \frac{\theta_1'\lambda_1 - \theta_2'\lambda_2}{\lambda_1 - \lambda_2} =$$

$$= \theta_1'\left(1 - \frac{\lambda_1 - \frac{\theta_2'}{\theta_1'}\lambda_2}{\lambda_1 - \lambda_2}\right) > 0, \tag{4.66}$$

since $\theta_2' > \theta_1'$, and thus we conclude, consistently with the above insights regarding equity holdings, that the ambiguity-averse agent *increases* his bond holdings relative to the $\upsilon = 0$ zero-bonds case: his portfolio composition moves away from equity and into bonds because he is more pessimistic than person 2 in his perception of the return (performance) of equity.

There are two sources of uncertainty in this economy: (i) the payoff of equity and (ii) the price of the bond. The price of the bond depends on θ, the relative wealth of consumer 1, and this variable is random. In particular, since $\theta_2' > \theta > \theta_1'$, the price of the bond, q, increases if state 2 occurs and it decreases if state 1 occurs.

Below we numerically compute solutions for $\upsilon(\theta)$, $\theta_1'(\theta)$, $\theta_2'(\theta)$, $e(\theta)$, $\frac{p(\theta)e(\theta)}{p(\theta)e(\theta)+q(\theta)b(\theta)}$, $q(\theta)$, and $b(\theta)$ for agent 1. Once more, the parameter values are $\lambda_1 = 1.02$, $\lambda_2 = 1.01$, and $\beta = 0.98$.

As we see from the graphs in figure 4.8, the ambiguity-averse consumer short-sells equity for most values of θ. The reason for this is the following. State 2 is bad for the ambiguity-averse consumer for two reasons: (i) the payoff from equity is low and (ii) the price of the bond increases so that it makes the good next period more expensive (this consumer does not own any goods next period). Therefore, to provide protection against the former type of uncertainty, the ambiguity-averse consumer buys bonds and to provide protection against the latter type of uncertainty, the ambiguity-averse consumer sells equity short.

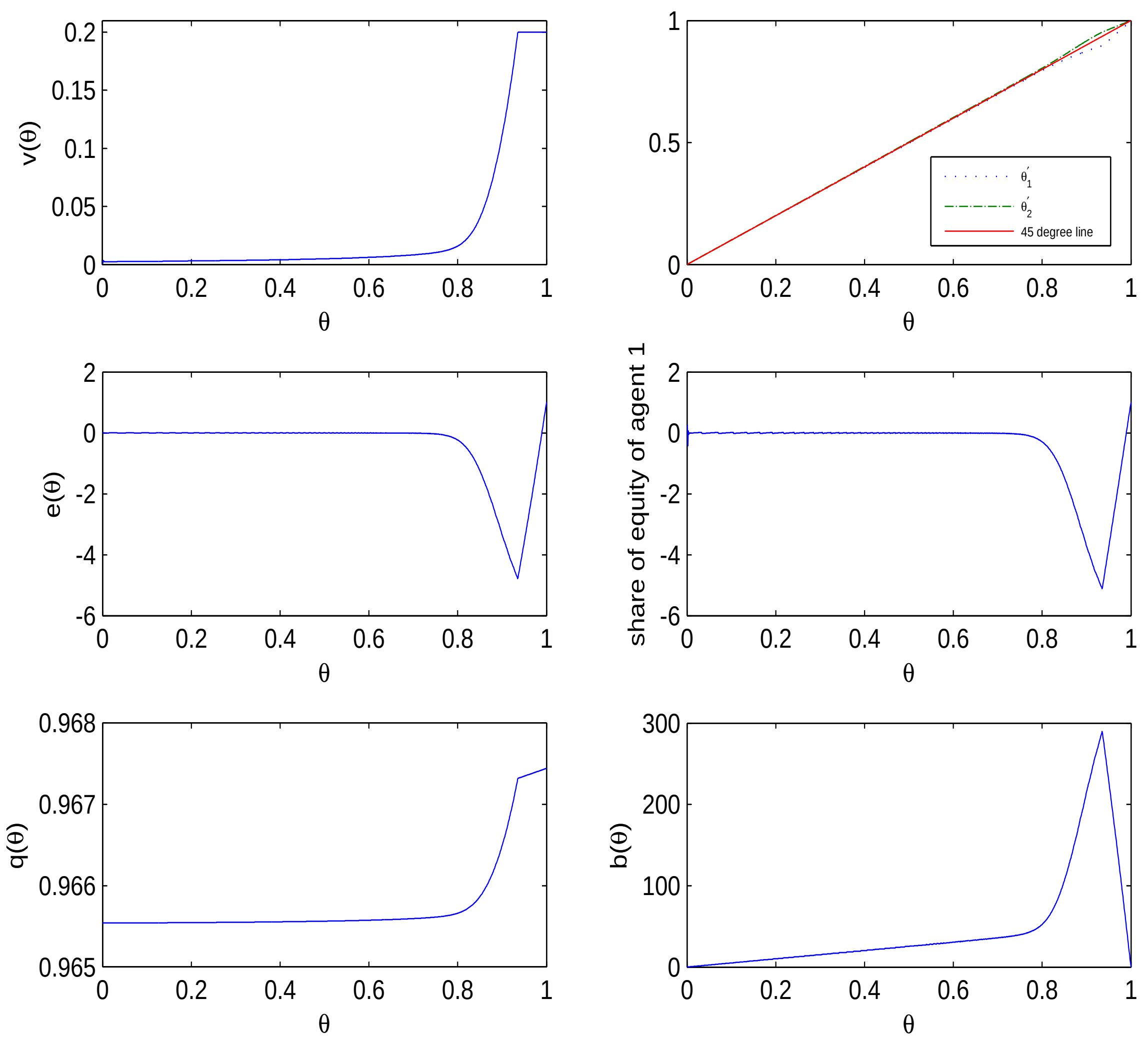

Figure 4.8: From left to right from top to bottom: (a) $v(\theta)$, (b) law of motion for θ (θ_1 below the 45 degree line; θ_2 above the 45 degree line), (c) $e_1(\theta)$, (d) share of equity of agent 1, (e) $q(\theta)$, (f) $b(\theta)$

The behavior of the ambiguity-averse consumer can be separately described for different ranges of θ. First, when θ is zero, the ambiguity-averse consumers have zero aggregate wealth. In this case, the price of bonds is solely determined by the "standard" agents and it does not fluctuate. Since there is no uncertainty on q, ambiguity-averse consumers only hold bonds. As shown in the below section, a very small amount of pessimism rationalizes this choice.

If θ is positive but small, changes in θ do not have any considerable effects on

q, so the randomness in q is not so important. Then, ambiguity-averse consumers mainly hold bonds and short-sell equity somewhat to protect against the uncertainty in p. This asset choice makes

$$V(w_1', \theta_1') = V(w_2', \theta_2') \tag{4.67}$$

for a small value of the belief v; that is, v is still an interior solution.

When θ is high, ambiguity aversion makes the fluctuations in q very large. Agents buy bonds and short-sell equity more heavily. The value of v is larger, reflecting more pessimism about state 2. Since V is decreasing in θ and increasing in w (the former is true because q is increasing in θ), and since θ_1' is much larger than θ_2', w_2' needs to be much larger than w_1' in order to equate $V(w_1', \theta_1')$ and $V(w_2', \theta_2')$ – and hence still make v an interior solution. This is achieved by short-selling equity even more heavily.

When θ is very close to 1, v is a corner solution since the ambiguity-averse agents need to hold most of the stock and they are pessimistic about state 1. The fluctuations in θ have become very small, and the uncertainty resulting from changes in q is therefore also very small and ambiguity-averse agents consequently do not need to short-sell the stock.

5.3 Relative consumption and wealth in the long run

We can analytically show[2] that

$$E(\theta'|\theta) < \theta, \tag{4.68}$$

i.e., that over time, the relative wealth of the ambiguity-averse agents decreases toward zero: these agents disappear, economically speaking.

However, it can also be shown that

$$E\left(\frac{\theta'}{\theta}\right) \rightarrow_{\theta \rightarrow 0} 1, \tag{4.69}$$

so the rate at which they disappear goes to zero: they remain with positive wealth

[2] The proofs of expressions (4.68) and (4.69) are in the appendix. This result and the following discussion are reminiscent of the analysis in Coen-Pirani (2004).

for a long, long time.

6 Conclusion

In this essay, we have studied asset pricing and evaluated the welfare costs of fluctuations in consumption for an economy where consumers are ambiguity-averse. First, we have shown parameter configurations under which the equity premium is quite large (and the riskfree rate is small); the ability to match these return features comes from the ability of ambiguity aversion to generate *first-order* effects on prices, which sets it apart from risk aversion, which operates through second-order effects. Ambiguity aversion has first-order effects, in essence, because consumers behave as if they believed that the good return outcomes to be less likely than they really are.

Second, using the calibrations that deliver realistic asset prices, we have shown that the welfare benefits of eliminating consumption fluctuations need not be as small as those in Lucas's (2003) calculations. This is not to say that the benefits are large: the numbers we obtain are, just like Lucas's numbers, upper bounds, and these upper bounds leave open what the costs of stabilization (say, in the form of distortions) might be, and also leave open whether full stabilization is even feasible. Nevertheless, it is valuable to note that these bounds can be as large as 15% of consumption when asset prices are matched by the model.

Third, by exploring an economy where some consumers are ambiguity-averse and others are not, we find an important qualification to the above findings: it appears that, by making consistently "bad bets", ambiguity-averse consumers will see their relative wealth decline over time, and thereby asset prices will be increasingly dominated by standard consumers. Note also that these bad bets are not bad in the sense of "crazy portfolios", but simply in the sense of delivering a lower return on average by not investing enough in stock. In particular, if ambiguity aversion is sufficiently large, the ambiguity-averse consumers choose to not participate at all in the stock market: the other, standard consumers hold all risk (and get all the high returns on average).[3] To make this wealth distribution not converge to an extreme outcome,

[3] It is interesting to note that there is (close to) non-participation for a large range of values for θ. Thus, without having to assume that there are costs of transacting/investing in stock, we can use this setting to derive conditions under which a large fraction of the population—the ambiguity-averse—(almost) do not have any stock. This kind of result was also derived in Epstein

one could consider an overlapping-generations structure, where in each generation of newborns with zero debt, some are ambiguity-averse; that way, a significant part of aggregate wealth will always belong to ambiguity-averse consumers.

and Schneider's (2007) work. Exact non-participation cannot be obtained here because the risk-free rate fluctuates with the endowment shock; because the ambiguity-averse agents hold bonds, it is optimal for them to use equity to hedge against the interest-rate risk. This risk, however, is very small for a large range of (low) values of θ: when θ is zero, the risk-free rate is constant, and thus not until the ambiguity-averse agents have a significant fraction of total wealth will these fluctuations be large enough to induce significant equity holdings for these agents.

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Appendix

A1 Heterogeneity in ambigiuty aversion and serial correlation

The planning problem

In this case, we have

$$V_s(d,\theta) = \max_{c_1,c_2,\theta'_{s'}} \theta \log c_1 + (1-\theta)\log c_2 +$$

$$+\beta \min_{v_s} \left\{ \sum_{s'=1}^{2} \phi_{ss'}(\theta v) V_{s'}(d\lambda_{s'}, \theta'_{s'}) \right\}$$

subject to

$$\theta'_{s'} = \theta \frac{\phi_{ss'}(v_s)}{\phi_{ss'}(\theta v_s)}, \tag{4.70}$$

and

$$c_1 + c_2 = d. \tag{4.71}$$

Using the FOCs for consumption, we obtain $c_1 = \theta d$ and $c_2 = (1-\theta)d$ so we get

$$V_s(d,\theta) = \log d + \log \theta^{\theta}(1-\theta)^{1-\theta} +$$

$$+\beta \min_{v_s} \left[(\phi_{s1} - \theta v_s) V_1(d\lambda_1, \theta'_1) + (\phi_{s2} + \theta v_s) V_2(d\lambda_2, \theta'_2) \right]$$

with

$$\theta'_1 = \theta \frac{\phi_{s1} - v_s}{\phi_{s1} - \theta v_s}, \tag{4.72}$$

and

$$\theta'_2 = \theta \frac{\phi_{s2} + v_s}{\phi_{s2} + \theta v_s}. \tag{4.73}$$

Here, we conjecture that $V_s(d,\theta)$ takes the form $A\log d + W_s(\theta)$. This guess delivers

$$A\log d + W_s(\theta) = \log d + \log \theta^{\theta}(1-\theta)^{1-\theta} +$$

$$+\beta \min_{v_s} \left((\phi_{s1} - \theta v_s)(A\log(d\lambda_1) + W_1(\theta'_1)) + (\phi_{s2} + \theta v_s)(A\log(d\lambda_2) + W_2(\theta'_2)) \right).$$

Inspecting the above expression, it can be seen that $A = \frac{1}{1-\beta}$ works and leaves

$$W_s(\theta) = \log \theta^{\theta}(1-\theta)^{1-\theta} +$$

$$+\beta \min_{v_s} \left\{ (\phi_{s1} - \theta v_s) \left[\frac{\log \lambda_1}{1-\beta} + W_1 \left(\theta \frac{\phi_{s1} - v_s}{\phi_{s1} - \theta v_s} \right) \right] + (\phi_{s2} + \theta v_s) \left[\frac{\log \lambda_2}{1-\beta} + W_2 \left(\theta \frac{\phi_{s2} + v_s}{\phi_{s2} + \theta v_s} \right) \right] \right\}$$

for $s = 1, 2$. This is a two-dimensional dynamic programming problem that delivers optimal v_s, $s = 1, 2$, as a function of θ, and hence a law of motion for θ.

The decentralized economy

The problem of the consumer is

$$V_s(d, w, \theta) = \max_{c,b,e} \left\{ \log c + \beta \min_{v_s} \sum_{s'=1}^{2} \phi_{ss'}(v_s) V_{s'}(\lambda_{s'} d, w'_{s'}, \theta'_{s'}) \right\}$$

subject to the budget constraint

$$c + p_s(d, \theta) e + q_s(d, \theta) b = w, \tag{4.74}$$

$$w'_{s'} = b + e \left[\lambda_{s'} d + p_{s'}(d\lambda_{s'}, \theta'_{s'}) \right], \tag{4.75}$$

and the law of motion for $\theta'_{s'}$ given by

$$\theta'_{s'} = g_{s'}(d, \theta, s) \tag{4.76}$$

where p is the price of equity, e is the fraction of the equity share held by the consumer, q is the price today of a bond that pays one unit of the consumption good next period, and b is the holdings of the bond. (The argument d is included for g only for completeness; it will not be there under the log assumption.)

The consumers' decision rules for all (d, w, θ, s) are

$$c_{is}(d, w, \theta) \tag{4.77}$$

$$b_{is}(d, w, \theta) \tag{4.78}$$

$$e_{is}(d, w, \theta) \tag{4.79}$$

for $i \in \{1, 2\}$.

Total wealth in the economy when the state variable is (d, θ, s) is $d + p_s(d, \theta)$.

Thus, market clearing requires, for all values of the arguments,

$$c_{1s}(d, \theta\left[d + p_s(d,\theta)\right], \theta) + c_{2s}(d, (1-\theta)\left[d + p_s(d,\theta)\right], \theta) = d \tag{4.80}$$

$$b_{1s}(d, \theta\left[d + p_s(d,\theta)\right], \theta) + b_{2s}(d, (1-\theta)\left[d + p_s(d,\theta)\right], \theta) = 0 \tag{4.81}$$

$$e_{1s}(d, \theta\left[d + p_s(d,\theta)\right], \theta) + e_{2s}(d, (1-\theta)\left[d + p_s(d,\theta)\right], \theta) = 1, \tag{4.82}$$

The relative wealth dynamics, finally, are given by

$$g_{s'}(d,\theta,s) = \frac{w'_{1s'}(d,\theta,s)}{w'_{1s'}(d,\theta,s) + w'_{2s'}(d,\theta,s)}, \tag{4.83}$$

where

$$w'_{1s'}(d,\theta,s) \equiv b_{1s}(d,\theta\left[d+p_s(d,\theta)\right],\theta) + e_{1s}(d,\theta\left[d+p_s(d,\theta)\right],\theta)(d\lambda_{s'} + p_{s'}\left[d\lambda_{s'}, g_{s'}(d,\theta)\right])$$

and

$$w'_{2s'}(d,\theta,s) \equiv b_{2s}(d,(1-\theta)\left[d+p_s(d,\theta)\right],\theta) + e_{2s}(d,(1-\theta)\left[d+p_s(d,\theta)\right],\theta)(d\lambda_{s'} + p_{s'}\left[d\lambda_{s'}, g_{s'}(d,\theta)\right])$$

Now, we will show how to find prices and portfolio allocations in this economy. We use the planning problem and identify the θ in that problem with the corresponding variable here: the planning weight on agent 1 equals the relative fraction of total wealth held in equilibrium by this agent.

The prices of bonds, $q_s(d,\theta)$, and of equity, $p_s(d,\theta)$, then become

$$q_s(d,\theta) = \beta\left[\phi_{s1}(v)\frac{\phi_{s1}-\theta v}{(\phi_{s1}-v)\lambda_1} + \phi_{s2}(v)\frac{\phi_{s2}+\theta v}{(\phi_{s2}+v)\lambda_2}\right] \equiv \hat{q}_s(\theta), \tag{4.84}$$

and

$$p_s(d,\theta) = \beta\left\{\frac{(\phi_{s1}-\theta v)\left[\lambda_1 d + p_1(\lambda_1 d, \theta'_1)\right]}{\lambda_1} + \frac{(\phi_{s2}+\theta v)\left[\lambda_2 d + p_2(\lambda_2 d, \theta'_2)\right]}{\lambda_2}\right\} \tag{4.85}$$

We see that $p_s(d,\theta) = d\hat{p}_s(\theta)$ solves this equation, delivering

$$\hat{p}_s(\theta) = \beta\left\{(\phi_{s1}-\theta v)\left[1+\hat{p}_1(\theta'_1)\right] + (\phi_{s2}+\theta v)\left[1+\hat{p}_2(\theta'_2)\right]\right\} \tag{4.86}$$

This is a system of two functional equations.

Asset holdings are the following. First, his equilibrium holdings of bonds are

$$b_{1s}(d,\theta) = [d\lambda_1 + p_1(\lambda_1 d, \theta_1')]\,[\theta_1' - e_{1s}(d,\theta)] = d\lambda_1\,[1+\hat{p}_1(\theta_1')]\,[\theta_1' - \hat{e}_{1s}(\theta)]$$

It is interesting to note here that bond holdings are proportional to d. Naturally, they are zero in the special case $v = 0$, when $e = \theta$ and $\theta' = \theta$.

And his equilibrium holdings of equity are

$$e_{1s}(d,\theta) = \frac{\theta_1'\,[d\lambda_1 + p_1(\lambda_1 d,\theta_1')] - \theta_2'\,[d\lambda_2 + p_2(\lambda_2 d,\theta_2')]}{[d\lambda_1 + p_1(\lambda_1 d,\theta_1')] - [d\lambda_2 + p_2(\lambda_2 d,\theta_2')]} =$$

$$= \frac{\theta_1'\lambda_1\,[1+\hat{p}_1(\theta_1')] - \theta_2'\lambda_2\,[1+\hat{p}_2(\theta_2')]}{\lambda_1\,[1+\hat{p}_1(\theta_1')] - \lambda_2\,[1+\hat{p}_2(\theta_2')]} \equiv \hat{e}_{1s}(\theta) \tag{4.87}$$

This is once more a system of two functional equations.

Neither bond holdings nor equity holdings depend directly on s, but they do through the dependence of the θ's on s.

The special case where $\theta = 0$

We solve the problem for an ambiguity-averse agent who is measure zero in the economy. This agent solves the problem

$$V(w,d) = \max_{c,b,e} u(c) + \min_{v} \beta\,[(\phi - v)V(w_1', d\lambda_1) + (1-\phi+v)V(w_2', d\lambda_2)]$$

subject to

$$c + qb + pde = w \tag{4.88}$$

$$w_1' = b + (\lambda_1 d + p\lambda_1 d)e \tag{4.89}$$

$$w_2' = b + (\lambda_2 d + p\lambda_2 d)e \tag{4.90}$$

The FOCs with respect to b are

$$qu'(w - qb + pde) =$$

$$= \beta\,\{(\phi - v)u'\,[b + e\lambda_1 d(1+p) - qb' - pd\lambda_1 e'] + (1-\phi+v)u'\,[b + e\lambda_2 d(1+p) - qb' - pd\lambda_2 e']\}$$

and with respect to e, they are

$$pdu'(w - qb + pde) =$$

$$= \beta\{(\phi - v)u'\left[b + e\lambda_1 d(1+p) - qb' - pd\lambda_1 e'\right]\lambda_1 d(1+p) +$$

$$+(1 - \phi + v)u'\left[b + e\lambda_2 d(1+p) - qb' - pd\lambda_2 e'\right]\lambda_2 d(1+p)\}$$

Using logarithmic utility, we see that these equations become

$$q = \beta\left[\frac{\phi - v}{b + e\lambda_1 d(1+p) - qb' - pd\lambda_1 e'} + \frac{1 - \phi + v}{b + e\lambda_2 d(1+p) - qb' - pd\lambda_2 e'}\right](w - qb + pde)$$

$$\frac{p}{1+p} = \beta\left[\frac{(\phi - v)\lambda_1}{b + e\lambda_1 d(1+p) - qb' - pd\lambda_1 e'} + \frac{(1 - \phi + v)\lambda_2}{b + e\lambda_2 d(1+p) - qb' - pd\lambda_2 e'}\right](w - qb + pde).$$

We guess that

$$b = \alpha_b w \tag{4.91}$$

and

$$ed = \alpha_e w. \tag{4.92}$$

Then

$$q =$$

$$= \beta\left\{\frac{\phi - v}{\left[\alpha_b + \alpha_e\lambda_1(1+p)\right](1 - q\alpha_b - p\lambda_1\alpha_e)} + \frac{1 - \phi + v}{\left[\alpha_b + \alpha_e\lambda_2(1+p)\right](1 - q\alpha_b - p\lambda_2\alpha_e)}\right\}(1 - q\alpha_b - p\alpha_e)$$

$$\frac{p}{1+p} =$$

$$= \beta\left\{\frac{(\phi - v)\lambda_1}{\left[\alpha_b + \alpha_e\lambda_1(1+p)\right](1 - q\alpha_b - p\lambda_1\alpha_e)} + \frac{(1 - \phi + v)\lambda_2}{\left[\alpha_b + \alpha_e\lambda_2(1+p)\right](1 - q\alpha_b - p\lambda_2\alpha_e)}\right\}(1 - q\alpha_b - p\alpha_e)$$

The problem of the consumer can be rewritten as

$$V(w) = \max_{c,b,e} \log c + \min_{v} \beta\left[(\phi - v)V(w_1') + (1 - \phi + v)V(w_2')\right]$$

subject to

$$c + qb + p\hat{e} = w \tag{4.93}$$

$$w_1' = b + \hat{e}\lambda_1(1+p) \tag{4.94}$$

$$w_2' = b + \hat{e}\lambda_2(1+p), \tag{4.95}$$

where $\hat{e} \equiv de$. The variable v will be chosen (due to the envelope theorem) so that $V(w_1') = V(w_2')$, i.e., so that $w_1' = w_2'$. That means that $b = \alpha_b w$ and $\hat{e} = 0$ – the agent does not hold equity – and from the FOC above, that

$$q = \beta \left[\frac{\phi - v}{\alpha_b(1 - q\alpha_b)} + \frac{1 - \phi + v}{\alpha_b(1 - q\alpha_b)} \right] (1 - q\alpha_b). \tag{4.96}$$

This expression simplifies to

$$\alpha_b q = \beta \tag{4.97}$$

and then consumption is given by

$$c = (1 - \beta)w. \tag{4.98}$$

Since $\frac{p}{1+p} = \beta$ in the $\theta = 1$ case, this implies

$$\alpha_b = (\phi - v)\lambda_1 + (1 - \phi + v)\lambda_2. \tag{4.99}$$

Therefore,

$$v = \frac{\phi\lambda_1 + (1 - \phi)\lambda_2 - \alpha_b}{\lambda_1 - \lambda_2} = \phi - \frac{\alpha_b - \lambda_2}{\lambda_1 - \lambda_2} \tag{4.100}$$

and

$$v = \phi - \frac{\alpha_b - \lambda_2}{\lambda_1 - \lambda_2}. \tag{4.101}$$

For $\phi = 0.5$, $\beta = 0.98$ $\lambda_1 = 1.02$, and $\lambda_2 = 1.01$, $\alpha_b = \frac{2\lambda_1\lambda_2}{\lambda_1+\lambda_2}$, and

$$v = \phi - \frac{\lambda_2}{\lambda_1 + \lambda_2} = 0.00246. \tag{4.102}$$

A2 Proofs of subsection 5.3

We want to proof that

$$E(\theta' \mid \theta) < \theta. \tag{4.103}$$

Since

$$E(\theta' \mid \theta) = \phi \frac{\theta\,(\phi - v)}{\phi - \theta v} + (1 - \phi)\,\frac{\theta\,(1 - \phi + v)}{1 - \phi + \theta v}, \tag{4.104}$$

expression (4.103) becomes:

$$\phi\frac{\theta\left(\phi-v\right)}{\phi-\theta v}+\left(1-\phi\right)\frac{\theta\left(1-\phi+v\right)}{1-\phi+\theta v}<\theta. \tag{4.105}$$

Simplifying (4.105) yields:

$$\theta^2 v^2<\theta v^2. \tag{4.106}$$

Since $v\neq 0$,

$$\theta<1. \tag{4.107}$$

And condition (4.107) is always true in the case we study, otherwise we would be back to the case of one agent.

The proof that $\lim_{\theta\to 0}E\left(\frac{\theta'}{\theta}\right)=1$ is even simpler. First, consider expression for $E\left(\frac{\theta'}{\theta}\right)$:

$$E\left(\frac{\theta'}{\theta}\right)=\phi\frac{\phi-v}{\phi-\theta v}+\left(1-\phi\right)\frac{1-\phi+v}{1-\phi+\theta v}. \tag{4.108}$$

Then, the limit becomes:

$$\lim_{\theta\to 0}\phi\frac{\phi-v}{\phi-\theta v}+\left(1-\phi\right)\frac{1-\phi+v}{1-\phi+\theta v}=$$

$$\phi\frac{\phi-v}{\phi}+\left(1-\phi\right)\frac{1-\phi+v}{1-\phi}=\phi-v+1-\phi+v=1. \tag{4.109}$$