Real Effects of Monetary Regimes

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Abstract

This thesis consists of three essays on the real effects of monetary regimes.

Monetary Regimes, Labour Mobility and Equilibrium Employment analyses the impact of the monetary regime on labour markets in a small open economy by considering the game between large wage setters and an independent central bank in a two-sector model with potential labour mobility between sectors. Two monetary regimes are considered: membership in a monetary union and a national inflation target combined with a flexible exchange rate. A key result is that when there is perfect labour mobility between sectors, the monetary regime is of no importance for real wages, employment or profits. Labour mobility substantially reduces wages and increases employment. Other findings are that when labour is immobile between sectors: (i) the real wage in the tradables sector is higher under inflation targeting than in a monetary union, while the reverse applies to the non-tradables sector; (ii) inflation targeting generates higher employment and profits than membership in a monetary union; and (iii) both workers and firms in the two sectors in general prefer inflation targeting to membership in a monetary union.

Fiscal Activism under Inflation Targeting and Non-atomistic Wage Setting considers a game between the government, an independent central bank and nonatomistic wage setters in a model with monopolistic competition. The paper combines the literature on fiscal/monetary interactions with the literatures on strategic interaction between central banks and large wage setters and on strategic interaction between the government and centralised trade unions. I discuss the implications of inflation targeting and fiscal activism for the labour market outcome. The results suggest that while inflation targeting may discipline wage setters, activist fiscal policy generates higher real wages and lower employment. The main explanation is that unions exploit the government and make it assume responsibility for some of the cost of high wages. A key result is that the difference between regimes is greater if the government pursues activist fiscal policy, which suggests that inflation targeting is even more important in economies with a high degree of fiscal activism. Aggregate welfare is also higher the less activist is the government.

The Swedish Real Exchange Rate under Different Currency Regimes presents evidence on the behaviour of the Swedish real exchange rate relative to Germany under iv

different currency regimes 1973:1-2001:4. The results suggest that the real exchange rate is cointegrated with Swedish and German productivity, which is consistent with Balassa (1964) and Samuelson (1964). In the short run, the exchange rate regime has mattered for the dynamics of the real exchange rate. Deviations from long-run equilibrium have been adjusted more quickly when the nominal exchange rate has been allowed to float freely.

To my mother

Acknowledgments

This is a thesis on strategic interaction. Indeed, it is about other things as well, but when you start thinking about it and when you know what to look for, you realise that there is strategic interaction everywhere. During the five years that it took me to complete this thesis, I have had the great fortune to interact (more or less strategically) with some extraordinary people. I would like to take this opportunity to express my gratitude.

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Stockholm, December 2006

Anna Larsson

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Chapter 1

Introduction

This thesis consists of three self-contained essays on the real effects of monetary regimes. Below I describe the background, motivation and common themes of the thesis. I then briefly summarise the contents and results of each chapter.

Many European countries have been struggling with high unemployment since the early 1980s. As a consequence, policymakers and researchers alike have been trying to come up with plausible explanations and suitable remedies. Economists have suggested that differences in labour market institutions may explain some of the differences across countries. In particular, trade unions and the existence of collective bargaining have been targeted as possible culprits. It is well known that unions may have a negative effect on the aggregate labour market outcome if they are able to negotiate a higher wage than the one that would prevail in a perfectly competitive labour market. Excessively high wages in economies with high union density and/or high coverage of collective agreements may therefore help explain high unemployment. Although wage-setting systems differ substantially across Europe, collective bargaining covers on average two thirds of the European labour markets, suggesting that their impact on European unemployment may be substantial.

At the same time, monetary regime switches have been very common worldwide over the last fifteen years. Many countries have abandoned their systems of fixed exchange rates and taken one of the following two paths. Some countries, including Sweden, UK, Canada, New Zealand and Norway, have combined floating exchange rates with explicit inflation targeting. Others, including Germany, France, Italy, Finland and others, have chosen to irrevocably fix their exchange rates by joining the economic and monetary union (EMU). In the light of these developments, there has been an extensive debate on the impact of the monetary regime on macroeconomic performance, both among policymakers and researchers. In the academic debate, one strand of literature has challenged a conventional result known as the neutrality of money. The theory of the neutrality of money states that monetary policy merely has transitory effects on real variables such as real wages, employment and real output, but is unable to influence them in the long run. This recent literature explains inter alia how strategic interaction between large wage setters and the central bank may cause the monetary regime to be of importance for the labour market outcome; see, for instance, Cukierman and Lippi (1999), Soskice and Iversen (2000), Coricelli et al. (2000) and Calmfors (2001). Consider an economy where an independent central bank follows an inflation target. If large wage setters such as unions recognise that they influence the aggregate price level by their wage decisions, their behaviour will depend on the response by the central bank. If unions threaten the monetary target, the central bank will punish them by a monetary contraction that increases unemployment. A key result in the literature is therefore that an independent central bank may discipline wage setters and thus help reduce unemployment.

In addition to affecting labour markets, it is likely that the monetary regime affects real prices such as the real exchange rate. The real exchange rate is determined by the nominal exchange rate and relative price levels in national currencies and is perhaps the most important measure of the conditions facing firms engaging in international trade. Since the nominal exchange rate is a component of the real exchange rate, the currency regime, i.e. whether the exchange rate is fixed or floating, should affect real exchange rate dynamics and its short-run behaviour. Although there is some consensus among economists that this is the case (see, for instance, Mussa, 1986 and Taylor, 2002), arguments have yet to be theoretically formalised as well as empirically tested using recent data.

In this thesis, I elaborate on these issues and study how monetary regimes affect real economic variables such as real wages, employment and real exchange rates. The thesis consists of two theoretical papers on the impact of monetary regimes on the labour market outcome, and one empirical paper on real exchange rate determination under different currency regimes.

I address a number of questions with great policy relevance. These include:

- How does inflation targeting versus membership in a monetary union affect labour market outcomes in a small open economy?
- If the monetary regime is of importance for the labour market outcome, is this result robust to the more realistic assumption of labour mobility across

sectors?

- Given that an inflation target can discipline wage setters and thus promote employment, can the government achieve similar results by means of fiscal policy?
- In a setting where monetary policy is tied to the mast by inflation targeting, can fiscal policy help reduce unemployment?
- Should the government pursue activist fiscal policy in an economy with an inflation target and large wage setters or are there welfare gains to be made by following a fiscal rule?
- How are the conditions facing firms trading internationally affected by the exchange rate regime?
- Has the switch in currency regimes in recent years affected the ability of the real exchange rate to return to long-run equilibrium?

Below, I summarise each chapter of the thesis in turn.

Chapter 2: Monetary Regimes, Labour Mobility and Equilibrium Employment

The first essay analyses the impact of the monetary regime on labour markets in a small open economy, by considering the game between large wage setters and an independent central bank in a two-sector model with potential labour mobility between sectors. Two monetary regimes are considered: membership in a monetary union and national inflation targeting combined with a flexible exchange rate. The paper revisits some issues discussed in the emerging literature on money nonneutrality in the presence of large wage setters; see Calmfors (2001) for a review. Although most studies on the impact of the monetary regime on unionised labour markets consider closed economies, exceptions include Vartiainen (2002) and Holden (2003) which both model a small open economy consisting of two sectors. The main objective of the essay is to introduce the realistic feature of labour mobility in a theoretical model of a small open economy. The hypothesis is that the impact of the monetary regime on sectoral wages and employment may be offset by worker migration in the long run. Given the option to move, rational workers will migrate to sectors where their expected utility of looking for a job is higher. A key result of the analysis is that when there is perfect labour mobility between sectors, the monetary regime is indeed of no importance for real wages, employment or profits. Worker migration causes real wages to be equalised across sectors and regimes. Moreover, introducing labour mobility substantially reduces wages and increases employment. This suggests that labour mobility is an important factor that should be taken into account, both when modelling labour markets theoretically, but also when designing labour market policy.

Other findings are that when labour is immobile between sectors: (i) the real wage in the tradables sector is higher under inflation targeting than in a monetary union, while the reverse applies to the non-tradables sector; (ii) inflation targeting generates higher employment and profits than membership in a monetary union; and (iii) both workers and firms in the two sectors in general prefer inflation targeting to membership in a monetary union.

The essay thus lends support to the previous literature on the benefits of inflation targeting, but suggests that the regime may be of little importance in the long run.

Chapter 3: Fiscal Activism under Inflation Targeting and Non-atomistic Wage Setting

In the second essay, I consider a game between the government, an independent central bank and non-atomistic wage setters in a model with monopolistic competition. The paper combines the literature on fiscal/monetary interactions with the literatures on strategic interaction between central banks and large wage setters and strategic interaction between the government and a centralised trade union (see, for instance, Hersoug (1985), Driffill (1985) and Calmfors and Horn (1985,1986)). The essay is mainly motivated by the recent monetary regime switches across Europe and their associated challenges to economic policy. An important issue for most European countries is how to conduct fiscal policy when monetary policy is either relinquished by irrevocably fixing the exchange rate as in the EMU, or tied to the mast by a predetermined inflation target. Although there is a recent but growing literature on the interaction and substitutability between fiscal and monetary policy (see, for instance, Dixit and Lambertini 2001, 2003), we know very little about how fiscal policy should be set in the presence of inflation targeting *and* non-atomistic wage setters.

In this essay, I introduce a government that provides a public good and makes endogenous decisions about fiscal policy, in a setting where large unions and the

central bank interact strategically. The central bank has an objective function that encompasses inflation targeting as a special case. I discuss the implications of inflation targeting for the labour market outcome, and evaluate the implications of fiscal activism. The model is able to replicate key findings in the previous literature on monetary regimes and labour markets, but also offers some new and important insights. The results show that while inflation targeting may discipline wage setters, endogenising fiscal policy generates higher real wages and lower employment. The main explanation is that when the government strives to maintain high employment, unions exploit the government's policy response and make it assume responsibility for some of the cost of high wages. Real wages are increasing in the degree of fiscal activism, suggesting that there are welfare gains to be made by pursuing less activist fiscal policy, or in the extreme to implement a fiscal rule. A key result is that the differences between monetary regimes is greater if the government pursues activist fiscal policy. This suggests that if the objective is moderation in wage setting, inflation targeting is even more important in economies characterised by a high degree of fiscal activism.

Chapter 4: The Swedish Real Exchange Rate under Different Currency Regimes

In the third essay,¹ I present evidence on the behaviour of the Swedish real exchange rate relative to that of Germany under different currency regimes 1973:1-2001:4. Although there is a fair amount of empirical evidence on real exchange rate determination, evidence on how the real exchange rate behaves under different currency regimes is scant. Taylor (2002) studies the impact of the regime in a study of the related concept of purchasing power parity. One paper on the impact of the regime on the real exchange rate is Mussa (1986). However, due to the switches in exchange rate regimes over the last fifteen years, it is essential to revisit the issue using recent data. I model both the long-run behaviour and the short-run dynamics of the real exchange rate. Moreover, I study the dynamics of its three separate components: the Swedish and German price levels and the nominal exchange rate.

I find that the real exchange rate is cointegrated with Swedish and German productivity, which is consistent with Balassa (1964) and Samuelson (1964). The result suggests that the main determinant of the real exchange rate in the long run

¹ Published in *Review of World Economics* 140 (4), 2004.

is relative productivity growth: higher relative growth in total factor productivity at home than abroad induces a real appreciation. In the short run, demand-side factors, such as the interest rate differential, also affect the behaviour of the real exchange rate. The exchange rate regime is of importance for the dynamics of the real exchange rate: deviations from long-run equilibrium are more quickly adjusted when the nominal exchange rate is allowed to float freely. A much debated topic in recent years is whether the nominal exchange rate is a shock absorber or a source of shocks. Although I do not formally test for the stabilising properties of the nominal exchange rate, the finding that the nominal exchange rate is equilibrating may be interpreted as indicative evidence that the exchange rate tends to offset shocks. Finally, the study shows that while all components of the real exchange rate contribute to adjustments to the long-run equilibrium, the Swedish price level and the nominal exchange rate respond more forcefully than the German price level to deviations from equilibrium. This suggests that, being a smaller country than Germany, Sweden is forced to adapt to German conditions rather than the other way around.

Bibliography

- Balassa, B. (1964), "The Purchasing Power Doctrine: A Reappraisal", The Journal of Political Economy, Vol. 72, pp. 584-596
- [2] Calmfors, L. (2001), "Wages and Wage-Bargaining Institutions in the EMU -A Survey of the Issues", *Empirica 28*
- [3] Calmfors, L. and Horn, H., (1985), "Classical Unemployment, Accommodation Policies and the Adjustment of Real Wages", *Scandinavian Journal of Economics*, Vol. 87 (2), pp. 234-261
- [4] Calmfors, L. and Horn, H., (1986), "Employment Policies and Centralized Wage Setting", *Economica*, Vol. 53, pp. 281-302
- [5] Coricelli, F., Cukierman, A. & Dalmazzo, A.(2000), "Monetary Institutions, Monopolistic Competition, Unionized Labor Markets and Economic Performance", CEPR Discussion Paper 2407, Center for Economic Policy Research, London
- [6] Cukierman, A. and Lippi, F. (1999), "Central Bank Independence, Centralization of Wage bargaining, Inflation and Unemployment: Theory and Some Evidence", *European Economic Review* 43
- [7] Dixit, A. and Lambertini, L. (2001), "Monetary-fiscal Policy Interactions and Commitment Versus Discretion in a Monetary Union", *European Economic Review* 45, pp. 977-987
- [8] Dixit, A. and Lambertini, L. (2003), "Interactions of Commitment and Discretion in Monetary and Fiscal Policies", *American Economic Review* Vol. 93, pp. 1522-1542

- [9] Driffill, E. J. (1985), "Macroeconomic Stabilization Policy and Trade Union Behavior as a Repeated Game", *Scandinavian Journal of Economics*, Vol. 87 (2), pp. 300-326
- [10] Hersoug, T. (1985), "Workers versus Government Who adjusts to Whom?", Scandinavian Journal of Economics, Vol. 87 (2), pp. 270-292, pp. 270-292
- [11] Mussa, M. (1986), "Nominal Exchange Rate Regimes and the Behavior of Real Exchange Rates: Evidence and Implications", *Carnegie-Rochester Series on Public Policy* Vol. 25, pp. 117-214
- [12] Samuelson, P. A. (1964), "Theoretical Notes on Trade Problems", Review of Economics and Statistics, Vol. 46, pp. 145-154
- [13] Soskice, D. and T. Iversen (2000) "The Nonneutrality of Monetary Policy with Large Price or Wage Setters", *Quarterly Journal of Economics CXV*.
- [14] Taylor, Alan M. (2002), "A Century of purchasing-power Parity", Review of Economics and Statistics, Vol. 84, pp. 139-150

Chapter 2

Monetary Regimes, Labour Mobility and Equilibrium Employment^{*}

1 Introduction

Over the last decade, interest in the macroeconomic consequences of different monetary regimes has been unprecedented. In addition to the debate on optimal currency areas, spurred by the launch of the Economic and Monetary Union (EMU), advocates of price stability have suggested that inflation targeting may have desirable long-run effects on the economy by promoting sustainable growth and higher employment.

In light of this debate, several studies have considered the interaction between monetary authorities and labour markets. It has been shown that when wage setting is non-atomistic, a game between large wage setters and the central bank emerges. The monetary regime is then indeed of importance for labour market outcomes; see, for instance, Cukierman and Lippi (1999), Soskice and Iversen (2000), Coricelli et al. (2000) and Calmfors (2001). A simple mechanism modelled in the previous literature

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is that in the presence of a liberal central bank, inflation-averse trade unions may have an incentive to set wages at a low level in order to avoid inflation (Cukierman and Lippi 1999). Another, perhaps more plausible, mechanism suggested in the literature is that a conservative central bank may act as a deterrent to wage increases: by threatening to pursue contractionary monetary policy in response to high wage claims, the central bank creates an incentive for wage restraint because large unions will then face higher cost of increasing wages in terms of lower employment (Soskice and Iversen 2000, Corricelli et al. 2000 and Lippi 2003). Although the majority of previous studies model closed economies, exceptions include Vartiainen (2002) and Holden (2003) who model the game between large wage setters and an independent central bank in two-sector models of a small open economy where labour is sectorspecific. These studies show that inflation targeting is likely to generate higher employment and welfare than credible exchange rate targeting.

In this paper, I argue that it cannot be established whether there are sustainable effects of the monetary regime on labour markets in a framework with sector-specific labour. To analyse permanent effects, one needs to allow for worker migration across sectors of the economy. If wages differ across sectors, rational workers should move to sectors where their expected income is higher. Therefore, the impact of the monetary regime on labour markets may be exaggerated in models where labour is immobile.

This paper extends the previous theoretical literature on the interaction between large wage setters and the central bank in small open economies by considering a labour market set-up featuring the realistic assumption of labour mobility between the tradables and non-tradables sectors. I distinguish between national inflation targeting combined with a flexible exchange rate and membership in a monetary union and derive equilibrium implications of the regime on real wages and equilibrium employment.

A key result is that with perfect labour mobility between sectors, the monetary regime is of no importance for real wages, employment or profits. I also show that introducing labour mobility substantially reduces wages and increases employment. Other findings are that when labour is immobile between sectors: (i) real wages in the tradables sector are higher under inflation targeting than in a monetary union, while the reverse applies to the non-tradables sector; (ii) inflation targeting generates higher employment and profits than membership in a monetary union; and (iii) both workers and firms in the two sectors in general prefer inflation targeting to membership in a monetary union.

The rest of the paper is organised as follows: The basic model is presented and solved in Section 2. Results and numerical solutions are presented in Section 3. Section 4 concludes.

2 The Model

Consider a small open economy consisting of a tradables (T) and a non-tradables (N) sector, where subscript i = N, T indicates the sector. The economy is inhabited by a large number of identical households that consume the two goods and provide labour to two sets of identical firms. The sector-specific wage is set through Nash bargaining between one large union and one employer's federation in each sector. In the labour market, the individual takes wages as given.

The monetary target is given and credible to all players. The timing of events is as follows: In stage one, wages are set simultaneously in the two sectors under the assumption that wage setters take the nominal wage in the other sector as given. In stage two, the response of the central bank depends on the wage set in the previous stage. Under inflation targeting, the central bank sets the nominal exchange rate, E, to keep the aggregate price level, P, constant, i.e. $d \ln P = 0$. If the country is a member of a monetary union, then $d \ln E = 0$ by definition and there is no monetary policy response to wage setting. Finally, in stage three, production, consumption and employment are determined as a consequence of the wage setting outcome in stage two. In this stage, workers also decide in which sector to apply for a job if there is labour mobility. The model is solved by backward induction and the equilibrium is subgame perfect.

2.1 Production, Consumption and Employment

In the last stage of the model, profit-maximising firms decide how much to produce and utility-maximising households decide how much to consume. In the labour market, workers take wages as given and decide in which sector to apply for a job. Below, I model these choices of individual agents.

2.1.1 Firms

Firms in each sector produce a homogeneous good with labour and capital as inputs. A representative firm in sector i maximises real profits subject to a technology constraint and thus chooses employment solving the following optimisation problem

$$\max_{N_i} \left(P_i Y_i - W_i N_i \right) / P \tag{2.1}$$

subject to

$$Y_i = \frac{1}{\delta_i} N_i^{\delta_i}$$

where $i = N, T, \delta_i \in (0, 1)$. The first-order condition for profit maximisation gives labour demand in sector *i*:

$$N_i = \left(\frac{W_i}{P_i}\right)^{-\eta_i} \tag{2.2}$$

where $\eta_i = (1 - \delta_i)^{-1} > 1$. The corresponding supply function in sector i is given by:

$$Y_i = \frac{1}{\delta_i} \left(\frac{W_i}{P_i}\right)^{-\sigma_i} \tag{2.3}$$

where $\sigma_i = \frac{\delta_i}{1-\delta_i}$ is the output elasticity with respect to the real product wage. The profit function is

$$\Pi_i = \frac{1}{\eta_i - 1} \frac{W_i}{P} \left(\frac{W_i}{P_i}\right)^{-\eta_i}.$$
(2.4)

For simplicity, I assume that firms are owned by a group of capitalists in each sector who share profits equally among them.

2.1.2 Households

A household solves the following optimisation problem

$$\max_{C_N, C_T} C_N^{\gamma} C_T^{1-\gamma}$$

subject to

$$I/P = \left(P_N C_N + P_T C_T\right)/P.$$

where P is the aggregate price level. Real income is taken as given:

$$I/P = \begin{cases} w_i \text{ if employed in sector } i \\ \pi_i \text{ if capitalist in sector } i \end{cases}$$

where π_i is real income from profits of capitalists in sector *i*. Solving the problem yields the demand functions

$$C_N = \gamma \frac{I}{P_N}$$

$$C_T = (1 - \gamma) \frac{I}{P_T}.$$
(2.5)

The aggregate price level is given by

$$P = P_N^{\gamma} P_T^{1-\gamma}. \tag{2.6}$$

The budget share of non-traded goods can be seen as a measure of openness in the economy, or rather a measure of closedness, so that when $\gamma \to 1$, the economy is a completely closed economy with only production of non-tradables.

Market clearing for non-tradables together with the aggregate budget constraint imply that $C_i = Y_i$, where Y_i is aggregate supply.¹ In what follows, I make the simplifying assumption that production technology is the same in the two sectors, i.e. $\delta_N = \delta_T \equiv \delta$. Using $C_i = Y_i$, the demand functions (2.5) and the supply functions (2.3), I obtain the following condition for "relative" market clearing:

$$\frac{P_N}{P_T} = \left[\frac{\gamma}{1-\gamma} \left(\frac{W_N}{W_T}\right)^{\sigma}\right]^{\frac{1}{1+\sigma}}.$$
(2.7)

2.1.3 The Labour Market

Below, I model the case with no labour mobility and the case with perfect labour mobility, respectively.² Throughout the paper, the case of no labour mobility will be treated as the benchmark case when investigating how labour mobility affects the impact of the monetary regime on real wages and employment.

¹ Note that market clearing in the non-traded sector $C_N = Y_N$ implies balanced trade. To see this, use the fact that nominal output is equal to aggregate nominal income, i.e $P_N Y_N + P_T Y_T = P_N C_N + P_T C_T$. Since $C_N = Y_N$, it follows that $C_T = Y_T$.

 $^{^{2}}$ Note that the first case is equivalent to the static models in Holden (2003) and Vartiainen (2002).

No Labour Mobility

Consider first the case with no labour mobility. There is a fixed labour force M in the economy, which without loss of generality is normalised to one. Workers take wages as given and jobs are randomly assigned among workers. Let M_i be the number of union members (the labour force) in sector i and let N_i be the number of employed workers in sector i. Consequently, the number of unemployed workers in sector i, U_i , is given by $U_i = M_i - N_i$. When referring to real wages, I let lower case letters denote real variables, i.e. $w_i = \frac{W_i}{P}$. I let b denote the utility of unemployment and assume it to be exogenously given. b can be thought of as the value of home production. A representative union member cares about expected income, i.e. a weighted average of income in the two states employment and unemployment. The expected utility of a representative member in sector i is thus given by

$$V_i = \frac{N_i}{M_i} w_i + \left(1 - \frac{N_i}{M_i}\right) b \tag{2.8}$$

for i = N, T. To ensure that a worker prefers employment to unemployment, I assume that $w_i > b$ always holds in equilibrium.

Perfect Labour Mobility

Next, consider the case of perfect labour mobility. Union members take wages as given when deciding in which sector to apply for a job. A job seeker can only apply for a job in one of the sectors. Let f be the stock of workers who have migrated from sector T to sector N. The expected income of a worker looking for a job in sector N and T, respectively is:

$$V_N = \frac{N_N}{M_N + f} w_N + \left(1 - \frac{N_N}{M_N + f}\right) b \tag{2.9}$$

$$V_T = \frac{N_T}{M_T - f} w_T + \left(1 - \frac{N_T}{M_T - f}\right) b.$$
 (2.10)

Since there is perfect labour mobility, I impose a no-arbitrage condition stating that in equilibrium, there will be no utility gains from moving to the other sector, that is

$$V_N = V_T. (2.11)$$

Using expressions (2.9) and (2.10), the no-arbitrage condition can be written as

$$\frac{N_N}{M_N+f}w_N + \left(1 - \frac{N_N}{M_N+f}\right)b = \frac{N_T}{M_T-f}w_T + \left(1 - \frac{N_T}{M_T-f}\right)b$$

Solving for f I obtain:

$$f = \frac{M_T N_N (w_N - b) - M_N N_T (w_T - b)}{N_N (w_N - b) + N_T (w_T - b)}.$$
(2.12)

When membership, wages and employment levels are equal in the two sectors, i.e. when $M_N = M_T$, $w_N = w_T$ and $N_N = N_T$, there is no worker migration, i.e. f = 0. In this situation, workers receive the same utility from being a job seeker in either of the sectors and thus, have no incentive to move to the other sector to look for employment.

2.2**Monetary Policy**

In stage two, the central bank maintains $d \ln P = 0$ under national inflation targeting by adjusting the nominal exchange rate, E^{3} . Since the model is static, I cannot distinguish between price level targeting and inflation targeting, but use the term inflation targeting throughout the paper. The central bank recognises that the law of one price holds for tradable goods, i.e. $P_T = E P_T^*$ where P_T is the price of the tradable good in domestic currency, E is the nominal exchange rate in domestic currency per unit of foreign currency and P_T^* is the foreign price of tradable goods in foreign currency. P_T^* is taken as exogenously given. In what follows, I do not evaluate in detail how the central bank sets the nominal exchange rate, but merely recognise that it always succeeds in its attempts, so that the monetary target is attained.⁴ Membership in a monetary union can be modelled as an irrevocably fixed nominal exchange rate, i.e. $d \ln E = 0$.

Let subindex m denote the monetary regime, m = M, I for the regimes monetary union and inflation targeting, respectively.⁵ To evaluate the regime-specific impact

³ In theory, I might consider some other policy instrument than the exchange rate for the central bank, such as the nominal interest rate, but I would then have to model an explicit link between the interest rate and domestic demand, which would complicate the model.

⁴ Differentiating the law of one price and the consumer price index, it follows that $d \ln E =$ $-\frac{1}{(1-\gamma)} \left[\gamma d \ln P_N + (1-\gamma) d \ln P_T^* \right] \text{ under inflation targeting.}$

Note that all endogenous variables are regime-specific.

of wages on prices, I derive closed-form expressions for how supply and demand mechanisms in the goods markets determine the responsiveness of price levels to wage changes under the two regimes. For future reference, I shall refer to the elasticities of the producer prices with respect to nominal wages, i.e. $\left(\frac{\partial \ln P_i}{\partial \ln W_j}\right)_m$ and $\left(\frac{\partial \ln P_i}{\partial \ln W_j}\right)_m$, as "producer price effects", and the elasticity of the consumer price level with respect to nominal wages, $\left(\frac{\partial \ln P}{\partial \ln W_i}\right)_m$ and $\left(\frac{\partial \ln P}{\partial \ln W_j}\right)_m$, as "consumer price effects".

Taking logs of the relative goods market equilibrium condition for relative prices (2.7) and differentiating the expression with respect to P_N, P_T, W_N, W_T gives:

$$d\ln P_N - d\ln P_T = \frac{\sigma}{1+\sigma} \left(d\ln W_N - d\ln W_T \right).$$
 (2.13)

Together with the expression for the aggregate price level (2.6), (2.13) determines the elasticities of prices with respect to wages under the two monetary regimes. Taking logs and differentiating (2.6) I obtain

$$d\ln P = \gamma d\ln P_N + (1 - \gamma) d\ln P_T.$$
(2.14)

Under inflation targeting, the consumer price effects are zero by definition i.e. $d \ln P = 0$. However, nominal wage changes induce changes in producer prices. Setting $d \ln P = 0$ and substituting, in turn, $d \ln P_N = -\frac{1-\gamma}{\gamma} d \ln P_T$ and $d \ln P_T = -\frac{\gamma}{1-\gamma} d \ln P_N$ into (2.13) and rearranging gives the following producer price elasticities for the tradable and non-tradable sectors, respectively:

$$\begin{pmatrix} \frac{d\ln P_T}{d\ln W_T} \end{pmatrix}_I = \frac{\gamma\sigma}{1+\sigma} \\ \begin{pmatrix} \frac{d\ln P_T}{d\ln W_N} \end{pmatrix}_I = -\frac{\gamma\sigma}{1+\sigma} \\ \begin{pmatrix} \frac{d\ln P_N}{d\ln W_N} \end{pmatrix}_I = \frac{(1-\gamma)\sigma}{1+\sigma} \\ \begin{pmatrix} \frac{d\ln P_N}{d\ln W_T} \end{pmatrix}_I = -\frac{(1-\gamma)\sigma}{1+\sigma}.$$

The price elasticities are computed under the assumption $\frac{d \ln W_i}{d \ln W_j} = 0$. This follows because the equilibrium concept is Nash. When the wage is set in sector *i*, the nominal wage in sector *j* is taken as given.

In a Monetary Union, $d \ln E = 0$ by definition. As long as there is no foreign inflation, this implies $d \ln P_T = 0$ according to the law of one price. Imposing $d \ln P_T = 0$ on (2.13) implies:

$$\begin{pmatrix} \frac{d \ln P_N}{d \ln W_N} \end{pmatrix}_M = \frac{\sigma}{1+\sigma} \\ \begin{pmatrix} \frac{d \ln P_N}{d \ln W_T} \end{pmatrix}_M = -\frac{\sigma}{1+\sigma}$$

Inserting these expressions into (2.14) gives the consumer price effects in a monetary union:

$$\left(\frac{d \ln P}{d \ln W_N} \right)_M = \frac{\gamma \sigma}{1 + \sigma} \\ \left(\frac{d \ln P}{d \ln W_T} \right)_M = -\frac{\gamma \sigma}{1 + \sigma}.$$

Summing up, the regime-specific elasticities of prices with respect to wages, i.e. the consumer and producer price effects, are given by the expressions in Table 2.1. Column 1 displays the elasticities under inflation targeting and column 2 the elasticities in a monetary union.

Under inflation targeting, the consumer price effect is always zero by definition, i.e. $\left(\frac{d\ln P}{d\ln W_i}\right)_I = 0$. The mechanisms at work are as follows. Suppose that there is a wage increase in the non-tradables sector. This negative supply shock generates price pressure, which the central bank offsets by appreciating the nominal exchange rate. The appreciation leads to lower prices in the tradables sector, and the inflation target is attained. Similarly, if there is a wage increase in the tradables sector, there is a reduction in output, leading to lower aggregate income and lower demand for nontradable goods. The fall in demand for non-tradables causes deflationary pressure on both the price of non-tradables and the consumer price index. Therefore, the central bank depreciates the nominal exchange rate to raise the price of tradables in domestic currency. Hence, the aggregate price level is unchanged and the inflation target attained.

In a monetary union, there is no exchange-rate response to domestic wage changes. If the nominal wage in the non-tradables sector is raised by one percent, the price of non-tradables increases with a factor $\left(\frac{d\ln P_N}{d\ln W_N}\right)_M = \frac{\sigma}{1+\sigma}$ due to the

negative effect on supply. The aggregate price level increases with a factor proportional to the producer-price effect, with the proportionality coefficient given by the budget share of non-tradables.

In the tradables sector, the producer price effect is zero.⁶ However, a wage increase in the tradables-sector causes a negative supply shift, which reduces tradables-sector output. This, in turn, leads to a fall in aggregate real income, which generates a fall in demand for non-tradables. This reduces the price of non-tradables and consequently, also the consumer price, so that $\left(\frac{d \ln P}{d \ln W_T}\right)_M = -\frac{\gamma\sigma}{1+\sigma}$.

2.3 Wage Setting

In the first stage of the game, wages are set through Nash bargaining between one large union and one employers' federation in each sector. Wages are set simultaneously in the two sectors, and when bargaining over the wage in sector i, wage setters assume that the wage in the other sector W_{jm} does not respond to W_{im} , as discussed above. The union cares about the utility of its own members, taking into account that it is large enough to influence employment, as given by the labour demand function, the producer price of the own sector and the aggregate price level. In the case of perfect labour mobility, the union in sector i recognises that some of its members may move to sector j and maximisation is then subject to an additional constraint: the no-arbitrage condition governing the allocation of the labour force.

No Labour Mobility

I assume that the union in sector *i* is utilitarian and cares about the sum of expected utilities of its members, i.e. $M_i V_i$. If the bargaining parties fail to reach an agreement, workers will obtain the value of unemployment so that the fall-back utility is $\Lambda_{i0} =$ $M_i b$. Union rents from reaching an agreement can then be written:

$$\Lambda_i - \Lambda_{i0} = M_i \left(\frac{N_{im}}{M_i} w_{im} + \left(1 - \frac{N_{im}}{M_i} \right) b \right) - M_i b$$

= $N_{im} \left(w_{im} - b \right)$

⁶ According to the law of one price, $d \ln P_T = d \ln E + d \ln P_T^*$ and hence, $d \ln E = d \ln P_T^* = 0$ implies $d \ln P_T = 0$.

The objective of the employers' federation is to maximise real profits of the representative firm as given by (2.4). I assume that fall-back profits are zero, i.e. $\Pi_0 = 0$. Letting λ_i be the relative bargaining power of the union in sector *i*, I define the Nash-product to be maximised in bargaining as:

$$\Omega_i = [N_{im} (w_{im} - b)]^{\lambda_i} [\Pi_{im}]^{1-\lambda_i}$$

The nominal wage in sector i is given by the solution to

$$\max_{\ln W_{im}} \lambda_i \ln \left[N_{im} \left(\frac{W_{im}}{P_m} - b \right) \right] + (1 - \lambda_i) \ln \left[(\eta - 1)^{-1} \frac{W_{im}}{P_m} \left(\frac{W_{im}}{P_{im}} \right)^{-\eta} \right]$$

subject to

$$N_{im} = \left(\frac{W_{im}}{P_{im}}\right)^{-\eta}$$
$$P_m = P(W_{Nm}, W_{Tm})$$
$$P_{im} = P_i(W_{Nm}, W_{Tm}).$$

Let $\varphi_{im} = \left(1 - \frac{d \ln P_i}{d \ln W_i}\right)_m$ and $\epsilon_{im} = \left(1 - \frac{d \ln P}{d \ln W_i}\right)_m$. The first-order condition for maximisation is

$$\lambda_i \left[-\eta \varphi_{im} + \frac{w_{im} \epsilon_{im}}{(w_{im} - b)} \right] + (1 - \lambda_i) \left(\epsilon_{im} - \eta \varphi_{im} \right) = 0$$
(2.15)

The first-order condition states that the union's marginal gain of a wage increase must balance the marginal loss of the employers' federation. Note that both parties benefit from a positive producer price effect: the union's employment loss generated by a marginal wage is partly offset and so is the profit loss of firms. Similarly, both parties lose from a positive consumer price effect since it decreases real wages and real profits. Solving for the real wage I obtain:

$$w_{im} = \left[1 + \frac{\lambda_i \epsilon_{im}}{\eta \varphi_{im} - \epsilon_{im}}\right] b.$$
(2.16)

Note that (2.16) represents two equations since i = N, T. The regime-specific priceelasticities φ_{im} and ϵ_{im} , show how the monetary regime influences wage setting, and are therefore key parameters of interest. They display how wage setters may be constrained by the central bank, as it sets the nominal exchange rate in order to offset wage pressure threatening the monetary target. Consequently, equilibrium wages are governed by the regime-specific elasticities displayed in Table 2.1.

Perfect Labour Mobility

When there is perfect labour mobility, the union still seeks to maximise the sum of expected utilities of its members. Consider the union in sector *i*. Let M_{ii} denote the number of workers who stay in sector *i* and seek employment. Then $M_i - M_{ii}$ workers move to sector *j* to look for a job in that sector. Consequently, the union in sector *i* seeks to maximise:

$$M_{ii}V_i + (M_i - M_{ii})V_i$$

But since the union recognises that in equilibrium, it will always hold that $V_i = V_j$ because of worker migration, the objective function of the union is still given by

$$\Lambda_i = M_i V_i$$

If the parties fail to reach an agreement, members obtain the value associated with being unemployed, so that fall-back utility is $\Lambda_{i0} = M_i b$. This presumes that if parties fail to reach an agreement, workers in sector *i* cannot apply for a job in sector *j*. This can be taken to represent an implicit agreement between employers not to undermine each others' bargaining positions by hiring from the workforce of other employers during a conflict. The objective functions of the two unions may be written:

$$\Lambda_N - \Lambda_{N0} = M_N \left[\frac{N_{Nm}}{M_N + f_m} (w_{Nm} - b) \right]$$

$$\Lambda_T - \Lambda_{T0} = M_T \left[\frac{N_{Tm}}{M_T - f_m} (w_{Tm} - b) \right]$$

The maximisation is now also subject to (2.12), which determines f_m .

The nominal wage solves:

$$\max_{\ln W_{im}} \lambda_i \ln \left[\Lambda_i - \Lambda_{i0} \right] + (1 - \lambda_i) \ln \left[(\eta - 1)^{-1} \frac{W_{im}}{P_m} \left(\frac{W_{im}}{P_{im}} \right)^{-\eta} \right]$$

subject to

$$N_{im} = \left(\frac{W_{im}}{P_{im}}\right)^{-\eta}$$

$$f_m = \frac{M_T N_{Nm} (w_{Nm} - b) - M_{Nm} N_{Tm} (w_{Tm} - b)}{N_{Nm} (w_{Nm} - b) + N_{Tm} (w_{Tm} - b)}$$

$$P_m = P(W_{Nm}, W_{Tm})$$

$$P_{im} = P_i(W_{Nm}, W_{Tm}).$$

When there is perfect labour mobility, wage setters in sector *i* internalise the fact that their wage decision affects the distribution of the labour force across sectors. They also take into account that their wage decisions affect prices in sector *j*. Therefore, it will prove useful to introduce some additional notation. Let $(1 - \frac{d \ln P_j}{d \ln W_i})_m = \psi_{im}$ (and $(1 - \frac{d \ln P_i}{d \ln W_j})_m = \psi_{jm}$). The first-order conditions for the union in the non-tradables and tradables sector, respectively, are:

$$\lambda_{N} \left[-\eta \varphi_{Nm} - \frac{\frac{\partial f_{m}}{\partial \ln W_{Nm}}}{(M_{N} + f_{m})} + \frac{w_{Nm} \epsilon_{Nm}}{(w_{Nm} - b)} \right] + (1 - \lambda_{N}) \left(\epsilon_{Nm} - \eta \varphi_{Nm} \right) = 0 \quad (2.17)$$
$$\lambda_{T} \left[-\eta \varphi_{Tm} + \frac{\frac{\partial f_{m}}{\partial \ln W_{Tm}}}{(M_{T} - f_{m})} + \frac{w_{Tm} \epsilon_{T}}{(w_{Tm} - b)} \right] + (1 - \lambda_{N}) \left(\epsilon_{Tm} - \eta \varphi_{Tm} \right) = 0 \quad (2.18)$$

where $\frac{\partial f_m}{\partial \ln W_{im}}$ is the effect on worker flows. The first term within brackets is the marginal effect on union rents of a one percent wage increase. When the assumption of immobile labour is relaxed, the additional terms $\frac{\partial f_m}{\partial \ln W_{Nm}}/(M_N + f_m)$ and $\frac{\partial f_m}{\partial \ln W_{Tm}}/(M_N - f_m)$ enter the first-order conditions of the unions in sectors N and T, respectively. The intuition is that when the wage in sector *i* increases, there will ceteris paribus be an inflow of workers to that sector, increasing the stock of workers competing for employment there and thus, reducing union rents in the sector.

It will prove useful to consider the following relationship between the producer price effects under regime m: inserting the price elasticities from table 1 under the

different regimes, it can be shown that

$$\varphi_{im} - \psi_{im} = -\frac{(\eta - 1)}{\eta} < 0 \tag{2.19}$$

 $\forall i, m$. This is equivalent to

$$\frac{\partial \ln P_{jm}}{\partial \ln W_{im}} < \frac{\partial \ln P_{im}}{\partial \ln W_{im}}.$$

The pass-through to *j*-sector prices, P_{jm} , is always smaller than the direct impact of a wage increase on prices in the own sector. The reason is that the effect on *j*-sector prices stems from an indirect effect on aggregate income, while the effect on *i*-sector prices is the direct result of a negative shift in supply. The result (2.19) implies that $(1 - \eta (1 + \varphi_{im} - \psi_{im})) = 0 \forall i, m$. Using this result, the effect on net worker migration from the tradables sector to the non-tradables sector of a wage increase in the two sectors can be written:

$$\frac{\partial f_m}{\partial \ln W_{Nm}} = (M_N + M_T) N_{Nm} N_{Tm} b \frac{\left[(w_{Nm} - b) \left(1 - \epsilon_{Nm} \right) + \left(w_{Tm} - b \right) \epsilon_{Nm} \right]}{\left[N_{Nm} \left(w_{Nm} - b \right) + N_{Tm} \left(w_{Tm} - b \right) \right]^2} > 0$$
(2.20)

and

$$\frac{\partial f_m}{\partial \ln W_{Tm}} = -(M_N + M_T) N_{Nm} N_{Tm} b \frac{\left[(w_{Tm} - b)(1 - \epsilon_{Tm}) + (w_{Nm} - b)\epsilon_{Tm}\right]}{\left[N_{Nm}(w_{Nm} - b) + N_{Tm}(w_{Tm} - b)\right]^2} < 0.$$
(2.21)

Ceteris paribus, a wage increase in the non-tradables sector causes a net inflow of workers to the sector, since there is utility to be gained by migrating to that sector. In analogy, the reverse holds true for an increase in wages in the tradables sector. Henceforth, let $\tilde{}$ denote the case of perfect mobility. The first-order conditions for wage setting imply

$$\widetilde{w}_{Nm} = \left[1 + \frac{\lambda_N \epsilon_{Nm}}{\eta \varphi_{Nm} - \epsilon_{Nm} + \lambda_N \frac{\partial f_m / \partial \ln W_{Nm}}{(M_N + f_m)}}\right] b$$
$$\widetilde{w}_{Tm} = \left[1 + \frac{\lambda_T \epsilon_{Tm}}{\eta \varphi_{Tm} - \epsilon_{Tm} - \lambda_T \frac{\partial f_m / \partial \ln W_{Tm}}{(M_T - f_m)}}\right] b$$

where $\partial f_m / \partial \ln W_{im}$ is given by the above expressions. Once more, the price elasticities φ_{im} and ϵ_{im} display the influence of the monetary regime on wage setting. Unions and employers' federations bargaining with each other internalise the impact of wage increases on aggregate price levels and take into account that the extent to which prices are allowed to increase is influenced by the monetary regime. Unions now also internalise the impact of their wage claims on prices in sector j since they take into account that some of their members may move to that sector, but since these effects and the producer price effects in the own sector cancel out according to (2.19), ψ_{im} does not matter for the optimal wage.

Substituting for equilibrium net migration, f_m , and $\partial f_m / \partial \ln W_{im}$, I obtain the following expressions for real wages in the two sectors:

$$\widetilde{w}_{Nm} = \left[1 + \frac{\lambda_N \epsilon_{Nm}}{(\eta \varphi_{Nm} - \epsilon_{Nm})}\right] b - \frac{\widetilde{N}_{Tm}}{\widetilde{N}_{Nm}} \left[\frac{\lambda_N (1 - \epsilon_{Nm})}{(\eta \varphi_{Nm} - \epsilon_{Nm})} b + (\widetilde{w}_{Tm} - b)\right] (2.22)$$

$$\widetilde{w}_{Tm} = \left[1 + \frac{\lambda_T \epsilon_{Tm}}{(\eta \varphi_{Tm} - \epsilon_{Tm})}\right] b - \frac{\widetilde{N}_{Nm}}{\widetilde{N}_{Tm}} \left[\frac{\lambda_T (1 - \epsilon_{Tm})}{(\eta \varphi_{Tm} - \epsilon_{Tm})} b + (\widetilde{w}_{Nm} - b)\right] (2.23)$$

Real wages are now functions of employment in the two sectors and of wages in the other sector. Wages are increasing in employment in the own sector and the wage curves are concave in employment-real wage space:

$$\frac{\partial \widetilde{w}_{im}}{\partial \widetilde{N}_{im}} = \frac{\widetilde{N}_{jm}}{\widetilde{N}_{im}^2} \left[\frac{\lambda_i \left(1 - \epsilon_{im}\right)}{\left(\eta \varphi_{im} - \epsilon_{im}\right)} b + \left(\widetilde{w}_{jm} - b\right) \right] > 0$$

$$\frac{\partial^2 \widetilde{w}_{im}}{\partial \widetilde{N}_{im}^2} = -2 \frac{\widetilde{N}_{jm}}{\widetilde{N}_{im}^3} \left[\frac{\lambda_i \left(1 - \epsilon_{im}\right)}{\left(\eta \varphi_{im} - \epsilon_{im}\right)} b + \left(\widetilde{w}_{jm} - b\right) \right] < 0.$$

The wage in sector *i* is a decreasing function of wages and employment in sector *j*. This may, at first, seem counterintuitive. Consider, for instance, the union in the non-tradables sector. From the first-order condition of wage setters, it follows that if $\frac{\partial f_m}{\partial \ln W_{Nm}}$ increases, the union chooses a lower wage since the marginal gain of a wage increase decreases. Therefore, studying the sensitivity of f_m with respect to w_{Nm} , $\frac{\partial f_m}{\partial \ln w_{Nm}}$, and the level of f_m is key to understanding why w_{Nm} is a decreasing function of N_{Tm} and w_{Tm} . I next consider, in turn, the effects of an increase in wages or employment in the tradables sector on $\frac{\partial f_m}{\partial \ln w_{Nm}}$ and on f_m , respectively.

It may be shown that

$$\frac{\partial}{\partial w_{Tm}} \left(\frac{\partial f_m}{\partial \ln W_{Nm}} \right) > 0 \text{ if and only if}$$

$$\epsilon_{Nm} N_{Nm} \left(w_{Nm} - b \right) - 2N_{Tm} \left(1 - \epsilon_{Nm} \right) \left(w_{Nm} - b \right) - N_{Tm} \epsilon_{Nm} \left(w_{Tm} - b \right) > 0$$

and

$$\frac{\partial}{\partial N_{Tm}} \left(\frac{\partial f_m}{\partial \ln W_{Nm}} \right) > 0 \text{ if and only if}$$
$$N_{Nm} \left(w_{Nm} - b \right) - N_{Tm} \left(w_{Tm} - b \right) > 0.$$

The sensitivity of flows to w_{Tm} and N_{Tm} may be positive or negative depending on

wage levels, employment rates and the consumer price effects. If employment and wages are higher in the non-tradables sector than in the tradables sector, an increase in N_{Tm} increases the sensitivity of flows to an increase in w_{Nm} . This, in turn, causes wage setters to lower the wage.

Turning to the level of net flows, it is straightforward to show that

$$\begin{array}{lll} \displaystyle \frac{\partial f_m}{\partial w_{Tm}} & < & 0 \\ \displaystyle \frac{\partial f_m}{\partial N_{Tm}} & < & 0. \end{array}$$

When the wage or employment in the tradables sector increases, f_m decreases, i.e. there will be a lower stock of workers in the non-tradables sector and for a given level of $\frac{\partial f_m}{\partial \ln W_{Nm}}$, the term $\frac{\partial f_m}{(M_N + f_m)}$ increases. This means that the marginal gain of a wage increase to unions in the non-tradables sector decreases and a lower wage is set. In other words, there will be a higher percentage increase in union rents of a one percent wage increase for a given change in the labour stock, $\frac{\partial f_m}{\partial \ln W_{Nm}}$, if the stock f_m is small to begin with. Since the net effect is uncertain, it is useful to consider the total effect on the term $\frac{\partial f_m}{\partial \ln W_{Nm}}$. It may be shown that

$$\frac{\partial}{\partial N_{Tm}} \left(\frac{\partial f_m / \partial \ln W_{Nm}}{M_N + f_m} \right) > 0$$

and

$$\frac{\partial}{\partial w_{Tm}} \left(\frac{\partial f_m / \partial \ln W_{Nm}}{M_N + f_m} \right) > 0 \text{ if and only if} \\ N_{Nm} \epsilon_{Nm} - N_{Tm} \left(1 - \epsilon_{Nm} \right) > 0$$

This means that even if wage levels and employment rates are such that $\frac{\partial}{\partial N_{Tm}} \left(\frac{\partial f_m}{\partial \ln W_{Nm}} \right) < 0$, the effect on $\frac{\partial f_m}{\partial N_{Tm}}$ always dominates so that when N_{Tm} increases, the union's marginal gain of increasing w_{Nm} decreases, which causes them to set a lower wage. Thus, w_{Nm} is a decreasing function of N_{Tm} . The net effect on the term of an increase in w_{Tm} is uncertain. Due to symmetry, a similar argument applies to unions in the tradables sector.

When comparing wage levels with and without mobility, respectively, it is trivial to prove the following proposition:

Proposition 1 Wages are always lower when there is perfect mobility than when labour is immobile between sectors, i.e.

$$w_{im} > \widetilde{w}_{im}$$

 $\forall i, m.$

Proof. The wage curves (2.16) and (2.22)-(2.23) imply that $\widetilde{w}_{im} = w_{im} - \left[\frac{\lambda_i(1-\epsilon_{im})}{(\eta\varphi_{im}-\epsilon_{im})}b + (\widetilde{w}_{jm}-b)\right]$. Thus $w_{im} > \widetilde{w}_{im}$ if and only if $\frac{\lambda_i(1-\epsilon_{im})}{(\eta\varphi_{im}-\epsilon_{im})}b + (\widetilde{w}_{jm}-b) > 0$. In equilibrium, this holds true $\forall j, m$ and the proposition follows.

Key to understanding the above proposition is recalling that unions take into account that some of their members may move to the other sector, but also that some of the members of the other union may move to their sector. This provides an incentive for wage restraint since unions know that if they set wages too high, there will be an inflow of workers from the other sector (i.e. workers who are members of the other union) competing for jobs in their own sector, thus reducing employment probabilities and utility of their own members.

2.3.1 Decentralised Wage Setting

When solving the model numerically, it will prove useful to derive the decentralised outcome and use it as a benchmark. Therefore, I next derive the wage curve under the assumption of atomistic wage setting. Suppose that wages are negotiated in bargaining units that are so small that the wages set are unable to influence prices. Clearly, the monetary regime will be of no importance for real wages or employment under this assumption. When wage setters are small, they do not need to take into account that their wage decisions will affect price levels and the equilibrium wage is therefore obtained by imposing $\epsilon_{im} = \varphi_{im} = 1$ on (2.16), and the wage in the case with no mobility reads

$$\widehat{w}_{im} = \left[1 + \frac{\lambda_i}{\eta - 1}\right]b. \tag{2.24}$$

The real wage is a constant mark-up on the real value of unemployment. I will use this expression when calibrating b in the numerical solutions to the model.

2.4 Non-neutrality of the Monetary Regime

Why is the monetary regime potentially of importance when wage setting is nonatomistic? The first-order conditions for wage setting in sector i, (2.15) and (2.17), respectively, define reaction functions: the real wage in sector i, w_{im} , as functions of the real wage in sector j, w_{jm} . Moreover, the consumer price level, P_m , is a function of nominal wages in the two sectors and of the monetary regime. Thus, the aggregate price level differs across regimes, $P_I \neq P_M$. In Nash equilibrium, wage setters in sector i take the *nominal* wage in sector j as given, but since the consumer price level will differ across regimes, so will the perceived consumer *real* wage in sector j. Wage setters in sector i therefore perceive that they are solving different maximisation problems under the two regimes. Consequently, the real wage in sector i will also be regime specific according to the reaction function in sector i.

For future reference, note the asymmetric features of a monetary union: When the exchange rate is fixed, wages in the tradables sector may increase infinitely without any increase in the price of tradables and without any reaction from the central bank. Moreover, a wage increase in the non-tradables sector generates an increase in the price for non-tradables, also with no response from the central bank. Under inflation targeting, however, the central bank is equally concerned with inflationary pressure from both sectors and ensures that the inflation target is attained by adjusting the nominal exchange rate.

To evaluate the impact of the monetary regime in this setting, I need to derive

the general equilibrium. This is done in the next section.

2.5 Equilibrium

To simplify, I need to get rid of the producer real wage that enters the labour demand functions. By using the definition of the aggregate price level (2.6) and inserting the equilibrium relative price (2.7), I can rewrite the labour demand equations in terms of consumer real wages as:

$$N_{Nm} = w_{Nm}^{-\eta} \left(\frac{w_{Nm}}{w_{Tm}}\right)^{(1-\gamma)\sigma} \left(\frac{\gamma}{1-\gamma}\right)^{(1-\gamma)}$$
(2.25)

$$N_{Tm} = w_{Tm}^{-\eta} \left(\frac{w_{Nm}}{w_{Tm}}\right)^{-\gamma\sigma} \left(\frac{\gamma}{1-\gamma}\right)^{-\gamma}.$$
 (2.26)

In equilibrium, four equations determine the four endogenous variables: w_{Nm} , w_{Tm} , n_{Nm} and n_{Tm} . The equations are the labour demand equations in each sector, (2.25) and (2.26), the sectoral wage equations (2.16) or (2.22) and (2.23) (evaluated for i = N, T and the equilibrium price elasticities in Table 2.1).

No Labour Mobility

In the case with immobile labour, the wage in a sector is independent of the employment rate and wage in the other sector. Thus, wages are given by:

$$w_{Nm} = \left[1 + \frac{\lambda_N \epsilon_{Nm}}{\eta \varphi_{Nm} - \epsilon_{Nm}}\right] b \tag{2.27}$$

$$w_{Tm} = \left[1 + \frac{\lambda_T \epsilon_{Tm}}{\eta \varphi_{Tm} - \epsilon_{Tm}}\right] b.$$
(2.28)

Wages in the two sectors are a positive mark-up on the value of unemployment. Given wages, employment rates are determined according to (2.25) and (2.26).

Perfect Labour Mobility

Next, consider the case of perfect labour mobility. Dividing (2.25) by (2.26) implies:

$$\frac{\widetilde{N}_{Nm}}{\widetilde{N}_{Tm}} = \left(\frac{\widetilde{w}_{Tm}}{\widetilde{w}_{Nm}}\right) \left(\frac{\gamma}{1-\gamma}\right).$$
(2.29)

Substituting the expression for relative employment (2.29) into the wage equations (2.22) and (2.23) gives two linear equations in two unknowns, \tilde{w}_{Tm} and \tilde{w}_{Nm} . I may therefore solve for equilibrium real wages on reduced form:

$$\widetilde{w}_{Nm} = \left[\frac{\eta\varphi_{Tm} - \epsilon_{Tm} + \eta\varphi_{Nm} - \epsilon_{Nm} - \lambda\left(1 - \epsilon_{Nm} - \epsilon_{Tm}\right)}{(\eta\varphi_{Nm} - \epsilon_{Nm})\epsilon_{Tm} + (\eta\varphi_{Tm} - \epsilon_{Tm})\left(1 - \epsilon_{Nm}\right)}\right]\gamma b$$
(2.30)

$$\widetilde{w}_{Tm} = \left[\frac{\eta \varphi_{Tm} - \epsilon_{Tm} + \eta \varphi_{Nm} - \epsilon_{Nm} - \lambda \left(1 - \epsilon_{Nm} - \epsilon_{Tm}\right)}{\left(\eta \varphi_{Nm} - \epsilon_{Nm}\right) \left(1 - \epsilon_{Tm}\right) + \left(\eta \varphi_{Tm} - \epsilon_{Tm}\right) \epsilon_{Nm}} \right] (1 - \gamma) b. (2.31)$$

Wages are still a markup on the value of unemployment, but the markup is now interacted with sector sizes. As in the case with immobile labour, employment rates are determined according to (2.25) and (2.26).

3 Analysis

In this section, I first compare the equilibria under the two different regimes analytically and then solve the model numerically. Finally, I address the issue of which interest groups in the economy benefit from the two regimes.

3.1 Real Wage Rankings Across Regimes and Sectors

Inserting the equilibrium price elasticities in Table 2.1 and simplifying, gives the reduced-form expressions for regime-specific consumer real wages given in Table 2.2. It is easy to verify Proposition 1 by looking at reduced-form wages: wages are always lower when labour is mobile.

Next, I evaluate the impact of different regimes by comparing different wage levels under inflation targeting and in a monetary union. I start by looking at the ranking of different regimes in a given sector and then look at how wages differ across sectors under a given regime.

Proposition 2 When labour is immobile between sectors, the ranking of regimes within each sector is as follows:

$$w_{TI} > w_{TM}$$

 $w_{NM} > w_{NI}$

Proof. Given in Appendix A1. ■

The wage ranking stated in the proposition is consistent with previous literature; see Holden (2003). In the tradables sector, the result is explained by the fact that the positive producer price effect under inflation targeting is stronger than the negative consumer price effect in a monetary union. In the non-tradables sector, the positive producer price effect is so much stronger in a monetary union than under inflation targeting that the mitigating consumer price effect in a monetary union is neutralised.

Proposition 3 When labour is immobile between sectors, the ranking of sectoral wages under a given regime is as follows:

$$w_{TI} \geq w_{NI} \text{ if and only if } \gamma \geq \frac{1}{2}$$

 $w_{NM} \geq w_{TM} \text{ if and only if } \gamma \leq \frac{(1+\sigma)}{2}$

Proof. Given in Appendix A1. ■

Under inflation targeting, the wage will be higher in the tradables sector than in the non-tradables sector if the latter is larger. Recall that there are no consumer price effects present to deter wage setters from raising the wage under this regime, only producer price effects. Perhaps the easiest way to think about this result is by considering wage setters in the tradables sector. When they raise wages, the resulting fall in aggregate income causes deflationary pressure in the non-tradables sector, threatening the inflation target. The central bank therefore depreciates the exchange rate, which raises prices in the tradables sector. This, in turn, increases profits for the employer's federation in the tradables sector, which mitigates the negative impact on *T*-sector employment caused by the wage increase. The passthrough to consumer prices from the decrease in the price of non-tradables is larger the larger is the *N*-sector, as is the depreciation that is triggered. Therefore, the larger the non-tradables sector (the larger γ), the larger the depreciation and thus, the stronger the incentives for wage setters in the tradables sector to raise wages.

The intuition in a monetary union is as follows. Since there is no central bank to deter wage setters in the non-tradables sector from raising the wage infinitely, they will do so until the consumer price effect (decreasing real wages and profits) becomes sufficiently damaging to them. Wage setters in the non-tradables sector exploit their strategic advantage of being small as long as the size of the sector is below the threshold value $\frac{(1+\sigma)}{2}$.

Proposition 4 When there is perfect labour mobility, there is wage equality across sectors and regimes, *i.e.*

$$\widetilde{w}_{NI} = \widetilde{w}_{TI} = \widetilde{w}_{NM} = \widetilde{w}_{TM}.$$

Proof. The proposition follows directly from Table 2.2. \blacksquare

When there is perfect labour mobility between sectors, there is always wage equality across sectors and regimes, i.e. the regime is of no importance. This is not a self-evident result, since it is the expected utility of a worker that should be the same in the two sectors and not wages - according to the no-arbitrage condition (2.11). But it turns out that the first-order conditions for wage setters are the same in the two sectors regardless of regime. This can be seen by substituting for equilibrium relative labour demand (2.29) into (2.17) and (2.18).

3.1.1 A Crucial Assumption

The result that the regime is of no importance as stated in Proposition 4, hinges on the assumption of the utility of the unemployed being exogenously given in *real* terms. In contrast, consider the case where the utility of the unemployed is interpreted as an unemployment benefit.⁷ Under the assumption that the *nominal* unemployment benefit, B, is exogenously given, there is an additional effect present in the unions' first-order conditions arising from the fact that the real unemployment benefit, defined as $b = \frac{B}{P}$, is regime-specific due to the impact of wages on prices.⁸ The first-order conditions for wage setting then define the following reaction functions: the wage in sector *i* as a function of the real wage in sector *j* and

⁷ One way of modelling the financing of such an unemployment benefit would be to introduce a constant tax rate levied on all labour income (both wage income and unemployment benefits). A term equal to $(1 - \tau)$, where τ is the tax rate, would then enter multiplicatively in equation (2.8), and thus not affect the maximisation problem of wage setters. One way of closing the model in this case would be to introduce an exogenously given number of pensioners (with the same Cobb-Douglas utility function as workers) in the model and assume that the tax on labour income is used to finance both unemployment benefits and pensions. The pension level could then be taken to be determined residually so that budget balance is always obtained.

⁸ I focused on this case at length in an earlier version of the paper. The derivations are now given in Appendix A2.

the real unemployment benefit b. When setting the wage, unions perceive a given nominal benefit B. Since the response of the price level is regime-specific, so is the real unemployment benefit, b, and consequently the wage set in sector i. It turns out that with an exogenous nominal benefit level and perfect mobility, wages are equalised across sectors under inflation targeting, but not in a monetary union. The intuition is that under inflation targeting, the consumer price effects are zero, and since they are the effects governing the real unemployment benefit, the additional effect in the unions' first-order conditions described above disappears. In a monetary union, however, there are consumer price effects present which cause the wage outcome under this regime to differ from the outcome under inflation targeting.

3.2 Numerical Solutions

The aim of this section is to compute equilibrium employment rates and assess the quantitative importance of the mobility assumption by means of numerical illustrations.

3.2.1 Parameters

First, consider the parameters governing the labour demand curves. The labour share in production, δ , is set to 0.5. Due to the asymmetric features of a monetary union, I suspect that the relative size of the non-tradables sector, γ , is crucial for which sector performs better under the two regimes. This is also shown to be true for models with immobile labour; see Larsson and Zetterberg (2003). Therefore, it is important to let this parameter assume many different values ranging from 0 (a super-open economy with only production of tradables) to 1 (a super-closed economy with only production of non-tradables). As a benchmark, I will consider the completely symmetric case when the two sectors are equally sized, i.e. letting $\gamma = 0.5$.

I need to calibrate the value of being unemployed so that it generates reasonable unemployment rates. Consider the case when wage setting is completely decentralised and labour is immobile between sectors so that the wage equation is given by (2.24). Then, I impose complete symmetry across sectors, i.e., $\lambda_N = \lambda_T = \lambda$ and $\gamma = 0.5$, i.e. I assume the sectors to be equally large. From (2.24), it follows that there is then real wage equality across sectors, i.e. $w_N = w_T$. Moreover, the labour demand curves in the two sectors are identical and given by:

$$n_i = w^{-\eta}.$$

Next, I impose five percent unemployment in the economy, i.e. set $n_i = 0.475$. Substituting for w from the wage curve I obtain:

$$0.475 = \left[\frac{\eta - 1 + \lambda}{\eta - 1}\right]^{-\eta} b^{-\eta}.$$

Solving for *b*:

$$b = (0.475)^{-\frac{1}{\eta}} \left[\frac{\eta - 1}{\eta - 1 + \lambda} \right].$$

Finally, the relative bargaining power of unions is set equal to 0.5, i.e. I consider symmetric bargaining.

3.2.2 Results

Numerical solutions to the model are given in Table 2.3. A key result is that with immobile labour, aggregate employment is always higher under inflation targeting than in a monetary union. This suggests that an independent central bank may discipline wage setters, i.e. provide incentive for wage restraint by targeting inflation.

As can be seen, there is always wage equality across regimes and sectors under perfect mobility. However, employment rates differ across sectors since they are governed by labour demand which, in turn, depends on the relative size of the non-tradables sector. When there is labour mobility, the monetary regime is of no importance for aggregate employment.

Proposition 1 is easily verified: wages in the two sectors are always lower under perfect labour mobility. The results suggest that as a consequence of the reduction in wages, employment in both sectors is higher with mobile labour. The model generates unrealistically low employment levels with immobile labour, but aggregate employment rates are much improved and reach much more realistic values under perfect labour mobility. This suggests that labour mobility is quantitatively important and that worker migration should be taken into account, both when modelling labour markets theoretically and when designing labour market policy.

The results with immobile labour demonstrate the symmetric properties of in-

flation targeting: Wage levels and employment rates are symmetric around $\gamma = 0.5$ in the sense that the wage in the non-tradables sector when $\gamma = 0.4$ is equal to the wage in the tradables sector when $\gamma = 0.6$. The intuition is related to the fact that the central bank is equally concerned with wage pressure from the two sectors under this regime and γ and $1-\gamma$ constitute a measure of the magnitude of the inflationary pressure generated by a wage increase in the non-tradables and tradables sectors, respectively. Thus, the extent to which wage setters in the two sectors are punished by the central bank for excessive wage claims is directly proportional to γ . The pattern in a monetary union is not as symmetric, but nevertheless quite clear. In accordance with Proposition 2, wages in the non-tradables sector are always higher in a monetary union than under inflation targeting, while the reverse holds true for the tradables sector.

Turning to the ranking across sectors under a given regime and with immobile labour, note that under inflation targeting, wages in the tradables sector are only higher than wages in the non-tradables sector when $\gamma = 0.6$, which is consistent with Proposition 3. For the parameterisation considered here, the results in Table 2.3 suggest that in a monetary union, wages are always higher in the non-tradables sector than in the tradables sector. This is also consistent with Proposition 3 since with $\delta = 0.5$ I obtain $\frac{1+\sigma}{2} = 1$, which implies $w_{NM} > w_{TM}$ for all $\gamma < 1$.

3.3 Political Economy Analysis

In this section I perform some political economy analysis by evaluating which groups benefit from an inflation target and membership in a monetary union, respectively. Table 2.4 displays implied expected income levels of a worker and profits of firms in the two sectors under different regimes and assumptions about mobility.

Consider first the results with immobile labour. The results suggest that when there is no labour mobility, the expected income of a worker in the non-tradables sector is higher in a monetary union than under inflation targeting. This is mainly due to the fact that wages are always higher under that regime. Employment in the non-tradables sector is lower in a monetary union than under inflation targeting, but the difference across regimes is not sufficiently large to offset the difference in wages. Similarly, the expected income of a worker in the tradables sector is always higher under inflation targeting than in a monetary union. Profits in both sectors are higher under inflation targeting than in a monetary union, when labour is immobile between sectors. The result that firms in both sectors would prefer inflation targeting to membership in a monetary union may seem inconsistent with the notion that many advocates of a monetary union are found among firms and entrepreneurs in the tradables sector. However, the arguments typically made in favour of a monetary union, such as elimination of exchange rate risk, reduction of transaction costs and so forth, are not present in the model. Moreover, I analyse the incentives for wage restraint under the two regimes, and the subsequent effects on employment and profits but do not analyse shocks or evaluate the stabilising properties of the two monetary regimes.

Introducing mobility, expected income is equal in both sectors. Since expected income is also equalised across regimes, a worker in any of the two sectors is indifferent between the two regimes. Moreover, profits are equalised due to wage equality. Hence, firms in both sectors are indifferent between inflation targeting and membership in a monetary union.

Finally, Table 2.4 suggests that labour mobility raises profits due to the reduction in wages and the subsequent increase in employment, which was established in Table 2.3.

Summing up, the model suggests that with immobile labour, all groups prefer inflation targeting to membership in a monetary union except workers in the nontradables sector who prefer a monetary union to inflation targeting. With labour mobility, workers as well as firms in the two sectors are indifferent between the two regimes, since they generate the same expected income and profits. This is a key result.

4 Concluding Remarks

In this paper, I have presented a theoretical model of the impact of the monetary regime on wage setting and employment in a small open economy with and without labour mobility between sectors. I compare the outcomes under inflation targeting and in a monetary union when the exchange rate is irrevocably fixed. The monetary regime affects equilibrium wages and employment rates since wage setters take into account whether or not the central bank will react to their wage claims under a given monetary regime.

The main result is that with perfect labour mobility, the monetary regime is of no importance for equilibrium real wages, profits or employment. As a consequence, workers as well as firms in the two sectors are indifferent between the two regimes, since they generate the same expected income and profits. Labour mobility substantially increases aggregate employment as a consequence of the moderation induced in wages.

With immobile labour, the consumer real wage in the tradables sector is higher under inflation targeting than in a monetary union, while the consumer real wage in the non-tradables sector is higher in a monetary union than under inflation targeting. Moreover, the real wage is higher in the larger sector under inflation targeting, while in a monetary union the wage is higher in the non-tradables sector than in the tradables sector, provided that the economy is sufficiently open (i.e. the nontradables sector is not too large). The numerical solutions to the model suggest that, with immobile labour, aggregate employment levels are higher under inflation targeting than in a monetary union.

When investigating which interest groups in the economy benefit from which regimes under no labour mobility, a striking result is that the only group that prefers a monetary union to inflation targeting is workers in the non-tradables sector, who benefit from a fixed exchange rate since it generates higher expected income than inflation targeting. The fact that inflation targeting is preferred also by workers and firms in the tradables sector may at first seem contradictory to the notion that firms and employees exposed to international trade often provide arguments in favour of membership in the EMU. However, in the policy debate, advocates of a monetary union generally refer to features not included in my model, such as elimination of exchange rate risk and transaction costs.

There are several interesting extensions to the model to be considered. First, one could allow for the fact that a large country in a monetary union may not treat the response of the nominal exchange rate as exogenous. This feature could be accounted for in the model by letting the response of the nominal exchange rate in the economy be proportional to the size of the country. Second, it would be interesting to consider complete centralisation in the model, i.e. a setting in which one union and one employers' federation bargain over wages in both sectors. Finally,

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a setting that has a great deal of real-world relevance is the case where unions set wages sequentially, i.e. where one of the unions acts as a Stackelberg leader relative to the other.

Bibliography

- [1] Blanchflower, D.G. and Oswald, A.J. (1995), The Wage Curve, MIT Press
- [2] Calmfors, L. (2001), "Wages and Wage-Bargaining Institutions in the EMU A Survey of the Issues", *Empirica 28*
- [3] Calmfors, L. and Lang, H. (1995), "Macroeconomic Effects of Active Labour Market Programmes in a Union Wage-Setting Model", *The Economic Journal* 105, pp. 601-619
- [4] Coricelli, F., Cukierman, A. & Dalmazzo, A.(2000), "Monetary Institutions, Monopolistic Competition, Unionized Labor Markets and Economic Performance", CEPR Discussion Paper 2407, Center for Economic Policy Research, London.
- [5] Cukierman, A. and Lippi, F. (1999), "Central Bank Independence, Centralization of Wage bargaining, Inflation and Unemployment: Theory and Some Evidence", *European Economic Review* 43
- [6] Holden, S. (2003), "Wage Setting under Different Monetary Regimes", *Economica*, 70, pp. 251-265
- [7] Larsson, A. and Zetterberg, J. (2003), "Does Inflation Targeting Matter for Labour Markets? - Some Empirical Evidence", FIEF Working Paper Series 191, FIEF, Stockholm
- [8] Lippi, F. (2003), "Strategic Monetary Policy with Non-atomistic Wage Setters", CEIS Tor Vergata Research Paper Series, Vol. 6, No. 17, Centre for International Studies on Economic Growth

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- [9] Manning, A. (1993), "Wage Bargaining and the Phillips Curve: The Identification and Specification of Aggregate Wage Equations", *The Economic Journal*, 103, pp. 98-118
- [10] Soskice, D. and T. Iversen (2000) "The Nonneutrality of Monetary Policy with Large Price or Wage Setters", *Quarterly Journal of Economics CXV*.
- [11] Vartiainen, J. (2002), "Relative Prices in Monetary Union and Floating". Scandinavian Journal of Economics, vol 104, No.2

Appendix

A1 Proofs

Proof of Proposition 2 When labour is immobile between sectors, the ranking of regimes within each sector is as follows:

$$w_{TI} > w_{TM}$$

 $w_{NM} > w_{NI}$

Proof. According to Table 2.2 $w_{TI} > w_{TM}$ if and only if

$$\begin{pmatrix} \frac{\lambda + (1 - \gamma) \sigma}{(1 - \gamma) \sigma} \end{pmatrix} b > \begin{pmatrix} \frac{(1 + \sigma) (\sigma + \lambda) - \gamma \sigma (1 - \lambda)}{\sigma (1 - \gamma + \sigma)} \end{pmatrix} b \Leftrightarrow \\ (\lambda + (1 - \gamma) \sigma) (\sigma (1 + \sigma) - \gamma \sigma) > (1 - \gamma) \sigma ((1 + \sigma) (\sigma + \lambda) - \gamma \sigma (1 - \lambda)) \Leftrightarrow \\ \lambda (\sigma (1 + \sigma) - \gamma \sigma) > (1 - \gamma) \sigma ((1 + \sigma) - \gamma \sigma) (1 - \gamma) - \sigma (1 + \sigma) + \gamma \sigma) \Leftrightarrow \\ \lambda (\sigma (1 + \sigma) - \gamma \sigma) > (1 - \gamma) \sigma \lambda (1 + \sigma + \gamma \sigma) \Leftrightarrow \\ 1 + \sigma - \gamma > (1 - \gamma) (1 + \sigma + \gamma \sigma) \Leftrightarrow \\ \sigma > (1 - \gamma) (\sigma + \gamma \sigma) \Leftrightarrow \\ 1 > (1 - \gamma) (1 + \gamma) \Leftrightarrow \\ 1 > 1 - \gamma^2 \Leftrightarrow \\ \gamma^2 > 0$$

Similarly: $w_{NM} > w_{NI}$ if and only if

$$\begin{pmatrix} \frac{\lambda (1+\sigma) + \gamma \sigma (1-\lambda)}{\gamma \sigma} \end{pmatrix} b > \begin{pmatrix} \frac{\lambda + \gamma \sigma}{\gamma \sigma} \end{pmatrix} b \Leftrightarrow \\ \lambda (1+\sigma) + \gamma \sigma (1-\lambda) > \lambda + \gamma \sigma \Leftrightarrow \\ \lambda \sigma (1-\gamma) > 0$$

which holds true $\forall \gamma \in (0, 1)$ and the proposition follows.

Proof of Proposition 3 When labour is immobile between sectors, the ranking of sectoral wages under a given regime is as follows:

$$w_{TI} \geq w_{NI}$$
 if and only if $\gamma \geq \frac{1}{2}$
 $w_{NM} \geq w_{TM}$ if and only if $\gamma \leq \frac{(1+\sigma)}{2}$

Proof. According to Table 2.2 $w_{TI} \ge w_{NI}$ if and only if

$$\begin{pmatrix} \frac{\lambda + (1 - \gamma) \sigma}{(1 - \gamma) \sigma} \end{pmatrix} b \geq \left(\frac{\lambda + \gamma \sigma}{\gamma \sigma} \right) b \Leftrightarrow \gamma \left(\lambda + (1 - \gamma) \sigma \right) \geq \left(1 - \gamma \right) \left(\lambda + \gamma \sigma \right) \Leftrightarrow \lambda \gamma + (1 - \gamma) \sigma \gamma \geq \lambda \left(1 - \gamma \right) + (1 - \gamma) \gamma \sigma \Leftrightarrow \gamma \geq \left(1 - \gamma \right) \Leftrightarrow \gamma \geq \frac{1}{2}$$

Moreover, $w_{NM} \ge w_{TM}$ if and only if

$$\begin{pmatrix} \frac{\lambda\left(1+\sigma\right)+\gamma\sigma\left(1-\lambda\right)}{\gamma\sigma} \end{pmatrix} b \geq \left(\frac{\left(1+\sigma\right)\left(\sigma+\lambda\right)-\gamma\sigma\left(1-\lambda\right)}{\sigma\left(1+\sigma-\gamma\right)}\right) b \Leftrightarrow \\ \left(1+\sigma-\gamma\right)\left(\lambda\left(1+\sigma\right)+\gamma\sigma\left(1-\lambda\right)\right) \geq \gamma\left(\left(1+\sigma\right)\left(\sigma+\lambda\right)-\gamma\sigma\left(1-\lambda\right)-\gamma\sigma\left(1-\lambda\right)\right) \Leftrightarrow \\ \left(1+\sigma\right)\left(\lambda\left(1+\sigma\right)+\gamma\sigma\right) \geq \gamma\left(\left(1+\sigma\right)\left(\sigma+\lambda\right)-\gamma\sigma\left(1-\lambda\right)+\lambda\left(1+\sigma\right)+\gamma\sigma\left(1-\lambda\right)\right) \Leftrightarrow \\ \lambda\left(1+\sigma\right)+\gamma\sigma \geq \gamma\left(\sigma+\lambda+\lambda\right) \Leftrightarrow \\ 1+\sigma \geq 2\gamma \Leftrightarrow \\ \frac{1+\sigma}{2} \geq \gamma$$

and the proposition follows. \blacksquare

A2 The Nominal Value of Being Unemployed Exogenously Given

The nominal wage solves:

$$\max_{\ln W_{im}} \lambda_i \ln \left[\Lambda_i - \Lambda_{i0} \right] + (1 - \lambda_i) \ln \left[(\eta - 1)^{-1} \frac{W_{im}}{P_m} \left(\frac{W_{im}}{P_{im}} \right)^{-\eta} \right]$$

subject to

$$N_{im} = \left(\frac{W_{im}}{P_{im}}\right)^{-\eta}$$

$$f_m = \frac{M_T N_{Nm} (w_{Nm} - b) - M_{Nm} N_{Tm} (w_{Tm} - b)}{N_{Nm} (w_{Nm} - b) + N_{Tm} (w_{Tm} - b)}$$

$$P_m = P(W_{Nm}, W_{Tm})$$

$$P_{im} = P_i(W_{Nm}, W_{Tm}).$$

The first-order conditions for the union in the non-tradables and tradables sector, respectively, are:

$$\lambda_{N} \left[-\eta \varphi_{Nm} - \frac{\frac{\partial f_{m}}{\partial \ln W_{Nm}}}{(M_{N} + f_{m})} + \frac{w_{Nm} \epsilon_{Nm} + b\left(1 - \epsilon_{Nm}\right)}{(w_{Nm} - b)} \right] + (1 - \lambda_{N}) \left(\epsilon_{Nm} - \eta \varphi_{Nm} \right) = 0$$

$$\lambda_{T} \left[-\eta \varphi_{Tm} + \frac{\frac{\partial f_{m}}{\partial \ln W_{Tm}}}{(M_{T} - f_{m})} + \frac{w_{Tm} \epsilon_{T} + b\left(1 - \epsilon_{T}\right)}{(w_{Tm} - b)} \right] + (1 - \lambda_{N}) \left(\epsilon_{Tm} - \eta \varphi_{Tm} \right) = 0$$

$$(2.32)$$

where

$$\frac{\partial f_m}{\partial \ln W_{Nm}} = \frac{(M_N + M_T) N_{Nm} N_{Tm} (w_{Tm} - b) b}{[N_{Nm} (w_{Nm} - b) + N_{Tm} (w_{Tm} - b)]^2} > 0$$

$$\frac{\partial f_m}{\partial \ln W_{Tm}} = -\frac{(M_N + M_T) N_{Nm} N_{Tm} (w_{Nm} - b) b}{[N_{Nm} (w_{Nm} - b) + N_{Tm} (w_{Tm} - b)]^2} < 0.$$

The first-order conditions (2.32) and (2.33) show why there is wage equality under inflation targeting but not in a monetary union. Under inflation targeting, $\epsilon_{NI} = \epsilon_{TI} = 1$ and the additional term stemming from the effect on *b* vanishes. Thus, one may think of the case with an exogenous *B* under inflation targeting as a special case of the value of unemployment being given in real terms. In a monetary union

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 $\epsilon_{iM} \neq 1$ and this difference in consumer price outcomes across regimes also causes real wages under inflation targeting to differ from real wages in a monetary union.

Substituting for equilibrium net flows, f_m , and $\partial f_m / \partial \ln W_{im}$ and letting $\hat{}$ denote the case with perfect mobility under the assumption of an exogenous B, I obtain the following expressions for the wage curves in the two sectors:

$$\widehat{w}_{Nm} = \left[1 + \frac{\lambda_N}{\eta \varphi_{Nm} - \epsilon_{Nm}}\right] b - \frac{\widehat{N}_{Tm}}{\widehat{N}_{Nm}} \left(\widehat{w}_{Tm} - b\right)$$
$$\widehat{w}_{Tm} = \left[1 + \frac{\lambda_T}{\eta \varphi_{Tm} - \epsilon_{Tm}}\right] b - \frac{\widehat{N}_{Nm}}{\widehat{N}_{Tm}} \left(\widehat{w}_{Nm} - b\right).$$

Substituting the expression for relative employment (2.29) into the above wage curves yields equilibrium real wages on reduced form:

$$\widehat{w}_{Nm} = \left[1 + \frac{\lambda_N}{\eta\varphi_{Nm} - \epsilon_{Nm}} \left(1 + \frac{\eta\varphi_{Tm} - \epsilon_{Tm}}{\lambda_T}\right)\right] \gamma b$$
$$\widehat{w}_{Tm} = \left[1 + \frac{\lambda_T}{\eta\varphi_{Tm} - \epsilon_{Tm}} \left(1 + \frac{\eta\varphi_{Nm} - \epsilon_{Nm}}{\lambda_N}\right)\right] (1 - \gamma) b.$$

Evaluating the wage curves for the equilibrium price elasticities I obtain:

$$\widehat{w}_{NI} = \widehat{w}_{TI} = \left[\frac{\lambda + \sigma}{\sigma}\right] b$$

$$\widehat{w}_{NM} = \left[\frac{\lambda + \sigma}{\sigma}\right] [1 + \sigma] b$$

$$\widehat{w}_{TM} = \left[\frac{\lambda + \sigma}{1 - \gamma + \sigma}\right] \left[\frac{1 + \sigma}{\sigma}\right] [1 - \gamma] b$$

Thus, $w_{iI} \neq w_{iM}$ for i = N, T, i.e. the monetary regime matters for equilibrium real wages when the value of being unemployed is treated as exogenously given in nominal terms.

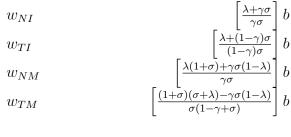
A3 Tables

$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	n (M)	Monetary Union	Inflation Target (I)	Regime (m)	
$ \begin{array}{ccccc} 1 - \varphi_{Nm} & \left(\frac{d\ln P_N}{d\ln W_N}\right)_m & \frac{(1-\gamma)\sigma}{1+\sigma} \\ 1 - \psi_{Nm} & \left(\frac{d\ln P_T}{d\ln W_N}\right)_m & -\frac{\gamma\sigma}{1+\sigma} \\ 1 - \epsilon_{Nm} & \left(\frac{d\ln P}{d\ln W_N}\right)_m & 0 \\ 1 - \varphi_{Tm} & \left(\frac{d\ln P_T}{d\ln W_T}\right)_m & \frac{\gamma\sigma}{1+\sigma} \\ 1 - \varphi_{Mm} & \left(\frac{d\ln P_T}{d\ln W_T}\right)_m & \frac{1-\gamma\sigma}{1+\sigma} \\ \end{array} $					
$ \begin{array}{cccc} 1 - \psi_{Nm} & \left(\frac{d\ln P_T}{d\ln W_N}\right)_m^{\text{in}} & -\frac{\gamma\sigma}{1+\sigma} \\ 1 - \epsilon_{Nm} & \left(\frac{d\ln P}{d\ln W_N}\right)_m^{\text{in}} & 0 \\ 1 - \varphi_{Tm} & \left(\frac{d\ln P_T}{d\ln W_T}\right)_m^{\text{in}} & \frac{\gamma\sigma}{1+\sigma} \\ 1 - d\nu & \left(d\ln P_N\right)^{\text{in}} & (1-\gamma)\sigma \end{array} $	$rac{\sigma}{1+\sigma}$		$rac{(1-\gamma)\sigma}{1+\sigma}$	$\left(\frac{d\ln P_N}{d\ln W_N}\right)_m$	$1 - \varphi_{Nm}$
$ \begin{array}{ll} 1 - \epsilon_{Nm} & \left(\frac{d\ln P}{d\ln W_N}\right)_m^m & 0 \\ 1 - \varphi_{Tm} & \left(\frac{d\ln P_T}{d\ln W_T}\right)_m & \frac{\gamma\sigma}{1+\sigma} \\ 1 - q_{Mn} & \left(d\ln P_N\right) & (1-\gamma)\sigma \end{array} $	0		$-rac{\gamma\sigma}{1+\sigma}$	$\left(\frac{d\ln P_T}{d\ln W_N}\right)_m^m$	$1-\psi_{Nm}$
$ \begin{array}{ccc} 1 - \varphi_{Tm} & \left(\frac{d\ln P_T}{d\ln W_T}\right)_m & \frac{\gamma\sigma}{1+\sigma} \\ 1 & dn & (d\ln P_N) \end{array} $	$rac{\gamma\sigma}{1+\sigma}$		0	$\left(\frac{d\ln P}{d\ln W_N}\right)_m$	$1 - \epsilon_{Nm}$
$1 \qquad d\ln P_N \qquad (1-\gamma)\sigma$	0			$\left(\frac{d\ln P_T}{d\ln W_T}\right)_m$	$1 - \varphi_{Tm}$
$1 - \psi_{Tm} \left(\frac{d \ln W_T}{d \ln W_T}\right)_m \qquad - \frac{1 - \sigma}{1 + \sigma}$	$-rac{\sigma}{1+\sigma}$	-	$-\frac{(1-\gamma)\sigma}{1+\sigma}$	$\left(\frac{d\ln P_N}{d\ln W_T}\right)_m$	$1 - \psi_{Tm}$
$1 - \epsilon_{Tm} \left(\frac{d\ln P}{d\ln W_T}\right)_m \qquad \qquad$	$-\frac{\gamma\sigma}{1+\sigma}$	-	0	$\left(\frac{d\ln P}{d\ln W_T}\right)_m$	$1 - \epsilon_{Tm}$

Table 2.1: Producer and consumer price effects under the two regimes

Table 2.2: Equilibrium real wages under the two regimes

No Mobility



Perfect Mobility

$ \widetilde{w}_{NI} \\ \widetilde{w}_{TI} \\ \widetilde{w}_{NM} \\ \widetilde{\omega}$	$\begin{bmatrix} \frac{\lambda + \sigma}{\sigma} \\ \frac{\lambda + \sigma}{\sigma} \end{bmatrix} b \\ \begin{bmatrix} \frac{\lambda + \sigma}{\sigma} \\ \lambda + \sigma \end{bmatrix} b \\ \begin{bmatrix} \lambda + \sigma \\ \lambda + \sigma \end{bmatrix} b$
\widetilde{w}_{TM}	$\left[\frac{\lambda+\sigma}{\sigma}\right]b$

	Regime	Regime Monetary Union		Inflation Targeting	
		No mobility	Perfect mobility	No mobility	Perfect mobility
γ					
0.2	n_N	0.0311	0.1567	0.0532	0.1567
	n_T	0.425	0.6268	0.4581	0.6268
	n^{-}	0.4561	0.7835	0.4888	0.7835
	w_N	5.3202	1.451	3.3856	1.451
	w_T	1.5584	1.451	1.5719	1.451
	f		-0.3		-0.3
0.3	n_N	0.0717	0.2625	0.1132	0.2625
	n_T	0.3826	0.6125	0.4107	0.6125
	n	0.4543	0.875	0.5239	0.875
	w_N	3.708	1.451	2.5795	1.451
	w_T	1.6217	1.451	1.6582	1.451
	f		-0.2		-0.2
0.4	n_N	0.1287	0.3724	0.1872	0.3724
	n_T	0.3308	0.5586	0.3446	0.5586
	\overline{n}	0.4595	0.931	0.5318	0.931
	w_N	2.9019	1.451	2.1764	1.451
	w_T	1.6928	1.451	1.7734	1.451
	f		-0.1		-0.1
0.5	n_N	0.1997	0.475	0.2672	0.475
	n_T	0.2723	0.475	0.2672	0.475
	\overline{n}	0.472	0.95	0.5344	0.95
	w_N	2.4183	1.451	1.9346	1.451
	w_T	1.7734	1.451	1.9346	1.451
	f		0.0		0.0
0.6	n_N	0.2805	0.5586	0.3446	0.5586
	n_T	0.2101	0.3724	0.1872	0.3724
	n^{-}	0.4906	0.931	0.5318	0.9310
	w_N	2.0958	1.451	1.7734	1.451
	w_T	1.8655	1.451	2.1764	1.451
	f		0.1		0.1

Table 2.3: Numerical solutions of the model, $\delta=0.5,\,\lambda=0.5$

	Regime	Monetary Union	_	Inflation Targeting	
		No mobility	Perfect mobility	No mobility	Perfect mobility
γ					
0.2	V_N	1.2383	1.3462	1.2245	1.3462
0.2	V_T	1.4698	1.3462	1.5212	1.3462
	Π_N	0.1656	0.2274	0.18	0.2274
	Π_N Π_T	0.6623	0.9094	0.72	0.9094
0.3	V_N	1.3604	1.3905	1.3322	1.3905
	V_T	1.4680	1.3905	1.5348	1.3905
	Π_N	0.2659	0.3809	0.2919	0.3809
	Π_T	0.6204	0.3809	0.6811	0.8887
0.4	V_N	1.4651	1.4176	1.4199	1.4176
	V_T	1.4473	1.4176	1.5228	1.4176
	Π_N	0.3733	0.5404	0.4073	0.5404
	Π_T	0.5600	0.8106	0.611	0.8106
0.5	V_N	1.5468	1.4268	1.4842	1.4268
	V_T	1.4063	1.4268	1.4842	1.4268
	Π_N	0.4829	0.6892	0.5169	0.6892
	Π_T	0.4829	0.6892	0.5169	0.6892
0.6	V_N	1.6004	1.4176	1.5228	1.4176
0.0	V_T	1.3447	1.4176	1.4199	1.4176
	Π_N	0.5879	0.8106	0.611	0.8106
	Π_N Π_T	0.5879	0.5404	0.4073	0.5404

Table 2.4: Expected income and firm profits in the two sectors, $\delta = 0.5, \lambda = 0.5$

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Chapter 3

Fiscal Activism under Inflation Targeting and Non-atomistic Wage Setting^{*}

1 Introduction

The existence of a potential conflict of interest between fiscal and monetary authorities is well known. If the government pursues expansionary fiscal policy in an attempt to promote employment, the central bank is likely to respond with a monetary contraction if the fiscal expansion threatens price stability. Such disagreement between governments and central banks could be particularly severe if the central bank maintains an explicit inflation target. Therefore, the introduction of inflation targeting in many countries over the last fifteen years or so constitutes sufficient motivation for revisiting these issues. Another question, much debated by policymakers in recent years (within as well as outside the EMU), is to what extent fiscal policy will substitute for monetary policy when the latter is tied to the mast. How does the role of fiscal policy depend on the monetary regime? Should the government follow a policy rule or act under discretion?

The implications of monetary-fiscal interactions may be even more far-reaching in economies with collective bargaining, where the objectives of the central bank are likely to differ from the objectives of trade unions. Thus, in an economy with

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large unions and inflation targeting, the central bank must attempt to counteract inflationary pressure stemming from both expansionary fiscal policy and wage claims by unions. Moreover, the choice of fiscal policy may not be independent of the choices of unions: large unions are likely to anticipate how fiscal policy will react to wage changes and exploit government responses when trading off wages against employment.

This paper considers the multi-player game between the government, an independent central bank and non-atomistic trade unions in a closed economy. I mainly combine three strands of literature. The first strand is the literature on strategic interaction between large wage setters and independent central banks.¹ This literature explains, inter alia, how independent central banks may discipline large wage setters; see, for example, Soskice and Iversen (2000), Corricelli et al. (2000) and Lippi (2003). The second strand of literature considers fiscal-monetary policy interactions. Dixit and Lambertini (2001 and 2003) consider the strategic interaction between fiscal and monetary authorities under different assumptions about commitment and discretion. They alternate which authority is given first-mover advantage and show that non-cooperation between authorities may yield inferior equilibria. This may be remedied by joint commitment or actors agreeing on the optimal levels of output and inflation. The third strand of literature analyses the potential interaction between the government and large wage setters. Early contributions include Calmfors (1982), Calmfors and Horn (1985, 1986) and Driffill (1985). Hersoug (1985) considers the game between an all-embracing trade union and the government when the government is concerned with employment and the trade balance, which are both affected by union wage decisions. Conversely, government tax and expenditure decisions affect employment and the real disposable wage of union members. Hersoug analyses equilibrium implications of various game structures and argues that the most plausible case is when the union acts as a Stackelberg leader, deciding on irreversible wage contracts, while the government may adjust fiscal policy as a response to wages set by the union.

None of these studies considers the simultaneous interaction between the government (the fiscal authority), the central bank (the monetary authority) and unions (wage setters). An exception is Cukierman and Dalmazzo (2005) who construct a model where fiscal and monetary authorities have different objectives in a framework with unionised labour markets. The different objectives of fiscal and monetary

¹ Calmfors (2001) provides a review.

authorities give rise to strategic interaction between these two agents. Since unions recognise that they are sufficiently large to influence the aggregate price level, a game also arises between unions and the monetary and fiscal authorities. However, in the setting of Cukierman and Dalmazzo, fiscal policy only consists of labour taxes levied to finance unemployment benefits and such a setting fails to incorporate aggregate demand as a channel through which expansionary fiscal policy may affect the real economy.

I draw somewhat on the work by Cukierman and Dalmazzo (2005), but extend the model in four dimensions. First, I expand on the role of fiscal policy by introducing a public good, the provision of which affects labour demand. Second, I let the central bank pursue a flexible strategy that encompasses inflation targeting as a special case. The objective function is consistent with "the two-pillar strategy" of the ECB, which focuses on both inflation and the money supply. Third, I provide some microfoundations for the objective functions of unions and to some extent also the government instead of postulating entirely ad hoc loss functions for the main players. The government objective function is formulated such that the degree of fiscal activism can be varied within the model. Finally, I extend the analysis further, by assigning the first-mover advantage to different players.

The main findings are as follows. While inflation targeting by an independent central bank may discipline wage setters, activist fiscal policy tends to raise real wages and decrease equilibrium employment. The main intuition for this result is that when unions internalise the government's response to their wage claims, they exploit the fact that the government cares about workers' utility: fiscal policy responses enable unions to raise wages with lower costs in terms of unemployment than would otherwise be the case. However, a fiscal rule induces moderation in wage setting. The difference in real wages and employment across monetary regimes is greater when fiscal policy is endogenous than when it is exogenous, and the difference is increasing in the degree of activism. Finally, the results suggest that inflation targeting may provide incentives also for fiscal restraint.

The paper is organised as follows. Section 2 presents the general model when unions are Stackelberg leaders to the government. To make the model as transparent as possible, in Section 3, I consider special cases of the general model, where I focus on the interaction between only two players at a time. Therefore, I consider the following three cases: (i) the strategic interaction between large unions and the central bank when fiscal policy is exogenously set; (ii) the interaction between the fiscal and the monetary authorities when wage setting is decentralised; and (iii) the interaction between the government and large wage setters when the money supply is exogenously given. In Section 4, I study the interaction between fiscal and monetary authorities under non-atomistic wage setting, but assume that the government is a Stackelberg leader to unions. Numerical solutions to the general model are presented in Section 5. Section 6 concludes.

2 The Model

Consider a small, closed economy, consisting of a large number of firms and workers organised by trade unions. There is monopolistic competition in the goods market and there is an independent central bank conducting monetary policy. In Stage 1, unions set nominal wages, taking the nominal wages set by all other unions as given, but anticipating the reactions of the government, the central bank and firms. In Stage 2, the government decides how much to provide of a public good, taking wages as given, but anticipating the response of the central bank, households and firms. In Stage 3, the central bank takes wages and public spending as given when setting monetary policy, taking the expected responses of households and firms into account. Finally, in Stage 4, households maximise utility and firms set prices, taking wages, public spending and monetary policy as given: markets clear and employment is determined. The model is solved by backward induction and the equilibrium is subgame perfect.

2.1 Stage 4: Households' and Firms' Decisions and Market Clearing

There is a continuum of households in the economy, indexed h and with mass one. Households provide labour supply to the firms and consume the goods supplied by the firms. The economy consists of a number of regions, with mass one and indexed by j. In each region, there is one private firm, owned by capitalists living abroad. The assumption that firms are owned by foreign capitalists is made for simplicity and does not affect the results. In addition, there is also a branch of the government sector in each region, producing a collective good which is consumed by all households in the economy, independent of where they live.² There is a fixed quantity of workers, L_0 , attached to each region, and both the private firm and

 $^{^{2}}$ We can think of the public good as, for instance, national defense.

the local government branch hire workers from this regional pool of labour. The setup is equivalent to considering the economy as consisting of many local labour markets with mass one. There are N equally sized unions, indexed i, imposing the same wage in the region for private and government employees. Each union covers a fraction 1/N of the regions. Without loss of generality, firms are ordered such that all regions with a labour force represented by union i are located on the subinterval $\left(\frac{i}{N}, \frac{i+1}{N}\right)$, i = 0, 1, ..., N - 1.

2.1.1 Households

Each household h has the utility function:³

$$U_h = \left(\frac{C_h}{\alpha}\right)^{\alpha} \left(\frac{M_h/P}{1-\alpha}\right)^{1-\alpha} + \eta G - \frac{\beta}{2}G^2,$$

where C_h is private consumption defined below, M_h/P is real money balances and P is the aggregate price level defined as:

$$P = \left(\int_0^1 P_{ij}^{1-\theta} dj\right)^{\frac{1}{1-\theta}},\tag{3.1}$$

where P_{ij} is the price set by firm j, covered by union i. G is the provision of a public good: the functional form is chosen to ensure concavity of utility with respect to G and $\eta, \beta > 0$. C_h is a measure of all goods provided by the different firms and is defined as

$$C_h = \left(\int_0^1 C_{hij}^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}},$$

where C_{hij} is household h's consumption of the good produced by firm j covered by union i. The budget restriction of household h is:

$$\int_0^1 C_{hij} P_{ij} dj + M_h \equiv PC_h + M_h = X_h$$

where X_h is the nominal income of household h. The household chooses C_{hij} and M_h so as to maximise its utility. The solution to the maximisation problem can be

³ The microstructure of the goods market draws on Blanchard and Fischer (1989), although they consider a yeoman farmer economy, while I explicitly assume that firms are owned by capitalists. Moreover, I introduce a public good in the utility function of individual households.

written in the following way:⁴

$$C_{hij} = \left(\frac{P_{ij}}{P}\right)^{-\theta} C_h. \tag{3.2}$$

$$\frac{M_h}{P} = (1-\alpha)\frac{X_h}{P}, \qquad (3.3)$$

$$C_h = \alpha \frac{X_h}{P}. \tag{3.4}$$

Combining (3.3) and (3.4) gives the following relationship between household consumption and real money holdings:

$$C_h = \frac{\alpha}{(1-\alpha)} \frac{M_h}{P}.$$
(3.5)

Finally, aggregate demand facing firm j is obtained by integrating (3.2) over all households on the unit interval:

$$Y_{ij}^{D} \equiv \int_{0}^{1} C_{hij} dh = \left(\frac{P_{ij}}{P}\right)^{-\theta} \int_{0}^{1} C_{h} dh.$$
(3.6)

Substituting (3.5) in (3.6), I obtain:

$$Y_{ij}^{D} \equiv \frac{\alpha}{(1-\alpha)} \left(\frac{P_{ij}}{P}\right)^{-\theta} \int_{0}^{1} \frac{M_{h}}{P} dh.$$

This implies that demand facing firm j can be written:

$$Y_{ij}^D = \frac{\alpha}{(1-\alpha)} \left(\frac{P_{ij}}{P}\right)^{-\theta} \frac{M}{P},\tag{3.7}$$

where $M/P = \int_0^1 \frac{M_h}{P} dh$ in equilibrium. Evaluating the utility function in the optimal choices, (3.3) and (3.4), I obtain the indirect utility function of household h:

$$V_{h} = \frac{X_{h}}{P} + \eta G - \frac{\beta}{2}G^{2}.$$
 (3.8)

⁴ Details are given in Appendix A1.

2.1.2 Private Production

In each private firm, output is produced with labour as the only input. The production function of private firm j, associated with union i, is given by:

$$Y_{ij} = L_{ij}^{\gamma}, \tag{3.9}$$

where $\gamma \in (0, 1)$. Each firm faces a demand for its output given by (3.7). Firms set prices, P_{ij} so as to maximise real profits, and thus consider:

$$\max_{P_{ij}} \prod_{ij} = \frac{P_{ij}}{P} Y_{ij}^D - \frac{W_i}{P} L_{ij}$$

subject to (3.9) and (3.7), taking P, M and W_i as given. Solving the maximisation problem and re-arranging, I obtain the firm's price setting rule:

$$\left(\frac{P_{ij}}{P}\right) = \zeta \left(\frac{W_i}{P}\right)^{\gamma \phi} \left(\frac{M}{P}\right)^{(1-\gamma)\phi}, \qquad (3.10)$$

where $\zeta \equiv \left(\frac{\alpha}{1-\alpha}\right)^{(1-\gamma)\phi} \left(\frac{\theta}{\gamma(\theta-1)}\right)^{\gamma\phi}$ and $\phi \equiv \frac{1}{\gamma+\theta(1-\gamma)}$. It should be noted that P_{ij} is linearly homogenous in W_i , M and P.⁵ For future reference, I need to derive each firm's demand for labour. Equations (3.9) and (3.7) together imply:

$$L_{ij} = \left[\frac{\alpha}{(1-\alpha)} \left(\frac{P_{ij}}{P}\right)^{-\theta} \frac{M}{P}\right]^{\frac{1}{\gamma}}.$$
(3.11)

Inserting the firm's optimal pricing rule (3.10), I obtain:

$$L_{ij} = \vartheta \left(\frac{M}{P}\right)^{\phi} \left(\frac{W_i}{P}\right)^{-\theta\phi}, \qquad (3.12)$$

where $\vartheta \equiv \left[\frac{\alpha}{1-\alpha}\right]^{\phi} \left[\frac{\theta}{\gamma(\theta-1)}\right]^{-\theta\phi}$.

⁵ Collecting terms, I obtain: $P_{ij} = \zeta W_i^{\gamma\phi} M^{(1-\gamma)\phi} P^{(1+\phi)}$. Increasing all inputs by a factor k implies that the right-hand side can be written: $\zeta (kW_i)^{\gamma\phi} (kM)^{(1-\gamma)\phi} (kP)^{(1+\phi)} = \zeta k^{\gamma\phi+(1-\gamma)\phi+(1+\phi)} W_i^{\gamma\phi} M^{(1-\gamma)\phi} P^{(1+\phi)} = k\zeta W_i^{\gamma\phi} M^{(1-\gamma)\phi} P^{(1+\phi)} = kP_{ij}$.

2.1.3 Public Good Production

The public good is produced with a constant-returns-to-scale technology with labour as the only input:

$$Y_j^G = L_j^G,$$

where superindex G denotes the government sector. I assume government demand for the public good to be symmetrically distributed across regions. Since regions have mass one, $\int_0^1 G dj = G$, it follows that $G_j = G$ and:

$$L_j^G = G,$$

where L_j^G is regional demand for labour for the production of the public good.

2.1.4 Aggregation

By aggregating the private firms' demand for labour (3.11) and using the definition of the aggregate price level (3.1), I obtain the aggregate demand for labour as⁶

$$L^{D} = \left[\frac{\alpha}{1-\alpha}\frac{M}{P}\right]^{\frac{1}{\gamma}},$$

where aggregate labour demand is defined as

$$L^{D} = \left(\int_{0}^{1} L_{ij}^{\frac{\gamma(\theta-1)}{\theta}} dj\right)^{\frac{\theta}{\gamma(\theta-1)}}$$

Since the total labour force equals the mass of households, the labour force has mass one. Aggregate labour demand for the production of the public good is given by G. The aggregate unemployment rate is therefore given by:

$$u = 1 - L^{D} - G = 1 - \left[\frac{\alpha}{1 - \alpha} \frac{M}{P}\right]^{1/\gamma} - G.$$
 (3.13)

The equation shows how unemployment is decreasing in the money supply. Averaging (3.10) over firms and rearranging gives the following expression for the aggregate price level:

$$P = \kappa W^{\gamma} M^{(1-\gamma)}, \qquad (3.14)$$

⁶ Details are given in Appendix A1.

where $\kappa \equiv \left(\frac{\alpha}{1-\alpha}\right)^{(1-\gamma)} \left(\frac{\theta}{\gamma(\theta-1)}\right)^{\gamma}$ and W is the aggregate wage index defined as:

$$W = \left(\frac{1}{N}\sum_{i=0}^{N-1} W_i^{1-\theta\phi}\right)^{\frac{1}{1-\theta\phi}}$$

where, as before, $\phi \equiv \frac{1}{\gamma + \theta(1-\gamma)}$. Note that a linear technology would imply $\gamma = 1$ and thus $\phi = 1$, which is the case studied in Gnocchi (2005). In my analysis, the elasticity of the aggregate wage to the individual union's wage decision depends - in addition to the number of unions - on the elasticity of substitution of goods, θ , and the elasticity of output with respect to employment.

2.2 Stage 3: Monetary Policy

Suppose that the objective of the Central Bank is to set the money supply in order to keep a weighted mean of the price level and the money supply constant:

$$M^{\varphi}P^{1-\varphi} = c$$

Since the model is static, I cannot distinguish between price level targeting and inflation targeting, but I shall use the term inflation targeting throughout the paper. Strict inflation targeting is the case when $\varphi = 0$, while $\varphi = 1$ is equivalent to fixing the money supply. Targeting the money supply is equivalent to assuming that the money supply is exogenously given, thus assuming away any policy action on the part of the central bank. The case when $\varphi = 1$ can therefore be considered as what would happen if wages were set without any strategic response of the central bank. $\varphi = 1/2$ is equivalent to targeting the real money supply. The objective function of the central bank can be considered as a general characterisation of "a two-pillar strategy" like that pursued by the ECB. Although the primary objective of the ECB is price stability, it recognises the need for taking a wide range of variables into account when assessing the level of economic activity and making policy decisions. Alternatively, the money supply can be considered as a proxy variable for the development of asset prices in the economy. Money growth can thus serve as a signal indicating risks of increases in asset prices, which may imply long-run risks of inflation, even though short-run inflation targets are met.

Throughout the paper, let subindices P and M denote the cases when the central bank fixes inflation and the money supply, respectively. Solving for the money

supply, I obtain:

$$M = c^{\frac{1}{\varphi}} P^{\frac{\varphi-1}{\varphi}}.$$

Substituting for the equilibrium price level (3.14) and rearranging, I obtain the reaction function of the central bank:

$$M = c^{\varpi} \kappa^{-(1-\varphi)\varpi} W^{-\gamma(1-\varphi)\varpi}, \qquad (3.15)$$

where $\varpi \equiv \frac{1}{\varphi + (1-\gamma)(1-\varphi)}$. Substituting this expression into the expression for the equilibrium price level implies:

$$P = \frac{W^{\gamma\varphi\varpi}}{\widetilde{\chi}},\tag{3.16}$$

where $\tilde{\chi} \equiv \left(\frac{\alpha}{1-\alpha}\right)^{-(1-\gamma)\varphi\varpi} \left(\frac{\theta}{\gamma(\theta-1)}\right)^{-\gamma\varphi\varpi} c^{-(1-\gamma)\varpi}$. The real money supply can be written:

$$\frac{M}{P} = \kappa^{-\varpi} \left(\frac{W}{c}\right)^{-\gamma\varpi}.$$
(3.17)

Substituting (3.17) into the expression for unemployment (3.13) implies:

$$u = 1 - \delta \left(\frac{W}{c}\right)^{-\varpi} - G, \qquad (3.18)$$

where $\delta \equiv \left(\frac{\alpha}{(1-\alpha)}\right)^{\varphi \varpi} \left(\frac{\theta}{\gamma(\theta-1)}\right)^{-\varpi}$. It follows that unemployment is increasing in the aggregate nominal wage. For future reference, note that by substituting for the equilibrium price level and the optimal response of the central bank, the aggregate real wage can be expressed as:

$$\frac{W}{P} = \chi \left(\frac{W}{c}\right)^{(1-\gamma)\varpi},\tag{3.19}$$

where $\chi = \kappa^{-\varphi \varpi}$.⁷ Similarly, the real wage for union *i* is given by:

$$\frac{W_i}{P} = \chi \frac{W_i}{c} \left(\frac{W}{c}\right)^{-\gamma \varphi \varpi}$$

⁷ Under inflation targeting $\varphi = 0$, $\varpi = \frac{1}{1-\gamma}$ and c = P shows that the right-hand side is indeed given by the real wage. Under money supply targeting $\varphi = \varpi = 1$ and c = M implies $\frac{W}{P} = \kappa^{-1} \left(\frac{W}{M}\right)^{(1-\gamma)}$.

2.3 Stage 2: Fiscal Policy

I assume that the government cares about the sum of indirect expected utilities of individual workers. I also assume that the government can run a budget deficit. I do not exactly specify how this is done, but one can think of a possibility for the government to borrow and lend abroad. It is as if the model were open in terms of capital mobility for the government, although the economy is closed in all other respects. The government, but no other agents, cares about the size of the deficit. By varying the weight placed by the government on the deficit in its objective function, I can characterise different degrees of fiscal policy activism. This is crude but serves as a simple way of introducing "demand management policy" in my framework. The interpretation of ϵ is analysed in detail below. The government objective function is thus assumed to be:

$$\Lambda = \int_{0}^{1} V_{h} dh + \epsilon \rho = \int_{0}^{1} \left(\frac{X_{h}}{P} + \eta G - \frac{\beta}{2} G^{2} \right) dh + \epsilon \rho$$

= $(1-u) \frac{W}{P} (1-t) + uB (1-t) + \eta G - \frac{\beta}{2} G^{2} + \epsilon \rho,$ (3.20)

where ρ is the budget surplus, defined below, and t is an exogenously given tax rate. I assume taxes to be levied on both wages and unemployment benefits. This assumption normally implies that the wage chosen by unions will be independent of the tax rate, which makes it possible to study pure demand-side effects of fiscal policy. The above formulation suggests that the government cares about unemployment, government spending and the price level, since inflation reduces the real wage and hence the indirect utility of households. The notion that the government cares about inflation and unemployment is common in the literature, but usually some ad hoc loss function with these variables as arguments is simply postulated. Here, I provide a microeconomic rationale for why the government cares about inflation and unemployment.

The parameter ϵ measures the weight the government assigns to the budget balance relative to the current utility of workers. The formulation of the objective function is a crude way of incorporating intertemporal aspects in the model. A more forward-looking government is likely to assign a large weight to the budget surplus, while a completely myopic government assigns very little weight to it. In what follows, I will refer to ϵ as a measure of the degree of myopia of the government. $\epsilon = 0$ represents a completely myopic government, caring about the current utility of workers only. The budget surplus is defined as:

$$\rho = t(1-u)\frac{W}{P} + tuB - \frac{W}{P}G - uB.$$

Since there is no direct effect on the price level of fiscal policy, I may express the problem of the government in real terms:

$$\max_{G} \Lambda = (1-u)\frac{W}{P}(1-t) + uB(1-t) + \eta G - \frac{\beta}{2}G^{2} + \epsilon\rho$$
(3.21)

subject to

$$u = 1 - \varsigma \left(\frac{W}{P}\right)^{-\frac{1}{1-\gamma}} - G$$

$$\rho = t(1-u)\frac{W}{P} + tuB - \frac{W}{P}G - uB$$

taking wages as given and where $\varsigma = \delta \chi^{\frac{1}{1-\gamma}}$. The first-order condition is:

$$-\frac{du}{dG}\frac{W}{P}\left[1-t+\epsilon t\right]+\frac{du}{dG}B\left(1-t\right)\left(1-\epsilon\right)+\eta-\beta G-\epsilon\frac{W}{P}=0,$$

where

$$\frac{du}{dG} = -1$$

Higher public spending has three effects on government utility: first, it reduces unemployment, thus increasing the current utility of workers. Second, it has a positive direct effect on current worker utility by providing more public goods. Third, it reduces the budget surplus.⁸ Solving for G, I obtain:

$$G = \frac{\eta}{\beta} + \frac{(1-\epsilon)\left(1-t\right)\left(\frac{W}{P} - B\right)}{\beta}.$$
(3.22)

The formulation shows that the only way through which the central bank can influence the government is through its impact on real wages. The reason is that the

⁸ In this setting, the government may set the consumption of the public good, G, such that the household's marginal utility of such consumption is negative, i.e. there is "over-provision" of public goods. The reason is that the negative marginal utility of government consumption is offset by the positive marginal utility from higher employment. It would thus be possible to increase government utility by letting the government increase public spending up to the point where the household's marginal utility is zero, and then increase unemployment benefits. For simplicity, I abstract from this possibility in the analysis as I want to model how fiscal policy is actually pursued. One way of ruling out benefit increases as a superior method of achieving higher government utility would be to introduce disutility from being unemployed.

partial derivative of unemployment with respect to G is equal to minus one and thus independent of the regime. More specifically, I have:

$$\frac{dG}{dW/P} = \frac{(1-\epsilon)(1-t)}{\beta} > 0,$$

which implies:

$$\frac{dG}{dW/P} = \begin{cases} \frac{(1-t)}{\beta} > 0 \text{ if } \epsilon = 0\\ \frac{(1-\epsilon)(1-t)}{\beta} > 0 \text{ if } \epsilon < 1\\ 0 \text{ if } \epsilon = 1 \end{cases}$$

Thus, if $\epsilon \in [0, 1)$ i.e. if the government puts a lower weight on the budget surplus than on the current utility of workers, $\frac{dG}{dW/P} > 0$. The higher is the real wage, the higher is public spending since when faced with a higher wage, the government counteracts the negative effect on unemployment by raising G. Moreover, the following ranking holds:

$$\left. \frac{dG}{dW/P} \right|_{\epsilon=1} < \left. \frac{dG}{dW/P} \right|_{\epsilon<1} < \left. \frac{dG}{dW/P} \right|_{\epsilon=0}$$

Given a real wage increase, the government raises G relatively more, the less it cares about the budget deficit.

To better understand the government's actions, note that when $\epsilon = 1$ the government objective function can be written:

$$\Lambda = L^D \frac{W}{P} + \eta G - \frac{\beta}{2} G^2,$$

i.e. only private-sector employment is included in the objective function, since public-sector labour demand enters both on the cost-side of the budget balance and in the current utility level of workers and the two expressions cancel out when $\epsilon = 1$. Since L^D is independent of G and the government takes the real wage level as given, the first-order condition is simply:

$$\eta - \beta G = 0,$$

which implies that the optimal choice of G is given by:

$$G = \frac{\eta}{\beta}.\tag{3.23}$$

Therefore, the case when $\epsilon = 1$ is equivalent to the government following a fiscal rule of balancing the budget. Alternatively, I could let the government meet some

surplus or deficit target. Regardless of the wage outcome, it sets G according to (3.23).

Comparing (3.23) to (3.22) shows that, given real wages, the following ranking applies:

$$G|_{\epsilon=1} < G|_{\epsilon<1} < G|_{\epsilon=0}$$

The intuition is straightforward. If the government cares relatively less about the budget balance than about the current utility of workers, government spending is higher than if the government were indifferent between the two.

In sum, there are thus two interpretations of ϵ : A low ϵ characterises a myopic government and a high level of fiscal activism. Analogously, a high level of ϵ characterises a more forward-looking government and a low level of fiscal activism. In the extreme case when $\epsilon = 1$, it is optimal for the government to follow a fiscal rule.

Since wage setters will internalise their impact on prices, I will solve the union's problems in nominal terms. Therefore, I need to derive the responsiveness of public spending with respect to the nominal wage. I then obtain:

$$\frac{dG}{dW} = \frac{\left(1-\epsilon\right)\left(1-t\right)\left(1-\gamma\right)\varpi\chi\left(\frac{W}{c}\right)^{(1-\gamma)\varpi-1}}{\beta}\frac{1}{c}$$

Thus, $\frac{dG}{dW} > 0$ if $\epsilon < 1$.

2.4 Stage 1: Wage Setting

In the first stage of the game, each union sets its nominal wage, taking the nominal wage decisions of all other unions, W_{-i} , as given. Thus, it anticipates the subsequent reactions of the government, the central bank and firms and households. The indirect utility function of a consumer belonging to the labour force of a firm bargaining with union *i* is according to (3.8) given by:

$$V_{hi} = \frac{X_{hi}}{P} + \eta G - \frac{\beta}{2}G^2,$$

where

$$X_{hi} = \begin{cases} W_i (1-t) \text{ if employed} \\ BP (1-t) \text{ if unemployed} \end{cases}$$

and B is the real value of unemployment benefits. To ensure that workers prefer employment to unemployment, I impose the restriction $B < W_i/P$ on all solutions to the model. I assume that the union cares about a weighted mean of expected utilities of its members. The objective function of union i is:

$$\Omega_i = \int_{\frac{i}{N}}^{\frac{i+1}{N}} \left(\frac{X_{hi}}{P}\right) dh = (1-u_i) \frac{W_i}{P} (1-t) + u_i B (1-t) + \eta G - \frac{\beta}{2} G^2, \quad (3.24)$$

where t is an exogenously given tax rate and u_i is the average unemployment rate among the members of union i. The average unemployment rate in each region j is given by:

$$u_{ij} = 1 - \frac{L_{ij}^D}{L_0} - \frac{G}{L_0}.$$
(3.25)

Since regions as well as the labour force (households) have mass one, i.e. $\int_0^1 L_0 dj = \int_0^1 dh \equiv 1$, it follows that $L_0 = 1$. Next, consider equation (3.10). Since all firms on the sub-interval $\left[\frac{i}{N}, \frac{i+1}{N}\right]$ face the same nominal wage, W_i , in equilibrium, it must hold that $P_i = P_{ij} \forall j \in \left[\frac{i}{N}, \frac{i+1}{N}\right]$. Substituting for (3.12) in (3.25) and substituting for the real money supply consistent with the optimal response of the central bank (3.17), unemployment facing union i is determined by:

$$u_i = 1 - \delta \left(\frac{W_i}{c}\right)^{-\phi\theta} \left(\frac{W}{c}\right)^{-\gamma \varpi \phi(1-\theta\varphi)} - G,$$

where $\delta \equiv \left(\frac{\alpha}{1-\alpha}\right)^{\varphi \varpi} \left(\frac{\theta}{\gamma(\theta-1)}\right)^{-\varpi}$. Eliminating the equilibrium price level in the objective function, union *i* solves the following optimisation problem:

$$\max_{W_i} (1-u_i) \chi \frac{W_i}{c} \left(\frac{W}{c}\right)^{-\gamma\varphi\varpi} (1-t) + u_i B (1-t) + \eta G - \frac{\beta}{2} G^2$$
(3.26)

subject to

$$u_{i} = 1 - \delta \left(\frac{W_{i}}{c}\right)^{-\phi\theta} \left(\frac{W}{c}\right)^{-\gamma\varpi\phi(1-\theta\varphi)} - G$$
$$W = \left(\frac{1}{N}\sum_{i=0}^{N-1}W_{i}^{1-\theta\phi}\right)^{\frac{1}{1-\theta\phi}}$$
$$G = g(W),$$

taking W_{-i} as given and where g(W) is defined by (3.22). Note that

$$\frac{dW}{dW_i} = \frac{1}{N} \frac{W}{W_i}.$$

The first-order condition is:

$$-\frac{du_i}{dW_i} \left(\chi \frac{W_i}{c} \left(\frac{W}{c} \right)^{-\gamma \varphi \varpi} - B \right) (1-t) + (1-u_i) \tau \chi \frac{1}{c} \left(\frac{W}{c} \right)^{-\gamma \varphi \varpi} (1-t) \\ + \left[\eta - \beta G \right] \frac{dG}{dW} \frac{dW}{dW_i} = 0,$$

where

$$\frac{du_i}{dW_i} = \delta\sigma \left(\frac{W_i}{c}\right)^{-\phi\theta-1} \left(\frac{W}{c}\right)^{-\gamma\varpi\phi(1-\theta\varphi)} \frac{1}{c} - \frac{dG}{dW} \frac{dW}{dW_i}$$

and $\sigma = \left[\phi\theta + \frac{\gamma \varpi \phi(1-\varphi\theta)}{N}\right]$ and $\tau = \left[1 - \frac{\gamma \varphi \varpi}{N}\right]$. The first-order condition shows that there are three effects associated with an incremental increase in the nominal wage:

- 1. Raising the wage increases unemployment among union members, which has a negative effect on union utility since $\frac{du_i}{dW_i} > 0$. Rasing the wage decreases labour demand for workers in the region, thus directly increasing unemployment. The negative effect on unemployment of increasing wages is mitigated by the fact that when the aggregate wage increases, so does government spending since $\frac{dG}{dW} > 0$, thus counteracting the increase in unemployment. This mitigating effect is proportional to the degree of centralisation in wage setting through $\frac{dW}{dW_i}$: If $\frac{dW}{dW_i}$ is large, i.e. if wage setting is highly centralised, the mitigating effect will also be large, and if wage setting is highly decentralised, the effect tends to zero.
- 2. Raising the wage makes the employed better off. The effect on union utility is positive and depends on the level of employment among union members. The magnitude of the effect also depends on the degree of centralisation as captured by the parameter τ : as wages are increased, so is the aggregate wage and also the price level. Therefore, the impact of a nominal wage increase on the real wage is partly offset if wage setters are large.
- 3. Raising the wage increases the aggregate wage, W, which, in turn, increases the provision of the public good, and thus, also consumption of the public good. This has a positive utility effect provided that the marginal utility of public good consumption is positive, i.e. if $\eta - \beta G > 0$.

In symmetric equilibrium, $W = W_i$ and thus $\frac{dW}{dW_i} = \frac{1}{N}$. Substituting for the unemployment rate, I obtain:

$$-\underbrace{\left(\varsigma\sigma\chi^{\frac{1}{(1-\gamma)\varpi}}\left(\frac{W}{P}\right)^{\frac{-(\varpi+1)}{(1-\gamma)\varpi}} - c\frac{dG}{dW}\frac{1}{N}\right)}_{\frac{du_i}{dW_i}}\left(\frac{W}{P} - B\right)(1-t) \qquad (3.27)$$

$$+\underbrace{\left(\varsigma\left(\frac{W}{P}\right)^{-\frac{1}{(1-\gamma)}} + G\right)}_{1-u_i}\tau\chi^{\frac{1}{(1-\gamma)\varpi}}\left(\frac{W}{P}\right)^{\frac{-\gamma\varphi}{(1-\gamma)}}(1-t) + \left[\eta - \beta G\right]c\frac{dG}{dW}\frac{1}{N} = 0.$$

This formulation shows that when N is large, $\frac{dG}{dW}$ is of no importance.

2.5 General Equilibrium

Rewriting also the expression for $\frac{dG}{dW}$ in terms of real wages, the general equilibrium is characterised by the following set of equations:

$$-\left(\varsigma\sigma\chi^{\frac{1}{(1-\gamma)\varpi}}\left(\frac{W}{P}\right)^{\frac{-(\varpi+1)}{(1-\gamma)\varpi}} - c\frac{dG}{dW}\frac{1}{N}\right)\left(\frac{W}{P} - B\right)(1-t)$$
(3.28)

$$+\left(\varsigma\left(\frac{W}{P}\right)^{-\frac{1}{(1-\gamma)}} + G\right)\tau\chi^{\frac{1}{(1-\gamma)\varpi}}\left(\frac{W}{P}\right)^{\frac{-\gamma\varphi}{(1-\gamma)}}(1-t) + \left[\eta - \beta G\right]c\frac{dG}{dW}\frac{1}{N} = 0,$$

where

$$c\frac{dG}{dW} = \frac{(1-\epsilon)(1-t)(1-\gamma)\varpi\chi^{\frac{1}{(1-\gamma)\varpi}}}{\beta} \left(\frac{W}{P}\right)^{-\gamma\varphi\varpi},\qquad(3.29)$$

$$G = \frac{\eta}{\beta} + \frac{(1-\epsilon)(1-t)\left(\frac{W}{P} - B\right)}{\beta}, \qquad (3.30)$$

$$u = 1 - \varsigma \left(\frac{W}{P}\right)^{-\frac{1}{(1-\gamma)}} - G.$$
(3.31)

Since the system is non-linear in wages, it cannot be solved analytically. Numerical solutions are presented in Section 6 below.

3 Simplified Games

To develop some intuition, I next consider simplified games of the general model. I "shut down" one player at a time in an attempt to clarify the different mechanisms at work. I start by exogenising fiscal policy, removing the strategic interaction between the government and the other players. By studying the strategic interaction between unions and the central bank only, I may examine the properties of the model in detail by investigating if the model generates results consistent with the previous literature. Then, I consider the interaction between fiscal and monetary authorities when wage setting is completely atomistic. Finally, in this section, I consider the game between large unions and the government when monetary policy is exogenously determined, i.e. when the money supply is fixed.

3.1 A Game Between Unions and the Central Bank

When fiscal policy is exogenously given, wage setters and the central bank take the unemployment benefit and the tax rate as given. There is no public good in the economy, so this case is obtained by setting G = 0 in the utility function of households, the expression for unemployment and the objective functions of unions. Union *i* thus faces the following problem:

$$\max_{W_i} (1 - u_i) \chi \frac{W_i}{c} \left(\frac{W}{c}\right)^{-\gamma \varphi \varpi} (1 - t) + u_i B (1 - t)$$

subject to

$$u_{i} = 1 - \delta \left(\frac{W_{i}}{c}\right)^{-\phi\theta} \left(\frac{W}{c}\right)^{-\gamma\varpi\phi(1-\theta\varphi)}$$
$$W = \left(\frac{1}{N}\sum_{i=0}^{N-1}W_{i}^{1-\theta\phi}\right)^{\frac{1}{1-\theta\phi}},$$

taking the average nominal wage set by all other unions, W_{-i} , as given. The first-order condition is:

$$-\frac{du_i}{dW_i} \left[\chi \frac{W_i}{c} \left(\frac{W}{c} \right)^{-\gamma \varphi \varpi} - B \right] + (1 - u_i) \chi \frac{1}{c} \left(\frac{W}{c} \right)^{-\gamma \varphi \varpi} \tau = 0,$$

where

$$\frac{du_i}{dW_i} = \sigma \delta \left(\frac{W_i}{c}\right)^{-\phi \theta - 1} \left(\frac{W}{c}\right)^{-\gamma \varpi \phi (1 - \theta \varphi)} \frac{1}{c}.$$

In symmetric equilibrium, $W_i = W$. After some algebra, I obtain the following expression for the real wage:

$$\frac{W}{P} = \left[1 + \frac{\tau}{(\sigma - \tau)}\right]B.$$

The real wage is a markup on the real unemployment benefit and the size of the markup depends on the objectives of the central bank.

In this simple version of the model, when there is strategic interaction between the central bank and unions only, it is possible to derive some general analytical results. First, the real wage is always lower under inflation targeting than under money supply targeting. I can prove the following proposition:

Proposition 1 If fiscal policy is exogenously given and there is no provision of the public good, the following real wage ranking applies: $\left(\frac{W}{P}\right)_M > \left(\frac{W}{P}\right)_P \forall N > 1.$

Proof. See Appendix A2.

A central bank pursuing an inflation target may discipline wage setters compared to the case of a fixed money supply. The intuition is that if a union increases its wage, thus threatening the inflation target, the central bank will respond by decreasing the nominal money supply. The consequence is a reduction in the real money supply, which punishes the union by reducing aggregate demand and thus increasing unemployment. This anticipated central bank policy response works as a deterrent to wage increases and will discipline unions. The result is consistent with the previous literature; see, for instance, Soskice and Iversen (2000) and Coricelli et al. (2002).

The real-wage ranking translates to unemployment rankings since unemployment is strictly increasing in the real wage and since the functional form of the unemployment equation is identical across regimes:

Proposition 2 $u_P < u_M \ \forall N > 1.$ **Proof.** See Appendix A2.

The proposition states that an inflation target may help promote employment in the economy, since it provides an incentive for wage restraint.

I also prove the following proposition regarding the degree of centralisation of wage setting:

Proposition 3 $\left(\frac{W}{P}\right)_{P,N=1} = \left(\frac{W}{P}\right)_{M,N=1} = \frac{1}{\gamma}B.$

Proof. See Appendix A2. ■

If there is complete centralisation, the monetary regime is of no importance for the real-wage outcome (and hence not for unemployment). This result is consistent with the previous literature.⁹ To understand the intuition, consider the decision by union *i*. The first-order condition for the union determines its optimal real wage as a function of the *real* wages of other unions and the *real* money supply. When wage setting is completely atomistic, each union knows that it is too small to influence the price level and hence, the monetary regime will be of no importance for the wage outcome. Taking the *money* wages of the other unions as exogenous, union i can infer both their real wages and the real money supply. When wage setting is non-atomistic, the real money supply governed by monetary policy will be regime-specific, as will the real wages of the other unions. In a Nash equilibrium with given *nominal* wages for the other unions, the reason is that the wage decision of union i will now result in different *real* wages for other unions and a different real money supply depending on the monetary regime. As a consequence, the real wage of union i will also be regime-specific. When there is complete centralisation, i.e. only one union in the economy, its first-order condition will not define its own real wage as a reaction function of other unions' real wages, simply because there are then, by definition, no other unions. The one union can always infer the money supply response of the central bank and hence calculate the price response to its own actions. A completely centralised union thus solves exactly the same maximisation problem when choosing the real wage independently of the monetary regime.

Proposition 4 If fiscal policy is exogenously given and there is no provision of the public good, the real wage is decreasing in the degree of centralisation.

Proof. See Appendix A2.

The larger are unions, the more they influence the aggregate price level and the stronger are the incentives for wage restraint. In other words, unions exploit the strategic advantage of being small. If wage setting is atomistic, they set wages relatively higher.

⁹ See, for instance, Larsson (2006) and references therein.

3.2 Fiscal-Monetary Interactions under Decentralised Wage Setting

I next consider the case when fiscal policy is endogenously determined but unions are too small to have any effect on aggregate variables, i.e. wage setting is atomistic. This means that there can be no strategic interaction between unions and the central bank, and I may isolate the interaction between the fiscal and the monetary authority.

The government faces the same problem as in the general model when wage setting is non-atomistic, i.e. (3.21). Optimal public spending is thus given by (3.22). The expression shows that wage setting is the only channel through which the monetary regime is of importance for fiscal policy.

The objective function of unions is the same as when wage setting is nonatomistic, but they now take aggregate variables as given. Therefore, union i solves the following problem:

$$\max \Omega_i = (1 - u_i) \chi \frac{W_i}{c} \left(\frac{W}{c}\right)^{-\gamma \varphi \varpi} (1 - t) + u_i B (1 - t) + \eta G - \frac{\beta}{2} G^2$$

subject to

$$u_i = 1 - \delta \left(\frac{W_i}{c}\right)^{-\phi\theta} \left(\frac{W}{c}\right)^{-\gamma \varpi \phi(1-\theta\varphi)} - G,$$

taking both W and G as given. The first-order condition is:

$$-\frac{du_i}{dW_i}\chi\frac{W_i}{c}\left(\frac{W}{c}\right)^{-\gamma\varphi\varpi}(1-t) + (1-u_i)\chi\frac{1}{c}\left(\frac{W}{c}\right)^{-\gamma\varphi\varpi}(1-t) + \frac{du_i}{dW_i}B(1-t) = 0,$$

where

$$\frac{du_i}{dW_i} = \phi\theta\delta\left(\frac{W_i}{c}\right)^{-\phi\theta-1} \frac{1}{c}\left(\frac{W}{c}\right)^{-\gamma\varpi\phi(1-\theta\varphi)}$$

In symmetric equilibrium, $W_i = W$ and the first-order condition reduces to:

$$-\underbrace{\phi\theta\delta\left(\frac{W}{c}\right)^{-(\varpi+1)}}_{\frac{du_i}{dW_i}} \left(\chi\left(\frac{W}{c}\right)^{(1-\gamma)\varpi} - B\right) + \underbrace{\left(\delta\left(\frac{W}{c}\right)^{-\varpi} + G\right)}_{1-u_i}\chi\left(\frac{W}{c}\right)^{-\gamma\varphi\varpi} = 0.$$

Using the relationship between real and nominal wages (3.19), the first-order

condition can be written in terms of real wages as follows:

$$-\phi\theta\varsigma\left(\frac{W}{P}\right)^{-\frac{(\varpi+1)}{(1-\gamma)\varpi}}\left(\frac{W}{P}-B\right) + \left(\varsigma\left(\frac{W}{P}\right)^{-\frac{1}{(1-\gamma)}}+G\right)\left(\frac{W}{P}\right)^{-\frac{\gamma\varphi}{(1-\gamma)}} = 0,$$

where $\varsigma \equiv \left(\frac{\theta}{\gamma(\theta-1)}\right)^{-\frac{1}{1-\gamma}}$. The equation shows that the monetary regime is of no importance for the real wage outcome under decentralised wage setting. The reason is that the real wages of other unions - given their money wages - are independent of the wage decision of union *i*, when it is too small to affect the aggregate price level.

The fact that real wages are independent of the monetary regime implies that there is no strategic interaction between fiscal and monetary authorities when wage setting is decentralised, since the only way the central bank may influence the government is through its impact on wage formation.

3.3 Money Supply Targeting

Next, consider a simplified game between wage setters and the government only. I assume the money supply to be exogenously given so that the central bank plays no active roll in the economy. This case is equivalent to imposing $\varphi = 1$ on the general equilibrium (3.28)-(3.31).

As established above, the monetary regime will only be of importance for the fiscal outcome via wage setting, so public spending is given by (3.30). However, since the price level will differ across regimes, the sensitivity of G with respect to the nominal wage will depend on the monetary regime. To see this, impose $\varphi = 0, 1$, respectively on (3.29). First, note that

$$\varphi = 0 \Rightarrow \varpi = \frac{1}{1 - \gamma} \text{ and } \chi = 1$$
 (3.32)

$$\varphi = 1 \Rightarrow \varpi = 1 \text{ and } \chi = \kappa^{-1}.$$
 (3.33)

I then obtain:

$$\frac{dG}{dW}\Big|_{\varphi=0} = \frac{(1-\epsilon)(1-t)}{\beta c}$$

$$\frac{dG}{dW}\Big|_{\varphi=1} = \frac{(1-\epsilon)(1-t)(1-\gamma)\kappa^{-\frac{1}{(1-\gamma)}}}{\beta c} \left(\frac{W}{P}\right)^{-\gamma} > 0.$$

Under inflation targeting, the derivative of G with respect to the nominal wage is constant. However, when the money supply is exogenously given, $\frac{dG}{dW}$ will depend on the real wage level. When wages are raised, the government raises public spending to offset the negative effect on unemployment. The effect is decreasing in the real wage level:

$$\frac{d}{d\left(W/P\right)}\left(\left.\frac{dG}{dW}\right|_{\varphi=1}\right) = -\gamma \frac{\left(1-\epsilon\right)\left(1-t\right)\left(1-\gamma\right)\kappa^{-\frac{1}{\left(1-\gamma\right)}}}{\beta c}\left(\frac{W}{P}\right)^{-\gamma-1} < 0.$$

As wages are increased, the government raises public spending by less and less. As the real wage increases, it becomes increasingly costly for the government to provide the public good, since the cost of producing it increases. The general-equilibrium expressions are too complicated to further analyse analytically and numerical solutions are needed. Therefore, I return to the impact of the monetary regime in Section 5.

4 The Government as a Stackelberg Leader

I now return to the general setting when there is strategic interaction between fiscal and monetary authorities and large wage setters. To evaluate the importance of the timing of the game, I next derive the case when the government acts as Stackelberg leader to unions and the central bank. Suppose now that the government sets fiscal policy prior to the decisions made by unions, while Stages 3 and 4 remain unchanged and do not need to be analysed again.

4.1 Stage 2: Wage Setting

Unions solve the same maximisation problem as in the benchmark model, but now they treat government spending as exogenously given. Union i thus faces:

$$\max_{W_i} (1 - u_i) \chi \frac{W_i}{c} \left(\frac{W}{c}\right)^{-\gamma\varphi\varpi} (1 - t) + u_i B (1 - t) + \eta G - \frac{\beta}{2} G^2$$

subject to

$$u_{i} = 1 - \delta \left(\frac{W_{i}}{c}\right)^{-\phi\theta} \left(\frac{W}{c}\right)^{-\gamma\varpi\phi(1-\theta\varphi)} - G$$
$$W = \left(\frac{1}{N}\sum_{i=0}^{N-1}W_{i}^{1-\theta\phi}\right)^{\frac{1}{1-\theta\phi}},$$

taking W_{-i} and G as given. I thus impose $\frac{dG}{dW} = 0$ in the union FOC evaluated in symmetric equilibrium, equation (3.27). I then obtain:

$$-\underbrace{\varsigma\sigma\left(\frac{W}{P}\right)^{-\frac{(\varpi+1)}{(1-\gamma)\varpi}}}_{\frac{du_i}{dW_i}}\left(\frac{W}{P}-B\right) + \underbrace{\left(\varsigma\left(\frac{W}{P}\right)^{-\frac{1}{(1-\gamma)}}+G\right)}_{1-u_i}\tau\left(\frac{W}{P}\right)^{-\frac{\gamma\varphi}{(1-\gamma)}} = 0. \quad (3.34)$$

The first-order condition shows how the unions trade off the same effects as when they are Stackelberg leaders but with the modification that there are no longer any effects of increased government spending when the wage is raised, neither directly or indirectly via the mitigating effect on unemployment. The expression highlights the effect of government spending on wages: if the government increases public spending, more union members are employed, and the utility to the union of increasing its wage is higher. In other words, the interaction with the government affects the union's trade off between wages and employment. In particular, more public spending implies a larger weight on the positive effect of higher wages relative to the negative effect of lower employment. This seems to imply that this setting would induce unions to increase wages. Differentiating this condition with respect to W/Pand G and solving for $\frac{dW/P}{dG}$, I obtain:

$$\frac{dW/P}{dG} = \frac{\tau \left(1 - \gamma\right)}{\xi\varsigma\sigma \left(1 - \gamma\right) \left(\frac{W}{P}\right)^{\xi - 1} \left(\frac{W}{P} - B\right) + \varsigma\sigma \left(1 - \gamma\right) \left(\frac{W}{P}\right)^{\xi} + \varsigma\tau \left(\frac{W}{P}\right)^{-\frac{1}{(1 - \gamma)} - 1}},$$

where $\xi = \frac{\gamma \varphi}{(1-\gamma)} - \frac{(\varpi+1)}{(1-\gamma)\varpi} < 0$. According to Samuelson's correspondence principle, the denominator must be positive in order to ensure convergence to equilibrium, and hence $\frac{dW/P}{dG} > 0$.

4.2 Stage 1: Fiscal Policy

The government now maximises the same objective function, but subject to an additional constraint since wages react to fiscal policy. Thus, the government solves:

$$\max_{G} \Lambda = (1-u)\frac{W}{P}\left[1-t+\epsilon t\right] + uB\left(1-t\right)\left(1-\epsilon\right) + \eta G - \frac{\beta}{2}G^2 - \epsilon \frac{W}{P}G$$

subject to

$$u = 1 - \varsigma \left(\frac{W}{P}\right)^{-\frac{1}{(1-\gamma)}} - G$$
$$\frac{W}{P} = h(G),$$

where the function h is implicitly defined by the union's first-order condition (3.34). The first-order condition is:

$$\begin{aligned} -\frac{du}{dG}\frac{W}{P}\left[1-t+\epsilon t\right] + (1-u)\frac{dW/P}{dG}\left[1-t+\epsilon t\right] + \frac{du}{dG}B\left(1-t\right)\left(1-\epsilon\right) \\ +\eta - \beta G - \epsilon G\frac{dW/P}{dG} - \epsilon \frac{W}{P} = 0, \end{aligned}$$

where

$$\frac{du}{dG} = \frac{\varsigma}{1-\gamma} \left(\frac{W}{P}\right)^{-\frac{1}{1-\gamma}-1} \frac{dW/P}{dG} - 1.$$

Solving for G gives:

$$G = \frac{\eta + (1-t)\left(1-\epsilon\right)\left(\frac{W}{P} - B\right) - \frac{\varsigma}{1-\gamma}\left(\frac{W}{P}\right)^{-\frac{1}{1-\gamma}}\frac{dW/P}{dG}\left[\gamma\left[1-t+\epsilon t\right] - (1-t)\left(1-\epsilon\right)B\left(\frac{W}{P}\right)^{-1}\right]}{\left[\beta - (1-t)\left(1-\epsilon\right)\frac{dW/P}{dG}\right]}$$

Obviously, imposing $\frac{dW/P}{dG} = 0$ yields the same expression for government spending as in the case when unions are Stackelberg leaders.

4.3 General Equilibrium

The general equilibrium in the case with the government as a Stackelberg leader is thus characterised by the following equations:

$$-\varsigma\sigma\left(\frac{W}{P}\right)^{-\frac{(\varpi+1)}{(1-\gamma)\varpi}}\left(\frac{W}{P}-B\right) + \left(\varsigma\left(\frac{W}{P}\right)^{-\frac{1}{(1-\gamma)}} + G\right)\tau\left(\frac{W}{P}\right)^{-\frac{\gamma\varphi}{(1-\gamma)}} = 0,$$

$$G = \frac{\eta + (1-t)\left(1-\epsilon\right)\left(\frac{W}{P} - B\right) - \frac{\varsigma}{1-\gamma}\left(\frac{W}{P}\right)^{-\frac{1}{1-\gamma}}\frac{dW/P}{dG}\left[\gamma\left[1-t+\epsilon t\right] - (1-t)\left(1-\epsilon\right)B\left(\frac{W}{P}\right)^{-1}\right]}{\left[\beta - (1-t)\left(1-\epsilon\right)\frac{dW/P}{dG}\right]}$$

where

$$\frac{dW/P}{dG} = \frac{\tau \left(1-\gamma\right)}{\xi\varsigma\sigma \left(1-\gamma\right) \left(\frac{W}{P}\right)^{\xi-1} \left(\frac{W}{P}-B\right) + \varsigma\sigma \left(1-\gamma\right) \left(\frac{W}{P}\right)^{\xi} + \varsigma\tau \left(\frac{W}{P}\right)^{-\frac{1}{(1-\gamma)}-1}},$$

and

$$u = 1 - \varsigma \left(\frac{W}{P}\right)^{-\frac{1}{(1-\gamma)}} - G.$$

Once more, the system is non-linear and cannot be solved analytically.

5 Numerical Solutions

Since the equilibrium systems are non-linear in wages, I need to solve the model numerically. I start by considering plausible parameterisations of the model.

5.1 Parameters

I have no prior of how to set the real unemployment benefit, but choose a value generating reasonable values of employment in the benchmark case with exogenous fiscal policy. Throughout the analysis, I then verify that the equilibria generate reasonable replacement rates. Three parameters are subject to policy experiments: the degree of centralisation, $\frac{1}{N}$, the relative weight assigned to the money supply by the central bank, φ , and the government's weight on the budget balance, ϵ . The tax rate used to cover unemployment benefits and public consumption, t, is set to 0.2 and the weight assigned to consumption in the household's utility function, α , to 0.5. The parameters η and β determine the functional form of the utility functions with respect to government spending and are simply calibrated to generate reasonable values of government spending. Following Gnocchi (2005), I set the elasticity of substitution across different goods, θ , equal to 11. Finally, γ is set to 0.7, capturing decreasing returns to scale in production. For tractability, Table 3.1 lists the parameters of the model.

5.2 Comparability and Welfare Analysis

I express government spending and the budget surplus as percentage shares of private sector output. To be able to compare the results, I normalise the real wage set under complete centralisation in the absence of a government to one, i.e.

$$\left. \frac{W}{P} \right|_{N=1,No\ gov} = 1.$$

Thus, all real wage comparisons will be relative to this regime. The fact that the real wage is regime-independent in this case simplifies comparisons across regimes and policy experiments. I next construct a measure of welfare in the economy. In the case with no government, the appropriate measure is obviously the indirect utility of a representative worker, i.e.

$$V_h = n \frac{W}{P} (1-t) + (1-n) B (1-t).$$

Also when evaluating welfare, I let the centralised case with no government constitute the norm so that

$$V_h|_{N=1,No\ gov} = 1.$$

When the public good is introduced, the indirect utility function of a representative household is expanded to:

$$V_h = n \frac{W}{P} (1-t) + (1-n) B (1-t) + \eta G - \frac{\beta}{2} G^2.$$
(3.35)

However, the government also has preferences over the budget balance and the welfare to the government thus depends on ϵ . To be able to compare the welfare outcome for different levels of ϵ , I need to construct a measure that is independent of ϵ . It is not obvious what value to choose. Therefore, I report two different measures, implied by the extreme values: $\epsilon = 0$ and $\epsilon = 1$, respectively. When $\epsilon = 0$, the government is completely myopic, caring about the current utility of workers only and the government objective function coincides with the indirect utility function (3.35). When $\epsilon = 1$, the government assigns equal weight to the budget balance and the current utility of workers, i.e.:

$$\Lambda|_{\epsilon=1} = n \frac{W}{P} (1-t) + (1-n) B (1-t) + \eta G - \frac{\beta}{2} G^2 + \rho.$$
(3.36)

Since this case may be interpreted as the government taking future generations into account, in what follows I will refer to this function as the *forward-looking measure* of welfare. When evaluating government welfare according to (3.36), I normalise the case with complete centralisation and $\epsilon = 0$ to 1, i.e.

$$\Lambda|_{N=1,\epsilon=0} = 1.$$

Thus, all comparisons are made relative to this regime.

5.3 Results

Tables 3.2-3.5 display numerical solutions to the model. In addition to equilibrium values for the endogenous variables in the model, I display the cross-elasticities (i.e. $\frac{dG}{dW}$ in the case when unions are Stackelberg leaders and $\frac{dW/P}{dG}$ when unions are Stackelberg followers) and the welfare measures described above. I first exclusively focus on the benchmark case when unions are Stackelberg leaders to the government and analyse the two key issues of the paper: the importance of the degree of fiscal activism and the importance of the monetary regime. Then, I end the section by discussing the case when the government is a Stackelberg leader to unions, thus assessing the issue of whether the timing of the game is of importance for the results.

The simple case when fiscal policy is exogenous and there is no public good is displayed in Table 3.2. The results confirm the analytical findings stated in Propositions 1-4. It takes at least two unions for the monetary regime to be of importance for the labour market outcome. For less than complete centralisation, real wages are always lower and employment rates always higher under inflation targeting. The difference between regimes is greater if wage setting is highly centralised, since the effects internalised by unions are greater the larger they are. For instance, employment is 7.3 percentage points higher under inflation targeting than under money supply targeting when N = 2, while the difference across regimes is reduced to 1.9 percentage points when N = 10. This shows that in terms of employment, the benefits of inflation targeting are higher with heavily centralised wage setting, provided that centralisation is not complete. The results also suggest that the current utility of workers is higher when the central bank targets the price level. Moreover, utility is increasing in the degree of centralisation, but the associated effects are small. The reason is that while real wages are decreasing, employment is increasing in the degree of centralisation, which implies that the net effect of centralisation on utility is small.

5.3.1 Unions as Stackelberg Leaders

In Tables 3.3-3.5, I introduce the public good and endogenise fiscal policy. The tables display numerical solutions to the model for different values of ϵ , the degree of myopia of the government. In Table 3.3, I set ϵ to zero, thus assuming that the government cares about workers' current utility only, i.e. that it is completely myopic. In Table 3.4, ϵ is set equal to 0.5, so that the government cares twice as much about workers' current utility as about the budget balance, and finally in Table 3.5, ϵ is set equal to 1, capturing the case when the government assigns equal weight to workers' current utility and the budgetary outcome. Recall that ϵ can alternatively be interpreted as the degree of fiscal activism and that when $\epsilon = 1$, it is optimal for the government to follow a fiscal rule of budget balance.

The Impact of Fiscal Activism

By comparing columns (1) and (2) in Tables 3.3-3.5 to Table 3.2, the first thing to note is that endogenising fiscal policy increases real wages and reduces employment for all values of ϵ . The intuition is that when there is an endogenous response of public spending to wage increases, unions exploit the fact that the government cares about employment and makes the government assume responsibility for some of the costs of high wages. Thus, while the central bank may discipline wage setters and provide incentives for wage restraint, the presence of a government acting under discretion in general does the opposite.

Turning to the impact of fiscal activism, the results suggest that real wages are increasing in the degree of activism as captured by ϵ . When the government is equally concerned with the budget balance and the current utility of workers, i.e. when $\epsilon = 1$, wages are lower than when the government assigns a relatively larger weight to the current utility of workers. The reason is that when the government sets public spending such that it accommodates wage increases, unions exploit the response of the government and set wages relatively higher. Recall that the case when $\epsilon = 1$ is equivalent to the government following a fiscal rule in optimum, and hence, no such mitigating effect on unemployment exists. Unions therefore set wages relatively lower when $\epsilon = 1$. This means that the government can discipline wage setters by pursuing less activist fiscal policy.

The Impact of the Monetary Regime

Next, consider the impact of the monetary regime. Recall that in the simple case with no public good and exogenous fiscal policy, inflation targeting imposes discipline on unions and generates lower real wages and higher employment. The disciplining effect of inflation targeting is robust to the introduction of endogenous fiscal policy. A key result is that when fiscal policy is endogenous, the difference between the two monetary regimes is larger. This holds true for all values of ϵ , which means that inflation targeting disciplines unions relatively more when fiscal policy is endogenous. To develop some intuition for this, suppose there to be no response from the government (the simplified game described in Section 3.1). Unions then know that if they increase the wage under inflation targeting, the central bank offsets the inflationary pressure by reducing the money supply, thereby decreasing employment. Unions thus have incentive for wage moderation. When the wage increase also triggers a policy response from the government (increasing public spending to partly offset the negative effect on unemployment), the inflationary pressure is even stronger, causing the central bank to reduce the money supply even more than in the case when there is no policy response from the government. The inflationary pressure stemming from a wage increase will be higher when it also triggers a policy response from the government, and as a consequence, the punishment in terms of a monetary contraction from the central bank will be larger. Therefore, inflation targeting is even more effective when fiscal policy is endogenous.

However, the difference between the two extreme monetary regimes in terms of real wages and employment is decreasing in ϵ . For instance, when N = 2 and $\epsilon = 0.5$, Table 3.5 shows that wages are 4.1 percent higher under money supply targeting than under inflation targeting. When $\epsilon = 1$ as in Table 3.6, the corresponding difference is 3.7 percent. This indicates that inflation targeting is even more important in economies where fiscal policy is characterised by a high degree of fiscal activism.

Real wages and employment only provide part of the picture, since consumers now derive utility also from government spending. Turning to evaluations of consumer utility as defined by (3.35), Tables 3.3-3.5 show that the current utility of households, V_h , is obviously always increased by introducing a public good, since the government sets G > 0 in equilibrium. Using the forward-looking measure of welfare with $\epsilon = 1$ as defined by (3.36), the results show that welfare is decreasing in the degree of fiscal activism and the degree of centralisation of wage setting.

There is one parameterisation that deserves special attention. Recall that when

there is no government, the real wage is regime-independent when there is complete centralisation, i.e. when there is one single union in the economy. This finding was analysed in detail in Section 3.1 above. Now, consider the case when there is complete centralisation and the one union is Stackelberg leader to the government. The first-order condition of the union will obviously not be a function of what other unions do since there are no other unions by definition. However, the condition will depend on the response of the government since G is regime-specific. Therefore, the monetary regime should be of importance for wage setting even under complete centralisation. Is this true? The numerical solutions show that this is the case when $\epsilon = 0.5$ (in Table 3.4). In this case, inflation targeting generates a slightly higher real wage than money supply targeting. However, Tables 3.3 and 3.5 show that money neutrality is restored when either $\epsilon = 0$ or $\epsilon = 1$. To understand this result, we need to return to the first-order condition of unions when they are Stackelberg leaders: When $\epsilon = 1$, the government follows a fiscal policy rule, keeping G constant, which implies $\frac{dG}{dW} = 0$. Therefore, the response of the government is regime-independent as is the wage set by the single union. In the case when $\epsilon = 0$, $\frac{dG}{dW}$ will not matter for the optimal wage, and the union faces the same first-order condition regardless of regime.¹⁰ The intuition is that when $\epsilon = 0$, the union and the government have identical objective functions and the government thus chooses the same level of G as the union would choose. Therefore, $\frac{dG}{dW}$ is of no importance for the union's decision.

As mentioned above, the analysis suggests that an inflation target may discipline unions and provide an incentive for wage restraint. Can the central bank also discipline the government? The results show that this is the case when the government acts as a Stackelberg follower when $\epsilon < 1$. When $\epsilon = 1$, the government follows a fixed rule and the monetary regime is clearly of no importance.¹¹

Summing up, the benchmark model when unions are Stackelberg leaders to the government generates some important insights. The main results are: (i) introducing a public good and endogenising fiscal policy causes unions to set wages relatively higher since they make the government bear some of the costs of unemployment; (ii) less activist policy corresponding to a fiscal rule of balancing the budget may

¹⁰ To see this, substitute for $\eta - \beta G = -(1-t)(\frac{W}{P} - B)$ from (3.22) in the union's first-order condition (3.27). The offsetting effect on unemployment and the direct effect on union utility of higher public spending cancel out, which implies that dG/dW does not enter into the first-order condition of the union when $\epsilon = 0$.

¹¹ The solution to the government's problem when $\epsilon = 1$ is to set G = 0.05 for this parameterisation, regardless of the degree of centralisation. The only reason why the numbers vary in columns (1) and (2) in Table 3.5 is because G is expressed as a percentage share of private sector production, varying with employment.

discipline unions to some extent; (iii) inflation targeting provides incentives for wage restraint and promotes employment also when fiscal policy is endogenous; and (iv) the difference in terms of real wages and employment between the two monetary regimes is greater when fiscal policy is endogenous and is increasing in the degree of fiscal activism.

5.3.2 The Government as Stackelberg Leader

Is timing of importance for the results? Columns (3) and (4) in Tables 3.3-3.5 display equilibrium solutions when the government is a Stackelberg leader to unions. The main results are robust to the reversed timing. Introducing a public good and endogenising fiscal policy increases real wages but the mechanism is different. When the government is a Stackelberg leader to unions and increases the provision of the public good, employment increases as does the value of wage increases to unions. Therefore, unions set wages relatively higher when the government provides a public good. Real wages are lower and employment higher under inflation targeting. Once more, the difference across monetary regimes is greater when fiscal policy is endogenised.

Turning to consumer welfare, the current utility of workers is, in general, increased by endogenising fiscal policy also when the government is a Stackelberg leader, since households derive positive utility from the public good. However, for some parameterisations ($N \ge 4$ in Table 3.4 and for all levels of centralisation in Table 3.5), there are corner solutions to the government's problem, rendering the current utility of households unaffected by endogenising fiscal policy.

So why do these corner solutions occur when the government is a Stackelberg leader? Recall that in Tables 3.4 and 3.5, the government assigns a rather large weight to the budget balance ($\epsilon = 0.5$ and $\epsilon = 1$, respectively). The solutions also show that as a consequence, the government is much more prone to run budget surpluses or, for lower levels of centralisation, negligible budget deficits. In other words, when assigning a large weight to the budget balance, the government adopts a much more restrictive policy stance, setting public spending very low or even equal to zero. This affects the current utility of households negatively, since households derive no utility from the budget surplus that arises. Despite the fact that the government enables unions to raise wages when they share the costs of unemployment, introducing a fiscal authority in the economy thus increases the current utility of workers only as long as the government does not care too much about the budget balance.

Table 3.3 suggests that an inflation target may also discipline the government when it does not care about the budget balance, i.e. when $\epsilon = 0$. The reason is that the government knows that if it increases G, the marginal benefit of increasing wages to the union is higher, thereby causing unions to raise wages even further. Under inflation targeting, the central bank will counteract the wage pressure stemming from wage hikes by reducing the money supply, thus decreasing employment. Caring only about the current utility of workers, this will decrease the utility of the government. Therefore, the central bank may indirectly discipline the government by disciplining wage setters. When $\epsilon = 0.5$, as in Table 3.4, the government sets government spending slightly higher under inflation targeting than under money supply targeting when N = 2, but the difference between regimes is negligible. For lower degrees of centralisation in wage setting, there are corner solutions to the government's problem, suggesting that the monetary regime is of no importance for public spending, when wage setting becomes increasingly atomistic. This is intuitively appealing since the effect of the regime on wages becomes smaller, the more decentralised is wage setting. When $\epsilon = 1$ in Table 3.5, there are corner solutions to the government's problem for all degrees of centralisation, and the monetary regime is of no importance.

5.3.3 First-mover-advantage and the Degree of Centralisation

All tables show that both unions and the government exploit their first mover advantages in the following sense. Unions always set higher wages when they are Stackelberg leaders to the government than when they are followers. The government, on the other hand, always sets public spending at a lower level when it is a Stackelberg leader, but using the forward-looking measure of welfare it achieves higher welfare when it is a Stackelberg leader than when it is a Stackelberg follower to unions.

Finally, wages are always decreasing in the degree of centralisation, regardless of the monetary regime and the timing of the game.

6 Concluding Remarks

I have presented a model of strategic interaction between the government, the central bank and trade unions. Consistent with the previous literature, I find that an inflation target in general provides an incentive for wage restraint, i.e. it disciplines wage setters. However, introducing a government pursuing endogenous fiscal policy affects the behaviour of wage setters. The analysis shows that when unions are Stackelberg leaders, they exploit the fact that the government cares about employment and lets the government assume responsibility for some of the costs associated with wage hikes. This implies that less activist fiscal policy corresponding to a fiscal rule of balancing the budget induces wage moderation.

The difference in real wages and employment across monetary regimes is greater when a public good is introduced and fiscal policy endogenised. This means that the benefits of inflation targeting are even greater when account is taken of endogenous fiscal policy. The difference between regimes is increasing in the degree of fiscal activism.

The model illustrates some empirically relevant mechanisms in an economy with collective bargaining and inflation targeting. I show that when fiscal policy is endogenised, unions have incentives to set wages relatively higher. However, both inflation targeting and a fiscal rule for the government may help achieve wage moderation in the economy. Thus, the model confirms earlier results in the literature on the benefits of inflation targeting but also shows that employment and aggregate welfare are decreasing in the degree of fiscal activism. Therefore, it seems that there are welfare gains to be made by pursuing less activist fiscal policy. An alternative interpretation of the results is that if the objective is to induce wage moderation in the economy, inflation targeting is even more important in countries where fiscal policy is characterised by a high degree of activism.

Bibliography

- Blanchard, O.J. and Fischer, S. (1989), *Lectures on Macroeconomics*, MIT Press, Cambridge, MA
- [2] Calmfors, L. (2001), "Wages and Wage-Bargaining Institutions in the EMU A Survey of the Issues", *Empirica 28*
- [3] Calmfors, L., (1982), "Employment Policies, Wage Formation and Trade Union Behavior in a Small Open Economy", Scandinavian Journal of Economics, Vol. 84, pp. 345-373
- [4] Calmfors, L. and Horn, H., (1985), "Classical Unemployment, Accommodation Policies and the Adjustment of Real Wages", *Scandinavian Journal of Economics*, Vol. 87 (2), pp. 234-261
- [5] Calmfors, L. and Horn, H., (1986), "Employment Policies and Centralized Wage Setting", *Economica*, Vol. 53, pp. 281-302
- [6] Coricelli, F. Cukierman, A. & Dalmazzo, A. (2000), "Monetary Institutions, Monopolistic Competition, Unionized Labor Markets and Economic Performance", CEPR Discussion Paper 2407, Center for Economic Policy Research, London
- [7] Cukierman, A. and Dalmazzo, A. (2005), "Fiscal-Monetary Policy Interactions in the Presence of Unionized Labour Markets", CEPR Discussion Paper No. 5282, Centre for Economic Policy Research, London
- [8] Dixit, A. and Lambertini, L. (2001), "Monetary-fiscal Policy Interactions and Commitment Versus Discretion in a Monetary Union", *European Economic Re*view 45, pp. 977-987
- [9] Dixit, A. and Lambertini, L. (2003), "Interactions of Commitment and Discretion in Monetary and Fiscal Policies", *American Economic Review* Vol. 93, pp. 1522-1542

- [10] Driffill, E. J. (1985), "Macroeconomic Stabilization Policy and Trade Union Behavior as a Repeated Game", *Scandinavian Journal of Economics*, Vol. 87 (2), pp. 300-326
- [11] Hersoug, T. (1985), "Workers versus Government Who adjusts to Whom?", Scandinavian Journal of Economics, Vol. 87 (2), pp. 270-292, pp. 270-292
- [12] Gnocchi, S., (2005), "Non-Atomistic Wage Setters and Monetary policy in a New Keynesian Framework", mimeo, Department of Economics, Universitat Pompeu Fabra
- [13] Larsson, A. (2006), "Monetary Regimes, Labour Mobility and Equilibrium Employment", IIES Seminar Paper No. 745
- [14] Lippi, F. (2003), "Strategic Monetary Policy with Non-atomistic Wage Setters", CEIS Tor Vergata Research Paper Series, Vol. 6, No. 17, Centre for International Studies on Economic Growth
- [15] Soskice, D. and Iversen, T. (2000), "The Nonneutrality of Monetary Policy with Large Price or Wage Setters", *Quarterly Journal of Economics* CXV

Appendix

A1 Derivations

Households

Household h solves:

$$\max_{C_{hij},M_h} U_h = \left(\frac{1}{\alpha} \left(\int_0^1 C_{hij}^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}}\right)^{\alpha} \left(\frac{M_h/P}{1-\alpha}\right)^{1-\alpha} + \eta G - \beta G^2$$

subject to

$$\int_0^1 C_{hij} P_{ij} dj + M_h \equiv PC_h + M_h = X_h$$

Denote the nominal expenditure spent on consumption by Z_h :

$$Z_h \equiv X_h - M_h = PC_h$$

The household's problem may be solved in two steps. Given a fixed nominal amount spent on consumption, Z_h , the household first chooses how much to consume of each good C_{hij} as a function of total household consumption C_h . The household then chooses how to allocate total nominal income, X_h , between consumption C_h and money holdings M_h .

In the first step, the Lagrangian can be written: ¹²

$$L\left(C_{hij},\widetilde{\xi}\right) = \left(\frac{1}{\alpha}\left(\int_{0}^{1} C_{hij}^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}}\right)^{\alpha} \left(\frac{M_{h}/P}{1-\alpha}\right)^{1-\alpha} + \eta G - \frac{\beta}{2}G^{2} + \widetilde{\xi}\left[Z_{h} - \int_{0}^{1} C_{hij}P_{ij}dj\right]$$

The first-order conditions are:

$$\alpha \left(\frac{1}{\alpha} \left(\int_{0}^{1} C_{hij}^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}}\right)^{\alpha-1} \left(\frac{M_{h}/P}{1-\alpha}\right)^{1-\alpha} \left(\int_{0}^{1} C_{hij}^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}-1} C_{hij}^{\frac{\theta-1}{\theta}-1} - \widetilde{\xi} P_{ij} = 0$$

$$(3.37)$$

$$Z_h - \int_0^1 C_{hij} P_{ij} dj = 0 \tag{3.38}$$

 $^{1^{2}}$ Let \sim denote intermediate parameters, i.e. parameters (such as multipliers) that are introduced for computational purposes only.

Simplifying the FOC with respect to C_{hij} :

$$\alpha C_h \left(\frac{1}{\alpha} \left(\int_0^1 C_{hij}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}} \right)^{-1} \left(\int_0^1 C_{hij}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}-1} C_{hij}^{\frac{\theta-1}{\theta}} = \widetilde{\xi} C_{hij} P_{ij} \left(\frac{M_h/P}{1-\alpha} \right)^{-(1-\alpha)}$$
(3.39)

Integrating both sides over (0, 1):

$$\alpha C_h \left(\frac{1}{\alpha} \left(\int_0^1 C_{hij}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}} \right)^{-1} \left(\int_0^1 C_{hij}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}-1} \int_0^1 C_{hij}^{\frac{\theta-1}{\theta}} dj$$
$$= \tilde{\xi} \left(\frac{M_h/P}{1-\alpha} \right)^{-(1-\alpha)} \int_0^1 C_{hij} P_{ij} dj$$

if and only if

$$\alpha C_h \left(\frac{1}{\alpha} \left(\int_0^1 C_{hij}^{\frac{\theta}{\theta}} dj \right)^{\frac{\theta}{\theta-1}} \right)^{-1} \left(\int_0^1 C_{hij}^{\frac{\theta}{\theta}} dj \right)^{\frac{\theta}{\theta-1}} = \widetilde{\xi} \left(\frac{M_h/P}{1-\alpha} \right)^{-(1-\alpha)} \int_0^1 C_{hij} P_{ij} dj$$
(3.40)

Imposing (3.38) on (3.40) and using the definition of C_h implies:

$$\alpha C_h \left(\frac{1}{\alpha} C_h\right)^{-1} C_h = \widetilde{\xi} \left(\frac{M_h/P}{1-\alpha}\right)^{-(1-\alpha)} Z_h \Leftrightarrow$$
$$\widetilde{\xi} = \frac{\alpha^2 C_h}{Z_h} \left(\frac{M_h/P}{1-\alpha}\right)^{(1-\alpha)}$$

Plugging this into the first-order condition (3.39) and using $Z_h = PC_h$:

$$\alpha C_h \left(\frac{1}{\alpha} \left(\int_0^1 C_{hij}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}} \right)^{-1} \left(\int_0^1 C_{hij}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}-1} C_{hij}^{\frac{\theta-1}{\theta}} = \tilde{\xi} C_{hij} P_{ij} \left(\frac{M_h/P}{1-\alpha} \right)^{-(1-\alpha)}$$

$$\alpha C_h \left(\frac{1}{\alpha} C_h \right)^{-1} \left(\int_0^1 C_{hij}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}-1} C_{hij}^{\frac{\theta-1}{\theta}} = \frac{\alpha^2 C_h}{Z_h} C_{hij} P_{ij} \left(\frac{M_h/P}{1-\alpha} \right)^{(1-\alpha)-(1-\alpha)} \Leftrightarrow$$

$$\left(\int_0^1 C_{hij}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}-1} C_{hij}^{\frac{\theta-1}{\theta}} = \frac{P_{ij}}{P} C_{hij} \Leftrightarrow$$

$$\left(\int_0^1 C_{hij}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{1}{\theta-1}} C_{hij}^{-\frac{1}{\theta}} = \frac{P_{ij}}{P}$$

Note that:

$$\left(\int_0^1 C_{hij}^{\frac{\theta-1}{\theta}} dj\right)^{\frac{1}{\theta-1}} = \left(\int_0^1 C_{hij}^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}\frac{\theta-1}{\theta}\frac{1}{\theta-1}} = C_h^{\frac{\theta}{\theta}-\frac{1}{\theta-1}} = C_h^{\frac{1}{\theta}}$$

I then obtain:

$$\left(\frac{C_h}{C_{hij}}\right)^{\frac{1}{\theta}} = \frac{P_{ij}}{P} \Leftrightarrow$$
$$C_h = \left(\frac{P_{ij}}{P}\right)^{\theta} C_{hij}$$

Rearranging implies:

$$C_{hij} = \left(\frac{P_{ij}}{P}\right)^{-\theta} C_h$$

Instead of explicitly deriving money demand from the above system, I want to derive the trade-off between total consumption of household h, C_h , and money holdings $\frac{M_h}{P}$. Therefore, I aggregate over goods and consider the following (equivalent problem):

$$\max_{C_h, M_h} \left(\frac{C_h}{\alpha}\right)^{\alpha} \left(\frac{M_h/P}{1-\alpha}\right)^{1-\alpha} + \eta G - \frac{\beta}{2}G^2$$

subject to

$$PC_h + M_h = X_h.$$

The Lagrangian is:

$$L(C_h, M_h, \widetilde{\lambda}) = \left(\frac{C_h}{\alpha}\right)^{\alpha} \left(\frac{M_h/P}{1-\alpha}\right)^{1-\alpha} + \eta G - \frac{\beta}{2}G^2 + \widetilde{\lambda}\left[X_h - PC_h - M_h\right].$$

The first-order conditions are:

$$\alpha \left(\frac{C_h}{\alpha}\right)^{\alpha-1} \frac{1}{\alpha} \left(\frac{M_h/P}{1-\alpha}\right)^{1-\alpha} - \widetilde{\lambda}P = 0$$
$$-(1-\alpha) \left(\frac{C_h}{\alpha}\right)^{\alpha} \frac{1}{(1-\alpha)} \frac{1}{P} \left(\frac{M_h/P}{1-\alpha}\right)^{1-\alpha-1} - \widetilde{\lambda} = 0$$
$$X_h - PC_h - M_h = 0.$$

Re-arranging means that the FOC with respect to C_h can be written:

$$\begin{pmatrix} \frac{M_h/P}{1-\alpha} \\ \frac{\overline{C_h}}{C_h} \end{pmatrix}^{1-\alpha} - \widetilde{\lambda}P = 0 \Leftrightarrow$$

$$\begin{pmatrix} \frac{M_h/P}{C_h} \frac{\alpha}{1-\alpha} \end{pmatrix}^{1-\alpha} - \widetilde{\lambda}P = 0 \Leftrightarrow$$

$$P = \frac{1}{\widetilde{\lambda}} \left(\frac{M_h/P}{C_h} \frac{\alpha}{1-\alpha} \right)^{1-\alpha}.$$

Similarly, the FOC with respect to money holdings can be written:

$$\frac{1}{P} \left(\frac{\frac{C_h}{\alpha}}{\frac{M_h/P}{1-\alpha}} \right)^{\alpha} - \widetilde{\lambda} = 0$$
$$\left(\frac{C_h}{M_h/P} \frac{1-\alpha}{\alpha} \right)^{\alpha} - \widetilde{\lambda}P = 0$$
$$\frac{1}{\widetilde{\lambda}P} \left(\frac{M_h/P}{C_h} \frac{\alpha}{1-\alpha} \right)^{-\alpha} = 1$$

Inserting these expressions into the FOC for $\widetilde{\lambda}$:

$$PC_{h} + M_{h} = X_{h} \Leftrightarrow$$

$$\frac{1}{\tilde{\lambda}} \left(\frac{M_{h}/P}{C_{h}} \frac{\alpha}{1-\alpha}\right)^{1-\alpha} C_{h} + \frac{1}{\tilde{\lambda}} \left(\frac{M_{h}/P}{C_{h}} \frac{\alpha}{1-\alpha}\right)^{-\alpha} \frac{M_{h}}{P} = X_{h} \Leftrightarrow$$

$$\left(\frac{M_{h}/P}{C_{h}}\right)^{1-\alpha} \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} C_{h} + \left(\frac{M_{h}/P}{C_{h}}\right)^{-\alpha} \left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} \frac{M_{h}}{P} = \tilde{\lambda}X_{h} \Leftrightarrow$$

$$\left(\frac{M_{h}/P}{C_{h}}\right) \left(\frac{M_{h}/P}{C_{h}}\right)^{-\alpha} \left(\frac{\alpha}{1-\alpha}\right) \left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} C_{h}$$

$$+ \left(\frac{M_{h}/P}{C_{h}}\right)^{-\alpha} \left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} \frac{M_{h}}{P} = \tilde{\lambda}X_{h}$$

If and only if:

$$\left(\frac{M_h/P}{C_h}\right)^{-\alpha} \left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} \left[\left(\frac{\alpha}{1-\alpha}\right) \left(\frac{M_h/P}{C_h}\right) C_h + \frac{M_h}{P} \right] = \tilde{\lambda} X_h \Leftrightarrow$$

$$\left(\frac{M_h/P}{C_h}\right)^{-\alpha} \left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} \frac{M_h}{P} \left[\frac{\alpha}{1-\alpha} + 1\right] = \tilde{\lambda} X_h \Leftrightarrow$$

$$\left(\frac{M_h/P}{C_h}\right)^{-\alpha} \left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} \frac{M_h}{P} \left[\frac{1}{1-\alpha}\right] = \tilde{\lambda} X_h \Leftrightarrow$$

$$\left(\frac{M_h/P}{C_h} \frac{\alpha}{1-\alpha}\right)^{-\alpha} \frac{M_h}{P} \left[\frac{1}{1-\alpha}\right] = \tilde{\lambda}.$$

Inserting this into the FOC for money holdings:

$$\begin{pmatrix} \frac{M_h/P}{C_h} \frac{\alpha}{1-\alpha} \end{pmatrix}^{-\alpha} = \tilde{\lambda}P \Rightarrow$$

$$\begin{pmatrix} \frac{M_h/P}{C_h} \frac{\alpha}{1-\alpha} \end{pmatrix}^{-\alpha} = \begin{pmatrix} \frac{M_h/P}{C_h} \frac{\alpha}{1-\alpha} \end{pmatrix}^{-\alpha} \frac{M_h}{P} \left[\frac{1}{1-\alpha} \right] \frac{1}{X_h}P \Leftrightarrow$$

$$1 = \frac{M_h}{P} \left[\frac{1}{1-\alpha} \right] \frac{1}{X_h}P \Leftrightarrow$$

$$\frac{M_h}{P} = (1-\alpha) \frac{X_h}{P}$$

Similarly, the FOC for household consumption implies:

$$\left(\frac{M_h/P}{C_h}\frac{\alpha}{1-\alpha}\right)^{1-\alpha} = \tilde{\lambda}P \Leftrightarrow$$

$$\left(\frac{M_h/P}{C_h}\frac{\alpha}{1-\alpha}\right) \left(\frac{M_h/P}{C_h}\frac{\alpha}{1-\alpha}\right)^{-\alpha} = \left(\frac{M_h/P}{C_h}\frac{\alpha}{1-\alpha}\right)^{-\alpha}\frac{M_h}{P}\left[\frac{1}{1-\alpha}\right]\frac{1}{X_h}P \Leftrightarrow$$

$$\left(\frac{\alpha}{C_h}\right) = \frac{1}{X_h}P \Leftrightarrow$$

$$C_h = \alpha\frac{X_h}{P}.$$

Combining the two expressions gives the following relationship between household consumption and real money holdings:

$$C_h = \frac{\alpha}{(1-\alpha)} \frac{M_h}{P}.$$

This means that household h's demand for goods provided by firm j can be written

$$C_{hij} = \left(\frac{P_{ij}}{P}\right)^{-\theta} C_h$$
$$= \left(\frac{P_{ij}}{P}\right)^{-\theta} \left[\frac{\alpha}{(1-\alpha)} \frac{M_h}{P}\right].$$

Finally, aggregate demand facing firm j, represented by union i, is obtained by integrating over all households on the unit interval:

$$Y_{ij}^{D} \equiv \int_{0}^{1} C_{hij} dh = \int_{0}^{1} \left(\frac{P_{ij}}{P}\right)^{-\theta} \left[\frac{\alpha}{(1-\alpha)} \frac{M_{h}}{P}\right] dh$$
$$= \frac{\alpha}{(1-\alpha)} \left(\frac{P_{ij}}{P}\right)^{-\theta} \frac{M}{P}.$$

Firms

Substituting for the constraints, the problem of the firm can be written:

$$\max_{P_{ij}} \Pi_{ij} = \frac{P_{ij}}{P} Y_{ij}^D - \frac{W_i}{P} \left(Y_{ij}^D \right)^{\frac{1}{\gamma}}$$
$$= \frac{P_{ij}}{P} \left[\frac{\alpha}{(1-\alpha)} \left(\frac{P_{ij}}{P} \right)^{-\theta} \frac{M}{P} \right] - \frac{W_i}{P} \left(\frac{\alpha}{(1-\alpha)} \left(\frac{P_{ij}}{P} \right)^{-\theta} \frac{M}{P} \right)^{\frac{1}{\gamma}}.$$

Taking P as given. The FOC is:

$$\frac{\partial \Pi_{ij}}{\partial P_{ij}} = \frac{1}{P} \left[\frac{\alpha}{(1-\alpha)} \left(\frac{P_{ij}}{P} \right)^{-\theta} \frac{M}{P} \right] - \theta \frac{P_{ij}}{P} \left[\frac{\alpha}{(1-\alpha)} \left(\frac{P_{ij}}{P} \right)^{-\theta-1} \frac{M}{P} \right] \frac{1}{P} - \frac{1}{\gamma} \frac{W_i}{P} \left(\frac{\alpha}{(1-\alpha)} \left(\frac{P_{ij}}{P} \right)^{-\theta} \frac{M}{P} \right)^{\frac{1}{\gamma}-1} (-\theta) \frac{\alpha}{(1-\alpha)} \left(\frac{P_{ij}}{P} \right)^{-\theta-1} \frac{M}{P} \frac{1}{P} = 0.$$

Simplifying implies:

$$\left(\frac{P_{ij}}{P}\right)^{-\theta} - \theta \left(\frac{P_{ij}}{P}\right)^{-\theta-1+1} = -\frac{\theta}{\gamma} \frac{W_i}{P} \left(\frac{\alpha}{(1-\alpha)} \frac{M}{P}\right)^{\frac{1}{\gamma}-1} \left(\frac{P_{ij}}{P}\right)^{-\theta\left(\frac{1}{\gamma}-1\right)-\theta-1} \Leftrightarrow \\ \left(\theta-1\right) \left(\frac{P_{ij}}{P}\right)^{-\theta} = \frac{\theta}{\gamma} \frac{W_i}{P} \left(\frac{\alpha}{(1-\alpha)} \frac{M}{P}\right)^{\frac{1}{\gamma}-1} \left(\frac{P_{ij}}{P}\right)^{-\frac{\theta}{\gamma}-1}$$

if and only if

$$\left(\frac{P_{ij}}{P}\right)^{1-\theta+\frac{\theta}{\gamma}} = -\frac{\theta}{\gamma\left(1-\theta\right)} \frac{W_i}{P} \left(\frac{\alpha}{\left(1-\alpha\right)} \frac{M}{P}\right)^{\frac{1-\gamma}{\gamma}} \Leftrightarrow \\ \left(\frac{P_{ij}}{P}\right)^{\frac{\gamma-\theta\gamma+\theta}{\gamma}} = \frac{\theta}{\gamma\left(\theta-1\right)} \frac{W_i}{P} \left(\frac{\alpha}{\left(1-\alpha\right)} \frac{M}{P}\right)^{\frac{1-\gamma}{\gamma}}.$$

This implies

$$\begin{pmatrix} \frac{P_{ij}}{P} \end{pmatrix} = \left[\frac{\theta}{\gamma (\theta - 1)} \frac{W_i}{P} \left(\frac{\alpha}{(1 - \alpha)} \frac{M}{P} \right)^{\frac{1 - \gamma}{\gamma}} \right]^{\frac{\gamma}{\gamma + \theta (1 - \gamma)}} \Leftrightarrow$$

$$\begin{pmatrix} \frac{P_{ij}}{P} \end{pmatrix} = \zeta \left(\frac{W_i}{P} \right)^{\frac{\gamma}{\gamma + \theta (1 - \gamma)}} \left(\frac{M}{P} \right)^{\frac{(1 - \gamma)}{\gamma + \theta (1 - \gamma)}},$$

where $\zeta \equiv \left[\frac{\theta}{\gamma(\theta-1)} \left(\frac{\alpha}{(1-\alpha)}\right)^{\frac{1-\gamma}{\gamma}}\right]^{\frac{\gamma}{\gamma+\theta(1-\gamma)}}$. Inserting the firm's optimal pricing rule into the expression for labour demand and simplifying, I obtain firm ij's demand for

labour:

$$\begin{split} L_{ij}^{P} &= \left[\frac{\alpha}{(1-\alpha)}\frac{M}{P}\right]^{\frac{1}{\gamma}} \left(\zeta \left(\frac{W_{i}}{P}\right)^{\frac{\gamma}{\gamma+\theta(1-\gamma)}} \left(\frac{M}{P}\right)^{\frac{(1-\gamma)}{\gamma+\theta(1-\gamma)}}\right)^{-\frac{\theta}{\gamma}} \\ &= \vartheta \left[\frac{M}{P}\right]^{\frac{1}{\gamma+\theta(1-\gamma)}} \left(\frac{W_{i}}{P}\right)^{-\frac{\theta}{\gamma+\theta(1-\gamma)}}, \end{split}$$

where $\vartheta \equiv \left[\frac{\alpha}{1-\alpha}\right]^{\frac{1}{\gamma}} \zeta^{-\frac{\theta}{\gamma}} = \left[\frac{\alpha}{(1-\alpha)}\right]^{\frac{1}{\gamma+\theta(1-\gamma)}} \left[\frac{\theta}{\gamma(\theta-1)}\right]^{-\frac{\theta}{\gamma+\theta(1-\gamma)}}$. Using this notation, the price setting rule of each firm can be written:

$$\left(\frac{P_{ij}}{P}\right) = \zeta \left(\frac{W_i}{P}\right)^{\gamma\phi} \left(\frac{M}{P}\right)^{(1-\gamma)\phi},$$

where $\zeta \equiv \left[\frac{\alpha}{1-\alpha}\right]^{(1-\gamma)\phi} \left[\frac{\theta}{\gamma(\theta-1)}\right]^{\gamma\phi}$.

Aggregation

To obtain an expression for aggregate unemployment, I need to derive an expression for aggregate labour demand. Recall that labour demand facing firm ij is given by:

$$L_{ij} = \left[\frac{\alpha}{(1-\alpha)} \left(\frac{P_{ij}}{P}\right)^{-\theta} \frac{M}{P}\right]^{\frac{1}{\gamma}}$$

Re-arranging and solving for $\frac{P_{ij}}{P}$:

$$L_{ij}^{\gamma} = \frac{\alpha}{(1-\alpha)} \left(\frac{P_{ij}}{P}\right)^{-\theta} \frac{M}{P} \Leftrightarrow$$
$$\left(\frac{P_{ij}}{P}\right)^{-\theta} = \left[\frac{\alpha}{(1-\alpha)} \frac{M}{P}\right]^{-1} L_{ij}^{\gamma} \Leftrightarrow$$
$$\frac{P_{ij}}{P} = \left[\frac{\alpha}{(1-\alpha)} \frac{M}{P}\right]^{\frac{1}{\theta}} L_{ij}^{-\frac{\gamma}{\theta}}.$$

Raising both sides to the power of $1 - \theta$:

$$\frac{P_{ij}^{1-\theta}}{P^{1-\theta}} = \left[\frac{\alpha}{(1-\alpha)}\frac{M}{P}\right]^{\frac{1-\theta}{\theta}}L_{ij}^{-\frac{\gamma(1-\theta)}{\theta}}.$$

Integrating both sides over (0, 1):

$$\frac{\int_{0}^{1} P_{ij}^{1-\theta} dj}{P^{1-\theta}} = \left[\frac{\alpha}{(1-\alpha)} \frac{M}{P}\right]^{\frac{1-\theta}{\theta}} \int_{0}^{1} L_{ij}^{-\frac{\gamma(1-\theta)}{\theta}} dj \Leftrightarrow$$

$$1 = \left[\frac{\alpha}{(1-\alpha)} \frac{M}{P}\right]^{\frac{1-\theta}{\theta}} \int_{0}^{1} L_{ij}^{-\frac{\gamma(1-\theta)}{\theta}} dj \Leftrightarrow$$

$$1 = \left[\frac{\alpha}{(1-\alpha)} \frac{M}{P}\right]^{-\binom{\theta-1}{\theta}} \int_{0}^{1} L_{ij}^{\frac{\gamma(\theta-1)}{\theta}} dj \Leftrightarrow$$

$$\int_{0}^{1} L_{ij}^{\frac{\gamma(\theta-1)}{\theta}} dj = \left[\frac{\alpha}{1-\alpha} \frac{M}{P}\right]^{\frac{\theta-1}{\theta}}$$

I next define aggregate labour demand, L^D :

$$L^{D} = \left(\int_{0}^{1} L_{ij}^{\frac{\gamma(\theta-1)}{\theta}} dj\right)^{\frac{\theta}{\gamma(\theta-1)}}$$

I then obtain:

$$\left(L^{D}\right)^{\frac{\gamma(\theta-1)}{\theta}} = \left[\frac{\alpha}{1-\alpha}\frac{M}{P}\right]^{\frac{(\theta-1)}{\theta}}$$

and thus

$$L^{D} = \left[\frac{\alpha}{1-\alpha}\frac{M}{P}\right]^{\frac{1}{\gamma}}$$

Since the total labour force equals the mass of households, the labour force has mass one. The aggregate unemployment rate is therefore given by:

$$u = 1 - (L^D + L^G).$$

Substituting for aggregate demand from the private and public sectors, I obtain:

$$u = 1 - \left[\frac{\alpha}{(1-\alpha)}\frac{M}{P}\right]^{\frac{1}{\gamma}} - G.$$

To obtain an expression for the aggregate price level, I need to aggregate the price setting rules of each firm:

$$\left(\frac{P_{ij}}{P}\right) = \zeta \left(\frac{W_i}{P}\right)^{\gamma\phi} \left(\frac{M}{P}\right)^{(1-\gamma)\phi}.$$

Raising both sides to the power of $1 - \theta$ implies:

$$\frac{P_{ij}^{1-\theta}}{P^{1-\theta}} = \zeta^{1-\theta} \left(\frac{W_i}{P}\right)^{\gamma\phi(1-\theta)} \left(\frac{M}{P}\right)^{(1-\gamma)\phi(1-\theta)}$$

Averaging over the intervals covered by each union implies:

$$\frac{1}{P^{1-\theta}} \frac{1}{N} \sum_{i=0}^{N-1} \frac{\int_{i}^{\frac{i+1}{N}} P_{ij}^{1-\theta} dj}{\int_{\frac{i}{N}}^{\frac{i+1}{N}} dj} = \zeta^{1-\theta} \left(\frac{1}{P}\right)^{\gamma\phi(1-\theta)} \frac{1}{N} \sum_{i=0}^{N-1} W_i^{\gamma\phi(1-\theta)} \left(\frac{M}{P}\right)^{(1-\gamma)\phi(1-\theta)}$$

Note that the left-hand side can be written:

$$\frac{1}{P^{1-\theta}} \frac{1}{N} \sum_{i=0}^{N-1} \frac{\int_{i}^{\frac{i+1}{N}} P_{ij}^{1-\theta} dj}{\int_{i}^{\frac{i+1}{N}} dj} = \frac{1}{P^{1-\theta}} \frac{1}{N} \sum_{i=0}^{N-1} \frac{\int_{i}^{\frac{i+1}{N}} P_{ij}^{1-\theta} dj}{[j]_{\frac{i}{N}}^{\frac{i+1}{N}}} = \frac{1}{P^{1-\theta}} \frac{1}{N} \sum_{i=0}^{N-1} \frac{\int_{i}^{\frac{i+1}{N}} P_{ij}^{1-\theta} dj}{1/N} = \frac{1}{P^{1-\theta}} \sum_{i=0}^{N-1} \int_{\frac{i}{N}}^{\frac{i+1}{N}} P_{ij}^{1-\theta} dj = \frac{\int_{0}^{1} P_{ij}^{1-\theta} dj}{P^{1-\theta}} = 1.$$

I thus obtain:

$$1 = \zeta^{1-\theta} \left(\frac{1}{P}\right)^{\gamma\phi(1-\theta)} \frac{1}{N} \sum_{i=0}^{N-1} W_i^{\gamma\phi(1-\theta)} \left(\frac{M}{P}\right)^{(1-\gamma)\phi(1-\theta)} \Leftrightarrow$$

$$1 = \zeta^{1-\theta} \left(\frac{1}{P}\right)^{\gamma\phi(1-\theta)} W^{\gamma\phi(1-\theta)} \left(\frac{M}{P}\right)^{(1-\gamma)\phi(1-\theta)}.$$

where W is the aggregate wage index defined as:

$$W = \left(\frac{1}{N}\sum_{i=0}^{N-1} W_i^{\gamma\phi(1-\theta)}\right)^{\frac{1}{\gamma\phi(1-\theta)}}.$$

By noting that $\gamma \phi (1 - \theta) = 1 - \theta \phi$, I can write:

$$W = \left(\frac{1}{N}\sum_{i=0}^{N-1} W_i^{1-\theta\phi}\right)^{\frac{1}{1-\theta\phi}}$$

Simplifying:

$$\begin{split} 1 &= \zeta^{1-\theta} \left(\frac{1}{P}\right)^{\gamma\phi(1-\theta)} W^{\gamma\phi(1-\theta)} \left(\frac{M}{P}\right)^{(1-\gamma)\phi(1-\theta)} \Leftrightarrow \\ P^{(\gamma\phi(1-\theta)+(1-\gamma)\phi(1-\theta))} &= \zeta^{1-\theta} W^{\gamma\phi(1-\theta)} M^{(1-\gamma)\phi(1-\theta)} \Leftrightarrow \\ P^{\phi(1-\theta)} &= \zeta^{1-\theta} W^{\gamma\phi(1-\theta)} M^{(1-\gamma)\phi(1-\theta)}. \end{split}$$

Simplifying this expression I obtain:

$$P = \kappa W^{\gamma} M^{(1-\gamma)},$$

where $\kappa \equiv \left[\frac{\alpha}{1-\alpha}\right]^{(1-\gamma)} \left[\frac{\theta}{\gamma(\theta-1)}\right]^{\gamma}$.

Monetary Policy

The objective function of the central bank is given by:

$$M^{\varphi}P^{1-\varphi} = c.$$

If and only if

$$M = c^{\frac{1}{\varphi}} P^{\frac{\varphi-1}{\varphi}}.$$

Substituting for the equilibrium price level:

$$M = c^{\frac{1}{\varphi}} P^{\frac{\varphi-1}{\varphi}} = c^{\frac{1}{\varphi}} \left[\kappa W^{\gamma} M^{(1-\gamma)} \right]^{\frac{\varphi-1}{\varphi}} = c^{\frac{1}{\varphi}} \kappa^{\frac{\varphi-1}{\varphi}} W^{\gamma\frac{\varphi-1}{\varphi}} M^{(1-\gamma)\frac{\varphi-1}{\varphi}}$$

This implies:

$$\begin{split} M^{1-^{(1-\gamma)\frac{\varphi-1}{\varphi}}} &= c^{\frac{1}{\varphi}} \kappa^{\frac{\varphi-1}{\varphi}} W^{\frac{\gamma(\varphi-1)}{\varphi}} \Leftrightarrow \\ M^{\frac{\varphi-(1-\gamma)(\varphi-1)}{\varphi}} &= c^{\frac{1}{\varphi}} \kappa^{\frac{\varphi-1}{\varphi}} W^{\frac{\gamma(\varphi-1)}{\varphi}} \Leftrightarrow \\ M &= \left[c^{\frac{1}{\varphi}} \kappa^{\frac{\varphi-1}{\varphi}} W^{\frac{\gamma(\varphi-1)}{\varphi}} \right]^{\frac{\varphi}{\varphi+(1-\gamma)(1-\varphi)}} \Leftrightarrow \\ M &= c^{\frac{1}{\varphi+(1-\gamma)(1-\varphi)}} \kappa^{\frac{\varphi-1}{\varphi+(1-\gamma)(1-\varphi)}} W^{-\frac{\gamma(1-\varphi)}{\varphi+(1-\gamma)(1-\varphi)}}. \end{split}$$

Thus:

$$M = \widetilde{\kappa} W^{-\gamma(1-\varphi)\varpi}$$

where $\tilde{\kappa} \equiv c^{\varpi} \kappa^{-(1-\varphi)\varpi}$ and $\varpi = \frac{1}{\varphi + (1-\gamma)(1-\varphi)} > 0$. Substituting this expression into the objective function of the central bank gives the following expression for the

equilibrium price level:

$$P = c^{\frac{1}{1-\varphi}} M^{\frac{-\varphi}{1-\varphi}} = c^{\frac{1}{1-\varphi}} \left[\widetilde{\kappa} W^{-\gamma(1-\varphi)\varpi} \right]^{\frac{-\varphi}{1-\varphi}}$$
$$= c^{\frac{1}{1-\varphi}} \widetilde{\kappa}^{\frac{-\varphi}{1-\varphi}} W^{\frac{\varphi}{1-\varphi}\gamma(1-\varphi)\varpi} = c^{\frac{1}{1-\varphi}} \widetilde{\kappa}^{\frac{-\varphi}{1-\varphi}} W^{\varphi\gamma\varpi}$$

Thus, the aggregate price level can be written:

$$P = \frac{W^{\gamma \varphi \varpi}}{\widetilde{\chi}},$$

where $\widetilde{\chi} \equiv c^{-(1-\gamma)\varpi} \left(\frac{\alpha}{1-\alpha}\right)^{-(1-\gamma)\varphi\varpi} \left(\frac{\theta}{\gamma(\theta-1)}\right)^{-\gamma\varphi\varpi}$ and $\varpi = \frac{1}{\varphi+(1-\gamma)(1-\varphi)}$. The aggregate unemployment rate is given by:

$$u = 1 - \left[\frac{\alpha}{(1-\alpha)}\frac{M}{P}\right]^{\frac{1}{\gamma}} - L^{G} = 1 - \left[\frac{\alpha}{(1-\alpha)}\kappa^{-\varpi}\left(\frac{W}{c}\right)^{-\gamma\varpi}\right]^{\frac{1}{\gamma}} - G$$
$$= 1 - \delta\left(\frac{W}{c}\right)^{-\varpi} - G.$$

where $\delta \equiv \left(\frac{\alpha}{(1-\alpha)}\kappa^{-\varpi}\right)^{\frac{1}{\gamma}} = \left(\frac{\alpha}{1-\alpha}\right)^{\varphi\varpi} \left(\frac{\theta}{\gamma(\theta-1)}\right)^{-\varpi}$.

The Unemployment Rate facing Union *i*

The expression for unemployment facing union i is:

$$u_i = 1 - \vartheta \left(\frac{M}{P}\right)^{\phi} \left(\frac{W_i}{P}\right)^{-\phi\theta} - G.$$

Moreover:

$$\frac{W_i}{P} = \kappa^{-\varphi\varpi} \frac{W_i}{c} \left(\frac{W}{c}\right)^{-\gamma\varphi\varpi},$$

and

$$\frac{M}{P} = \kappa^{-\varpi} \left(\frac{W}{c}\right)^{-\gamma\varpi}.$$

Substituting this into the expression for the unemployment rate implies:

$$u_{i} = 1 - \vartheta \left(\frac{M}{P}\right)^{\phi} \left(\frac{W_{i}}{P}\right)^{-\phi\theta} - G$$

$$= 1 - \vartheta \left(\kappa^{-\varpi} \left(\frac{W}{c}\right)^{-\gamma\varpi}\right)^{\phi} \left(\kappa^{-\varphi\varpi} \frac{W_{i}}{c} \left(\frac{W}{c}\right)^{-\gamma\varphi\varpi}\right)^{-\phi\theta} - G$$

$$= 1 - \delta \left(\frac{W_{i}}{c}\right)^{-\phi\theta} \left(\frac{W}{c}\right)^{-\gamma\varpi\phi(1-\theta\varphi)} - G,$$

where $\delta \equiv \left(\frac{\alpha}{1-\alpha}\right)^{\varphi \varpi} \left(\frac{\theta}{\gamma(\theta-1)}\right)^{-\varpi}$.

Real Wages

The consumer real wage is given by:

$$\frac{W}{P} = \widetilde{\chi} W \left(W^{\gamma \varphi \varpi} \right)^{-1} = \kappa^{-\varphi \varpi} \left(\frac{W}{c} \right)^{(1-\gamma) \varpi}.$$

And the real wage obtained by the members of union i can be written:

$$\frac{W_i}{P} = \kappa^{-\varphi\varpi} \frac{W_i}{c} \left(\frac{W}{c}\right)^{-\gamma\varphi\varpi}.$$

A2 Proofs

Proof of Proposition 1 If fiscal policy is exogenously given and there is no provision of the public good, the following real wage ranking applies: $\left(\frac{W}{P}\right)_M > \left(\frac{W}{P}\right)_P$ $\forall N>1.$

Proof.

$$\left(\frac{W}{P}\right)_{P} = \left[\frac{\phi\left[\theta + \left(\frac{\gamma}{1-\gamma}\right)\frac{1}{N}\right]}{\phi\left[\theta + \left(\frac{\gamma}{1-\gamma}\right)\frac{1}{N}\right] - 1}\right]B$$
$$\left(\frac{W}{P}\right)_{M} = \left[\frac{\phi\left[\theta + \gamma\left(1-\theta\right)\frac{1}{N}\right]}{\phi\left[\theta + \gamma\left(1-\theta\right)\frac{1}{N}\right] - \left[1-\frac{\gamma}{N}\right]}\right]B = \left[\frac{\phi\left[\theta + \gamma\left(1-\theta\right)\frac{1}{N}\right]}{\phi\left[\theta + \gamma\left(1-\theta\right)\frac{1}{N}\right] - \left[1-\frac{\gamma}{N}\right]}\right]B$$

$$\begin{pmatrix} \frac{W}{P} \end{pmatrix}_{P} < \begin{pmatrix} \frac{W}{P} \end{pmatrix}_{M} \Leftrightarrow \\ \left[\frac{\phi \left[\theta + \left(\frac{\gamma}{1 - \gamma} \right) \frac{1}{N} \right]}{\phi \left[\theta + \left(\frac{\gamma}{1 - \gamma} \right) \frac{1}{N} \right] - 1} \right] B < \left[\frac{\phi \left[\theta + \gamma \left(1 - \theta \right) \frac{1}{N} \right]}{\phi \left[\theta + \gamma \left(1 - \theta \right) \frac{1}{N} + \frac{\gamma}{\phi N} \right] - 1} \right] B$$

if and only if

$$\frac{\left[\theta + \left(\frac{\gamma}{1-\gamma}\right)\frac{1}{N}\right]}{\phi\left[\theta + \left(\frac{\gamma}{1-\gamma}\right)\frac{1}{N}\right] - 1} < \frac{\left[\theta + \gamma\left(1-\theta\right)\frac{1}{N}\right]}{\phi\left[\theta + \gamma\left(1-\theta\right)\frac{1}{N} + \frac{\gamma}{\phi N}\right] - 1}$$

if and only if

$$\begin{bmatrix} \theta + \left(\frac{\gamma}{1-\gamma}\right)\frac{1}{N} \end{bmatrix} \begin{bmatrix} \phi \left[\theta + \gamma \left(1-\theta\right)\frac{1}{N} + \frac{\gamma}{\phi N}\right] - 1 \end{bmatrix}$$

$$< \begin{bmatrix} \theta + \gamma \left(1-\theta\right)\frac{1}{N} \end{bmatrix} \begin{bmatrix} \phi \left[\theta + \left(\frac{\gamma}{1-\gamma}\right)\frac{1}{N}\right] - 1 \end{bmatrix} \Leftrightarrow$$

$$\begin{bmatrix} \theta + \left(\frac{\gamma}{1-\gamma}\right)\frac{1}{N} \end{bmatrix} \phi \begin{bmatrix} \theta + \gamma \left(1-\theta\right)\frac{1}{N} + \frac{\gamma}{\phi N} \end{bmatrix} - \begin{bmatrix} \theta + \left(\frac{\gamma}{1-\gamma}\right)\frac{1}{N} \end{bmatrix}$$

$$< \begin{bmatrix} \theta + \gamma \left(1-\theta\right)\frac{1}{N} \end{bmatrix} \phi \begin{bmatrix} \theta + \left(\frac{\gamma}{1-\gamma}\right)\frac{1}{N} \end{bmatrix} - \begin{bmatrix} \theta + \gamma \left(1-\theta\right)\frac{1}{N} \end{bmatrix}$$

if and only if

$$\begin{bmatrix} \theta + \left(\frac{\gamma}{1-\gamma}\right)\frac{1}{N}\end{bmatrix}\phi\left[\theta + \gamma\left(1-\theta\right)\frac{1}{N} + \frac{\gamma}{\phi N}\right] \\ < \left[\theta + \gamma\left(1-\theta\right)\frac{1}{N}\right]\phi\left[\theta + \left(\frac{\gamma}{1-\gamma}\right)\frac{1}{N}\right] - \gamma\left(1-\theta\right)\frac{1}{N} + \left(\frac{\gamma}{1-\gamma}\right)\frac{1}{N}.$$

Note that:

$$\gamma (1-\theta) \frac{1}{N} - \left(\frac{\gamma}{1-\gamma}\right) \frac{1}{N} = \frac{\gamma}{N} \left[(1-\theta) - \left(\frac{1}{1-\gamma}\right) \right]$$
$$= \frac{\gamma}{1-\gamma} \frac{1}{N} \left[(1-\theta) (1-\gamma) - 1 \right] = -\frac{\gamma}{1-\gamma} \frac{1}{N} \phi.$$

Thus, the condition reads:

$$\begin{bmatrix} \theta + \left(\frac{\gamma}{1-\gamma}\right)\frac{1}{N}\end{bmatrix}\phi\begin{bmatrix} \theta + \gamma\left(1-\theta\right)\frac{1}{N} + \frac{\gamma}{\phi N}\end{bmatrix}$$

<
$$\begin{bmatrix} \theta + \gamma\left(1-\theta\right)\frac{1}{N}\end{bmatrix}\phi\begin{bmatrix} \theta + \left(\frac{\gamma}{1-\gamma}\right)\frac{1}{N}\end{bmatrix} + \frac{\gamma}{1-\gamma}\frac{1}{N}\phi$$

if and only if

$$\begin{bmatrix} \theta + \left(\frac{\gamma}{1-\gamma}\right)\frac{1}{N} \end{bmatrix} \begin{bmatrix} \theta + \gamma \left(1-\theta\right)\frac{1}{N} + \frac{\gamma}{\phi N} \end{bmatrix}$$

$$< \begin{bmatrix} \theta + \gamma \left(1-\theta\right)\frac{1}{N} \end{bmatrix} \begin{bmatrix} \theta + \left(\frac{\gamma}{1-\gamma}\right)\frac{1}{N} \end{bmatrix} + \frac{\gamma}{1-\gamma}\frac{1}{N}$$

if and only if

$$\begin{split} \left[\theta + \left(\frac{\gamma}{1-\gamma}\right)\frac{1}{N}\right] \left[\theta + \gamma\left(1-\theta\right)\frac{1}{N} + \frac{\gamma}{\phi N} - \left[\theta + \gamma\left(1-\theta\right)\frac{1}{N}\right]\right] < \frac{\gamma}{1-\gamma}\frac{1}{N} \Leftrightarrow \\ \left[\theta + \left(\frac{\gamma}{1-\gamma}\right)\frac{1}{N}\right] \left[\frac{\gamma}{\phi N}\right] < \frac{\gamma}{1-\gamma}\frac{1}{N} \\ \left[\theta + \left(\frac{\gamma}{1-\gamma}\right)\frac{1}{N}\right] \left[\frac{1}{\phi}\right] < \frac{1}{1-\gamma} \\ \theta + \left(\frac{\gamma}{1-\gamma}\right)\frac{1}{N} < \frac{\gamma+\theta\left(1-\gamma\right)}{1-\gamma} \end{split}$$

if and only if

$$\begin{aligned} \theta + \left(\frac{\gamma}{1-\gamma}\right) \frac{1}{N} &< \frac{\gamma}{1-\gamma} + \theta \\ \left(\frac{\gamma}{1-\gamma}\right) \frac{1}{N} &< \frac{\gamma}{1-\gamma} \\ \frac{1}{N} &< 1 \end{aligned}$$

if and only if

N > 1

and the proposition follows. \blacksquare

Proof of Proposition 2 $u_P < u_M \ \forall N > 1$ **Proof.** The unemployment rate is given by:

$$u = 1 - \delta \left(\frac{W}{c}\right)^{-\varpi}$$

Substituting for $\frac{W}{c} = \left(\frac{1}{\chi}\frac{W}{P}\right)^{\frac{1}{(1-\gamma)\varpi}}$:

$$u = 1 - \delta \left(\frac{1}{\chi}\frac{W}{P}\right)^{-\frac{\varpi}{(1-\gamma)\varpi}} = 1 - \delta \left(\frac{1}{\chi}\frac{W}{P}\right)^{-\frac{1}{(1-\gamma)}} = 1 - \delta \chi^{\frac{1}{(1-\gamma)}} \left(\frac{W}{P}\right)^{-\frac{1}{(1-\gamma)}}$$

where $\delta \equiv \left(\frac{\alpha}{(1-\alpha)}\right)^{\varphi \varpi} \left(\frac{\theta}{\gamma(\theta-1)}\right)^{-\varpi}$, $\chi = \kappa^{-\varphi \varpi}$ and $\kappa \equiv \left(\frac{\alpha}{1-\alpha}\right)^{(1-\gamma)} \left(\frac{\theta}{\gamma(\theta-1)}\right)^{\gamma}$. This implies that

$$\delta \chi^{\frac{1}{(1-\gamma)}} = \left(\frac{\theta}{\gamma \left(\theta - 1\right)}\right)^{-\frac{1}{1-\gamma}} \equiv \varsigma$$

Thus, the functional form of the expression for unemployment is independent of the regime and I may rank unemployment rates according to wages. Since unemployment is strictly increasing in the real wage

$$\frac{du}{dW/P} = \frac{\varsigma}{(1-\gamma)} \left(\frac{W}{P}\right)^{-\frac{1}{(1-\gamma)}-1} > 0$$

and the proposition follows. \blacksquare

Proof of Proposition 3 $\left(\frac{W}{P}\right)_{P,N=1} = \left(\frac{W}{P}\right)_{M,N=1} = \frac{1}{\gamma}B$ **Proof.** Under complete centralisation:

$$(\sigma_P)_{N=1} = \frac{\phi}{(1-\gamma)} [\theta (1-\gamma) + \gamma] = \frac{1}{1-\gamma} (\sigma_P - \tau_P)_{N=1} = \frac{1}{1-\gamma} - 1 = \frac{1-(1-\gamma)}{1-\gamma} = \frac{\gamma}{1-\gamma}$$

Thus:

$$\left(\frac{W}{P}\right)_{P,N=1} = \frac{1}{\gamma}B$$

Under money supply targeting:

$$(\sigma_M)_{N=1} = [\phi\theta + \gamma\phi(1-\theta)] = \phi [\theta + \gamma(1-\theta)] = 1$$

$$(\tau_M)_{N=1} = [1-\gamma]$$

$$(\sigma_M - \tau_M)_{N=1} = 1 - [1-\gamma] = \gamma$$

and

$$\left(\frac{W}{P}\right)_{M,N=1} = \frac{1}{\gamma}B$$

and the proposition follows. \blacksquare

Proof of Proposition 4 If fiscal policy is exogenously given and there is no provision of the public good, the real wage is decreasing in the degree of centralisation.

Proof.
$$\frac{d}{dN} \left(\frac{W}{P}\right) = \frac{d}{dN} \left[\frac{\tau}{(\sigma-\tau)}\right] B = \left[\frac{\tau'(\sigma-\tau)-\tau(\sigma'-\tau')}{(\sigma-\tau)^2}\right] B = \left[\frac{\tau'\sigma-\tau\sigma'}{(\sigma-\tau)^2}\right] B$$
where $\tau' = \frac{d\tau}{dN} = \frac{\gamma\varphi\varpi}{N^2}$ and $\sigma' = \frac{d\sigma}{dN} = -\frac{\gamma\varpi\phi(1-\varphi\theta)}{N^2}$. This implies
 $\tau'\sigma - \tau\sigma' = \frac{\gamma\varphi\varpi}{N^2} \left(\phi\theta + \frac{\gamma\varpi\phi\left(1-\varphi\theta\right)}{N}\right) + \frac{\gamma\varpi\phi\left(1-\varphi\theta\right)}{N^2} \left(1-\frac{\gamma\varphi\varpi}{N}\right) = \frac{\gamma\varpi\phi}{N^2} > 0$

Thus $\frac{d}{dN}\left(\frac{W}{P}\right) > 0$ i.e. the real wage is increasing in the degree of decentralisation. Moreover, $\frac{d}{d(1/N)}\left(\frac{W}{P}\right) = -\frac{1}{N^2}\frac{d}{dN}\left(\frac{W}{P}\right) < 0$ and the proposition follows.

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A3 Tables

Table 3.1: Parameters of the model

	Value	Description
Policy variables		
В	0.45	The real unemployment benefit
t	0.2	The tax rate
ϵ	$\in (0,1)$	Marginal utility to the government
		of a higher budget surplus
N	≥ 1	The number of unions
φ	$\in (0,1)$	Relative weight assigned to the money supply
		in the CB objective function
Parameters		
α	0.5	Cobb-Douglas weight on household consumption
η,eta	0.35, 7	Household weights (concavity) on public goods
θ	11	Elasticity of substitution across private sector goods
γ	0.7	Decreasing returns to private sector production

Table 3.2: Numerical solutions to the model, no public good

	Monetary regime	Р	М
N = 1	W/P n $V_h _{No\ Gov.}$	1 0.967 1	1 0.967 1
N = 2	W/P n $V_h _{No\ Gov.}$	$1.043 \\ 0.841 \\ 0.998$	0.768
N = 4	W/P n $V_h _{No\ Gov.}$	$\begin{array}{c} 1.069 \\ 0.773 \\ 0.996 \end{array}$	$\begin{array}{c} 1.088 \\ 0.729 \\ 0.993 \end{array}$
N = 10	W/P n $V_h _{No\ Gov.}$	$\begin{array}{c} 1.087 \\ 0.732 \\ 0.993 \end{array}$	0.713

			berg lead		
		Unions		Govern	ment
		(1)	(2)	(3)	(4)
	Monetary regime	Р	M	Р	M
N = 1	W/P	1.039	1.039	1.039	1.039
	$G^{'}$	0.053	0.053	0.053	0.053
	n	0.925	0.925	0.925	0.925
	cross-elasticity	0.114	0.016	0.403	0.403
	ρ	0.049	0.049	0.049	0.049
	V_h	1.037	1.037	1.037	1.037
	Λ	1	1	1	1
N = 2	W/P	1.105	1.158	1.099	1.141
	G	0.068	0.082	0.062	0.068
	n	0.772	0.676	0.779	0.694
	cross-elasticity	0.114	0.015	0.632	0.853
	ρ	-0.035	-0.103	-0.026	-0.079
	V_h	1.034	1.029	1.034	1.030
	Λ	0.866	0.776	0.879	0.807
N = 4	W/P	1.154	1.196	1.138	1.167
	G	0.081	0.093	0.068	0.071
	n	0.684	0.619	0.700	0.648
	cross-elasticity	0.114	0.014	0.833	1.015
	ρ	-0.097	-0.152	-0.075	-0.11
	V_h	1.030	1.025	1.031	1.027
	Λ	0.783	0.719	0.812	0.768
N = 10	W/P	1.193	1.215	1.165	1.178
	G	0.092	0.099	0.071	0.072
	n	0.623	0.592	0.651	0.629
	cross-elasticity	0.114	0.014	1.003	1.095
	ρ	-0.148	-0.177	-0.109	-0.12
	V_h	1.025	1.023	1.027	1.025
	Λ	0.724	0.692	0.770	0.751

Table 3.3: Numerical solutions to the model, $\epsilon=0$

		Stackelberg leade Unions			er Government	
		(1)	(2)	(3)	(4)	
	Monetary regime	Р	М	Р	M	
V = 1	W/P	1.035	1.033	1.006	1.006	
	$G^{'}$	0.044	0.044	0.009	0.009	
	n	0.924	0.929	0.961	0.961	
	cross - elasticity	0.057	0.008	0.301	0.301	
	ρ	0.057	0.059	0.104	0.104	
	V_h	1.036	1.036	1.012	1.012	
	Λ	1.012	1.016	1.073	1.073	
V = 2	W/P	1.094	1.135	1.046	1.071	
	G	0.054	0.062	0.004	0.000	
	n	0.782	0.699	0.837	0.768	
	cross-elasticity	0.057	0.007	0.401	0.476	
	ρ	-0.017	-0.070	0.057	0.027	
	V_h	1.034	1.030	1.003	0.995	
	Λ	0.891	0.818	0.977	0.921	
V = 4	W/P	1.133	1.163	1.069	1.088	
	G	0.061	0.068	0.000	0.000	
	n	0.704	0.652	0.773	0.729	
	cross-elasticity	0.057	0.007	0.0469	0.532	
	ρ	-0.067	-0.105	0.030	0.006	
	V_h	1.030	1.027	0.995	0.993	
	Λ	0.822	0.774	0.925	0.888	
N = 10	W/P	1.161	1.175	1.087	1.096	
	G	0.067	0.070	0.000	0.000	
	n	0.654	0.632	0.732	0.713	
	cross-elasticity	0.057	0.007	0.528	0.559	
	ho	-0.103	-0.122	0.007	-0.004	
	V_h	1.027	1.025	0.993	0.992	
	Λ	0.777	0.755	0.890	0.874	

Table 3.4: Numerical solutions to the model, $\epsilon = 0$.	.5
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	Stackelberg leader				
		Unions		Govern	nment
		(1)	(2)	(3)	(4)
	Monetary regime	Р	M	Р	M
N = 1	W/P	1.025	1.025	1.000	1.000
	$G^{'}$	0.034	0.034	0.000	0.000
	\overline{n}	0.941	0.941	0.967	0.967
	cross - elasticity	0.000	0.000	0.285	0.285
	ρ	0.073	0.073	0.115	0.115
	V_h	1.033	1.033	1	1
	Λ	1.037	1.037	1.081	1.081
V = 2	W/P	1.078	1.115	1.043	1.071
	$G^{'}$	0.040	0.045	0.000	0.000
	n	0.803	0.722	0.841	0.768
	cross-elasticity	0.000	0.000	0.391	0.476
	ρ	0.007	-0.041	0.062	0.027
	V_h	1.031	1.027	0.998	0.995
	Λ	0.923	0.853	0.981	0.921
N = 4	W/P	1.112	1.138	1.069	1.088
	G	0.045	0.048	0.000	0.000
	n	0.728	0.677	0.773	0.729
	cross-elasticity	0.000	0.000	0.469	0.532
	ρ	-0.037	-0.071	0.030	0.006
	V_h	1.027	1.024	0.995	0.993
	Λ	0.858	0.814	0.925	0.888
N = 10	W/P	1.137	1.149	1.087	1.096
	G	0.048	0.049	0.000	0.000
	n	0.680	0.658	0.732	0.713
	cross-elasticity	0.000	0.000	0.528	0.559
	ρ	-0.068	-0.084	0.007	-0.004
	V_h	1.024	1.023	0.993	0.992
	Λ	0.817	0.797	0.890	0.874

Table 3.5: Numerical solutions to the model, $\epsilon=1$

Chapter 4

The Swedish Real Exchange Rate under Different Currency

$\mathbf{Regimes}^*$

1 Introduction

In light of the recent launch of the third step of the Economic and Monetary Union (EMU) there has been a vivid debate on exchange rate regimes. Many countries have abandoned fixed exchange rate regimes during the 1990s and in many cases combined floating exchange rates with inflation targets. Opponents to floating exchange rates argue that rather than absorbing shocks and stabilising the economy, the nominal exchange rate reflects noise from financial markets that may in fact be destabilising. The issue is clearly of great importance to countries contemplating joining the EMU. Obviously, if the nominal exchange rate causes shocks rather than offsets them, the cost of relinquishing it by joining the monetary union, is low.¹

At the same time, the real exchange rate is perhaps the most common measure of overall firm competitiveness, and to understand its behaviour over different horizons is to understand the conditions faced by firms engaging in international trade.²

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¹ Thomas (1997) finds evidence that, provided that demand shocks are controllable by economic policy, Sweden would face a low cost of relinquishing the nominal exchange rate. Artis and Ehremann (2002) suggest that the Swedish exchange rate is a source of shocks rather than a shock absorber.

² The real exchange rate is defined as $Q = SP^*/P$, where S is the nominal exchange rate in domestic currency per units of foreign currency (spot rate), P is the price level, and * henceforth

For small open economies in particular, the real exchange rate is therefore a key variable. In this paper I try to shed some light on the behaviour of the real exchange rate under different exchange rate regimes by addressing the following questions: What mechanisms ensure that the real exchange rate returns to equilibrium after a distortion? How are these mechanisms affected by the exchange rate regime?

The literature on long-run behaviour of real exchange rates is quite extensive. Since the introduction of cointegration in the empirical literature, interest in the real exchange rate has experienced a renaissance through its close connection to Purchasing Power Parity (PPP). A stationary real exchange rate implies that relative PPP must hold, while a non-stationary real exchange rate is a suitable object for cointegration analysis. Numerous tests for cointegration between the nominal exchange rate and various price combinations have therefore been presented in the literature; see Froot and Rogoff (1995) for a survey. In addition to tests for PPP, the literature on real exchange rates comprises various structural models of economic fundamentals. Seminal work by Balassa (1964) and Samuelson (1964) shows how the real exchange rate should be determined by the productivity growth differential between the traded and non-traded sectors. Extensions to the Balassa-Samuelson model have also been made and tested empirically. Evidence on how long-run real exchange rates depend on fundamentals is given in among others Alexius and Nilsson (2000) and, more recently, Bergvall (2002).

An interesting property of the real exchange rate is that it consists of variables exhibiting contrasting dynamic behaviour. As is well known, price levels tend to be sticky, while typically volatile nominal exchange rates can jump instantaneously under a floating rate regime, as in for instance Dornbusch (1976). Modelling the real exchange rate consequently implies modelling the slow adjustment of long-run relative prices, and the fast (instantaneous) adjustment of nominal exchange rates.

The research effort dedicated to investigating the short-run dynamics of the real exchange rate is far less extensive than the effort made to examine its long-run behaviour. Perhaps as a consequence, the impact of the exchange rate regime on the real exchange rate is often neglected.³ The choice between a fixed exchange rate and a floating exchange rate (perhaps combined with a price level or inflation

denotes a foreign country.

 $^{^{3}}$ One exception, although a little dated, is Mussa (1986), who finds that the exchange rate regime indeed matters for the real exchange rate. Since post-Bretton Woods time series only recently have become sufficiently long for results to be reliable, it is of great interest to re-examine the issue of regimes using recent data. Taylor (2002) distinguishes between regimes in a study of PPP reversion.

target) should be highly relevant in explaining the dynamic adjustment of the real exchange rate to its long-run path.

This paper analyses both the long-run behaviour and the short-run dynamics of the Swedish real exchange rate relative to Germany during the period 1973:1-2001:4, stretching from the collapse of the Bretton Woods system in March 1973 through various exchange rate regimes including the launch of the third step of the EMU on January 1 1999.

The contribution of the paper is as follows: Instead of focusing on the long run, I emphasise the analysis of dynamic models of the real exchange rate where the exchange rate regime is taken into account. Moreover, I investigate the question as to whether it is the Swedish price level, the German price level or the nominal exchange rate that has adjusted to deviations from long-run equilibrium, the hypothesis being that Sweden as a small country has been forced to adapt to German conditions rather than vice versa.

The main findings are that (i) there is support for the Balassa-Samuelson hypothesis according to which the real exchange rate is driven by productivity growth in the long run, (ii) the exchange rate regime has mattered for the dynamics of the real exchange rate and (iii) the Swedish price level and the nominal exchange rate account for most of the adjustment following a disturbance to long-run equilibrium.

The rest of the paper is organised as follows: Section 2 presents a simple theoretical framework for analysing the real exchange rate over different horizons and Section 3 describes the data. Section 4 discusses modelling strategies and empirical issues. Results are presented in Section 5 and Section 6 concludes.

2 Theoretical Framework

In two pioneering papers Balassa (1964) and Samuelson (1964) find that the productivity differential between the traded and non-traded sectors within countries, relative to other countries, should affect long-run real exchange rates.⁴ Alexius (2001) reports that 86 percent of the long-run variance of the Swedish real exchange rate is determined by supply-side factors. In line with this result, I follow Balassa and Samuelson and think of an economy where the supply side determines the long-

 $^{^4}$ De Gregorio & Wolf (1994) expand the theory of Balassa and Samuelson by considering the terms of trade, i.e. the ratio between export and import prices. This may however yield a simultaneity problem. Moreover, Chinn (1997) reports that the terms of trade are insignificant in the long run and they are therefore excluded in this study.

run equilibrium, but then consider the possibility that demand-side factors may affect the real exchange rate in the short run. Potential long-run and short-run determinants are considered in turn below.

2.1 Long-Run Determinants

Consider a small, open economy, consisting of two sectors: one sector producing tradable goods, indexed T, and one sector producing non-tradeable goods, indexed N. Since the economy is small, the rental cost of capital is exogenously given and in the traded sector the law of one price implies $P_T = SP_T^*$, where P_T and P_T^* are domestic and foreign T-sector prices and S is the nominal (spot) exchange rate in domestic currency per unit of foreign currency. There is perfect capital mobility between sectors as well as between countries, which implies the same rental price of capital in both sectors and countries. Moreover, there is perfect labour mobility domestically, which implies that the nominal wage is equal in the two sectors. Finally, there is perfect competition in the goods market and the labour market. The production technology is Cobb-Douglas and there are constant returns to scale in both sectors, so that production is given by $Y_i = A_i (L_i)^{\alpha_i} (K_i)^{1-\alpha_i}$, where Y_i is output, A_i total factor productivity (TFP), L_i and K_i are labour and capital input respectively, $\alpha_i \in (0, 1)$ and i = N, T. Taking logs and differentiating the first-order conditions for profit maximisation yields the central equation of the Balassa-Samuelson hypothesis:

$$\frac{\dot{P}_{rel}}{P_{rel}} = \left(\frac{\alpha_N}{\alpha_T}\right)\frac{\dot{A}_T}{A_T} - \frac{\dot{A}_N}{A_N} \tag{4.1}$$

where $P_{rel} \equiv P_N/P_T$, $\dot{X} = dX/dt$ and \dot{X}/X is the growth rate of X. The intuition behind hypothesis (4.1) is as follows: A productivity rise in the T-sector raises the aggregate wage level and prices on N-goods for a given level of productivity in the Nsector. Equally labour intensive production in the two sectors, ($\alpha_T = \alpha_N$) generates a growth rate in the relative price of non-tradeables that is equal to the inter-sector TFP growth differential.

Assume that the aggregate price level in the economy, i.e. the consumer price level, can be written as a weighted geometric mean of the price levels in the two sectors $P = (P_N)^{\beta} (P_T)^{1-\beta}$ where $\beta \in (0,1)$. Finally assume that the conditions stated above apply also to the foreign country so that there is complete symmetry in production and prices. By using the definition of the real exchange rate $Q = SP^*/P$, the definition of the aggregate price level and by imposing the law of one price for T-goods, the following cost-push hypothesis of the real exchange rate is obtained:

$$\frac{\dot{Q}}{Q} = \beta^* \left(\frac{\alpha_N^*}{\alpha_T^*} \frac{\dot{A}_T^*}{A_T^*} - \frac{\dot{A}_N^*}{A_N^*} \right) - \beta \left(\frac{\alpha_N}{\alpha_T} \frac{\dot{A}_T}{A_T} - \frac{\dot{A}_N}{A_N} \right)$$
(4.2)

Equation (4.2) states that higher relative TFP-growth in the traded sector at home than abroad induces a real appreciation, due to the effect of increased wage costs on domestic prices.

2.2 Short-Run Dynamics

In the short run, I expect changes in the nominal exchange rate to transmit fully to the real exchange rate. The time span is not long enough for prices to adjust. Since the nominal exchange rate is notoriously difficult to model, I use the simple uncovered interest rate parity (UIP) condition to capture some basic ideas. Letting R_t and R_t^* be the nominal domestic and foreign short-term interest rates, respectively, and $E_t(S_{t+1})$ the expectation at time t of the nominal exchange rate in the next period, UIP states that $R_t = R_t^* + [E_t(S_{t+1}) - S_t]/S_t$. A positive interest rate differential therefore reflects an expected depreciation of the nominal exchange rate. Rearranging the UIP-condition yields

$$S_{t} = \frac{E_{t} \left(S_{t+1} \right)}{1 + \left(R_{t} - R_{t}^{*} \right)}$$

If UIP holds, the nominal exchange rate is determined by the expected nominal exchange rate discounted by one plus the interest rate differential $(R_t - R_t^*)$. I treat Sweden as a small country taking German variables as given and assume that depreciation expectations are determined by Swedish unemployment, U and the Swedish Budget Balance BB. The intuition is that the Swedish central bank is more likely to let the exchange rate depreciate in a situation with high unemployment or a large budget deficit, since depreciating the exchange rate may stimulate the economy in such a way that these problems are offset. Historically, the Swedish exchange rate has also depreciated during periods of high unemployment in order to improve the economy's competitiveness.

In addition to variables operating through the nominal exchange rate, the demandside may have a short-run influence on the dynamics of the real exchange rate through the aggregate price levels. Chinn (1997) suggests that log relative government consumption, $(g - g^*)$, is significant over this horizon, since increased demand may cause demand-pull inflation and thereby a real appreciation. However, it may also be the case that an increase in log relative government consumption may generate inflation expectations, causing a nominal depreciation. The net effect on the real exchange rate is therefore ambiguous. Another hypothesis tested by Chinn (1997), is the possibility that the oil price (p_{oil}) affects the real exchange rate, representing a cost-push hypothesis. The sign of the effect is uncertain since it depends on how production technologies differ between countries.⁵ These variables will be included as short-run explanatory variables in the study at hand. The short-run dynamics suggested by theory may therefore be summarised by the following function

$$\Delta q = f \left(R - R^*, U, BB, g - g^*, p_{oil}, ECM \right)$$

where ECM is the error correction mechanism, to be explained in the next section. The above theory suggests $f_1 < 0$, $f_2 > 0$, $f_3 < 0$, $f_6 < 0$ while the signs of f_4 and f_5 are ambiguous.

3 Empirical Issues

I start by briefly considering the Engle-Granger one-step method (see Engle and Granger (1987) and Banerjee et al. (1993:157-161)) and proceed by modifying the specifications in an attempt to capture some aspects of the exchange rate regime. The section is ended with a description of the method used to test how the three components, s, p and p^* react to deviations from long-run equilibrium under different exchange rate regimes. Throughout the paper small letters denote logs unless otherwise stated.

3.1 Estimating the Cointegrating Vector and Obtaining the Error Correction Mechanism (ECM)

The Engle-Granger one-step method suggests estimating the short-run and longrun relationships in one step. Economic theory predicts that there is cointegration between the real exchange rate, q_t , and Swedish and German productivity, a_t and a_t^* respectively. Let x_j , j = 1, ..., J be the J short-run explanatory variables to be used

⁵ Since both Sweden and Germany are oil importers, the oil price should matter for the relative price only if the elasticities of output with respect to the oil price differ between the two countries. How the relative price is affected by the oil price is determined by which country is more dependent on oil as a factor of production.

in first differences if they are found to be non-stationary.⁶ The one-step procedure is then to estimate.

$$\Delta q_t = \gamma_0 + \sum_{j=1}^J \gamma_j \Delta x_j + \gamma_{J+1} q_{t-1} + \gamma_{J+2} a_{t-1} + \gamma_{J+3} a_{t-1}^* + \varepsilon_t$$
(4.3)

The error correction term is obtained from (4.3) as $ECM_t = q_t + \frac{\hat{\gamma}_{J+2}}{\hat{\gamma}_{J+1}}a_t + \frac{\hat{\gamma}_{J+3}}{\hat{\gamma}_{J+1}}a_t^*$, where $\hat{\gamma}_{J+1} < 0$. The reason $\hat{\gamma}_{J+1}$ should be non-positive is that if the real exchange rate at time t-1 was above its equilibrium value it has to appreciate, i.e. decrease, in period t in order to eventually return to equilibrium. Since first differences of all I(1) variables are stationary and there exists at least one linear combination of cointegrated variables that is stationary by definition, the residuals in (4.3) should be stationary as well. If the deviation from long run equilibrium adjusts proportionally to the current *level* of the deviation, 90 percent of the deviation has been adjusted after $t = \ln (0.10) / \hat{\gamma}_{J+1}$ periods.⁷

3.2 Modeling the Short-Run Dynamics Taking into Account the Exchange Rate Regime

Under a floating regime, the nominal exchange rate can adjust to shocks instantaneously, while the relative price has to absorb all shocks under an irrevocably fixed rate regime such as a monetary union.⁸ In the long run, the type of regime should not affect the real exchange rate, but the mechanisms through which the real exchange rate reaches its equilibrium value, i.e. the short-run dynamics, are likely to be affected by the regime. In this paper, an attempt to capture these features is

⁶ Throughout the paper weak (covariance) stationarity is intended. The stochastic process X_t is weakly stationary if $E(X_t) = \mu$, $Var(X_t) = \sigma^2$, where μ, σ^2 are constants, and $Cov(X_t, X_{t+j}) = \sigma_j \forall j$.

 $[\]sigma_j \forall j$. ⁷ To see this, let z = z(t) be the deviation from long run equilibrium, let $z(0) = z_0$ and assume that the deviation adjusts proportionally to its current level, so that $dz/dt = \mathring{\gamma}_{J+1} z$, where $\mathring{\gamma}_{J+1} < 0$. This yields an ordinary, separable differential equation with the general solution $lnz = \mathring{\gamma}_{J+1}t + C$, where C is a constant. The initial value implies $C = \ln z_0$, and the solution $z = z_o \exp(\mathring{\gamma}_{J+1}t)$. (100 ρ) percent of the deviation has consequently been adjusted after t periods according to the following: $(1 - \rho) z_0 = z_o \exp(\mathring{\gamma}_{J+1}t)$ implying $t = \ln (1 - \rho) / \mathring{\gamma}_{J+1}$, where $\rho \in$ (0, 1), and t > 0 since lnx < 0 for all x < 1. Note that the standard half-life measure is obtained by letting $\rho = 0.5$.

⁸ Throughout the paper, the ratio between the German and the Swedish price level in domestic currencies is intended when referring to the relative price if not stated otherwise.

made by using dummy variables.

First, two regime dummies are specified. The post-Bretton Woods period may in the Swedish case be roughly divided into four currency regimes: (i) The system referred to as the *Monetary Snake*, 1973-1977, (ii) the currency basket, 1977-1991, (iii) the peg relative the ECU, 1991-1992 and (iv) the floating rate regime 1992 onwards. Regimes (i) and (iii) are virtually equivalent to a fixed rate regime relative the D-mark, since the German currency dominated the Monetary Snake. Since the currency basket prevailed for the longest time period, it will be used as the norm and the following two dummy variables are used to capture regime-specific effects:

$$D_{fix} = \begin{cases} 1, \text{ if } 1973:1-1977:2, \text{ or } 1991:1-1992:4 \\ 0, \text{ otherwise} \end{cases}$$
$$D_{float} = \begin{cases} 1, \text{ if } 1993:1 \text{ onwards} \\ 0, \text{ otherwise} \end{cases}$$

Hence, D_{fix} indicates a fixed rate regime and D_{float} indicates a floating exchange rate regime. Dichotomous dummy variables will also be added to capture the devaluations during the fixed exchange rate regimes and the German reunification. These variables assume the value zero for all quarters except the quarter defined in each dummy according to the following: $D_1 = \{1, \text{ if } 1977 : 2\}, D_2 = \{1, \text{ if } 1977 : 3\},$ $D_3 = \{1, \text{ if } 1981 : 3\}, D_4 = \{1, \text{ if } 1982 : 3\}, D_5 = \{1, \text{ if } 1992 : 4\}$ and $D_6 =$ $\{1, 1990 : 3 \text{ onwards}\}$. The variable D_5 thus captures the interest rate turbulence in November 1992, the interest rate shock and the transition to a floating exchange rate regime. D_6 captures a possible structural break due to the German reunification.

It will prove useful to change the notation slightly by letting superscript r denote the exchange rate regime. Define $D^0 \equiv 1, D^1 = D_{fix}, D^2 = D_{float}$, so that the exchange rate regime is captured by $D^r, r = 0, 1, 2$. The unrestricted general model can then be formulated in the following way:

$$\Delta_{i}q_{t} = \mu_{0} + \sum_{j=1}^{J} \sum_{r=0}^{2} \Delta_{i}x_{j}\mu_{j}^{r}D^{r} + \sum_{r=0}^{2} \mu_{J+1}^{r}D^{r}ECM_{t-i}$$

$$+ \sum_{k=1}^{K} \mu_{J+1+k}D_{k} + \sum_{l=1}^{L} \mu_{J+K+1+l}\Delta_{i}q_{t-l} + \varepsilon_{t}$$

$$(4.4)$$

where i = 1, 4 since I will estimate both quarterly and annual differences. Moreover, j = 1, ..., J for the explanatory variables, k = 1, ..., 6 for the additive devaluation dummies and l = 1, ..., L indicate the L lagged dependent variables added in order to remove any serial correlation in the residuals. The devaluations are thus allowed to shift the intercept of the dynamic models, while the regime shifts are assumed to shift the slope coefficients. In this fashion, I allow for the possibility that the impact of the explanatory variables is regime-specific.

3.3 Estimating the Impact of the ECM on the Three Components of the Real Exchange Rate

In order to assess the effect of the ECM on the three components of the real exchange rate, I use each component as the dependent variable while keeping the right hand side virtually intact. By this method three models are obtained. Maintaining the notation from the previous section I obtain

$$\Delta_{i}p_{t} = \lambda_{0} + \sum_{j=1}^{J} \sum_{r=0}^{2} \Delta_{i}x_{j}\lambda_{j}^{r}D^{r} + \sum_{r=0}^{2} \lambda_{J+1}^{r}D^{r}ECM_{t-i}$$
$$+ \sum_{k=1}^{K} \lambda_{J+1+k}D_{k} + \sum_{l=1}^{L} \lambda_{J+K+1+l}\Delta_{i}p_{t-l} + \varepsilon_{t}$$
(4.5)

$$\Delta_{i}p_{t}^{*} = \varphi_{0} + \sum_{j=1}^{J} \sum_{r=0}^{2} \Delta_{i}x_{j}\varphi_{j}^{r}D^{r} + \sum_{r=0}^{2} \varphi_{J+1}^{r}D^{r}ECM_{t-i}$$
$$+ \sum_{k=1}^{K} \varphi_{J+1+k}D_{k} + \sum_{l=1}^{L} \varphi_{J+K+1+l}\Delta_{i}p_{t-l}^{*} + \varepsilon_{t}$$
(4.6)

$$\Delta_{i}s_{t} = \psi_{0} + \sum_{j=1}^{J} \sum_{r=0}^{2} \Delta_{i}x_{j}\psi_{j}^{r}D^{r} + \sum_{r=0}^{2} \psi_{J+1}^{r}D^{r}ECM_{t-i} + \sum_{k=1}^{K} \psi_{J+1+k}D_{k} + \sum_{l=1}^{L} \psi_{J+K+1+l}\Delta_{i}s_{t-l} + \varepsilon_{t}$$

$$(4.7)$$

where the *ECM* in the above equations is implied by the long run path of the real exchange rate, i.e. derived from (4.3).⁹ In order to assess the impact of the *ECM* on the various components I am interested in testing linear restrictions on the parameters. For instance, in the case of the Swedish price level as modeled in (4.5), letting i = 4 implies that $\hat{\lambda}_{J+1}^0$ is the annual percentage change in the real exchange

⁹ Enders (1988) uses a similar approach in a study testing for PPP.

rate triggered by a one percent increase in the deviation from long run equilibrium under the Currency Basket. Testing the hypothesis $\hat{\lambda}_{J+1}^0 + \hat{\lambda}_{J+1}^1 = 0$ is equivalent to testing the significance of the *ECM* under a fixed regime. Similarly, the null of $\hat{\lambda}_{J+1}^0 + \hat{\lambda}_{J+1}^2 = 0$ is equivalent to testing the significance of the *ECM* under a floating regime. Alternatively, the null of $\hat{\lambda}_{J+1}^1 = 0$ does not imply testing if the *ECM* is insignificant under a fixed regime, merely that the elasticity is the same under the Currency Basket and under a fixed exchange rate.

4 Data

Data consist of quarterly observations for the period 1973:1-2001:4.¹⁰ The variables are defined as follows: s is the nominal exchange rate (SEK/D-mark),¹¹ p and p^* are the CPI:s for Sweden and Germany respectively, p_{oil} is the price of oil, R and R^* are the nominal short interest rates, $g - g^*$ is the log ratio between Swedish and German government consumption and U is the open Swedish unemployment rate measured in percent. BB is the Swedish budget balance as a share of GDP. Since data on sectorial TFP is notoriously hard to find, I use as proxies Swedish and German labour productivity as measured by the ratio between industrial production and overall employment. Due to lack of data I am forced to use quarterly data on industrial production while the only data available on employment is annual data.

4.1 Data Properties

The series were tested for the order of integration using Augmented Dickey-Fuller (ADF) tests, see Engle and Granger (1987).¹² I could not reject the null hypothesis of a unit root, i.e. the series being I(1), for any variables except the nominal interest rate differential $(R - R^*)$. The remaining series will therefore, in the following, be treated as containing a unit root. Swedish unemployment behaves like a stationary series up to mid 1990 and is quite possibly exposed to a structural break in the early 1990s. It was, however, not possible to interpret the results of a Perron test for a structural break due to heavy auto correlation. The unemployment series will

¹⁰ All series are from the *International Financial Statistics* (IFS), IMF, except for data on employment obtained from *OECD Economic Outlook 68*, OECD and data on unemployment which was retrieved from *OECD Main Economic Indicators* (MEI), OECD.

¹¹ After 1999 I create an artifical D-mark/SEK exchange rate where the D-mark/EURO exchange rate is fixed at the conversion rate of 1.96.

¹² Test results for the order of integration are available on request.

therefore also be treated as potentially I(1).¹³ Graphs of Swedish and German productivity and the log real exchange rate are given in *Figure 1*. Indeed, the series tend to move together, and it seems that there is in fact a cointegrating relationship between them.

5 Results

First the long-run relationship is estimated by using the Engle-Granger one-step method as described by (4.3). The models are then reduced by Likelihood Ratio (LR) tests of linear restrictions and subject to testing until a preferred specification is obtained.

5.1 The Long-Run Relationship

The results obtained when estimating model (4.3) are displayed in column (1) in Table 4.1. All parameters have expected signs. Log relative government consumption is significant with a positive coefficient and lends some support to the hypothesis that an increase in $(g - g^*)$ gives rise to inflation expectations in Sweden, causing a nominal depreciation. Since I am estimating quarterly growth rates, the short horizon makes the nominal exchange rate a more probable channel than relative prices.

The long-run relationship is obtained by reducing the model by LR-tests of linear restrictions. The result is displayed in column (2) in Table 4.1. The residuals are clearly stationary with a t-value from the ADF-test of -4.34 (the critical value being equal to -2.89 at the five percent level),¹⁴ suggesting that the variables are cointegrated with a long run relationship equal to¹⁵

$$q_t = -.015a_t + .034a_t^* \tag{4.8}$$

Since all variables, except for $(R - R^*)$, U and BB, are expressed in logs, the coefficients of the model in Table 4.1 are elasticities.

In order to formally test for cointegration, I re-run the model in column (2),

¹³ Note that I could not reject the hypothesis that the real exchange rate is I(1) and hence that Swedish post-Bretton Woods data are inconsistent with the PPP-hypothesis.

¹⁴ Critical values in testing for stationary residuals are taken from MacKinnon (1991).

¹⁵ With greater precision, the estimated cointegrating vector is (-.04219, -.00065, .001435), and hence the normalised cointegrating vector is (1, .015431, -.03402).

using the obtained ECM as an explanatory variable. To test the significance of the ECM is then a test for cointegration as long as the conditioning variables are weakly exogenous to the cointegration parameters; see Banerjee et al (1993) and Kremers et al (1992). The results are shown in column (3) in Table 4.1. The ECM is highly significant with a p-value of .018, and I reject the null of no cointegration between the real exchange rate and Swedish and German productivity with long-run elasticities implied by (4.8). The long run path given by (4.8) is plotted against the actual log real exchange rate in Figure 4.2. The graph shows how the German reunification provided some space for a real depreciation of the Swedish real exchange rate.

5.2 Short-Run Dynamics

Throughout the analysis both quarterly differences using adjusted series, and annual differences using unadjusted series and seasonal dummies were used.¹⁶ Estimating quarterly differences implies that there may be a lot of short-run noise in the data that suppresses genuine relationships. This is confirmed by the empirical results. The goodness of fit is generally much higher when estimating annual rather than quarterly differences and the fitted values correspond better to the actual values. In what follows I therefore focus on annual difference estimation using unadjusted series and seasonal dummies in the main text.¹⁷

Estimation of the general unrestricted model (4.4) renders many variables insignificant. The model is therefore reduced using Lagrange Multiplier tests for linear restrictions on single variables and groups of variables, until an interpretable model of significant variables is obtained. Table 4.2 presents reduced versions of model (4.4) using annual differences, i = 4.

The model in column (1) shows that the dynamics of the real exchange rate is determined by the feedback from previous periods as captured by the autoregressive (AR) components, the change in the budget balance, and the interest rate differential. A positive interest rate differential causes a real appreciation of the Swedish real exchange rate under the currency basket and under a floating exchange rate. On conventional levels of significance, the model indicates that impact of the interest rate differential on the real exchange rate is the same under the currency

¹⁶ When estimating quarterly differences, series showing seasonal patterns (unemployment and government consumption) were seasonally adjusted using the X11-filter.

¹⁷ Tables of quarterly differences are available on request.

basket and when the exchange rate is fixed. However, on the rather low 12 percent level of significance the results suggest that the real exchange rate is unaffected by the interest rate differential under a fixed exchange rate regime. This is consistent with the hypothesis that the interest rate affects the real exchange rate through the nominal exchange rate and thus only matters under a floating regime.

Note that while log relative government consumption played a certain role when estimating quarterly differences in Table 4.1, it seems that the budget balance matters for annual differences in the real exchange rate. Recall the theoretical prediction that changes in log relative consumption might cause inflation expectations and therefore a nominal depreciation, while an improvement in the budget balance causes deflation and hence a real depreciation. Once again it seems that while the nominal exchange rate is the mechanism through which the real exchange rate is affected when estimating quarterly differences, the Swedish price level accounts for real exchange rate movements on an annual basis.

The error correction term has a significant negative effect on real exchange rate growth. A positive deviation from equilibrium by 1 percent causes a real appreciation of .051 percent under the currency basket and a fixed exchange rate, and an additional .142 percent under a floating exchange rate. In terms of long-term adjustment it means that under the currency basket it takes about 4.9 years for 90 percent of the deviation to adjust (half-life 1.5 years), while it takes only 1.3 years under a floating regime (half-life .4 years). Hence it seems that the real exchange rate responds more quickly to deviations from long-run equilibrium under a floating exchange rate regime. Removing the interaction term in column (2) renders both interest rate terms insignificant, which is rather re-assuring considering that the interest rate differential should be unable to affect the nominal exchange rate under a fixed exchange rate regime. Despite the low significance of the interaction term, I conclude that the model in column (1) is a good candidate for a preferred specification and I examine its properties in more detail. The fitted values, displayed in Figure 4.3, correspond rather well to the actual values. The graph shows that the largest deviation between actual and fitted values occurs around 1983. Incidentally, removing the dummy for the large devaluation 1982:3, i.e. D_4 in column (3) renders the interest rate term significant on the 10 percent level. Moreover, plotting recursive parameters indicates that the estimated coefficients are rather stable, converge as the sample size is increased and that none of the parameters change their signs.

Note that I initially allowed for regime-specific intercepts in all estimates, but they did not matter in any of the estimations. In addition some sensitivity analysis was made by estimating over 1973:1-1998:4 only, i.e. by suppressing the observations for which Germany is a member of the EMU. The main results were unaffected by this exercise.

To sum up, it seems that the regime indeed matters for the dynamics of the real exchange rate. More specifically, deviations from long-run equilibrium are corrected more quickly when the nominal exchange rate is allowed to float freely. This is a key result.¹⁸

5.3 The Component-specific Impact of the ECM

Table 4.3 presents unrestricted and restricted versions of models (4.5)-(4.7) using annual differences. According to Model 1 in Table 4.3, a positive deviation from long-run equilibrium causes inflation in Sweden under a fixed exchange rate, which is consistent with theory. Under a floating exchange rate however, the ECM does not affect prices (the hypothesis that the coefficients cancel out is not rejected at the 5 percent level). Other determinants of Swedish inflation include the oil price and log relative government consumption.

Using the German price level as the regressand as suggested in (4.6) yields Model 2. As for the interpretation, my main concern is the impact of the ECM under different regimes. According to theory, a positive deviation from long-run equilibrium should cause deflation in Germany, and thereby a real appreciation. The results in Table 4.3 indicate that this holds true regardless of regime.

However, when comparing the reaction of the Swedish and the German price level respectively the estimates suggest that the effect on the Swedish price level is almost four times as large (.031 compared to .008). It therefore seems that the Swedish price level responds much more forcefully to short-run deviations from longrun equilibrium than the German price level, which indicates that Sweden, being a small country, has to adapt to German conditions rather than the other way around. ¹⁹

¹⁸ The results show that the nominal exchange rate indeed reacts to deviations from a long run relationship between economic fundamentals. However, these deviations may be caused by productivity shocks or shocks to any of the components of the real exchange rate,. Since I do not attempt to test whether the nominal exchange rate reacts also to "irrelevant" shocks or whether it tends to overshoot, I cannot conclude from the analysis that the nominal exchange rate acts as a shock absorber.

¹⁹ The result is intuitively appealing, and one could, for instance, think of Sweden as being more sensitive to deviations from equilibrium than Germany since the impact on net exports from Sweden to Germany (net imports to Germany from Sweden) constitute a larger fraction of Swedish than German GDP, since Sweden is a smaller economy.

Finally, the nominal exchange rate in Model 3 is affected by the change in the interest rate differential under the currency basket and a floating regime, but not under a fixed exchange rate regime. This is completely in line with what one would expect. Moreover, under a floating rate, higher growth in log relative government consumption causes a nominal depreciation, probably due to inflation expectations. A positive deviation from long-run equilibrium triggers a nominal appreciation under the currency basket and a floating regime but not under a fixed exchange rate regime (the hypothesis that the coefficients cancel out is not rejected at the 5 percent level).²⁰ This result suggests that the nominal exchange rate indeed helps correct deviations from equilibrium when it is not constrained by a fixed exchange rate regime.

I conclude that all three components have contributed to the adjustment of the real exchange rate to its long-run path when the regime has allowed for such adjustments.

6 Concluding Remarks

This paper provides empirical evidence on the long-run behaviour and short-run dynamics of the Swedish real exchange rate relative to Germany during the post-Bretton Woods period. The results show that there is cointegration between the real exchange rate and Swedish and German labour productivity, supporting the Balassa-Samuelson hypothesis.

I conclude that the most important short-run explanatory variables are the interest rate differential, the feedback from previous periods and the deviation from long-run equilibrium. The regime has mattered for the dynamics of the real exchange rate. It seems that disturbances are adjusted more quickly when the exchange rate has been allowed to float freely.

Finally, the results provide some evidence that all three components have contributed to correcting disturbances, but that the Swedish price level and the nominal exchange rate have responded more forcefully than the German price level to deviations from long-run equilibrium. Therefore, it appears that Sweden, being a smaller country than Germany, is forced to adapt to German conditions and respond to disturbances rather than the other way around.

 $^{^{20}}$ Note that since we are looking at the bilateral Swedish-German exchange rate in isolation, there is theoretically no reason to expect the adjustment in the nominal exchange rate to differ under the currency basket and under a free float.

Bibliography

- Alexius, A. and Nilsson, J. (2000), "Real Exchange Rates and Fundamentals: Evidence from 15 OECD Countries", Open Economies Review, Vol. 11, pp.383-397
- [2] Alexius, A. (2001), "Sources of Real Exchange Rate Fluctuations in the Nordic Countries", Scandinavian Journal of Economics, Vol. 103, pp. 317-331
- [3] Artis, M.J. and Ehrmann, M. (2000), "The Exchange Rate as a Shock-absorber or Source of Shocks? A Study of Four Open Economies", CEPR Discussion Paper Series, No. 2550
- [4] Balassa, B. (1964), "The Purchasing Power Doctrine: A Reappraisal", The Journal of Political Economy, Vol. 72, pp. 584-596
- [5] Banerjee, A., Dolado, J., Galbraith, J.W., and Hendry, D.F (1993), Cointegration, Error correction, and the Econometric Analysis of Non-stationary Data, Oxford University Press, Oxford,
- [6] Bergvall, A. (2002), "What Determines Real Exchange Rates? The Nordic Countries", in *Essays on Exchange rates and Macroeconomic Stability*, Ph.D. Dissertation, Economic Studies 65, Uppsala University
- [7] Chinn, M.D (1997), "Sectoral Productivity, Government Spending and Real Exchange Rates: Empirical Evidence for OECD Countries", NBER Working Paper No. 6017
- [8] De Gregorio, J. and Wolf, H.C. (1994), "Terms of Trade, Productivity, and the Real Exchange Rate", NBER Working Paper No. 4807
- [9] Dornbusch, R. (1976), "Expectations and Exchange Rate Dynamics", The Journal of Political Economy, Vol.84, pp. 1161-1175

- [10] Enders, W. (1988), "ARIMA and Cointegration Tests of PPP under Fixed and Flexible Exchange Rate Regimes", *Review of Economics and Statistics*, Vol. 70, pp. 504-508
- [11] Engle, R.F and Granger, C.W.J. (1987), "Co-integration and Error Correction: Representation, Estimation and Testing", *Econometrica*, Vol. 55, pp. 251-276
- [12] Engle, R.F and Granger, C.W.J (1991) (Eds.), Long Run Economic Relationships, Oxford University Press, Oxford
- [13] Froot, K.A. and Rogoff, K. (1995), "Perspectives on PPP and Long-Run Real Exchange Rates", Chapter 32 in Grossman, G.M and Rogoff, K. (Eds.), Handbook of International Economics, Vol III, Elsevier Science B.V, Amsterdam
- [14] Kremers, J.J.M, Ericsson, N.R and Dolado, J.J. (1992), "The Power of Cointegration Tests", Oxford Bulletin of Economics and Statistics, Vol. 54, pp. 325-348
- [15] MacKinnon, J. (1991), "Critical values for Cointegration Tests", in Engle, R.F and Granger, C.W.J (Eds.) Long Run Economic Relationships, Oxford University Press, Oxford
- [16] Mussa, M. (1986), "Nominal Exchange Rate Regimes and the Behavior of Real Exchange Rates: Evidence and Implications", *Carnegie-Rochester Series on Public Policy* Vol. 25, pp. 117-214
- [17] Samuelson, P. A. (1964), "Theoretical Notes on Trade Problems", Review of Economics and Statistics, Vol. 46, pp. 145-154
- [18] Taylor, Alan M. (2002), "A Century of purchasing-power Parity", Review of Economics and Statistics, Vol. 84, pp. 139-150
- [19] Thomas, A. (1997), "Is the Exchange Rate a Shock Absorber? The Case of Sweden", IMF Working Paper 97/176

Appendix

Column	(1)	(2)	(3)
Dependent variable	Δq	Δq	Δq
Intercept	.111		•
	(.068)	•	•
$\Delta \left(R-R^{\ast }\right)$	003**	003**	003**
	(.001)	(.001)	(.001)
ΔU	008	•	
	(.016)	•	•
$\Delta BDef$	002	•	
	(.003)	•	•
$\Delta \left(g - g^* \right)$.288	.338*	.338*
	(.193)	(.190)	(.184)
Δp_{oil}	.016		•
	.021	•	•
q_{-1}	118**	042**	•
	(.049)	(.022)	•
a_{-1}	000	001	•
	(.001)	(.001)	•
a_{-1}^{*}	.001	.001**	•
	(.001)	(.001)	•
ECM_{-1}	•	•	042**
	•	•	(.018)
AR-components	0	0	0
Additive Dummies	No	No	No
Seasonal Dummies	No	No	No
R^2_{adj}	.071	.073	.090
N	115	115	115
Durbin-Watson	1.729	1.843	1.843

Table 4.1: Engle-Granger one-step estimation of model (3) by OLS, quarterly differences 1973:1-2001:4

Standard Errors in parenthesis

Significance codes: ***=1%, **=5%, *=10%

(1)	(2)	(3)
Λ , α	$\Delta \cdot a$	$\Delta_4 q$
$\Delta_4 q$	$\Delta_4 q$	$\Delta_4 q$
004*	•	004**
(.002)		(.002)
.004		.005*
(.003)		(.003)
.005**	.004**	.004**
(.002)		(.002)
· · · ·		(/
(/	· /	· · · ·
(.062)	(.001)	(.062)
3	3	3
Yes	Yes	Yes a
Yes	No	Yes
.728	.731	.728
107	107	107
4.939	2.622	3.046
	$\Delta_4 q$ 004* (.002) .004 (.003) .005** (.002)051* (.030)142** (.062) 3 Yes Yes .728 107	$\begin{array}{cccc} \Delta_4 q & \Delta_4 q \\ \hline004^* & . \\ (.002) & . \\ .004 & . \\ (.003) & . \\ .005^{**} & .004^{**} \\ (.002) & (.002) \\051^* &049^* \\ (.030) & (.030) \\142^{**} &158^{***} \\ (.062) & (.061) \\ \hline \\ $

Table 4.2: Estimation of dynamic real exchange rate models by OLS, annual differences, 1973:1-2001:4

Notes as in Table 4.1

^{*a*} Excluding D_4

Table 4.3: Estimation of dynamic models of the three components by OLS, annual differences, 1973:1-2001:4

Model 1	Dependent variable	$\Delta_4 p$		
	$\Delta_4 p_{oil}$.010*** (.003)		
	$\Delta_4 \left(g - g^* \right)$	(.003) .195*** (.047)		
	$\Delta_4 \left(g - g^* \right) \cdot D_{fix}$	(.047) 322^{***} (.080)		
	$\Delta_4 \left(R - R^* \right)$.001** (.000)		
	$ECM_{-4} \cdot D_{fix}$	(.000) $.031^{***}$ (.009)	$\begin{array}{c} \text{AR components} \\ N \end{array}$	1 111
	$ECM_{-4} \cdot D_{float}$	(.000) 036^{***} (.010)	R_{adj}^2 Breusch-Pagan LM-statistic, nR^2	.940 .065
Model 2	Dependent variable	$\Delta_4 p^*$		
	$\Delta_4 p_{oil}$.006***	$\begin{array}{l} {\rm AR \ components} \\ N \end{array}$	$\frac{1}{113}$
	ECM_{-4}	(.002) 008*** (.003)	R^2_{adj} Breusch-Pagan LM-statistic, nR^2	.909 4.606
Model 3	Dependent variable	$\Delta_4 s$		
	$\Delta_4 \left(g - g^* \right) \cdot D_{float}$	1.056^{**} (.508)		
	$\Delta_4 \left(R - R^* \right)$	003* (.002)		
	$\Delta_4 \left(R - R^* \right) \cdot D_{fix}$.002) .004* (.003)		
	ECM_{-4}	(.000) 136^{***} (.043)	$\begin{array}{c} \text{AR components} \\ N \end{array}$	$\frac{3}{107}$
	$ECM_{-4} \cdot D_{fix}$.152*** (.052)	R_{adj}^2 Breusch-Pagan LM-statistic, nR^2	.692 .368

Model 1 D ۸ 1 . .

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Notes as in Tables 4.1 and 4.2

Significant seasonal and additive dummies included

Figure 4.1: The log real exchange rate (LQ), Swedish labour productivity (PRODSV) and German labour productivity (PRODTY), 1973:1-2001:4.

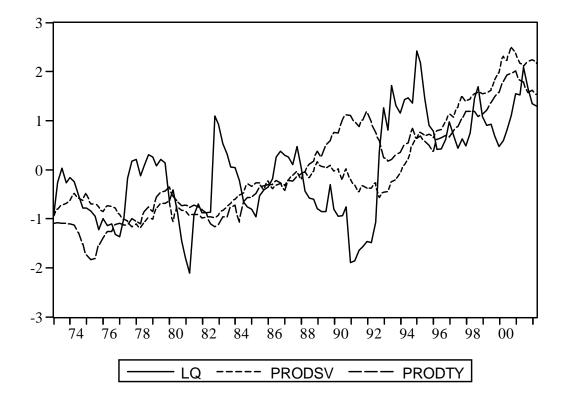


Figure 4.2: The log real exchange rate (LQ) and the estimated long-run path (LR path), 1973:1-2001:4

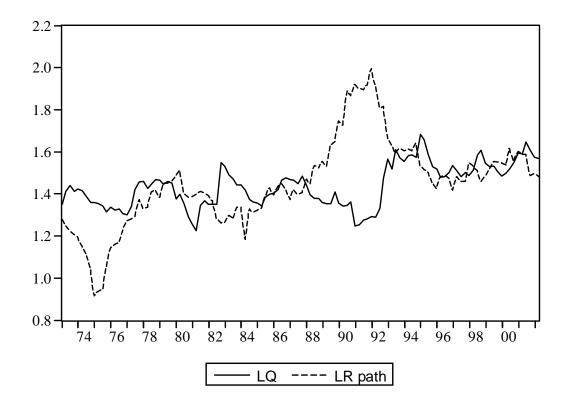


Figure 4.3: Actual and Fitted values for the model in Table 4.1, Column (2)

