Assignment Reversals:

Trade, Skill Allocation and Wage Inequality*

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Abstract

Understanding the allocation of skilled labor across industries is necessary to explain inter-industry wage differences and the effect of trade on wages. This paper develops an assignment model with both labor and non-labor inputs in which the assignment of heterogeneous labor across sectors is driven by variation in non-labor input productivity. A scale of operations effect causes high skill agents to work in high input productivity sectors where they can best leverage their talent. Growth in input productivity raises wage inequality by increasing the scope for leverage. If the ranking of sectors by input productivity differs across countries, their ranking by workforce skill and average wage also differs – this is an assignment reversal. In a two sector, two country model the existence of an assignment reversal implies that each country has a comparative advantage in its high skill sector. Consequently, trade integration causes both the relative wage of high skill workers, and wage inequality within the high skill sector, to increase in both countries. Evidence from industry wage data supports the existence of assignment reversals and shows that exogenous differences in capital productivity induced by a country's relative proximity to major capital exporters cause inter-industry wage variation consistent with the model's predictions.

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1 Introduction

What determines the allocation of skill across sectors? Answering this question is crucial for understanding the determinants of inter-industry wage differences[1] and how trade integration affects wages and inequality.[2] Existing theoretical work on the allocation of skill generally imposes functional form assumptions on the production technology that ensure the existence of an invariant ranking of sectors by workforce skill. In the Heckscher-Ohlin model this approach is embodied in the no factor intensity reversals assumption, while in comparative advantage based assignment models it follows from assuming that sectors can be ordered in such a way that output is strictly log-supermodular in labor skill and a sector index.[3]

However, industry wage data suggests that the ranking of sectors by workforce skill varies systematically both across countries and over time. Let the "wage rank correlation" be the rank correlation between industry wages in a country and industry wages in the US. Figure 1 shows wage rank correlations plotted against income levels (expressed as log differences from US income).[4] Although the wage rank correlation is always positive, it is strongly increasing in income. While industrialized countries have similar industry wage structures to the US, inter-industry wage differences in low income countries vary substantially from those in the US.[5]

The industry wage structure also varies over time within countries. Suppose US manufacturing industries are divided into quartiles based on growth in capital productivity between 1960 and 2000, where capital productivity growth is measured by the rate of decline in the quality-adjusted price of equipment used by an industry.[6] For each quartile, Figure 2 shows the log difference between the average wage of industries in the quartile and the average wage of all industries both in 1960 and in 2000. Industries in which capital productivity grew faster also experienced higher wage growth. The average wage of industries in the top quartile of capital productivity growth (quartile 4) increased by 6.5% relative to the mean manufacturing

[1] Abowd, Kramarz and Margolis (1999) show that worker fixed effects account for 90% of inter-industry wage differences in France, implying that inter-industry wage differences are primarily due to cross-industry variation in the composition of the labor force.

[2] For example, the classic Stolper-Samuelson theorem predicting the effect of trade integration on the skill premium relies on the assumption that the ranking of sectors by skill intensity is constant across countries.

[3] Comparative advantage based assignment models in which this assumption governs the equilibrium labor assignment include Sattinger (1975); Ohnsorge and Trefler (2007); Costinot and Vogel (2009), and; Acemoglu and Autor (2010).

[4] The wage data is taken from the UNIDO Industrial Statistics database and covers 42 countries and 127 ISIC 4 digit manufacturing industries in 2000. See Appendix C for a complete description of the data.

[5] An obvious concern is that this relationship is driven by measurement error being greater in low income countries. However, Section 6 shows that the same pattern is observed in the EU KLEMS data set which is specifically designed to provide high quality industry level data for growth accounting.

[6] See Section 5 for a complete description of the data.

wage between 1960 and 2000.

Inter-industry wage differences are mostly explained by variation in workforce composition. Krueger and Summers (1986) find that observable worker characteristics explain only around half of inter-industry wage differences in the US, but using a richer data set with matched employer-employee data Abowd, Kramarz and Margolis (1999) estimate that worker fixed effects account for 90% of inter-industry wage differences in France. Under the assumption that inter-industry wage differences are primarily due to variation in workforce skill, Figures 1 and 2 imply that the existing assignment literature overlooks an important phenomenon: assignment reversals. I define an assignment reversal to exist whenever the ranking of sectors by workforce skill differs either over time or across countries. Assignment reversals are analogous to skill intensity reversals in the Heckscher-Ohlin model.[7] However, Heckscher-Ohlin skill intensity reversals are driven by variation in the skill premium and in Section 6 I find no evidence indicating that the assignment reversals seen in Figure 1 are explained by differences in skill premia.

This paper develops and tests an assignment model of the labor market that can be used to study the causes and consequences of assignment reversals in both closed and open economies.[8] To build a tractable model of assignment reversals the paper introduces two new features to the assignment literature. First, it marries Roy (1951) to Becker (1973) by including both multiple sectors and matching between two factors of production with non-zero opportunity costs: heterogenous labor and an homogenous intermediate input.[9] Second, it explains the equilibrium labor allocation in terms of an observable sector level characteristic: intermediate input productivity.

Consider an economy with a continuum of agents, who differ along a single dimension of heterogeneity called skill and sort across a finite number of sectors. In comparative advantage based assignment models the production technology is assumed to take the Ricardian form:

$$y(\theta, k) = A_k g(\theta) F(\theta, k),$$

[7] See Minhas (1962) and Leontief (1964) for analysis of the conditions under which factor intensity reversals may occur and a debate over their existence. The extensive literature on factor intensity reversals tends to conclude that capital intensity reversals are of limited empirical relevance, but has largely overlooked skill intensity reversals.

[8] Murphy, Shleifer and Vishny (1991) discuss the possibility of cross-country assignment reversals in the allocation of talent between rent seeking and entrepreneurial activities.

[9] Sattinger (1979) considers the problem of matching heterogenous workers to machines of different quality when all worker-machine pairs produce the same output good and machines are in fixed supply. However, in existing models with multiple sectors either the production technology is Ricardian as in the comparative advantage based models discussed below or production combines different types of labor in fixed quantities (Grossman and Maggi 2000; Grossman 2004).

where y is the output of a skill θ agent working in sector k and A_k is a Hicks-neutral productivity term. Provided F is log-supermodular there is positive assortative matching of high skill agents to high k sectors. I extend this framework by assuming that production requires both labor and an intermediate input, which can be interpreted as machines, capital or materials. In particular, a production team consisting of one agent working with a quantity x of intermediate input in sector k produces output:

$$y(\theta, k) = A_k g(\theta) F(\theta, Q_k x),$$

where Q_k denotes intermediate input productivity in sector k, g is strictly increasing in θ and F exhibits constant returns to scale. Variation in Q induces changes in the cost per efficiency unit of intermediate input and is equivalent to variation in the intermediate input price. The restriction on g implies the existence of increasing returns to skill. Importantly, the quantity of intermediate input used by each agent is endogenous and is chosen to maximize profits under perfect competition.

In equilibrium, log-submodularity of the production function implies positive assortative matching between agent skill and sector intermediate input productivity. This reverses the condition on F required for positive assortative matching in comparative advantage based assignment models. The switch is a consequence of allowing the quantity of intermediate input used to be adjustable on the intensive margin. This adjustability enables higher skill agents to leverage their ability by working with larger quantities of intermediate input and when the production function is log-submodular there is sufficient substitutability between skill and intermediate inputs that the efficient allocation is for highly leveraged agents to work in sectors where the cost per efficiency unit of intermediate input is low.[10] This is an example of the scale of operations effect discussed in Sattinger (1993). If, instead, each agent must work with the same quantity of intermediate input, substitutability mandates that high skill agents work with low productivity intermediate inputs and log-submodularity of F implies negative assortative matching.[11]

Assignment reversals occur whenever the ranking of sectors by intermediate input productivity varies either over time or across countries. During the past decade assignment models have been used to study the determinants of comparative advantage and the impact of globalization on labor markets in economies

[10] I prove in Section 2 that a constant returns to scale function is strictly log-submodular if and only if the elasticity of substitution between factors exceeds unity.

[11] Similarly, if the production function is strictly log-supermodular the equilibrium assignment exhibits positive assortative matching if the intermediate input quantity is fixed and negative assortative matching if it is endogenous.

with multiple sectors.[12] However, this literature does not consider the possibility of assignment reversals. To understand the implications of assignment reversals for trade theory I develop a two sector, two country, general equilibrium version of the model. When the ranking of sectors by intermediate input productivity differs across countries: (i) both countries have a comparative advantage in their high skill, high wage sector; (ii) trade liberalization causes the high skill sector to expand in both countries and in the free trade equilibrium both countries export the output of their high skill sector, and; (iii) in both countries trade liberalization causes wage levels and wage inequality to increase in the high skill sector and decrease in the low skill sector. Therefore, assignment reversals can overturn the Stolper-Samuelson prediction and this offers a new mechanism to explain why trade liberalization episodes have been associated with increases in wage inequality in many unskilled labor abundant developing countries.[13]

The model also has important implications for the distribution of wages:

1. Labor's share of output is decreasing in worker skill and, therefore, in wages – a correlation that is observed empirically.

2. At any given skill level, the returns to skill (the elasticity of wages with respect to skill) are higher in sectors with greater intermediate input productivity. Moreover, holding workforce composition constant, an increase in the returns to skill implies higher wage inequality.[14]

3. Technological progress that increases intermediate input productivity is complementary to skills in two distinct senses. First, positive shocks to any sector's intermediate input productivity increase the skill level of agents assigned to that sector. Second, in a two sector general equilibrium model an increase in either sector's intermediate input productivity raises the payoff to agents in both sectors from leveraging their skills. Consequently, technological progress disproportionately benefits more highly leveraged agents and raises the returns to skill in both sectors. Since changes in intermediate input productivity are formally equivalent to variation in the intermediate input price, reductions in intermediate input trade costs will have the same effects as technological progress.[15]

By linking the ranking of sectors by workforce skill to their ranking by intermediate input productivity

[12] See, for example, Grossman and Maggi (2000); Ohnsorge and Trefler (2007); Costinot (2009), and; Costinot and Vogel (2009). A closely related literature uses assignment models of firm hierarchies to analyze the consequences of globalization in economies with a single output good (Antràs, Garicano and Rossi-Hansberg 2006; Burstein and Monge-Naranjo 2009).

[13] Of course, many other mechanisms have been suggested. See, for example, Davis (1996); Feenstra and Hanson (1996); Manasse and Turrini (2001); Yeaple (2005); Matsuyama (2007); Verhoogen (2008), and; Helpman, Itskhoki and Redding (2010). Goldberg and Pavcnik (2007) summarize empirical evidence on the relationship between trade liberalization and wage inequality.

[14] Gibbons et al. (2005) estimate that the returns to skill are higher in occupations which employ more skilled workers.

[15] Csillag and Koren (2009) and Parro (2010) provide evidence that capital imports increase the relative wage of high skilled labor.

the paper generates a concrete, empirically testable prediction relating the equilibrium assignment to parameters of the model. By contrast, the literature using a single heterogenous factor of production assumes that labor productivity is log-supermodular in skill and some variable that indexes sectors in order to guarantee comparative advantage based assignment of workers to sectors, but does not attempt to unpack the source of this log-supermodularity in terms of observable sector characteristics.[16] To test the model's empirical relevance I interpret the intermediate input as capital and analyze whether industry specific variation in the cost of capital investment over time or across countries explains differences in the inter-industry wage structure. Under this interpretation the paper offers an explanation for the existence of long-run capital-skill complementarity at the industry level.[17]

The empirical work uses two industry wage data sets: time series data for the US from the NBER manufacturing database and cross-country data for 2000 from UNIDO's Industrial Statistics database. I develop estimation strategies based on isolating exogenous variation in the cost of capital. In particular, I exploit two observations. First, the distribution of investment across equipment types varies by industry. Second, the prices of different types of equipment are plausibly exogenous to individual industries. For US manufacturing, the paper finds that wage growth between 1960 and 2000 was higher in industries which invest intensively in equipment types that experienced lower price growth (Figure 2 depicts this relationship). Looking across countries, I use geographic proximity to the major exporters of an equipment type as a proxy for low cost access to that equipment type and show that wages are higher in industries that invest intensively in types of equipment for which a country is geographically close to the main sources of export supply. This finding suggests that equipment trade plays an under appreciated role in shaping the inter-industry wage structure.

The results from both data sets are consistent with the assignment reversals model. Although more detailed within industry data on labor force composition and the assignment of workers to tasks is needed to fully disentangle how variation in capital productivity affects different types of workers, the empirical work provides evidence both that higher capital productivity increases average workforce skill and that assignment reversals are sufficiently common over time and across countries to recommend the study of their causes and consequences. This paper presents the first assignment model to tackle this challenge.

[16] See, for example, Sattinger (1975); Ohnsorge and Trefler (2007); Costinot (2009); Costinot and Vogel (2009), and; Acemoglu and Autor 2010).

[17] See Krusell et al. (2000) for evidence of capital-skill complementarity at the aggregate level in the US. Short-run capital-skill complementarity at the industry level can be rationalized by a specific factors model (Amano 1977).

The remainder of the paper is organized as follows. Section 2 develops and solves the assignment problem in partial equilibrium. Section 3 embeds a two sector version of the assignment problem in general equilibrium and solves the general equilibrium model for a closed economy. Section 4 extends the model to a two country open economy setting and compares the free trade and autarky equilibria. Section 5 studies the effect of the cost of capital on industry wages in US manufacturing, while Section 6 analyzes differences in the inter-industry wage structure across countries. Finally, Section 7 concludes.

2 Assignment problem

At the heart of this paper is an assignment problem. What explains the assignment of an heterogenous factor across alternative productive activities? To clarify the mechanism that drives the equilibrium assignment this section considers the assignment problem in partial equilibrium and discusses how it differs from assignment problems found in the previous literature.

Partial equilibrium model

Consider an economy facing the following two-sided matching problem. There exists an heterogenous factor that differs along a single dimension of heterogeneity indexed by θ. To be concrete, suppose the factor is labor and there are a continuum of agents with differing skill levels θ. Let $M(\theta)$ be the mass of agents with skill less than or equal to θ and suppose M has support on $\left(0, \overline{\theta}\right]$. Bounded support is the only restriction on the skill distribution M required to obtain the main results of the paper.

The economy contains K alternative productive activities in which labor can be employed. The appropriate interpretation of these productive activities depends on how the partial equilibrium assignment problem is embedded in general equilibrium. For consistency with the general equilibrium model in Section 3 I will refer to the productive activities as sectors, but they could also be tasks or occupations. Each sector produces a different output and there is variation across sectors in the production technology. Let Q_k denote the level of technology in sector k and suppose the sectors are ordered by ascending Q_k such that sector one is the least technologically advanced and sector K the most. The assignment problem is to characterize the mapping of agents to sectors.

To solve the assignment problem we need to know something about the production technology of each sector. Suppose that in all sectors output is produced by production teams, each of which consists of one

agent working with an intermediate input.[18] In particular, let the output of a skill θ agent working with x units of intermediate input in sector k, $y_k(\theta, x)$, be given by:

$$y_k(\theta, x) = g(\theta)F(\theta, Q_k x), \tag{1}$$

where g is non-negative, differentiable and strictly increasing and F is a twice differentiable, constant returns to scale function that is strictly increasing in both its arguments, strictly concave and satisfies $\lim_{\theta \to 0} \frac{\partial F}{\partial \theta} = \lim_{x \to 0} \frac{\partial F}{\partial x} = \infty$. This specification omits the Hicks-neutral productivity term A_k used in the introduction because, as shown below when discussing alternative sources of cross-sector heterogeneity, the level of A_k does not affect the equilibrium assignment. Within a sector all production teams produce an homogeneous output. Four features of the production function are particularly noteworthy. First, the labor input to production is indivisible. If, instead, agents with different skill levels were perfect substitutes within production teams, θ would simply measure an agent's efficiency units of labor and there would be no assignment problem. Second, skill enters production symmetrically in every sector. Holding $Q_k x$ fixed, the marginal effect of skill on output is constant across sectors.[19] Third, g captures the existence of increasing returns to ability. Fourth, the level of technology Q_k enters as an intermediate input augmenting productivity term. This is the only source of cross-sector heterogeneity. Note that $Q_k x$ measures the amount of intermediate input used in efficiency units.

Assume that there is perfect competition in all markets, that all sectors must produce positive aggregate output and that the intermediate input is in perfectly elastic supply at cost p. Provided x is a choice variable there is no loss of generality in assuming all sectors use the same intermediate input since allowing for variation in input cost across sectors is equivalent to varying Q_k. We can now proceed to solve the assignment problem in partial equilibrium. This is a partial equilibrium problem not because output prices are exogenous – they are not, but because the sources of intermediate input supply and output demand remain unspecified. The general equilibrium model in Section 3 shows that endogenizing intermediate input supply and explicitly specifying the nature of demand does not change the partial equilibrium assignment patterns.

Formally, the production function in (1) is similar to that used by Rosen (1982) in a single sector model of firm hierarchies. In theory, the intermediate input could represent materials, machines or an homogenous

[18] The model does not speak to where the boundaries of the firm may lie, so I will refer to the basic unit of activity as a production team.

[19] The implications of relaxing this assumption by allowing for cross-sector variation in a labor augmenting productivity term are discussed below.

labor input, but in the empirical section of the paper I will interpret the intermediate input as capital. Cross-sector technology heterogeneity then represents variation in the productivity, or equivalently the cost per efficiency unit, of capital across sectors. Such variation could result from sector specific capital augmenting technology investments or from sector specific differences in the price of capital. Following Rosen (1982) the form of the production function can be motivated by assuming that workers monitor machines and that a skill θ agent is endowed with θ units of monitoring time and produces output of quality $g(\theta)$, where quality and quantity of output are perfect substitutes. In this set-up $Q_k x$ denotes the number of machines monitored, expressed in efficiency units, and diminishing returns to capital result from spreading a fixed endowment of monitoring time over an increasing number of machines. However, the fact that higher skill agents produce higher quality output means there are increasing returns to skill.

Equilibrium assignment

The equilibrium assignment pattern depends crucially on whether or not the quantity of intermediate input used x is endogenous. Let us suppose that each agent can choose the optimal amount of input to work with. Since there is perfect competition each agent's income will equal the profit of her production team $\pi_k y_k(\theta, x) - px$, where π_k is the price of sector k output. Remembering that F has constant returns to scale we can define $f\left[s_k(\theta)\right] = \frac{1}{\theta} F(\theta, Q_k x)$, where $s_k(\theta) \equiv \frac{Q_k x}{\theta}$ is the span of control of a skill θ agent working in sector k. Note that f is strictly increasing and strictly concave. $s_k(\theta)$ measures the efficiency units of intermediate input used per unit of skill and captures the extent to which an agent can leverage her ability by working with large amounts of the intermediate input. Following this change of variables, profit maximization implies:

$$y_k(\theta) = \theta g(\theta) f\left[s_k(\theta)\right], \tag{2}$$

where,

$$f'\left[s_k(\theta)\right] = \frac{p}{\pi_k Q_k g(\theta)}. \tag{3}$$

Invoking the concavity of f we have that the span of control is strictly decreasing in the input cost p, but strictly increasing in the output price π_k, level of technology Q_k and skill θ. The span of control is increasing in θ only because of the existence of increasing returns to ability, that is because $g' > 0$.

$$\frac{ds_k}{d\left(\frac{\pi_k Q_k}{p}\right)} > 0, \qquad \frac{ds_k}{d\theta} > 0.$$

Henceforth, I will suppress the dependence of s_k on θ unless its inclusion is necessary to avoid confusion.

Choosing the optimal span of control solves the income maximization problem of an agent conditional on her sector, but how do agents sort across sectors? Using (3) we obtain that the wage $w_k(\theta)$ of an agent in sector k is:

$$w_k(\theta) = \pi_k \theta g(\theta) \left[f(s_k) - s_k f'(s_k) \right]. \tag{4}$$

In equilibrium each agent will select into the sector in which her wage is greatest and output prices will adjust to ensure a positive mass of agents is assigned to every sector.[20] Consider an agent choosing between two sectors k and l with $Q_k > Q_l$:

$$\frac{w_k(\theta)}{w_l(\theta)} = \frac{\pi_k}{\pi_l} \frac{f(s_k) - s_k f'(s_k)}{f(s_l) - s_l f'(s_l)}. \tag{5}$$

The requirement that neither sector offers a strictly higher income at all skill levels is sufficient to generate useful restrictions on permissible equilibrium output prices. Suppose $\pi_k \geq \pi_l$. Then, since $Q_k > Q_l$, we must also have $\pi_k Q_k > \pi_l Q_l$. However, noting that $f(s_k) - s_k f'(s_k)$ is strictly increasing in s_k and, therefore, in $\pi_k Q_k$, it follows that if both $\pi_k \geq \pi_l$ and $\pi_k Q_k > \pi_l Q_l$ sector k strictly dominates sector l. To avoid this possibility we must have $\pi_k < \pi_l$. The intuition is straightforward – if all sectors are selected it is not possible that one sector both uses the intermediate input more productively and has a higher output price than another. Similarly, to guarantee sector l does not dominate we must have $\pi_k Q_k > \pi_l Q_l$, which implies $s_k(\theta) > s_l(\theta)$. An agent's span of control is greater in the more technologically advanced sector.

To characterize the mapping of agents to sectors we can differentiate (5) obtaining:

$$\frac{d}{d\theta}\left[\frac{w_k(\theta)}{w_l(\theta)}\right] \propto \epsilon^g(\theta) \left[\epsilon^f(s_k) - \epsilon^f(s_l)\right], \tag{6}$$

where $\epsilon^g(\theta)$ is the elasticity of g with respect to skill and $\epsilon^f(s)$ is the elasticity of f, and consequently of output, with respect to the span of control. I will refer to $\epsilon^f(s)$ as the output elasticity. Equation (6) has two important implications. First, if $g' = 0$ (implying constant returns to ability) then all agents are indifferent

[20] This requirement follows from the demand assumption that all sectors produce positive aggregate output.

between sectors and there is no sorting. Second, given $g' > 0$, the sign of the right hand side of (6) depends on whether the output elasticity is increasing or decreasing in the span of control. The properties of the output elasticity can be determined using the following lemma. The proofs of all lemmas and propositions are in Appendix A.

Lemma 1. *The following are equivalent: (i) F is strictly log-submodular; (ii) F has elasticity of substitution greater than one; (iii) $\epsilon^f(s)$ is strictly increasing in the span of control s.*

Similarly, strict log-supermodularity of F is equivalent to F having elasticity of substitution σ less than one and to $\epsilon^f(s)$ being strictly decreasing in s.[21] Finally, if $\sigma = 1$ then $\epsilon^f(s)$ is independent of s. Following Acemoglu (2002) I will refer to labor and the intermediate input as gross complements if $\sigma < 1$ and gross substitutes if $\sigma > 1$. Note that σ need not be constant, but any restrictions on σ are assumed to hold globally.

We showed above that an agent's span of control is higher in sector k than in sector l. Consequently, if the output elasticity is strictly increasing in the span of control then $\epsilon^f(s_k) > \epsilon^f(s_l)$ and relative income in sector k is strictly increasing in ability. Moreover, to ensure neither sector dominates the other there must exist a threshold $\tilde{\theta} \in \left(0, \bar{\theta}\right]$ such that agents with skill below $\tilde{\theta}$ strictly prefer sector l and agents with skill above the threshold strictly prefer sector k. Therefore, appealing to Lemma 1 we have that when F is log-submodular there is positive assortative matching and high skill agents prefer the more technologically advanced sector. However, if F is log-supermodular the sorting pattern is reversed and high skill agents select into the low technology sector. If F has unit elasticity of substitution there is no sorting because all agents are indifferent between sectors.

What explains these assignment patterns? The higher an agent's skill, the larger her span of control in any given sector. Consequently, high skill agents select into the sector where the elasticity of output with respect to the span of control is greatest. In addition, the higher a sector's intermediate input productivity, the larger the span of control of any given agent. When the factors of production are gross substitutes, the output elasticity is increasing in the span of control and high skill agents select into the high productivity sector because the lower cost per efficiency unit of intermediate input allows them to exploit the substitutability between factors and leverage their ability by working with large quantities of the intermediate input. This is an example of a scale of operations effect (Sattinger 1993). However, if the factors of production are gross

[21] See Costinot (2009) for a definition and discussion of log-supermodularity and log-submodularity. In particular, I use the fact that F is strictly log-submodular if and only if $\frac{\partial^2 \log F}{\partial \theta \partial x} < 0$. Though alluded to by Sattinger (1975) and Kugler and Verhoogen (2008), I am not aware of the link between log-supermodularity, log-submodularity and the elasticity of substitution of a constant returns to scale production function having been made explicit in the previous literature.

complements having a greater span of control reduces the output elasticity because the complementarity between factors diminishes the value of working with large quantities of intermediate input when the labor input is fixed. Therefore, high skill agents are assigned to the low technology sector.

The preceding discussion is based on a comparison between only two sectors. However, by comparing all pairs of sectors it is straightforward to extend the results to encompass K sectors. The ranking of sectors by absolute advantage Q_k fully determines the ranking of output prices π_k and of $\pi_k Q_k$. With $Q_K > Q_{K-1} > \ldots > Q_1$, then in any equilibrium such that all sectors produce positive aggregate output:

(i) $\pi_1 > \pi_2 > \ldots > \pi_K$;

(ii) $\pi_1 Q_1 < \pi_2 Q_2 < \ldots < \pi_K Q_K$.

These orderings hold regardless of the value of σ. The role of the elasticity of substitution comes in determining how agents sort across sectors. As shown in the proof of Proposition 1, if $\sigma > 1$, meaning that F is strictly log-submodular, there is positive assortative matching:

(iii) $\exists\, 0 = \theta_0 \leq \theta_1 \leq \ldots \leq \theta_{K-1} \leq \theta_K = \bar{\theta}$ such that only agents with skill $\theta \in [\theta_{k-1}, \theta_k]$ are employed in sector k.[22]

This means that in equilibrium agents are partitioned by ability such that higher ability groups of agents select into higher technology sectors. If $\sigma < 1$ the sorting pattern is reversed and there is negative assortative matching.

Proposition 1. *If the production function is strictly log-submodular then the equilibrium assignment of agents to sectors exhibits positive assortative matching. High skill agents are assigned to sectors with high levels of technology. If the production function is strictly log-supermodular then the equilibrium displays negative assortative matching.*

Proposition 1 characterizes how the distribution of skilled labor across sectors is endogenous to cross-sector variation in intermediate input productivity. It is standard in the assignment literature to assume that any technological characteristics which affect the ranking of sectors by workforce skill do not vary across countries.[23] In this paper I will analyze what happens when this assumption is violated – when the ranking of sectors by intermediate input productivity differs across countries leading to assignment reversals. With its simple link between intermediate input productivity and sorting the model offers a tractable and empirically

[22] The inequalities in (iii) will be strict if there are no mass points in the distribution of θ.

[23] See, for example, Ohnsorge and Trefler (2007) and Costinot and Vogel (2009), or consider the no factor intensity reversals assumption in the Heckscher-Ohlin model.

testable framework within which to address this question.

The link between log-submodularity and positive assortative matching may seem surprising to readers familiar with the comparative advantage based assignment literature (Sattinger 1975; Ohnsorge and Trefler 2007; Costinot 2009; Costinot and Vogel 2009; Acemoglu and Autor 2010). In this literature there is a single heterogenous factor of production, the production function is Ricardian and log-supermodularity of the production function leads to positive assortative matching between the heterogenous factor and sectors. For example, if the factor is labor then log-supermodularity of labor productivity in skill and some variable that indexes sectors implies that in equilibrium more skilled labor is assigned to sectors where the marginal effect of skill on labor productivity is greater. However, this prediction is consistent with Proposition 1 provided we interpret the production function used in comparative advantage based models as a reduced form representation of the revenue function net of all non-labor input costs. This net revenue function is equivalent to the wage function $w_k(\theta)$ discussed above and differentiation of (4) shows that the wage is log-supermodular in θ and Q_k if and only if output is log-submodular and there are increasing returns to ability. An important contribution of this paper is to establish a link between the properties of the net revenue function used in previous assignment models and the properties of the production technology when there are two factors of production.[24]

The key assumption that leads to log-submodularity being required for positive assortative matching is not simply the inclusion of intermediate inputs, but the fact that the input level x is endogenously chosen. If each agent must work with a fixed quantity of intermediate input then the equilibrium assignment is reversed and a log-submodular production function implies negative assortative matching between agents and sectors. This occurs because if all sectors use the same quantity of intermediate input then each agent is assigned to the sector where she generates the greatest revenue, in exactly the same manner as happens when there is only a single factor of production. By contrast, when the input quantity is endogenous the equilibrium assignment maximizes the value of output net of input costs, meaning that each agent selects into the sector that maximizes her wage $w_k(\theta)$.

To understand why input choice reverses the sorting pattern it is useful to consider the degree of substitutability between labor and the intermediate input. When F is log-submodular the inputs are gross

[24]Note that when production uses intermediate inputs a distinction must be made between the primitive production function given in (1) and the equilibrium output function $y_k(\theta, Q_k) = \theta g(\theta) f\left[s_k(\theta, Q_k)\right]$ which gives output conditional on the optimal input choice. When F is log-submodular, the equilibrium output function can be either log-submodular or log-supermodular. However, the wage function will always be log-supermodular, which ensures positive assortative matching.

substitutes and, if the input quantity is fixed, efficiency requires matching high skill agents with low technology sectors to take advantage of this substitutability. However, when input choice is endogenous high skill agents leverage their ability by using greater quantities of input and, if there are increasing returns to ability and the inputs are gross substitutes, the leveraging effect is sufficiently strong that skill and technology become complementary and this leads to positive assortative matching. In the absence of increasing returns to ability the leveraging effect is weaker and, in equilibrium, agents are indifferent between sectors.

The switch between positive and negative assortative matching triggered by allowing for input adjustability has interesting implications for how institutional development affects the labor market. For example, consider an economy with a log-submodular production function. Suppose initially financial institutions are under-developed and borrowing constraints force all agents to work with a fixed quantity of the intermediate input. Under these circumstance high skill agents will work in low technology sectors. However, if credit markets develop to the point where agents can pledge some fraction of their income as collateral then more skilled agents will be able to work with greater quantities of input, sorting will reverse and financial development will precipitate dramatic changes in the labor market and the distribution of income.

Wage distribution

Before embedding the assignment problem in general equilibrium two further properties of the model are worth noting. First, from (2) and (4), labor's share of output is given by:

$$\frac{w_k(\theta)}{\pi_k y_k(\theta)} = 1 - \epsilon^f(s_k). \tag{7}$$

When the production function is strictly log-submodular labor's output share is strictly decreasing in labor skill and, therefore, wages both within and across sectors. However, if output is strictly log-supermodular labor's output share is increasing in wages within sectors, but has discontinuous downward jumps at the thresholds for sector assignment, meaning that the cross-sector correlation is in general ambiguous. Empirically, there exists a negative correlation across industries between wages and labor's share of output.[25] For example, the elasticity of labor's output share with respect to the average wage, estimated using the NBER manufacturing database for 2000, is -0.39.[26] Guided by this observation I will henceforth restrict atten-

[25] See Slichter (1950) for some early evidence.

[26] This elasticity is estimated using 4 digit SIC 1987 industries and is significant at the 1% level. There is a similar negative correlation between labor's share of value-added and the industry wage.

tion to the case where F is strictly log-submodular, meaning there is positive assortative matching between agents and sectors.[27]

Assumption 1. *The production function is strictly log-submodular in labor skill θ and intermediate input quantity x.*

Since expenditure on intermediate inputs is equal to $\pi_k \theta g(\theta) f(s_k) \epsilon^f(s_k)$ Assumption 1 also implies that intermediate input expenditure per worker is positively correlated with wages and negatively correlated with labor's share of output. Both these correlations are observed in the NBER manufacturing database for 2000, regardless of whether intermediate input expenditure is measured by expenditure on materials or by capital investment.[28]

The second important property of the model comes from differentiating (4) which gives a very simple expression for the within-sector returns to skill:

$$\epsilon^{w_k}(\theta) = 1 + \frac{\epsilon^g(\theta)}{1 - \epsilon^f(s_k)}, \tag{8}$$

where $\epsilon^{w_k}(\theta) \equiv \frac{\theta w_k'(\theta)}{w_k(\theta)}$. Equation (8) implies that, holding θ constant, the span of control is a sufficient statistic for the returns to skill. Given Assumption 1 the output elasticity is increasing in s_k, implying that a higher span of control raises the returns to skill. Since equation (3) implies that an increase in $\pi_k Q_k$ leads to a higher span of control, it follows that the within-sector returns to skill are strictly increasing in $\pi_k Q_k$. Intuitively, when labor and intermediate inputs are gross substitutes, high skill agents are better able to take advantage of positive technology or output price shocks to increase production levels by working with more intermediate inputs.

If $\epsilon^g(\theta)$ is non-decreasing then (8) also implies that the returns to skill are strictly increasing in ability within sectors and are strictly higher in more technologically advanced sectors. Consistent with this prediction Gibbons et al. (2005) find that returns to skill are higher in more skilled occupations. Let $w(\theta) \equiv \max_{1 \leq k \leq K} \{w_k(\theta)\}$ be an agent's equilibrium wage. Then, since log-submodularity implies positive assortative matching, we also have that $\epsilon^w(\theta)$ is strictly increasing in θ with discontinuous upward jumps at the thresholds $\theta_1, \ldots, \theta_{K-1}$.

[27] The empirical work in Sections 5 and 6 provides further support for this assumption.

[28] When the intermediate input is interpreted as capital, I estimate the correlation using labor's share of value-added instead of output.

The wage distribution depends on both the wage function $w(\theta)$ and the distribution of skills across agents. The model places no restrictions on the shape of the skill distribution, but equation (8), in combination with Lemma 2 below, will allow us to characterize how shocks, such as technological progress and trade liberalization, affect within-group wage inequality when the distribution of skills across agents is held constant.

Lemma 2. *Let $w(\theta)$ and $\tilde{w}(\theta)$ be wage functions such that $\epsilon^{w}(\theta) > \epsilon^{\tilde{w}}(\theta) \, \forall \, \theta \in (\theta_a, \theta_b) \subseteq \left(0, \overline{\theta}\right]$. Then wage inequality among any subset of agents with skill levels in $[\theta_a, \theta_b]$ is higher under $w(\theta)$ than under $\tilde{w}(\theta)$ for any measure of inequality that respects scale independence and second-order stochastic dominance.*

Lemma 2 tells us that within-group wage inequality rises whenever both the returns to skill increase at all skill levels and membership of the group is unchanged. Adapting an approach used by Helpman, Itskhoki and Redding (2010) the proof of Lemma 2 relies on showing that, after a change in means, the wage distribution implied by $\tilde{w}(\theta)$ second-order stochastically dominates the distribution implied by $w(\theta)$. Combining Lemma 2 and equation (8) implies that the sign of the change in wage inequality among any group of agents is fully determined by variation in the span of control. This result will be used repeatedly below to characterize how technological progress and trade liberalization affect wage inequality. In addition, since in the general equilibrium model wages are the only source of income, the income distribution will be identical to the wage distribution.

Cross-sector heterogeneity

It is straightforward to modify the production technology in (1) to allow for sources of cross-sector heterogeneity other than differences in intermediate input productivity. Suppose that production in sector k requires a team of N_k workers and that if each worker has skill θ output is given by:[29]

$$y_k(\theta, x) = g(\theta) A_k \left[\lambda_k \left(B_k \theta\right)^{\frac{\sigma-1}{\sigma}} + (1-\lambda_k) \left(Q_k x\right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}. \tag{1'}$$

Given Assumption 1 we must have $\sigma > 1$. This formulation allows for cross-sector heterogeneity in team size N_k, Hicks-neutral productivity A_k, labor augmenting productivity B_k, intermediate input productivity Q_k and the labor intensity of production λ_k. I restrict the production function to be a constant elasticity

[29] This specification assumes that in equilibrium all members of a team have the same skill level. This will necessarily be the case if, for example, a team inherits the skill level of its least able member.

of substitution (CES) technology in order to introduce the CES parameter λ_k. If λ_k is not included in the analysis then the results below hold without imposing functional form restrictions on F.

The same reasoning employed to derive Proposition 1 can be used to characterize the equilibrium assignment when output is given by $(1')$. The structure of equilibrium is unchanged, but agents sort across sectors based not on the ranking of sectors by absolute advantage Q_k, but on the ranking of sectors by V_k where:

$$V_k \equiv \left(\frac{1-\lambda_k}{\lambda_k}\right)^{\frac{\sigma}{\sigma-1}} \frac{N_k Q_k}{B_k}.$$

Higher ability agents are assigned to sectors with higher V_k. Consequently, skill levels and wages are higher, ceteris paribus, in sectors with: (i) higher intermediate input productivity; (ii) lower labor augmenting productivity; (iii) larger production teams, and; (iv) lower labor intensity.

Interestingly, different forms of technological progress have different implications for sorting across sectors. Whereas increases in intermediate input productivity tend to draw more skilled workers into a sector, labor augmenting technological change has the opposite effect. To understand this result, remember that when Assumption 1 holds and output is given by (1) higher ability agents sort into sectors with higher spans of control. Provided we redefine the span of control to equal the number of efficiency units of intermediate input used per efficiency unit of skill provided, $s_k(\theta) \equiv \frac{Q_k x}{B_k \theta}$, this insight remains true under the production technology $(1')$. By driving down the output price labor augmenting technological progress reduces an agent's optimal span of control and, therefore, has the opposite effect to increases in intermediate input productivity. Similarly, higher labor intensity is equivalent to a simultaneous rise in labor augmenting productivity and fall in intermediate input productivity and decreases the optimal span of control. Meanwhile, higher team size increases the output price by raising labor costs, thereby leading to a greater optimal span of control. Finally, the equilibrium sorting pattern does not depend on Hicks-neutral productivity A_k because A_k is multiplicatively separable from the production function.

For the remainder of the paper I will revert to working with the production function given by (1) where sectors differ only in terms of intermediate input productivity.

3 General equilibrium

To embed the assignment problem in general equilibrium I need to specify how the intermediate input is produced and the source of demand for each sector's output. The assignment problem is sufficiently tractable to permit multiple possible general equilibrium settings. For example, the productive activities agents undertake could be occupations, industries or tasks that produce different industry inputs. Likewise, the intermediate input could be produced using labor, capital or an aggregate output good. In this section I develop a general equilibrium model in which each productive activity constitutes a separate sector and there is an aggregate output good that can be used either for consumption or as the intermediate input. These assumptions are chosen primarily for their simplicity, allowing the paper to focus on the new insights arising directly from the assignment problem. However, in Appendix B I show that the paper's main results continue to hold in a more complex model where agents are assigned to tasks and task outputs are used as factor inputs in a Heckscher-Ohlin model. This alternative set-up gives a version of the Heckscher-Ohlin model in which the ranking of industries by workforce skill is endogenous to the distribution of intermediate input productivity across tasks.

Assumptions

Suppose there are two sectors, $K = 2$, with $Q_2 > Q_1$ and assume the skill distribution has continuous support on $\left(0, \bar{\theta}\right]$ and no mass points.[30] Output from the two sectors is combined to produce a final good using a Cobb-Douglas technology:

$$Z = \left(\frac{Y_1}{\beta}\right)^{\beta} \left(\frac{Y_2}{1-\beta}\right)^{1-\beta}, \qquad \beta \in (0,1), \tag{9}$$

where Z is final good output and Y_k is aggregate output of sector k:

$$Y_k = \int_{\theta_{k-1}}^{\theta_k} \theta g(\theta) f(s_k) dM(\theta). \tag{10}$$

This technology guarantees that all sectors must produce positive aggregate output. The final good can be used either for consumption or as the intermediate input. This completes the specification of the economy. The use of a Cobb-Douglas final good technology simplifies solving the model, but all the closed and open

[30] This assumption is for ease of exposition. It is straightforward to solve the model when the skill distribution is discrete, but the notation is more cumbersome due to the necessity of keeping track of where agents work when they are indifferent between sectors.

economy results obtained below continue to hold if the final good is produced using a general constant returns to scale technology. See Appendix B for details.

Equilibrium

Given Assumption 1 we know there is positive assortative matching between agents and sectors. Therefore, there exists a skill threshold θ_1 such that agents with skill below θ_1 work in sector one and agents with skill above θ_1 work in sector two.

To solve the model it is convenient to let the final good be the numeraire. This immediately implies that $p = 1$ and, from cost minimization using (9), that:

$$1 = \pi_1^{\beta} \pi_2^{1-\beta}. \tag{11}$$

Since $Q_2 > Q_1 \Rightarrow \pi_2 < \pi_1$ we must have that $\pi_2 < 1 < \pi_1$. In addition, (11) implies that $\frac{d\pi_1}{d\pi_2} < 0$. If the price of sector two output rises, then the price of sector one output falls. Cost minimization using (9) also gives the market clearing equations:

$$\beta Z = \pi_1 Y_1, \qquad (1-\beta) Z = \pi_2 Y_2. \tag{12}$$

Equations (3), (4), (10), (11) and (12) are sufficient to reduce the model to a system of two equations in the two unknowns, θ_1 and π_2. First, the income equalization (IE) condition requires that an agent with ability θ_1 be indifferent between the two sectors. From (4) and (11) this implies:

$$f\left[s_1(\theta_1)\right] - s_1(\theta_1) f'\left[s_1(\theta_1)\right] = \pi_2^{\frac{1}{\beta}} \left(f\left[s_2(\theta_1)\right] - s_2(\theta_1) f'\left[s_2(\theta_1)\right]\right). \tag{IE}$$

Second, the output markets must clear. Using (10), (11) and (12) gives the market clearing (MC) condition:

$$\int_0^{\theta_1} \theta g(\theta) f(s_1) dM(\theta) = \frac{\beta}{1-\beta} \pi_2^{\frac{1}{\beta}} \int_{\theta_1}^{\bar{\theta}} \theta g(\theta) f(s_2) dM(\theta). \tag{MC}$$

In both equilibrium conditions s_1 and s_2 are defined by (3) and depend implicitly on π_2.

Figure 3 shows the (IE) and (MC) conditions in θ_1-π_2 space. The (IE) curve is downward sloping because an increase in π_2 makes sector two more profitable and, since $\frac{w_2(\theta)}{w_1(\theta)}$ is increasing in θ, this decreases the skill level at which agents are indifferent between sectors. The (MC) curve is upward sloping because

a higher π_2 reduces the relative demand for sector two output, which necessitates the reallocation of labor to sector one. Together the two conditions define a unique equilibrium – see the proof of Proposition 2 for details.

Proposition 2. *Given Assumption* 1 *there exists a unique closed economy equilibrium with a threshold skill level* θ_1 *such that agents with skill above* θ_1 *work in the high technology sector and agents with skill below* θ_1 *work in the low technology sector.*

Technological change

Technological progress takes the form of growth in intermediate input productivity. It has its most dramatic effect when it changes the ranking of sectors by input productivity. For example, an increase in intermediate input productivity in sector one from $Q_1 < Q_2$ to $Q_1' > Q_2$ precipitates an assignment reversal which makes sector one the high skill, high wage sector. I will estimate the effect of higher intermediate input productivity on a sector's rank in the industry wage distribution in the empirical part of the paper.

Regardless of whether or not technological progress changes the sector technology ranking the equilibrium conditions can be used to show:[31]

$$\frac{d\left[\pi_k Q_k\right]}{dQ_j} > 0, \qquad \frac{d\pi_j}{dQ_j} < 0, \qquad \frac{d\pi_l}{dQ_j} > 0, \qquad j,k,l = 1,2, \quad l \neq j. \tag{13}$$

Unsurprisingly, technological progress in a given sector is accompanied by a price decline in that sector and a price rise in the other sector. More interestingly, technological progress in either sector always increases $\pi_k Q_k$ in both sectors. Remembering equations (3) and (8) this implies that, holding θ constant, the span of control $s_k(\theta)$ and returns to skill $\epsilon^{w_k}(\theta)$ rise in both sectors. Appealing to Lemma 2, the higher returns to skill increase within-group wage inequality among any group of agents who all work in the same sector and who do not switch sectors following the technology shock. Technological progress raises the returns to skill in both sectors because it causes all agents to increase their spans of control, which disproportionately benefits high skill agents for whom the elasticity of output with respect to the span of control is greater.

Proposition 3. *Technological progress raises the returns to skill in both sectors. An increase in intermediate input productivity in either sector increases within-group income inequality among any group of agents who all work in the same sector and who do not switch sectors following the productivity increase.*

[31] See the proof of Proposition 3 for details.

There are two distinct senses in which technological progress is complementary to skill in this model. First, it increases the within-sector returns to skill in both sectors. Second, any sector which experiences a sufficiently large positive technology shock becomes the high skill sector, regardless of the skill level of its workers prior to the shock.

Without placing restrictions on the shape of the skill distribution, or the functional form of the production technology, the effect of technological change on the skill threshold θ_1 and on inequality among groups that include agents who are induced to switch sectors by the technological shock is, in general, ambiguous. In particular, at skill levels such that agents switch from the high skill to the low skill sector following a technology shock the returns to skill can decrease. However, I show in the proof of Proposition 3 that any productivity increase which causes the high skill sector to expand on the extensive margin ($d\theta_1 < 0$) leads to higher wage inequality among all subgroups of the population.

4 Open economy

Let us now extend the model to include two countries: home and foreign. I will use an asterisk to denote foreign variables. Suppose the two countries are identical along all dimensions except: (i) intermediate input productivity; (ii) skill distribution, and; (iii) population size. Assumption 1 ensures that the production function F is log-submodular in both countries. The aim of this section is to analyze the effects of globalization when intermediate input productivity differs across both sectors and countries. I assume that each country's skill distribution has continuous bounded support, but I allow the functional form and upper bound of the skill distribution to differ across countries. I also assume that neither country is a small economy.

In the open economy each sector's output is freely traded, implying that both sectoral output prices and the price of the final good are equalized across countries. As above, I let the final good be the numeraire. To pin down the location of final good production, which is necessary to ensure trade flows are well-determined, assume there is some positive cost to trading the final good. It immediately follows that the final good is non-traded in equilibrium. When comparing the closed and open economy equilibria I will use a tilde to denote autarky outcomes.

Assignment reversals

Consider the case where there is an assignment reversal across countries. In particular, suppose that home has higher productivity in sector two, $Q_1 < Q_2$, but foreign has higher productivity in sector one, $Q_1^* > Q_2^*$.[32] This means that in autarky sector two is the high skill, high wage sector at home, while sector one is the high skill, high wage sector abroad. In addition, diversified production requires $\tilde{\pi}_2 < 1 < \tilde{\pi}_1$ and $\tilde{\pi}_1^* < 1 < \tilde{\pi}_2^*$ meaning:

$$\frac{\tilde{\pi}_2}{\tilde{\pi}_1} < 1 < \frac{\tilde{\pi}_2^*}{\tilde{\pi}_1^*},$$

which implies home has a comparative advantage in sector two and foreign has a comparative advantage in sector one. Therefore, when the ranking of sectors by intermediate input productivity differs across countries, each country has a comparative advantage in its high productivity sector, which is also its high skill, high wage sector.

We know from Section 2 that if $\pi_2 \geq \pi_1$ then at home sector two offers a strictly higher wage than sector one at all skill levels. Similarly, if $\pi_2 \leq \pi_1$ then sector one is strictly preferred to sector two by all foreign agents. Since free trade equalizes output prices across countries it follows that in the open economy at least one of the countries must specialize in its high skill sector. Let us suppose that $\pi_2 \leq \pi_1$.[33] Then foreign specializes in sector one and equation (11) implies $\pi_2 \leq 1 \leq \pi_1$.

In the open economy output prices must satisfy (11) and equilibrium spans of control and incomes are given by (3), (4) and their foreign equivalents. As in the closed economy, the open economy equilibrium can be reduced to a system of two equations in two unknowns, θ_1 and π_2. The income equalization (IE) condition, which determines the skill threshold above which home agents select into sector two, is unchanged from the closed economy case. The difference is that output markets clear at the global, not the national, level. From (12) and its foreign equivalent global output market clearing requires:

$$Y_1 + Y_1^* = \frac{\beta}{1-\beta}\pi_2^{\frac{1}{\beta}}(Y_2 + Y_2^*),$$

and using (10), (11) and that foreign is specialized in sector one we obtain the open economy market clearing (MC′) condition:

[32] I consider below the case where both countries have higher productivity in the same sector.

[33] Equation (14) below gives a necessary and sufficient condition for this to be the equilibrium outcome.

$$\int_0^{\theta_1} \theta g(\theta) f(s_1) dM(\theta) + \int_0^{\bar{\theta}^*} \theta g(\theta) f(s_1^*) dM^*(\theta) = \frac{\beta}{1-\beta} \pi_2^{\frac{1}{\beta}} \int_{\theta_1}^{\bar{\theta}} \theta g(\theta) f(s_2) dM(\theta). \qquad \text{(MC}'\text{)}$$

The only difference from the closed economy market clearing condition is the second term on the left hand side of (MC′), which represents foreign's sector one output. As in the closed economy, the (IE) curve is downward sloping and the (MC′) curve is upward sloping in θ_1–π_2 space and together they define a unique equilibrium. However, foreign production shifts the (MC′) curve upwards relative to the (MC) curve in the closed economy (see Figure 4). Therefore, globalization reduces the skill threshold above which home agents work in sector two, $\theta_1 < \tilde{\theta}_1$ and increases the home price of sector two output, $\pi_2 > \tilde{\pi}_2$.

For $\pi_2 \leq 1 \leq \pi_1$ to be the equilibrium outcome we must have that when $\pi_1 = \pi_2 = 1$, which implies both countries are specialized in their high productivity sector, there is not an excess supply of good one. Consequently, a necessary and sufficient condition for output prices to satisfy $\pi_2 \leq 1 \leq \pi_1$ in equilibrium is:

$$\int_0^{\bar{\theta}^*} \theta g(\theta) f(s_1^*) dM^*(\theta) \leq \frac{\beta}{1-\beta} \int_0^{\bar{\theta}} \theta g(\theta) f(s_2) dM(\theta), \tag{14}$$

where the spans of control are defined by (3) with $\pi_1 = \pi_2 = 1$. This condition tells us that if foreign is economically "small" relative to home then in the open economy equilibrium foreign specializes in its high productivity sector. In this context, an economy's size depends on how much output it can produce in its high productivity sector and smallness can result from having a relatively low population, relatively unskilled agents or relatively low intermediate input productivity in the high technology sector. Proposition 4 summarizes the structure of production in the open economy equilibrium.

Proposition 4. *When the ranking of sectors by intermediate input productivity differs across countries there exists a unique open economy equilibrium such that: (i) each country exports the output of its high skill sector; (ii) the smaller economy specializes in its high skill sector, and; (iii) compared to autarky the skill threshold above which agents select into the high skill sector is lower in both countries.*

Since each country has a comparative advantage in its high technology sector, and high skill agents are matched to the high technology sector, the model predicts that the export sector and the high skill sector

coincide in both countries. This prediction is absent from models that do not include assignment reversals.[34] In addition, trade integration causes an expansion of the high skill sector on the extensive margin in both countries.

Comparing the open economy equilibrium to autarky outcomes we have $\tilde{\pi}_2 < \pi_2 < \tilde{\pi}_2^*$ and $\tilde{\pi}_1^* < \pi_1 < \tilde{\pi}_1$. Following trade integration, each country experiences an increase in the price of its high skill sector and a decrease in the price of its low skill sector. From (3) and (4), in each country these price changes increase the wages of agents in the high skill sector and decrease the wages of agents in the low skill sector. Whether agents who switch into the high skill sector following globalization obtain a higher wage than in autarky is ambiguous, but in each country there exists a skill threshold such that, following trade liberalization, the wage of all agents with skill below the threshold falls and the wage of all agents with skill above the threshold rises.[35] Therefore, in stark contrast to the implications of the Stolper-Samuelson theorem, trade liberalization benefits high skill labor in both countries.

From equation (8) and Lemma 2 the price changes triggered by globalization increase the returns to skill in the high skill sector and decrease the returns to skill in the low skill sector. Consequently, in both countries, moving from autarky to free trade increases wage inequality among any group of agents employed in the high skill sector following trade liberalization and decreases wage inequality among any group of agents employed in the low skill sector following trade liberalization.[36] Since the smaller economy specializes in its high skill sector it experiences a pervasive rise in wage inequality – wage inequality increases among any subset of the population containing at least two agents with different skill levels. In addition, if equation (14) holds with equality, meaning the two economies are the same size, then both countries are fully specialized in the open economy equilibrium and trade integration causes a pervasive increase in wage inequality in both countries.

Proposition 5. *When the ranking of sectors by intermediate input productivity differs across countries moving from autarky to free trade causes each country to experience an increase in the price of its high skill good and a decrease in the price of its low skill good. Consequently, in both countries, wage levels and*

[34] For example, the Heckscher-Ohlin model with no factor intensity reversals, Ohnsorge and Trefler (2007), Costinot and Vogel (2009) and Costinot (2009). Matsuyama (2007) presents a model in which export sectors are always more skill intensive than import sectors because, by assumption, export production uses a more skill intensive technology than production for domestic consumption.

[35] See the proof of Proposition 5 for details.

[36] Autor, Katz and Kearney (2006) document that during the 1990s there was a compression of the bottom half of the US wage distribution and a dispersion of the top half.

wage inequality increase in the high skill sector and decrease in the low skill sector.

The observation that trade liberalization has coincided with increases in wage inequality in many unskilled labor abundant developing countries has prompted an extensive theoretical literature on alternatives to the Stolper-Samuelson theorem.[37] Explanations of how trade integration may raise inequality in both developed and developing countries have invoked intra-industry offshoring (Feenstra and Hanson 1996), the existence of multiple cones of diversification in a Heckscher-Ohlin model (Davis 1996), trade induced intra-industry input quality upgrading (Verhoogen 2008; Kugler and Verhoogen 2008), higher skill intensity of export production (Matsuyama 2007) and intra-industry selection of high skill or high wage firms into exporting (Manasse and Turrini 2001; Yeaple 2005; Helpman, Itskhoki and Redding 2010). This paper suggests a new mechanism – assignment reversals. In contrast to papers that focus solely on intra-industry effects, the model can explain both increased wage inequality across sectors and increasing returns to skill within export sectors. In addition, the model predicts that trade integration between two economies will benefit high skill labor in both countries only when the countries are sufficiently dissimilar that the ranking of sectors by intermediate input productivity differs across countries. This could explain why the effects of trade integration on inequality have varied across developing countries.

Productivity rankings match

When both countries have higher input productivity in the same sector there are no assignment reversals and the effects of trade on income inequality are more conventional. Suppose that in both countries intermediate input productivity is higher in sector two than in sector one. In this case, the pattern of comparative advantage will depend on the intermediate input productivities and skill distributions in both countries according to the autarky equilibrium conditions (IE) and (MC) and their foreign equivalents. The autarky price of sector two output is lower, ceteris paribus, in the country with: (i) higher relative productivity in sector two; (ii) higher absolute productivity levels, or; (iii) a greater proportion of high skill agents. Without loss of generality, let us assume $\tilde{\pi}_2 < \tilde{\pi}_2^*$, meaning home has a comparative advantage in sector two, while foreign has a comparative advantage in sector one. Therefore, in the open economy equilibrium home exports output from its high skill sector, while foreign exports output from its low skill sector.

Since the ranking of sectors by average employee skill is invariant across countries, trade-induced price

[37] See Goldberg and Pavcnik (2007) for an overview of the empirical literature.

changes cannot increase the price of the high skill good in both countries. Only the country with a comparative advantage in the high skill sector experiences an increase in the price of its high skill output. Open economy market clearing requires $\tilde{\pi}_2 < \pi_2 < \tilde{\pi}_2^*$ and from (11) this also implies $\tilde{\pi}_1^* < \pi_1 < \tilde{\pi}_1$. At home trade liberalization has similar effects to those experienced by both countries when there is an assignment reversal: the high skill sector expands on the extensive margin and the price changes benefit high skill labor. However, in foreign the price of high skill output declines, the price of low skill output increases and the low skill sector expands on the extensive margin. Consequently, trade liberalization benefits low skill labor – there exists a threshold such that the wage of all foreign agents with skill below the threshold is higher in the open economy than in autarky and the wage of all foreign agents with skill above the threshold is lower. Moreover, the returns to skill increase in the low skill sector and decrease in the high skill sector, meaning that trade liberalization increases wage inequality among any group of foreign agents employed in the low skill sector in autarky and decreases wage inequality among any group of foreign agents employed in the high skill sector following integration.

Proposition 6. *When the ranking of sectors by intermediate input productivity is the same in both countries there exists a unique open economy equilibrium such that: (i) one country exports its high skill, high productivity good and the other country exports its low skill, low productivity good, and; (ii) in both countries moving from autarky to free trade increases wage levels and the returns to skill in the export sector and decreases wage levels and the returns to skill in the import sector.*

Note that although trade liberalization benefits high skill labor in both countries only when there are assignment reversals, it increases the returns to skill in the export sector of both countries regardless of the patterns of input productivity or comparative advantage.[38]

Intermediate input trade

In the closed economy differences in intermediate input cost across sectors are formally equivalent to variation in intermediate input productivity. However, in the open economy this equivalence breaks down if the intermediate input is tradable. Suppose the home economy can produce $\frac{1}{P_k}$ units of sector k intermediate input from one unit of the final good. Then the cost per efficiency unit of sector k intermediate input at

[38] Brambilla et al. (2010) argue using Latin American data that the skill premium is higher in industries with a greater exports to output ratio.

home is $\frac{P_k}{Q_k}$. Here, Q_k is a reduced form representation of disembodied productivity and measures the efficiency with which each unit of intermediate input is used. For example, variation in Q_k could result from the interaction of cross-country differences in contract enforcement institutions with cross-sector variation in the contract intensity of intermediate input utilization. By contrast, variation in P_k is embodied in the intermediate input and can be transferred across countries through intermediate input trade.

If intermediate inputs are non-tradable, then the equilibrium assignment in the home economy depends on the ranking of sectors by $\frac{Q_k}{P_k}$. However, if intermediate inputs are tradable then each country-sector pair will source its input from the lowest cost supplier of the intermediate input (inclusive of trade costs). If there is no cross-country heterogeneity in disembodied productivity (i.e. $Q_k = Q_k^* \forall\, k$) then reductions in trade costs will lead to cross-country convergence in the cost per efficiency unit of intermediate input. Under free trade in intermediate inputs the ranking of sectors by intermediate input cost and, consequently, by workforce skill will be the same in all countries and there will be no assignment reversals. Effectively, when all technological differences are embodied in intermediate inputs, free trade is a perfect substitute for cross-country technology diffusion and leads to global convergence in the inter-industry wage structure. However, if there are costs associated with trade in intermediate inputs then assignment reversals can occur even when there is no cross-country variation in disembodied productivity. In Section 6 the paper exploits geographically induced differences in trade costs to identify cross-country variation in the relative cost of different types of equipment inputs.

It is also interesting to consider how intermediate input trade costs affect wage inequality. From the perspective of the importing country, reductions in intermediate input trade costs are equivalent to growth in intermediate input productivity. Proposition 3 above shows that productivity growth increases within-sector wage inequality in the closed economy. Similar effects are observed in the open economy. In particular, at any given skill level, the equilibrium span of control will increase in any country-sector pair that experiences growth in intermediate input productivity. Moreover, if input productivity in either sector grows at the same rate in both countries, then the equilibrium span of control will increase in both sectors of both countries.[39] These results hold regardless of whether productivity rankings differ or match across countries. This implies that the returns to skill will increase in any sector which experiences a fall in the cost of importing its intermediate input, regardless of whether the reduction in trade costs is country specific or global. As in

[39]Formally, for $k = 1, 2$, it can be shown that $\frac{d[\pi_k Q_k]}{dQ_k} > 0$, $\frac{d[\pi_k Q_k]}{dQ} > 0$ when $\hat{Q} = \hat{Q}_j = \hat{Q}_j^*$, $j = 1, 2$ and that analogous results hold for $\pi_k Q_k^*$. I omit the proof of these results since it follows directly from differentiating the open economy income equalization and market clearing conditions and then applying the reasoning used in the proof of Proposition 3.

the closed economy a shock that increase workers' opportunity to leverage their ability disproportionately benefits high skill agents.

If the intermediate input is interpreted as capital the model implies that reductions in the cost of trading capital goods will increase within-sector returns to skill. This prediction receives support from two recent papers that estimate the impact of capital imports on wages. Csillag and Koren (2009) undertake structural estimation of a single sector model of worker assignment, similar to Sattinger (1979), using a rich matched employer-employee-imports data set from Hungary. They find that on average imported machines are more productive than domestic machines and are matched with higher skill workers. In addition, the returns to skill on the median productivity imported machine are 26% higher than on the median productivity domestic machine. Parro (2010) estimates the impact of capital imports on the skill premium using a calibrated version of the Eaton and Kortum (2002) model in which production uses skilled labor, unskilled labor and capital and there is capital-skill complementarity. The paper finds that from 1990-2007 reductions in capital trade costs and productivity growth in capital production each increased the skill premium by an average across countries of around 2 percentage points.

5 US manufacturing wages

The model developed above offers a rich, yet still tractable, framework within which to understand worker assignment and the effects of technological change and trade liberalization. Central to the model is the prediction, contained in Proposition 1, that higher skill workers are assigned to sectors with greater intermediate input productivity. The remainder of the paper provides evidence first, that this assignment mechanism operates and second, that there is sufficient variation over time and across countries in the ranking of industries by workforce skill to recommend using a framework in which the pattern of worker sorting is endogenous to parameters of the model. Since the prediction being tested follows from the partial equilibrium model, it requires no restrictions either on the number of sectors considered or on the sources of output demand or intermediate input supply and, consequently, applies directly to all active industries in any economy.

The empirical work uses two data sets. This section studies the impact of changes in equipment prices on the structure of wages in US manufacturing using the NBER manufacturing database, while the next section analyzes cross-country variation in inter-industry wage differences using UNIDO's Industrial Statistics database. In both cases I treat the mean wage per employee in an industry as an observable measure of the

average skill of the industry's workforce. The assumption that inter-industry wage differences reflect differences in workforce skill, rather than variation in rent sharing across industries, is not only fully consistent with the model, but is supported by the empirical literature on inter-industry wage differences. Krueger and Summers (1986) find that observable worker characteristics only explain around half of inter-industry wage differences in the US, but using a richer dataset with matched employer-employee data Abowd, Kramarz and Margolis (1999) estimate that worker fixed effects account for 90% of inter-industry wage differences in France.

To obtain observable measures of intermediate input productivity, I interpret the intermediate input as capital. Since the model predicts that lower input costs and higher input productivity have equivalent implications for worker assignment, differences in intermediate input productivity across sectors can then be mapped to variation in the cost per efficiency unit of capital. The main challenge in testing the model is to find an exogenous source of variation in the cost of capital investment.

Empirical strategy

This section estimates the impact of industry specific changes in the cost of capital investment on wage growth in US manufacturing industries. The NBER manufacturing database includes a measure of the price of capital investment at the 4 digit SIC industry level. However, this measure may not be exogenous with respect to the industry wage. For example, offshoring of low skill intensive production activities could lead to both a higher industry wage and a change in the composition and, therefore, the price of an industry's capital investment. Alternatively, industries may be subject to shocks, unobservable to the econometrician, that affect both wages and the investment price. Consequently, an instrument for the investment price is needed.

To construct an instrument I will exploit the fact that technological restrictions cause variation in the composition of capital investment across industries. In particular, suppose capital is produced as a Cobb-Douglas aggregate of I different varieties of equipment with expenditure shares that vary across industries. Then the price of investment P_k is given by:

$$P_k \propto \prod_{i=1}^{I} p_i^{\alpha_i^k} \tag{15}$$

where p_i is the price of equipment variety i, α_i^k is the share of industry k's capital expenditure allocated to

equipment variety i and all equipment prices are expressed per efficiency unit. Since equipment prices do not vary by industry they are plausibly exogenous to industry level variables. Therefore, heterogeneity in price changes across equipment varieties generates exogenous shocks to the cost of capital at the industry level and I will use changes in equipment prices, weighted by industry specific equipment variety expenditure shares, as an instrument for changes in the price of investment.

This identification strategy could fail if industries with similar demand for different equipment varieties are subject to correlated shocks. For example, suppose a group of industries that all invest heavily in metalworking machinery experience a positive demand shock. Increased production by these industries may lead to both higher industry wages, through either rent sharing or skill upgrading, and increased demand for metalworking machinery. In turn, higher demand for metalworking machinery could stimulate cost reducing investments in metalworking machinery production. To control for this possibility I include 2 digit industry dummy variables in all regressions.

To be specific, I estimate the following equation:

$$\Delta \log \omega_{kt} = \gamma \Delta \log IP_{kt} + \phi X_{kt} + \delta_l + \epsilon_{kt}, \tag{16}$$

where ω is the average industry wage, IP is the investment price which will be treated as endogenous, X is a vector of controls, k is a 4 digit SIC industry, t denotes the period, δ_l is a 2 digit industry dummy variable and Δ denotes the annualized change over the period. The controls used include industry characteristics in 1960, research and development (R&D) expenditures as a share of output and growth in output, total factor productivity (TFP) and the ratios of imports and exports to industry output.

Figure 2, discussed in the introduction, shows that the US industry wage structure is not time invariant. However, as pointed out by Krueger and Summers (1986), inter-industry wage differences in the US do display a high degree of stability over time. For instance, the rank correlation between 4 digit SIC industry wages in 1960 and 2000 in the NBER manufacturing database is 0.83. Motivated by this observation I estimate equation (16) using a cross-section of changes between 1960 and 2000. The long difference specification is designed to capture long-run changes in the industry wage structure that may not be evident over shorter time periods. It also reduces the fraction of wage variation caused by short-run rent sharing between firms and workers.[40]

[40] See Blanchflower, Oswald and Sanfey (1996) for evidence supporting the existence of short-run rent sharing.

Data

Equipment price data is taken from Cummins and Violante (2002), who extend Gordon's (1990) quality-adjusted price indices for categories of equipment in the US National Income and Product Accounts (NIPA) to cover 26 types of equipment and software from 1947-2000.[41] These estimates are intended to measure the price of a constant quality unit of equipment and represent the most thorough existing attempt to adjust official price indices to account for changes in the quality of capital over time. To compute capital expenditure shares for each industry I use the 1982 US capital flow table and the 1982 Census of Manufactures. The capital flow table gives capital expenditures by equipment variety for 51 manufacturing industries, which map to either 2 or 3 digit SIC industries, while the Census of Manufactures measures investment in computers by 4 digit SIC industries.[42] I use capital investment data from 1982 because it is the earliest year for which computer investment data is widely available at the 4 digit level. Although, computer investment was also collected during the 1977 Census of Manufactures 38% of industries reported zero computer investment in 1977. By 1982 this proportion had fallen to 10%.

By combining the equipment price and expenditure share data I construct two instruments for the investment price change. First, the equipment price change ΔEP_k which is computed as the weighted average of the log price change for all equipment varieties other than Computers, office and accounting machinery, Software and Vehicles, where the weights are given by the expenditure shares from the capital flow table. Second, the computer price change ΔCP_k which is the change in the log price of computers weighted by the share of new capital expenditure allocated to computers from the Census of Manufactures. The formulae used to construct these instruments come from taking the log difference of the expression for the price of investment in (15) holding the expenditure share weights constant over time. Note that the variation in both instruments is generated solely by cross-industry differences in the share of capital expenditure allocated to different varieties of equipment.

Import and export data for SIC industries in 1960 are taken from Feenstra's US trade data available from the Center for International Data, while imports and exports in 2000 are from Schott (2008). R&D expenditures for 1974 at the 3 digit level are from Scherer (1984). All other variables are from the NBER manufacturing database. The final data set covers 451 SIC manufacturing industries at the 4 digit level.

[41] See Appendix C for further details on the data set. I am grateful to Giovanni Violante for sharing the equipment price data with me.

[42] I am grateful to Eli Berman for providing me with the computer investment data originally used in Berman, Bound and Griliches (1994).

Results

Estimating equation (16) using OLS shows that lower growth in the investment price is associated with faster wage growth between 1960 and 2000 (Table 1, column a). However, the possible endogeneity of the investment price means this association may not be causal. To check the validity of the identification strategy column (b) shows the results from the first stage regression of the investment price change on the equipment and computer price change instruments. As desired, the coefficients on both instruments are positive and significant and 2SLS estimation of the model confirms the OLS results (column c). Industries in which the cost of capital increased more slowly experienced faster wage growth. Based on the estimate in column (c), a one standard deviation decrease in the change in the investment price between 1960 and 2000 leads to a 4.0% higher industry wage in 2000, which is equivalent to increasing the log wage in 2000 by 0.15 standard deviations.

If changes in the cost of capital cause assignment reversals then they must effect not only industry wages, but also the ranking of industries by average wages. By contrast, skill-biased technical change could generate the results in column (c) without affecting the industry wage ranking. For example, suppose that there are two types of homogenous labor: skilled and unskilled, and let the ranking of industries by the ratio of skilled workers to unskilled workers be exogenously fixed. If skill-biased technical change causes an increase in the relative wage of skilled labor and, simultaneously, leads to falls in the price of capital goods used mainly by skill intensive industries then we will observe faster wage growth in industries with slower increases in the cost of capital, but the ranking of industries by average wages will remain unchanged. In columns (d) and (e) the model is estimated using the change in an industry's percentile rank in the wage distribution as the dependent variable. Regardless of whether the model is estimated using OLS (column d) or 2SLS (column e), we find that industries in which the investment price grows more slowly move up the industry wage ranking. This is consistent with the existence of assignment reversals. Note also that the skill-biased technical change story implies wage growth will be greater in skill intensive industries where wages are high initially. Figure 2 shows that there is in fact a negative correlation between wages in 1960 and subsequent wage growth (see also Table 2, column a). It follows that, in models which do not admit the possibility of assignment reversals, skill-biased technical change cannot explain the observed variation in wage growth across industries.

The model also predicts that labor's share of output will be lower in industries with high intermediate

input productivity. To test this prediction I estimate how changes in the investment price affect labor's share of value-added. I use labor's share of value-added, rather than labor's share of output, because when the intermediate input is interpreted as capital the output concept used in the model corresponds to value-added in the data. Both the OLS results (column f) and the 2SLS results (column g) are consistent with the model. Slower growth in the cost of capital leads to decreases in labor's share of value-added.

Could the results in Table 1 be driven by an omitted variable that is correlated with both wage growth and changes in the cost of capital, or by trends affecting particular types of industries? Controlling for observed industry characteristics in 1960 does not affect the estimated impact of investment price changes on wage growth (Table 2, column a). Neither does controlling for R&D expenditures as a fraction of industry output (column b), although the estimates do show a positive relationship between R&D expenditures and wage growth. In column (c) I include controls for growth in output, TFP, and imports and exports as a share of output. Since these variables may be endogenous to wage growth, this regression should be interpreted with caution, but given the possibility of omitted variable bias it provides a useful sensitivity check on the robustness of the results. Although inclusion of these controls causes the investment price change to become marginally insignificant (p-value of 0.11), the magnitude of the coefficient is similar to previous estimates and the results do not obviously suggest the existence of omitted variable bias.

Finally, I estimate the model using only the equipment price change as an instrument (column d). The Information and Communication Technology (ICT) revolution that has swept the economy since IBM introduced the first Personal Computer in 1981 has precipitated dramatic changes in technologies, labor markets and the organization of production.[43] Within the assignment reversals framework we can conceptualize the ICT revolution as causing industry specific shocks to input productivity that lead to changes in the allocation of workers across industries.[44] However, since there exist many alternative interpretations of how the ICT revolution has affected production technologies, it is useful to check that the investment price effect is not driven solely by cross-industry variation in computer investment. Reassuringly, column (d) shows that the investment price coefficient is essentially unchanged when the equipment price change is the only instrument used.[45]

Table 3 investigates further the mechanism through which changes in the investment price affect wage

[43] See Jorgenson (2001) for an overview of the ICT revolution.

[44] For evidence that investment in computers is associated with increased demand for skilled labor see Berman, Bound and Griliches (1994); Autor, Katz and Krueger (1998), and; Bresnahan, Brynjolfsson and Hitt (2002).

[45] In fact, all the 2SLS investment price coefficients shown in Tables 1, 2 and 3 are robust to estimating the model without using the computer price change as an instrument.

growth. Lower equipment price growth leads to faster growth in capital investment per worker (columns a and b), but the relationship between the computer price change and investment is insignificant (column a). When the equipment price change is used as an instrument for investment per worker, we find that the increased investment caused by negative shocks to the price of equipment leads to higher wages (column c). This finding is consistent with the prediction, documented in Section 2, that investment per worker is increasing in worker skill in the assignment reversals model.

We can also examine how investment price changes affect wages for different types of worker. Lower investment price growth does not affect the wages of production workers (column d), but leads to increased wages for non-production workers (column e) and an increase in the share of non-production workers in employment (column f). These results imply that a lower cost of capital leads to a shift in the composition of employment towards non-production workers and to employing higher skilled non-production workers. More detailed data on labor force composition and the allocation of workers to tasks within industries is needed to disentangle precisely how investment price changes affect different types of worker. However, a plausible interpretation of these findings is that higher productivity capital substitutes for low skill production workers and is matched with more skilled non-production workers who can use larger quantities of capital more efficiently. This interpretation is consistent with the labor input in the assignment reversals model being non-production labor.

Under this interpretation in particular, the model bears a resemblance to theories of capital-skill complementarity. However, there are important differences. First, capital-skill complementarity is typically modeled either at the aggregate level[46] or in terms of a specific factors framework, while this paper provides an explanation for why capital accumulation is associated with wage increases at the sector level over time horizons long enough to allow for factor mobility across sectors. Second, because it allows for a continuum of skill levels, the assignment reversals model can be used to study the effect of growth in capital productivity on within-group wage inequality, rather than just on the relative wages of skilled and unskilled labor. Third, if the labor input is interpreted as non-production labor, then the model predicts that a fall in the investment price will lead to a decrease in non-production labor's income relative to capital's income. By contrast, if capital and skill are gross complements then a lower investment price will increase this ratio. Column (g) shows that lower investment price growth reduces total wages paid to non-production workers

[46] See, for example, Krusell et al. (2000).

relative to value-added net of all labor costs, as predicted by the assignment reversals model.[47]

6 International wage structure comparisons

Studies of inter-industry wage differences have generally concluded that the pattern of industry wages is highly correlated across countries. For example, Krueger and Summers (1986) find that in eight of the thirteen countries they consider the correlation of log wages with the US exceeds 0.8,[48] leading them to conclude that the "wage structure is amazingly parallel in looking at data for different countries" (p.1). However, the consensus found in the literature has emerged primarily from comparisons between industrialized economies. Noting that four of the five countries with correlations below 0.8 are non-industrialized economies Krueger and Summers (1986) caution that the wage structure in mature capitalist economies is "different from that of Communist or less developed economies" (p.2).

Figure 1, discussed previously in the introduction, shows that this claim continues to hold when looking at industry wage data for a broader sample of countries than considered by Krueger and Summers (1986). Remember that Figure 1 shows wage rank correlations (the correlation between the ranking of industries by wage levels in a given country and the ranking in the US) plotted against income levels (expressed as log differences from US income). Regressing the wage rank correlation on the income difference gives an intercept of 0.75 and a slope of 0.13 (the robust t-statistic for the slope coefficient is 7.9).[49] Figure 5 shows the proportion of industry pairs in which the ranking of industries by wage levels is the same as in the US. For a country such as France the proportion exceeds 80%, but for the poorest country in the sample, Bangladesh, it is only 61%. Industry wage data also implies that income convergence with the US is associated with convergence towards the US inter-industry wage structure. Figure 6 plots the change in the wage rank correlation against the change in income relative to the US for 70 countries between 1965 and 1995.[50] Regressing the change in the wage rank correlation on the change in the income difference gives an

[47] Krusell et al. (2000) estimate an elasticity of substitution between capital and skilled labor of approximately two-thirds, whereas I assume an elasticity of substitution between intermediate inputs and labor greater than one. The likely explanation for this difference is that Krusell et al. (2000) fit their model to aggregate data, while I use only manufacturing industries. The share of labor in aggregate GDP stayed roughly constant during their sample period and the share of skilled labor actually increased. However, the share of labor costs in manufacturing value-added fell from 53% in 1960 to 36% in 2000 and the share of non-production labor fell from 17% to 14%.

[48] The correlations are calculated using wage data for around 20 manufacturing industries in 1981 or 1982.

[49] The positive association in Figure 1 is robust to weighting observations by industry employment shares when calculating the correlations and to calculating the correlations using wages instead of wage rankings.

[50] The wage data covers 28 ISIC 3 digit manufacturing industries. Changes are expressed in annualized terms. See Appendix C for further details about the data.

intercept of 0.005 and a slope of 0.12 (the robust t-statistic for the slope coefficient is 2.3).

If poorer countries report less reliable data, the results in Figures 1, 5 and 6 could be caused by measurement error. To allay this concern Figure 7 shows wage rank correlations plotted against income relative to the US using industry wage data for 1995 taken from the EU KLEMS database. The EU KLEMS database is designed to provide accurate industry level data for use in growth accounting exercises. The database covers 29 countries (the 25 EU countries plus Australia, Japan, South Korea and the US) and, at its most disaggregated level, 57 market-based industries, which together compose the entire market economy. Again, the wage rank correlation is strongly increasing in income, but the slope of the relationship is larger than in the UNIDO data. Regressing the wage rank correlation on the income difference gives an intercept of 0.69 and a slope of 0.26 (the robust t-statistic for the slope coefficient is 4.1). The slope estimate is basically unchanged if the wage rank correlations are computed either using only the 29 manufacturing industries in the sample or using a more aggregated set of 23 industries.[51]

These results support Krueger and Summers' (1986) hypothesis that although developed countries have strikingly similar industry wage structures, this similarity does not extend to developing economies. Under the maintained assumption that inter-industry wage differences stem from variation in workforce skill, the cross-country variation in wage rank correlations provides direct evidence that the ranking of industries by workforce skill is endogenous to a country's level of development.

Could the cross-country variation in wage rank correlations be caused by Heckscher-Ohlin style skill intensity reversals? Consider a multi-sector Heckscher-Ohlin economy in which production uses two types of labor: skilled and unskilled. In each industry the skill intensity of production will depend on the skill premium and the elasticity of substitution between skilled and unskilled labor. If skill intensity reversals occur, then industries in which the elasticity of substitution is relatively high will be skilled labor intensive in countries with low skill premia and unskilled labor intensive in countries with high skill premia. In particular, if all industries use constant elasticity of substitution production technologies it is simple to show that the number of skill intensity reversals between any two countries is an increasing function of the difference between their skill premia.[52] Therefore, if variation in wage rank correlations is caused by Heckscher-Ohlin skill intensity reversals, it should be strongly correlated with variation in skill premia.

Internationally comparable measures of the skill premium are not available for the majority of the coun-

[51] When the manufacturing industries are used the estimated slope coefficient is 0.26 and in the aggregated sample it is 0.23.

[52] See Reshef (2007) for a theoretical analysis of the causes and consequences of skill intensity reversals in such a model.

tries in the UNIDO sample used in Figure 1. However, differences in skill premia across countries are well explained by variation in human capital levels.[53] Therefore, to crudely examine whether cross-country differences in the inter-industry wage structure are due to Heckscher-Ohlin skill intensity reversals I regress the wage rank correlations shown in Figure 1 on countries' stocks of physical and human capital per capita.[54] There is a strong positive association between the capital stock and the wage rank correlation (slope 0.09; robust t-statistic 3.3), but the human capital variable is insignificant (robust t-statistic 0.01). This finding does not support the conjecture that Heckscher-Ohlin skill intensity reversals are driving cross-country variation in wage rank correlations. In addition, the link between wage rank correlations and relative capital abundance suggests that capital may play a role in shaping the inter-industry wage structure, but it is unclear how exactly this association maps to the assignment reversals model. Therefore, to test the model explicitly I next analyze whether inter-industry wage variation in the UNIDO data set is caused by differences in the cost of capital.

Empirical strategy

Model identification requires a source of exogenous variation in the cost per efficiency unit of capital that varies across both countries and industries. To construct a variable that captures such variation I will make use of three empirical regularities documented in Eaton and Kortum (2001) and Caselli and Wilson (2004). First, the production of equipment capital is highly concentrated in a handful of countries that invest heavily in research and development. Second, equipment imports from the major equipment producers account for over half of equipment investment in most countries.[55] Third, trade costs generate variation in the cost of equipment across countries. In addition to these observations, I document below that there is substantial variation across equipment varieties in the export market shares of major equipment producers, implying that the relative productivity of different producers varies across equipment varieties. Based on these four facts, I conjecture that the price of any given variety of equipment is lower, ceteris paribus, in countries that are geographically close to major exporters of that equipment variety. Therefore, in each country the cost of

[53] See, for example, Fernàndez, Guner and Knowles (2005) and Brambilla et al. (2010).

[54] The physical and human capital variables are expressed as the absolute value of the log deviation from US physical and human capital per capita, respectively. See Appendix C for a description of how these variables are constructed. The human capital measure is computed from the Barro and Lee (2001) educational attainment data set and is only available for 32 of the 42 countries. However, very similar results are obtained when human capital is measured using the secondary school enrollment rate, which is available for 41 countries.

[55] In fact, Caselli and Wilson (2004) argue that "for most countries, imports of capital of a certain type are an adequate proxy for overall investment in that type of equipment" (p.2).

capital will be relatively low in industries that use intensively equipment varieties for which the country is geographically close to exporters with large export market shares. If valid, this conjecture provides a source of variation in the cost of capital that is both measurable and plausibly exogenous.

To capture this source of variation, suppose there are D major equipment exporters and let the revealed advantage RA_{di} of exporter d in equipment variety i be d's share of the total exports of equipment variety i by the D major equipment exporters. Then I define the cost of imported equipment CIE for equipment variety i in country c as:

$$CIE_{ic} = \left(\sum_{d=1}^{D} \frac{RA_{di}}{GD_{dc}} \right)^{-1}, \tag{17}$$

where GD measures the gravity-adjusted distance between country c and country d. The gravity-adjusted distance is computed as:

$$GD_{dc} = -\hat{\eta}_0 \log dist_{dc} - \hat{\eta}_1 \, lang_{dc} - \hat{\eta}_2 \, bord_{dc} - \hat{\eta}_3 \, col_{dc},$$

where $dist$ denotes distance and $lang$, $bord$ and col are dummy variables indicating whether countries c and d share a common language, share a border or were ever in a colonial relationship, respectively. The $\hat{\eta}$ coefficients are obtained from estimating a gravity model of equipment trade including $dist$, $lang$, $bord$ and col, together with importer and exporter fixed effects, as regressors.[56]

By aggregating the cost of imported equipment across equipment varieties, using the share of industry investment allocated to different equipment varieties as weights, we can then define the cost of imported capital CIC in industry k as:

$$CIC_{kc} = \prod_{i=1}^{I} CIE_{ic}^{\alpha_i^k}, \tag{18}$$

where α_i^k is the share of industry k capital expenditure allocated to equipment variety i. Note that the capital expenditure shares do not vary across countries. I will use investment data for US industries to compute the capital expenditure shares. Equation (18) is based on the expression for the price of investment given in (15), but the unobserved equipment prices are replaced by the cost of imported equipment measure.

To the extent that the cost of imported capital variable succeeds in capturing variation in the investment

[56] The estimated coefficients are: $\hat{\eta}_0 = -1.21; \hat{\eta}_1 = 0.59; \hat{\eta}_2 = 0.54; \hat{\eta}_3 = 0.91$.

price, the model predicts that wages will be higher in industries where the cost of imported capital is lower. To test this prediction I estimate the following equation:

$$\log \omega_{kc} = \gamma \log CIC_{kc} + \phi X_{kc} + \alpha_k + \delta_c + \epsilon_{kc}, \tag{19}$$

where ω is the average industry wage, X is a vector of controls, α_k is an industry dummy variable and δ_c is a country fixed effect. As controls I use the log of industry level measures of capital, skill and contract intensity computed from US data interacted with the log of country level measures of capital abundance, skill abundance and the rule of law, respectively.

Data

To compute the cost of imported capital, I divide equipment into 15 varieties and use the 1997 US capital flow table to compute the share of equipment investment allocated to each variety by 4 digit ISIC manufacturing industries. Since for most industries the capital expenditure shares only vary at the 2 or 3 digit level I estimate equation (19) with the standard errors clustered by country-2 digit industry groups.

I define the major equipment exporters to be the eight largest equipment exporters between 1995 and 2000: US, Japan, Germany, France, UK, Canada, Italy and China. Each of these countries accounted for more than 3.5% of world exports of the 15 equipment varieties between 1995 and 2000 and collectively they accounted for 64% of equipment exports. Trade data is taken from the NBER-United Nations world trade data set and I use a concordance from SITC Rev. 2 product categories to US BEA industries obtained from the Center for International Data to identify trade in each of the 15 equipment varieties.

Wage and investment data for 4 digit ISIC industries are from UNIDO's Industrial Statistics database. Since country coverage in the Industrial Statistics database varies from year to year, I use wage data for 2000 whenever possible and from the latest year between 1995 and 1999 in which wage data is available otherwise. For each country, observations for all non-wage variables are taken from the same year as the wage data.[57]

The capital, skill and contract intensity variables are defined as the capital stock per worker, the share of non-production workers in employment and the fraction of inputs neither sold on an exchange nor reference priced, respectively. The capital and skill intensities are computed using the NBER manufacturing database

[57] See Appendix C for a complete description of the data set.

for 2000, while contract intensity is taken from Nunn (2007) and is based on US input-output tables in 1997. Capital abundance is defined as capital stock per capita computed from the Penn World Tables 6.3, skill abundance is defined as the secondary school enrollment rate from the World Bank's World Development Indicators and the rule of law is taken from the World Bank's World Governance Indicators for 2000.

The final data set includes 36 countries and 120 industries. The eight major equipment exporters are not included in the final data set since the cost of imported capital is endogenous for these countries.

Results

To generate within-country, cross-industry variation in the cost of imported capital, the revealed advantage of the eight major exporters must vary across equipment types for a given exporter. Table 4 shows summary statistics on revealed advantages in 2000 for each of the eight major exporters. There is substantial within-exporter variation in revealed advantage. The average coefficient of variation across the eight exporters is 64% and each exporter has a revealed advantage below 9% in at least one equipment variety and above 16% in at least one equipment variety. The identification strategy outlined above is based on the assumption that this variation in revealed advantage reflects differences in the relative export price of different equipment varieties.

The results of estimating equation (19) are shown in Table 5. Wages are higher when the cost of imported capital is lower and this effect is observed regardless of whether the capital, skill and contract intensity interactions are excluded (column a) or included (column b). As discussed in Section 5, the theory of assignment reversals predicts that variation in the cost of capital should affect not only industry wages, but also the industry wage ranking. When an industry's percentile rank in the wage distribution is used as the dependent variable the cost of imported capital is insignificant when it is the only explanatory variable (column c), but continues to have a significant effect when the interaction controls are included (column d).

If cross-country variation in the industry wage ranking is caused by Heckscher-Ohlin skill intensity reversals then, as discussed above, decreasing the skill premium will tend to increase the wage rank percentile of industries in which the elasticity of substitution between skilled and unskilled labor is high. We can test for this effect by including in equation (19) the interaction of log skill abundance, which acts as an inverse proxy for the skill premium, with the log of an industry level estimate of the elasticity of substitution between skilled and unskilled labor.[58] If Heckscher-Ohlin skill reversals are widespread then the estimated

[58] I use estimates of the elasticity of substitution at the 2 digit SIC level from Reshef (2007). See Appendix C for further details.

coefficient on this skill reversal interaction should be positive. Since the elasticities of substitution I use are somewhat imprecisely estimated the results from this exercise should be treated with caution, but column (e) shows that the skill reversal interaction has an insignificant negative effect on the wage rank percentile.[59] Therefore, these results do not support the hypothesis that cross-country differences in the inter-industry wage structure are driven by Heckscher-Ohlin skill intensity reversals.

The higher wages observed when the cost of imported capital is lower are not accompanied by an increase in labor's share of value-added (columns f and g). However, the estimates also fail to support the model's prediction that labor's share of value-added should be lower when the cost of imported capital is lower.

Table 6 examines more closely the validity of the identification strategy. If geographic proximity to a major equipment exporter lowers the relative cost of equipment varieties in which the exporter has a high market share, then it should also increase imports of such equipment varieties. Regressing imports by equipment variety in 2000 on the cost of imported equipment defined in equation (17), we do indeed find that imports are significantly higher when the cost of imported equipment is lower (column a). The model also predicts that investment per worker will be higher in industries where the cost of imported capital is lower and columns (b) and (c) confirm this effect. These results support the assumptions underlying the identification strategy. In addition, they suggest an alternative approach to verifying that the effect of the cost of imported capital proxy on wages is accounted for by changes in capital investment – treat investment per worker as an explanatory variable and use the cost of imported capital as an instrument. Adopting this approach we find that the increased capital investment per worker caused by a lower cost of imported capital leads to increases in wages (column d) and the industry wage ranking (column e), but has an insignificant effect on labor's share of value-added (column f).

The estimation results show that wages are higher in industries that have access to relatively cheap sources of imported capital. This finding is not only consistent with the model's assignment mechanism, but also indicates that, through its impact on the cost of capital, equipment trade may play a significant role in shaping the inter-industry wage structure. Remembering the discussion on intermediate input trade above, the model implies that reductions in barriers to equipment trade will: (i) lead to convergence across countries in the inter-industry wage structure by reducing cross-country variation in the relative cost of equipment

[59] If the skill reversal interaction is the only explanatory variable included, the estimated effect becomes positive, but remains insignificant.

varieties, and; (ii) increase within-industry returns to skill in all countries and industries by lowering the cost per efficiency unit of capital. This second prediction is consistent with the empirical findings of Csillag and Koren (2009) and Parro (2010) discussed in Section 4.

7 Conclusions

The distribution of skilled labor across sectors is commonly treated as an outcome to be assumed, not explained. However, industry wage data suggests that the assignment of skill to industries varies systematically across countries. This paper represents a first attempt at extending the assignment literature to make the allocation of skill endogenous to observable parameters with concrete empirical interpretations and at exploring the consequences of allowing the ranking of sectors by workforce skill to differ across countries. To achieve these goals the paper develops a new assignment model which combines two distinct strands of the existing literature: (i) multiple sectors, and; (ii) matching between two factors of production with non-zero opportunity costs. The model generates a rich new set of predictions linking technologies and input costs to labor assignment, the distribution of wages and trade patterns. In particular, the model implies that technical progress in capital production and variation in barriers to equipment trade play an important role in shaping the inter-industry wage structure and the distribution of wages. Labor market outcomes cannot be understood without considering the supply of non-labor inputs.

In future work I plan to extend the model in two directions. First, to allow for endogenous technical change in intermediate input productivity and analyze under what conditions profit maximizing R&D will lead countries to have different patterns of absolute advantage. Second, if the intermediate input is taken to be homogenous unskilled labor, the partial equilibrium model can be reinterpreted as a model of firm hierarchies. Consequently, the assignment reversals framework could be used to extend the single sector literature on globalization and firm hierarchies (Antràs, Garicano and Rossi-Hansberg 2006; Burstein and Monge-Naranjo 2009) to a multi-sector world.

The paper's empirical analysis shows that, when the non-labor input is interpreted as capital, variation in industry wages both over time in the US and across countries is consistent with the model's prediction of positive assortative matching between worker skill and non-labor input productivity. However, much remains to be done. The model should be tested using data sets that include within-industry data on worker characteristics and the tasks performed by different workers. Such data would permit a test of the hypothesis,

suggested by the results on wage growth in US manufacturing, that the model applies primarily to the allocation of high skill workers across industries. In addition, it could be used to check the validity of assuming that the average industry wage is a suitable proxy for workforce skill. It would also be interesting to test the model's success when the non-labor input is interpreted as intermediate inputs, instead of capital. Goldberg et al. (2008) use cross-industry heterogeneity in imported intermediate input price declines to show that India's 1991 trade liberalization caused a large increase in product innovation by domestic firms. Do such shocks also affect worker assignment? Finally, future work could look directly at whether the existence of assignment reversals can explain the observed cross-country heterogeneity in the effect of trade integration on wage inequality.

References

Abowd, John M., Francis Kramarz, and David N. Margolis. (1999) "High Wage Workers and High Wage Firms." *Econometrica* 67 (2): 251-333.

Acemoglu, Daron and David Autor. (2010) "Skills, Tasks and Technologies: Implications for Employment and Earnings." NBER Working Paper.

Acemoglu, Daron. (2002) "Directed Technical Change." *Review of Economic Studies* 69 (4): 781-809.

Amano, Akihiro. (1977) "Specific Factors, Comparative Advantage and International Investment." *Economica* 44 (174): 131-44.

Antràs, Pol, Luis Garicano, and Esteban Rossi-Hansberg. (2006) "Offshoring in a Knowledge Economy." *Quarterly Journal of Economics* 121 (1): 31-77.

Autor, David H., Lawrence F. Katz, and Melissa S. Kearney. (2006) "The Polarization of the U.S. Labor Market." *American Economic Review* 96 (2): 189-94.

Autor, David H., Lawrence F. Katz, and Alan B. Krueger. (1998) "Computing Inequality: Have Computers Changed the Labor Market?" *Quarterly Journal of Economics* 113 (4): 1169-1213.

Barro, Robert J., and Jong-Wha Lee. (2001) "International Data on Educational Attainment: Updates and Implications." *Oxford Economic Papers* 53 (3): 541-63.

Becker, Gary. (1973) "A Theory of Marriage: Part 1." *Journal of Political Economy* 81 (4): 813-46.

Berman, Eli, John Bound, and Zvi Griliches. (1994) "Changes in the Demand for Skilled Labor within U.S. Manufacturing: Evidence from the Annual Survey of Manufactures." *Quarterly Journal of Economics* 109 (2): 367-397.

Blanchflower, David G., Andrew J. Oswald, and Peter Sanfey. (1996) "Wages, Profits, and Rent-Sharing." *Quarterly Journal of Economics* 111 (1): 227-251.

Brambilla, Irene, Rafael Dix Carneiro, Daniel Lederman, and Guido Porto. (2010) "Skills, Exports, and the Wages of Five Million Latin American Workers." The World Bank, Policy Research Working Paper Series.

Bresnahan, Timothy F., Erik Brynjolfsson, and Lorin M. Hitt. (2002) "Information Technology, Workplace Organization, and the Demand for Skilled Labor: Firm-Level Evidence." *Quarterly Journal of Eco-*

nomics 117 (1): 339-376.

Burstein, Ariel T. and Alexander Monge-Naranjo. (2009) "Foreign Know-how, Firm Control, and the Income of Developing Countries." *Quarterly Journal of Economics* 124 (1): 149-195.

Caselli, Francesco. (2005) "Accounting for Cross-country Income Differences." In The Handbook of Economic Growth, Vol. 1, Amsterdam: Elsevier.

Caselli, Francesco and Daniel J. Wilson. (2004) "Importing Technology." *Journal of Monetary Economics* 51 (1): 1-32.

Costinot, Arnaud and Jonathan Vogel. (2009) "Matching and Inequality in the World Economy." NBER Working Paper.

Costinot, Arnaud. (2009) "An Elementary Theory of Comparative Advantage." *Econometrica* 77 (4): 1165-1192.

Csillag, Márton and Miklós Koren. (2009) "Machines and Machinists: The Effect of Imported Capital on Wage Inequality." Mimeo, Central European University.

Cummins, Jason G. and Giovanni L. Violante. (2002) "Investment-Specific Technical Change in the United States (1947-2000): Measurement and Macroeconomic Consequences." *Review of Economic Dynamics* 5 (2): 243-284.

Davis, Donald R. (1996) "Trade Liberalization and Income Distribution." NBER Working Paper.

Eaton, Jonathan and Samuel Kortum. (2001) "Trade in Capital Goods." *European Economic Review* 45 (7): 1195-1235.

Eaton, Jonathan, and Samuel Kortum. (2002) "Technology, Geography and Trade." *Econometrica* 70 (5): 1741-79.

Feenstra, Robert C. and Gordon H. Hanson. (1996) "Foreign Investment, Outsourcing and Relative Wages." In The Political Economy of Trade Policy: Papers in Honor of Jagdish Bhagwati, 89-127. Cambridge and London: MIT Press.

Fernandez, Raquel, Nezih Guner, and John Knowles. (2005) "Love and Money: A Theoretical and Empirical Analysis of Household Sorting and Inequality." *Quarterly Journal of Economics* 120 (1): 273-344.

Gibbons, Robert, Lawrence F. Katz, Thomas Lemieux, and Daniel Parent. (2005) "Comparative Advantage, Learning, and Sectoral Wage Determination." *Journal of Labor Economics* 23 (4): 681-723.

Goldberg, Pinelopi Koujianou and Nina Pavcnik. (2007) "Distributional Effects of Globalization in Developing Countries." *Journal of Economic Literature* 45 (1): 39-82.

Goldberg, Pinelopi K., Amit Khandelwal, Nina Pavcnik, and Petia Topalova. (2008) "Imported Intermediate Inputs and Domestic Product Growth: Evidence from India." NBER Working Paper.

Gordon, Robert J. (1990) "The Measurement of Durable Goods Prices." National Bureau of Economic Research Monograph series Chicago and London: University of Chicago Press.

Grossman, Gene M. and Giovanni Maggi. (2000) "Diversity and Trade." *American Economic Review* 90 (5): 1255-1275.

Grossman, Gene M. (2004) "The Distribution of Talent and the Pattern and Consequences of International Trade." *Journal of Political Economy* 112 (1): 209-239.

Helpman, Elhanan, Oleg Itskhoki, and Stephen Redding. (2010) "Inequality and Unemployment in a Global Economy." *Econometrica* 78 (4): 1239-1283.

Jorgenson, Dale W. (2001) "Information Technology and the U.S. Economy." *American Economic Review* 91 (1): 1-32.

Krueger, Alan B. and Lawrence H. Summers. (1986) "Reflections on the Inter-Industry Wage Structure." NBER Working Paper.

Krusell, Per, Lee E. Ohanian, José-Victor Ríos-Rull, and Giovanni L. Violante. (2000) "Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis." *Econometrica* 68 (5): 10291053.

Kugler, Maurice and Eric Verhoogen. (2008) "The Quality-Complementarity Hypothesis: Theory and Evidence from Colombia." NBER Working Paper.

Leontief, Wassily. (1964) "International Factor Costs and Factor Use." *American Economic Review* 54 (4): 335-45.

Manasse, Paolo and Alessandro Turrini. (2001) "Trade, Wages, and 'Superstars'." *Journal of International Economics* 54 (1): 97-117.

Matsuyama, Kiminori. (2007) "Beyond Icebergs: Towards a Theory of Biased Globalization." *Review of Economic Studies* 74 (1): 237-253.

Minhas, Bagicha. (1962) "The Homohypallagic Production Function, Factor Intensity Reversals and the Heckscher-Ohlin Theorem." *Journal of Political Economy* 70 (2): 138-156.

Murphy, Kevin M., Andrei Shleifer, and Robert W. Vishny. (1991) "The Allocation of Talent: Implications for Growth." *Quarterly Journal of Economics* 106 (2): 503-530.

Nunn, Nathan. (2007) "Relationship-specificity, Incomplete Contracts and the Pattern of Trade." *Quarterly Journal of Economics* 122 (2): 569-600.

Ohnsorge, Franziska and Daniel Trefler. (2007) "Sorting it Out: International Trade with Heterogeneous Workers." *Journal of Political Economy* 115 (5): 868-892.

Parro, Fernando. (2010) "Capital-Skill Complementarity and the Skill Premium in a Quantitative Model of Trade." Mimeo, University of Chicago.

Reshef, Ariell. (2007) "Heckscher-Ohlin and the Global Increase of Skill Premia: Factor Intensity Reversals to the Rescue." Mimeo, New York University.

Rosen, Sherwin. (1982) "Authority, Control, and the Distribution of Earnings." *Bell Journal of Economics* 13 (2): 311-323.

Roy, Andrew. (1951) "Some Thoughts on the Distribution of Earnings." *Oxford Economic Papers* 3: 135-146.

Sattinger, Michael. (1975) "Comparative Advantage and the Distributions of Earnings and Abilities." *Econometrica* 43 (3): 455-468.

Sattinger, Michael. (1979) "Differential Rents and the Distribution of Earnings." *Oxford Economic Papers* 31 (1): 60-71.

Sattinger, Michael. (1993) "Assignment Models of the Distribution of Earnings." *Journal of Economic Literature* 31 (2): 831-880.

Scherer, Frederic M. (1984) "Using Linked Patent and R&D Data to Measure Interindustry Technology Flows." In R&D, Patents, and Productivity, 417-461. NBER Conference Report. Chicago and London: University of Chicago Press.

Schott, Peter K. (2008) "The Relative Sophistication of Chinese Exports." *Economic Policy* 53: 5-40.

Slichter, Sumner H. (1950) "Notes on the Structure of Wages." *Review of Economics and Statistics* 32 (1): 80-91.

Verhoogen, Eric A. (2008) "Trade, Quality Upgrading, and Wage Inequality in the Mexican Manufacturing Sector." *Quarterly Journal of Economics* 123 (2): 489-530.

Yeaple, Stephen Ross. (2005) "A Simple Model of Firm Heterogeneity, International Trade, and Wages." *Journal of International Economics* 65 (1): 1-20.

Appendix A – Proofs

Proof of Lemma 1

Since F is twice differentiable it is strictly log-submodular if and only if $\frac{\partial^2 \log F}{\partial\theta\partial x} < 0$. Differentiating F gives:

$$\begin{aligned}\frac{\partial^2}{\partial\theta\partial x}\log F(\theta, Q_k x) &= \frac{Q_k}{F^2}(FF_{\theta x} - F_\theta F_x),\\ &= \frac{Q_k}{F^2}FF_{\theta x}(1-\sigma),\end{aligned}$$

where the second line uses the fact that the elasticity of substitution of a twice differentiable, constant returns to scale function F is given by $\sigma = \frac{F_\theta F_x}{FF_{\theta x}}$. Since F has constant returns to scale and is strictly concave we must have $F_{\theta x} > 0$. Therefore, F is strictly log-submodular if and only if $\sigma > 1$.

Finally, to prove that $\sigma > 1$ is equivalent to $\epsilon^f(s)$ being strictly increasing in s differentiate $\epsilon^f(s)$ to obtain:

$$\begin{aligned}\frac{\partial}{\partial s}\epsilon^f(s) &= \frac{1}{f^2}(ff' + sff'' - sf'^2),\\ &= \frac{1}{f^2}(F_\theta F_x - FF_{\theta x}),\\ &= \frac{FF_{\theta x}}{f^2}(\sigma - 1).\end{aligned}$$

Proof of Proposition 1

Consider the case where F is strictly log-submodular. For any $k \in \{1, \ldots, K\}$ the requirement that sector k produces positive aggregate output implies there exists $\theta \in (0, \bar{\theta}]$ such that agents with skill θ weakly prefer sector k to any other sector. Suppose the equilibrium assignment does not exhibit positive assortative matching. Then there exists $l' < l$ and $\theta_a, \theta_b \in (0, \bar{\theta}]$ with $\theta_b > \theta_a$ such that $w_l(\theta_a) \geq w_k(\theta_a)\,\forall k$ and $w_l'(\theta_b) \geq w_k(\theta_b)\,\forall k$.

However, $l > l' \Rightarrow Q_l > Q_{l'} \Rightarrow \pi_l Q_l > \pi_{l'} Q_{l'} \Rightarrow s_l > s_{l'}$. Since F is strictly log-submodular, $\epsilon^f(s)$ is strictly increasing in s and, therefore, it follows from equation (6) above that $s_l > s_{l'} \Rightarrow \frac{d}{d\theta}\left[\frac{w_l(\theta)}{w_l'(\theta)}\right] > 0\,\forall\theta$.

Consequently, $w_l(\theta_a) \geq w_l'(\theta_a) \Rightarrow w_l(\theta_b) > w_l'(\theta_b)$, which contradicts the assumption that there is not positive assortative matching.

An analogous argument can be used to prove that there is negative assortative matching when F is strictly log-supermodular.

Proof of Lemma 2

Let Ω be an arbitrary subset of agents with skill levels in $[\theta_a, \theta_b]$. If the mass of agents in Ω is concentrated at a single point, then there is no inequality between members of Ω. Assume this is not the case and let $\theta_{\min} = \inf\{\theta \in \Omega\}$ and $\theta_{\max} = \sup\{\theta \in \Omega\}$. Clearly, $\theta_{\max} > \theta_{\min}$.

Let $\hat{w}(\theta) = C\tilde{w}(\theta)$ where C is chosen to ensure $\mathbb{E}_\Omega \hat{w}(\theta) = \mathbb{E}_\Omega w(\theta)$ and $\mathbb{E}_\Omega$ denotes an expectation taken over the subset Ω. Obviously, $\epsilon^{\hat{w}}(\theta) = \epsilon^{\tilde{w}}(\theta)\,\forall\theta$. Since $\epsilon^{w}(\theta) > \epsilon^{\hat{w}}(\theta)\,\forall\theta \in (\theta_{\min}, \theta_{\max})$ we have that if $w(\theta') = \hat{w}(\theta')$ with $\theta' \in \Omega$ then $w(\theta) > \hat{w}(\theta)\,\forall\theta > \theta', \theta \in \Omega$ and $w(\theta) < \hat{w}(\theta)\,\forall\theta < \theta', \theta \in \Omega$. Remembering that $\mathbb{E}_\Omega \hat{w}(\theta) = \mathbb{E}_\Omega w(\theta)$ it immediately follows that $w(\theta)$ and $\hat{w}(\theta)$ satisfy a single-crossing property on $[\theta_{\min}, \theta_{\max}]$ with $w(\theta_{\min}) < \hat{w}(\theta_{\min})$ and $w(\theta_{\max}) > \hat{w}(\theta_{\max})$.

Consequently, the wage distribution over Ω induced by $\hat{w}(\theta)$ second-order stochastically dominates the distribution induced by $w(\theta)$. Since $\hat{w}(\theta)$ and $\tilde{w}(\theta)$ are identical up to a change in scale it follows that for any measure of inequality that respects scale independence and second-order stochastic dominance wage inequality among members of Ω is higher when wages are given by $w(\theta)$ than when wages are given by $\tilde{w}(\theta)$.

Proof of Proposition 2

Since the skill distribution has no mass points the (MC) condition implies that $\pi_2 \to 0$ as $\theta_1 \to 0$ and $\pi_2 \to \infty$ as $\theta_1 \to \bar{\theta}$. The (IE) condition implies that $\pi_2 < 1\,\forall\theta_1 \in \left(0, \bar{\theta}\right]$ since if $\pi_2 \geq 1$ all agents obtain a strictly higher wage in sector two than in sector one. Differentiating the (IE) condition gives:

$$\left(\pi_2^{\frac{1}{\beta}} f\left[s_2(\theta_1)\right] - f\left[s_1(\theta_1)\right]\right)\epsilon^g(\theta_1)\hat{\theta}_1 - s_1(\theta_1) f'\left[s_1(\theta_1)\right]\hat{Q}_1 =$$
$$-\pi_2^{\frac{1}{\beta}} s_2(\theta_1) f'\left[s_2(\theta_1)\right]\left(\hat{\pi}_2 + \hat{Q}_2\right) - \left(f\left[s_1(\theta_1)\right] - \beta s_1(\theta_1) f'\left[s_1(\theta_1)\right]\right)\frac{\hat{\pi}_2}{\beta}, \tag{20}$$

where $\hat{\theta}_1 \equiv \frac{d\theta_1}{\theta_1}$ and analogous definitions hold for other variables. Differentiating the (MC) condition gives:

$$C_1\hat{\theta}_1 = C_2\left(\frac{1-\beta}{\beta}\hat{\pi}_2 - \hat{Q}_1\right) + C_3\left(\hat{\pi}_2 + \hat{Q}_2\right) + C_4\hat{\pi}_2, \tag{21}$$

where:

$$\begin{aligned}
C_1 &\equiv \left(f\left[s_1(\theta_1)\right] + \frac{\beta\pi_2^{\frac{1}{\beta}}}{1-\beta}f\left[s_2(\theta_1)\right]\right)\theta_1^2 g(\theta_1)dM(\theta_1) > 0, \\
C_2 &\equiv \int_0^{\theta_1}\theta g(\theta)\frac{f'\left[s_1(\theta)\right]^2}{-f''\left[s_1(\theta)\right]}dM(\theta) > 0, \\
C_3 &\equiv \frac{\beta\pi_2^{\frac{1}{\beta}}}{1-\beta}\int_{\theta_1}^{\bar{\theta}}\theta g(\theta)\frac{f'\left[s_2(\theta)\right]^2}{-f''\left[s_2(\theta)\right]}dM(\theta) \geq 0, \\
C_4 &\equiv \frac{\pi_2^{\frac{1}{\beta}}}{1-\beta}\int_{\theta_1}^{\bar{\theta}}\theta g(\theta)f\left[s_2(\theta)\right]dM(\theta) \geq 0.
\end{aligned} \tag{22}$$

The derivations of (20) and (21) use $\hat{\pi}_1 = -\frac{1-\beta}{\beta}\hat{\pi}_2$, which follows from differentiating (11). Note that (20) and (21) allow for variation in Q_1 and Q_2. This is not necessary to prove Proposition 2, but will be needed for the proof of Proposition 3.

Let $\hat{Q}_1 = \hat{Q}_2 = 0$. Note that: (i) $\frac{\pi_2^{\frac{1}{\beta}} f[s_2(\theta_1)]}{f[s_1(\theta_1)]} = \frac{1-\epsilon^f[s_1(\theta_1)]}{1-\epsilon^f[s_2(\theta_1)]} > 1$ since the span of control is higher in sector two, and; (ii) $f\left[s_1(\theta_1)\right] > s_1(\theta_1)f'\left[s_1(\theta_1)\right]$. Therefore, it follows from (20) that the (IE) curve is strictly downwards sloping on $\left(0, \bar{\theta}\right]$. In addition, equation (21) implies that the (MC) curve is strictly upward sloping on $\left(0, \bar{\theta}\right]$. Combining these results with the boundary conditions above proves that the (IE) and (MC) curves have a unique intersection on $\left(0, \bar{\theta}\right)$.

Proof of Proposition 3

Suppose $\hat{Q}_1 = 0$, but $\hat{Q}_2 > 0$. Then, if $\hat{\pi}_2 \geq 0$ equation (20) implies $\hat{\theta}_1 < 0$, but equation (21) implies $\hat{\theta}_1 > 0$ – a contradiction. Therefore, we must have $\hat{\pi}_2 < 0 \Rightarrow \hat{\pi}_1 > 0$. Now suppose $\hat{\pi}_2 < 0$ and $\hat{\pi}_2 + \hat{Q}_2 \leq 0$. Then equation (20) implies $\hat{\theta}_1 > 0$, but equation (21) implies $\hat{\theta}_1 < 0$ – a contradiction. Therefore, we must have $\hat{\pi}_2 + \hat{Q}_2 > 0$. Similar reasoning shows that if $\hat{Q}_1 > 0$ and $\hat{Q}_2 = 0$ then (20) and (21) together imply $\hat{\pi}_2 > 0, \hat{\pi}_1 < 0$ and $\hat{\pi}_1 + \hat{Q}_1 = -\frac{1-\beta}{\beta}\hat{\pi}_2 + \hat{Q}_1 > 0$. This proves the claims made in equation (13).

Given $\frac{d[\pi_k Q_k]}{dQ_j} > 0, j, k = 1, 2$ equations (3) and (8) together imply that $\frac{d\epsilon^{w_k}(\theta)}{dQ_j} > 0 \, \forall \theta, j, k = 1, 2$. Lemma 2 is then sufficient to conclude that technological progress increases within-group inequality among any group of agents who all work in the same sector and do not switch sectors following the technology shock.

If $\frac{d\theta_1}{dQ_j} < 0$, agents switch from sector one to sector two following an increase in Q_j. Since $s_2(\theta) > s_1(\theta) \, \forall \theta$, equation (8) implies $\epsilon^{w_2}(\theta) > \epsilon^{w_1}(\theta) \, \forall \theta$. Remembering that $\frac{d\epsilon^{w_k}(\theta)}{dQ_j} > 0 \, \forall \theta, j, k = 1, 2$ this means that an increase in Q_j unambiguously increases $\epsilon^w(\theta)$ at any value of θ such that agents switch from sector one to sector two following the shock. It immediately follows that $\frac{d\epsilon^w(\theta)}{dQ_j} > 0 \, \forall \theta$. Therefore, Lemma 2 implies that income inequality increases among all subsets of agents.

Proof of Proposition 4

The (IE) condition is the same as in the closed economy. It is a strictly downward sloping curve on $\left(0, \bar{\theta}\right]$. Let π_2^{IE} be the value of π_2 at which the (IE) curve intersects the $\theta_1 = 0$ axis. Obviously, $\pi_2^{IE} \leq 1$. The (MC′) condition implies $\pi_2 \to \infty$ as $\theta_1 \to \bar{\theta}$. Differentiating the (MC′) condition gives:

$$C_1 \hat{\theta}_1 = C_2 \left(\frac{1-\beta}{\beta} \hat{\pi}_2 - \hat{Q}_1 \right) + C_3 \left(\hat{\pi}_2 + \hat{Q}_2 \right) + C_4 \hat{\pi}_2 + C_5 \left(\frac{1-\beta}{\beta} \hat{\pi}_2 - \hat{Q}_1^* \right), \tag{23}$$

where C_1, C_2, C_3 and C_4 are defined by (22) and:

$$C_5 \equiv \int_0^{\bar{\theta}^*} \theta g(\theta) \frac{f' \left[s_1^*(\theta) \right]^2}{-f'' \left[s_1^*(\theta) \right]} dM^*(\theta) > 0.$$

Equation (23) implies that the (MC′) curve is strictly upward sloping on $\left(0, \bar{\theta}\right]$. Let $\pi_2^{MC'}$ be the value of π_2 at which the (MC′) curve intersects the $\theta_1 = 0$ axis. Equation (14) implies $\pi_2^{MC'} \leq 1$. If $\pi_2^{MC'} < \pi_2^{IE}$ then the (IE) condition and the (MC′) condition must have a unique intersection on $\left(0, \bar{\theta}\right)$ and this gives the open economy equilibrium. If $\pi_2^{MC'} \geq \pi_2^{IE}$ then equilibrium is given by $\theta_1 = 0$ and $\pi_2 = \pi_2^{MC'}$ and in equilibrium both countries specialize in their high productivity sector. This proves the existence of a unique open economy equilibrium.

The remainder of Proposition 4 follows immediately from the discussion in the main body of the paper.

Proof of Proposition 5

Consider the home country and assume home is not specialized in equilibrium. Since $\tilde{\pi}_2 < \pi_2$ and $\tilde{\pi}_1 > \pi_1$, equations (3) and (4) imply that $w_2(\theta) > \tilde{w}_2(\theta)$ and $w_1(\theta) < \tilde{w}_1(\theta)\,\forall\theta$. In addition, $0 < \theta_1 < \tilde{\theta}_1$ and the continuity of w and $\tilde{w}$ imply $w(\theta_1) < \tilde{w}(\theta_1)$ and $w(\tilde{\theta}_1) > \tilde{w}(\tilde{\theta}_1)$. Moreover, $\epsilon^w(\theta) > \epsilon^{\tilde{w}}(\theta)\,\forall\theta \in (\theta_1, \tilde{\theta}_1)$. Therefore, invoking continuity once more, w and $\tilde{w}$ must intersect exactly once on $(\theta_1, \tilde{\theta}_1)$. Trade liberalization reduces the wage of all agents with skill below the intersection and increases the wage of all agents with skill above the intersection.

From (3) we have that $\tilde{\pi}_1 > \pi_1 \Rightarrow \tilde{s}_1(\theta) > s_1(\theta)\,\forall\theta$. Equation (8) then implies $\epsilon^{\tilde{w}}(\theta) > \epsilon^w(\theta)\,\forall\theta < \theta_1$. Applying Lemma 2 this means that income inequality among any subset of agents who work in sector one after trade liberalization is higher in the open economy than in the closed economy. By contrast, $\tilde{\pi}_1 Q_1 < \tilde{\pi}_2 Q_2 < \pi_2 Q_2 \Rightarrow \epsilon^{\tilde{w}}(\theta) < \epsilon^w(\theta)\,\forall\theta > \theta_1$. Consequently, trade liberalization increases income inequality among any subset of agents who work in sector two in the open economy.

Similar reasoning can be used to prove the analogous results for the home country when $\theta_1 = 0$ and for the foreign country.

Proof of Proposition 6

Equilibrium is defined by the income equalization (IE) condition and its foreign equivalent, which are the same as in autarky, and by the global output market clearing condition:

$$\int_0^{\theta_1} \theta g(\theta) f(s_1) dM(\theta) + \int_0^{\theta_1^*} \theta g(\theta) f(s_1^*) dM^*(\theta) = \frac{\beta}{1-\beta}\pi_2^{\frac{1}{\beta}} \left[\int_{\theta_1}^{\bar{\theta}} \theta g(\theta) f(s_2) dM(\theta) + \int_{\theta_1^*}^{\bar{\theta}^*} \theta g(\theta) f(s_2^*) dM^*(\theta) \right].$$

From the foreign income equalization condition, θ_1^* is strictly decreasing in π_2. Given this relationship it is easy to differentiate the market clearing condition, as was done in the proofs of Propositions 2 and 4, and show that it defines a strictly upward sloping relationship between θ_1 and π_2. The market clearing condition also implies that when $\theta_1 = \bar{\theta}, \pi_2 > \tilde{\pi}_2^* > \tilde{\pi}_2$ implying that in θ_1–π_2 space the market clearing curve sits above the home (IE) curve when $\theta_1 = \bar{\theta}$. Let π_2^{IE} be the value of π_2 at which the home (IE) curve intersects the $\theta_1 = 0$ axis. Let $\pi_2^{MC''}$ be the value of π_2 at which the market clearing curve intersects the $\theta_1 = 0$

axis. If $\pi_2^{MC''} < \pi_2^{IE}$ then the home (IE) condition and the market clearing condition must have a unique intersection on $\left(0, \bar{\theta}\right)$ and this gives the open economy equilibrium. If $\pi_2^{MC''} \geq \pi_2^{IE}$ then equilibrium is given by $\theta_1 = 0$ and $\pi_2 = \pi_2^{MC''}$. This proves the existence of a unique open economy equilibrium.

In addition, since the global market clearing condition is simply the sum of the home autarky market clearing condition (MC) and its foreign equivalent we cannot have $\pi_2 \leq \tilde{\pi}_2$ or $\pi_2 \geq \tilde{\pi}_2^*$. In the former case there is excess global supply of good one, and in the later there is excess global supply of good two. Therefore, $\tilde{\pi}_2 < \pi_2 < \tilde{\pi}_2^*$. The remainder of the proof follows from the discussion in the main body of the paper and from using reasoning analogous to that applied in the proof of Proposition 5 to characterize the effect of moving from autarky to free trade on wage levels and wage inequality.

Appendix B – Theoretical extensions

Generalized final good technology

Suppose that instead of equation (9), the final good production function is given by:

$$Z = H(Y_1, Y_2), \tag{24}$$

where H is a constant returns to scale function that is strictly increasing in both its arguments, strictly concave and satisfies $\lim_{Y_k \to 0} \frac{\partial H}{\partial Y_k} = \infty$, $k = 1, 2$. Obviously, introducing this final good technology does not affect the existence of positive assortative matching between high skill agents and high technology sectors.

Let $\zeta \equiv \frac{Y_2}{Y_1}$. Then cost minimization in final good production, together with the choice of the final good as numeraire, imply $\frac{d\pi_2}{d\pi_1} = -\frac{1}{\zeta} < 0$ and:

$$\frac{h'(\zeta)}{h(\zeta) - \zeta h'(\zeta)} = \frac{\pi_2}{\pi_1}, \tag{25}$$

where $h(\zeta) \equiv H(1, \zeta)$. Since H is strictly concave, (25) implies that ζ is a strictly decreasing function of $\frac{\pi_2}{\pi_1}$.

As in the Cobb-Douglas case, equilibrium reduces to an income equalization condition and a market clearing condition. The income equalization condition is still given by equation (IE) above, while the market clearing condition is:

$$\int_0^{\theta_1} \theta g(\theta) f(s_1) dM(\theta) = \frac{1}{\zeta} \int_{\theta_1}^{\bar{\theta}} \theta g(\theta) f(s_2) dM(\theta).$$

By differentiating this expression and using that $\zeta' \left(\frac{\pi_2}{\pi_1} \right) < 0$, it is straightforward to show that the market clearing condition defines an upward sloping curve in θ_1-π_2 space and that Propositions 2 and 3 continue to hold when the final good technology is given by (24).

To solve the open economy model with a general constant returns to scale final good technology note that the open economy market clearing condition is:

$$Y_1 + Y_1^* = \frac{1}{\zeta}(Y_2 + Y_2^*),$$

where ζ is given by (25). In addition, when productivity rankings differ across countries foreign will specialize in good one if and only if:

$$\bar{\zeta}\int_0^{\bar{\theta}^*}\theta g(\theta)f(s_1^*)dM^*(\theta) \leq \int_0^{\bar{\theta}}\theta g(\theta)f(s_2)dM(\theta)$$

where $\frac{h'(\bar{\zeta})}{h(\bar{\zeta})-\bar{\zeta}h'(\bar{\zeta})} = 1$ and $\pi_1 = \pi_2 = h'(\bar{\zeta})$.

Using these expressions we can solve for the open economy equilibrium following the same reasoning applied in the Cobb-Douglas case and Propositions 4, 5 and 6 continue to hold.

Heckscher-Ohlin general equilibrium

Consider the following variant of the Heckscher-Ohlin model. There are two industries and two factors of production and each industry has a Cobb-Douglas technology:

$$Z_j = \left(\frac{Y_{1j}}{\mu_j}\right)^{\mu_j}\left(\frac{Y_{2j}}{1-\mu_j}\right)^{1-\mu_j}, \qquad \mu_j \in (0,1), \quad j = 1,2,$$

where Z_j is output of industry j and Y_{kj} is the quantity of factor k used in industry j. Assume $\mu_1 > \mu_2$ meaning that industry one is factor one intensive. Now, suppose that the factors of production do not represent the economy's endowments, but must be produced. Factor k is the output of task k and task production is governed by the assignment problem in Section 2. Finally, suppose that output from the two industries is combined to produce a final good, which can either be consumed or used as the intermediate input in task production. Output of the final good is given by:

$$Z = \left(\frac{Z_1}{\beta}\right)^{\beta}\left(\frac{Z_2}{1-\beta}\right)^{1-\beta}, \qquad \beta \in (0,1).$$

In this set-up factor supplies are endogenous to the equilibrium of the assignment problem. Suppose task two has higher intermediate input productivity than task one, $Q_2 > Q_1$. Then, given Assumption 1, high skill agents will be assigned to task two and low skill agents will perform task one.

Following the same logic used to solve for equilibrium in Section 3, it is easy to show that the closed economy equilibrium of this Heckscher-Ohlin model can be characterized by the same (IE) and (MC) conditions derived in Section 3, except that the parameter β must be replaced by $\mu_1\beta + \mu_2(1-\beta)$. Consequently, the model has a unique closed economy equilibrium featuring positive assortative matching between agents

and tasks and the effects of technological progress on the returns to skill and wage inequality are as described in Section 3.

In the baseline model all workers in the high productivity sector have higher ability than any worker in the low technology sector. However, in this Heckscher-Ohlin model each industry must employ both high skill workers to perform task two and low skill workers to perform task one. The equilibrium wage function ensures that employers are indifferent between all workers assigned to a particular task. Therefore, I will assume that the skill distribution of workers employed to perform each task is the same in both industries. Under this assumption the average wage w_j in industry j is:

$$w_j = \frac{\bar{w}_1 + \nu_j \bar{w}_2}{1 + \nu_j},$$

where $\bar{w}_k$ is the average wage of agents assigned to task k and:

$$\nu_j \equiv \frac{1 - \mu_j}{\mu_j} \frac{\mu_1 \beta + \mu_2 (1 - \beta)}{1 - \mu_1 \beta - \mu_2 (1 - \beta)} \frac{M(\bar{\theta}) - M(\theta_1)}{M(\theta_1)}.$$

Unsurprisingly, the mean industry wage is a weighted average of the mean task wages. Note that $\mu_1 > \mu_2 \Rightarrow \nu_1 < \nu_2$. Therefore, the mean industry wage is higher in the industry that uses intensively the high skill task. As in the baseline model, shocks to intermediate input productivity that change the productivity ranking across tasks will change the ranking of industries by average wages and average employee skill. It can also be shown that labor's share of output is lower in the industry that uses the high skill task intensively.

Appendix C – Data

US manufacturing wages

The Cummins and Violante (2002) equipment price data covers 23 categories of equipment and 3 categories of software. The US capital flow table for 1982 includes 23 equipment categories. In the absence of any data on industry software investment in 1982, I do not use the software price data. The equipment price change instrument is computed using price data for 19 equipment categories.[60]

The US Census Bureau collected data on investment in "Computers and peripheral data processing equipment" and "Autos, trucks, etc. for highway use" by 4 digit SIC industries as part of the Census of Manufactures in 1982.[61] I converted the data from SIC 1972 industries to SIC 1987 industries using a concordance from the NBER manufacturing database website. I experimented with using the share of new capital expenditure allocated to vehicles to construct a third instrument for the investment price change, but the instrument was insignificant in the first stage regression.

Scherer (1984) gives industry R&D expenditures in 1974, at approximately the SIC 3 digit level, calculated from the Federal Trade Commission's Line of Business Survey. I use the R&D by industry of use variable, which Scherer (1984) computes from R&D by industry of origin using patent data.

All wage variables are defined as the average wage per employee in an industry. The non-production employment share is the ratio of non-production employees to total employment. Investment per worker is defined as total capital expenditures per employee. The TFP measure used in the analysis is the 4-factor TFP index from the NBER manufacturing database, but results are robust to using the 5-factor TFP index instead.

International wage structure comparisons

UNIDO's Industrial Statistics database contains employment and compensation data for 127 ISIC Revision 3 manufacturing industries at the 4 digit level. The database starts in 1990, but country coverage varies over time. The wage variable is defined as the ratio of Wages and salaries to Employment. The sample used in the

[60] The equipment categories are: Communication equipment; Instruments and photocopies; Fabricated metal products; Engines and turbines; Metalworking machinery; Special industry machinery, n.e.c.; General industrial equipment; Electrical transmission, distribution and industrial apparatus; Aircraft; Ships and boats; Railroad equipment; Furniture and fixtures; Tractors; Agricultural machinery, except tractors; Construction machinery, except tractors; Mining and oilfield machinery; Service industry machinery; Electrical equipment, n.e.c., and; Other equipment.

[61] Although the survey coincided with the Census of Manufactures, data was only collected from firms in the Annual Survey of Manufactures sample.

paper is selected as follows: (i) for each country the data used is from the latest year between 1995 and 2000 for which wage data is reported; (ii) all industries reporting negative wages and salaries, or with fewer than 10 employees, were dropped; (iii) only countries with data on at least 60% of industries were included.[62] The final sample covers 43 countries including the US.[63] Wage data for the US is available from 1997-2000. The statistics shown in Figure 1 and Figure 5 are calculated using US data for the same year in which a country reported data, unless the data is from 1995 or 1996, in which case US data from 1997 is used. The log wage variable used in the regression analysis was cleaned using a Winsorization procedure. To be specific, I demeaned log wages by country and industry, performed a 98% Winsorization on the demeaned residual and then added the country and industry means to the Winsorized residual to obtain the cleaned log wage variable. The log investment per worker variable was cleaned in the same manner.

UNIDO's Industrial Statistics database does not include long time series of industry data at the 4 digit level. Consequently, the changes in wage rank correlations shown in Figure 6 are computed using wage data for 3 digit ISIC Revision 2 manufacturing industries. The 3 digit data covers 28 industries and I drop country-year observations with wage data for fewer than 80% of industries. I use data from 1965-1995 and compute annualized changes between the first and the last year in which a country is included in the data set. Only countries for which the first and the last year are at least 10 years apart are included.

The EU KLEMS data is taken from the March 2008 release of the database. The industry wage rate is defined as the ratio of Compensation of employees to Total hours worked by employees. The data for 1995 covers 29 countries[64] and, at the most disaggregated level available, 57 market-based industries which together compose the entire market economy. I use the NAICS-based data for the US and drop Luxembourg from the sample since it has a higher income than the US.

Capital stock per capita is computed from the Penn World Tables 6.3 using the perpetual inventory method as implemented by Caselli (2005). Human capital per capita is computed from the Barro and Lee (2001) educational attainment data set. Average years of schooling for the population 25 and over is converted to human capital following the methodology in Caselli (2005).

[62] Informal examination of the data suggests that there is substantial noise in the Industrial Statistics database. The 60% coverage cut-off is designed to select for countries that produce relatively comprehensive industrial statistics, since such countries are likely to report higher quality data. It also reduces selection bias that may arise if there is endogeneity in which industries report data. The results in the paper do not depend on the exact value of the cut-off.

[63] The sample countries are: Austria, Azerbaijan, Bangladesh, Belgium, Canada, Colombia, Denmark, Ecuador, Egypt, Finland, France, Germany, Hungary, India, Indonesia, Iran, Italy, Japan, Kyrgyzstan, Latvia, Lebanon, Lithuania, Malaysia, Mexico, Morocco, Netherlands, New Zealand, Norway, Peru, Portugal, Singapore, Slovakia, Slovenia, South Korea, Spain, Sweden, Thailand, Turkey, Ukraine, United Kingdom, US, Vietnam and Zimbabwe.

[64] The 25 EU countries plus Australia, South Korea, Japan and the US.

The fifteen equipment types used to compute the cost of imported capital are: Computers, office and accounting equipment; Communication equipment; Instruments and medical equipment; Fabricated metal products; Engines and turbines; Metalworking machinery; Special industry machinery, n.e.c.; General industrial equipment; Electrical equipment; Autos and trucks; Aircraft; Ships and boats; Railroad equipment; Furniture and fixtures, and; Agricultural machinery. The 1997 US capital flow table gives equipment investment for 53 manufacturing industries which I map to ISIC industries by combining the concordance from capital flow industries to NAICS industries in the capital flow table and a concordance from NAICS industries to ISIC industries from the US Census Bureau.

The geographic variables used to estimate the gravity equation are from the CEPII. The population weighted arithmetic mean distance between major cities is used to measure distance.

To obtain measures of capital, skill and contract intensity for ISIC industries I used concordances between NAICS and ISIC Revision 3 industries from the US Census Bureau and Statistics Canada to construct a concordance that mapped each NAICS manufacturing industry to its primary ISIC counterpart. I then used this concordance to: (i) convert the NBER manufacturing database for 2000 to ISIC industries, allowing me to compute capital and skill intensity, and; (ii) map the contract intensity data classified using the BEA's 1997 input-output industries from Nunn (2007) to ISIC industries.

Estimates of the elasticity of substitution between skilled and unskilled labor are from Reshef (2007). I use the elasticities estimated from US data using the stock adjustment model and reported in Table 6 of Reshef (2007). The estimates are for 2 digit SIC industries, which map naturally to 2 digit ISIC industries. I assume all 4 digit ISIC industries within a given 2 digit industry have the same elasticity of substitution.

Table 1: Cost of Capital and US Manufacturing Wages

Dependent variable:	Wage	Investment price	Wage	Wage rank percentile		Labor's share of value-added	
	OLS	OLS	2SLS	OLS	2SLS	OLS	2SLS
	(a)	(b)	(c)	(d)	(e)	(f)	(g)
Δ log Investment price	-0.139 ***		-0.191 ***	-0.091 *	-0.177 **	0.372 ***	0.616 ***
	(0.043)		(0.061)	(0.050)	(0.075)	(0.130)	(0.161)
Δ Equipment price		0.013 ***					
		(0.002)					
Δ Computer price		0.088 ***					
		(0.023)					
2 digit industry dummies	Yes	Yes	Yes	Yes	Yes	Yes	Yes
F-test (p-value)			0.00		0.00		0.00
Overidentification test (p-value)			0.86		0.58		0.18
R^2	0.25	0.81	0.25	0.20	0.19	0.19	0.17
N	451	451	451	451	451	451	451

Sample is SIC 4 digit US manufacturing industries 1960-2000.
All dependent variables, except Wage rank percentile, expressed as log differences between 1960 and 2000.
Wage rank percentile expressed as difference between 1960 and 2000.
Robust standard errors in parentheses.
* indicates coefficient statistically significant at 10% level; ** at 5% level, and; *** at 1% level.
The 2SLS estimates use Δ Equipment price and Δ Computer price as instruments for Δ log Investment price. Column (b) shows the first stage results.
F-test reports the p-value from an F-test of the joint significance of the instruments in the first stage.
Overidentification test reports the p-value from Wooldridge's robust score test for overidentification.

Table 2: Robustness Checks on Investment Price Effect

Dependent variable:	Wage			
	2SLS	2SLS	2SLS	2SLS
	(a)	(b)	(c)	(d)
Δ log Investment price	-0.234 **	-0.144 **	-0.164	-0.195 ***
	(0.100)	(0.059)	(0.103)	(0.070)
Log Wage 1960	-0.0086 ***			
	(0.0013)			
Log Non-production employment share 1960	0.0002			
	(0.0006)			
Log Output 1960	0.0003 *			
	(0.0002)			
Log Investment per worker 1960	0.0015 ***			
	(0.0003)			
Log TFP 1960	0.0004			
	(0.0006)			
Log (R&D expenditure/Output) 1974		0.081 ***		
		(0.027)		
Δ log Output			0.026 ***	
			(0.010)	
Δ log TFP			0.001	
			(0.024)	
Δ Imports/Output			0.001	
			(0.004)	
Δ Exports/Output			0.011	
			(0.021)	
2 digit industry dummies	Yes	Yes	Yes	Yes
F-test (p-value)	0.00	0.00	0.00	0.00
Overidentification test (p-value)	0.21	0.55	0.94	
R^2	0.35	0.28	0.28	0.25
N	451	392	395	451

Sample is SIC 4 digit US manufacturing industries 1960-2000.
Dependent variable expressed as log difference between 1960 and 2000.
Robust standard errors in parentheses.
* indicates coefficient statistically significant at 10% level; ** at 5% level, and; *** at 1% level.
The 2SLS estimates in columns (a)-(c) use Δ Equipment price and Δ Computer price as instruments for Δ log Investment price. In column (d) Δ Equipment price is the only instrument.
F-test reports the p-value from an F-test of the joint significance of the instruments in the first stage.
Overidentification test reports the p-value from Wooldridge's robust score test for overidentification.

Table 3: Investment, Worker Types and the Cost of Capital

Dependent variable:	Investment per worker		Wage	Production wage	Non-production wage	Non-production employment share	Non-production to capital income ratio
	OLS	OLS	2SLS	2SLS	2SLS	2SLS	2SLS
	(a)	(b)	(c)	(d)	(e)	(f)	(g)
Δ log Investment price				0.018	-0.223 **	-0.462 ***	0.763 **
				(0.063)	(0.099)	(0.132)	(0.377)
Δ log Investment per worker			0.462 *				
			(0.238)				
Δ Equipment price	-0.008 *	-0.006 *					
	(0.004)	(0.004)					
Δ Computer price	0.070						
	(0.096)						
2 digit industry dummies	Yes	Yes	Yes	Yes	Yes	Yes	Yes
F-test (p-value)			0.09	0.00	0.00	0.00	0.00
Overidentification test (p-value)				0.84	0.77	0.68	0.66
R^2	0.13	0.13	0.04	0.28	0.09	0.23	0.10
N	451	451	451	451	451	451	450

Sample is SIC 4 digit US manufacturing industries 1960-2000.
All dependent variables expressed as log differences between 1960 and 2000.
Non-production to capital income ratio denotes the ratio of total wages received by non-production workers to value-added net of all wages.
Robust standard errors in parentheses.
* indicates coefficient statistically significant at 10% level; ** at 5% level, and; *** at 1% level.
The IV estimates in column (c) use Δ Equipment price as an instrument for Δ log Investment per worker.
The IV estimates in columns (d)-(f) use Δ Equipment price and Δ Computer price as instruments for Δ log Investment price.
F-test reports the p-value from an F-test of the joint significance of the instruments in the first stage.
Overidentification test reports the p-value from Wooldridge's robust score test for overidentification.

Table 4: Revealed Advantage in Equipment Exports

	Revealed Advantage			
Exporter	Mean	Std. dev.	Min.	Max.
Canada	6.7%	4.6%	2.4%	16.7%
			(Electrical apparatus)	(Railroad equipment)
China	8.1%	9.5%	0.1%	31.8%
			(Aircraft)	(Furniture and fixtures)
France	8.4%	4.7%	3.2%	22.8%
			(Metalworking machinery)	(Aircraft)
Germany	16.8%	6.8%	8.4%	27.9%
			(Computers, office and accounting equipment)	(Agricultural machinery)
Italy	7.8%	5.3%	1.8%	18.1%
			(Computers, office and accounting equipment)	(Furniture and fixtures)
Japan	18.7%	14.2%	1.5%	53.0%
			(Furniture and fixtures)	(Ships and boats)
United Kingdom	7.4%	3.9%	3.7%	16.8%
			(Furniture and fixtures)	(Aircraft)
United States	26.1%	10.0%	5.0%	44.7%
			(Ships and boats)	(Engines and turbines)

Revealed advantage computed for 15 equipment types in 2000.
Revealed advantage is exporter's share of total exports of equipment type by the eight exporters listed above.
The equipment types in which each country has its minimum and maximum revaled advantage are listed in parentheses.

Table 5: Cost of Imported Capital and Cross-Country Variation in the Inter-Industry Wage Structure

Dependent variable:	Wage		Wage rank percentile			Labor's share of value-added	
	(a)	(b)	(c)	(d)	(e)	(f)	(g)
log Cost of imported capital	-3.151 ***	-3.639 ***	-0.565	-2.234 **	-2.189 **	0.687	-0.280
	(1.080)	(1.235)	(0.841)	(0.891)	(0.898)	(1.806)	(1.998)
Capital interaction		-0.011		0.035 ***	0.036 ***		-0.001
		(0.009)		(0.005)	(0.005)		(0.013)
Skill interaction		-0.071		0.099 ***	0.109 *		0.011
		(0.056)		(0.036)	(0.036)		(0.069)
Contract interaction		0.087 *		0.073 ***	0.077 ***		-0.126 **
		(0.045)		(0.024)	(0.024)		(0.064)
Skill reversal interaction					-0.049		
					(0.036)		
Industry dummies	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Country fixed effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.39	0.39	0.46	0.48	0.48	0.25	0.25
N	3656	3444	3668	3454	3454	3228	3030

Sample is ISIC 4 digit manufacturing industries in 36 countries in 2000.
All dependent variables, except Wage rank percentile, expressed as logs.
Standard errors, clustered by country and 2 digit industry, in parentheses.
* indicates coefficient statistically significant at 10% level; ** at 5% level, and; *** at 1% level.

Table 6: Imports, Investment and the Cost of Capital

Dependent variable:	Imports	Investment per worker		Wage	Wage rank percentile	Labor's share of value-added
			First stage	2SLS	2SLS	2SLS
	(a)	(b)	(c)	(d)	(e)	(f)
log Cost of imported equipment	-3.395 **					
	(1.672)					
log Cost of imported capital		-9.818 ***	-9.396 ***			
		(3.019)	(3.277)			
log Investment per worker				0.346 **	0.262 **	0.022
				(0.165)	(0.120)	(0.187)
Capital interaction			0.014	-0.012	0.036 ***	-0.006
			(0.024)	(0.009)	(0.007)	(0.011)
Skill interaction			-0.080	-0.077	0.128 ***	0.048
			(0.136)	(0.063)	(0.046)	(0.071)
Contract interaction			0.242	0.008	0.006	-0.115 *
			(0.151)	(0.060)	(0.044)	(0.065)
Equipment variety dummies	Yes					
Industry dummies		Yes	Yes	Yes	Yes	Yes
Country fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.74	0.33	0.33	0.19	0.20	0.24
N	514	2891	2707	2707	2707	2657

Except for column (a), sample is ISIC 4 digit manufacturing industries in 36 countries in 2000.
In column (a) sample covers 15 equipment types and 36 countries in 2000.
All dependent variables, except Wage rank percentile, expressed as logs.
Standard errors in parentheses. In column (a) standard errors are clustered by country. In columns (b) and (c) standard errors are clustered by country and 2 digit industry.
* indicates coefficient statistically significant at 10% level; ** at 5% level, and; *** at 1% level.
The 2SLS estimates use log Cost of imported capital as an instrument for log Investment per worker.

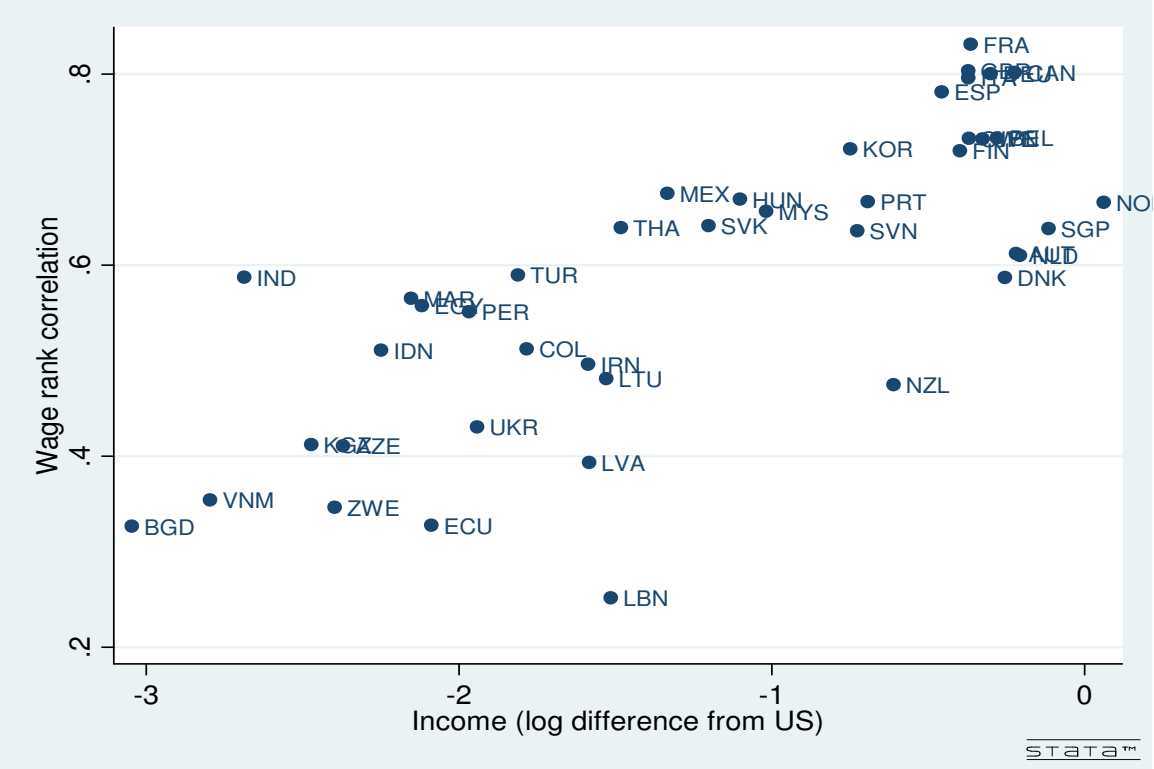

Figure 1: Wage rank correlations – UNIDO 2000

Figure 2: Equipment prices and wages – NBER manufacturing database 1960-2000

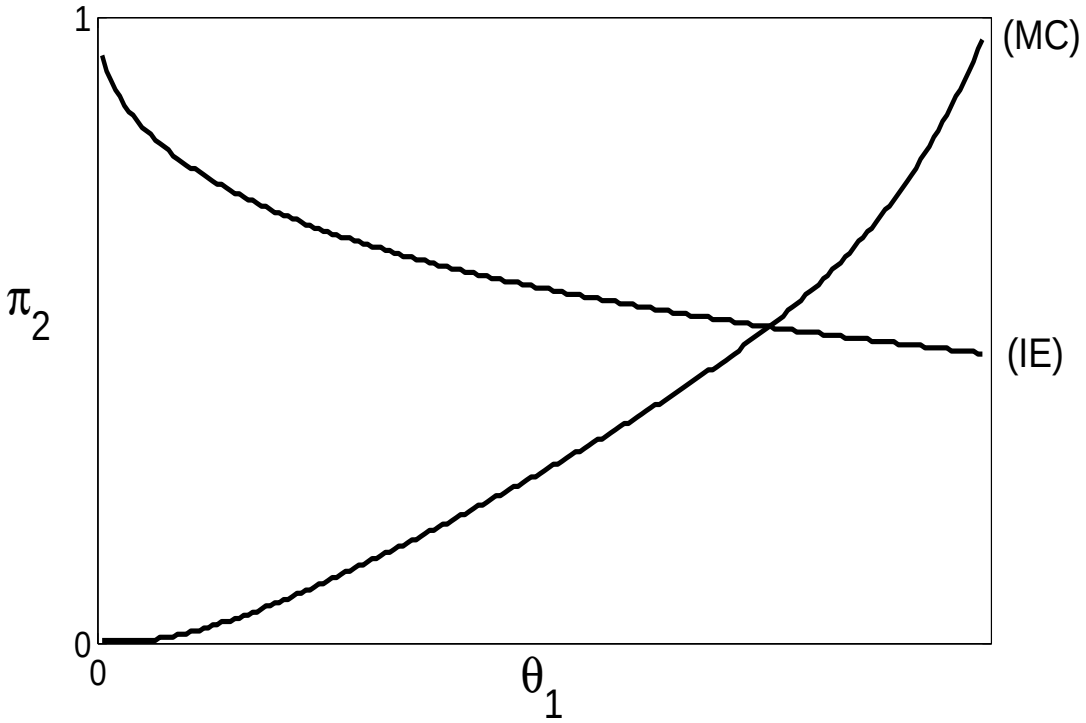

Figure 3: Closed economy equilibrium

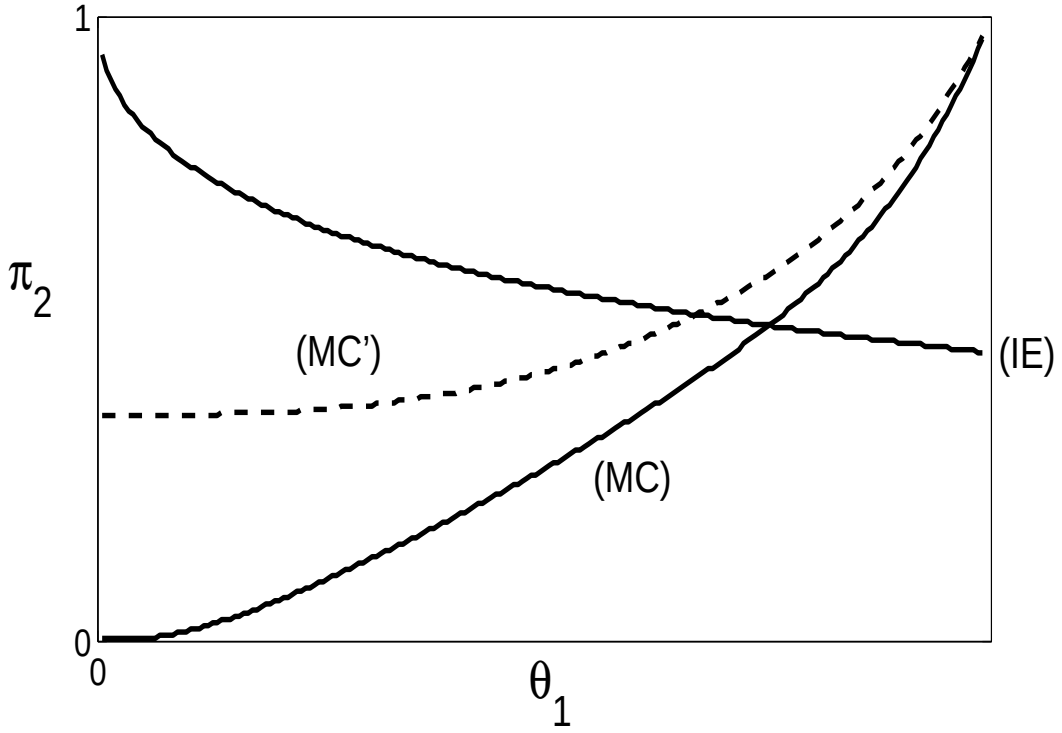

Figure 4: Open economy equilibrium

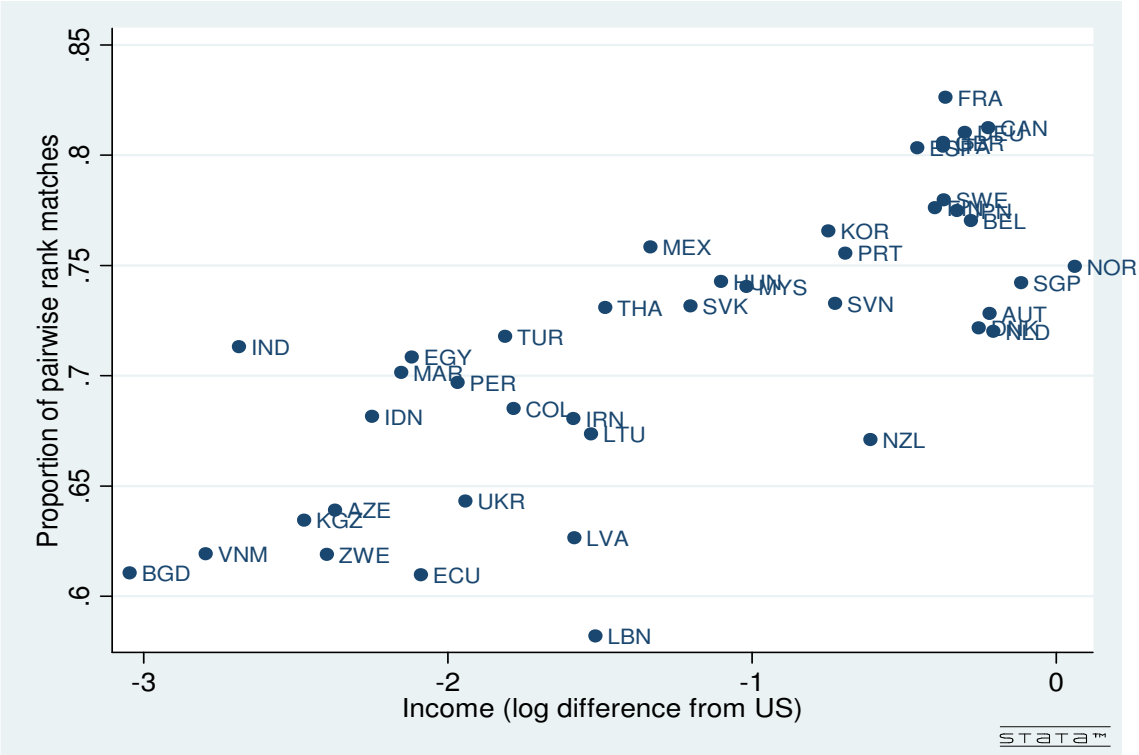

Figure 5: Proportion of pairwise rank matches – UNIDO 2000

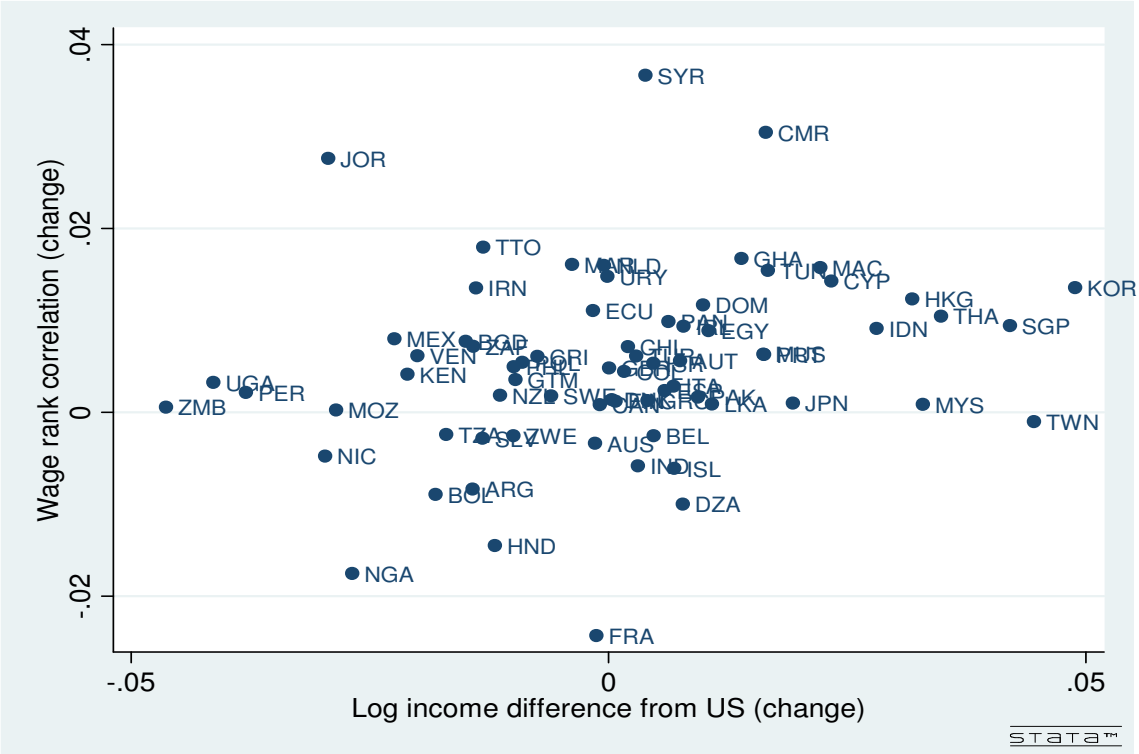

Figure 6: Changes in wage rank correlations – UNIDO 1965-95

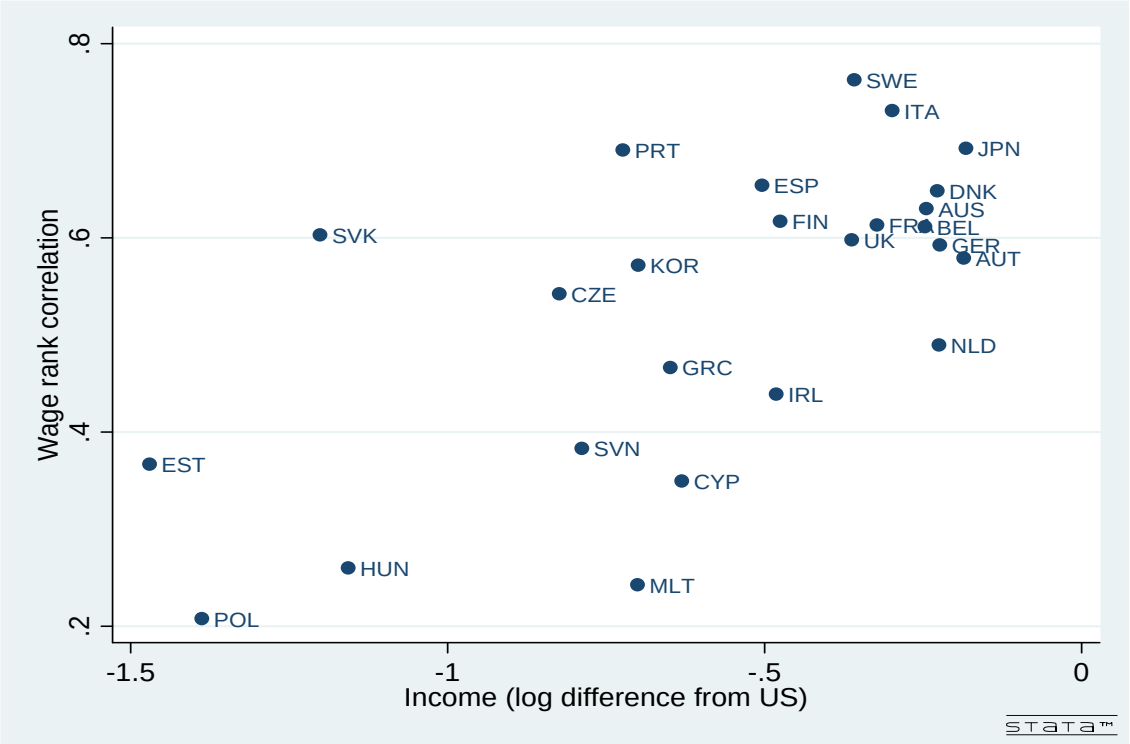

Figure 7: Wage rank correlations – EU KLEMS 1995